



Constraining the nuclear matter equation of state around twice saturation density

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¹GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt, Germany

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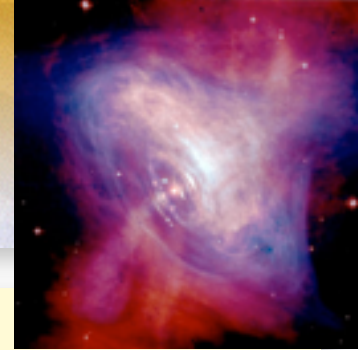
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- ▶ Introduction.
- ▶ Analysis and results.
- ▶ Simulations: the scenario.
- ▶ Summary and discussion.



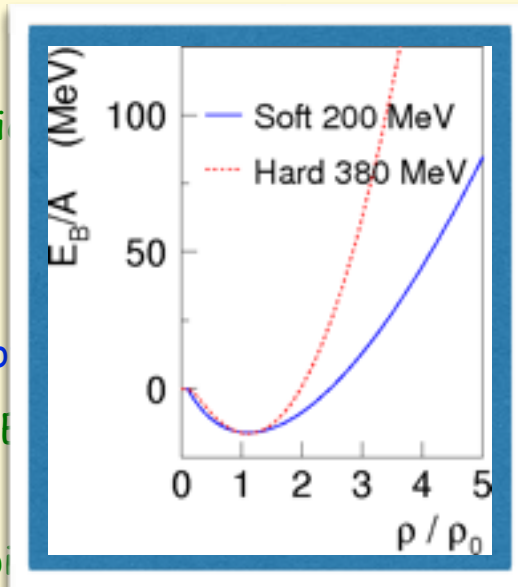
Introduction



NGC 1952, Crab Nebula pulsar neutron star imaged by the NASA/ESA Hubble Space Telescope

▶ The equation of state (EOS) of nuclear matter:

- ▶ of fundamental interest
- ▶ object of intense theoretical studies
- ▶ an important ingredient in the study of:
 - ▶ compact stars ^[1]
 - ▶ core collapse supernovae
- ▶ The calculation of the nuclear EOS ^[2], is a very complex task.
- ▶ Nuclear physics based on empirical data and the most 'fundamental' theory of nuclear forces requires a confrontation with empirical facts.
- ▶ **1st method**, from astrophysicists: from 'neutron' star masses and radii. **But missing:**
 - ▶ precise model-independent radii,
 - ▶ composition of the matter in the centre of the stars.



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[1] J. M. Lattimer, Ann. Rev. Nucl. Part. Sci. 62 (2012) 485.

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Introduction



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- ▶ The equation of state (EOS) of nuclear matter:
 - ▶ of fundamental interest
 - ▶ object of intense theoretical efforts since several decades
 - ▶ an important ingredient in modeling fascinating astrophysical phenomena such as:
 - ▶ compact stars^[1]
 - ▶ core collapse supernovae^[2]
- ▶ The calculation of the nuclear EOS from first principles, such as very recently attempted in [3], is a very complex task.
- ▶ Nuclear physics based on empirical observations => even the most 'fundamental' theory of nuclear forces requires a confrontation with empirical facts.
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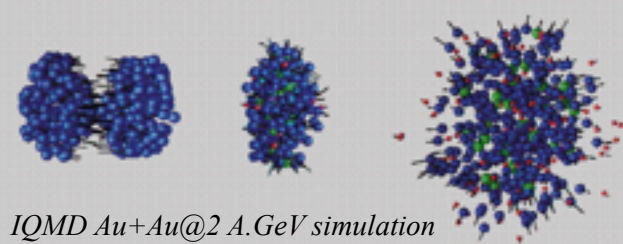
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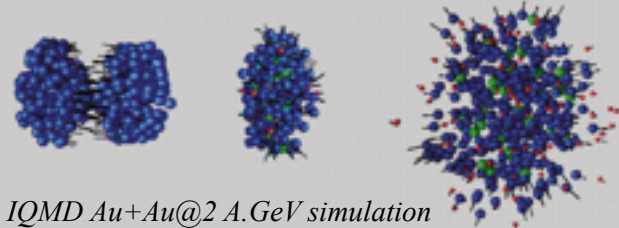
IQMD Au+Au@2 A.GeV simulation

- ▶ **Alternative method:** in earth laboratories, heavy ion collisions over a wide range of incident energies, system sizes and compositions.
 - ▶ flow method: limited to $E_{\text{beam}} < 10 \text{ A.GeV}$ ← some kind of a clock is available (sound velocity versus participant-spectator interaction).
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 - ▶ FOPI (2005), Au+Au @ 0.09-1.5 A.GeV, $Z=1$ elliptic flow, versus 4 different transport codes → 'no strong constraint on the EOS can be derived at this stage'.
 - ▶ BEVALAC & AGS accelerators, proton flows versus transport theories → $K_0 = 167\text{-}200 \text{ MeV}$ (soft) from V_1 , $K_0 = 300 \text{ MeV}$ (semi-stiff) from V_2 → contradictions.



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Flows at high density in heavy-ion collisions

$$\frac{dN}{d(\phi - \phi_R)}(y, p_t) = \frac{N_0}{2\pi} \left(1 + 2 \sum_{n \geq 1} v_n \cos n(\phi - \phi_R) \right)$$

Y = rapidity

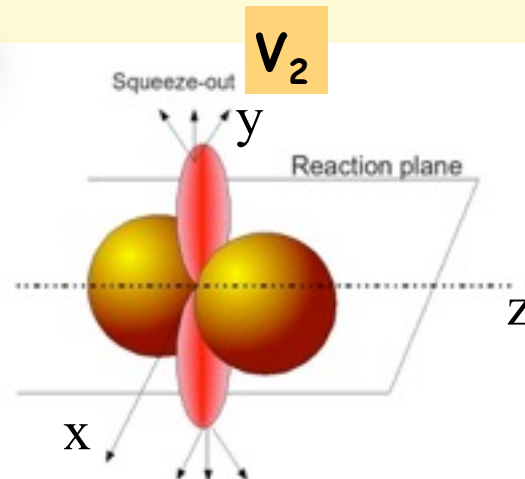
p_t = transverse momentum

Φ_R = reaction plane azimuthal angle

V_1 = 'side/directed flow', $\cos(\Phi - \Phi_R)$ mode

$$V_2(y, p_t) = \left\langle \frac{p_x^2 - p_y^2}{p_t^2} \right\rangle$$

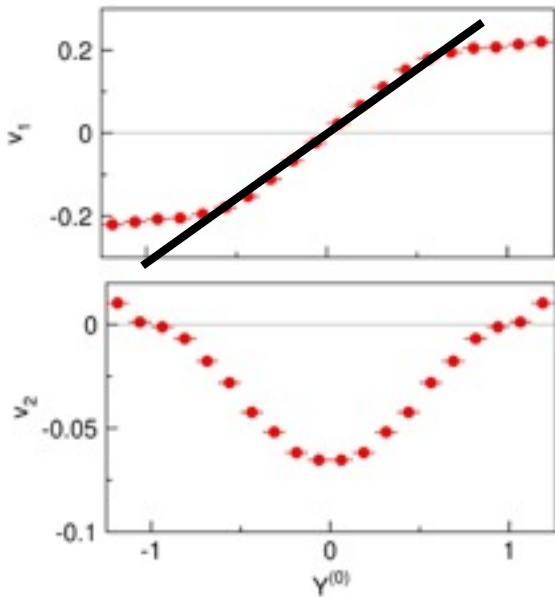
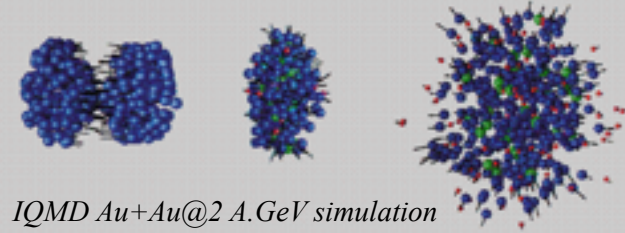
'Elliptic flow': $\cos(2(\Phi - \Phi_R))$ mode, competition between 'in-plane' ($V_2 > 0$) and 'out-of-plane' ejection ($V_2 < 0$).



under control in
 squeeze-out versus
 different transport
 → $K_0 = 167-200$

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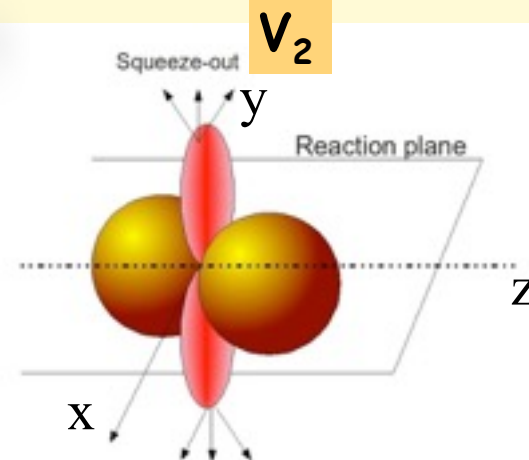
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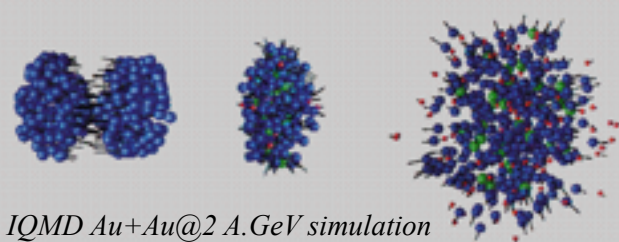
different transport

→ $K_0 = 167-200$

- ▶ Eo
- ▶ QI
- ▶ FC
- ▶ co
- ▶ BE
- ▶ Me



Introduction

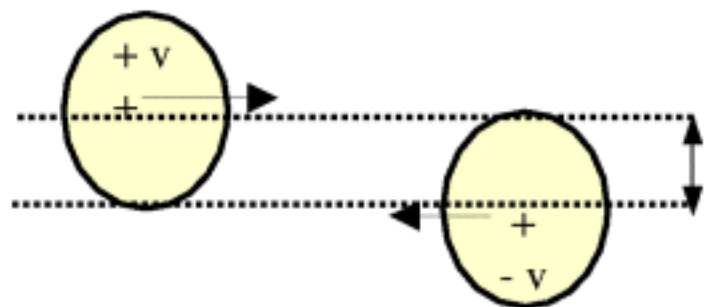


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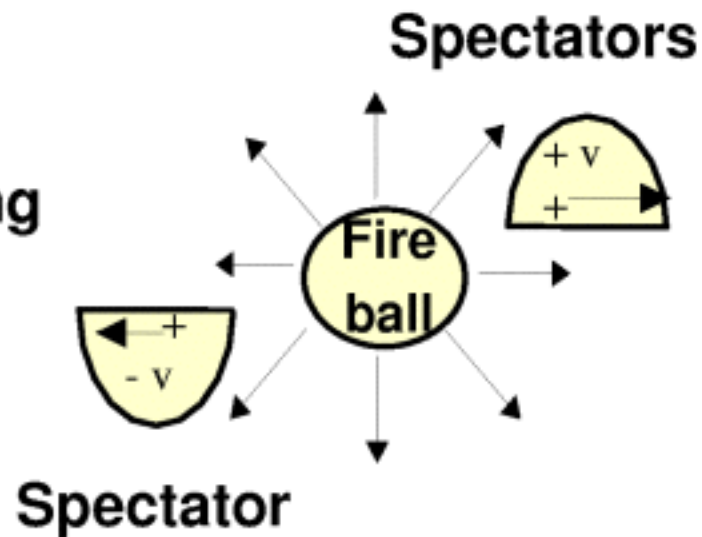
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Before the collision

after the collision

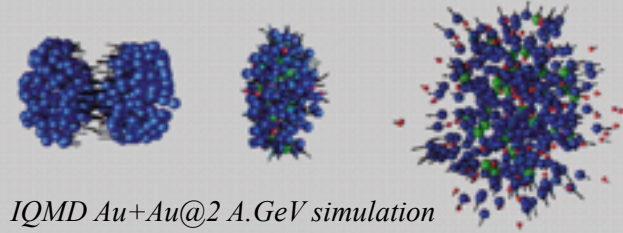


**Overlapping
zone**



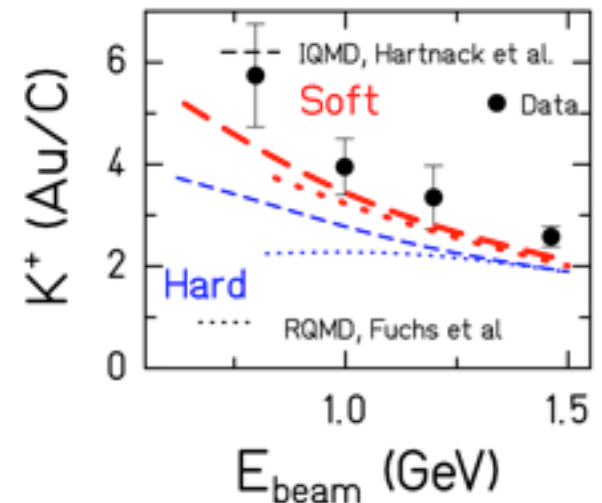


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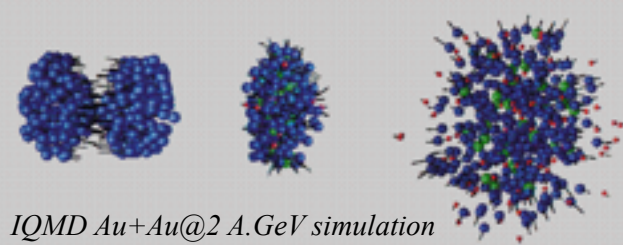
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 - ▶ BEVALAC & AGS accelerators, proton flows versus $\sqrt{s_{NN}}$ (soft) from V_1 , $K_0 = 300 \text{ MeV}$ (semi-stiff) from V_2



Data: C. Sturm et al., PRL 86 (2001)



Introduction

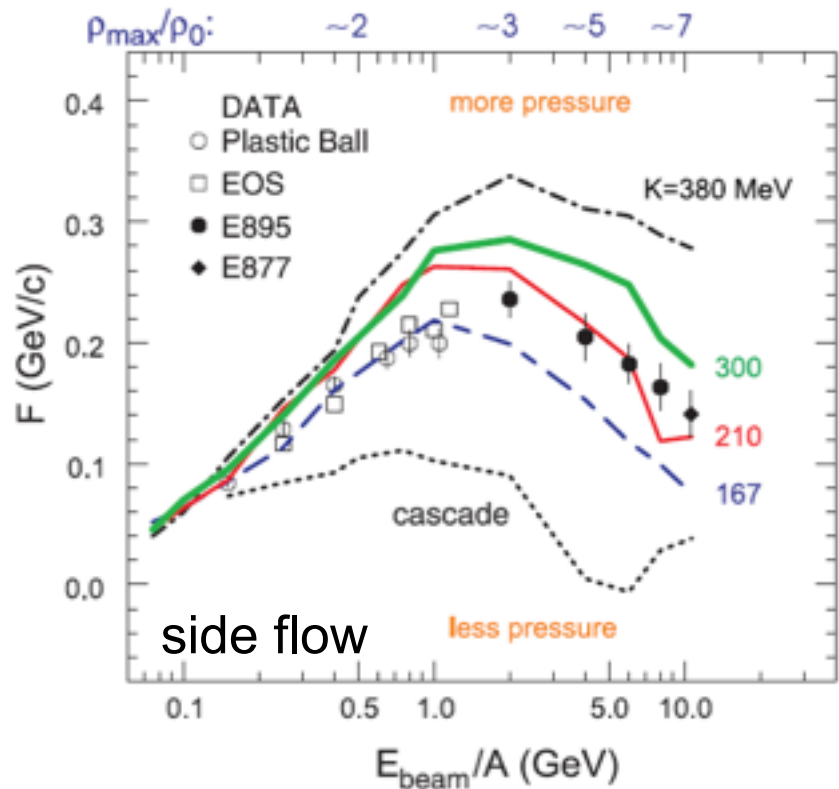
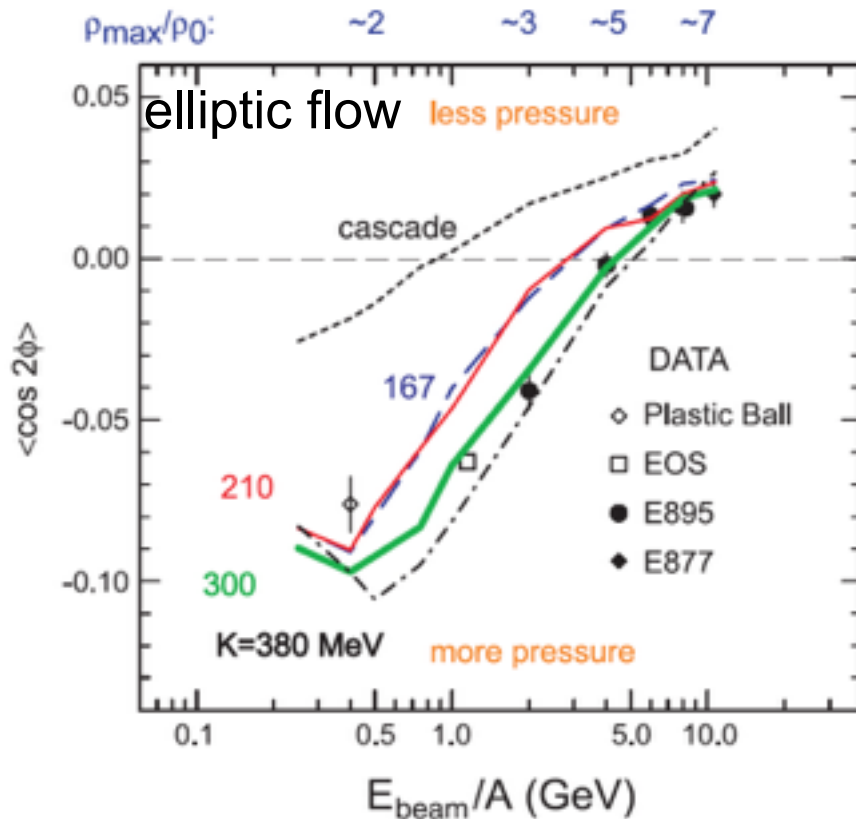


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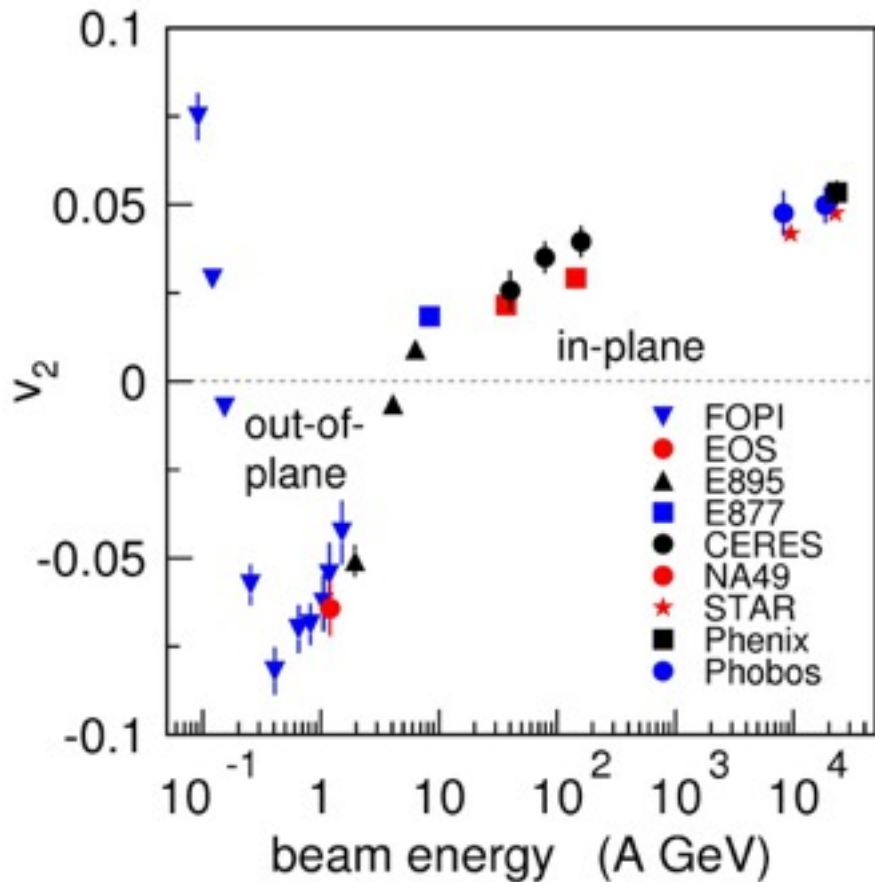
Elliptic flow and the nuclear matter EOS



P. Danielewicz et al.
Science 298, 1592 (2002)



Beam energy dependence of elliptic flow



elliptic flow

- pressure gradient of compression zone
- shadowing of spectators
 - attraction due to mean field of nucleons
- at low energies
- at high energies
 - lacking shadowing of spectators



Introduction

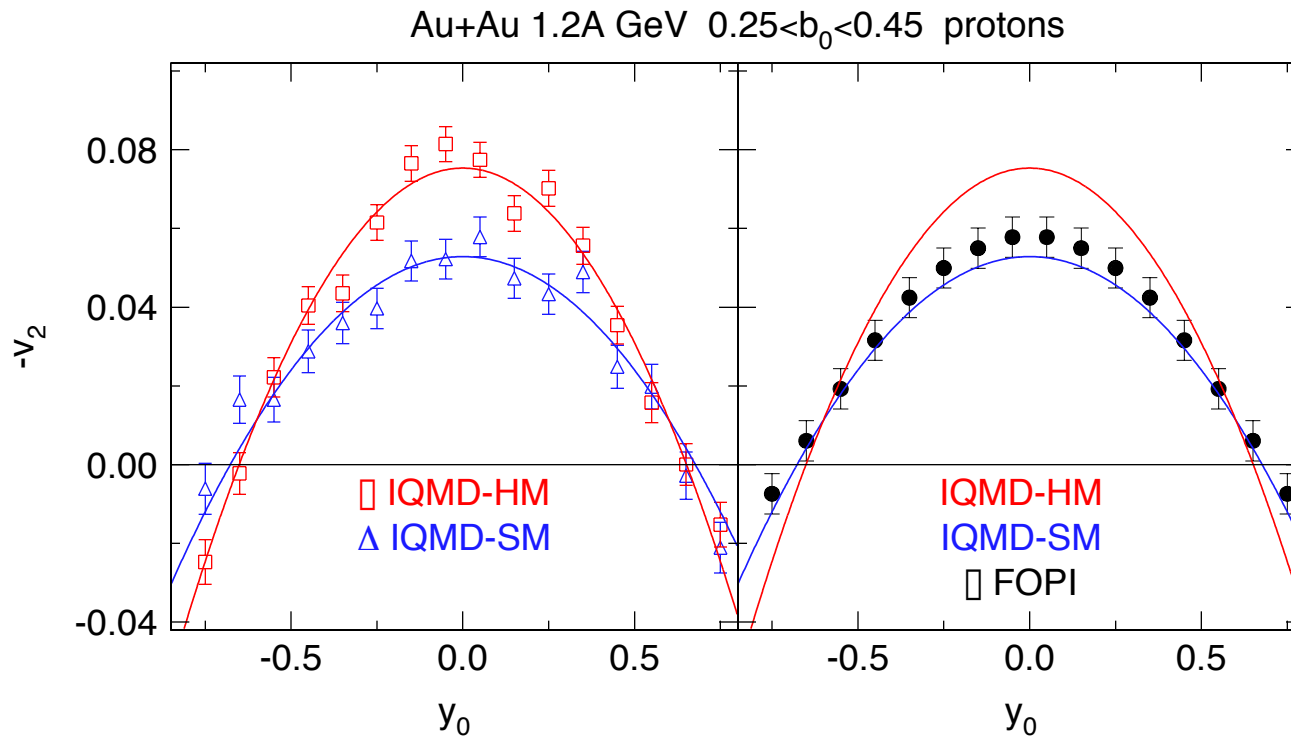
- ▶ **Present work:** improve the situation in the 1 A.GeV regime, from extensive flow data published recently by the **FOPI Collaboration** (Au+Au @ 0.4-1.5 A.GeV) [4]
 - close look at the **elliptic flow data with improvements:**
 - ▶ 1) not only **protons:** d, t, ^3He ^4He having larger flow signals than single nucleons.
 - ▶ 2) not only **mid-rapidity data:** 80% of the target- projectile rapidity gap.

[4] W. Reisdorf, et al. (FOPI Collaboration), Nucl. Phys. A 876 (2012) 1.



Analysis and results

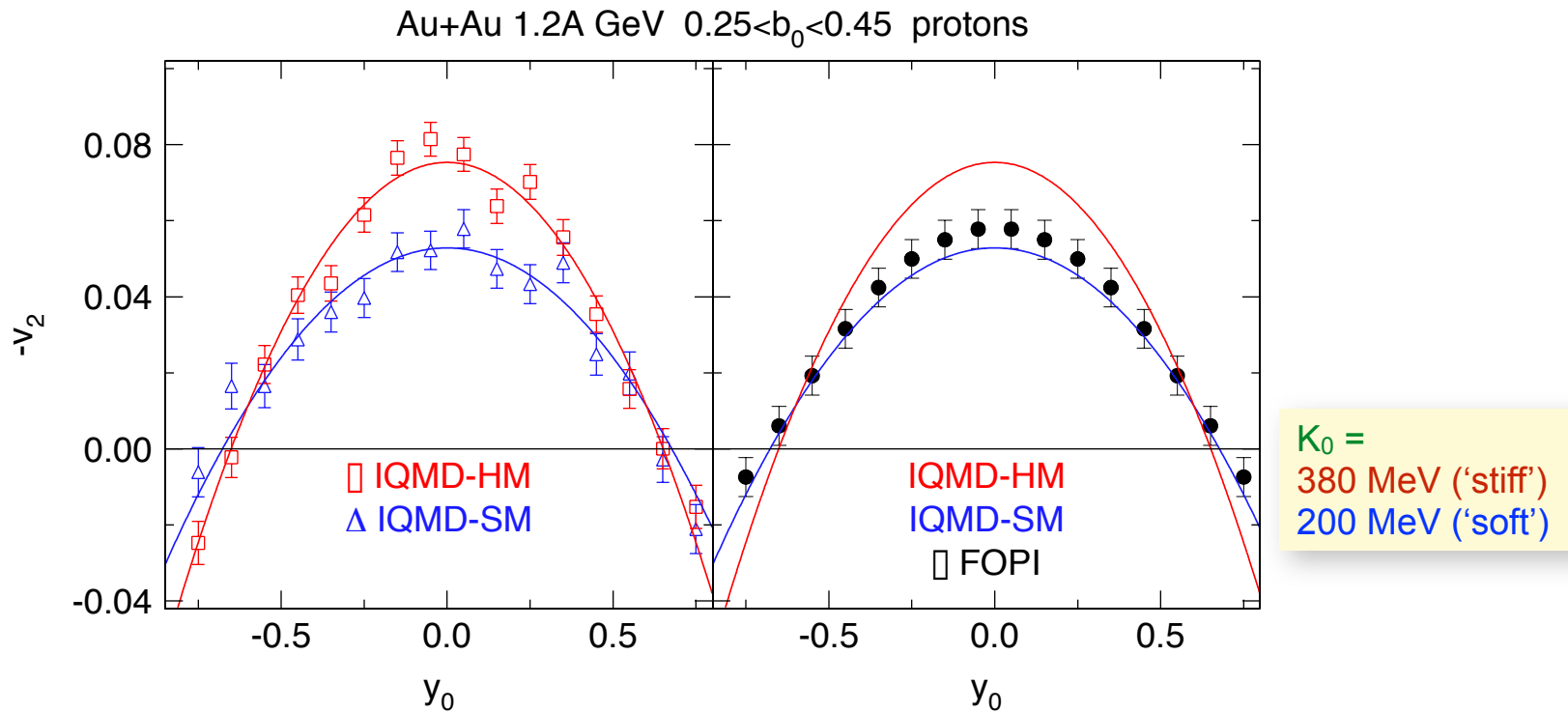
Elliptic flow





Analysis and results

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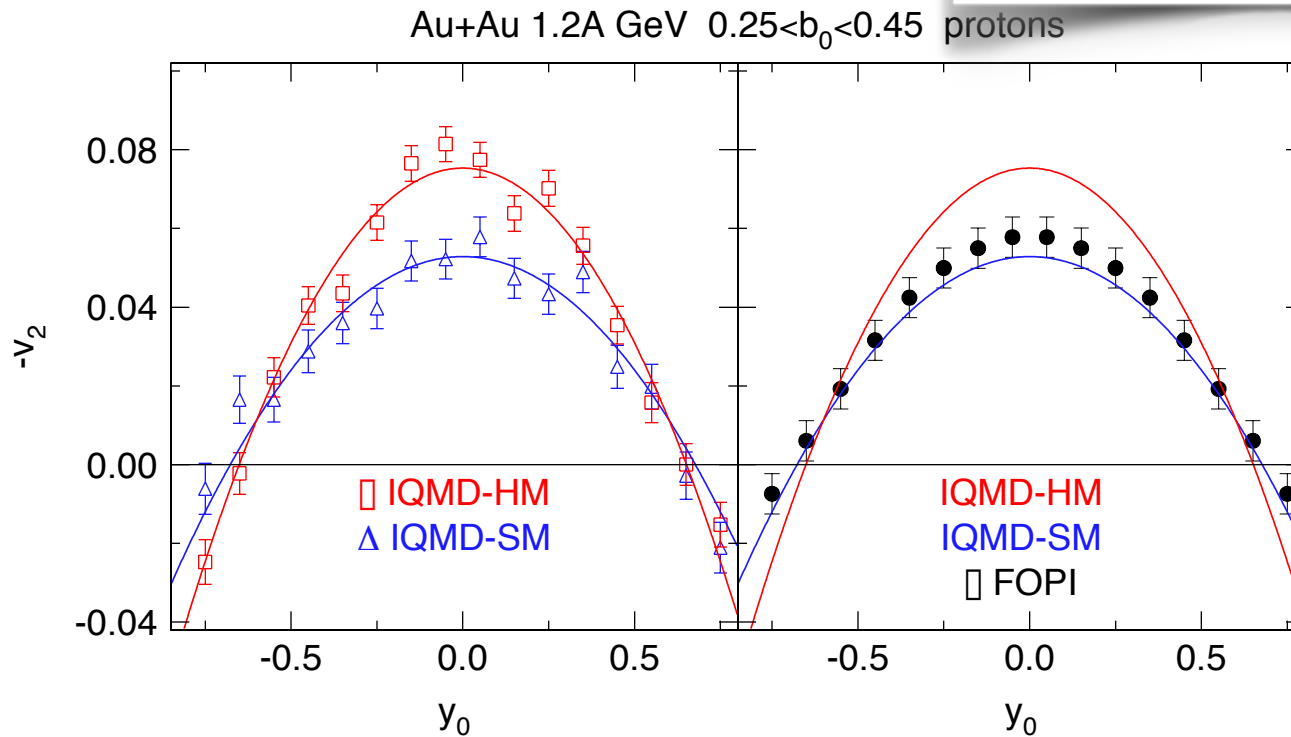
Complete shape of $v_2(y_0)$:

a new observable:

$$v_{2n} = |v_{20}| + |v_{22}|,$$

from fit

$$v_2(y_0) = v_{20} + v_{22} \cdot y_0^2$$

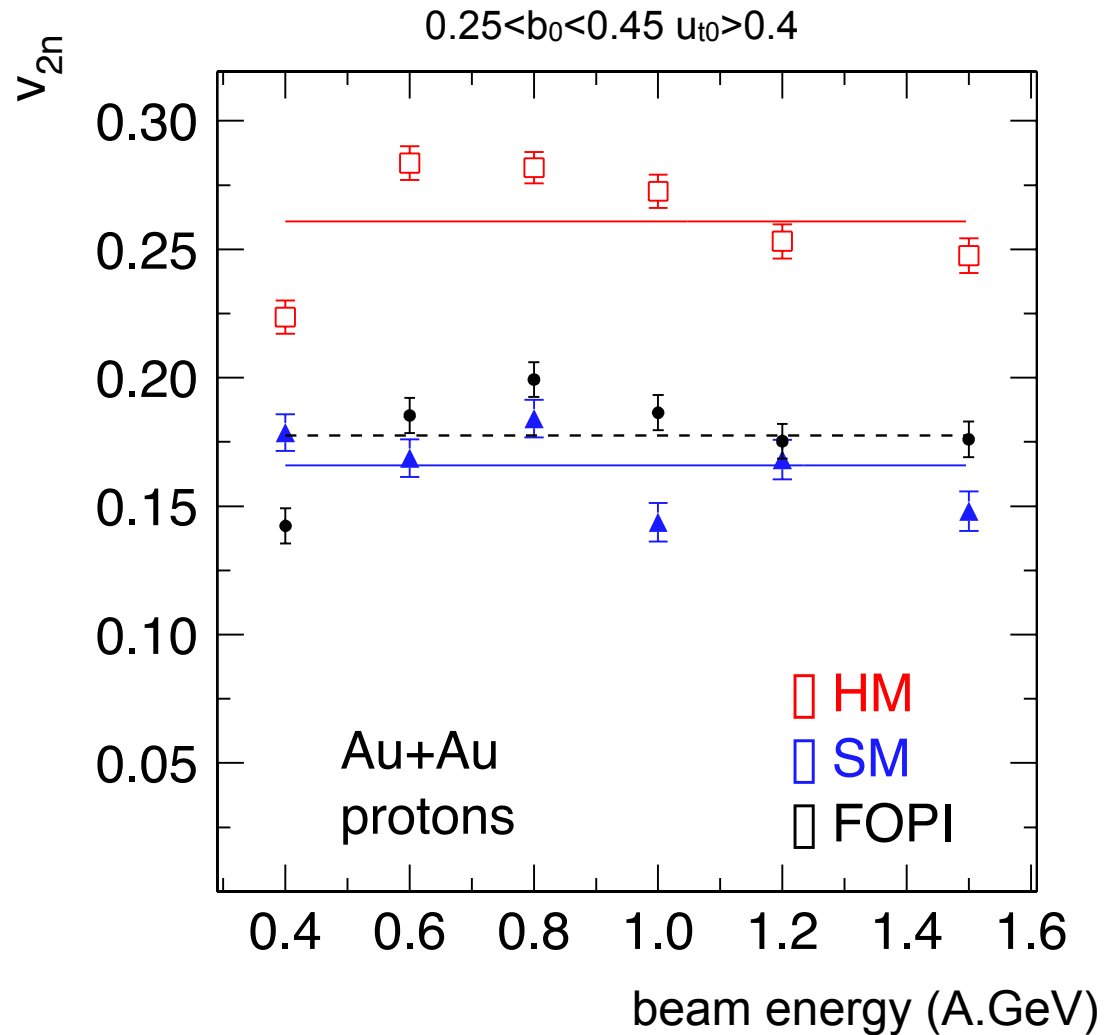


$K_0 =$
380 MeV ('stiff')
200 MeV ('soft')



Analysis and results

→ $v_{2n}(E_{\text{beam}})$ varies by a factor ≈ 1.6 , \gg measured uncertainty (≈ 1.1)

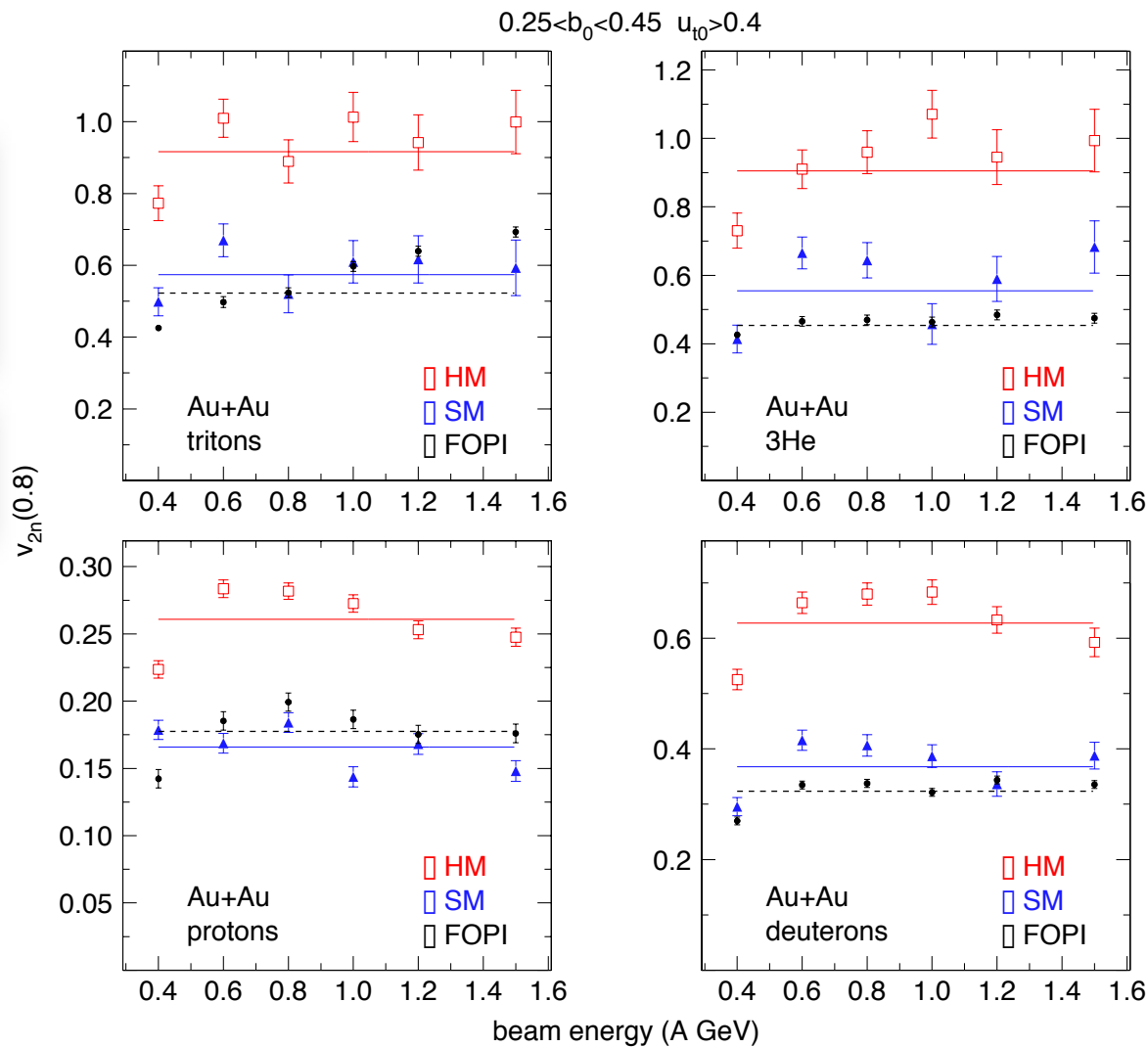




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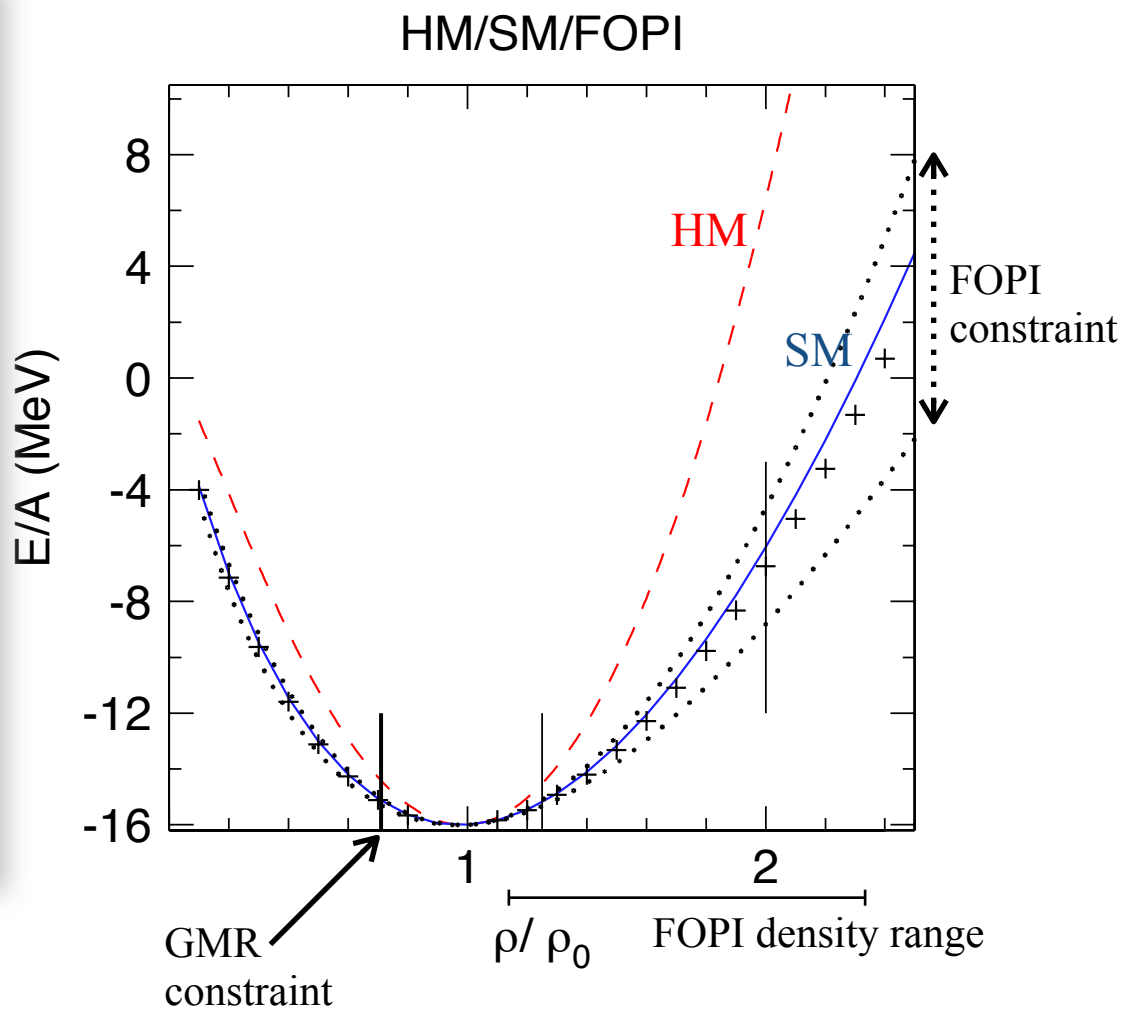
→ clearly favors a 'soft' EOS : $K_0 = 190 \pm 30 \text{ MeV}$





Analysis and results

- ▶ Phenomenological EOS
HM and SM include the saturation point at $\rho/\rho_0 = 1$,
 $E/A = -16$ MeV by construction.
- ▶ → fixes the absolute position of the curves:
- ▶ the heavy ion data are only sensitive to the shape, i.e. the pressure (derivative).
- ▶ → a stiff EOS, characterised by $K_0 = 380$ MeV is not in agreement with the flow data in the incident energy range 0.4 - 1.5 A.GeV.





Simulations: the scenario



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Purpose = characterise
which 'typical' densities
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IQMD transport model^[5,6]
various phenomenological
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Here: protons in Au+Au at
1.5 A.GeV, $b=3$ fm

[5] J. Aichelin, Phys. Rep. 202 (1991) 233.

[6] C. Hartnack, et al., Eur. Phys. J. A 1 (1998) 151.

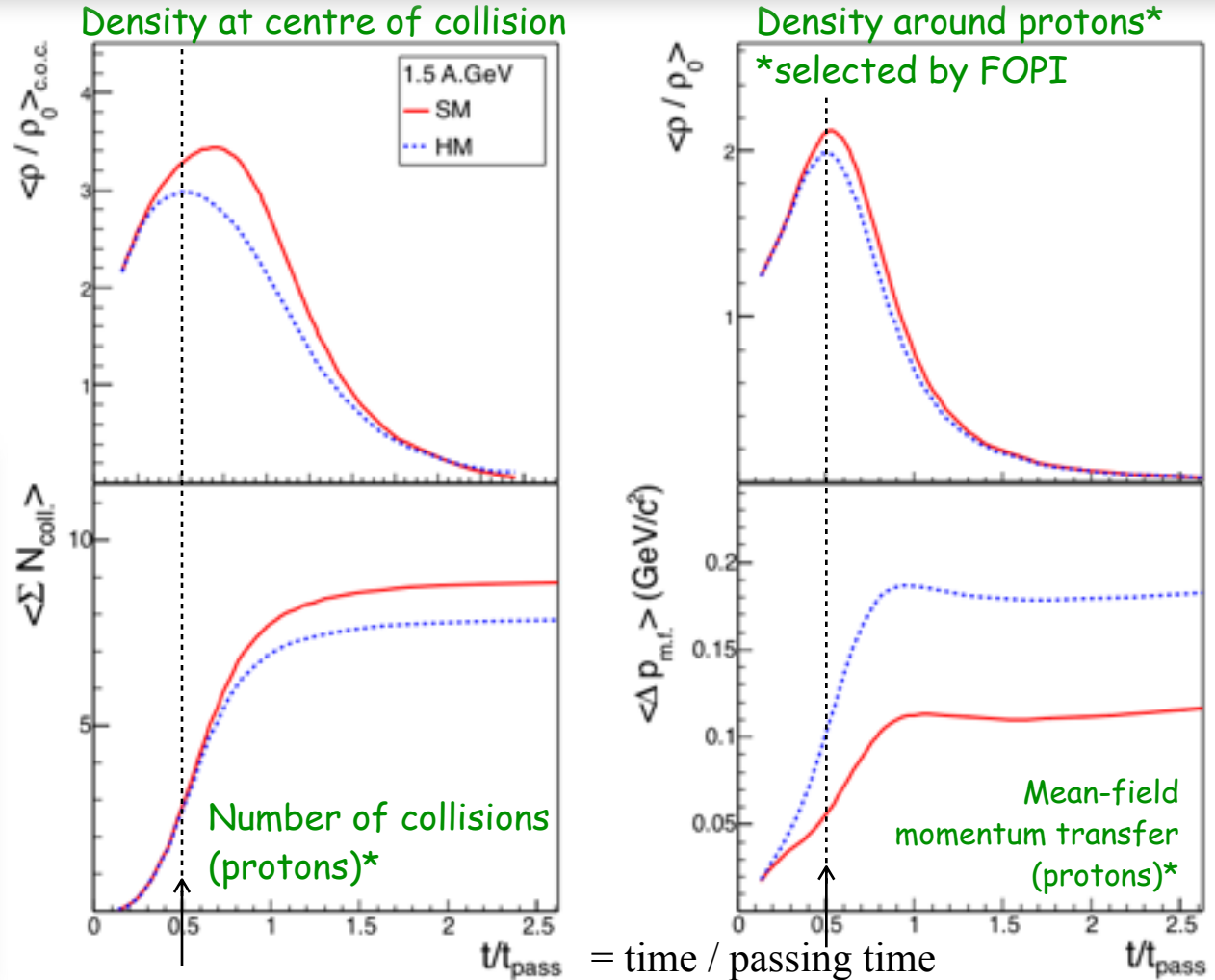
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full target-projectile overlap

[5] J. Aichelin, Phys. Rep. 202 (1991) 233.

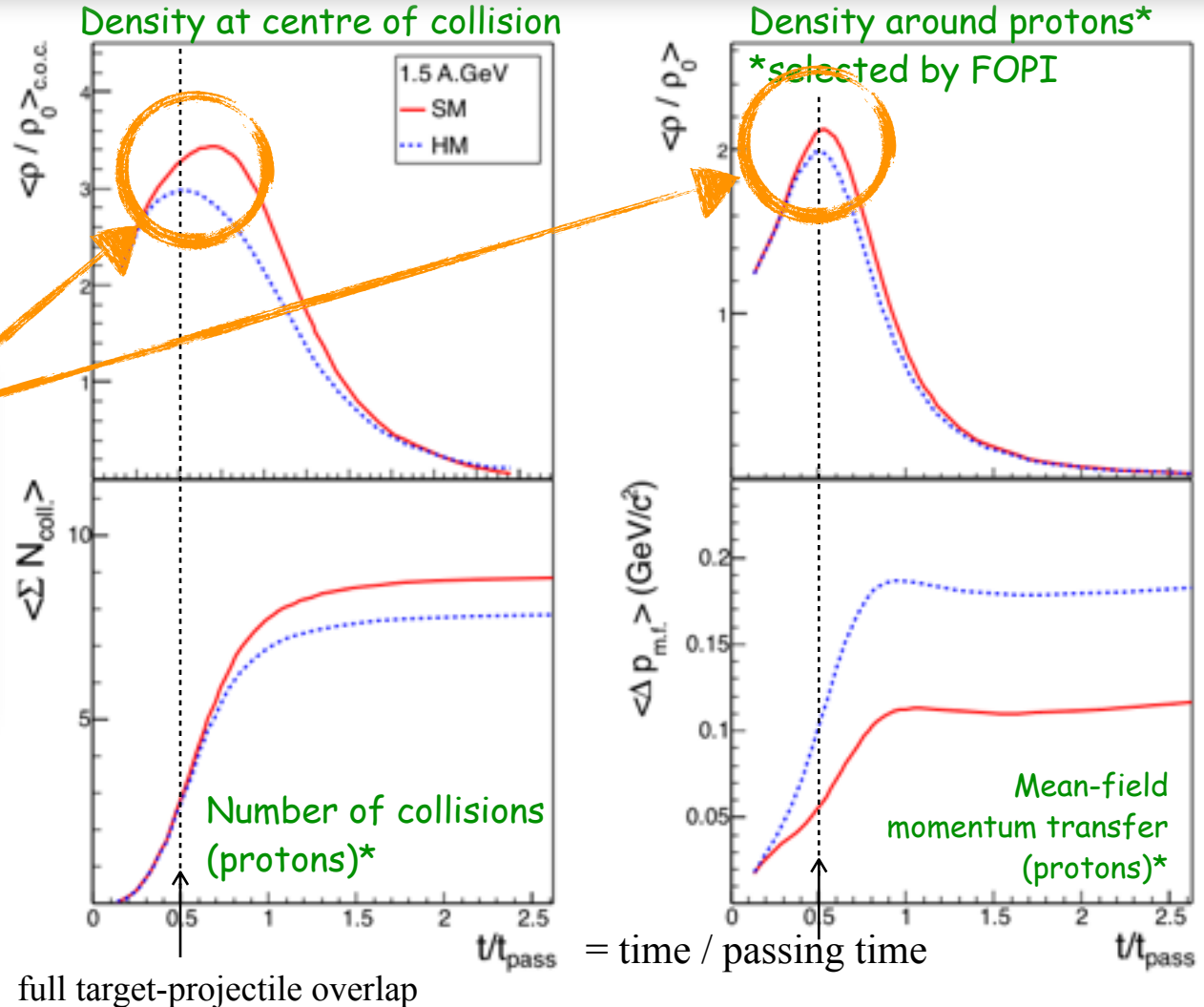
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The highest density phase initiates the high pressure, hence the flow.

Tested: a high density cut-off in the EOS => no elliptic flow.



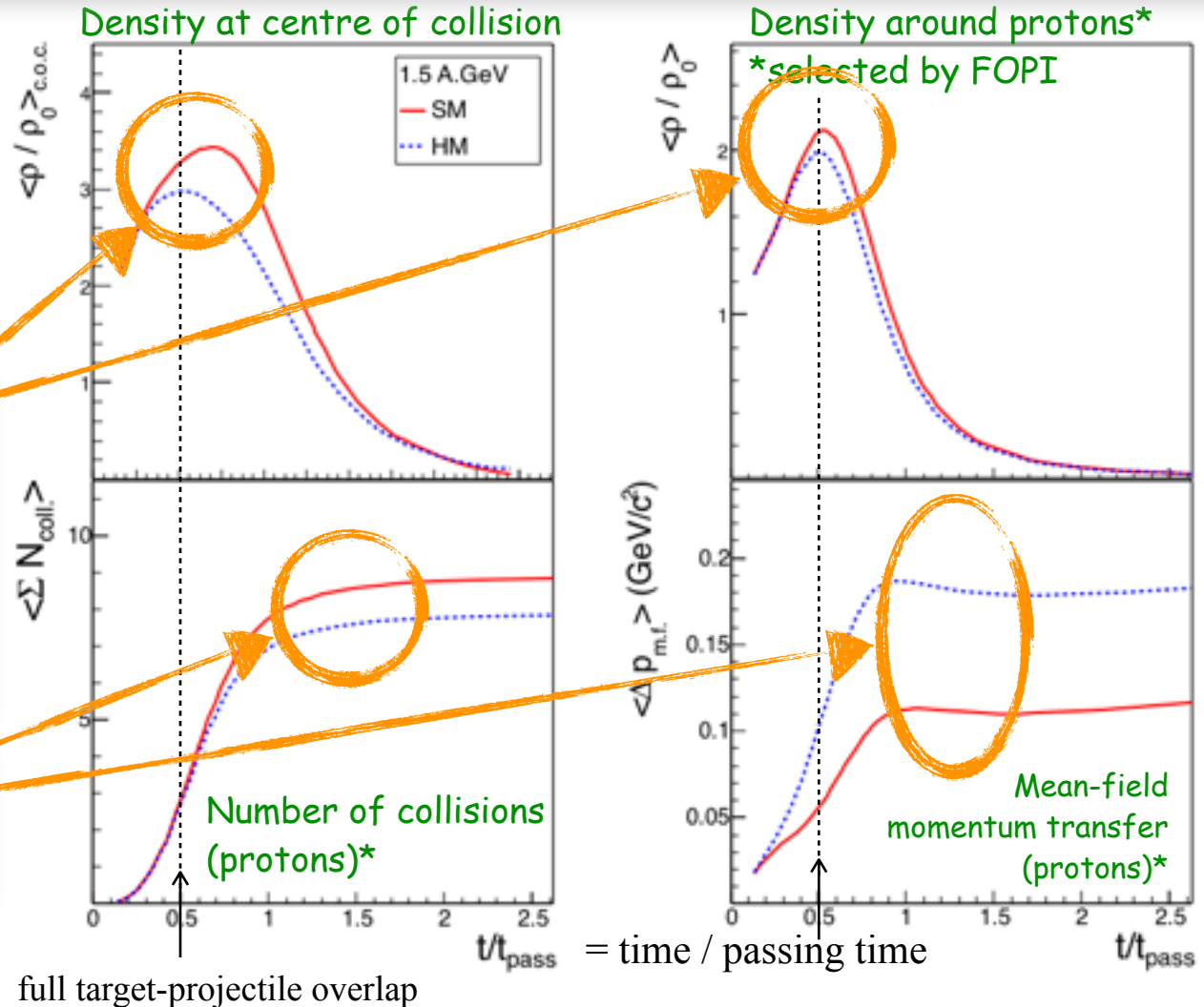
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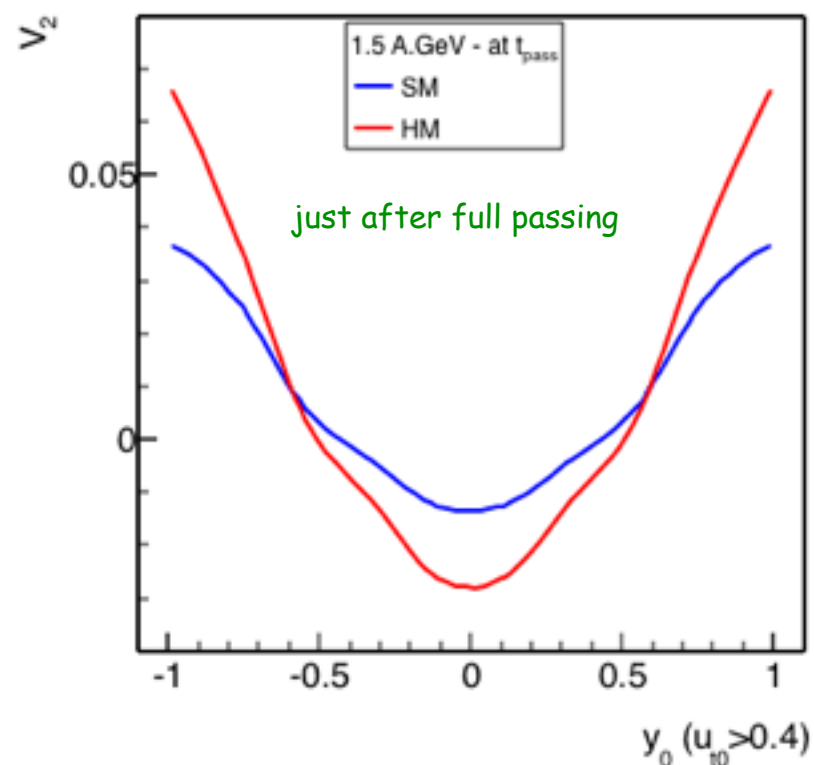
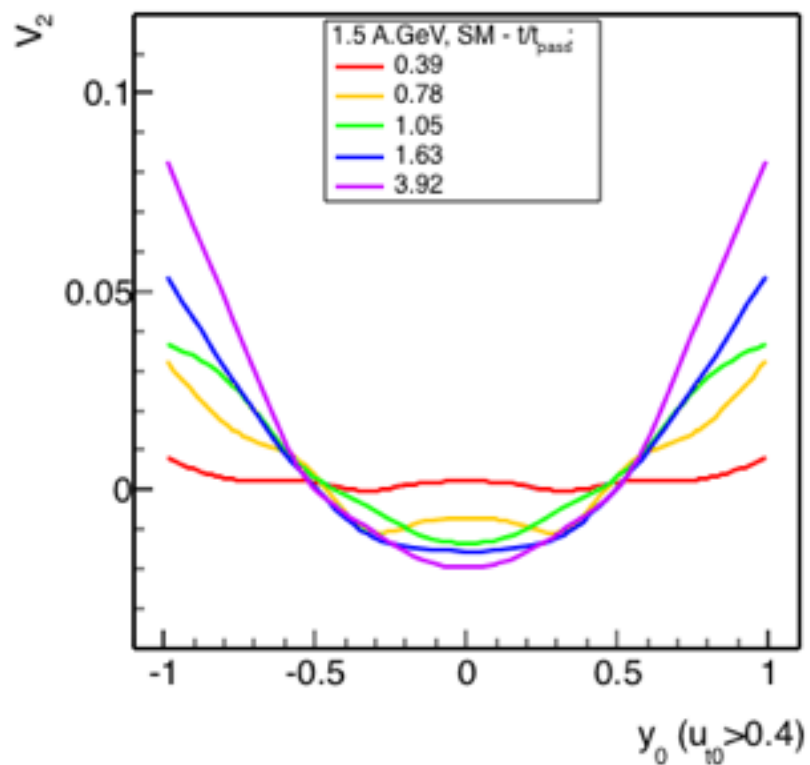
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The (flow) dynamics develops up to later times, hence lower densities.



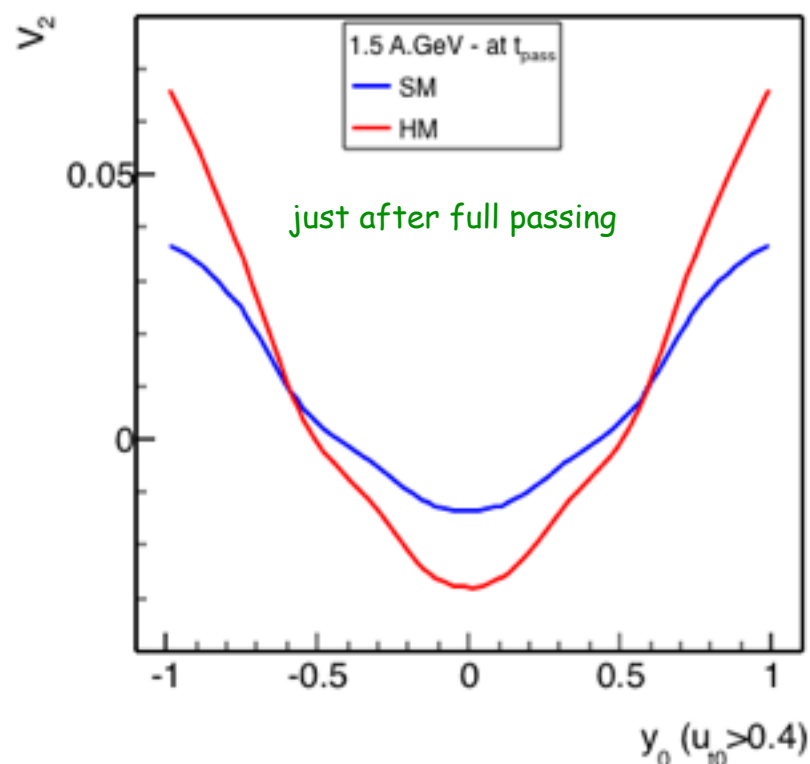
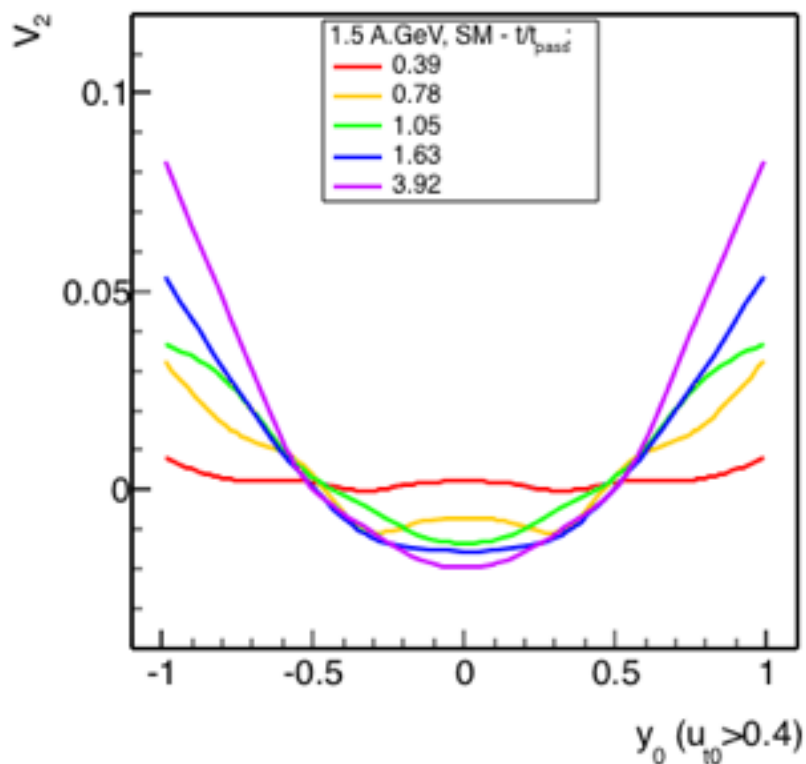


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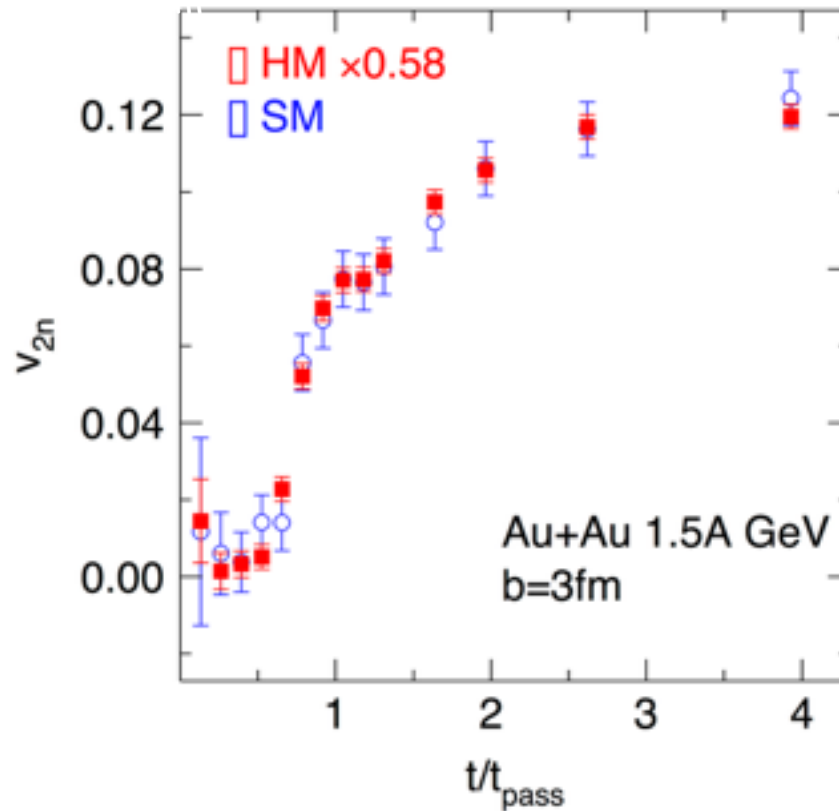
Simulations: the scenario



- ▶ The elliptic flow at mid-rapidity develops fast: already **stabilised at the passing time**.
- ▶ **At t_{pass}** , the elliptic flow, in its rapidity dependence, depends already strongly on the EOS.
- ▶ The elliptic flow **around the spectators** ($|y_0|$ close to 1) stabilises **twice slower**.



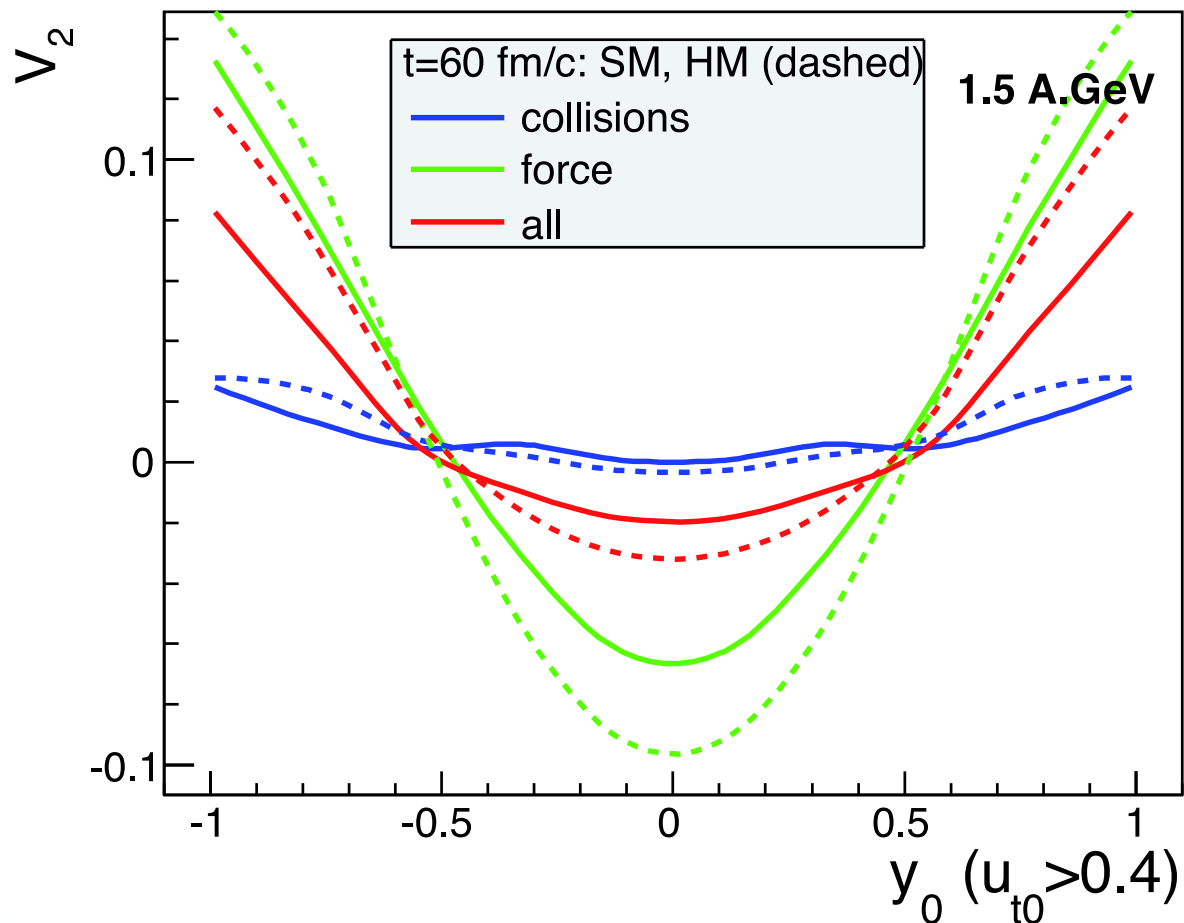
Simulations: the scenario



- ▶ The **shape** of its rapidity dependence shows a **universality** with the EOS's (through scaling).



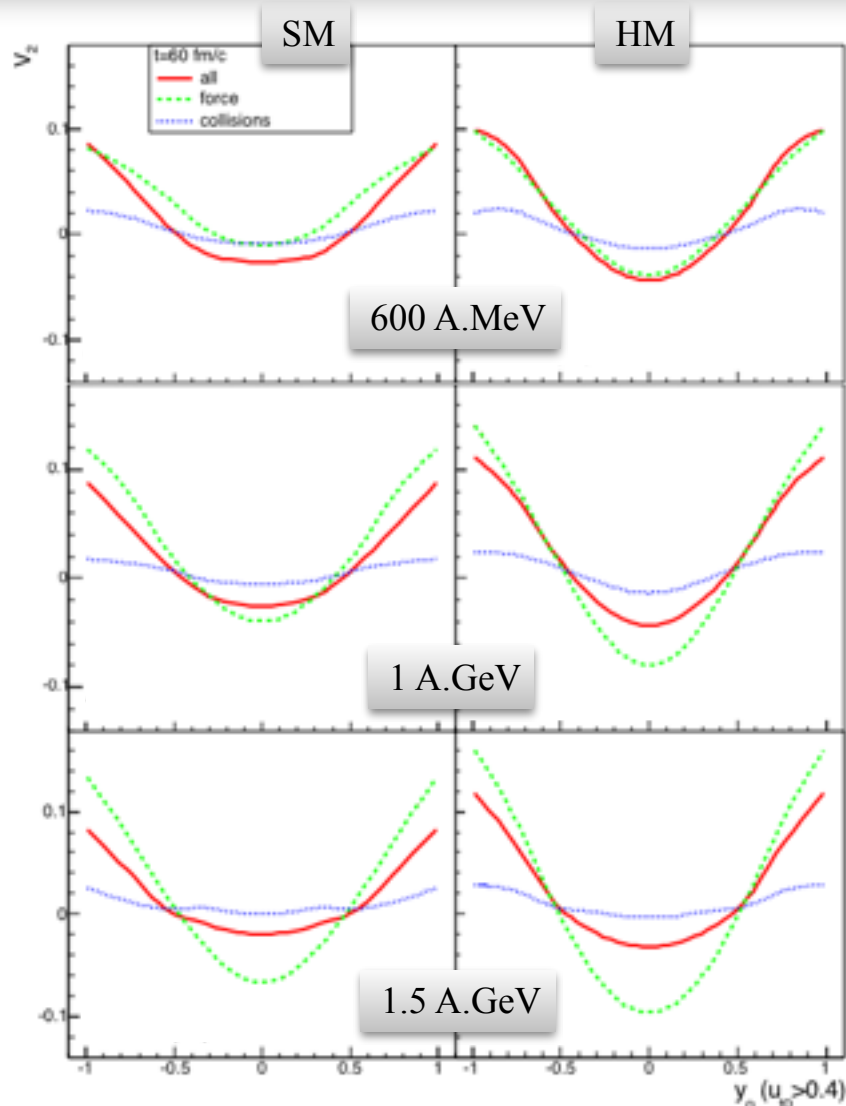
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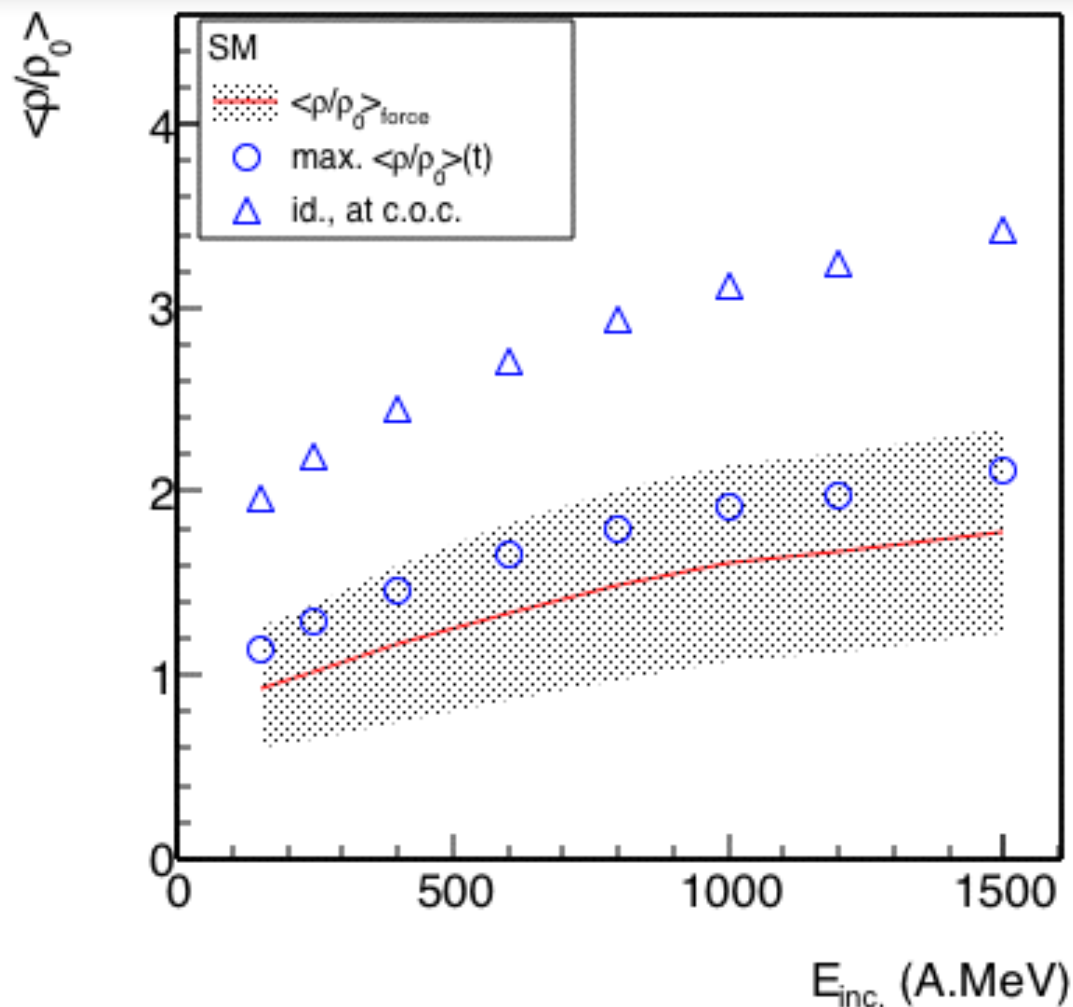
Simulations: the scenario

- ▶ The elliptic flow in strength and shape is mostly influenced by the force of the mean field (hence EOS).
- ▶ A 'mean' density characterising the development of the elliptic flow can be built from the mean value weighted by this force up to around the passing time.

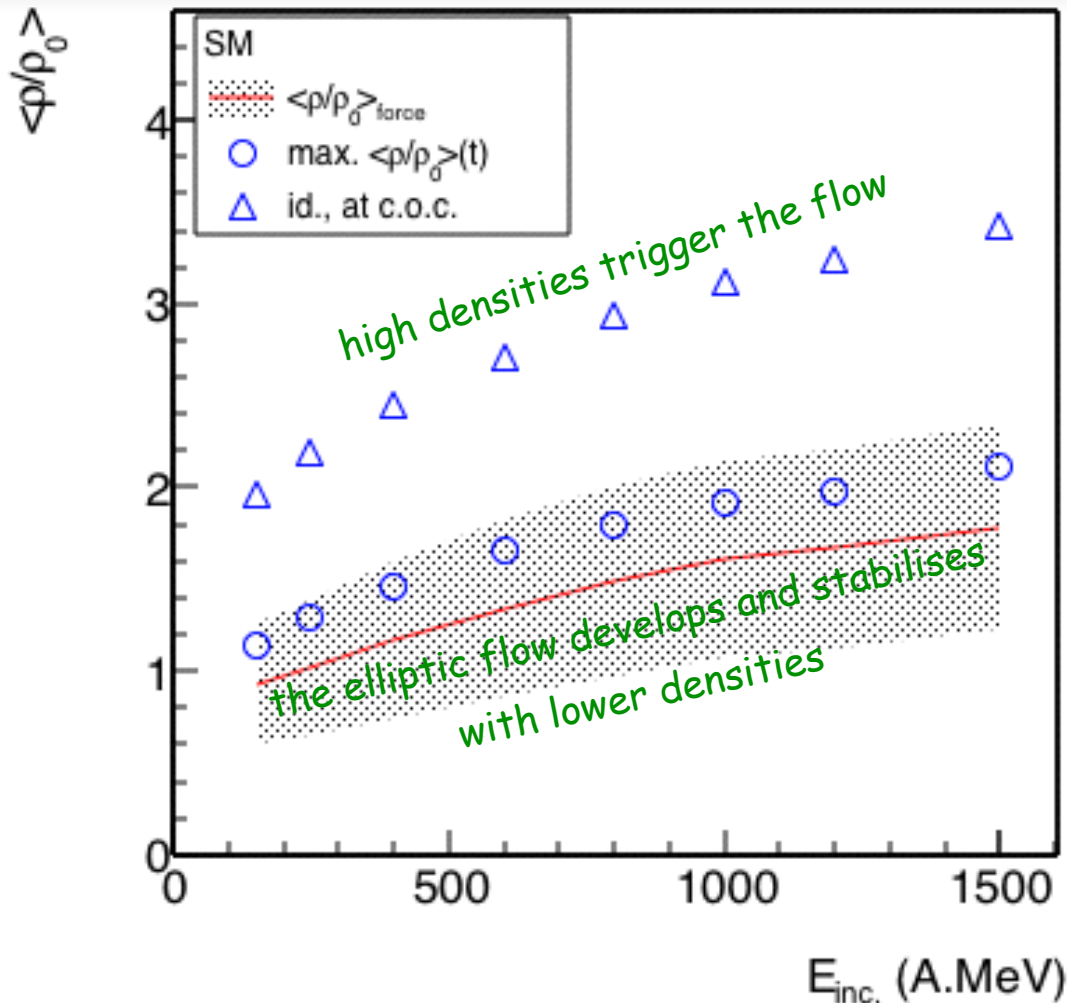




Simulations: the scenario



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► In the QMD model, the EOS must be correct over a broad range of densities in order to predict the observed elliptic flow.

► The density range, relevant to the EOS evidenced by the FOPI Collaboration, spans in the range

$$\rho \approx (1 - 3) \rho_0.$$



Summary and discussion



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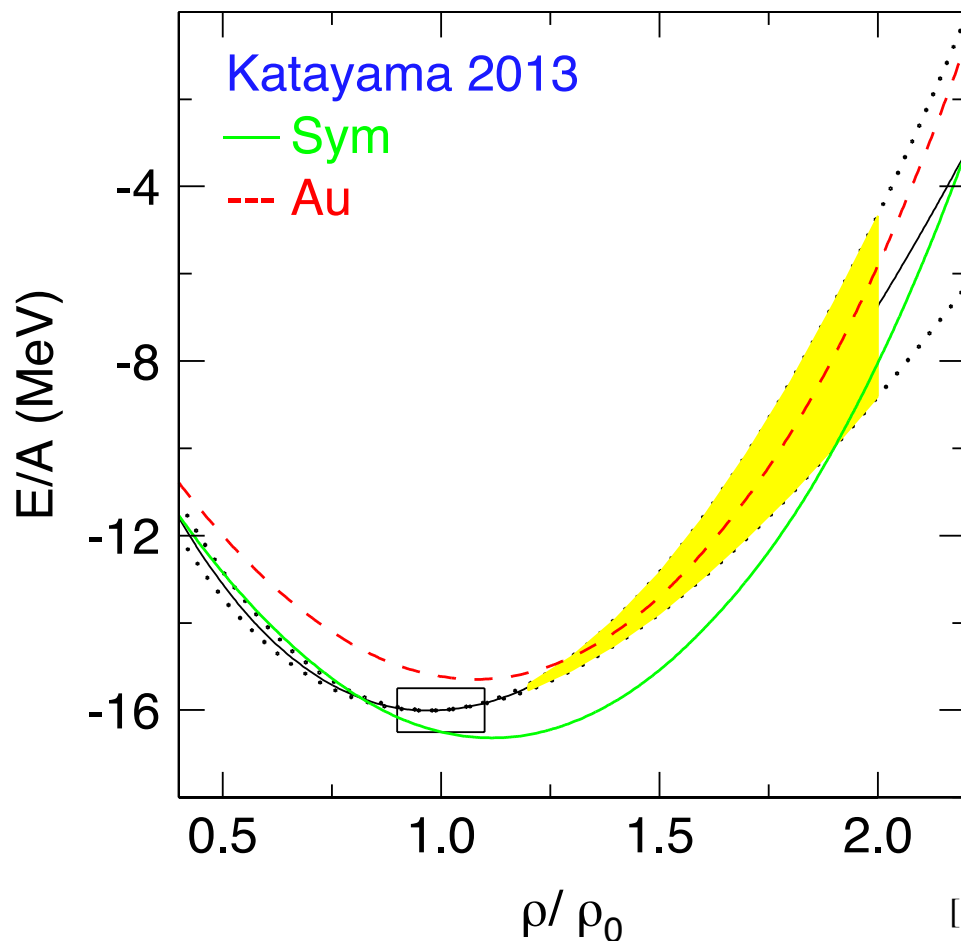
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- Thank you for your attention!*
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Comparison to microscopic calculations

(three representative microscopic calculations compared with our new constraints)



Dirac-Brueckner-Hatree-Fock (DBHF) calculation^[10] using the Bonn A^[11] nucleon-nucleon potential

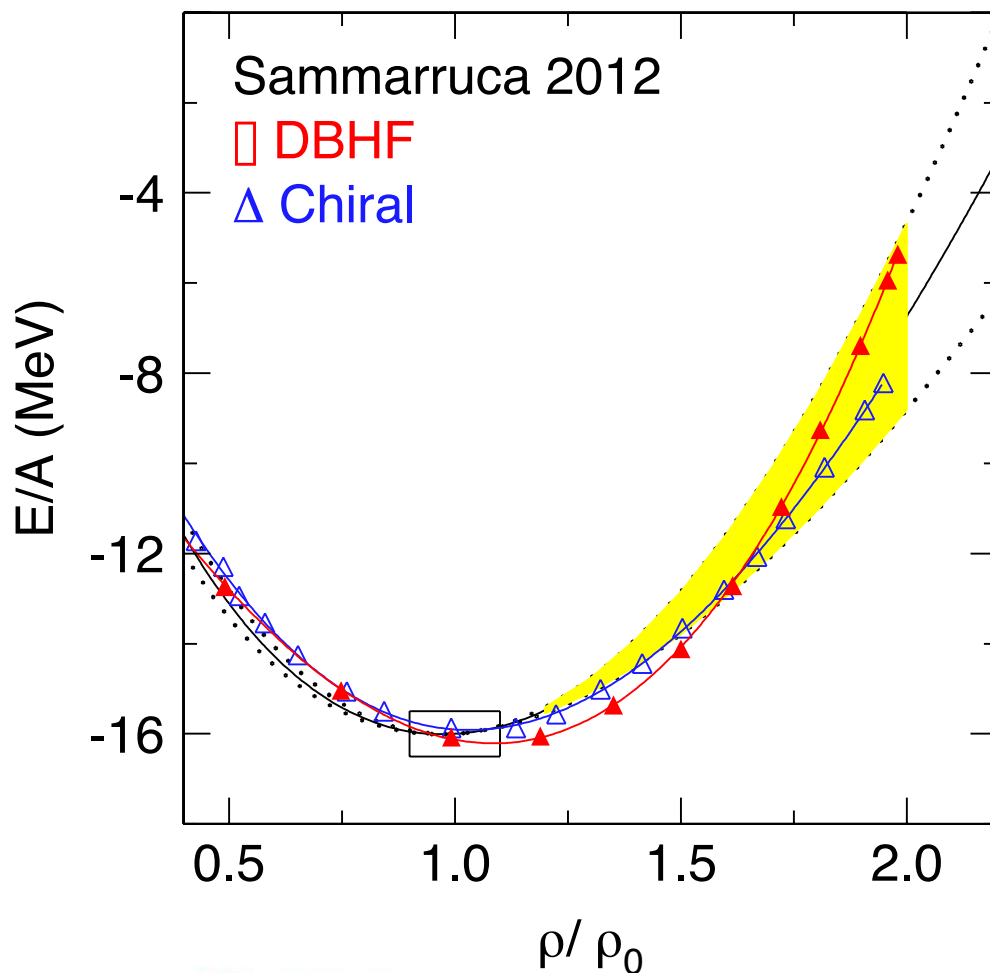
[10] R. Brockmann, R. Machleidt, Phys. Rev. C 42 (1990) 1965.

[11] T. Katayama, K. Saito, Phys. Rev. C 88 (2013) 035805.



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2 symmetric nuclear matter EOS's from [12]:

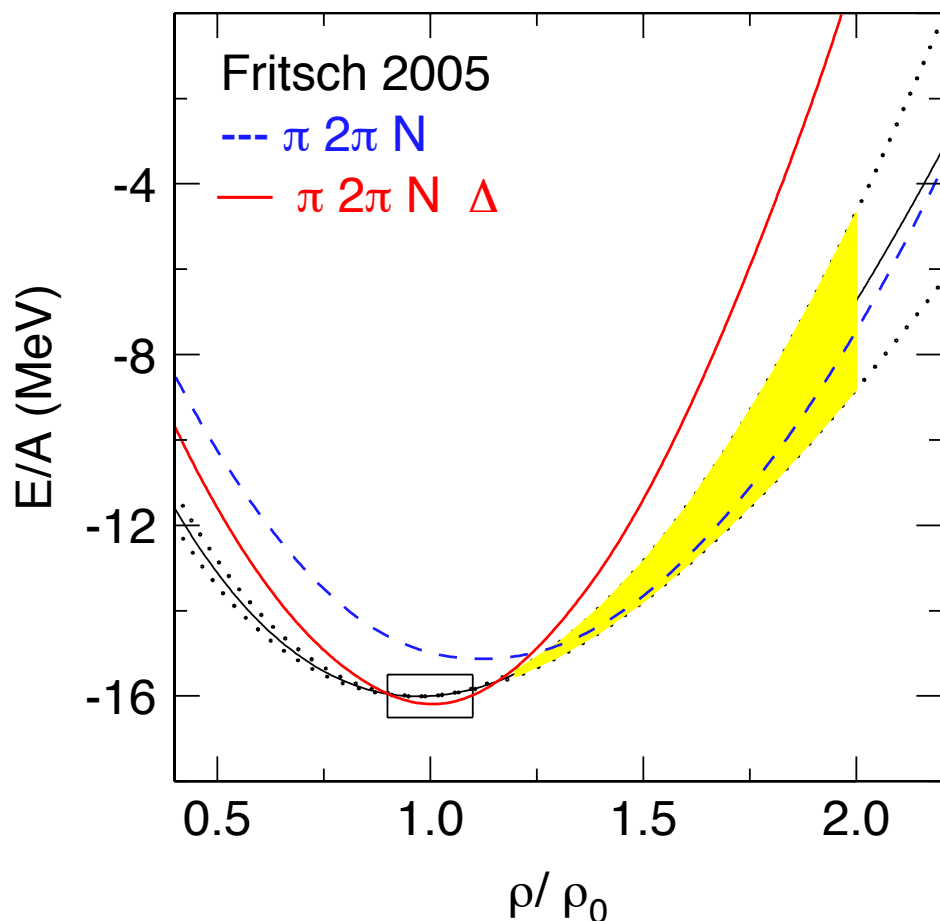
- 1) 'DBHF' = meson theoretic potential together with the DBHF method
- 2) 'Chiral' = use of effective field theory (EFT) with density dependent interactions derived from leading order chiral three-nucleon forces.

[12] P. Danielewicz, G. Odyniec, Phys. Lett. B 157 (1985) 168.



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Using the chiral approach^[13]: 2 rather different EOS's including or not virtual Δ excitations.

- » the virtual Δ -excitations help locate the EOS at the right horizontal place around $\rho = 0.16 \text{ fm}^{-3}$.
- » the Δ leads to a rather marked stiffening of the EOS ($K_0 = 304 \text{ MeV}$)
- » because 'cold' EOS ?
- » finite temperature in the reaction \Rightarrow the Δ are real rather than virtual. The theoretical ' Δ stiffness' could then be a dispersion effect rapidly changing with temperature.

[13] S. Fritsch, N. Kaiser, W. Weise, Nucl. Phys. A 750 (2005) 259.