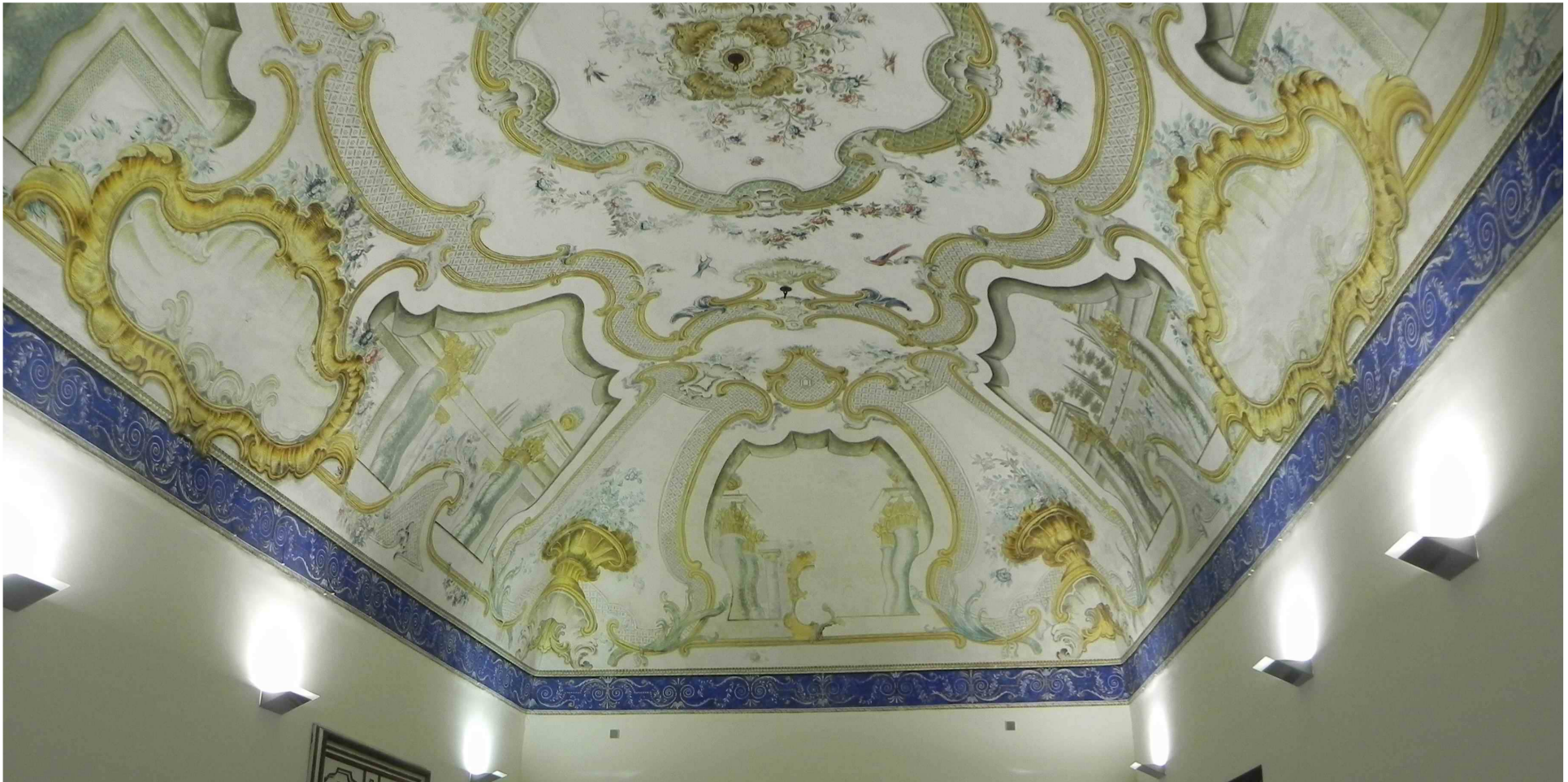




## Concluding Remarks

Hermann Wolter, Univ. of Munich

Asy-EoS 2015 Int. Workshop on the Nuclear Symmetry Energy and Reaction Mechanisms,  
Piazza Armerina, Sicily, March 3-6, 2015



We thank the organizers for bringing us to such a beautiful and interesting place and spectacular lecture hall

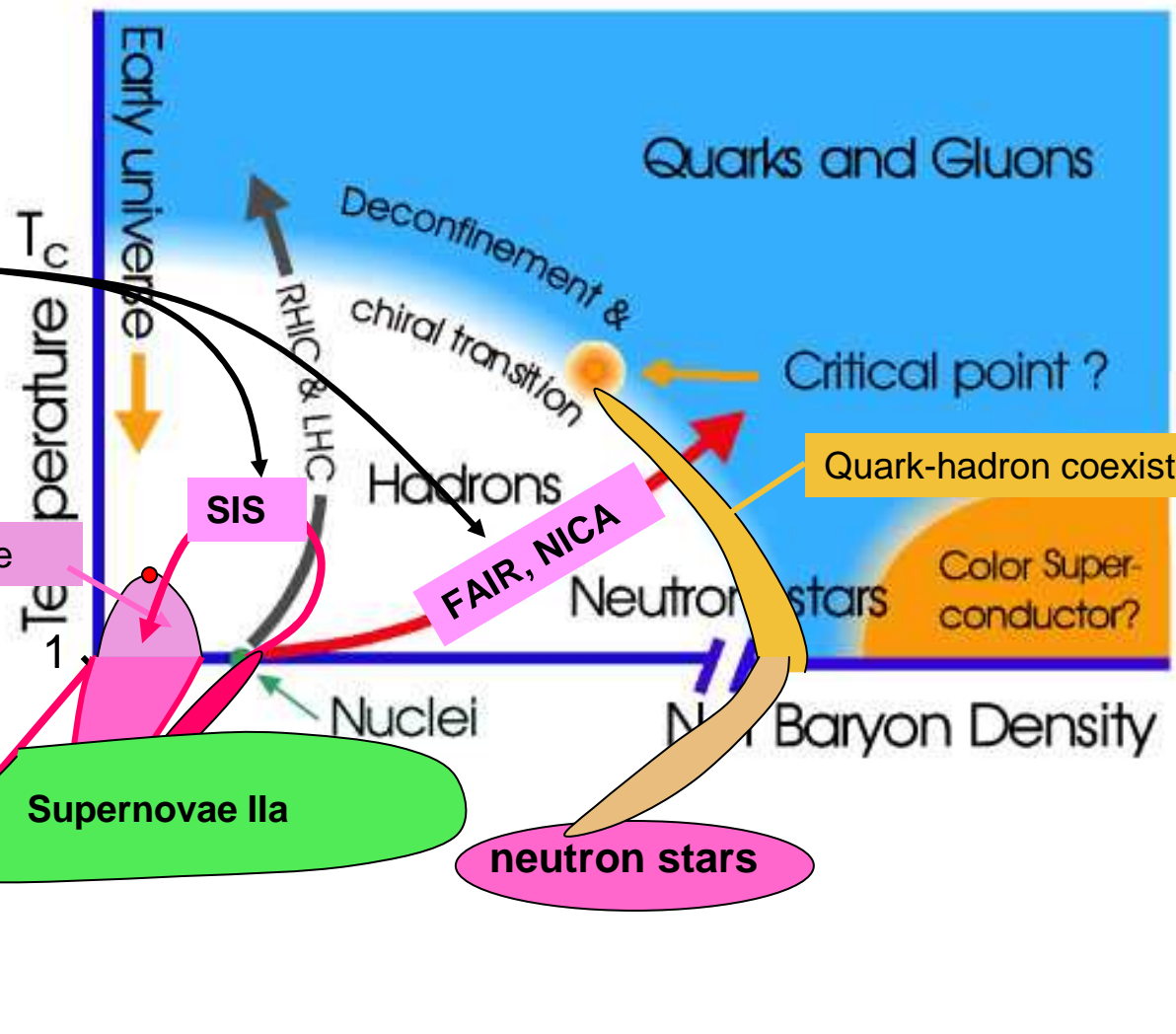


Our general aim in Heavy Ion Reactions:  
 The Phase Diagram of **Strongly Interacting Matter**

**Caveat:**  
 HIC trajectories are non-equilibrium processes, and are not in the plane of the diagram  
 → **transport theory is necessary**

Liquid-gas coexistence

**Isospin degree of freedom**



Quark-hadron coexistence

FAIR, NICA

SIS

Supernovae IIa

neutron stars

Z/N

# Transport theory: 2 families

## 1. Boltzmann-Uehling-Uhlenbeck (BUU)

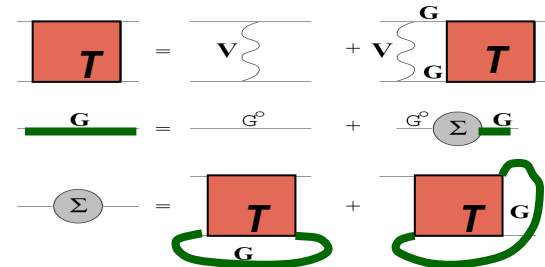
$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}^{(r)} f - \vec{\nabla} U(r) \vec{\nabla}^{(p)} f(\vec{r}, \vec{p}; t) = \int d\vec{v}_2 d\vec{v}_1 d\vec{v}_2' v_{21} \sigma_{12}(\Omega) (2\pi)^3 \delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_1' - \vec{p}_2') [f_1' f_2' (1-f_1)(1-f_2) - f_1 f_2 (1-f_1')(1-f_2')]$$

Derived:

→ Classically or semiclassically from **THDF**, collision term added (and fluctuations)

→ From non-equilibrium theory (**Kadanoff-Baym**); collision term included mean field and in-medium cross sections consistent, e.g. from BHF

$$\Sigma_{s,\mu}(k; \rho) \approx \text{Tr}(T f); \quad \sigma_{NN}^{(in-med)}(k; \rho) \approx |T^2|$$



Spectral fcts, off-shell transport, quasi-particle approx. (QPA)

$$A(x, p) \propto \frac{2\Gamma(x, p)}{(p^{*2} - m^{*2}) + \Gamma^2(x, p)}$$

$$\Gamma(x, p) = m^* \text{Im} \Sigma_s^+ - p_\mu^* \text{Im} \Sigma^{+\mu}$$



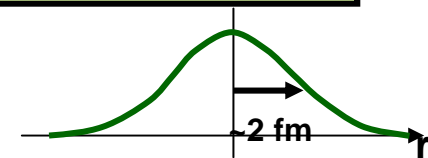
$$\propto \delta(p^{*2} - m^{*2}) \Theta(p^{*0})$$

## 2. Molecular Dynamics (QMD)

classical molecular Dynamics with Gaussian particles to reduce fluctuations + Boltzmann collision term

2b) Antisymmetrized MD (AMD, FMD) Gaussians are antisymmetrized w/ collision term with stochastic features (wave packet splitting), CoMD

$$\frac{dr_i}{dt} = \{r_i, \mathcal{H}\}; \quad \frac{dp_i}{dt} = \{p_i, \mathcal{H}\}; \quad H = \sum_i t_i + \sum_{i<j} V_{ij}$$



Transport theory is on a well defined footing, **in principle – but in practice?**

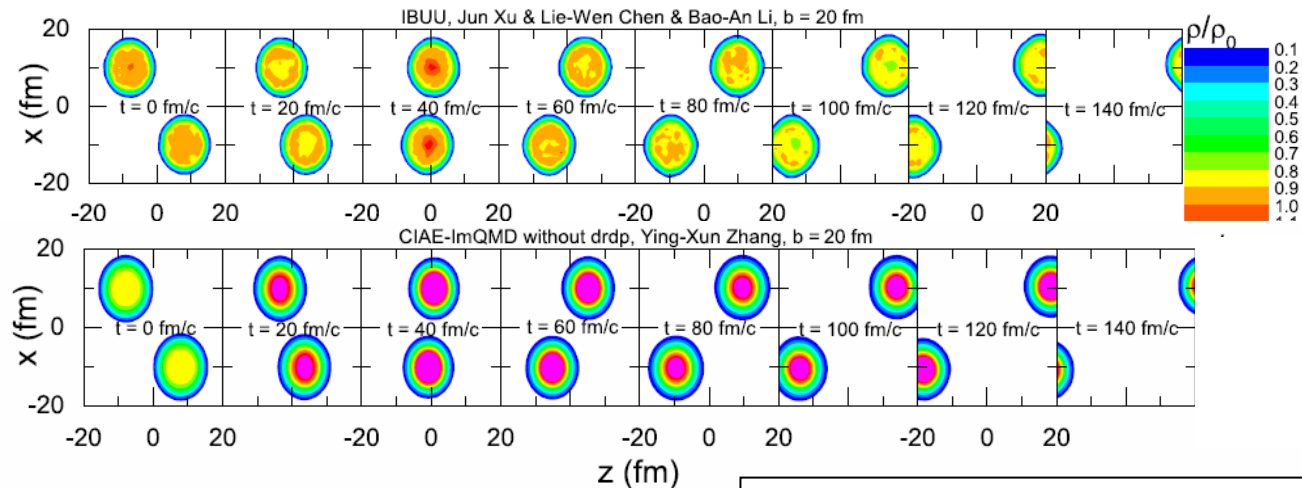
# Code Comparison Project: Trento, ECT\*, 2006 and 2009 Shanghai, Jan. 2014, Lanzhou 2014

check consistency of transport codes in calculations with same system (Au+Au),  $E=100,400$  AMeV, identical (simple) physical input (mean field (EOS) and cross sections)

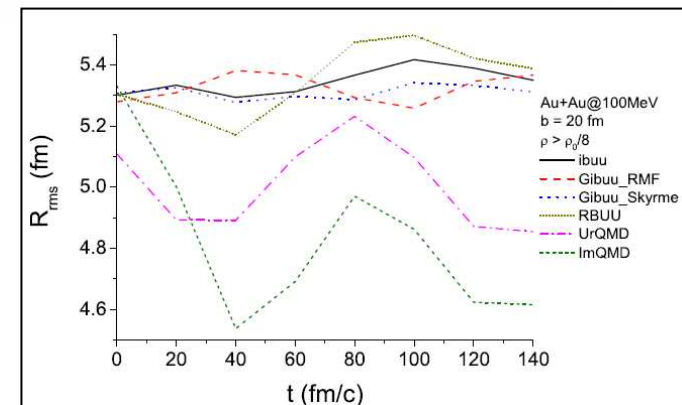
idea: establish sort of **theoretical systematic error or transport calculations** (and hopefully to reduce it )

## 1. step: Initialize colliding nuclei. usually not exact ground states

free  
propagation  
(large  
impact  
parameter)

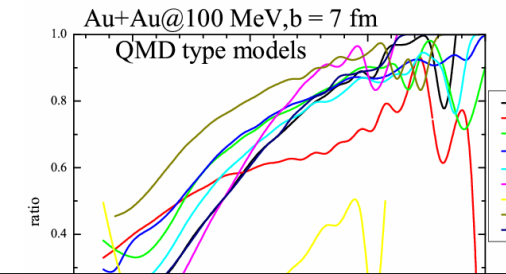
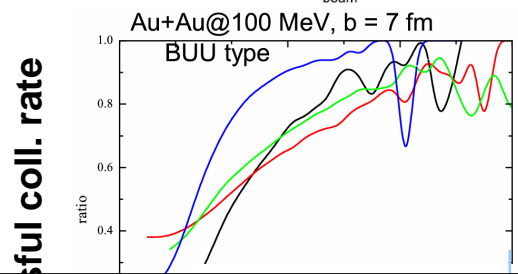
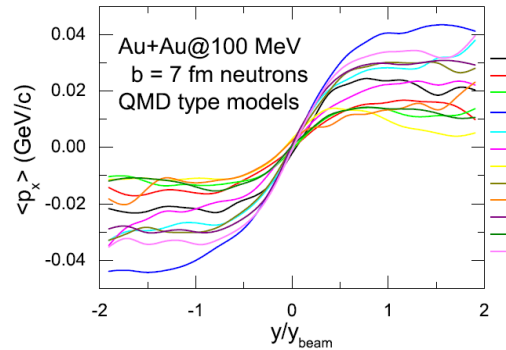
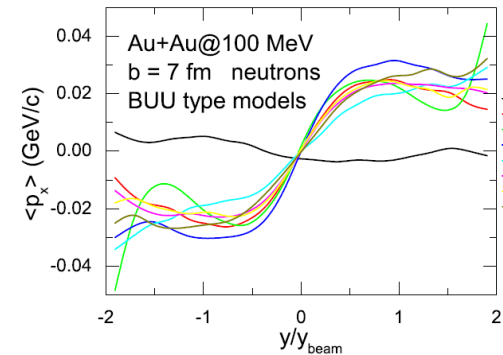
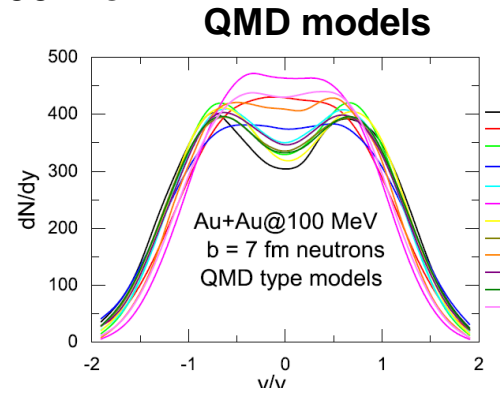
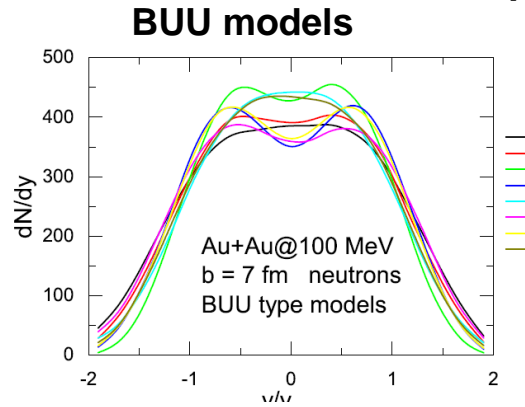


- nuclei oscillate, influences dynamical evolution in collision, part. at lower incident energies
- construct better gs, e.g. Thomas-Fermi

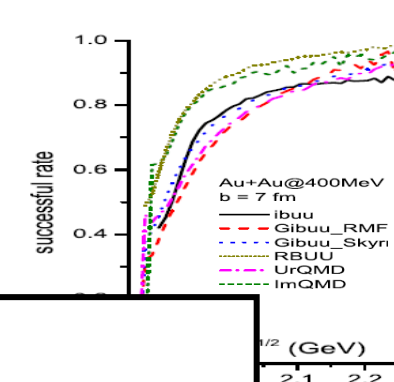
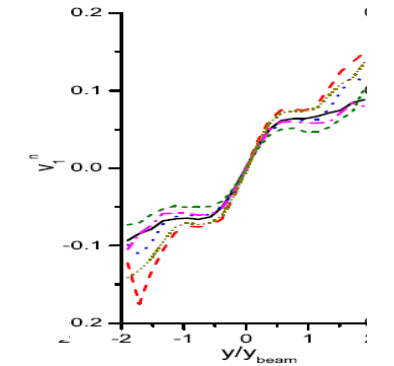
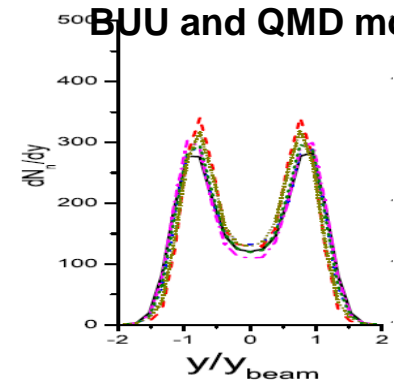


# Examples of results: Au+Au, **PRELIMINARY**

E/A=100 MeV



E/A=400 MeV  
BUU and QMD models

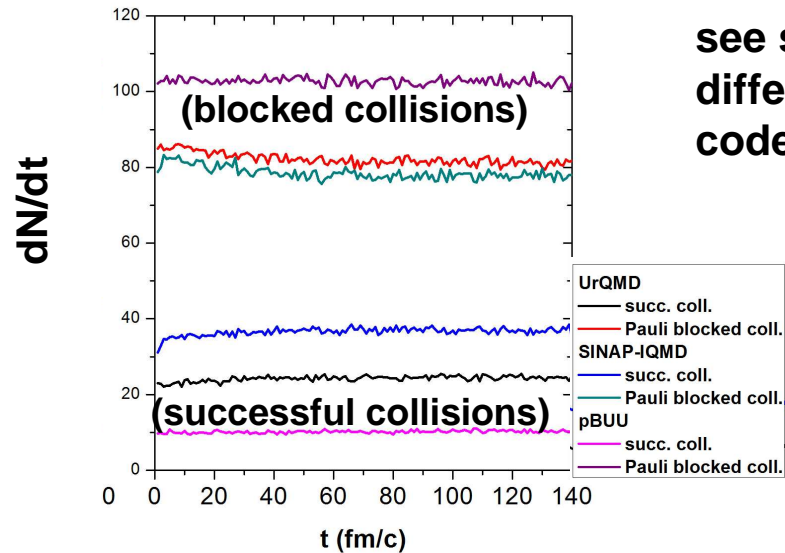
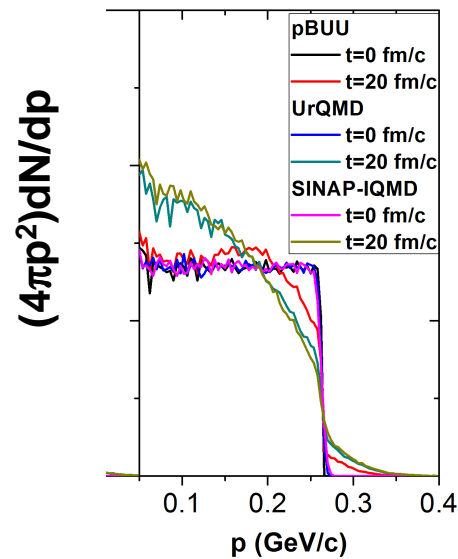


- considerable differences
- partly due to initialization, but mainly to collision term
- no essential difference between BUU and QMD models
- 100 MeV sensitive region for flow because of competition between mf and collisions, better at higher energy

# Transport Calculations in a (periodic) Box

test collision routine and Pauli blocking under controlled conditions;  
 reveal important features of the semiclassical approach.

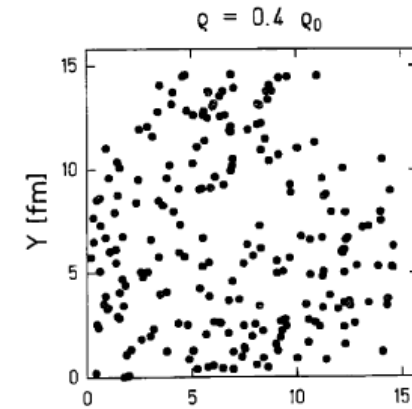
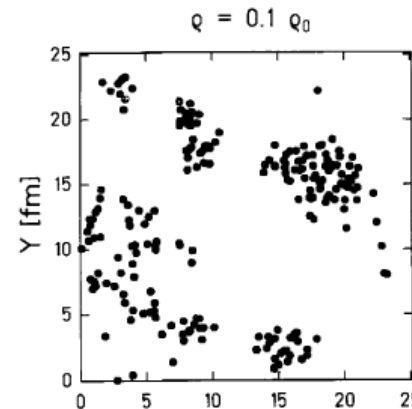
One effect: Initialization,  $T=0$ .  $\rightarrow$  Fermi statistics is lost quickly!



see significant differences between codes!

Broader applications:

1. Initialization in spinodal region: fragment formation, check of fluctuations
2. Initialize not in ground state: check of equilibration



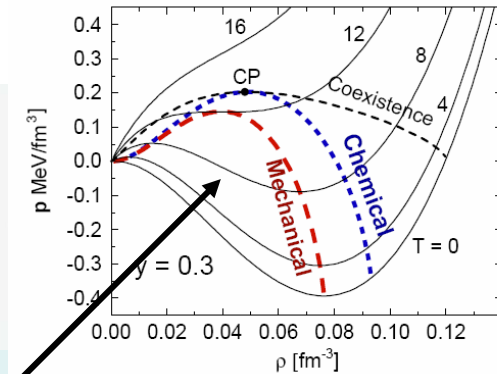
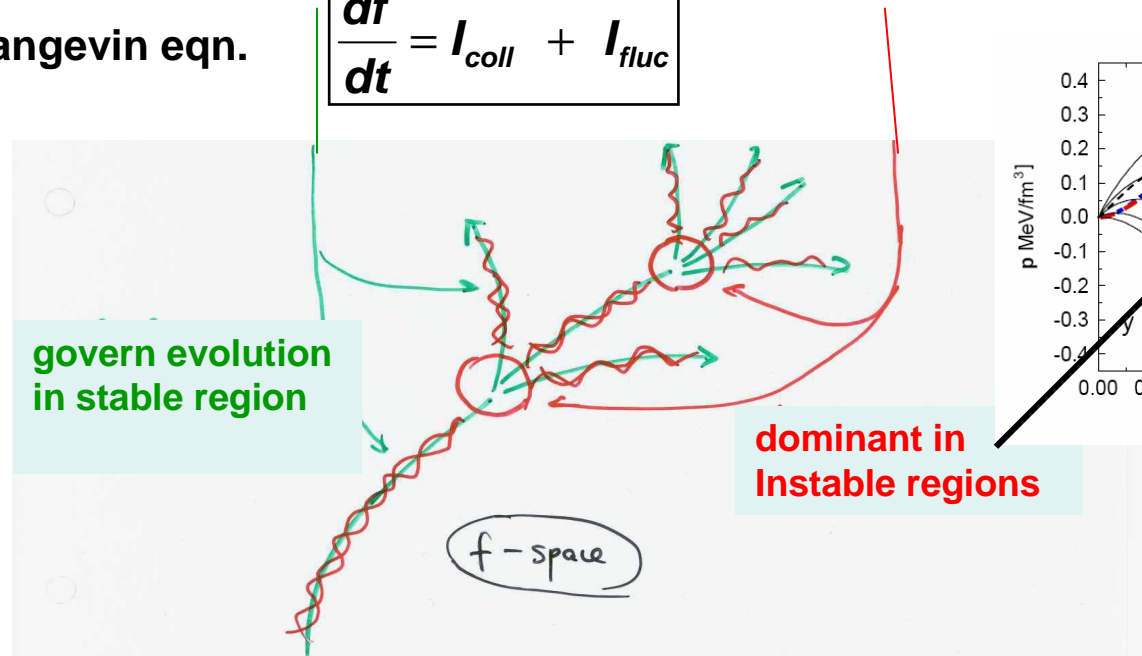
# Fluctuations in Phase Space

$$f(r, p, t) = \bar{f}(r, p, t) + \delta f(r, p, t)$$

Mean field evolution  
(dissipative)      Fluctuations

Boltzmann-Langevin eqn.

$$\frac{df}{dt} = I_{coll} + I_{fluc}$$



- Origin of fluctuations:**
- initial state correlations (how important and realistic?)
  - higher order correlations
  - collisions (diss.-fluct. theorem)

The last two are not contained in BUU and have to be reintroduced, i.e. the Boltzmann-Langevin eq. has to be solved, at least approximatively



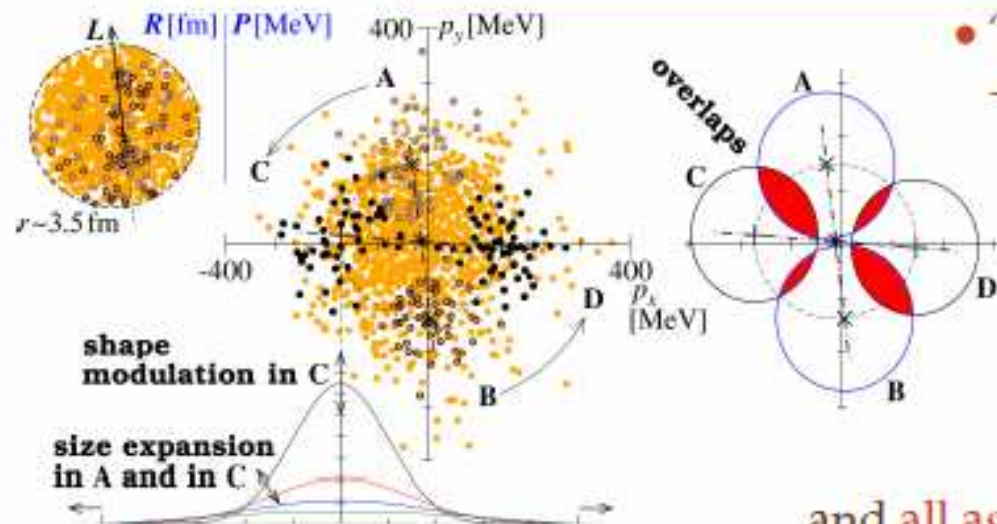
Stochastic Mean-Field (SMF) model :  
Fluctuations are projected on the coordinate space

P. Napolitani  
M. Colonna

Boltzmann-Langevin One Body (**BLOB**) model :  
*fluctuations implemented in full phase space*

At a given time  $t$ ,  
in  $(\mathbf{r}_a, \mathbf{p}_a)$ ,  
for elastic coll. :

$$\dot{f}_a(\mathbf{r}_a, \mathbf{p}_a) = g \int \frac{d\mathbf{p}_b}{h^3} \int d\Omega W(AB \leftrightarrow CD) F(AB \rightarrow CD)$$



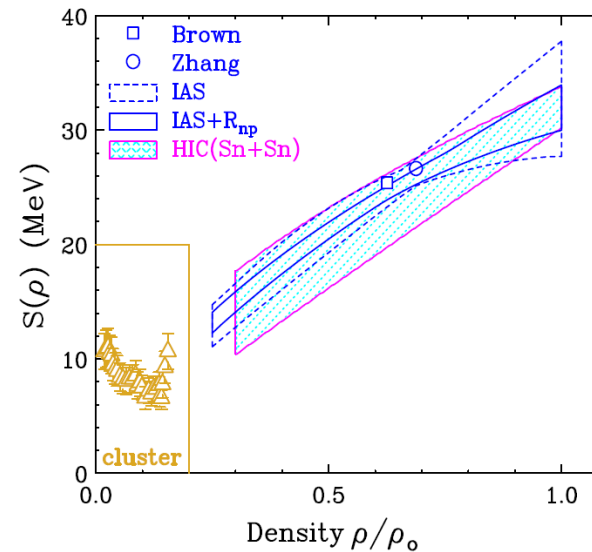
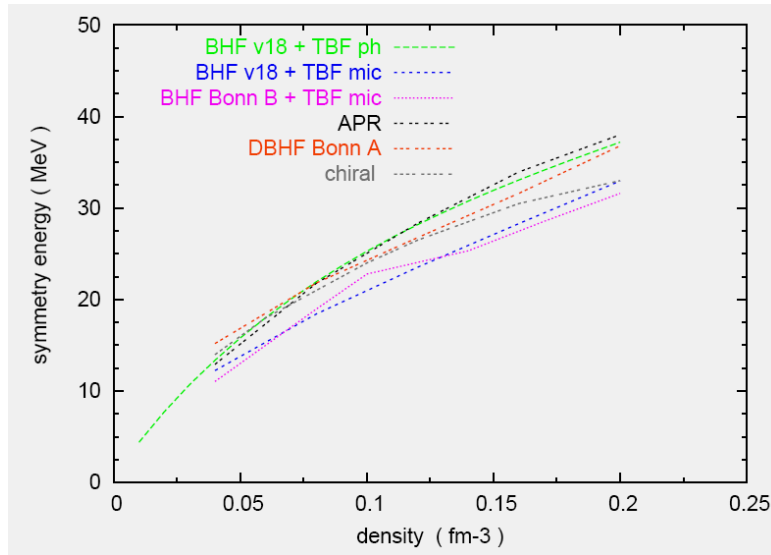
• “nucleon wave packets” →  
→ phase-space agglomerates  
of  $N_{\text{test}}$  test-particles  
of equal isospin  
( $a \in A, b \in B \dots$ )

• at each  $\Delta t$ :  
all phase space is  
scanned for collisions

and all agglomerates are redefined

- Clouds of test particles (nucleons) are moved once a collision happens
- Shape modulation of the packet ensures Pauli blocking is respected

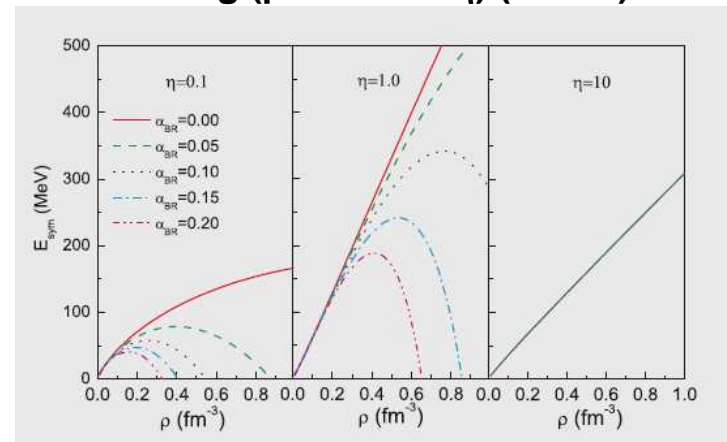
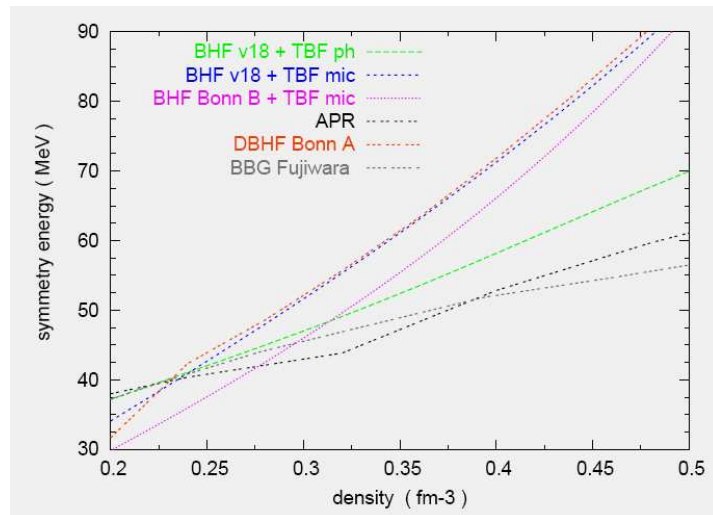
# SE: Microscopic many-body calculations: Marcello Baldo, NuSYM14



J. Phys. G: Nucl. Part. Phys. 41 (2014) 093001

Low density symmetry energy behave similarly and are consistent with analyses from nuclear structure and HIC.

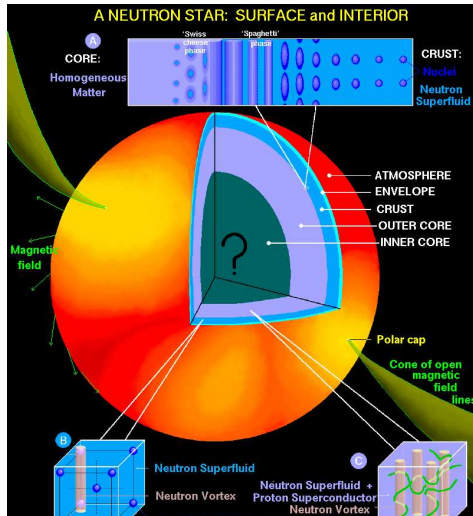
However, at high densities large differences. -- 3-body forces? (Baldo); scaling with density?  
 -- short range tensor force (cut-off  $r_c$ ) and in-medium mass scaling (parameter  $\eta$ ) (B.A.Li)



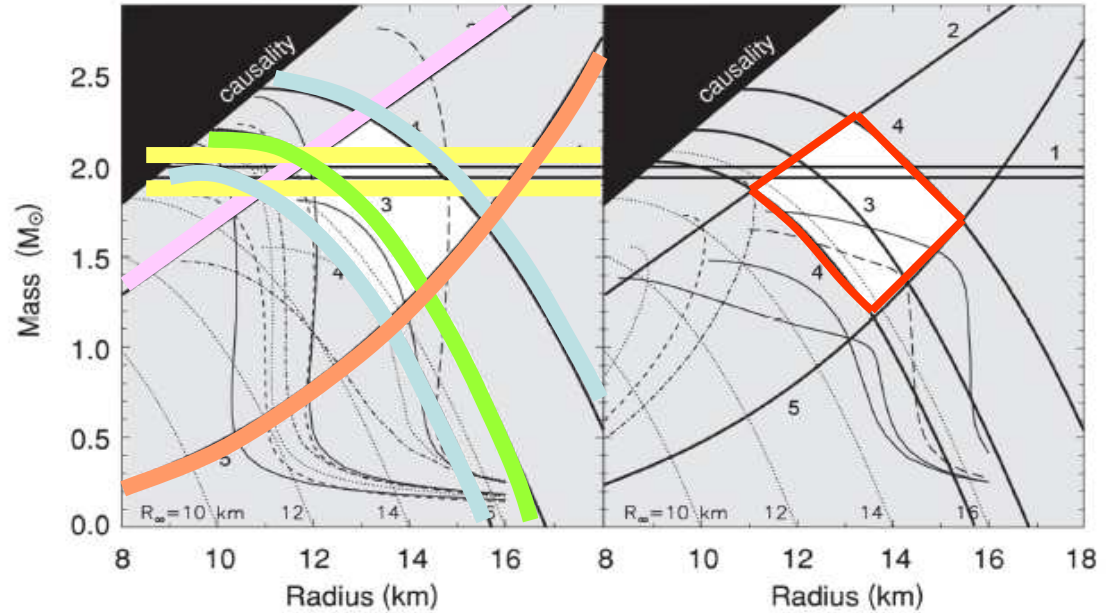
further work required! Investigation heavy ion collisions

# Constraints on EoS from Astrophysical Observation

Observations of:  
 masses  
 radii (X-ray bursts)  
 rotation periods

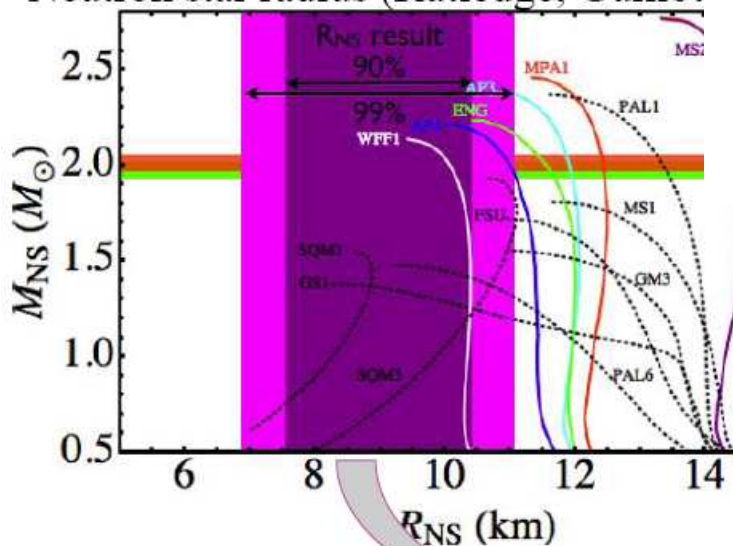


## Trümper Constraints (Universe Cluster, Irsee 2012)



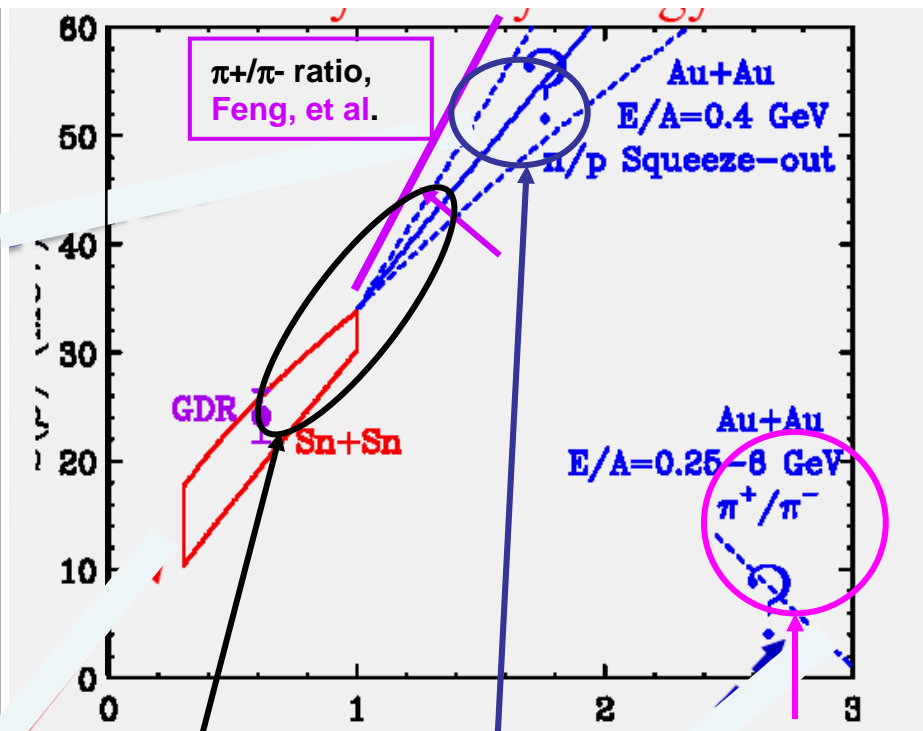
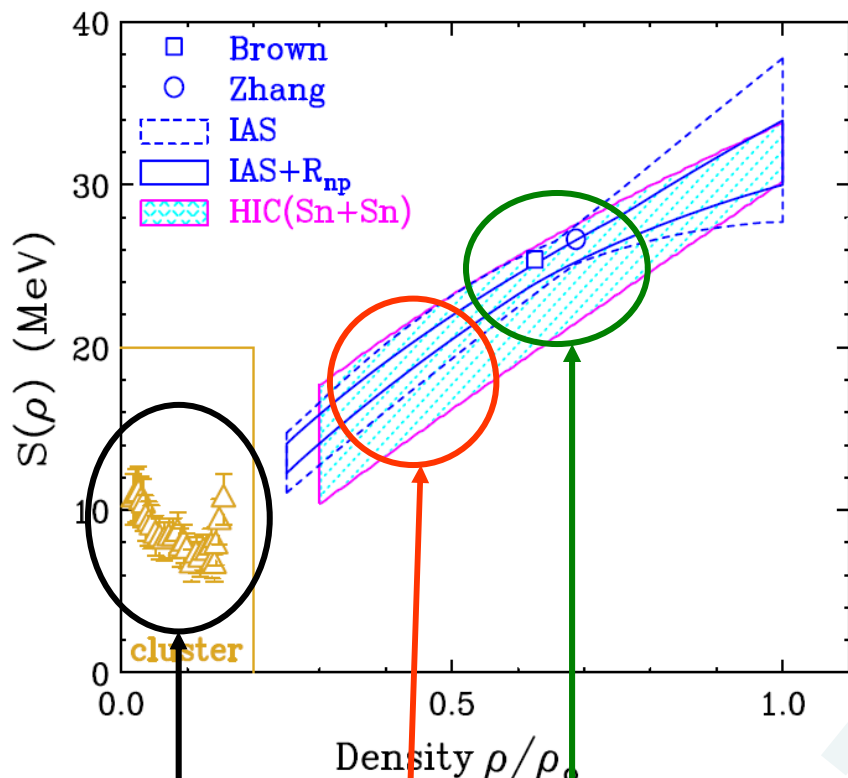
- 1 Largest mass J1614 - 2230 (Demorest et al. 2010)
- 2 Maximum gravity XTE 1814 - 338 (Bhattacharyya et al. 2005)
- 3 Minimum radius RXJ1856 - 3754 (Trümper et al. 2004)
- 4 Radius, 90% confidence limits LMXB 47 Tuc (Heinke et al. 2006)
- 5 Largest spin frequency J1748 - 2446 (Hessels et al. 2006)

## Neutron star radius (Rutledge, Gulliot)



**Increasingly stringent constraint on many EoS models**

# Present constraints on the symmetry energy from HIC discuss for different density regions



very low density, clusterization

around  $\rho_0$ , masses, collective excitations

fragmentation, liquid-gas PT

pre-equilibrium emission of nucleons and light clusters

collective flow

pion, kaon, ... production

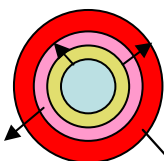
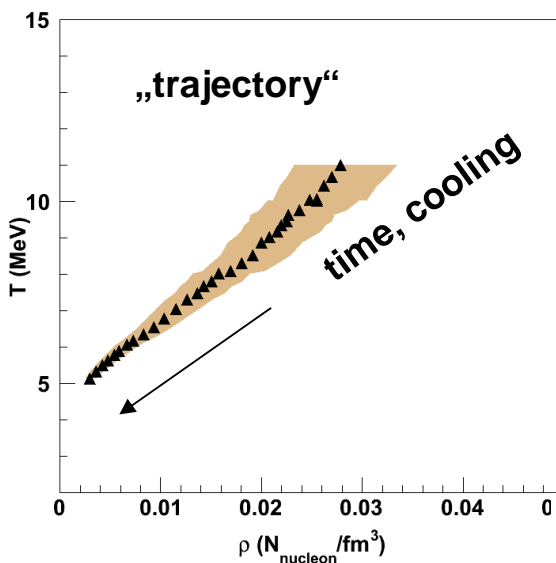
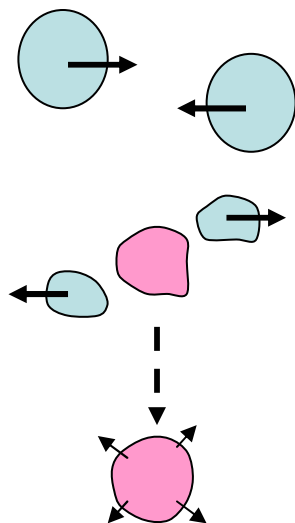
talks C. Hagel, C. Horowitz

Contributions to the workshop from all these fields!  
almost impossible to summarize in a finite time

# Investigation of very low density NSE in Heavy ion collisions

C. Hagel

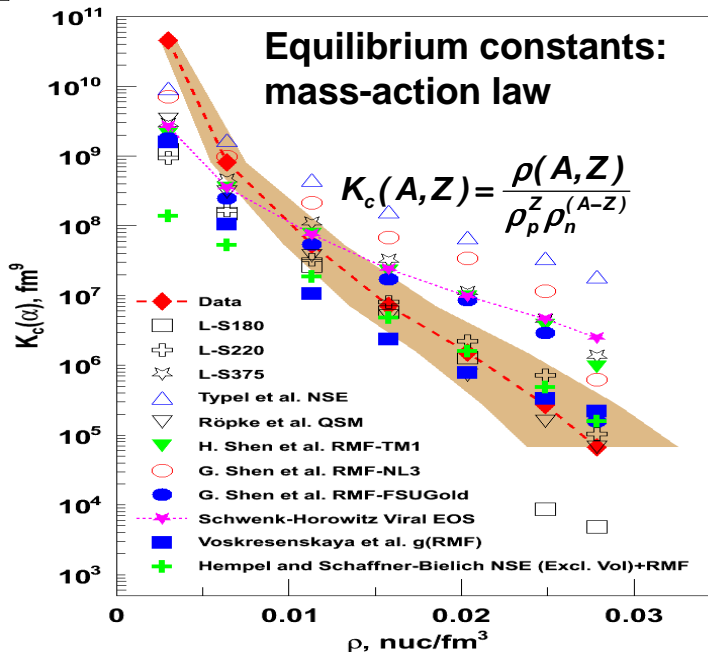
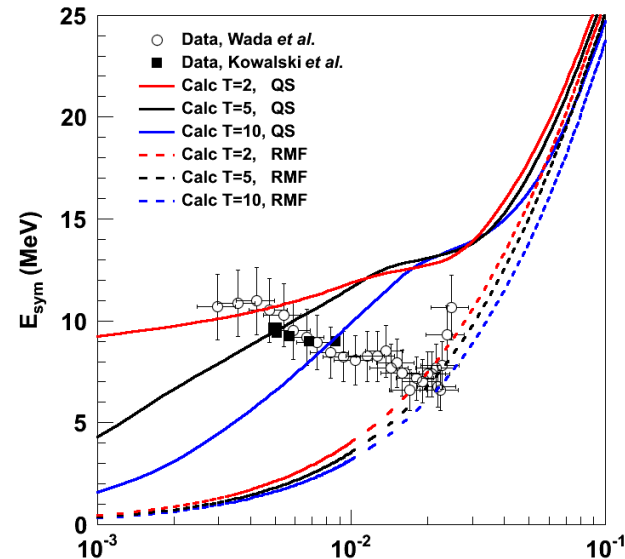
e.g. experiment 64Zn+(92Mo,197Au) at 35 A MeV  
 S. Kowalski, J. Natowitz, et al., PRC75 014601 (2007)  
 J. Natowitz, G. Röpke, S. Typel, .. PRL 104, 202501 (2010)



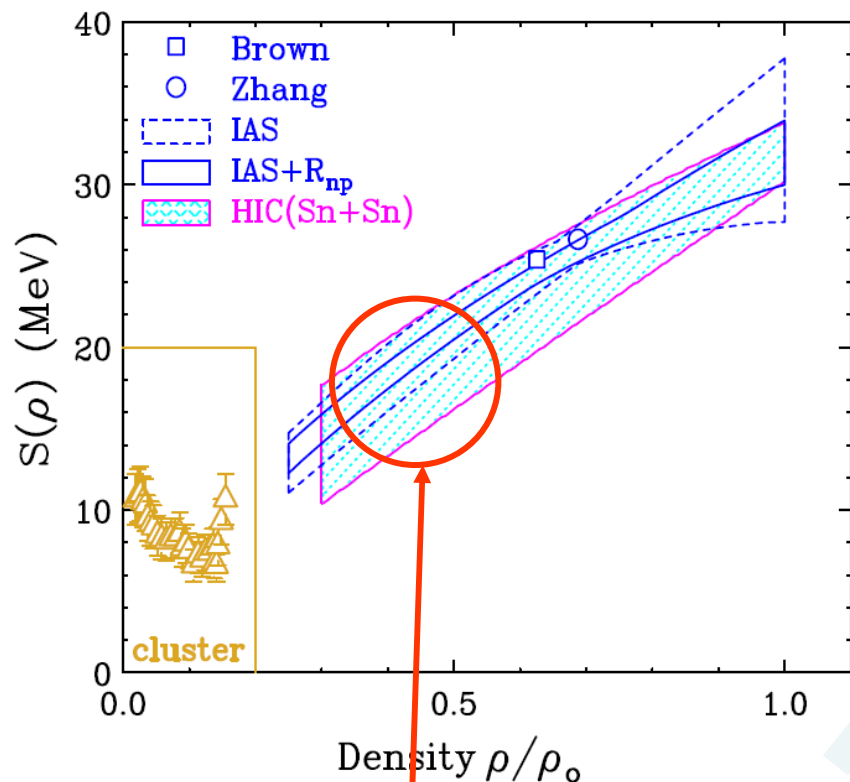
„differential“ freeze-out analysis:  
 source reconstruction,  
 analysis in terms of  $v_{surf} \sim$  time of emission  
 determination of thermodyn. properties as fct of  $v_{surf}$   
 determination of symmetry energy

**Assumptions need to be checked in transport calculations.**

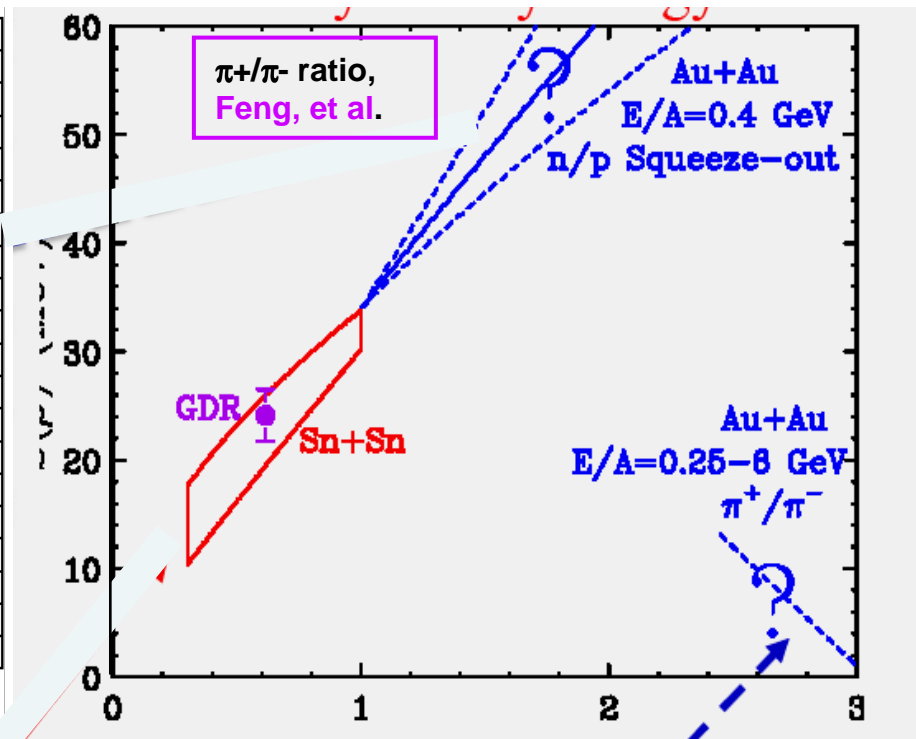
## symmetry energy



# Present constraints on the symmetry energy from HIC discuss for different density regions



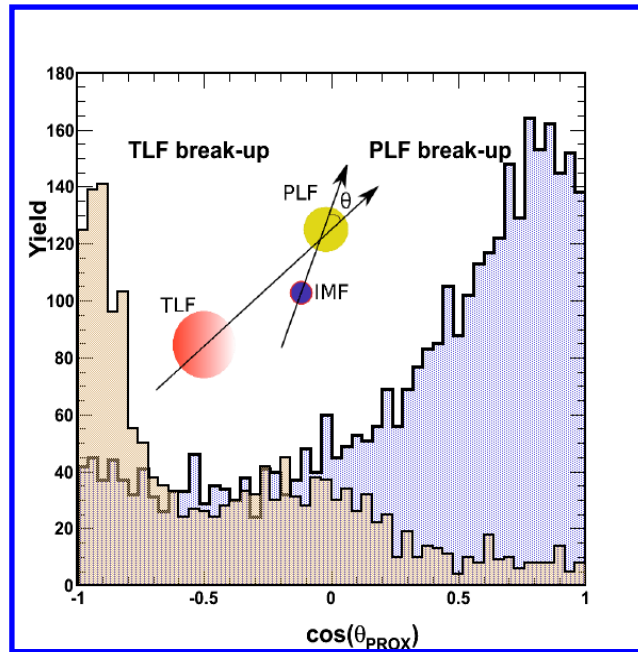
fragmentation,  
liquid-gas PT



Region of low density symmetry energy:  
 →importance of fluctuations: Langevin terms  
 Colonna, Napolitani

- isospin transport (migration, fractionation): Colonna
- isospin diffusion, neck neutron enrichment
- multifragmentation, freeze out approximation
- dynamical fission

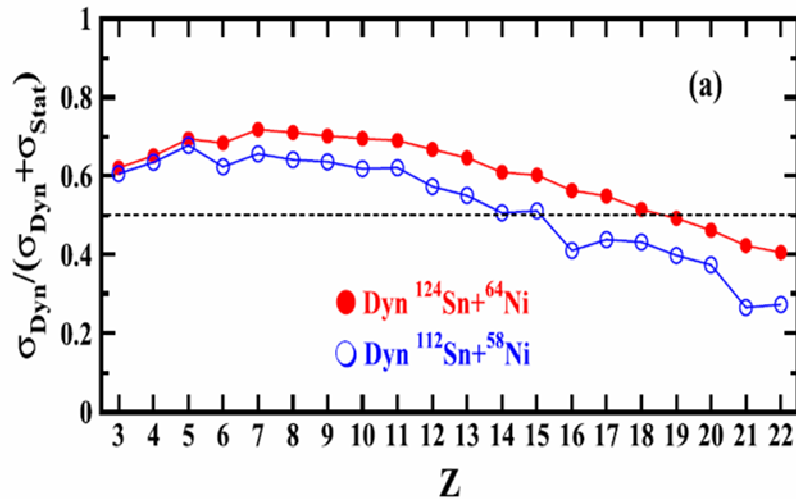
E. De Filippo: Dynamical fission at low energies  
Break-up of PLF in vicinity of TLF.



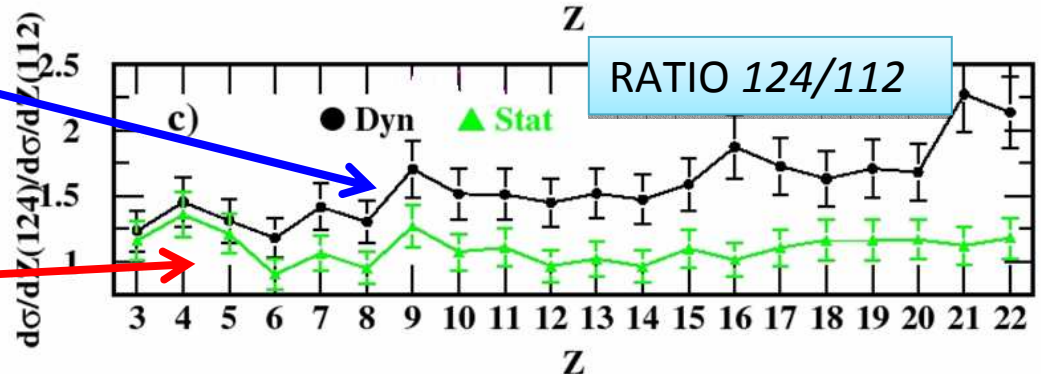
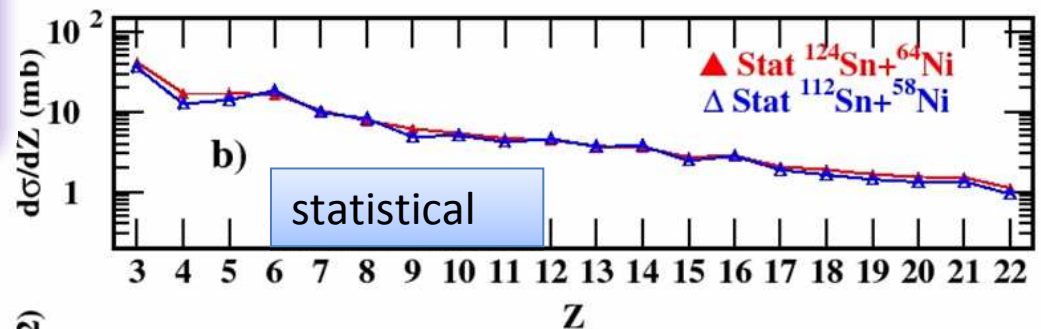
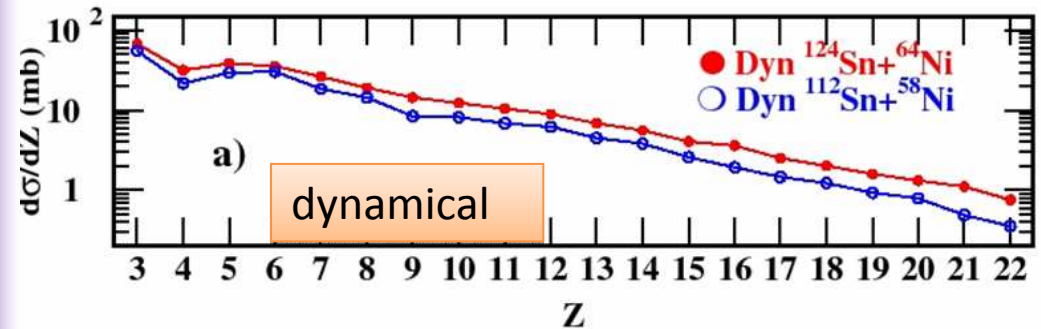
# Comparison of IMFs cross sections for $^{124}\text{Sn}+^{64}\text{Ni}$ and $^{112}\text{Sn}+^{58}\text{Ni}$

P. Russotto et al. , Phys. Rev. C91, 014610 (2015)

Ratio of  $\sigma_{\text{dyn}}/(\sigma_{\text{dyn}}+\sigma_{\text{stat}})$  as a function of IMFs charge Z for the two systems.



## Cross-sections



• **Dynamical component:** enhanced for the neutron rich

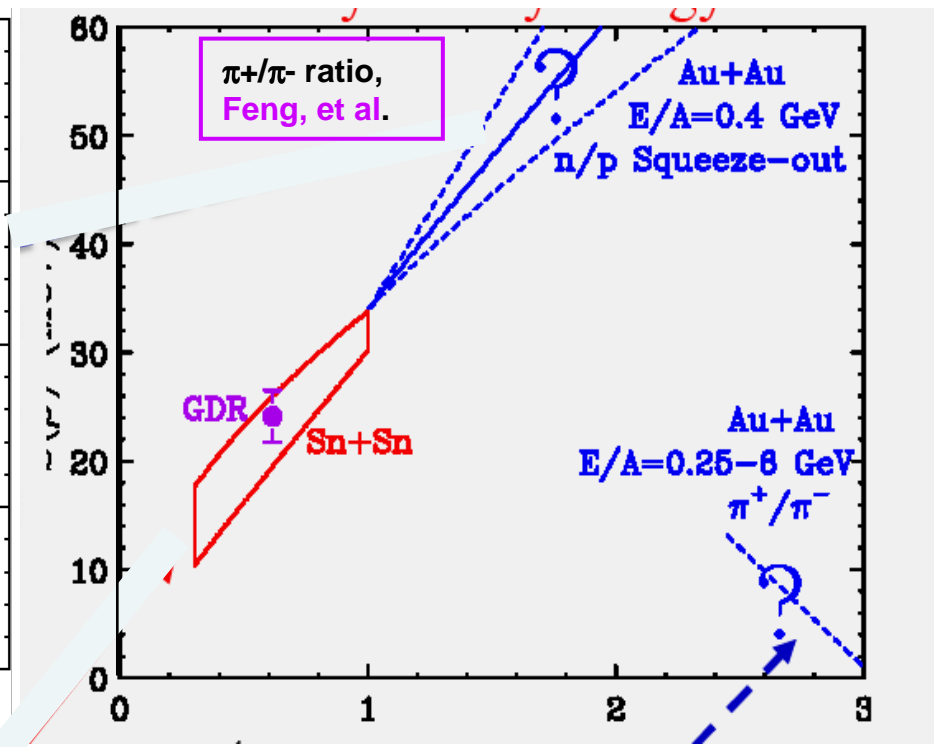
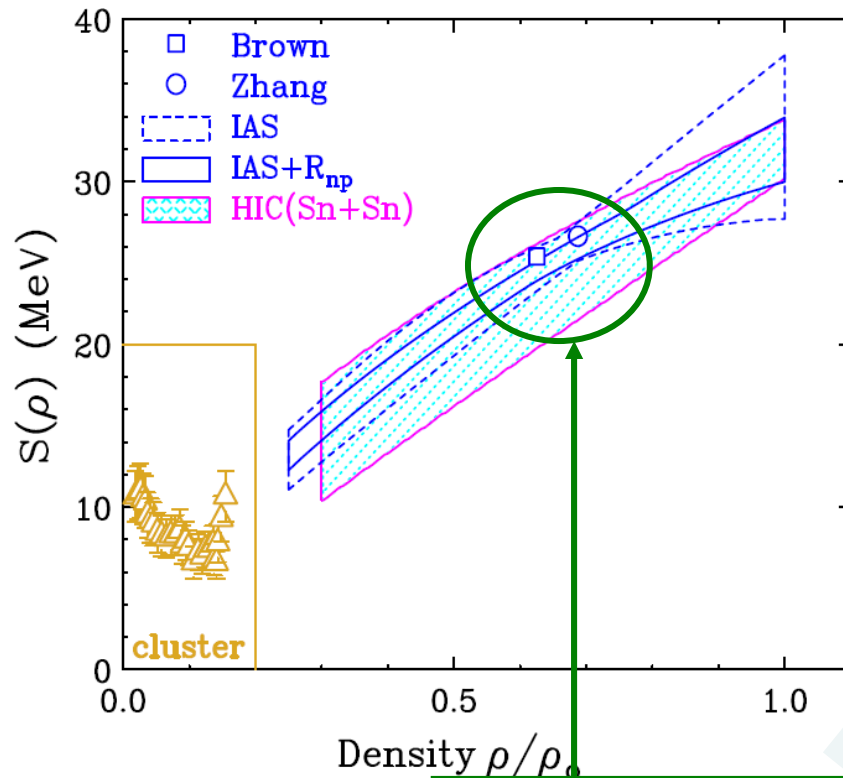
• **Statistical component:** almost equal (A ratio:  $\sim 1.1$  close to the mass ratio between the systems)

see also A.B. McIntosh et al. PRC 81 034603 (2010)

DeFilippo



# Present constraints on the symmetry energy from HIC discuss for different density regions



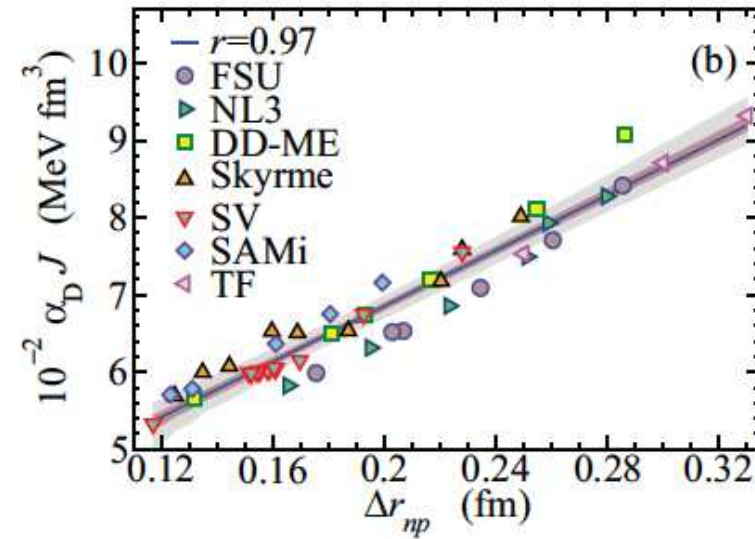
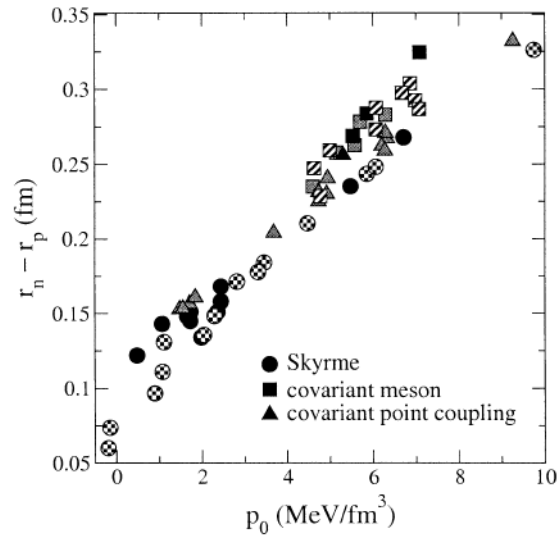
around  $\rho_0$ : Masses,  
Giant resonances: IVGDR, PDR, ISGDR  
Dipole radiation  
Clustering in skin  
Correlations ( $S \leftrightarrow L$ )

# Correlations

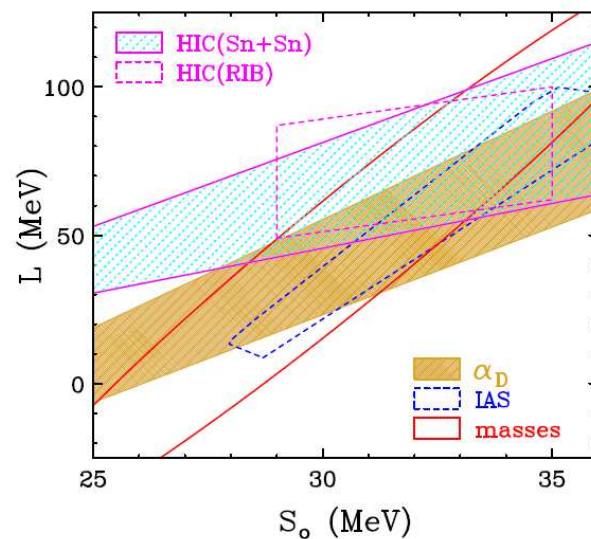
(X. Roca-Maza)

distinguish between

1) correlations between model parameters and an observable, or between observables e.g.

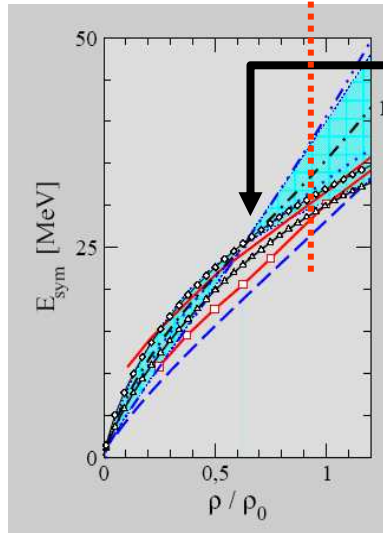


2) and correlations between model parameters, e.g.  $S_0$  and  $L$



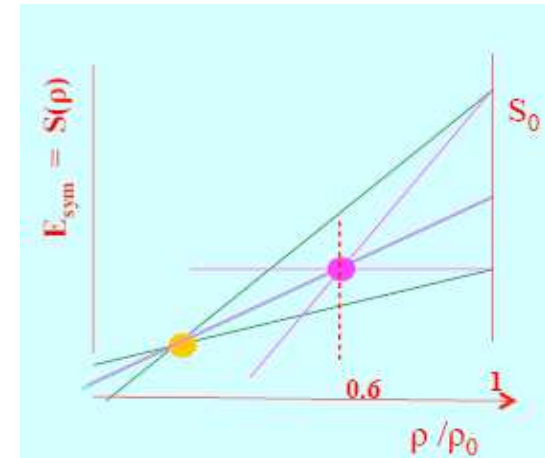
C.J.Horowitz, et al., J. Phys. G: Nucl. Part. Phys. 41 (2014) 093001

## Correlations between model parameters, e.g.

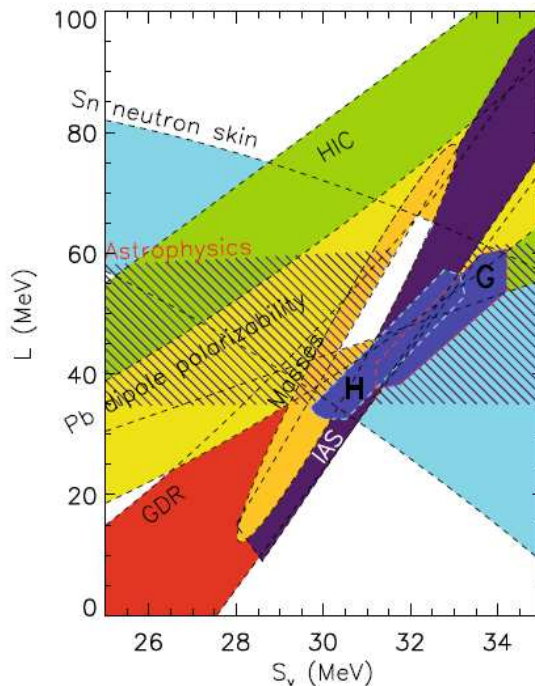


- e.g. SE that fit nuclear masses cross below saturation density, (some average density of a finite nucleus)
- induces a correlation between value and slope at  $\rho_0$ , within the model., eg. in lin. approx.

$$S(\rho = 2\rho_0/3) = J - \frac{L}{9} + \frac{K_{\text{sym}}}{162} + \dots \approx J - \frac{L}{9}$$



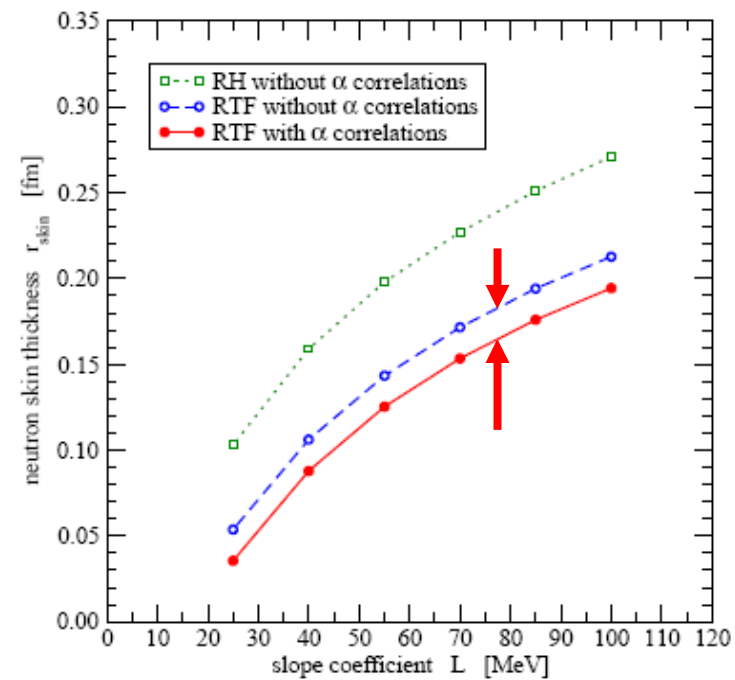
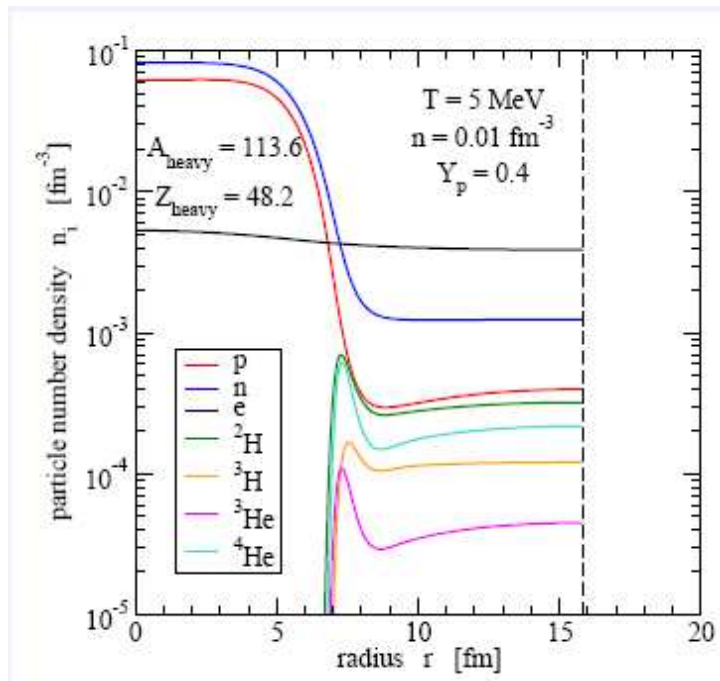
(from M. Colonna)



- different observables are sensitive to different densities (or ranges of densities) and thus induce different correlations
- crossing point will hopefully fix S and L, which are independent
- Represents an extrapolation using a model with different density dependences in some cases a wide extrapolation, eg. for neutron star

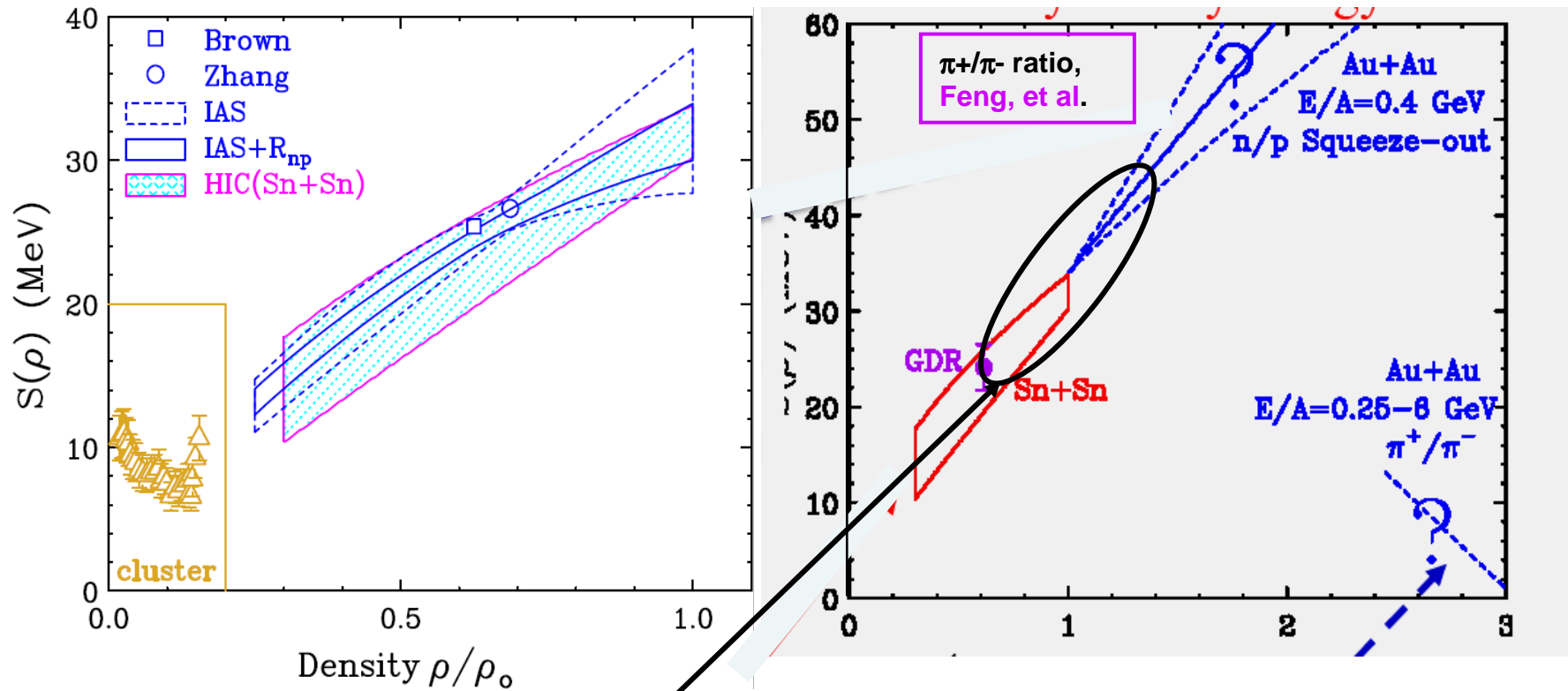
# Neutron Skins with Cluster Correlations

- finite temperature gRDF calculations in spherical Wigner-Seitz cell, extended Thomas-Fermi approximation
  - ⇒ enhanced cluster probability at surface of heavy nuclei, effects for **heavy nuclei in vacuum at zero temperature?**



important effect for determination of  $L$  via measurement of skin thickness

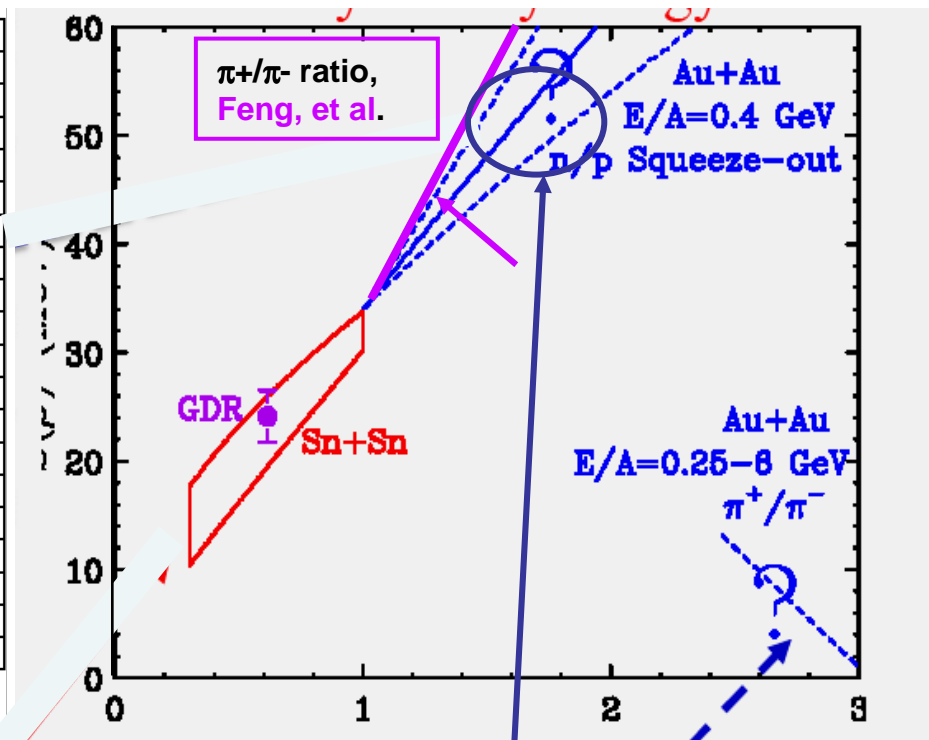
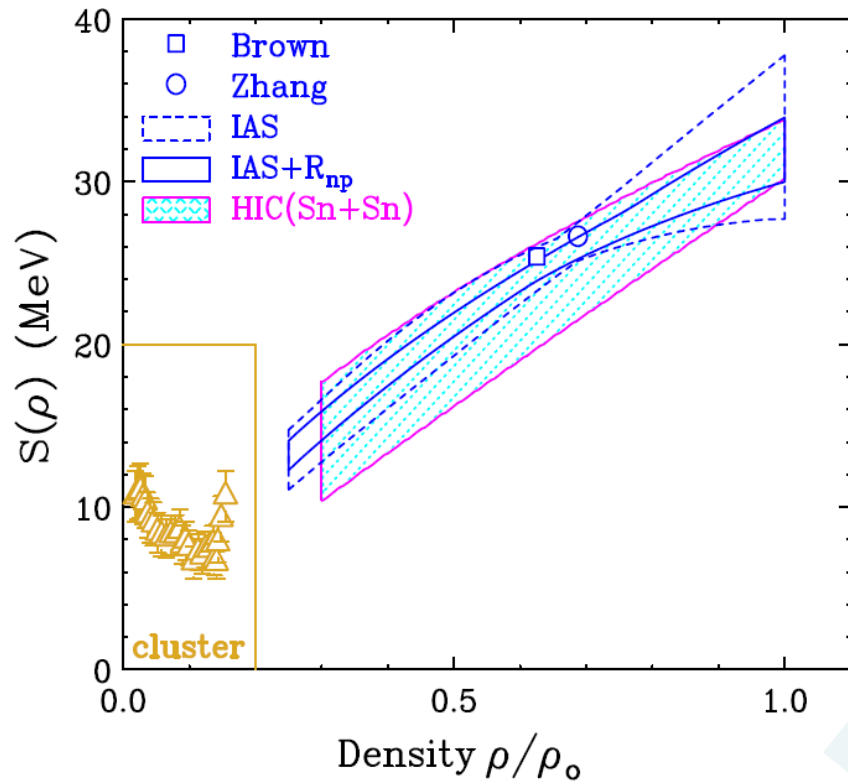
# Present constraints on the symmetry energy from HIC discuss for different density regions



pre-equilibrium  
emission of  
nucleons and  
light clusters

- discussed in my previous talk and extensively in open discussion,
- a promising probe of the symmetry energy and the effective mass splitting
- but needs a reliable treatment of light cluster production in collisions

# Present constraints on the symmetry energy from HIC discuss for different density regions

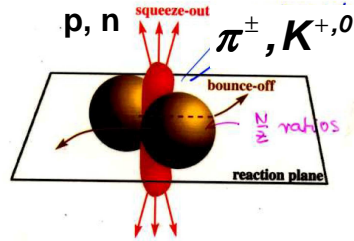


collective n/p flow  
 ASYEoS Experiment  
 symmetric EOS: LeFevre

# Momentum distributions, "Flow"

Au+Au @ 400 AMeV, FOPI-LAND

prediction

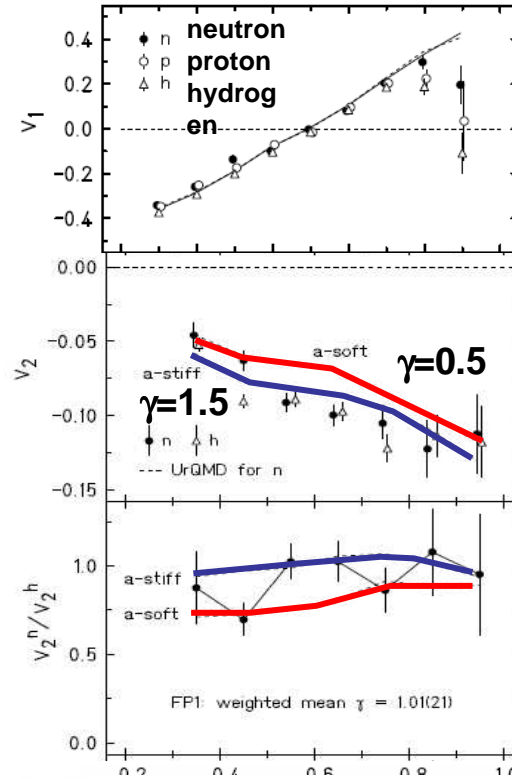


- Directed flow not very sensitive to SE (involves many different densities)

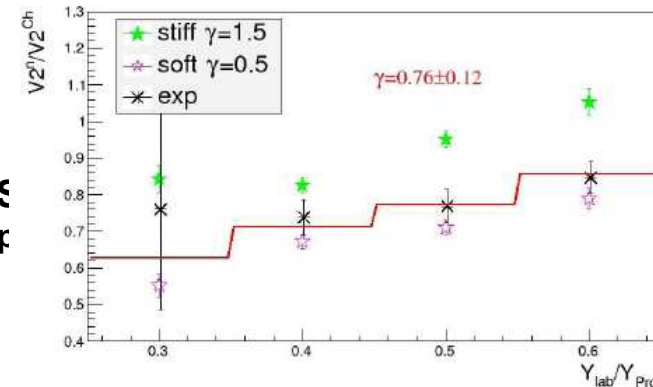
- Elliptic flow in this energy region probe of high density

preliminary result from new experiment ASY-EOS (Rusotto, IWM\_EC workshop Catania 2014)

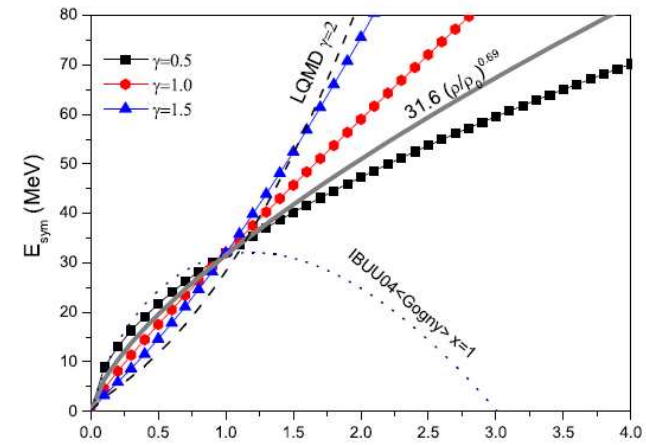
$\gamma = 0.7 \pm 0.2$   
 $L = 70 \pm 13$  (FP1)



(Rusotto, et al., PLB 697, 471 (11))



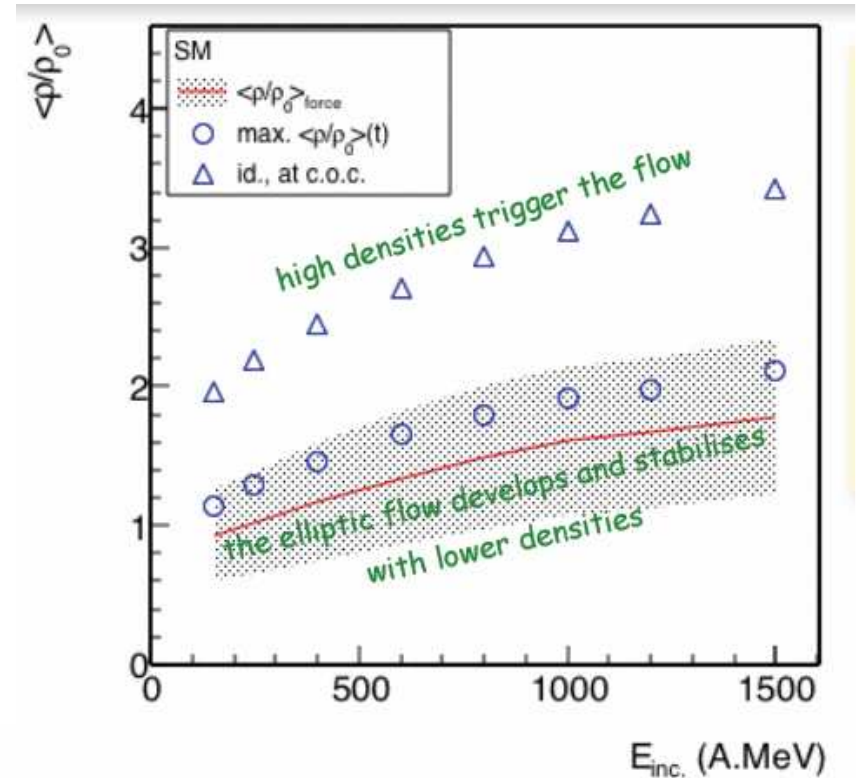
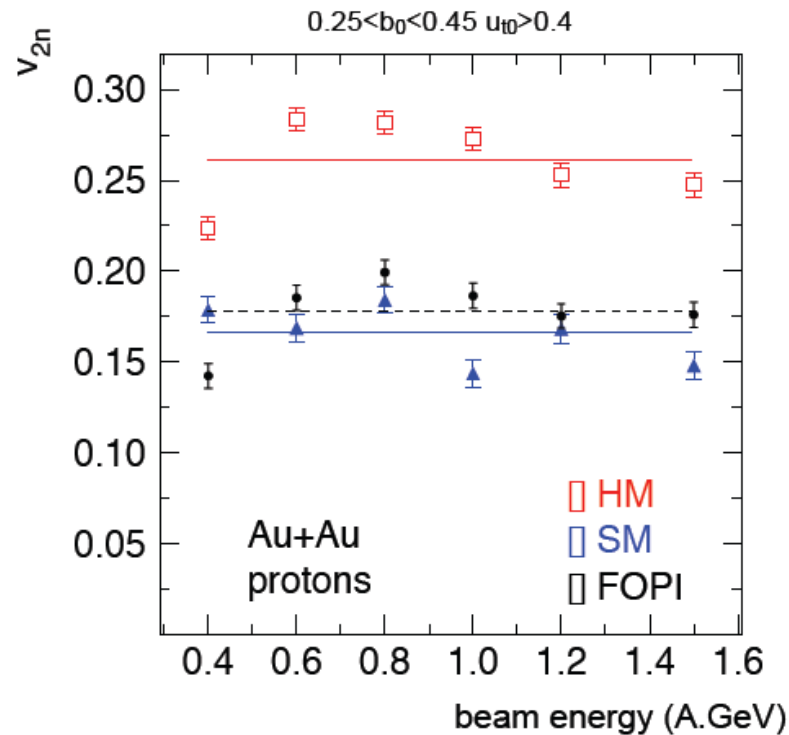
not very precise (yet) but indicates rather stiff SE,  $\gamma \sim 1$



A major advance in the search for the high density Symmetry Energy!

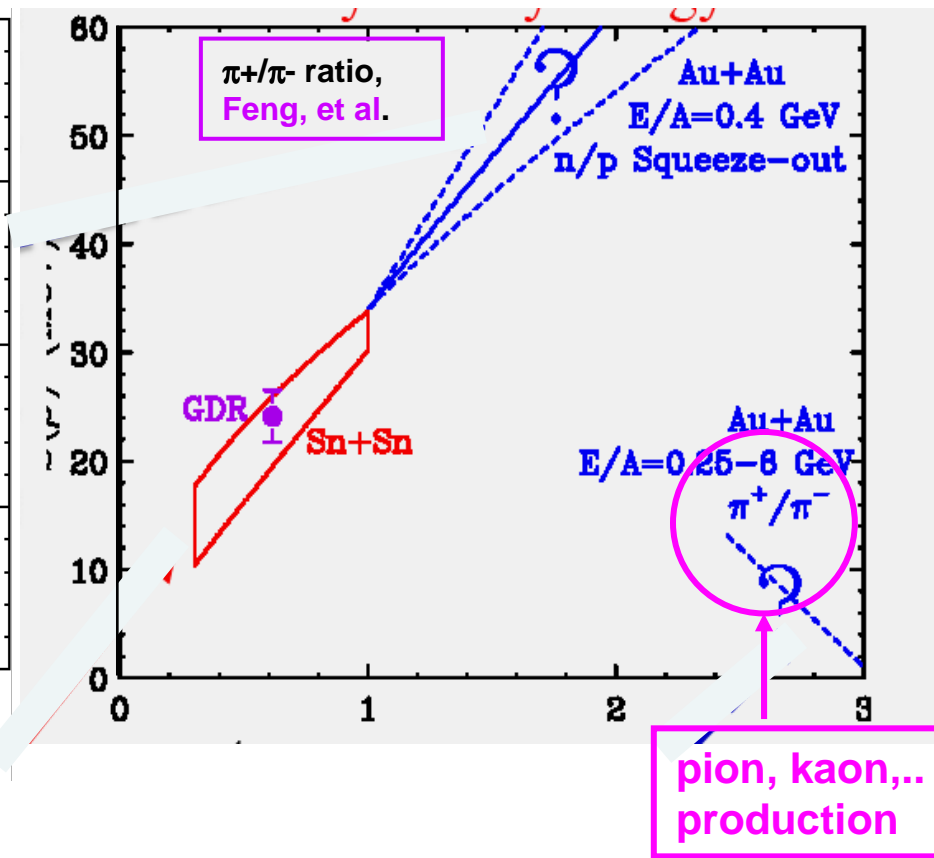
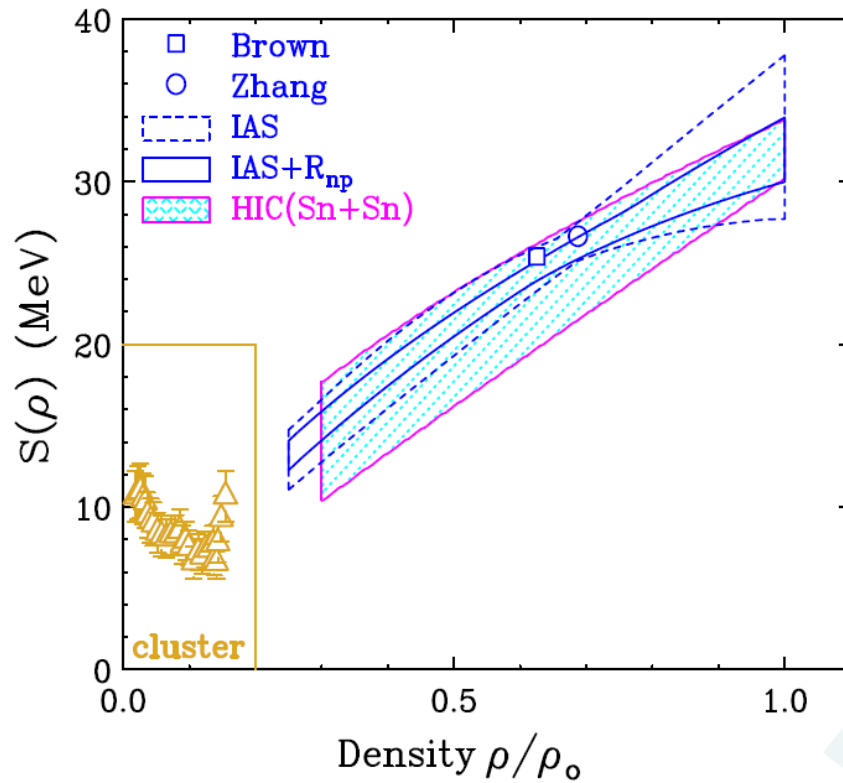
A. Le Fevre: Analysis of extensive FOPI data on flow with IQMD:  
 strong constraint of the symmetric EOS  
 extension of earlier work by P. Danielewicz, et al., Science

Complete shape of  $v_2(y_0)$ :  
 a new observable:  
 $v_{2n} = |v_{20}| + |v_{22}|$ ,  
 from fit  
 $v_2(y_0) = v_{20} + v_{22} \cdot y_0^2$





# Present constraints on the symmetry energy from HIC discuss for different density regions



# Particle Production

Inelastic collisions: Production of particles and resonances: Coupled transport equations

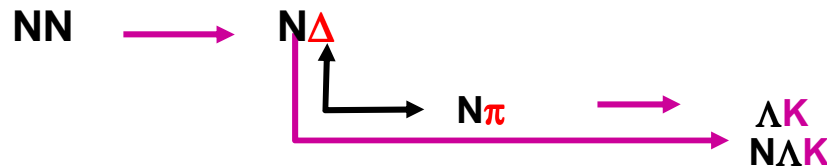
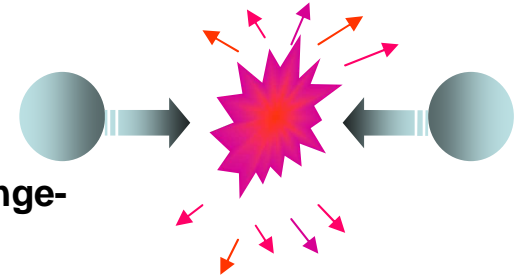
$$\frac{d}{dt} f_N(\mathbf{x}_\mu) = I_{coll}(\sigma_{NN \rightarrow NN'} f_N; \sigma_{NN \rightarrow N\Delta} f_\Delta; \dots)$$

$$\frac{d}{dt} f_\Delta(\mathbf{x}_\mu) = I_{coll}(\sigma_{\Delta N \rightarrow NYK} f_Y f_K; \dots)$$

etc.

e.g. pion and kaon production;

coupling of  $\Delta$  and strangeness channels.

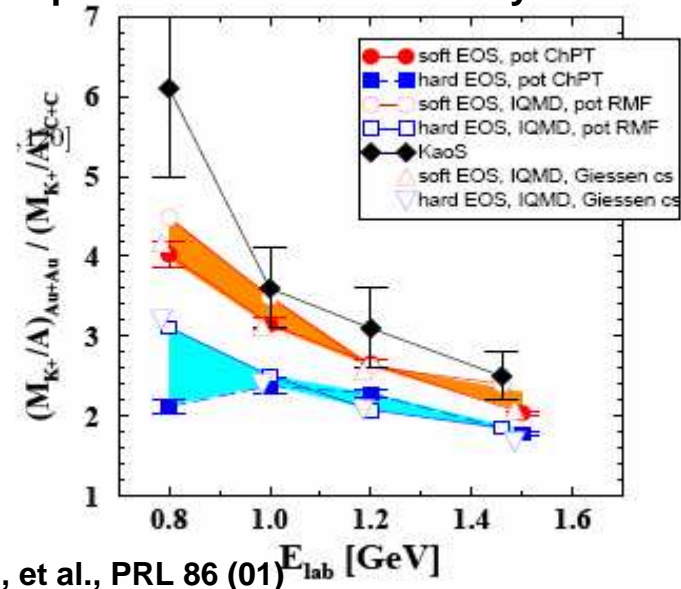


Many new potentials, elastic and inelastic cross sections needed,  $\Delta$  dynamics in medium

## What can one learn from different species?

- **pions**: production at all stages of the evolution via the  $\Delta$ -resonance
- **kaons** (strange mesons with high mass): subthreshold production, probe of high density phase
- **ratios** of  $\pi^+/\pi^-$  and  $K^0/K^+$ :  
→ **probe for symmetry energy**

Important to fix the EOS of symm. nucl. matter



# Particle production as probe of symmetry energy

Two effects:

G.Ferini et al., PRL 97 (2006) 202301

1. Mean field effect:  $U_{\text{sym}}$  more repulsive for neutrons, and more for asystiff  
 → pre-equilibrium emission of neutron, reduction of asymmetry of residue

$$\frac{n}{p} \downarrow \Rightarrow \frac{Y(\Delta^{0,-})}{Y(\Delta^{+,++})} \downarrow \Rightarrow \frac{\pi^-}{\pi^+} \downarrow$$

*decrease with asy – stiffness*

2. Threshold effect, in medium effective masses:

Canonical momenta have to be conserved. To convert to kinetic momenta, the self energies enter

$$I_{\text{coll}} = \int d\vec{v}_2 d\vec{v}_1 d\vec{v}_2' v_{12} \sigma_{\text{inel}}(\Omega) (2\pi)^3 \delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_1' - \mathbf{p}_2')$$

$$\times [f_1' f_2' (1 - f_1)(1 - f_2) - f_1 f_2 (1 - f_1')(1 - f_2')]$$

In inelastic collisions, like  $nn \rightarrow p\Delta^-$ , the selfenergies may change. Simple assumption about self energies of  $\Delta$ .

$$\Sigma_i(\Delta^-) = \Sigma_i(n),$$

$$\Sigma_i(\Delta^0) = \frac{2}{3} \Sigma_i(n) + \frac{1}{3} \Sigma_i(p),$$

$$\Sigma_i(\Delta^+) = \frac{1}{3} \Sigma_i(n) + \frac{2}{3} \Sigma_i(p),$$

$$\Sigma_i(\Delta^{++}) = \Sigma_i(p),$$

Yield of pions depends on

$$\sigma = \sigma_{\text{inel}} \left( \sqrt{\mathbf{s}_{in}} - \sqrt{\mathbf{s}_{in}'} \right)$$

Detailed analysis gives

$$\frac{\pi^-}{\pi^+} \uparrow \text{ increase with asy – stiffness}$$

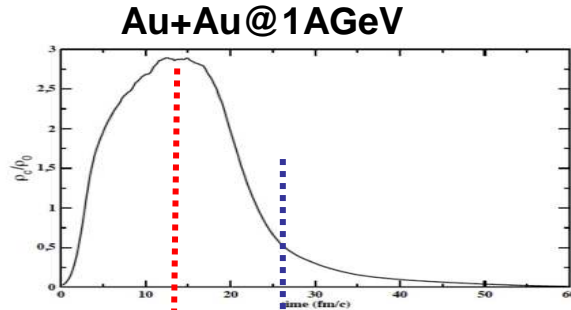
Competing effects!

Not clear, how taken into account in all studies

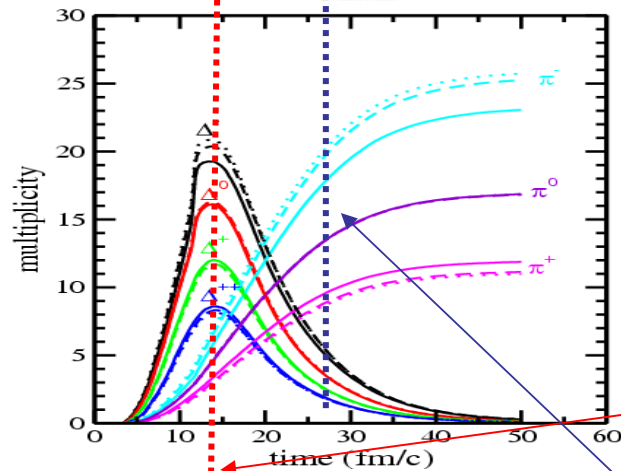
Assumptions may also be too simple.

# Dynamics of particle production ( $\Delta, \pi, K$ ) in heavy ion collisions

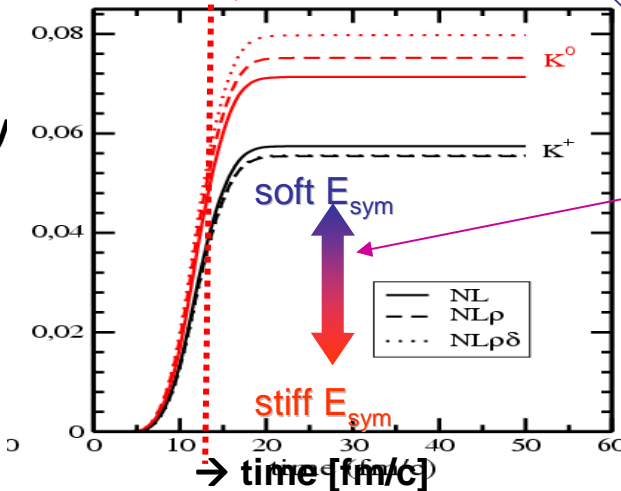
Central density



$\pi$  and  $\Delta$  multiplicity



$K^{0,+}$  multiplicity



Dependence of ratios on asy-stiffness

$n/p$

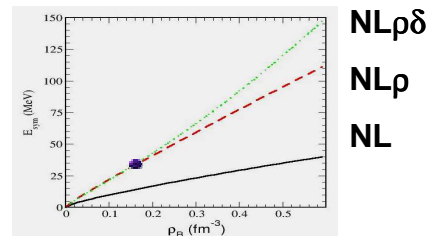
$\rightarrow \Delta^{0,-}/\Delta^{+,++}$

$\rightarrow \pi/\pi^+$

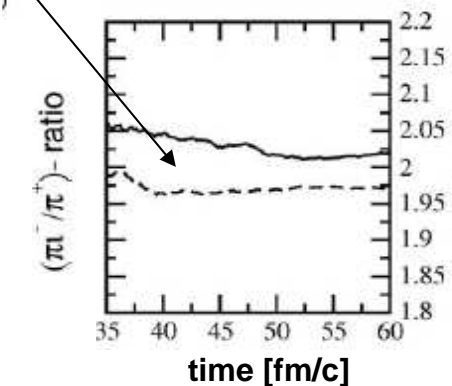
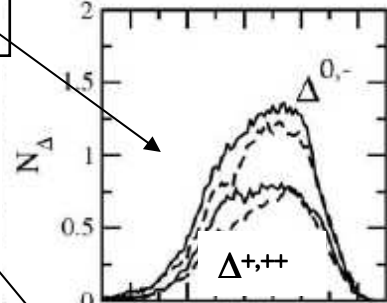
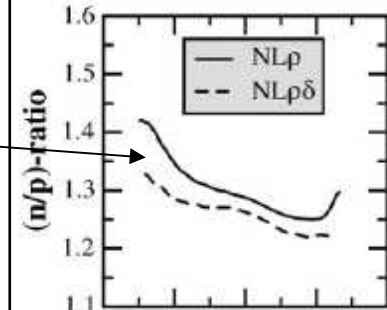
$\Delta$  and  $K$ : production in high density phase

Pions: low and high density phase

Sensitivity to asy-stiffness

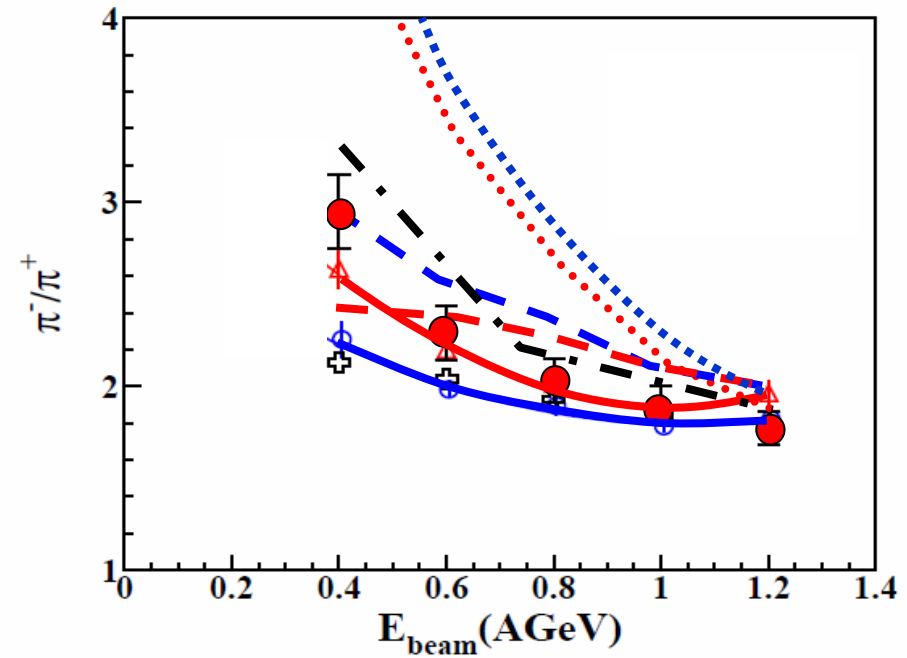
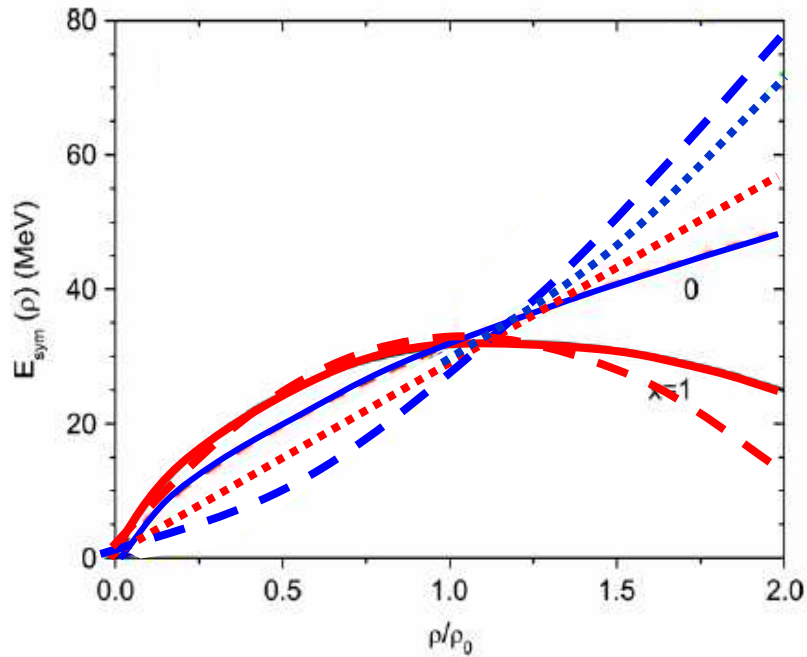
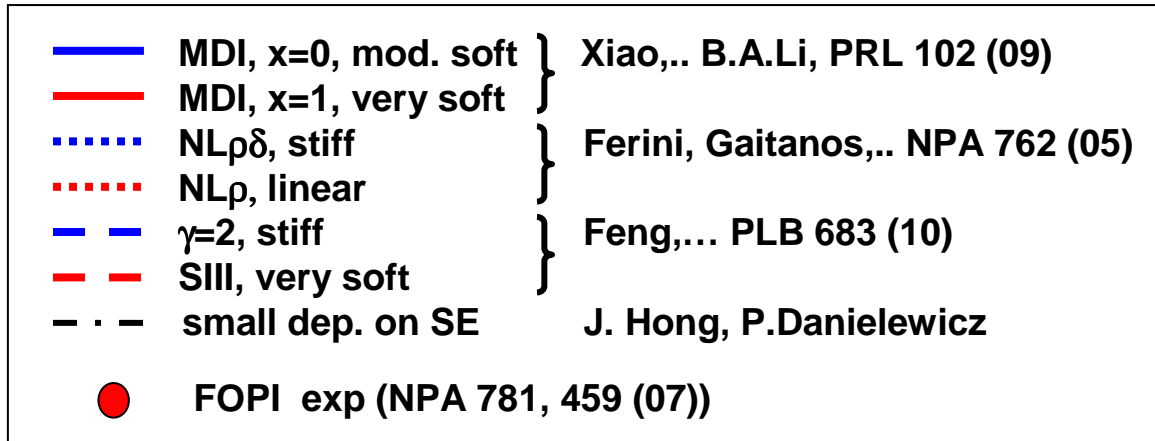


Au+Au, 0.6 A MeV



# Pion ratios in comparison to FOPI data

Au+Au, semi-central



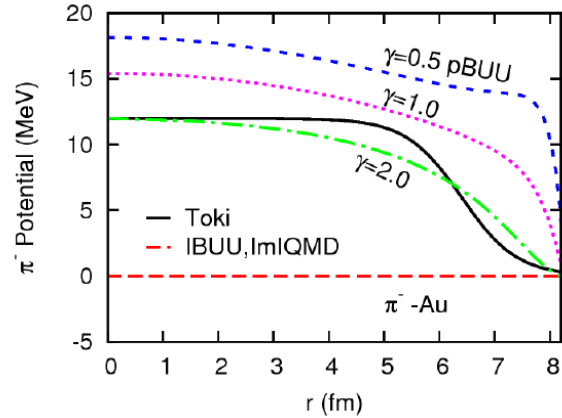
**Contradictory results, trend with asy-stiffness differs**

P. Danielewicz

$\pi$  vs Baryon Optical Potentials

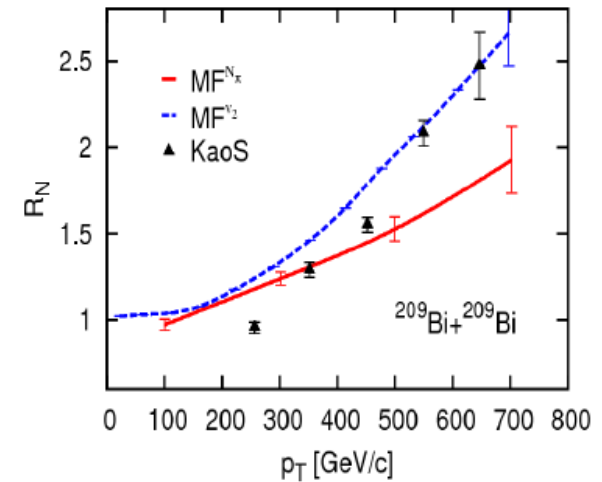
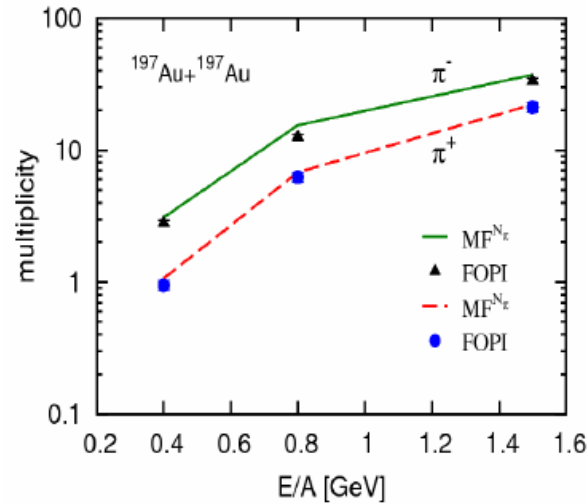
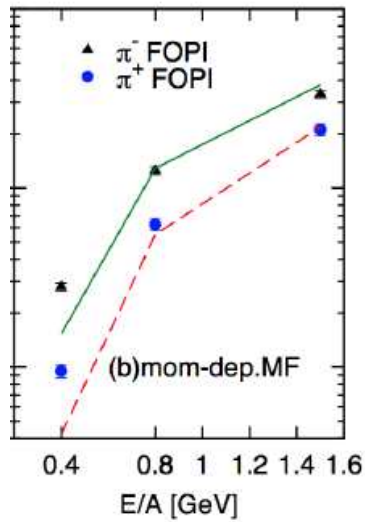
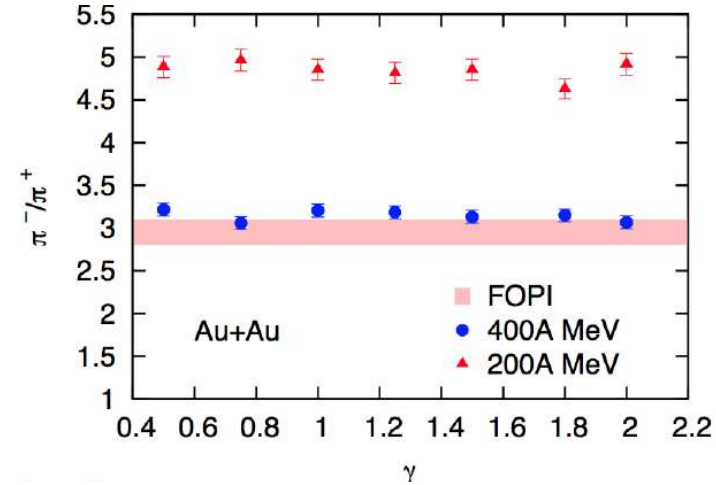
$$\Delta \longleftrightarrow N + \pi \quad U_{\Delta} \stackrel{?}{=} U_N + U_{\pi}$$

Symmetry-Energy Derived  $\pi$  Potential



Jun Hong&PD PRC90(14)024605 Nucl density: Thomas-Fermi

→ independence of pion ratio on SE

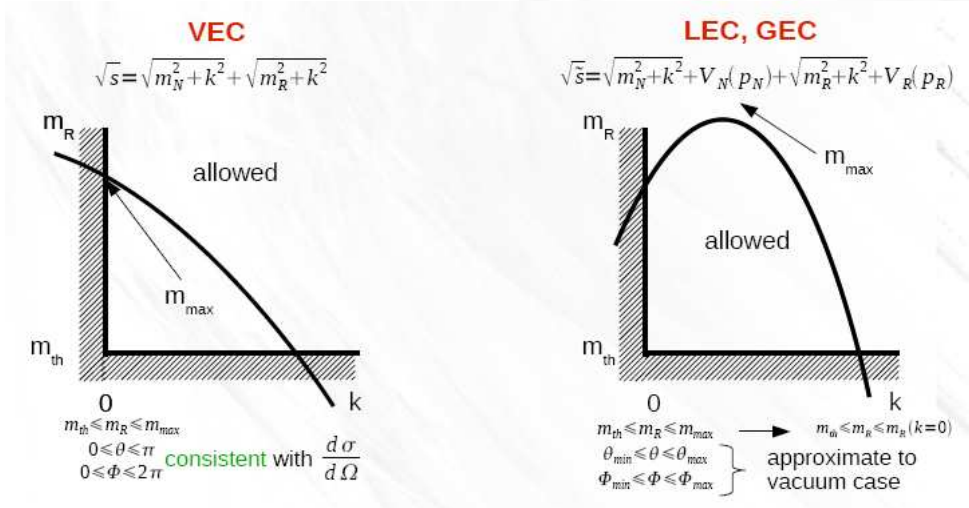


but does not fit pion data →  $U_{\pi}(p)$  modified (softened) for low momenta → inconsistent with ell. flow

# D. Cozma

## Threshold condition for NN→NR

$$\sqrt{p_1^2 + m_1^2} + U(p_1) + \sqrt{p_2^2 + m_2^2} + U(p_2) = \sqrt{p_1'^2 + m_1'^2} + U(p_1') + \sqrt{p_2'^2 + m_2'^2} + U(p_2')$$



$$V(\Delta^{**}) = V_s + \frac{3}{2} V_v$$

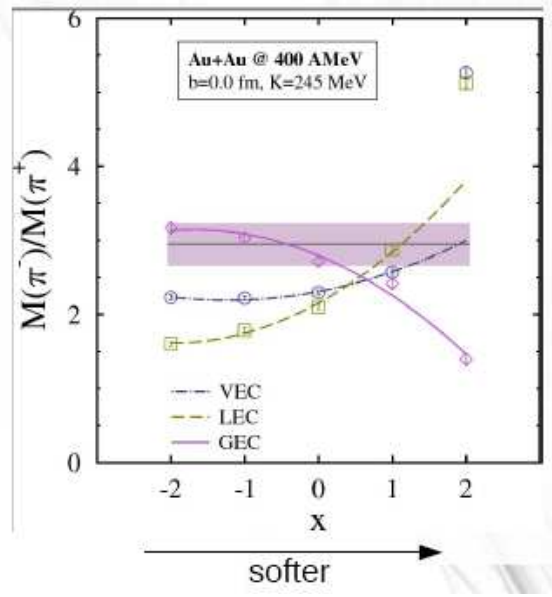
$$V(\Delta^*) = V_s + \frac{1}{2} V_v$$

$$V(\Delta^0) = V_s - \frac{1}{2} V_v$$

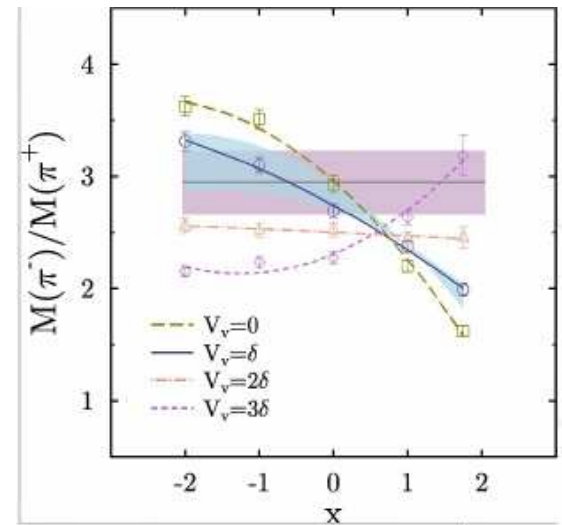
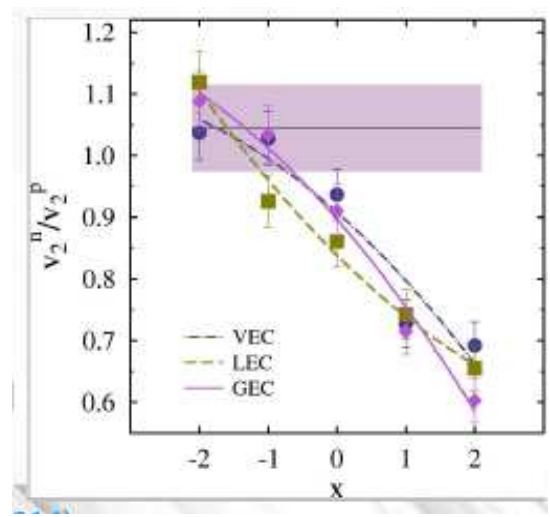
$$V(\Delta^-) = V_s - \frac{3}{2} V_v$$
  

$$V_s = \frac{1}{2}(V_n + V_p)$$

$$\delta = \frac{1}{3}(V_n - V_p)$$



strong dependence on assumption on threshold condition  
 consistency with flow can be achieved for stiff SE and GEC

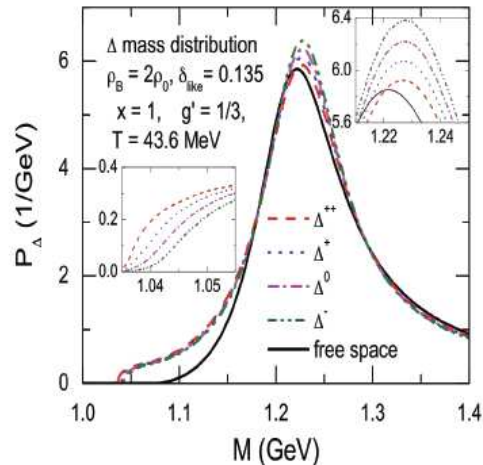


however, strongly dependent on assumption on isovector Δ potential

**pion production decisively depending on implementation of threshold condition and in-medium potentials of  $\pi$  and  $\Delta$**

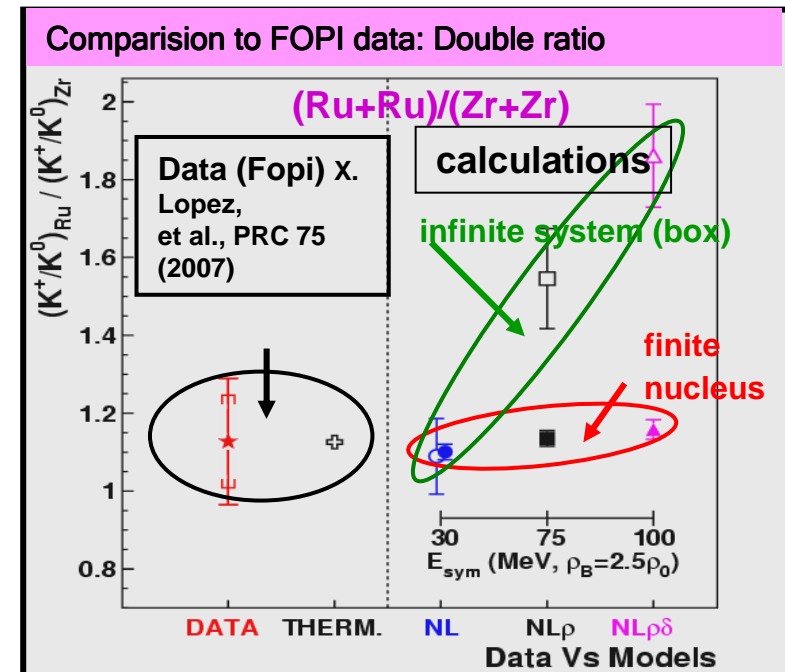
**more generally: transport theory of particles with finite width,  
„off-shell“ transport,**

**see Mosel (GiBUU) and Cassing (HSE) groups; not systematically investigated**



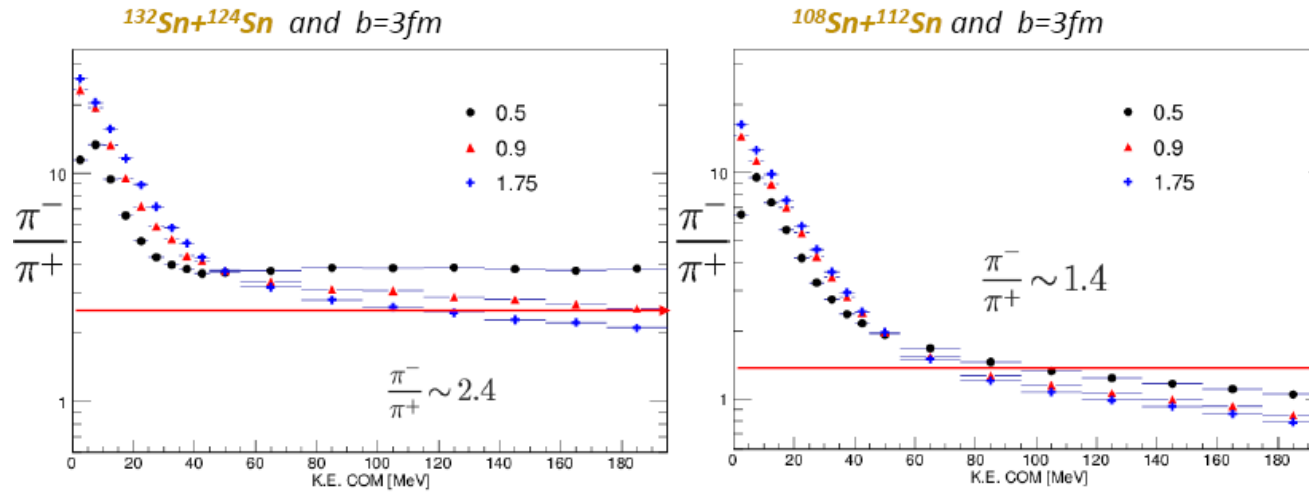
$\Delta$  spectral function

**antistrange Kaons should be considered as an alternative, since small interaction with nuclear medium  
single ratios sensitive to SE**



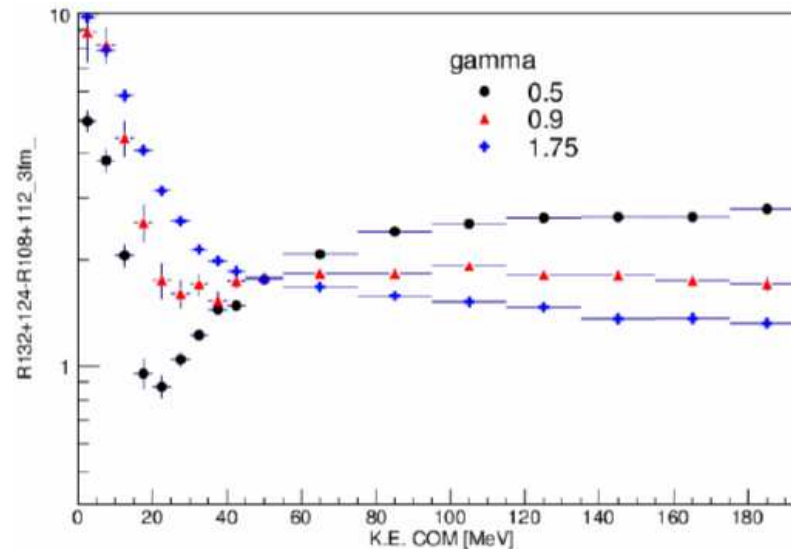


**Spectra of pion ratios seem to be more sensitive:**  
**Planned experiment at S $\pi$ rit (MSU, Riken) 300 MeV: R. Shane, J. Estee**  
**calculations with pBUU (P. Danielewicz)**



**new sugg.  
observable**

$$\Delta R_{(132+124)-(108+112)}(\pi^-/\pi^+) = R_{132+124}(\pi^-/\pi^+) - R_{108+112}(\pi^-/\pi^+)$$



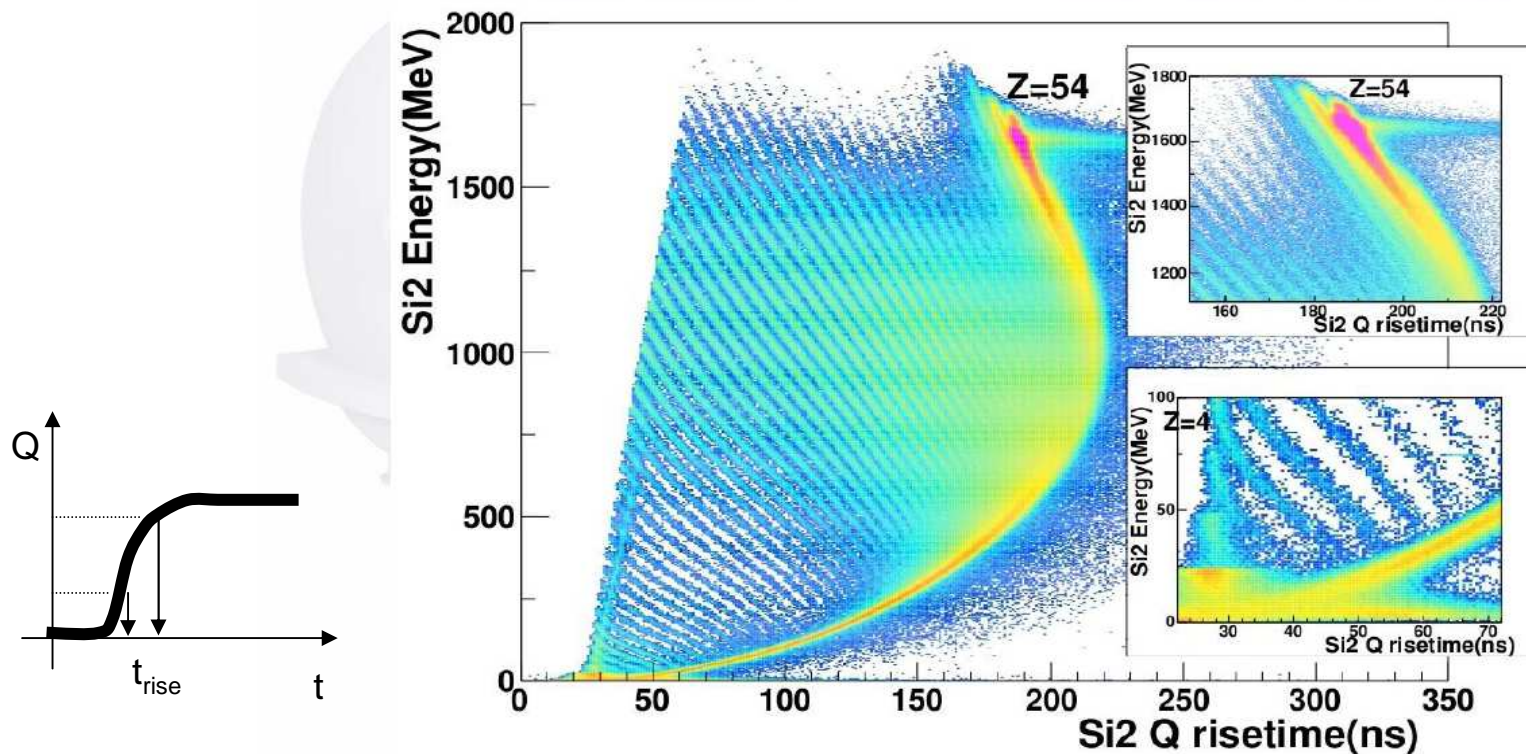
*High energy pions  
(Better understood)*

- produced early in high density regions
- less likely to be absorbed and exchange charge

Impressive experimental developments: ex.: FAZIA (O. Lopez); FARCOS (Tririro)  
VAMOS-INDRA (Fable)

## FAZIA Phase 1 : Pulse Shape Analysis

**Some results at the end of phase 1:  
Pulse Shape Analysis from E – Charge Rise Time**



S.Carboni et al., NIMA 664 (2012) 251

## **Final Remarks:**

**ASY-EoS: a field of strong exchange between theory and experiment**

**largest uncertainties at very low ( $\rho < 0.1\rho_0$ ) and at high density ( $\rho > 2\rho_0$ )**  
**(clusters) (strongly correlated)**

**mapping of microscopic models to phenomenological approaches**  
**(both in nuclear structure and in transport calculation):**  
**(e.g. effective masses, mean field potentials, kinetic energies, medium**  
**cross sections, medium modification of clusters)**

**development of transport approaches:**  
**fluctuations and fragmentation**  
**dynamical role of light clusters**

**Direct confrontation with astrophysical questions:**  
**NS, e.g. mass radius relation and other observables**  
**CC-SN: neutrino opacities in the  $\nu$ -sphere**

**many things to do in the future**



with many thanks to the organizers for the great organization and nice treatment