# Influence of the Symmetry Energy on the $\pi^-/\pi^+$ Multiplicity Ratio in Heavy-Ion Collisions

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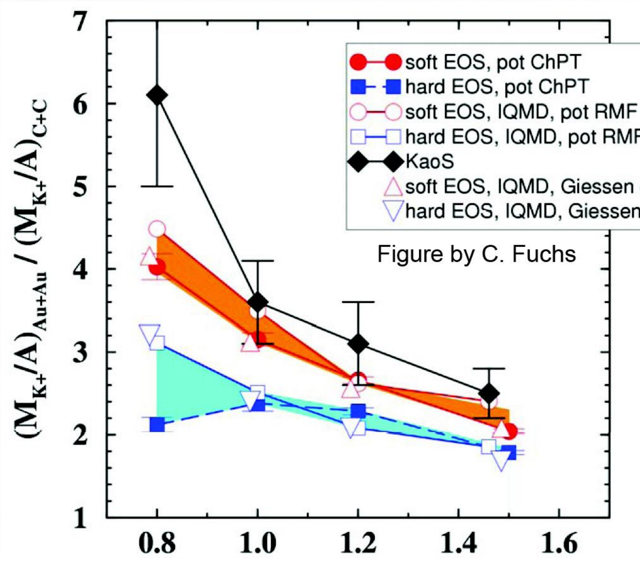
# Symmetry Energy

## EoS of Asymmetric Nuclear Matter

$$E(\rho,\beta)=E(\rho,\beta=0)+S(\rho)\beta^2 \quad \beta=\frac{\rho_n-\rho_p}{\rho}$$

$$S(\rho)=S(\rho_0)+\frac{L_{sym}}{3}\frac{\rho-\rho_0}{\rho_0}+\frac{K_{sym}}{18}\frac{(\rho-\rho_0)^2}{\rho_0^2}$$

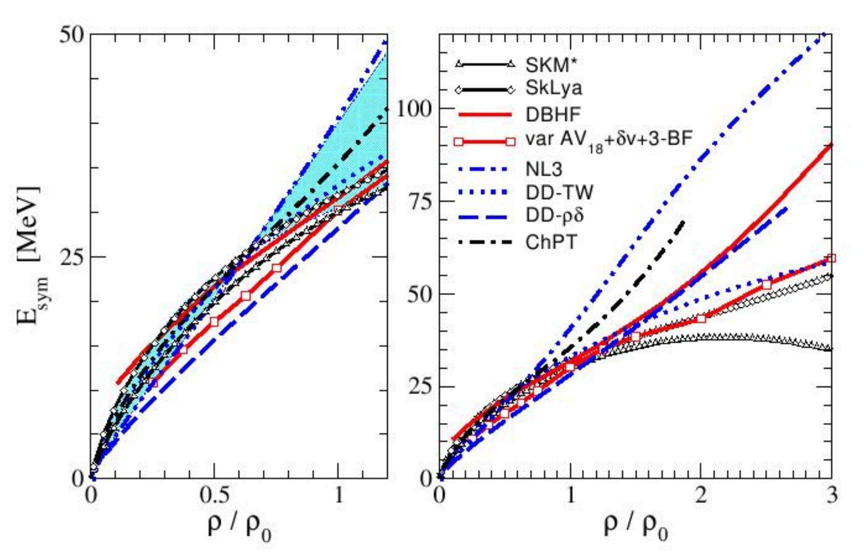
C. Fuchs *et al.* PRL 86, 1974 (2001)




## Theoretical estimates of L and K

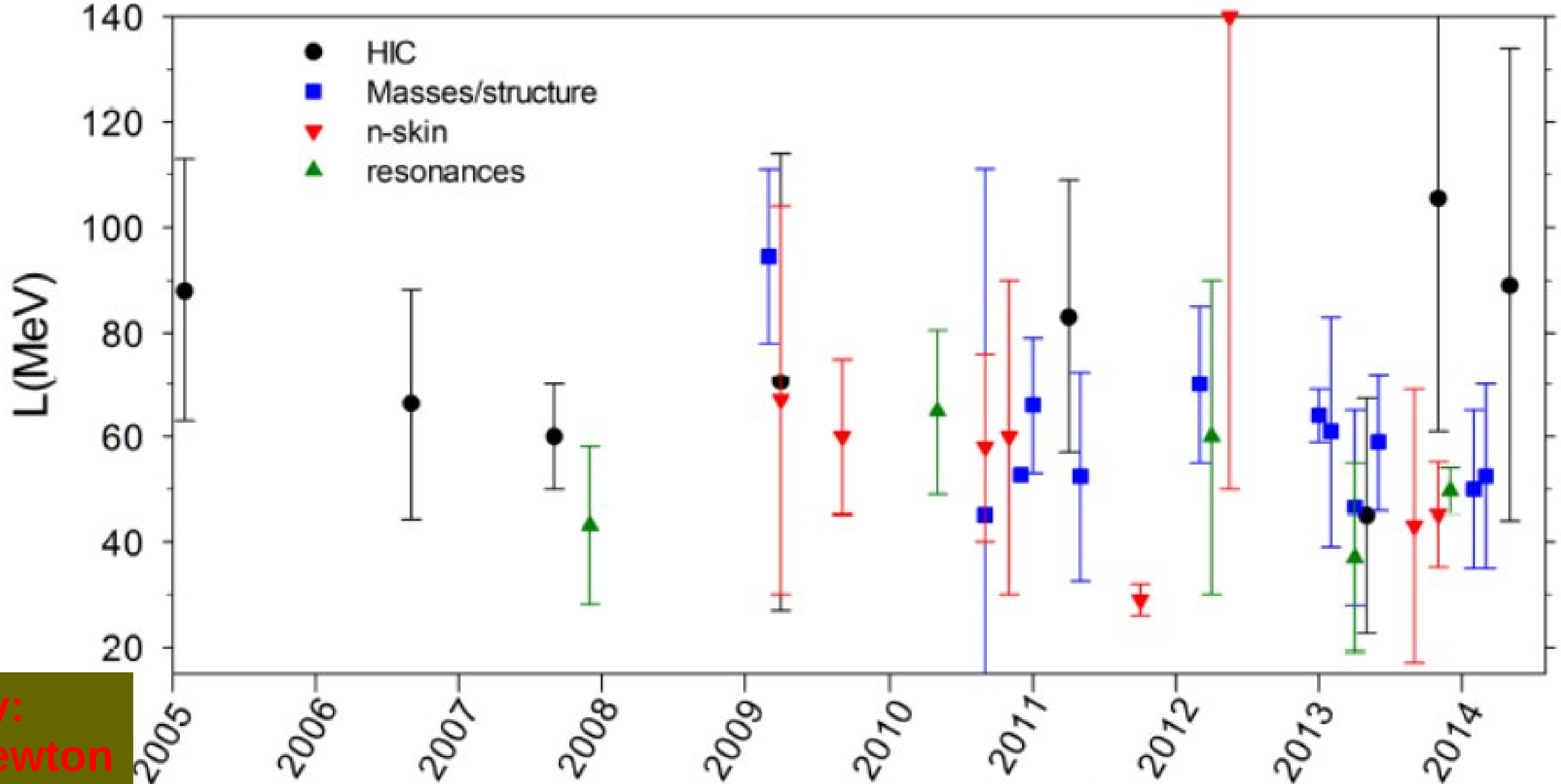
B.A. Li *et al.* Int.J.Mod.Phys. E7, 147 (1998)

| Force | Paris | SKM* | SI′ | SIII | DHF (b) | DHF (e) |
|---|---|---|---|---|---|---|
| $L$ | 68.8 | 45.78 | 35.34 | 9.91 | 132 | 138 |
| $K_{sym}$ | 37.56 | -155.9 | -259.1 | -393.7 | 466 | 276 |




See also: M.B. Tsang *et al.* PRC86, 015803 (2012)

# Nuclear experimental constraints on L




**Slide by: W.G. Newton Nusym 14**
http://npg.dl.ac.uk/NuSYM2014-Workshop/

n-skins
Centelles et al PRL102 (2009)
Warda et al PRC80 (2009)
Chen et al PRC82 (2010)
Zenihiro et al PRC82 (2010)
Gaidarov et al PRC84 (2011)
Roca-Maza et al PRC88(2013)
Zhang and Chen, PLB726 (2013)
Abrahamyan et al

resonances
Klimkiewicz et al, PRC76(2007)
Carbone et al PRC81, (2010)
Chen, Gu JPhG39 (2012)
Roca-Maza et al PRC87, (2013)
Krashnahorkay et al, arxiv:1311.1456

Masses/structure
Kortelainen et al, PRC82 (2010)
Danielewicz, Lee NphysA818 (2009)
Xu et al, PRC82 (2010)
Liu et al PRC82 (2010)
Chen, PRC82 (2011)
Moller et al PR108 (2012)
Agrawal et la PRL109 (2012)
Dong et al, PRC87 (2013)
Wang et al PRC87 (2013)
Agrawal et al PRC87 (2013)
Fan et al PRC89 (2014)
Danielewicz,Lee NPhysA922 (2014)

HIC
Chen, Ko, Li PRL94(2005)
Famiano et al PRL97(2006)
Shetty et al PRC76(2007)
Tsang et al PRL102 (2009)
Russotto et al PLB697 (2011
Li et al PLB721 (2013)
Cozma et al PRC88 (2013)
Wang et al PRC89 (2014)

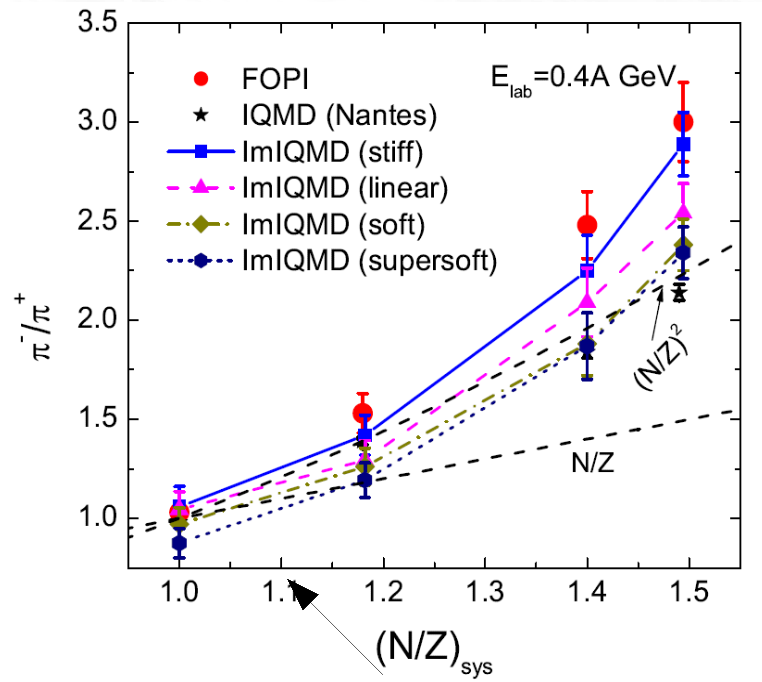
# Pion ratios

Isobar model (no symmetry potential)

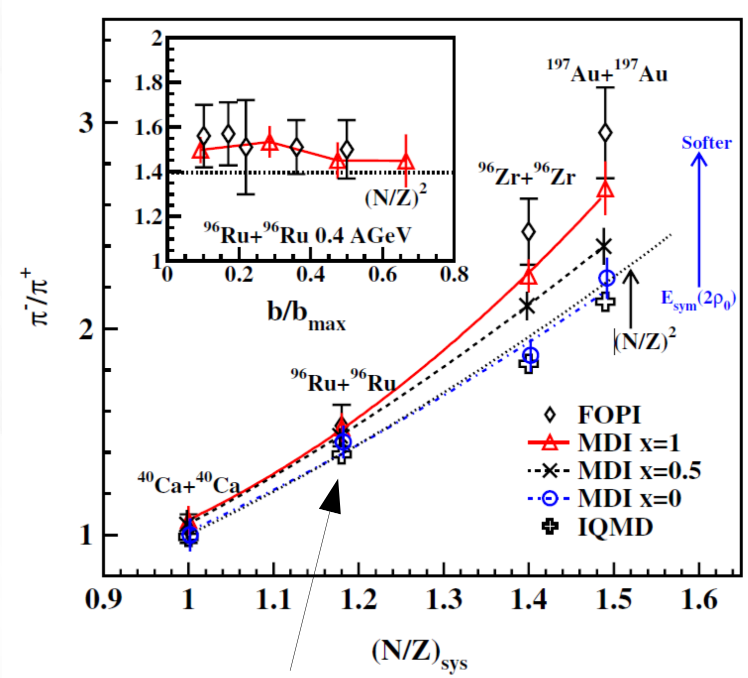
$$\pi^-/\pi^+=(5N^2+NZ)/(5Z^2+NZ)$$

Z.-Q. Feng et al., PLB 683, 140 (2010)

Z.Xiao et al. PRL 102,062502 (2009)




IQMD




IBUU

Boltzmann-Langevin approach - super-soft W.-J. Xie et al., PLB 718, 1510 (2013)

pBUU – no sensitivity to stiffness of SE J. Hong et al. PRC 90, 024605 (2014)

TuQMD – super-soft scenario favored (opposite to elliptic flow constraint)

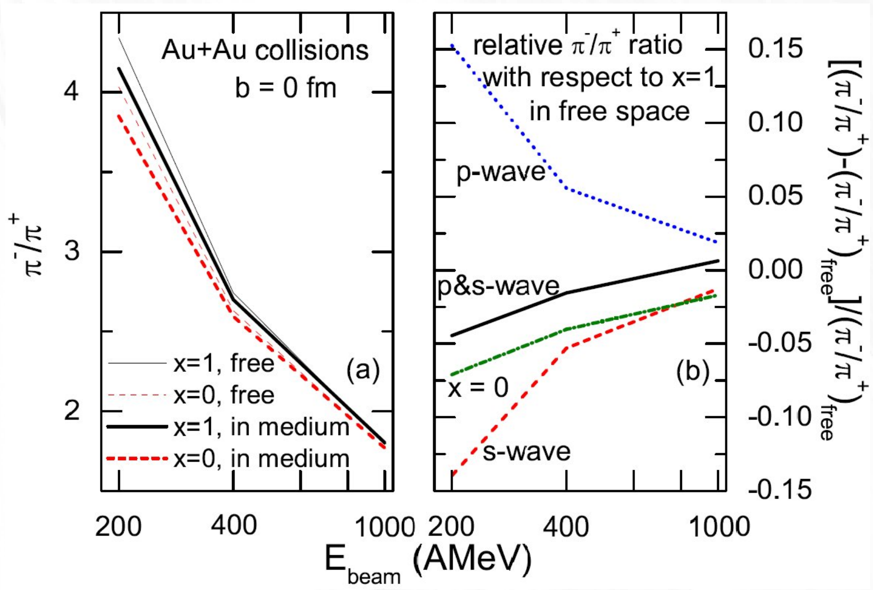
# Medium Effects

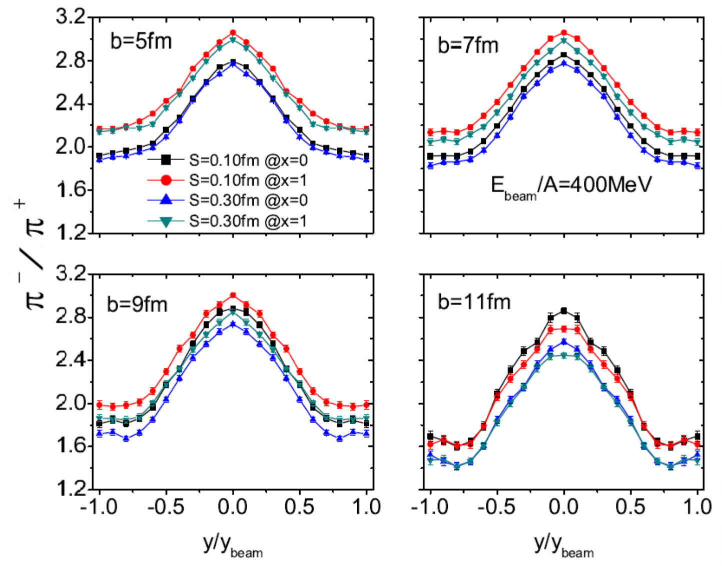
-pion production threshold in asymmetric & hot nuclear matter

C.L. Korpa et al. PLB 446, 15 (1999)

-charge dependent pion mass shift driven by S wave πN interaction (ChPT)

N. Kaiser et al. PLB 512, 283 (2001)




J. Xu et al. PRC 87, 067601 (2013)

G.-F. Wei et al. PRC 90, 014610 (2014)

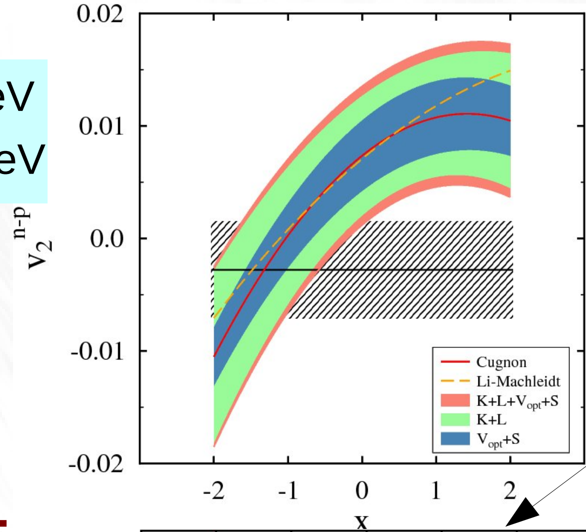
Pitfalls: - S wave pion potential needed to explain pionic atoms 2x more repulsive
- isospin breaking effects in the Δ(1232) mass/decay widths

$$\frac{dN}{d\phi} \sim 1+2v_1 \cos\phi + 2v_2 \cos 2\phi$$

# Constraints using elliptic flow

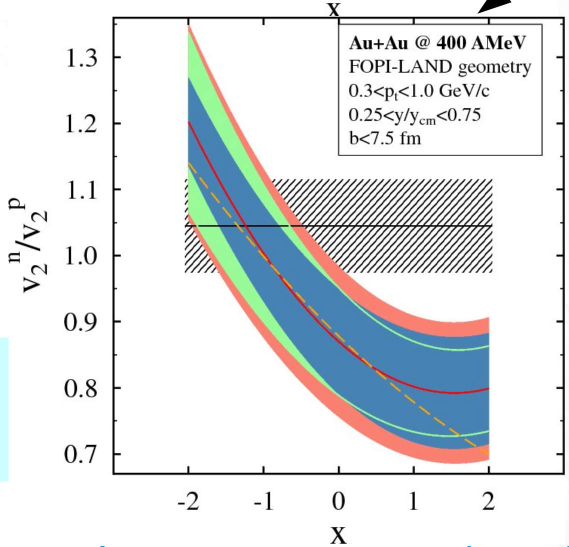
**flow difference:**

$L_{sym}=129^{+46}_{-80}$ MeV
$K_{sym}=272^{+291}_{-508}$ MeV




**flow ratio:**

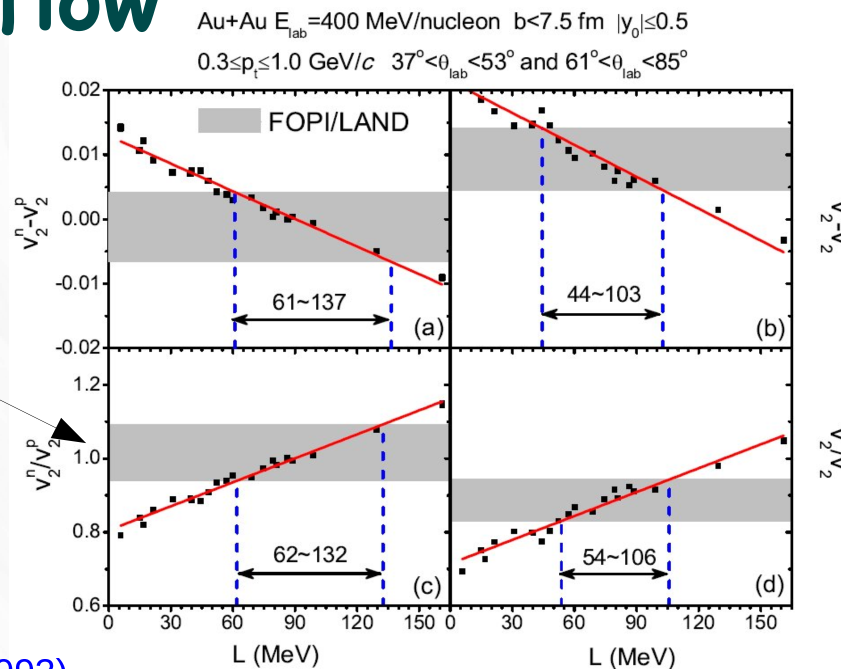
$L_{sym}=118^{+45}_{-57}$ MeV
$K_{sym}=199^{+291}_{-362}$ MeV



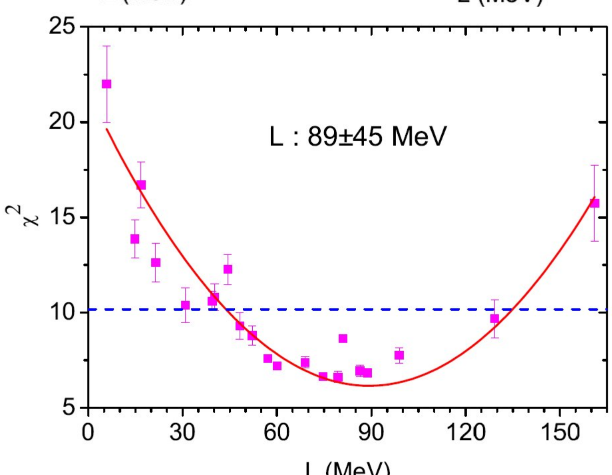

M.D. Cozma et al. PRC 88, 044912 (2013)

**FOPI-LAND**

Y.Leifels et al.,
PRL 71, 963 (1993)




$L_{sym}=89\pm45$ MeV

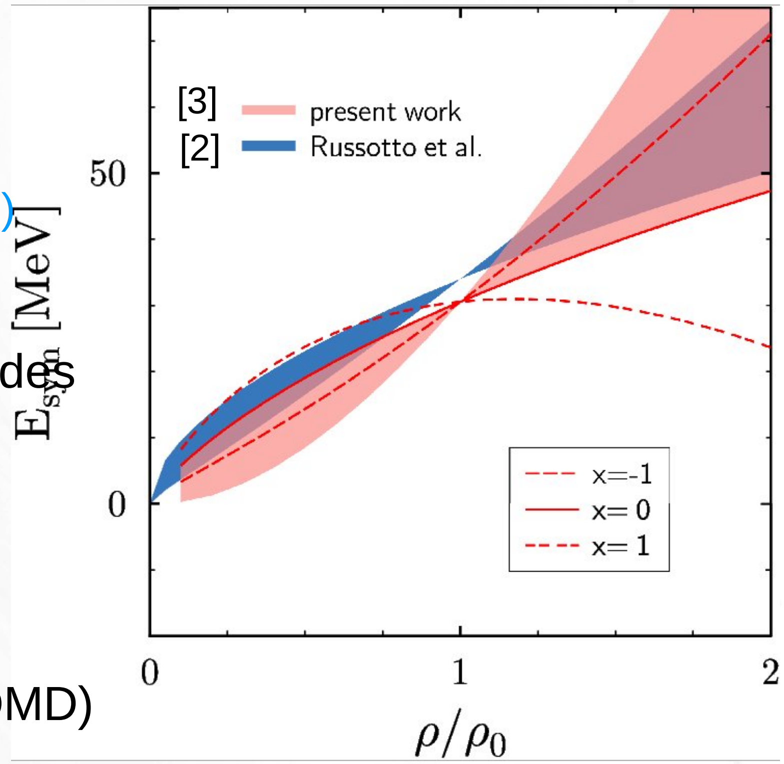



Y. Wang et al. PRC 89, 044603 (2014)

# Density dependence of SE

**studies** presented in

[1] M.D. Cozma, PLB 700, 139 (2011)
[2] P. Russotto *et al.,* PLB 697, 471 (2011)
[3] M.D. Cozma *et al.,* PRC 88, 044912(2013)
[4] W.-M. Guo *et al.,* PLB 726, 211 (2013)
[5] Y. Wang *et al.,* PRC 89, 044603 (2014)




-**Independently developed** transport codes and upgrades
QMD – Tuebingen/ upgraded in Bucharest
UrQMD – Frankfurt/ upgraded Q.Li et al. Huzhou
IBUU – G.-C. Yong et al. (IMP, Lanzhou, China)

-**different parametrizations** of the symmetry energy
- momentum dependent (QMD/IBUU) /
momentum independent, Skyrme interactions (UrQMD)

-inclusion of **in-medium effects**
-in medium NN cross-section (QMD,IBUU,UrQMD)

-**thorough study** of various model parameters:
- width of nucleon wave-function (L)
- compressibility modulus of nuclear matter (K)
- impact of optical potential

similar result

[2,3] $L_{sym}=106\pm46\,MeV$
$K_{sym}=127\pm290\,MeV$

[5] $L_{sym}=89\pm45\,MeV$

# Transport Model

**Quantum Molecular Dynamics** (TuQMD):

Monte Carlo cascade + Mean field + Pauli-blocking+ in medium cross section

all 4∗ resonances below 2 GeV - 10 $\Delta^*$ and 11 $N^*$

**baryon-baryon collisions:**

all elastic channels

inelastic channels $NN \to NN^*$, $NN \to N\Delta$, $NN \to \Delta N^*$, $NN \to \Delta\Delta^*$, $NR \to NR'$

**pion-absorption ⇋ resonance-decay channels:** $\Delta \leftrightarrow N\pi$, $\Delta^* \leftrightarrow \Delta\pi$, $N^* \leftrightarrow N\pi$

**meson production/absorption:** $\eta(547)$, $\rho(770)$, $\omega(782)$, $\eta'(958)$, $f_0(980)$, $a_0(980)$, $\Phi(1020)$

**previously applied to study:**

- -dilepton emission in HIC: K.Shekter, PRC 68, 014904 (2003);D. Cozma, PLB640,170 (2006); E.Santini PRC78,03410 (2008)
- -EoS of symmetric nuclear matter: C. Fuchs, PRL 86, 1974 (2001); Z.Wang NPA 645, 177 (1999)
- -In-medium effects and HIC dynamics: C. Fuchs, NPA 626,987 (1997); U. Maheswari NPA 628,669 (1998)

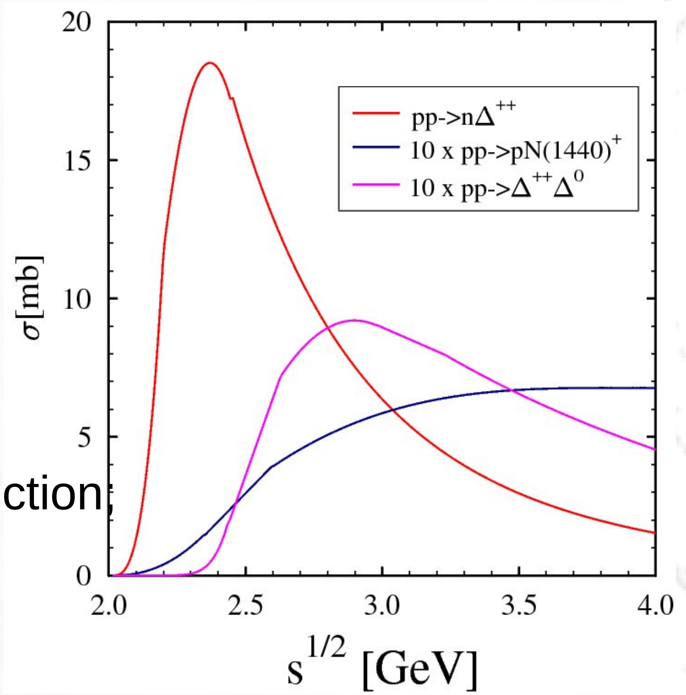
**upgrades implemented in Bucharest:**

- -various parametrizations for the EoS: optical potential, symmetry energy(powerlaw, Gogny)
- -threshold effects for baryon resonance reaction emission absorption, π emission/absorption
- -in-medium pion potential
- -clusterization algorithms (MST, SACA): promising preliminary results
- -**planned**:account for threshold effects for reactions involving strangeness degrees of freedom

# Pion production

**two step process:**

- resonance excitation in baryon-baryon collisions
  parametrization of the OBE model of
  S.Huber et al., NPA 573, 587 (1994)
- resonance decay:
  Breit-Wigner shape of the resonance spectral function;
  parameters -> K. Shekhter, PRC 68, 014904 (2003)
  decay channels: $R \to N\pi,\quad R \to N\pi\pi$
  $R \to \Delta(1232)\pi,\quad R \to N(1440)\pi$




**pion absorption:**

-resonance model (all 4* resonances below 2 GeV)
K. Shekhter, PRC 68, 014904 (2003)

**additional channels:**

vector meson production/absorption

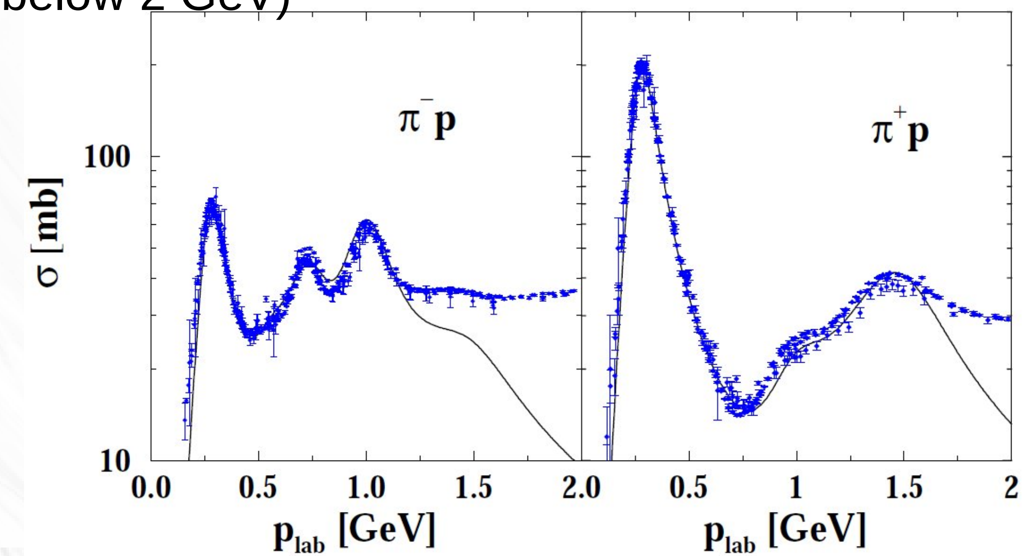
$V+B \to \pi+B', \pi+\pi+B'$

$\pi+B \to V+B'$

vector meson decay $\rho \to \pi+\pi$

$\omega \to 3\pi,\ 2\pi^0$

pion annihilation $\pi+\pi \to \rho$



# Isospin dependence of EoS

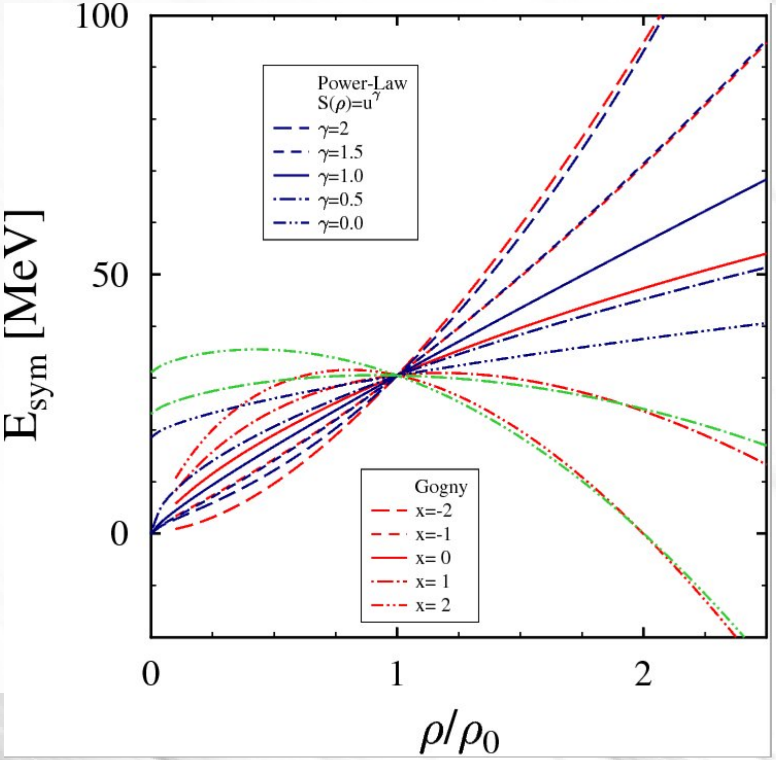
a) momentum dependent – generalization of the Gogny interaction:

Das, Das Gupta, Gale, Li PRC67, 034611 (2003)

$$U(\rho,\beta,p,\tau,x) = A_u(x)\frac{\rho_{\tau'}}{\rho_0} + A_l(x)\frac{\rho_\tau}{\rho_0} + B(\rho/\rho_0)^\sigma(1-x\beta^2) - 8\tau x\frac{B}{\sigma+1}\frac{\rho^{\sigma-1}}{\rho_0^\sigma}\beta\rho_{\tau'}$$
$$+\frac{2C_{\tau\tau}}{\rho_0}\int d^3p'\,\frac{f_\tau(\vec{r},\vec{p'})}{1+(\vec{p}-\vec{p'})^2/\Lambda^2} + \frac{2C_{\tau\tau'}}{\rho_0}\int d^3p'\,\frac{f_{\tau'}(\vec{r},\vec{p'})}{1+(\vec{p}-\vec{p'})^2/\Lambda^2}$$

$$S(\rho)=S(\rho_0)+\frac{L_{sym}}{3}\frac{\rho-\rho_0}{\rho_0}+\frac{K_{sym}}{18}\frac{(\rho-\rho_0)^2}{\rho_0^2}$$

| x | $L_{sym}$ [MeV] | $K_{sym}$ [MeV] |
|---|---|---|
| -2 | 152 | 418 |
| -1 | 106 | 127 |
| 0 | 61 | -163 |
| 1 | 15 | -454 |
| 2 | -301 | -745 |




b) momentum dependent – power law

$$U_{sym}(\rho,\beta)=\begin{cases} S_0(\rho/\rho_0)^\gamma - linear, stiff \\ a+(18.5-a)(\rho/\rho_0)^\gamma - soft, supersoft \end{cases}$$

# Energy conservation (in-medium)

$$\sqrt{p_1^2+m_1^2}+U(p_1)+\sqrt{p_2^2+m_2^2}+U(p_2)=\sqrt{p'^2_1+m'^2_1}+U(p'_1)+\sqrt{p'^2_2+m'^2_2}+U(p'_2)$$

- rarely considered in transport models below 1 AGeV, with a few exceptions: G. Ferini et al. PRL 97, 202301 (2006), C.Fuchs et al. PRC 55, 411 (1997), T. Song, C.M. Ko PRC 91, 014901 (2015)
- Ansatz for the isospin 3/2 resonance potential motivated by decay channel – see also S.A. Bass et al., PRC 51, 3343 (1995)
- imposed in the CM of the colliding nuclei (not in Eckart frame)
- reactions: NN↔NR, R↔Nπ (R↔Nππ not corrected)

$$U(\Delta^{++})=U^p$$
$$U(\Delta^{+})=\frac{2}{3}U^p+\frac{1}{3}U^n$$
$$U(\Delta^{0})=\frac{1}{3}U^p+\frac{2}{3}U^n$$
$$U(\Delta^{-})=U^n$$

B.-A. Li, NPA 708 (365) 2002

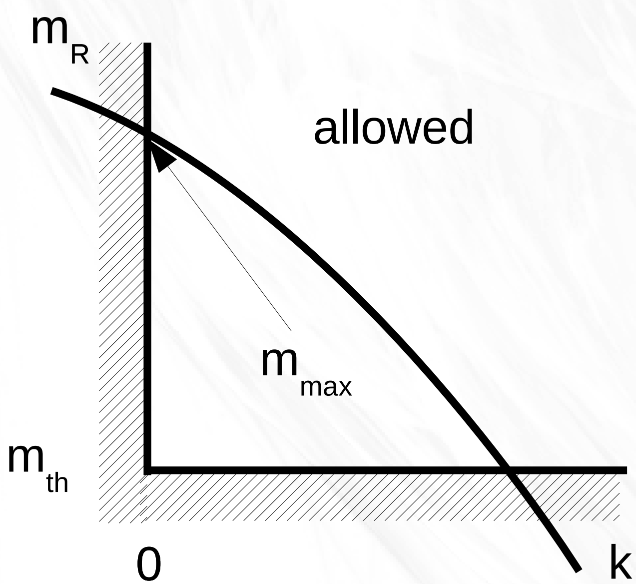
| Reaction | $\Delta U=U^f-U^i$ | Effect |
|---|---|---|
| nn → p$\Delta^-$ | $U^p-U^n<0$ | more $\pi^-$ |
| nn → n$\Delta^0$ | $1/3(U^p-U^n)<0$ | more $\pi^-$, $\pi^0$ |
| np → p$\Delta^0$ | $1/3(U^p-U^n)<0$ | more $\pi^-$, $\pi^0$ |
| np → n$\Delta^+$ | $1/3(U^n-U^p)>0$ | less $\pi^+$, $\pi^0$ |
| pp → p$\Delta^+$ | $1/3(U^n-U^p)>0$ | less $\pi^+$, $\pi^0$ |
| pp → n$\Delta^{++}$ | $U^n-U^p>0$ | less $\pi^+$ |

# Approximations

final state phase space in NN->NR

## VEC

$$\sqrt{s}=\sqrt{m_N^2+k^2}+\sqrt{m_R^2+k^2}$$

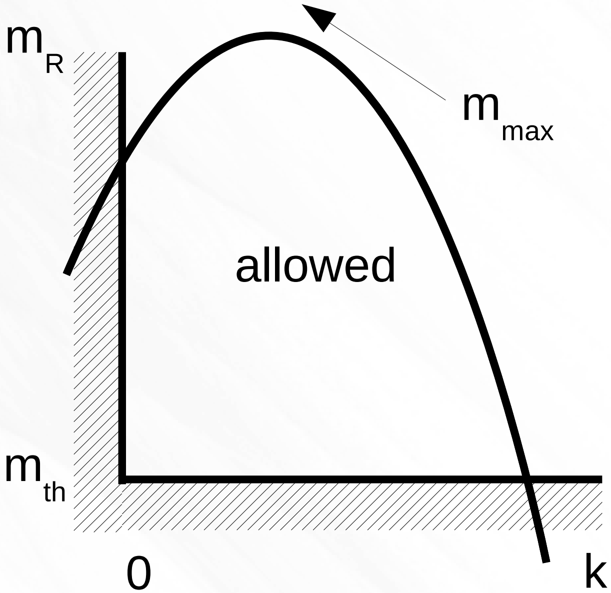



$$m_{th} \leqslant m_R \leqslant m_{max}$$
$$0 \leqslant \theta \leqslant \pi$$
$$0 \leqslant \Phi \leqslant 2\pi$$

consistent with $\frac{d\sigma}{d\Omega}$

## LEC, GEC

$$\sqrt{\tilde{s}}=\sqrt{m_N^2+k^2}+V_N(p_N)+\sqrt{m_R^2+k^2}+V_R(p_R)$$




$$m_{th} \leqslant m_R \leqslant m_{max} \longrightarrow m_{th} \leqslant m_R \leqslant m_R(k=0)$$

$$\theta_{min} \leqslant \theta \leqslant \theta_{max}$$
$$\Phi_{min} \leqslant \Phi \leqslant \Phi_{max}$$

} approximate to vacuum case

# Different “scenarios”

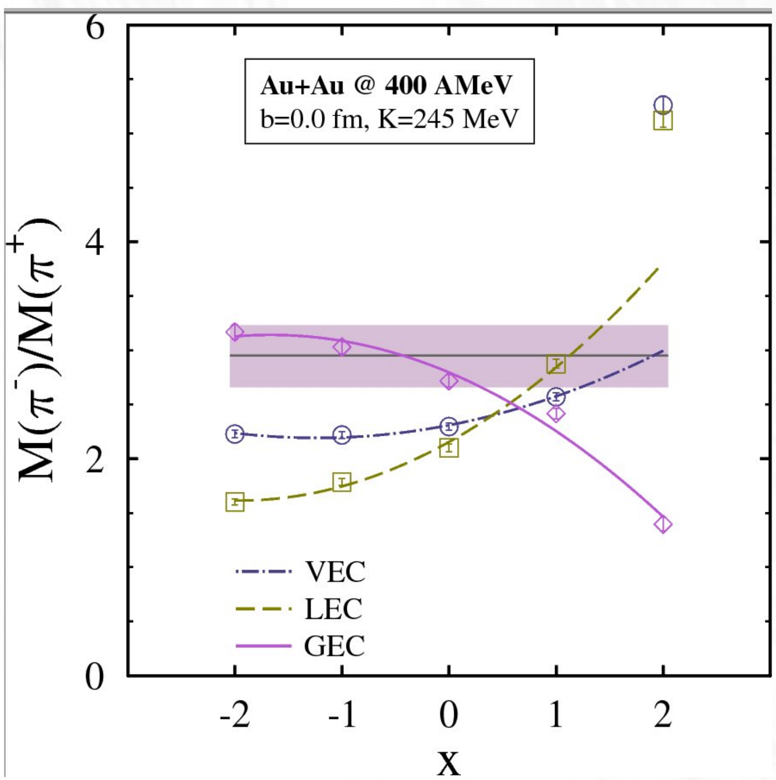
VEC – vacuum energy conservation constraint

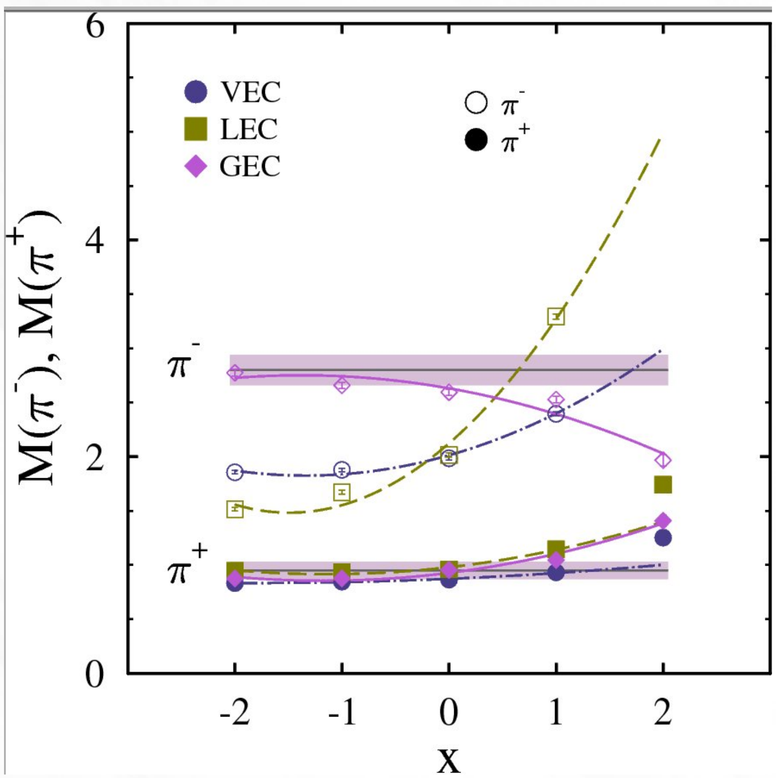
T. Song, C.M. Ko PRC 91, 014901 (2015) (similar)

LEC - “local” energy conservation – limited impact on multiplicities and ratios

GEC- “global” energy conservation – conserve energy of the entire system

-in-medium cross-sections above pion production threshold (mass scaling)




softer

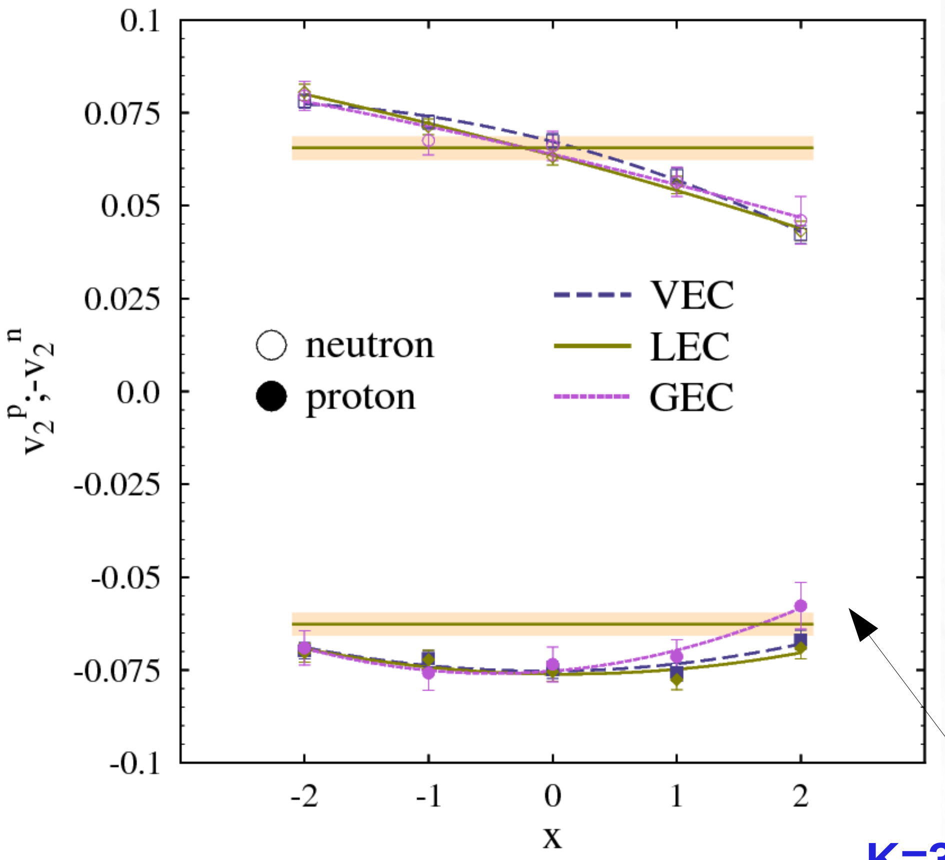
Experimental data: W. Reisdorf et al. (FOPI) NPA 848, 366 (2010)

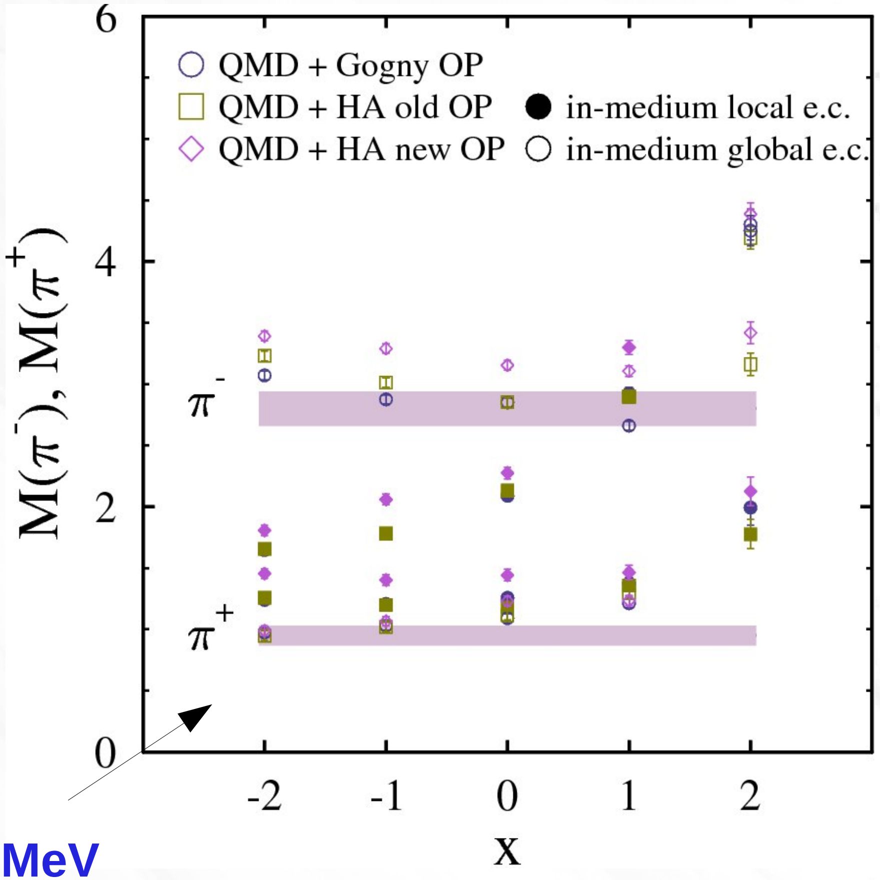
# Arguments for in-medium cross-sections

-vacuum cross-section – difficult to describe exp. FOPI-LAND $v_2$ and FOPI π data

-detailed analysis of FOPI light particle flow data: K=230±30 MeV

Y.Wang et al., PRC 89, 034606 (2014)

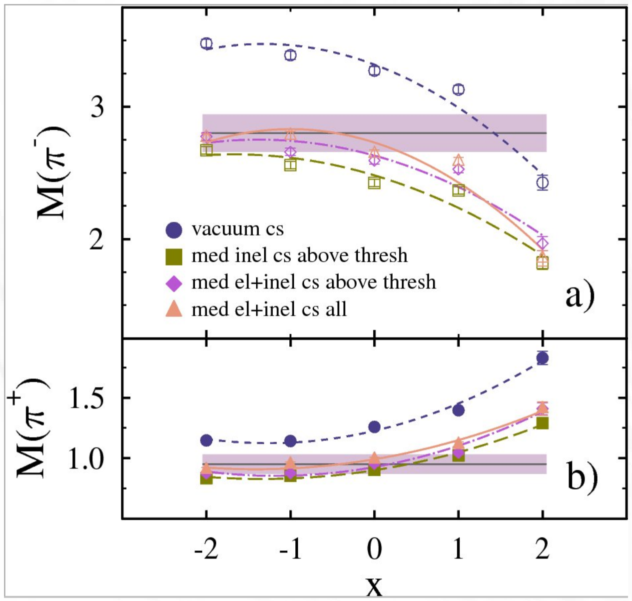


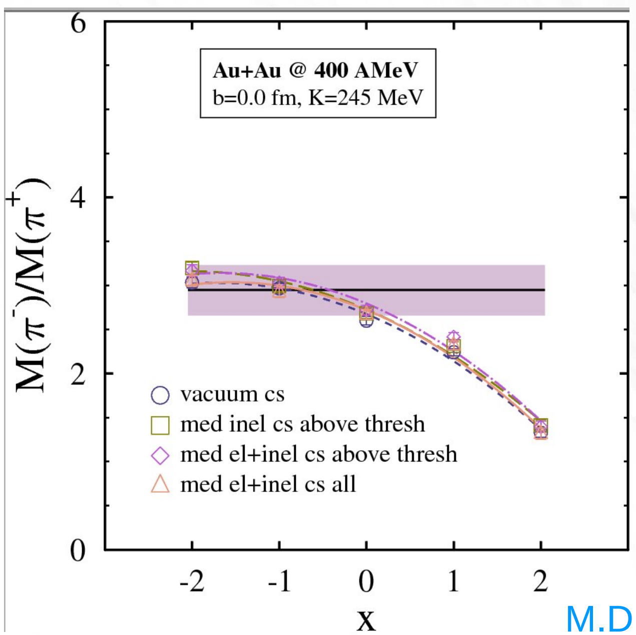




K=300 MeV

softer

# Pion ratio vs Elliptic Flow



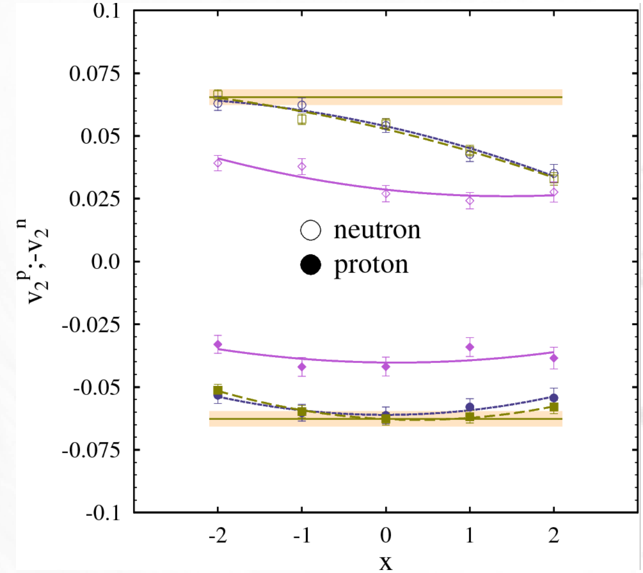



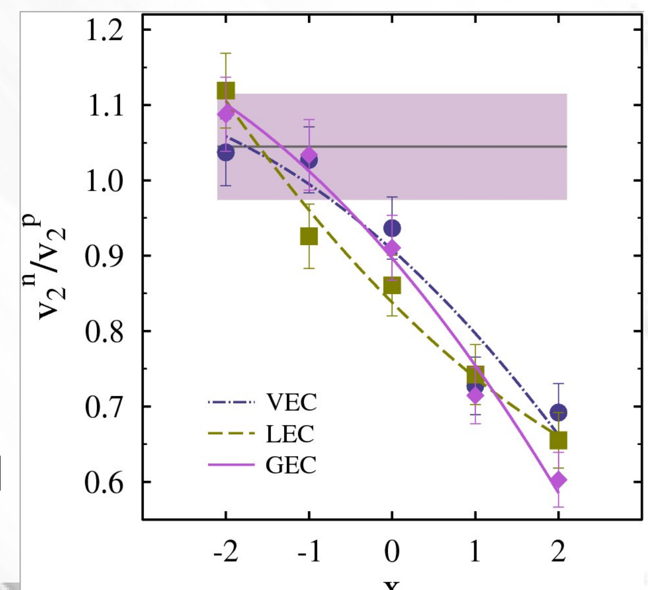

consistency between pion ratio and elliptic flow constraints **can** be achieved

conservation of the total energy of the system

\+

finer details: medium modified cross-sections, delta isobar potential

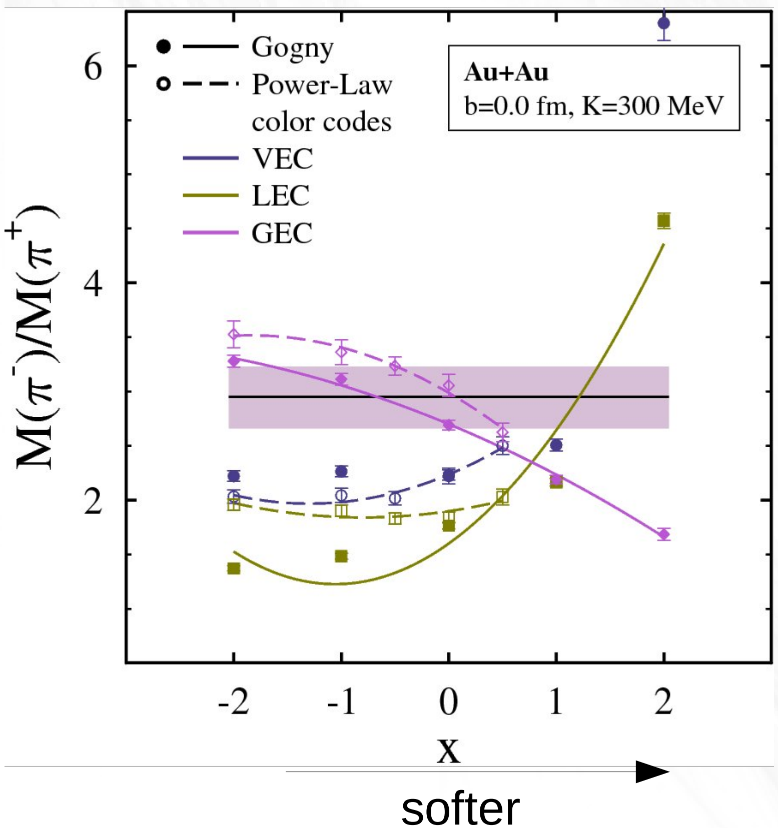


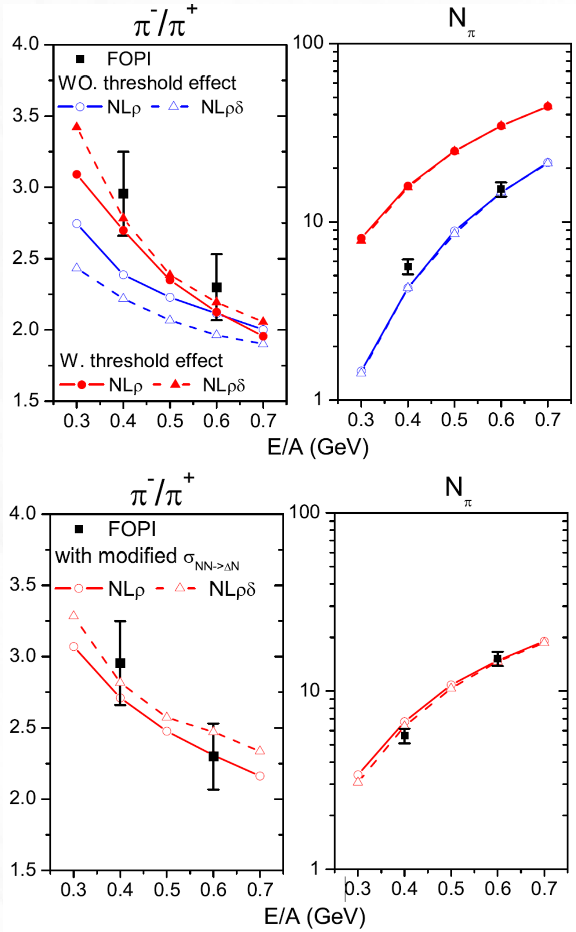




M.D. Cozma arXiv:1409.3110 (2014)

# Comparison with RVUU

- recent similar calculation (LEC): T. Song, C.M. Ko PRC 91, 014901 (2015)
- QMD: impact of neutron-proton effective mass splitting



# Energy dependence 1

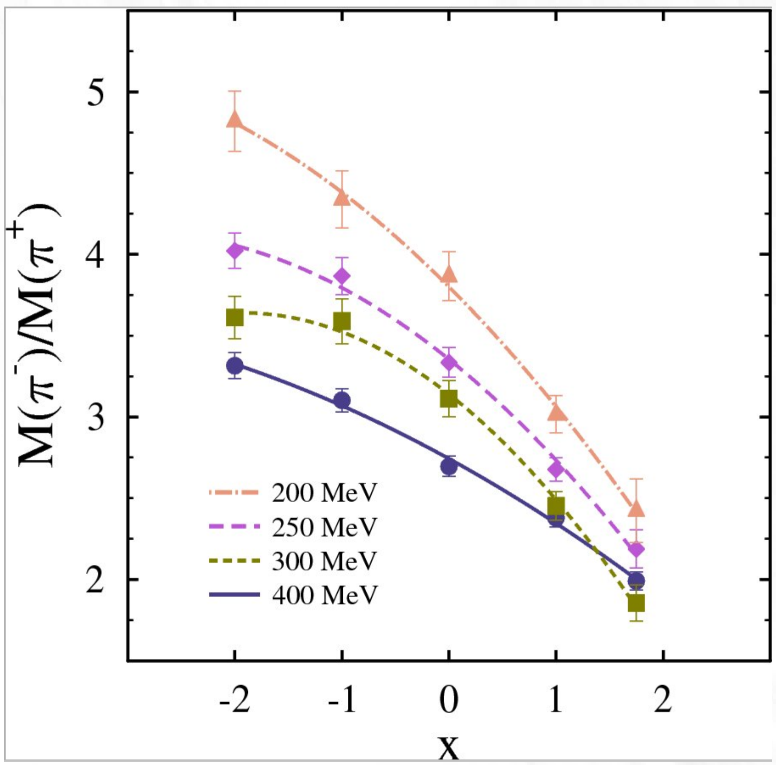
**close or below threshold**

Samurai (TPC+Nebula Collaboration)

-<u>pion production</u>, flow (including neutrons)

-energies 285-350 MeV

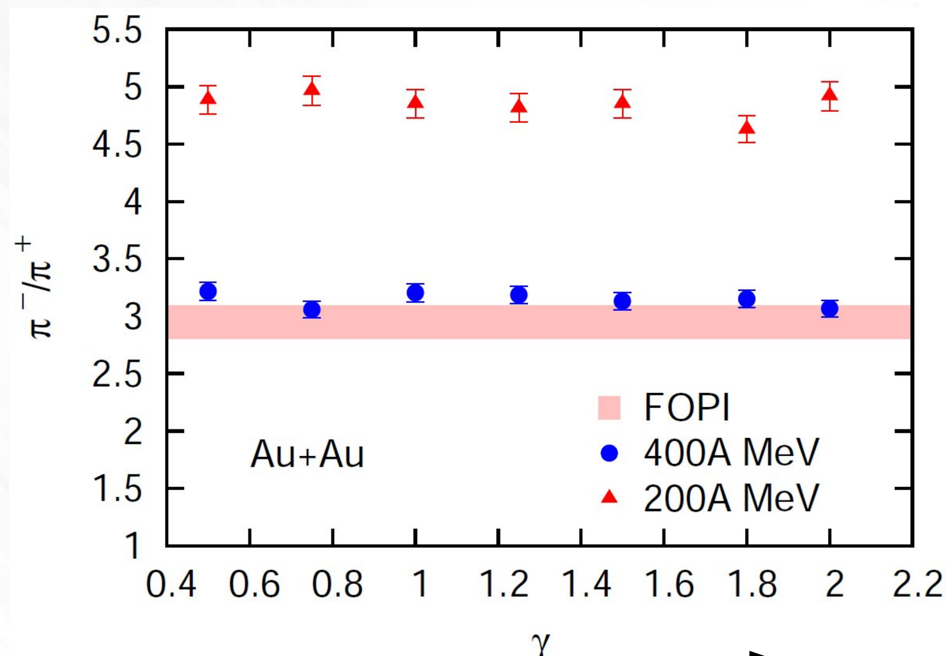
$^{132}Sn+^{124}Sn$, $^{105}Sn+^{112}Sn$




softer

M.D. Cozma arXiv:1409.3110 (2014)

$$S(u) = S_0\, u^{\gamma} \quad u = \rho/\rho_0$$

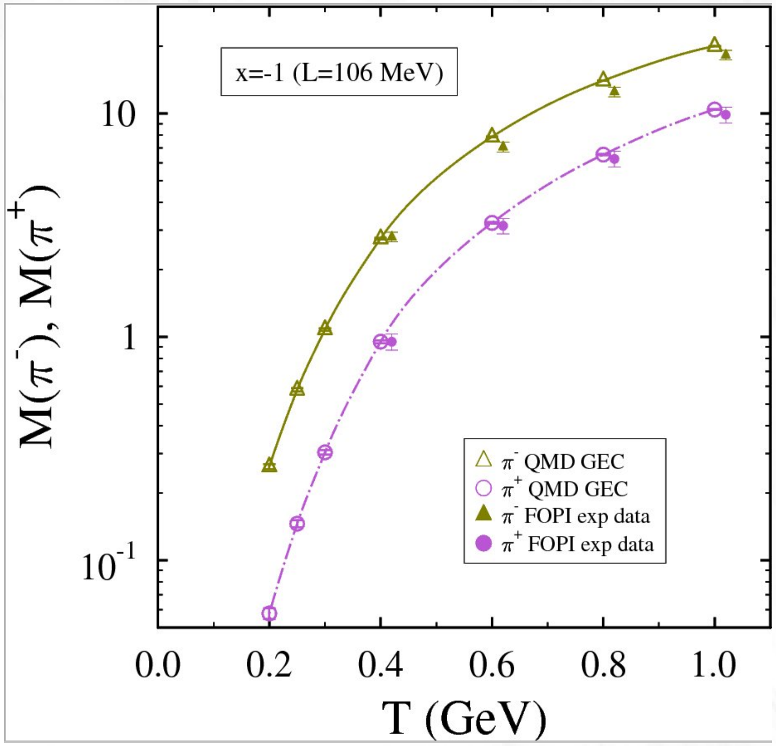


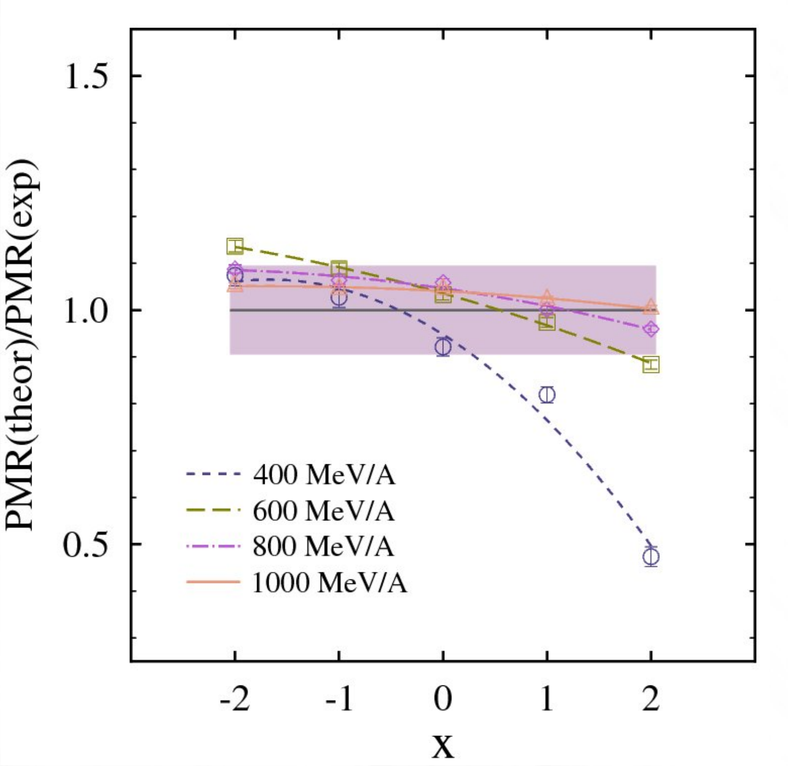

stiffer

J. Hong & P. Danielewicz, PRC 90, 024605 (2014)

# Energy dependence 2

**Comparison with available FOPI data T>0.4 GeV/A**




Experimental data: W. Reisdorf et al. (FOPI) NPA 848, 366 (2010)

# Impact of the Δ(1232) potential

Phenomenology – inclusive electron nucleus scattering (He,C,Fe) attractive

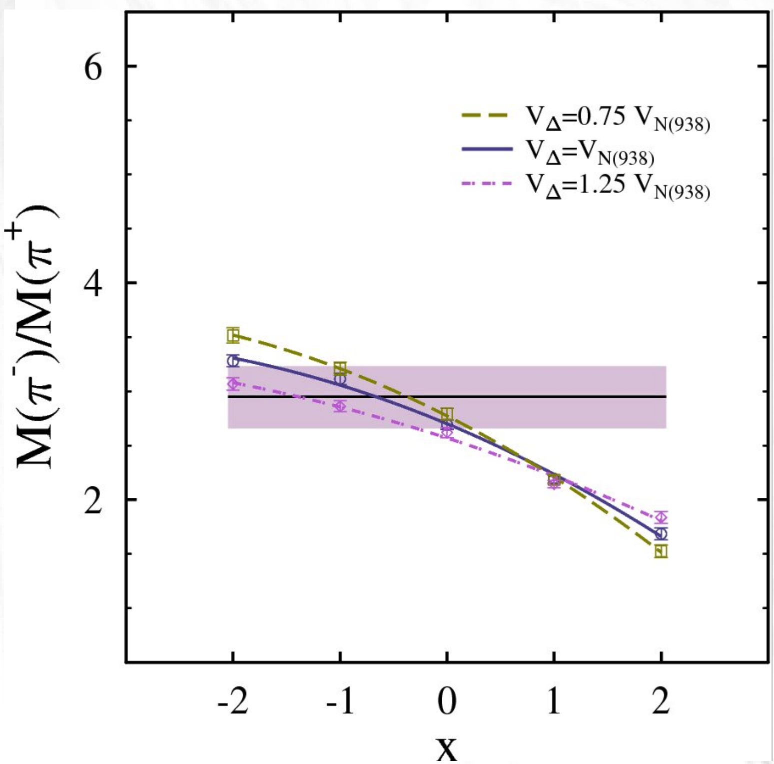
- Δ-nucleus potential deeper than the nucleon-nucleus potential

O'Connel, Sealock PRC 42, 2290 (1990)

Ab initio calculations – Argonne $v_{28}$ interaction (BBG)=> repulsive Δ potential

Baldo, Ferreira NPA 569, 645 (1994) - 3D reduction of Bethe-Salpeter equation similar (DB)

Malfliet, de Jong PRC 46, 2567 (1992) - strong dominant repulsive contribution from T=2 sector




$$V(\Delta^{++}) = V_s + \frac{3}{2} V_v$$

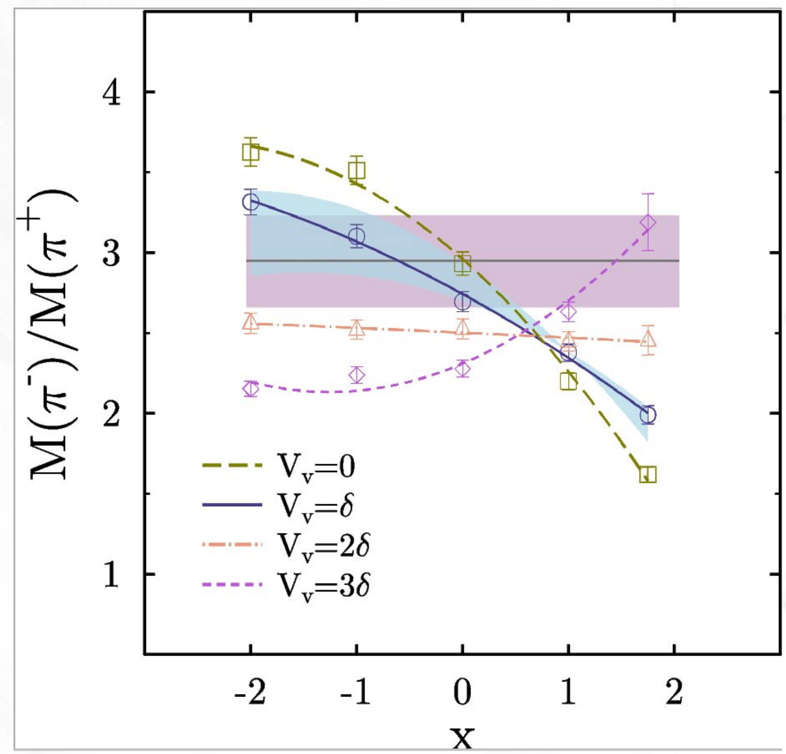
$$V(\Delta^{+}) = V_s + \frac{1}{2} V_v$$

$$V(\Delta^{0}) = V_s - \frac{1}{2} V_v$$

$$V(\Delta^{-}) = V_s - \frac{3}{2} V_v$$

$$V_s = \frac{1}{2}(V_n + V_p)$$

$$\delta = \frac{1}{3}(V_n - V_p)$$




softer

M.D. Cozma arXiv:1409.3110 (2014)

# Pion S wave potential

**Microsopical approaches: Hadronic Models** J.Nieves et al., NPA 554, 509 (1993)

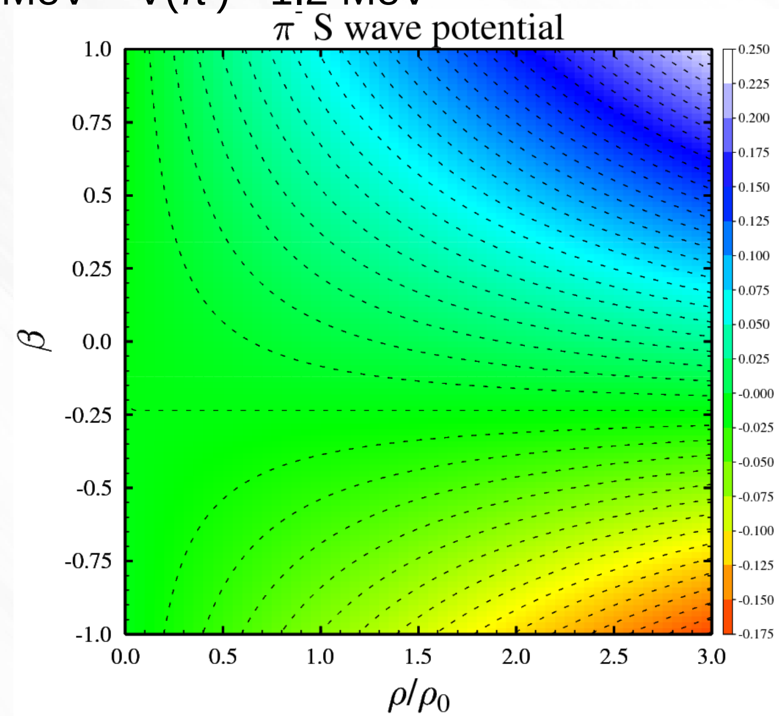
$V(\pi^+)$=18.9 MeV $V(\pi^0)$=8.8 MeV $V(\pi^-)$=-1.3 MeV

M.Doring et al, PRC 77, 024602 (2008)

can accomodate repulsion needed by phenomelogical fits; large theoretical uncertainties

**ChPT**: two loop approximation N. Kaiser et al. PLB 512, 283 (2001)

$V(\pi^+)$=13.8 MeV $V(\pi^0)$=6.1 MeV $V(\pi^-)$=-1.2 MeV




**Parametrization:**

$$V_S(r) = -4\pi[b(r) + \epsilon_2 B_0 \rho^2(r)]$$
$$b(r) = \epsilon_1[b_0\rho(r) + b_1(\rho_n(r) - \rho_p(r))]$$

$$\epsilon_1 = \frac{1}{2\mu} + \frac{1}{2M} \qquad \epsilon_1 = \frac{1}{2\mu} + \frac{1}{4M}$$

$$b_0 = 0.0283\, m_\pi^{-1} \qquad b_1 = -0.120\, m_\pi^{-1}$$

$$B_0 = 0.042\, i\, m_\pi^{-4}$$

R. Seki, K. Masutani, PRC 27, 2799 (1983)

$V(\pi^+)$=28.8 MeV $V(\pi^0)$=15.6 MeV $V(\pi^-)$=2.4 MeV

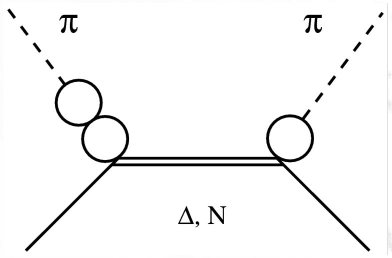
(at $\rho=\rho_0$, $\beta$=0.20)

# Pion P wave potential

**Three level model (3LM)**: allows analytical calculation (self-energies)

M.Urban et al.
NPA 641, 433 (1988)

**Approximations:** only Δ and non-resonant scattering
perform a non-relativistic reduction
pion momentum larger than Fermi momentum




$$\Pi(k)=\frac{\Pi_{Nh}+\Pi_{\Delta h}-(g_{11}-2g_{12}+g_{22})\Pi_{Nh}\Pi_{\Delta h}}{1-g_{11}\Pi_{Nh}-g_{22}\Pi_{\Delta h}+(g_{11}g_{22}-g_{12}^2)\Pi_{Nh}\Pi_{\Delta h}}$$

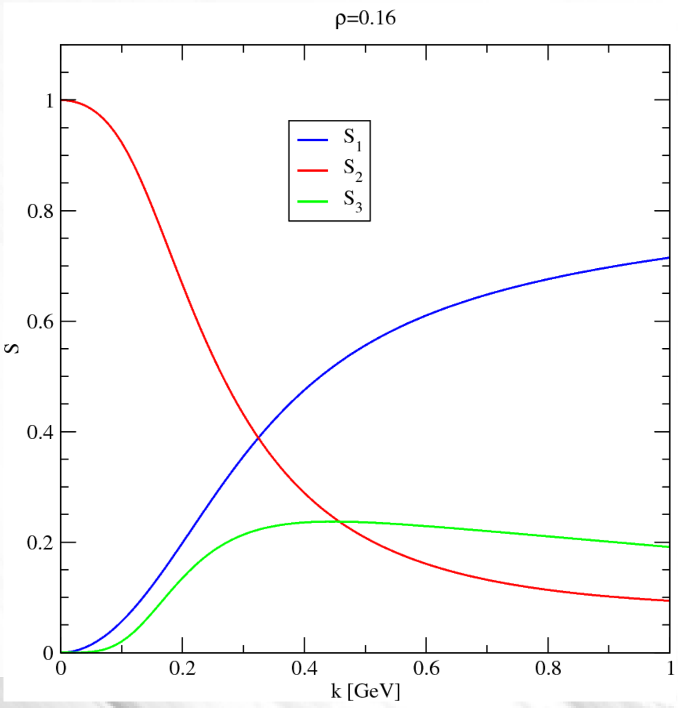
$$\Delta_\pi(k)=\frac{1}{k^2-m_\pi^2-\vec{k}^2\Pi(k)}$$

$$\Delta_\pi(k)\stackrel{3LM}{=}\frac{S_1(\vec{k})}{k_0^2-\omega_1^2(\vec{k})}+\frac{S_2(\vec{k})}{k_0^2-\omega_2^2(\vec{k})}+\frac{S_3(\vec{k})}{k_0^2-\omega_3^2(\vec{k})}$$

Effective dispersion relation:

$$\omega_{eff}(\vec{k})=S_1(\vec{k})\omega_1(\vec{k})+S_2(\vec{k})\omega_2(\vec{k})+S_3(\vec{k})\omega_3(\vec{k})$$

W. Ehehalt et al., PLB 298, 31 (1993)
C. Fuchs et al., PRC 55, 411 (1997)




Effective potential:

$$V_\pi^{eff}=\omega^{eff}-\sqrt{m_\pi^2+\vec{k}^2}$$

# Pion Potential

**P wave (continued)**

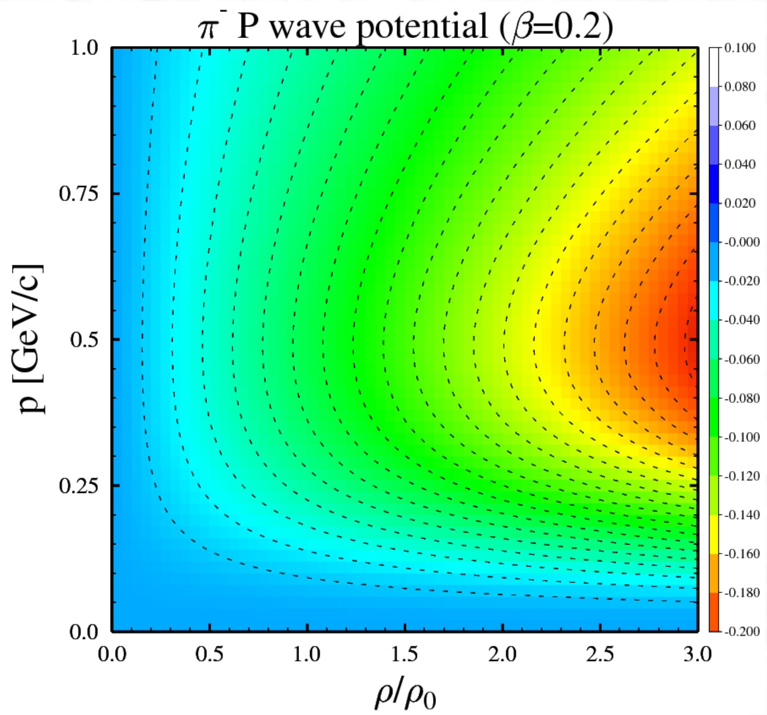
**Parametrization:**

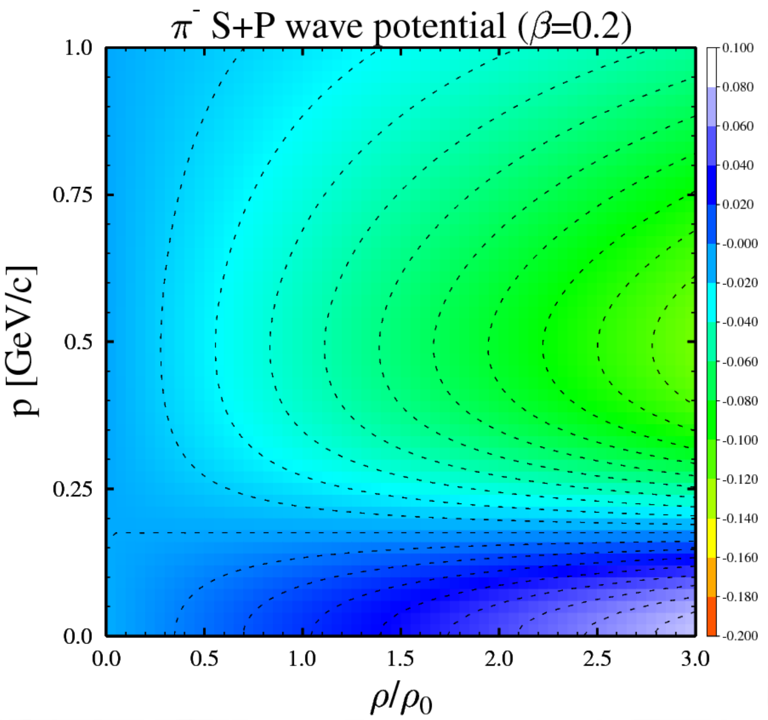
$$V(\pi^- ; u, \beta, k) = \frac{k^2}{1 + k^2/\Lambda_1^2 + k^4/\Lambda_2^4}(b_{11} u + b_{21} u \beta)$$

$$\Lambda_1 = 2.138 \mathrm{m}_\pi \qquad \Lambda_2 = 3.551 \mathrm{m}_\pi$$

$$b_{11} = -0.180 \mathrm{m}_\pi^{-1} \qquad b_{21} = 0.011 \mathrm{m}_\pi^{-1}$$

fitted in the region: 0 < u < 3.0; -0.5 < β < 0.5; 0.0 GeV/c < k < 0.75 GeV/c



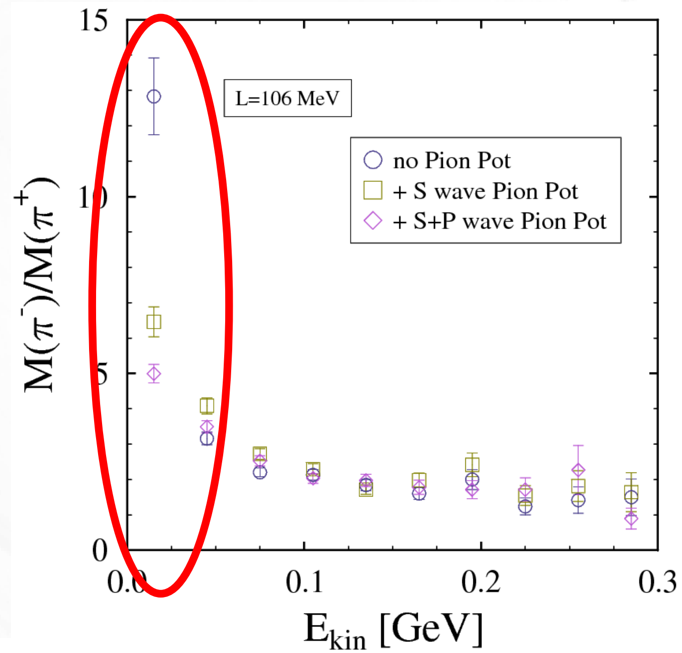
# Impact on π multiplicities

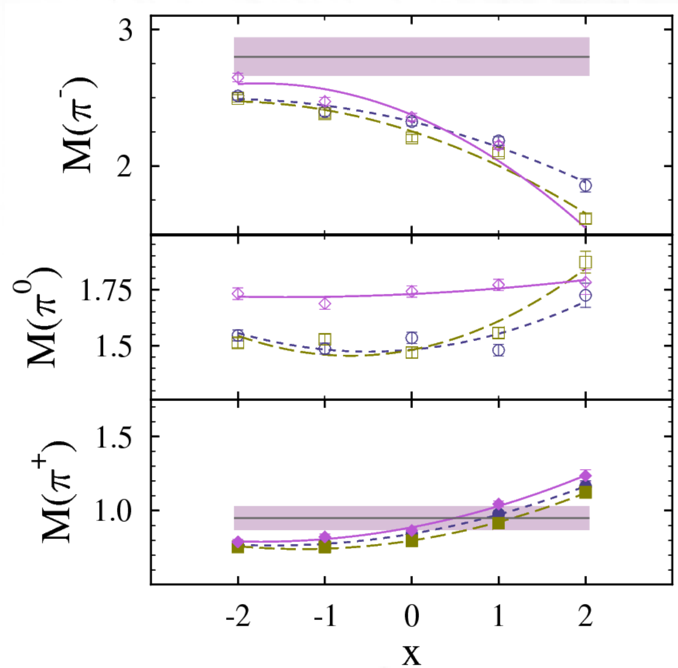
inelastic channels – modified via mass scaling scenario

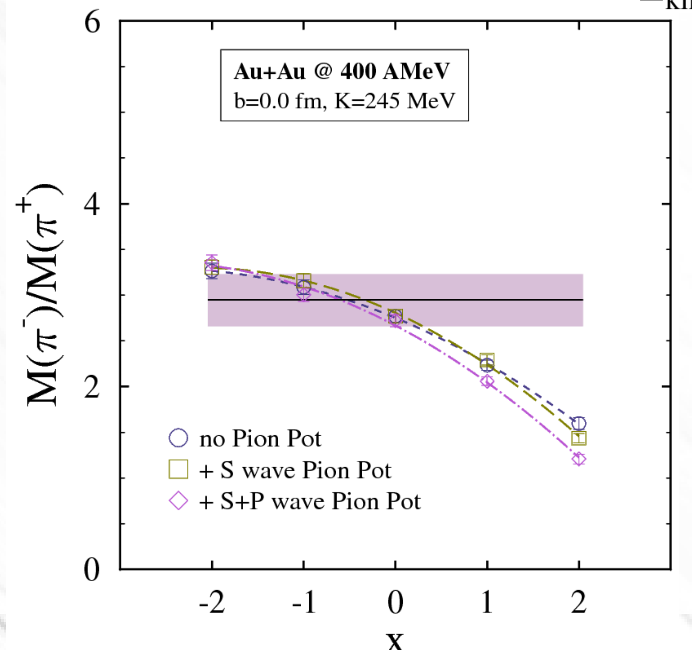
initialization – improved description of rms and diffuseness parameters

$p_T$ spectra - fail to reproduce FOPI $\langle p_T^{\pi^-} \rangle / \langle p_T^{\pi^+} \rangle$ ratio

W. Reisdorf et al., NPA 781, 459 (2007)




softer

# Conclusions

**Conservation of Energy:** important impact on pion multiplicities in heavy-ion collisions at few hundred MeV impact energy

**$\pi^-/\pi^+$:** - confirmed as sensitive to the stiffness of asy-EoS
- increased sensitivity below production threshold
- pion potential moderate impact on multiplicities and small on ratios

However:
- isovector part of the in-medium Δ potential has a large/decisive impact
- precise density/asymmetry dependence of elastic/inelastic channels cross-sections important

**Good news:** -consistency between pion ratio and elliptic flow constraints can be achieved (GEC scenario only!)

**To do list**: - retardation and relativistic corrections
- in-medium baryon potentials (particularly Δ(1232))
- in-medium modification of cross-sections

**Worst case scenario:** test of our understanding of hadronic interaction in the few hundred MeV energy domain