The Lamb shift in hydrogen and muonic hydrogen and the proton charge radius

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MAX-PLANCK-INSTITUTE OF QUANTUM OPTICS GARCHING

Pulkovo Observatory

Outline

- Electromagnetic interaction and structure of the proton
- Atomic energy levels and the proton radius
  - *Brief story of hydrogenic energy levels*
  - *Brief theory of hydrogenic energy levels*
- Different methods to determine the proton charge radius
  - *spectroscopy of hydrogen (and deuterium)*
  - *the Lamb shift in muonic hydrogen*
  - *electron-proton scattering*
- The proton radius: the state of the art
  - *electric charge radius*
  - *magnetic radius*
- What is the next?

Electromagnetic interaction and structure of the proton

Quantum electrodynamics:

- kinematics of photons;
- kinematics, structure and dynamics of leptons;
- hadrons as compound objects:

- *hadron structure*
  - *affects details of interactions;*
  - *not calculable, to be measured;*
  - *space distribution of charge and magnetic moment;*
  - *form factors (in momentum space).*

Electromagnetic interaction and structure of the proton

Quantum electrodynamics:

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- *hadron structure*
  - *affects details of*
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Electromagnetic interaction and structure of the proton

Quantum electrodynamics:

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- hadrons as compound objects:

- *hadron structure*
  - *affects details of interactions;*
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  - *space distribution of charge and magnetic moment;*
  - *form factors (in momentum space).*

Atomic energy levels and the proton radius

- Proton structure affects
  - the Lamb shift
  - the hyperfine splitting
- The Lamb shift in hydrogen and muonic hydrogen
  - splits $2s_{1/2}$ & $2p_{1/2}$
  - The proton finite size contribution

    $\sim (Z\alpha)\, R_p^2\, |\Psi(0)|^2$
  - shifts all s states

Different methods to determine the proton charge radius

- *Spectroscopy of hydrogen (and deuterium)*

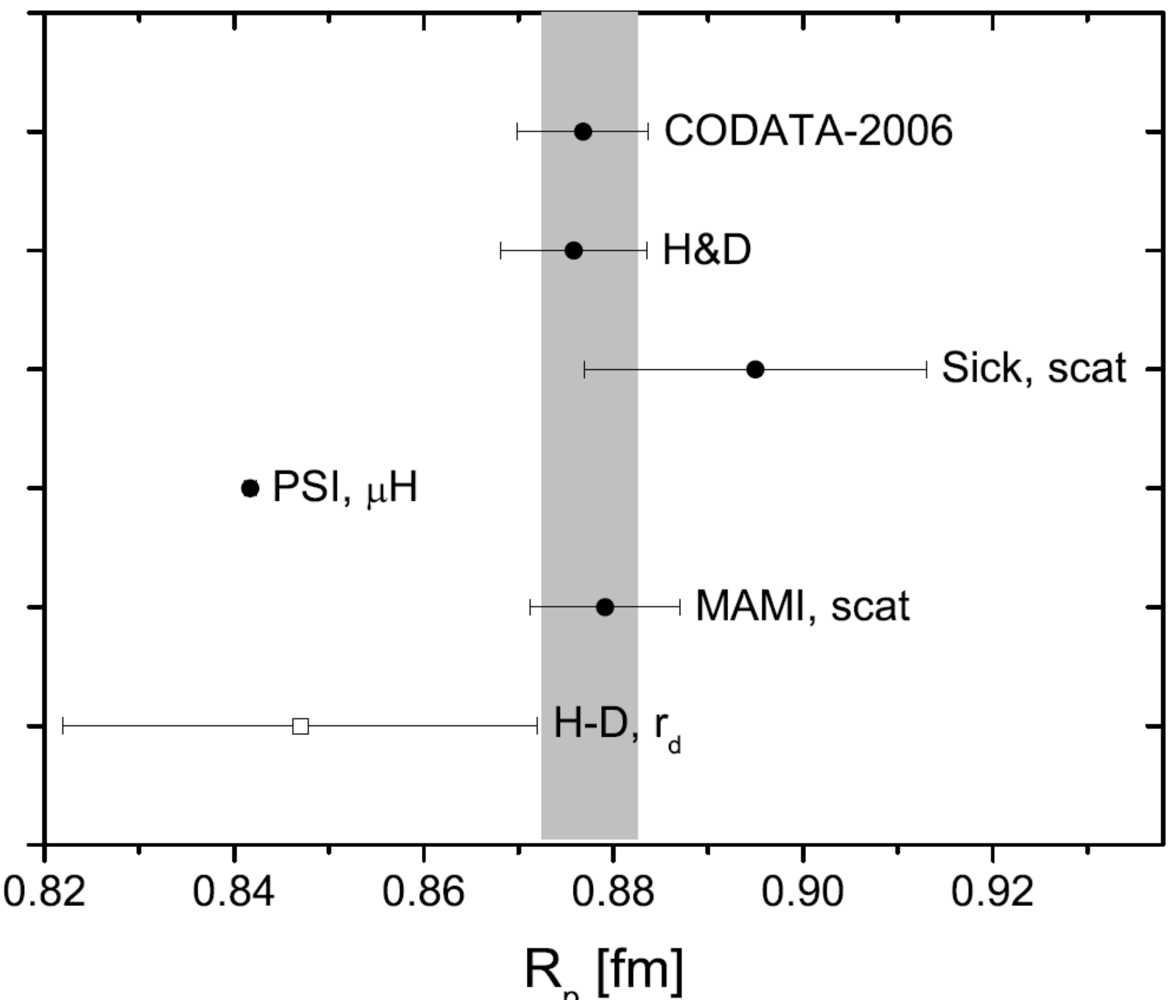
- *The Lamb shift in muonic hydrogen*

Spectroscopy produces a model-independent result, but involves a lot of theory and/or a bit of modeling.

- *Electron-proton scattering*

Studies of scattering need theory of radiative corrections, estimation of two-photon effects; the result is to depend on model applied to extrapolate to zero momentum transfer.

Different methods to determine the proton charge radius



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Brief story of hydrogenic energy levels

- First, there were the Bohr levels...
- *That as a rare success of numerology (Balmer series).*

Brief story of hydrogenic energy levels

- First, there were the Bohr levels.
- The energies were OK, the wave functions were not. Thus, nonrelativistic quantum mechanics appeared.
- *That was a pure non-relativistic theory.*
- *Which was not good after decades of enjoying the special relativity.*
- *Without a spin there were no chance for a correct relativistic atomic theory.*

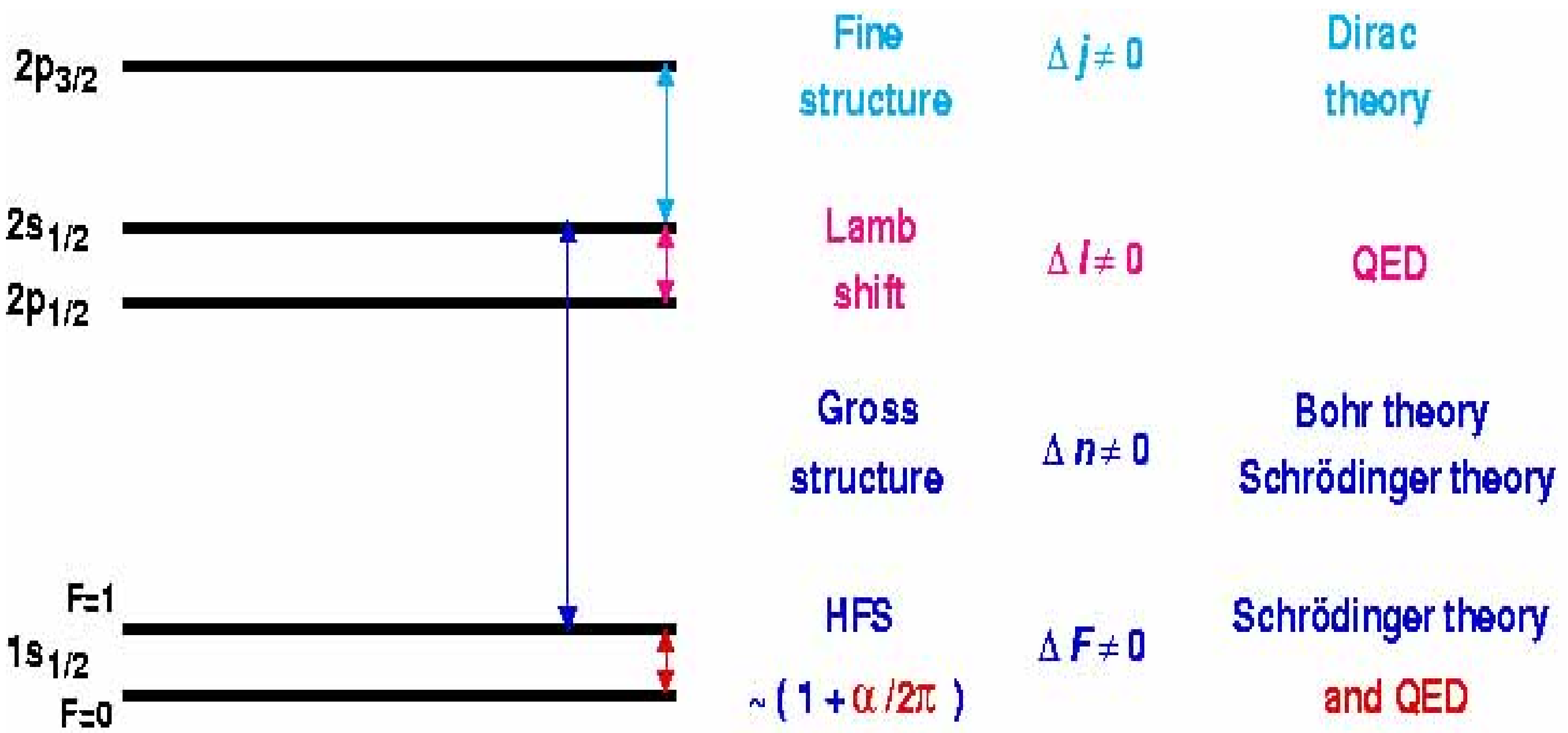
Brief story of hydrogenic energy levels

- First, there were the Bohr levels..
- Next, nonrelativistic quantum mechanics appeared.
- Later on, the Dirac theory came.
- *The Dirac theory predicted:*
  - *the fine structure*
  - *$g = 2$ (that was expected also for a proton)*
  - *positron*

Brief story of hydrogenic energy levels

- First, there were the Bohr levels..
- Next, nonrelativistic quantum mechanics appeared.
- Later on, the Dirac theory came.
- *The Dirac theory predicted:*
  - *the fine structure*
  - *$g = 2$.*
- *Departures from the Dirac theory and their explanations were the beginning of practical quantum electrodynamics.*

Energy levels in the hydrogen atom



Brief theory of hydrogenic energy levels

- The Schrödinger-theory energy levels are

  $E_n = -\frac{1}{2}\,\alpha^2 mc^2/n^2$

– no dependence on momentum (j, l).

Brief theory of hydrogenic energy levels: without QED

- The Schrödinger-theory energy levels are

$$E_n = -\tfrac{1}{2}\ \alpha^2 mc^2/n^2$$

– no dependence on momentum (j, l).

- The Dirac theory of the energy levels:

- The $2p_{1/2}$ and $2p_{3/2}$ are split (j=1/2 & 3/2) = fine structure;
- The $2p_{1/2}$ and $2s_{1/2}$ are degenerated (j=1/2; l=0 & 1).

Brief theory of hydrogenic energy levels: still without QED

- The nuclear spin:

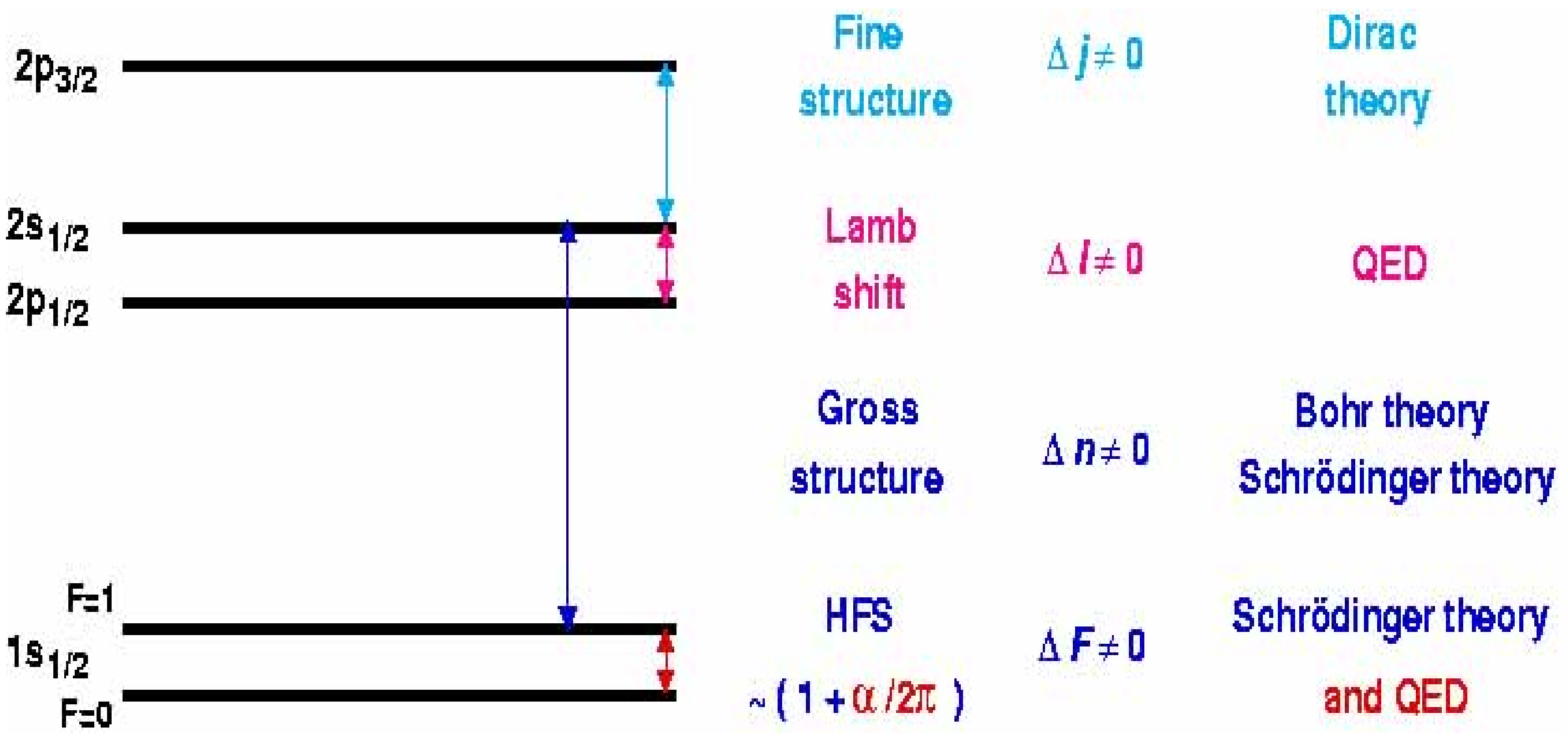
Hyperfine structure is due to splitting of levels with the same total angular momentum (electron’s + nucleus);

In particular, 1s level in hydrogen is split into two levels.

- Quantum mechanics + emission:

all states, but the ground one, are metastable, i.e. they decay via photon(s) emission.

Energy levels in the hydrogen atom



Brief theory of hydrogenic energy levels: now with QED

- Radiative width
- Self energy of an electron and Lamb shift
- Hyperfine structure and Anomalous magnetic moment of the electron
- Vacuum polarization
- Annihilation of electron and positron
- Recoil corrections

Brief theory of hydrogenic energy levels: now with QED

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- The leading channel is E1 decay. Most of levels (and 2p) go through this mode. The E1 decay width ~ $\alpha(Z\alpha)^4mc^2$.
- The 2s level is metastable decaying via two-photon 2E1 mode with width ~ $\alpha^2(Z\alpha)^6mc^2$.
- Complex energy with the imaginary part as decay width.
- Difference in width of $2s_{1/2}$ and $2p_{1/2}$ is a good reason to expect a difference in their energy in order $\alpha(Z\alpha)^4mc^2$.

Brief theory of hydrogenic energy levels: now with QED

- Radiative width
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- Recoil corrections

- A complex energy for decaying states is with its real part as energy and its imaginary part as decay width.
- The E1 decay width is an imaginary part of the electron self energy while its real part is responsible for the Lamb shift ~ $\alpha(Z\alpha)^4mc^2\log(Z\alpha)$ and a splitting of $2s_{1/2} - 2p_{1/2}$ is by about tenfold larger than the $2p_{1/2}$ width.
- Self energy dominates.

Brief theory of hydrogenic energy levels: now with QED

- Radiative width
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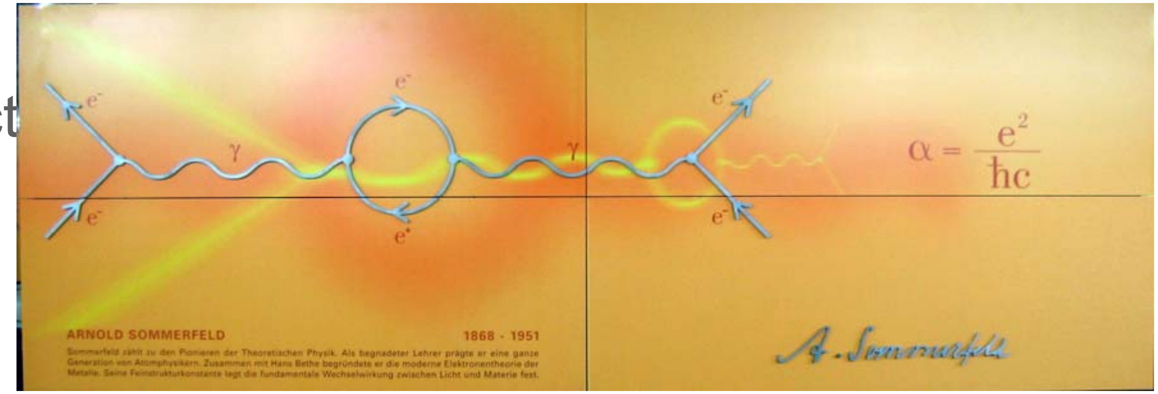
- The `electron magnetic moment anomaly' was first observed studying HFS.

Brief theory of hydrogenic energy levels: now with QED

- Radiative width
- Self energy of an electron and Lamb shift
- Hyperfine structure and Anomalous magnetic moment of the electron
- Vacuum polarization
- Annihilation of electron and positron
- Recoil correct

- dominates in muonic atoms:

$$\alpha(Z\alpha)^2 m_\mu c^2 \times F(Z\alpha m_\mu/m_e)$$



Three fundamental spectra: $n = 2$

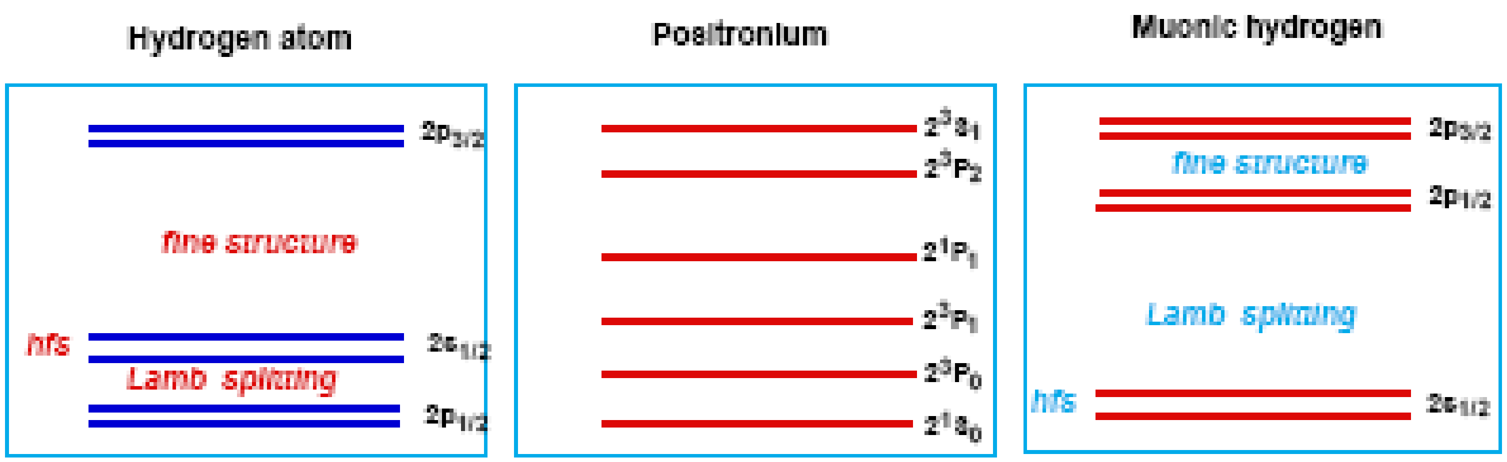
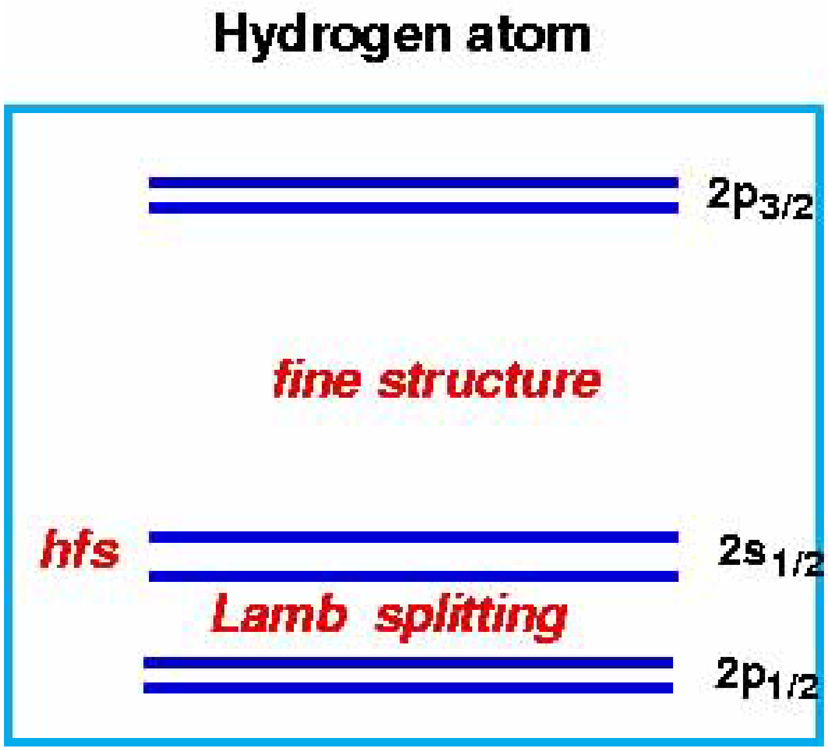


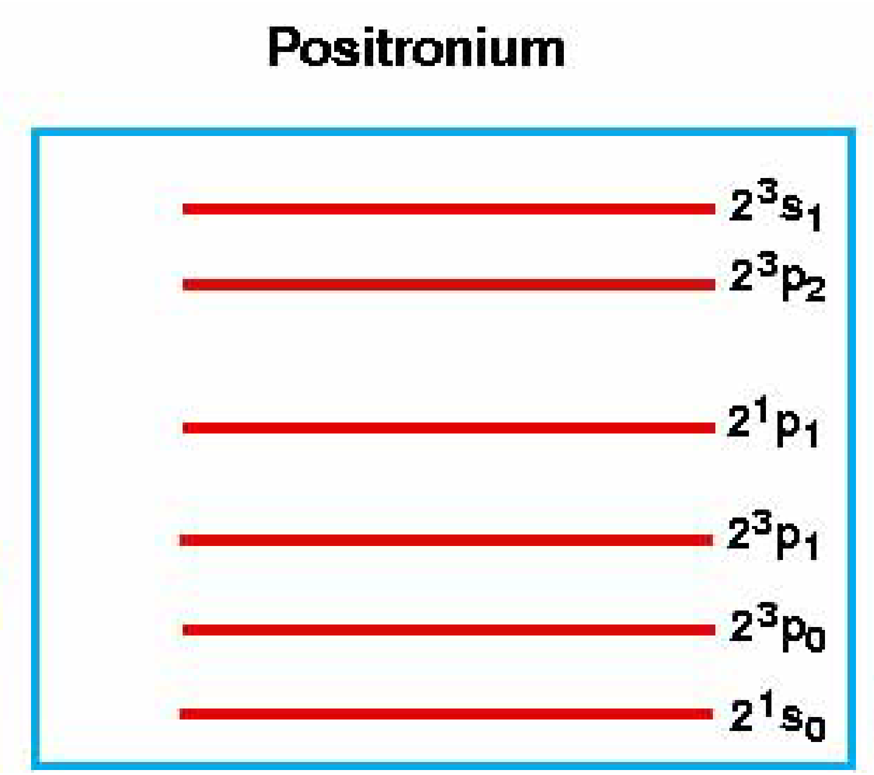
Fig. 6. Scheme of the lowest excited levels ($n = 2$) in different simple atoms (not to scale).

Three fundamental spectra: *n* = 2



- The dominant effect is the fine structure.
- The Lamb shift is about 10% of the fine structure.
- The 2p line width (not shown) is about 10% of the Lamb shift.
- The 2s hyperfine structure is about 15% of the Lamb shift.

Three fundamental spectra: $n = 2$

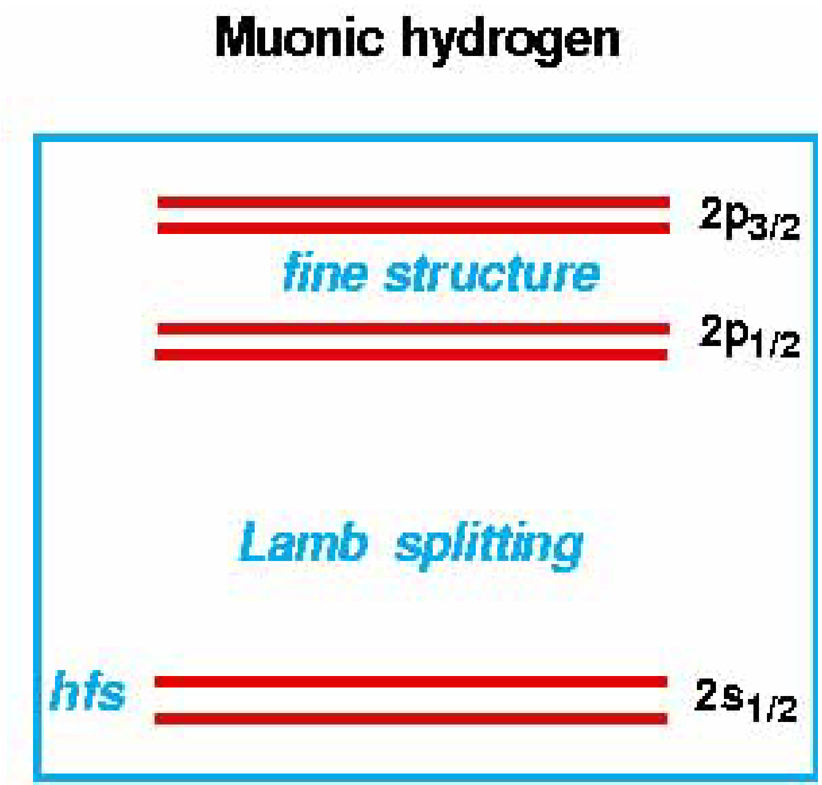


In posirtonium a number of effects are of the same order:

- fine structure;
- hyperfine structure;
- shift of 2^3S_1 state (orthopositronium) due to virtual annihilation.

There is no strong hierarchy.

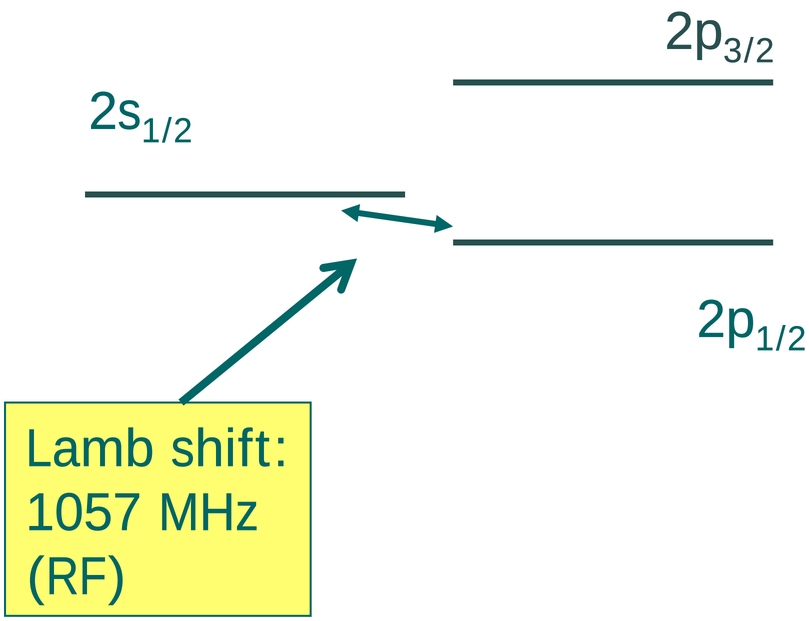
Three fundamental spectra: *n* = 2



- The Lamb shift originating from vacuum polarization effects dominates over fine structure (4% of the Lamb shift).
- The fine structure is larger than radiative line width.
- The HFS is larger than fine structure ~ 10% of the Lamb shift (because $m_\mu/m_p \sim 1/9$).

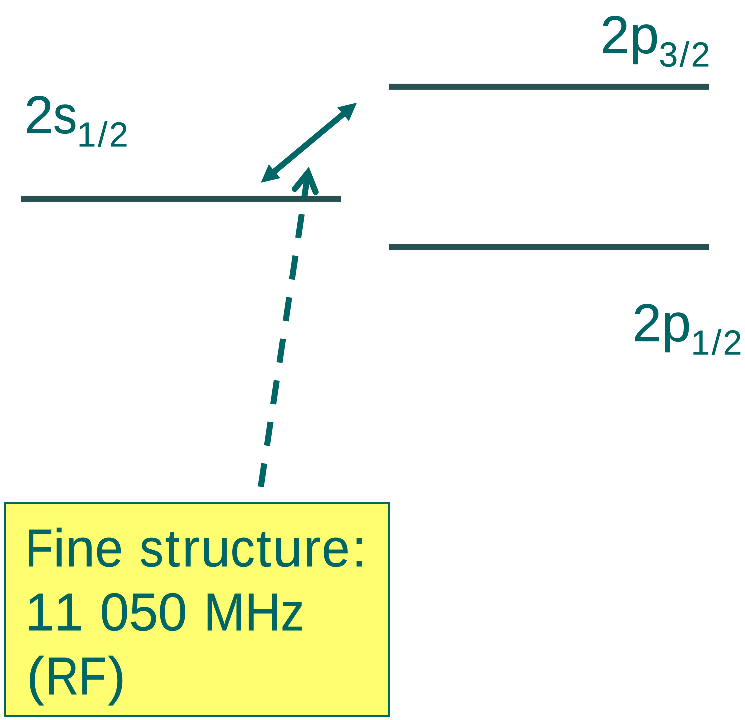
QED tests in microwave

- Lamb shift used to be measured either as a splitting between $2s_{1/2}$ and $2p_{1/2}$ (1057 MHz)



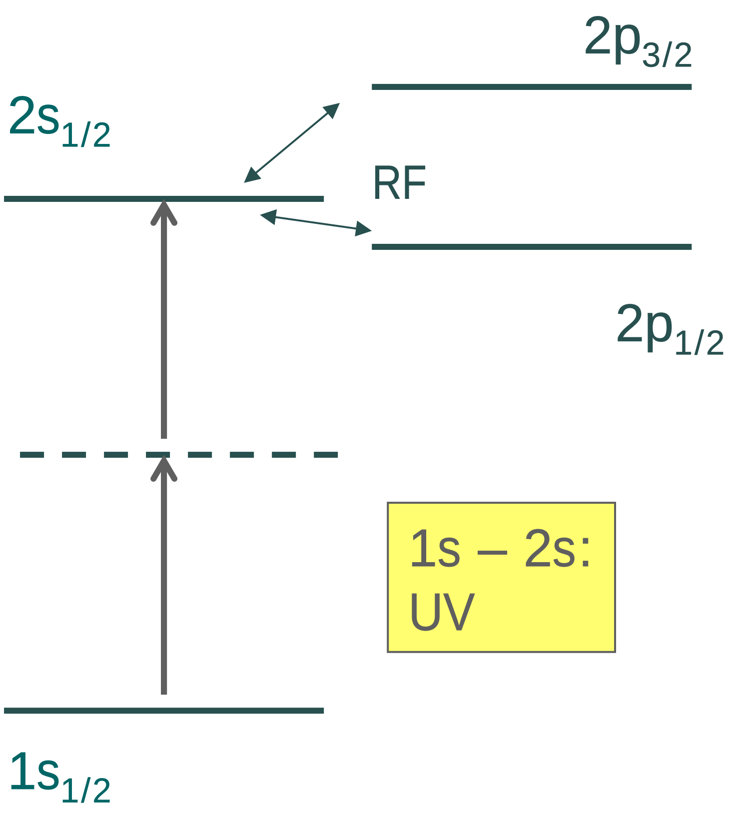
QED tests in microwave

- Lamb shift used to be measured either as a splitting between $2s_{1/2}$ and $2p_{1/2}$ (1057 MHz) or a big contribution into the fine splitting $2p_{3/2} - 2s_{1/2}$ 11 THz (fine structure).



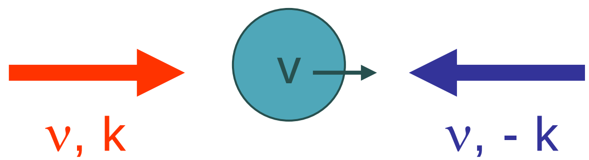
QED tests in microwave & optics

- Lamb shift used to be measured either as a splitting between $2s_{1/2}$ and $2p_{1/2}$ (1057 MHz) or a big contribution into the fine splitting $2p_{3/2}$ – $2s_{1/2}$ 11 THz (fine structure).
- *However, the best result for the Lamb shift has been obtained up to now from UV transitions (such as 1s – 2s).*



Two-photon Doppler-free spectroscopy of hydrogen atom

Two-photon spectroscopy



is free of linear Doppler effect.

That makes cooling relatively not too important problem.

All states but 2s are broad because of the E1 decay.

The widths decrease with increase of n.

However, higher levels are badly accessible.

Two-photon transitions double frequency and allow to go higher.

Spectroscopy of hydrogen (and deuterium)

Two-photon spectroscopy involves a number of levels strongly affected by QED.

In “old good time” we had to deal only with 2s Lamb shift.

Theory for p states is simple since their wave functions vanish at $r=0$.

Now we have more data and more unknown variables.

Spectroscopy of hydrogen (and deuterium)

- Two-photon spectroscopy involves a number of levels strongly affected by QED.
- In “old good time” we had to deal only with 2s Lamb shift.
- Theory for p states is simple si functions
- *Now we hav*

The idea is based on theoretical study of

$$\Delta(2) = L_{1s} - 2^3 \times L_{2s}$$

which we understand much better since any short distance effect vanishes for $\Delta(2)$.

The Lamb shift in the hydrogen atom

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D.I. Mendeleyev Russian Metrology Research Institute, 198005 St. Petersburg, Russia
(Submitted 6 April 1994)
Zh. Eksp. Teor. Fiz. **106**, 414–424 (August 1994)

A theoretical expression is derived for the difference $\Delta E_L(1s_{1/2}) - 8\Delta E_L(2s_{1/2})$ in Lamb shifts

variables to determine: the 1s Lamb shift L_{1s} & R_∞.

Z. Phys. D 39, 109–113 (1997)

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The Lamb shift of excited *S*-levels in hydrogen and deuterium atoms

Savely G. Karshenboim*

Spectroscopy of hydrogen (and deuterium)

Two-photon spectroscopy involves a number of levels strongly affected by QED.

In “old good time” we had to deal only with 2s Lamb shift.

Theory for p states is simple since their wave functions vanish at $r=0$.

Now we have more data and more unknown variables.

The idea is based on theoretical study of

$$\Delta(2) = L_{1s} - 2^3 \times L_{2s}$$

which we understand much better since any short distance effect vanishes for $\Delta(2)$.

Theory of p and d states is also simple.

That leaves only two variables to determine: the 1s Lamb shift L_{1s} & R_∞.

Spectroscopy of hydrogen (and deuterium)

TABLE V. Summary of measured transition frequencies ν considered in the present work for the determination of the Rydberg constant R_∞ (H is hydrogen and D is deuterium).

Authors	Laboratory[a]	Frequency interval(s)	Reported value ν/kHz	Rel. stand. uncert. u_r
Niering *et al.* (2000)	MPQ	$\nu_H(1S_{1/2}-2S_{1/2})$	2 466 061 413 187.103(46)	1.9×10^{-14}
Weitz *et al.* (1995)	MPQ	$\nu_H(2S_{1/2}-4S_{1/2})-\frac{1}{4}\nu_H(1S_{1/2}-2S_{1/2})$	4 797 338(10)	2.1×10^{-6}
		$\nu_H(2S_{1/2}-4D_{5/2})-\frac{1}{4}\nu_H(1S_{1/2}-2S_{1/2})$	6 490 144(24)	3.7×10^{-6}
		$\nu_D(2S_{1/2}-4S_{1/2})-\frac{1}{4}\nu_D(1S_{1/2}-2S_{1/2})$	4 801 693(20)	4.2×10^{-6}
		$\nu_D(2S_{1/2}-4D_{5/2})-\frac{1}{4}\nu_D(1S_{1/2}-2S_{1/2})$	6 494 841(41)	6.3×10^{-6}
Huber *et al.* (1998)	MPQ	$\nu_D(1S_{1/2}-2S_{1/2})-\nu_H(1S_{1/2}-2S_{1/2})$	670 994 334.64(15)	2.2×10^{-10}
de Beauvoir *et al.* (1997)	LKB/SYRTE	$\nu_H(2S_{1/2}-8S_{1/2})$	770 649 350 012.0(8.6)	1.1×10^{-11}
		$\nu_H(2S_{1/2}-8D_{3/2})$	770 649 504 450.0(8.3)	1.1×10^{-11}
		$\nu_H(2S_{1/2}-8D_{5/2})$	770 649 561 584.2(6.4)	8.3×10^{-12}
		$\nu_D(2S_{1/2}-8S_{1/2})$	770 859 041 245.7(6.9)	8.9×10^{-12}
		$\nu_D(2S_{1/2}-8D_{3/2})$	770 859 195 701.8(6.3)	8.2×10^{-12}
		$\nu_D(2S_{1/2}-8D_{5/2})$	770 859 252 849.5(5.9)	7.7×10^{-12}
Schwob *et al.* (1999)	LKB/SYRTE	$\nu_H(2S_{1/2}-12D_{3/2})$	799 191 710 472.7(9.4)	1.2×10^{-11}
		$\nu_H(2S_{1/2}-12D_{5/2})$	799 191 727 403.7(7.0)	8.7×10^{-12}
		$\nu_D(2S_{1/2}-12D_{3/2})$	799 409 168 038.0(8.6)	1.1×10^{-11}
		$\nu_D(2S_{1/2}-12D_{5/2})$	799 409 184 966.8(6.8)	8.5×10^{-12}
Bourzeix *et al.* (1996)	LKB	$\nu_H(2S_{1/2}-6S_{1/2})-\frac{1}{4}\nu_H(1S_{1/2}-3S_{1/2})$	4 197 604(21)	4.9×10^{-6}
		$\nu_H(2S_{1/2}-6D_{5/2})-\frac{1}{4}\nu_H(1S_{1/2}-3S_{1/2})$	4 699 099(10)	2.2×10^{-6}
Berkeland *et al.* (1995)	Yale	$\nu_H(2S_{1/2}-4P_{1/2})-\frac{1}{4}\nu_H(1S_{1/2}-2S_{1/2})$	4 664 269(15)	3.2×10^{-6}
		$\nu_H(2S_{1/2}-4P_{3/2})-\frac{1}{4}\nu_H(1S_{1/2}-2S_{1/2})$	6 035 373(10)	1.7×10^{-6}
Hagley and Pipkin (1994)	Harvard	$\nu_H(2S_{1/2}-2P_{3/2})$	9 911 200(12)	1.2×10^{-6}
Lundeen and Pipkin (1986)	Harvard	$\nu_H(2P_{1/2}-2S_{1/2})$	1 057 845.0(9.0)	8.5×10^{-6}
Newton *et al.* (1979)	U. Sussex	$\nu_H(2P_{1/2}-2S_{1/2})$	1 057 862(20)	1.9×10^{-5}

[a]MPQ: Max-Planck-Institut für Quantenoptik, Garching. LKB: Laboratoire Kastler-Brossel, Paris. SYRTE: Systèmes de référence Temps Espace, Paris, formerly Laboratoire Primaire du Temps et des Fréquences (LPTF).

Spectroscopy of hydrogen (and deuterium)

TABLE XIX. Observational equations that express the input data related to R_∞ in Table XI as functions of the adjusted constants in Table XVIII. The numbers in the first column correspond to the numbers in the first column of Table XI. The expressions for the energy levels of hydrogenic atoms are discussed in Appendix A. As pointed out in Sec. A.12 of that Appendix, $E_X(nL_j)/h$ is in fact proportional to cR_∞ and independent of h, hence h is not an adjusted constant in these equations. [The notation for hydrogenic energy levels $E_X(nL_j)$ and for additive corrections $\delta_X(nL_j)$ in this table have the same meaning as the notations $E^X_{nL_j}$ and $\delta^X_{nL_j}$ in Appendix A, Sec. A.12.] See Sec. IV.B for an explanation of the symbol $\doteq$.

Type of input datum	Observational equation
$A1-A6$ $A13, A14$	$\nu_H(n_1L_{1j_1}-n_2L_{2j_2}) \doteq [E_H(n_2L_{2j_2};R_\infty,\alpha,A_r(e),A_r(p),R_p,\delta_H(n_2L_{2j_2}))$ $-E_H(n_1L_{1j_1};R_\infty,\alpha,A_r(e),A_r(p),R_p,\delta_H(n_1L_{1j_1}))]/h$
$A7-A12$	$\nu_H(n_1L_{1j_1}-n_2L_{2j_2})-\frac{1}{4}\nu_H(n_3L_{3j_3}-n_4L_{4j_4}) \doteq \{E_H(n_2L_{2j_2};R_\infty,\alpha,A_r(e),A_r(p),R_p,\delta_H(n_2L_{2j_2}))$ $-E_H(n_1L_{1j_1};R_\infty,\alpha,A_r(e),A_r(p),R_p,\delta_H(n_1L_{1j_1}))$ $-\frac{1}{4}[E_H(n_4L_{4j_4};R_\infty,\alpha,A_r(e),A_r(p),R_p,\delta_H(n_4L_{4j_4}))$ $-E_H(n_3L_{3j_3};R_\infty,\alpha,A_r(e),A_r(p),R_p,\delta_H(n_3L_{3j_3}))]\}/h$
$A15$	$R_p \doteq R_p$
$A16-A20$	$\nu_D(n_1L_{1j_1}-n_2L_{2j_2}) \doteq [E_D(n_2L_{2j_2};R_\infty,\alpha,A_r(e),A_r(d),R_d,\delta_D(n_2L_{2j_2}))$ $-E_D(n_1L_{1j_1};R_\infty,\alpha,A_r(e),A_r(d),R_d,\delta_D(n_1L_{1j_1}))]/h$
$A21-A22$	$\nu_D(n_1L_{1j_1}-n_2L_{2j_2})-\frac{1}{4}\nu_D(n_3L_{3j_3}-n_4L_{4j_4}) \doteq \{E_D(n_2L_{2j_2};R_\infty,\alpha,A_r(e),A_r(d),R_d,\delta_D(n_2L_{2j_2}))$ $-E_D(n_1L_{1j_1};R_\infty,\alpha,A_r(e),A_r(d),R_d,\delta_D(n_1L_{1j_1}))$ $-\frac{1}{4}[E_D(n_4L_{4j_4};R_\infty,\alpha,A_r(e),A_r(d),R_d,\delta_D(n_4L_{4j_4}))$ $-E_D(n_3L_{3j_3};R_\infty,\alpha,A_r(e),A_r(d),R_d,\delta_D(n_3L_{3j_3}))]\}/h$
$A23$	$R_d \doteq R_d$
$A24$	$\nu_D(1S_{1/2}-2S_{1/2})-\nu_H(1S_{1/2}-2S_{1/2}) \doteq \{E_D(2S_{1/2};R_\infty,\alpha,A_r(e),A_r(d),R_d,\delta_D(2S_{1/2}))$ $-E_D(1S_{1/2};R_\infty,\alpha,A_r(e),A_r(d),R_d,\delta_D(1S_{1/2}))$ $-[E_H(2S_{1/2};R_\infty,\alpha,A_r(e),A_r(p),R_p,\delta_H(2S_{1/2}))$ $-E_H(1S_{1/2};R_\infty,\alpha,A_r(e),A_r(p),R_p,\delta_H(1S_{1/2}))]\}/h$
$A25-A40$	$\delta_H(nL_j) \doteq \delta_H(nL_j)$
$A41-A49$	$\delta_D(nL_j) \doteq \delta_D(nL_j)$

The Rydberg constant R_∞

The Rydberg constant is important for a number of reasons. It is a basic atomic constant.

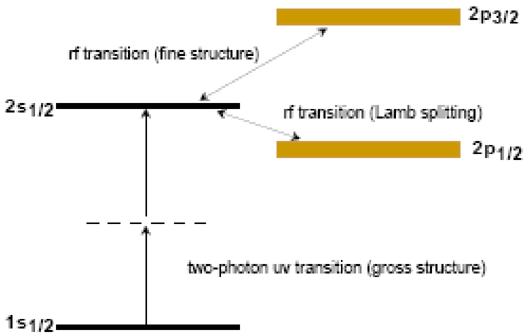
Meantime that is the most accurately measured fundamental constant.

The improvement of accuracy is nearly 4 orders in 30 years.

There has been no real progress since that.

1973	10 973 731.77(83) m^{-1} [7.5×10^{-8}]
1986	10 973 731.534(13) m^{-1} [1.2×10^{-9}]
1998	10 973 731.568 549(83) m^{-1} [7.6×10^{-12}]
2002	10 973 731.568 525(73) m^{-1} [6.6×10^{-12}]
2006	10 973 731.568 527(73) m^{-1} [6.6×10^{-12}]

Spectroscopy of hydrogen (and deuterium)



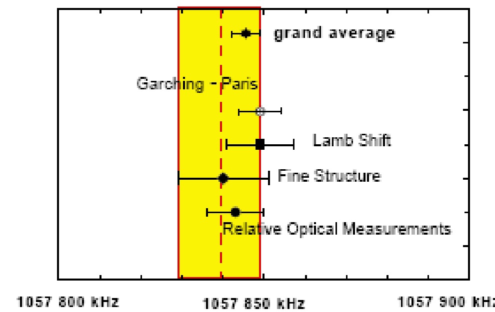


Fig. 9. Measurement of the Lamb shift in the hydrogen atom. The most accurate experimental result comes from a comparison of the $1s-2s$ interval measured at MPQ (Garching) [38] and the $2s-ns/d$ intervals at LKB (Paris) [39], where $n=8,10,12$ (see also [33] for detail). Three more results are shown for the average values extracted from direct *Lamb shift* measurements, measurements of the *fine structure* and a comparison of *two optical* transitions within a *single* experiment (i.e., a relative optical measurement). The filled part is for theory. Theory and evaluation of the experimental data are presented according to Ref. [36].

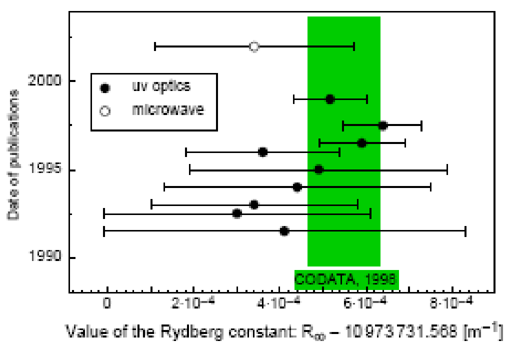


Fig. 8. Progress in determination of the Rydberg constant by means of two-photon Doppler-free spectroscopy of hydrogen and deuterium. The label *CODATA* stands for the recommended value of the Rydberg constant $R_\infty(1998)$ [21] from Eq. (12). The most recent original value is a preliminary result from MIT obtained by microwave means [37].

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Precision physics of simple atoms: QED tests, nuclear structure and fundamental constants

Savely G. Karshenboim[a, b, *]

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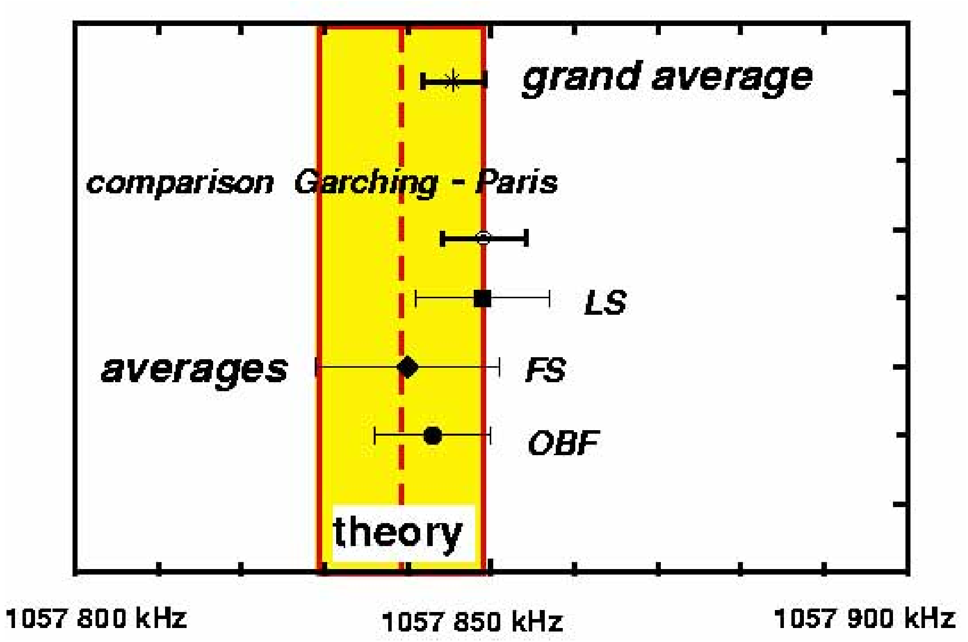
[b] Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany

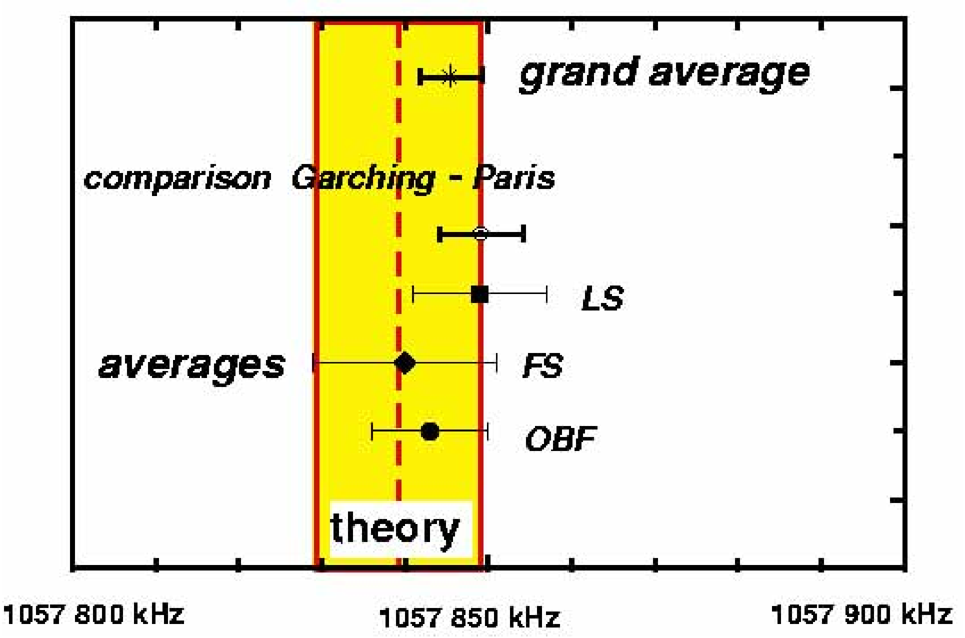
Лэмбовский сдвиг ($2s_{1/2}$–$2p_{1/2}$) в атоме водорода

theory vs. experiment



Lamb shift ($2s_{1/2} - 2p_{1/2}$) in the hydrogen atom

theory vs. experiment



Lamb shift ($2s_{1/2} - 2p_{1/2}$) in the hydrogen atom

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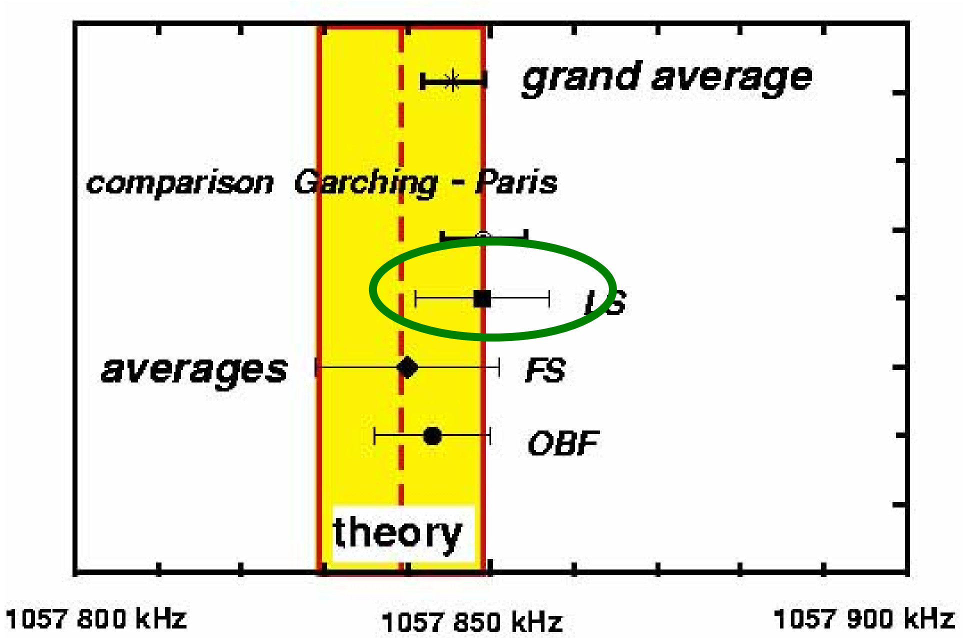
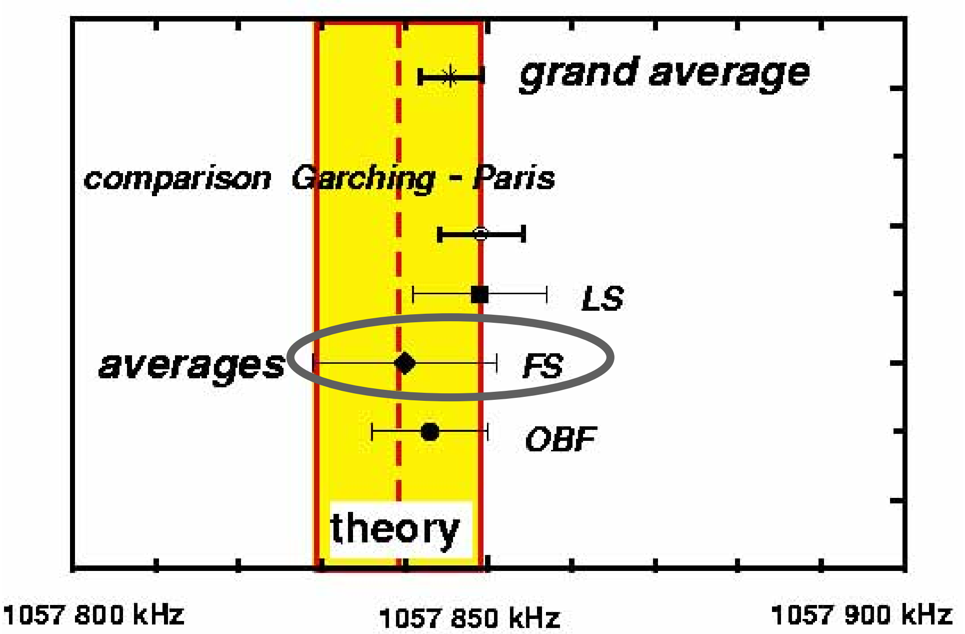


- LS: direct measurements of the $2s_{1/2} - 2p_{1/2}$ splitting.
  - Sokolov-&-Yakovlev's result (2 ppm) is excluded because of possible systematic effects.
  - The best included result is from Lundeen and Pipkin (~10 ppm).

Lamb shift ($2s_{1/2} - 2p_{1/2}$) in the hydrogen atom

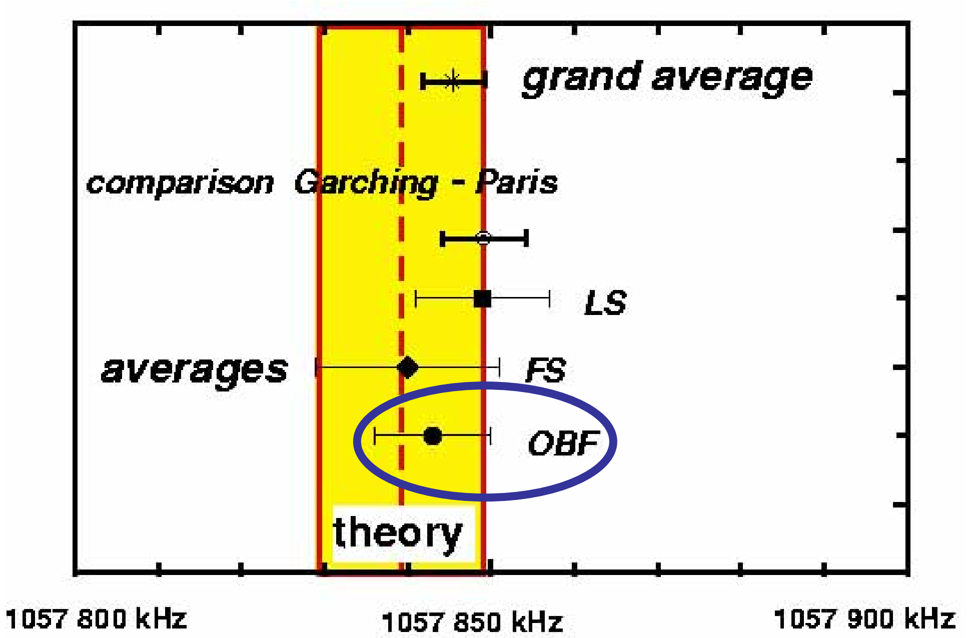
theory vs. experiment



- FS: measurement of the $2p_{3/2} - 2s_{1/2}$ splitting. The Lamb shift is about of 10% of this effects.
  - The best result (Hagley & Pipkin) leads to uncertainty of ~ 10 ppm for the Lamb shift.

Lamb shift ($2s_{1/2} - 2p_{1/2}$) in the hydrogen atom

theory vs. experiment



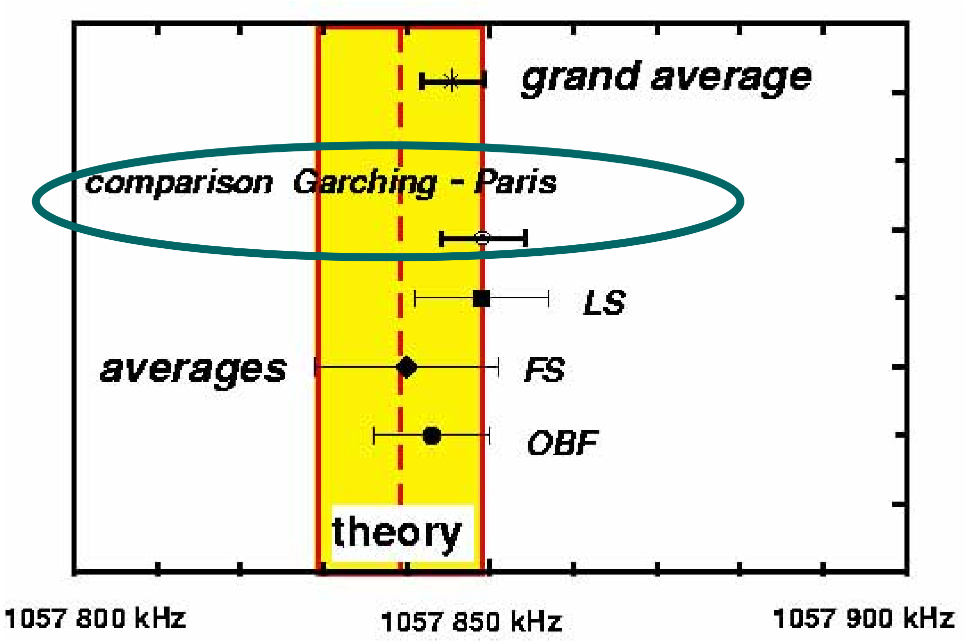
- OBF: the first generation of optical measurements. They were relative measurements with two frequencies different by an almost integer factor.
  - Yale: 1s-2s and 2s-4p
  - Garching: 1s-2s and 2s-4s
  - Paris: 1s-3s and 2s-6s

The result was reached through measurement of a `beat frequency' such as

f(1s-2s)-4×f(2s-4s).

Lamb shift ($2s_{1/2} - 2p_{1/2}$) in the hydrogen atom

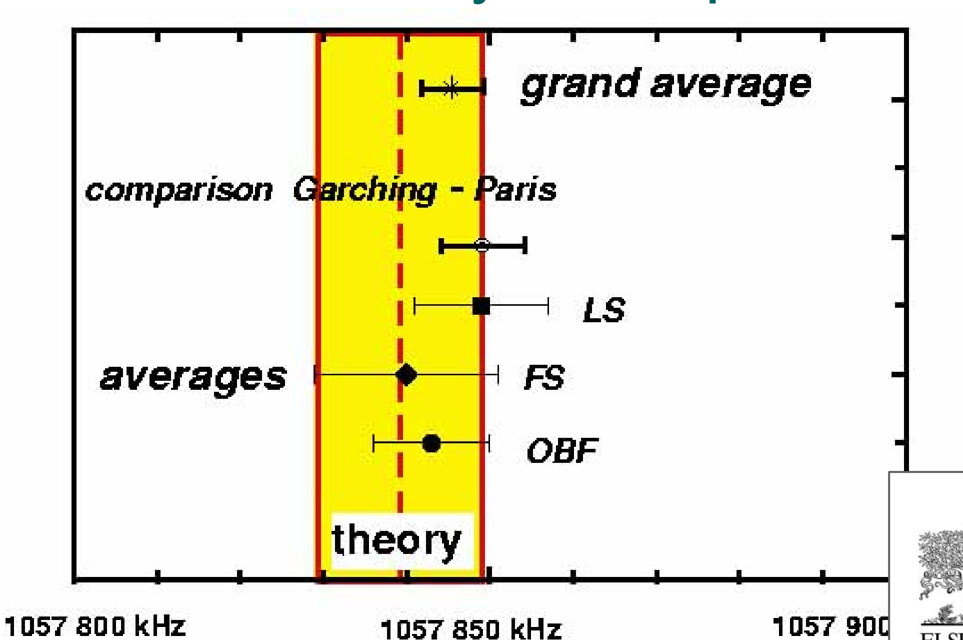
theory vs. experiment



- The most accurate result is a comparison of independent absolute measurements:
  - Garching: 1s-2s
  - Paris: 2s → n=8-12

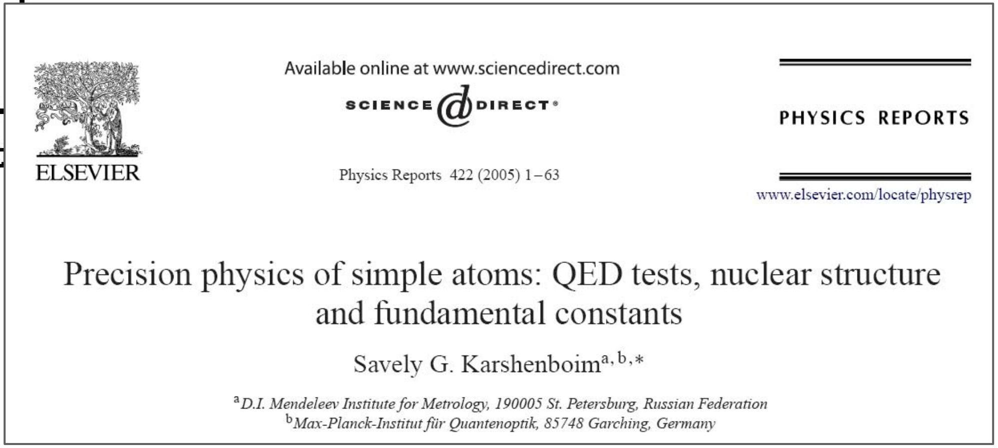
Lamb shift ($2s_{1/2} – 2p_{1/2}$) in the hydrogen atom

theory vs. experiment



Uncertainties:

- Experiment: 2 ppm
- QED: 2 ppm
- Proton size 10 ppm



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[b] Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany

Lamb shift ($2s_{1/2} – 2p_{1/2}$) in the hydrogen atom

Uncertainties:

- Experiment: 2 ppm
- QED: 2 ppm
- Proton size 10 ppm

There are data on a number of transitions, but most of them are correlated.

Lamb shift ($2s_{1/2} – 2p_{1/2}$) in the hydrogen atom

Uncertainties:

- Experiment: 2 ppm
- QED: 2 ppm
- Proton size 10 ppm

At present, it used to be believed that the theoretical uncertainty is well below 1 ppm.

However, we are in a kind of g_e-2 situation: the most important two-loop corrections have not been checked independently.

Lamb shift ($2s_{1/2} - 2p_{1/2}$) in the hydrogen atom

Uncertainties:

- Experiment: 2 ppm
- QED: 2 ppm
- Proton size 10 ppm

Accuracy of the proton-radius contribution suffers from estimation of uncertainty of scattering data evaluation and of proper estimation of higher-order QED and two-photon effects.

Lamb shift ($2s_{1/2} - 2p_{1/2}$) in the hydrogen atom

Uncertainties:

- Experiment: 2 ppm
- QED: 2 ppm
- Proton size 10 ppm

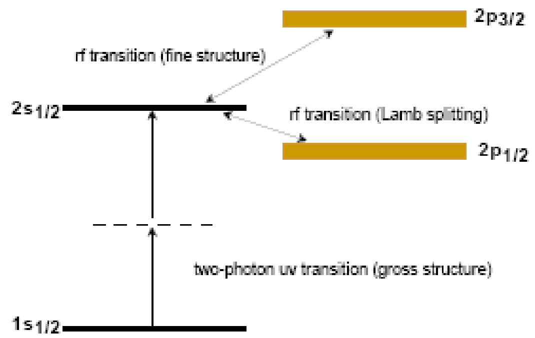
The scattering data claimed a better accuracy (3 ppm), however, we should not completely trust them.

It is likely that we need to have proton charge radius obtained in some other way (e.g. via the Lamb shift in muonic hydrogen – in the way at PSI).

The Lamb shift in muonic hydrogen

- Used to believe: *since a muon is heavier than an electron, muonic atoms are more sensitive to the nuclear structure.*
- Not quite true. What is important: scaling of various contributions with m.

- Scaling of contributions
  - nuclear finite size effects: $\sim m^3$;
  - standard Lamb-shift QED and its uncertainties: $\sim m$;
  - width of the 2p state: $\sim m$;
  - nuclear finite size effects for HFS: $\sim m^3$



The Lamb shift in muonic hydrogen: experiment

The size of the proton

Randolf Pohl[1], Aldo Antognini[1], François Nez[2], Fernando D. Amaro[3], François Biraben[2], João M. R. Cardoso[3], Daniel S. Covita[3,4], Andreas Dax[5], Satish Dhawan[5], Luis M. P. Fernandes[3], Adolf Giesen[6]†, Thomas Graf[6], Theodor W. Hänsch[1], Paul Indelicato[2], Lucile Julien[2], Cheng-Yang Kao[7], Paul Knowles[8], Eric-Olivier Le Bigot[2], Yi-Wei Liu[7], José A. M. Lopes[3], Livia Ludhova[8], Cristina M. B. Monteiro[3], Françoise Mulhauser[8]†, Tobias Nebel[1], Paul Rabinowitz[9], Joaquim M. F. dos Santos[3], Lukas A. Schaller[8], Karsten Schuhmann[10], Catherine Schwob[2], David Taqqu[11], João F. C. A. Veloso[4] & Franz Kottmann[12]

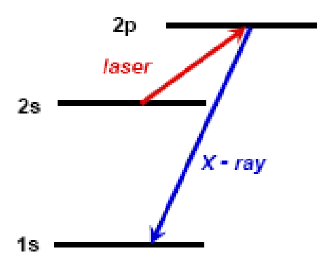


Fig. 16. Level scheme of the PSI experiment on the Lamb shift in a muonic hydrogen [88] (not to scale). The hyperfine structure is not shown.

The Lamb shift in muonic hydrogen: experiment

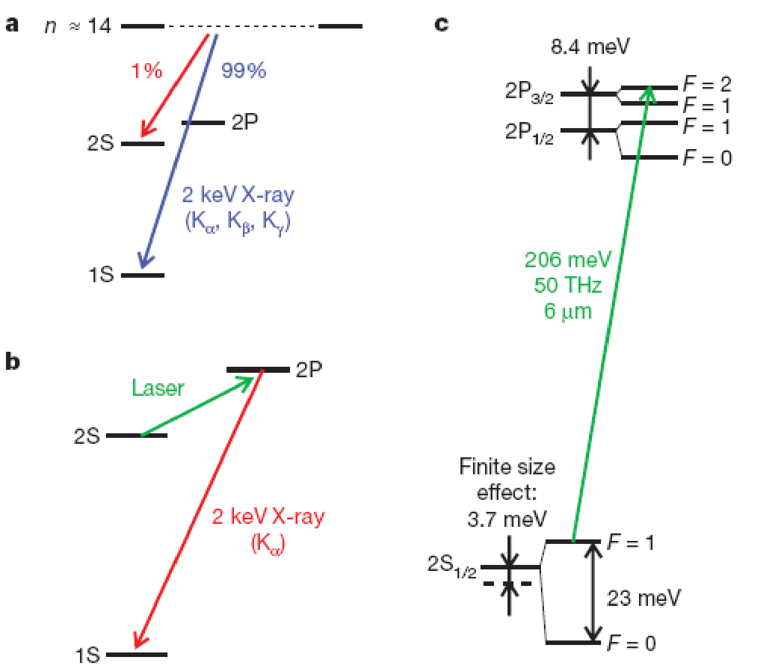


Figure 1 | Energy levels, cascade and experimental principle in muonic hydrogen. a, About 99% of the muons proceed directly to the 1S ground state during the muonic cascade, emitting 'prompt' K-series X-rays (blue). 1% remain in the metastable 2S state (red). **b**, The μp(2S) atoms are illuminated by a laser pulse (green) at 'delayed' times. If the laser is on resonance, delayed K_α X-rays are observed (red). **c**, Vacuum polarization dominates the Lamb shift in μp. The proton's finite size effect on the 2S state is large. The green arrow indicates the observed laser transition at $\lambda = 6$ μm.

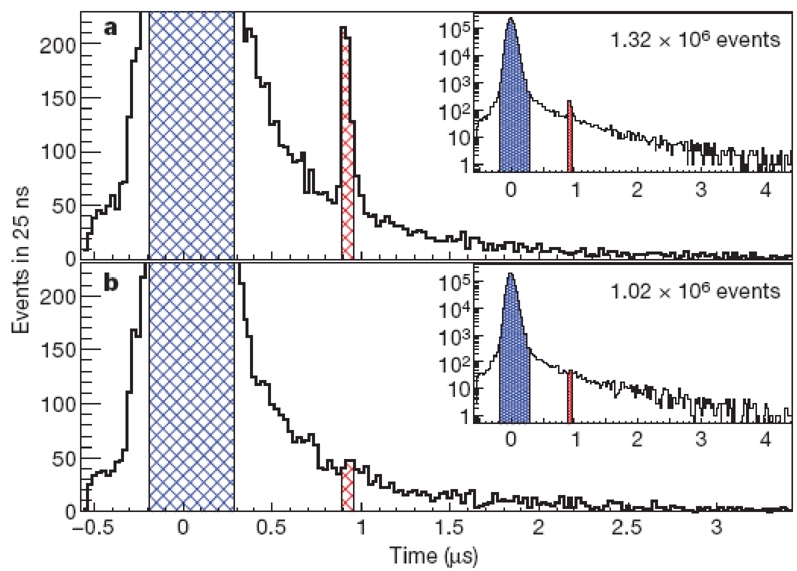


Figure 4 | Summed X-ray time spectra. Spectra were recorded on resonance (**a**) and off resonance (**b**). The laser light illuminates the muonic atoms in the laser time window $t \in [0.887, 0.962]$ μs indicated in red. The 'prompt' X-rays are marked in blue (see text and Fig. 1). Inset, plots showing complete data; total number of events are shown.

The Lamb shift in muonic hydrogen: experiment

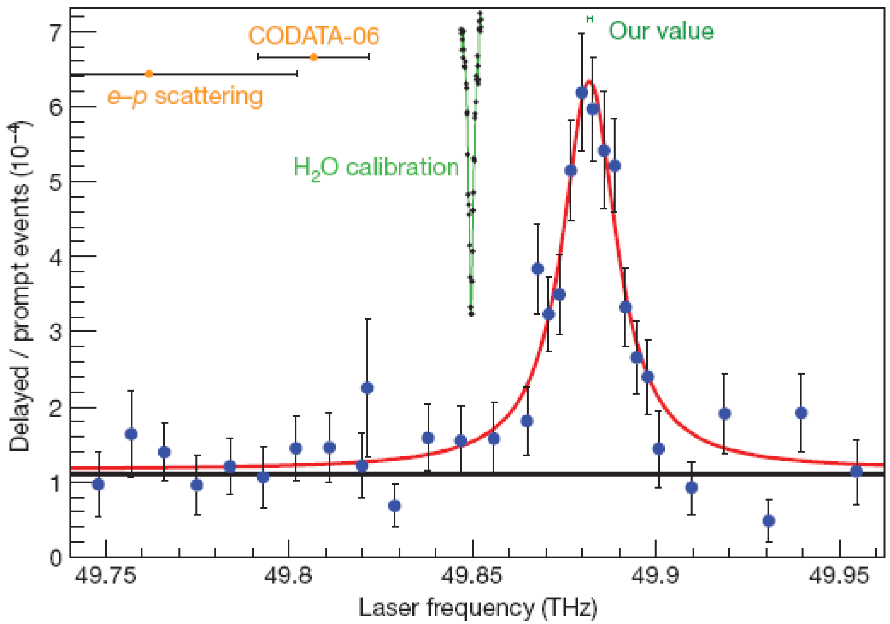


Figure 5 | Resonance. Filled blue circles, number of events in the laser time window normalized to the number of 'prompt' events as a function of the laser frequency. The fit (red) is a Lorentzian on top of a flat background, and gives a χ^2/d.f. of 28.1/28. The predictions for the line position using the proton radius from CODATA[3] or electron scattering[1,2] are indicated (yellow data points, top left). Our result is also shown ('our value'). All error bars are the ±1 s.d. regions. One of the calibration measurements using water absorption is also shown (black filled circles, green line).

The Lamb shift in muonic hydrogen: theory

#	Contribution	Ref.	Our selection Value	Our selection Unc.	Pachucki[1–3] Value	Pachucki[1–3] Unc.	Borie[5] Value	Borie[5] Unc.
1	NR One loop electron VP	1, 2			205.0074			
2	Relativistic correction (corrected)	1–3, 5			0.0169			
3	Relativistic one loop VP	5	205.0282				205.0282	
4	NR two-loop electron VP	5, 14	1.5081		1.5079		1.5081	
5	Polarization insertion in two Coulomb lines	1, 2, 5	0.1509		0.1509		0.1510	
6	NR three-loop electron VP	11	0.00529					
7	Polarisation insertion in two and three Coulomb lines (corrected)	11, 12	0.00223					
8	Three-loop VP (total, uncorrected)				0.0076		0.00761	
9	Wichmann-Kroll	5, 15, 16	−0.00103				−0.00103	
10	Light by light electron loop contribution (Virtual Delbrück scattering)	6	0.00135	0.00135			0.00135	0.00015
11	Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$	1, 2	−0.00500	0.0010	−0.006	0.001	−0.005	
12	Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$	17–19	−0.00150					
13	Mixed electron and muon loops	20	0.00007				0.00007	
14	Hadronic polarization $\alpha(Z\alpha)^4 m_r$	21–23	0.01077	0.00038	0.0113	0.0003	0.011	0.002
15	Hadronic polarization $\alpha(Z\alpha)^5 m_r$	22, 23	0.000047					
16	Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m_r$	22, 23	−0.000015					
17	Recoil contribution	24	0.05750		0.0575		0.0575	
18	Recoil finite size	5	0.01300	0.001			0.013	0.001
19	Recoil correction to VP	5	−0.00410				−0.0041	
20	Radiative corrections of order $\alpha^n(Z\alpha)^k m_r$	2, 7	−0.66770		−0.6677		−0.66788	
21	Muon Lamb shift 4th order	5	−0.00169				−0.00169	
22	Recoil corrections of order $\alpha(Z\alpha)^5 \frac{m}{M} m_r$	2, 5–7	−0.04497		−0.045		−0.04497	
23	Recoil of order α^6	2	0.00030		0.0003			
24	Radiative recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m_r$	1, 2, 7	−0.00960		−0.0099		−0.0096	
25	Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability contribution)	2, 5, 22, 25	0.015	0.004	0.012	0.002	0.015	0.004
26	Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	0.00019					
27	Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	−0.00001					
	Sum		206.0573	0.0045	206.0432	0.0023	206.05856	0.0046

Table 1: All known radius-**independent** contributions to the Lamb shift in μp from different authors, and the one we selected. We follow the nomenclature of Eides *et al.*[7] Table 7.1. Item # 8 in Refs.[2,5] is the sum of items #6 and #7, without the recent correction from Ref.[12]. The error of #10 has been increased to 100% to account for a remark in Ref.[7]. Values are in meV and the uncertainties have been added in quadrature.

The Lamb shift in muonic hydrogen: theory

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25	Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability contribution)	2,5,22,25	0.015	0.004	0.012	0.002	0.015	0.004
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- Numerous errors, underestimated uncertainties and missed contributions …

The Lamb shift in muonic hydrogen: theory

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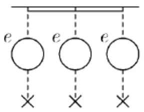

FIG. 5. The $\alpha^5 m$ correction to the Lamb shift in muonic hydrogen: the only third-order contribution of nonrelativistic perturbation theory [see (3)].

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Second-order corrections to the wave function at the origin in muonic hydrogen and pionium

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D. I. Mendeleev Institute for Metrology (VNIIM), St. Petersburg 198005, Russia

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D. I. Mendeleev Institute for Metrology (VNIIM), St. Petersburg 198005, Russia and Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany

The Lamb shift in muonic hydrogen: theory

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17	Recoil contribution	24	0.05750		0.0575		0.0575	
18	Recoil finite size	5	0.01300	0.001			0.013	0.001
19	Recoil correction to VP	5	−0.00410				−0.0041	
20	Radiative corrections of order $\alpha^n(Z\alpha)^k m_r$	2,7	−0.66770		−0.6677		−0.66788	
21	Muon Lamb shift 4th order	5	−0.00169				−0.00169	
22	Recoil corrections of order $\alpha(Z\alpha)^5 \frac{m}{M} m_r$	2,5–7	−0.04497		−0.045		−0.04497	
23	Recoil of order α^6	2	0.00030		0.0003			
24	Radiative recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m_r$	1,2,7	−0.00960		−0.0099		−0.0096	
25	Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability contribution)	2,5,22,25	0.015	0.004	0.012	0.002	0.015	0.004
26	Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	0.00019					
27	Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	−0.00001					
	Sum		206.0573	0.0045	206.0432	0.0023	206.05856	0.0046

Table 1: All known radius-**independent** contributions to the Lamb shift in μp from different authors, and the one we selected. We follow the nomenclature of Eides *et al.*[7] Table 7.1. Item # 8 in Refs.[2,5] is the sum of items #6 and #7, without the recent correction from Ref.[12]. The error of #10 has been increased to 100% to account for a remark in Ref.[7]. Values are in meV and the uncertainties have been added in quadrature.

- Numerous errors, underestimated uncertainties and missed contributions …

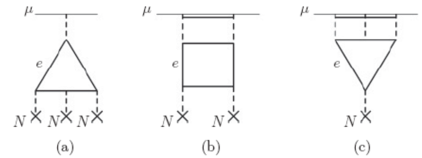

FIG. 1. Characteristic diagrams for three basic contributions of LbL scattering effects to the Lamb shift in muonic hydrogen of order $\alpha^5 m_\mu$. Here, N stands for a nucleus, which may be a proton, a deuteron, etc. The horizontal double line is for the muon propagator in the Coulomb field.

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Contribution of Light-by-Light Scattering to Energy Levels of Light Muonic Atoms¶

S. G. Karshenboim[a,b], E. Yu. Korzinin[a], V. G. Ivanov[a,c], and V. A. Shelyuto[a]
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[c] Central Astronomical Observatory of the Russian Academy of Sciences at Pulkovo, St. Petersburg, 196140 Russia

The Lamb shift in muonic hydrogen: theory

#	Contribution	Ref.	Our selection Value	Our selection Unc.	Pachucki[1–3] Value	Pachucki[1–3] Unc.	Borie[5] Value	Borie[5] Unc.
1	NR One loop electron VP	1,2			205.0074			
2	Relativistic correction (corrected)	1–3,5			0.0169			
3	Relativistic one loop VP	5	205.0282				205.0282	
4	NR two-loop electron VP	5,14	1.5081		1.5079		1.5081	
5	Polarization insertion in two Coulomb lines	1,2,5	0.1509		0.1509		0.1510	
6	NR three-loop electron VP	11	0.00529					
7	Polarisation insertion in two and three Coulomb lines (corrected)	11,12	0.00223					
8	Three-loop VP (total, uncorrected)				0.0076		0.00761	
9	Wichmann-Kroll	5,15,16	−0.00103				−0.00103	
10	Light by light electron loop contribution (Virtual Delbrück scattering)	6	0.00135	0.00135			0.00135	0.00015
11	Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$	1,2	−0.00500	0.0010	−0.006	0.001	−0.005	
12	Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$	17–19	−0.00150					
13	Mixed electron and muon loops	20	0.00007				0.00007	
14	Hadronic polarization $\alpha(Z\alpha)^4 m_r$	21–23	0.01077	0.00038	0.0113	0.0003	0.011	0.002
15	Hadronic polarization $\alpha(Z\alpha)^5 m_r$	22,23	0.000047					
16	Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m_r$	22,23	−0.000015					
17	Recoil contribution	24	0.05750		0.0575		0.0575	
18	Recoil finite size	5	0.01300	0.001			0.013	0.001
19	Recoil correction to VP	5	−0.00410				−0.0041	
20	Radiative corrections of order $\alpha^n(Z\alpha)^k m_r$	2,7	−0.66770		−0.6677		−0.66788	
21	Muon Lamb shift 4th order	5	−0.00169				−0.00169	
22	Recoil corrections of order $\alpha(Z\alpha)^5 \frac{m}{M} m_r$	2,5–7	−0.04497		−0.045		−0.04497	
23	Recoil of order α^6	2	0.00030		0.0003			
24	Radiative recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m_r$	1,2,7	−0.00960		−0.0099		−0.0096	
25	Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability contribution)	2,5,22,25	0.015	0.004	0.012	0.002	0.015	0.004
26	Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	0.00019					
27	Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	−0.00001					
	Sum		206.0573	0.0045	206.0432	0.0023	206.05856	0.0046

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$$\Delta E_{LS} = 206.0573(45) - 5.2262\, r_p^2 + 0.0347\, r_p^3 \text{ meV}, \quad (1)$$

- Discrepancy ~ 0.300 meV.
- Only few contributions are important at this level.
- They are reliable.

The Lamb shift in muonic hydrogen: theory

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12	Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$	17–19	−0.00150					
13	Mixed electron and muon loops	20	0.00007				0.00007	
14	Hadronic polarization $\alpha(Z\alpha)^4 m_r$	21–23	0.01077	0.00038	0.0113	0.0003	0.011	0.002
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The Lamb shift in muonic hydrogen: theory

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Discrepancy ~ 0.300 meV.

- \`Rescaled' hydrogen-Lamb-shift contributions
  - *well established.*
- Specific muonic contributions.

The Lamb shift in muonic hydrogen: theory

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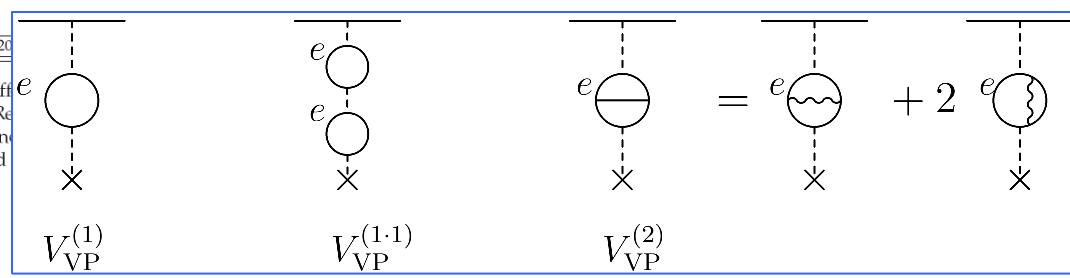
account for a remark in Ref.[7]. Values are in meV and the uncertainties have been added

Discrepancy ~ 0.300 meV.

- Specific muonic contributions
  - 1st and 2nd order perturbation theory with VP potential



The Lamb shift in muonic hydrogen: theory

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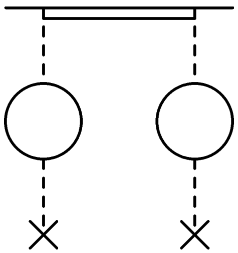
Discrepancy ~ 0.300 meV.

- Specific muonic contributions
  - The only relevant contribution of the 2nd order PT



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25	Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability contribution)	2,5,22,25	0.015	0.004	0.012	0.002	0.015	0.004
26	Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	0.00019					
27	Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	−0.00001					
	Sum		206.0573	0.0045	206.0432	0.0023	206.05856	0.0046

Table 1: All known radius-**independent** contributions to the Lamb shift in μp from different authors, and the one we selected. We follow the nomenclature of Eides *et al.*[7] Table 7.1. Item # 8 in Refs.[2,5] is the sum of items #6 and #7, without the recent correction from Ref.[12]. The error of #10 has been increased to 100% to account for a remark in Ref.[7]. Values are in meV and the uncertainties have been added in quadrature.

Discrepancy ~ 0.300 meV.

- Specific muonic contributions

  - *well established.*

The Lamb shift in muonic hydrogen: theory

#	Contribution	Ref.	Our selection Value	Unc.	Pachucki[1–3] Value	Unc.	Borie[5] Value	Unc.
1	NR One loop electron VP	1,2			205.0074			
2	Relativistic correction (corrected)	1–3,5			0.0169			
3	Relativistic one loop VP	5	205.0282				205.0282	
4	NR two-loop electron VP	5,14	1.5081		1.5079		1.5081	
5	Polarization insertion in two Coulomb lines	1,2,5	0.1509		0.1509		0.1510	
6	NR three-loop electron VP	11	0.00529					
7	Polarisation insertion in two and three Coulomb lines (corrected)	11,12	0.00223					
8	Three-loop VP (total, uncorrected)				0.0076		0.00761	
9	Wichmann-Kroll	5,15,16	−0.00103				−0.00103	
10	Light by light electron loop contribution (Virtual Delbrück scattering)	6	0.00135	0.00135			0.00135	0.00015
11	Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$	1,2	−0.00500	0.0010	−0.006	0.001	−0.005	
12	Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$	17–19	−0.00150					
13	Mixed electron and muon loops	20	0.00007				0.00007	
14	Hadronic polarization $\alpha(Z\alpha)^4 m_r$	21–23	0.01077	0.00038	0.0113	0.0003	0.011	0.002
15	Hadronic polarization $\alpha(Z\alpha)^5 m_r$	22,23	0.000047					
16	Hadronic polarization in the radiative	22,23	−0.000015					

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Sixth-Order Vacuum-Polarization Contribution to the Lamb Shift of Muonic Hydrogen

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PHYSICAL REVIEW D **80**, 027702 (2009)

Second-order corrections to the wave function at the origin in muonic hydrogen and pionium

Vladimir G. Ivanov
Pulkovo Observatory, 196140, St. Petersburg, Russia and D. I. Mendeleev Institute for Metrology (VNIIM), St. Petersburg 198005, Russia

Evgeny Yu. Korzinin
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Savely G. Karshenboim*
D. I. Mendeleev Institute for Metrology (VNIIM), St. Petersburg 198005, Russia and Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany

Discrepancy ~ 0.300 meV.

- Specific muonic contributions

  - well established.

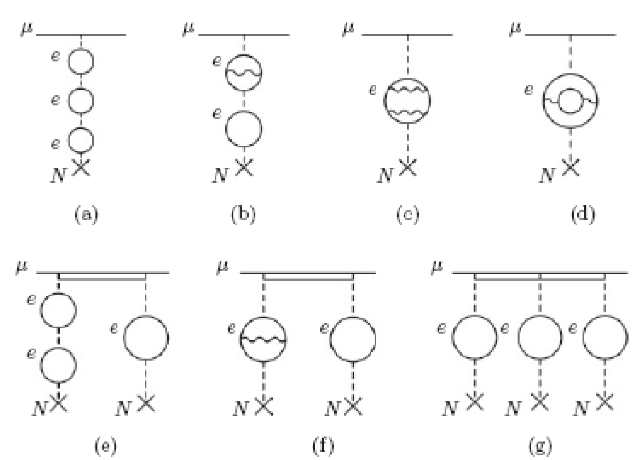


FIG. 2. Characteristic diagrams for free-electron-VP contributions to the Lamb shift in muonic hydrogen of order $\alpha^5 m_\mu$.

Electron-proton scattering: early experiments

- Rosenbluth formula for electron-proton scattering.
- Corrections are introduced
  - QED
  - two-photon exchange
- `Old Mainz data' dominates.

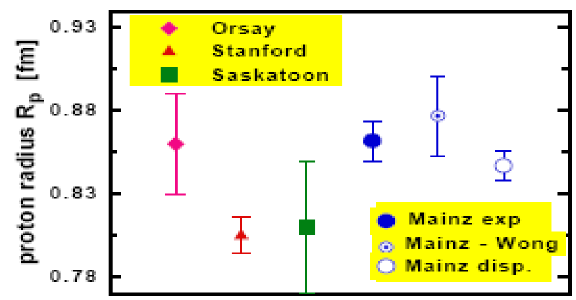


Fig. 13. Proton charge radius determined from the scattering experiments. The presented results are phenomenologically extracted from the scattering at Orsay [81], Stanford [79], Saskatoon [82] and Mainz [80] or found with more sophisticated analysis from Mainz data by Wong [83] and by Mainz theoretical group from a multi-parameter dispersion fit of all available data [84].

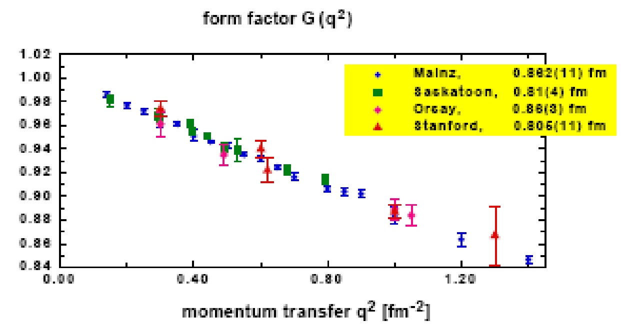


Fig. 14. Data for the electric form factor of the proton from the electron-proton elastic scattering experiments performed at Orsay [81], Stanford [79], Saskatoon [82] and Mainz [80].

Electron-proton scattering: old Mainz experiment

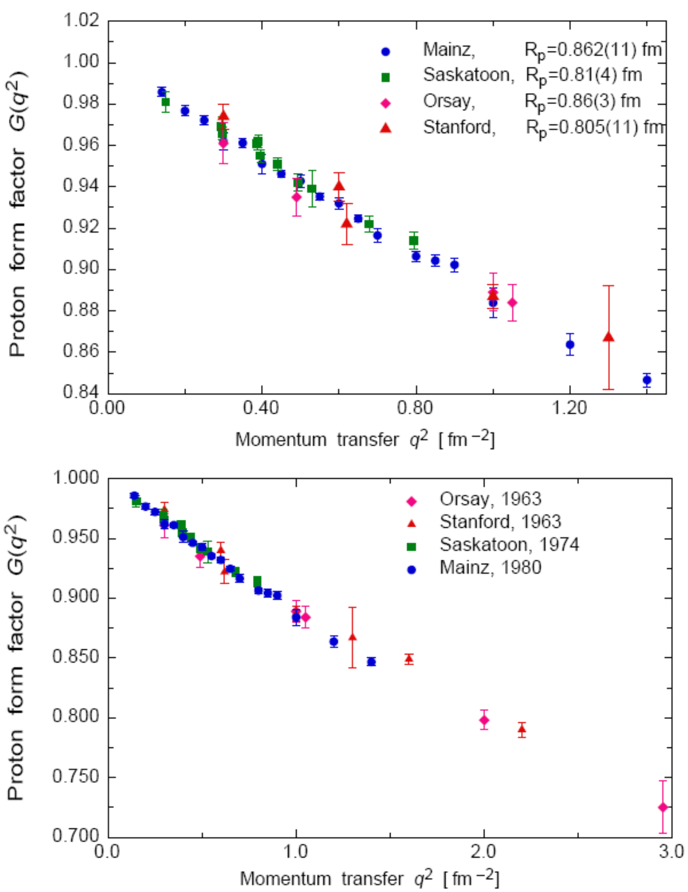


Figure 3: Experimental data of the elastic electron-proton scattering are given for the whole low energy range (*a*), and they are only for the Mainz-1980 fitting momentum range (*b*)

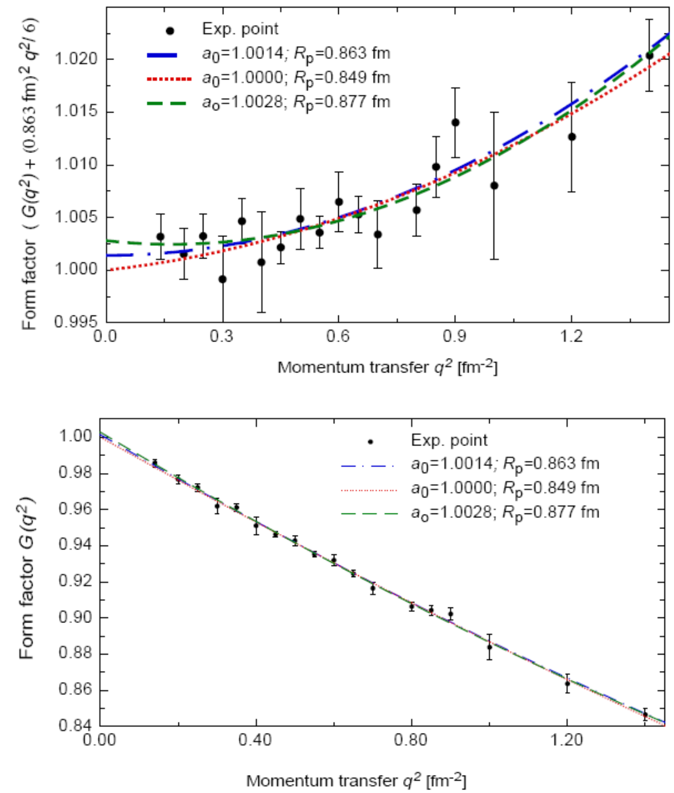


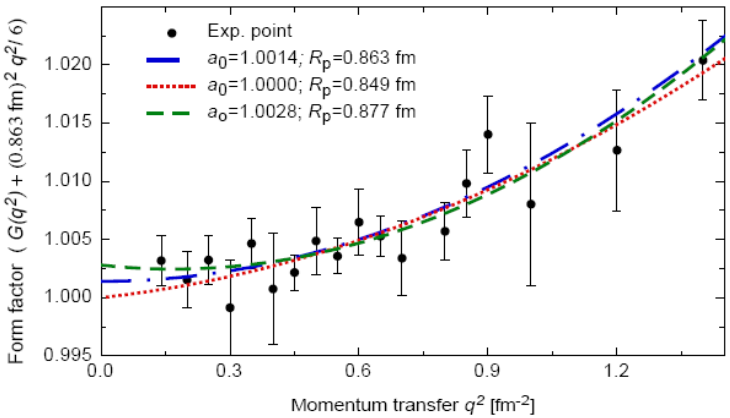
Figure 4: The Wong fits [58] of the Mainz-1980 data [2]

241

What do we actually know about the proton radius?

Savely G. Karshenboim

Electron-proton scattering: old Mainz experiment



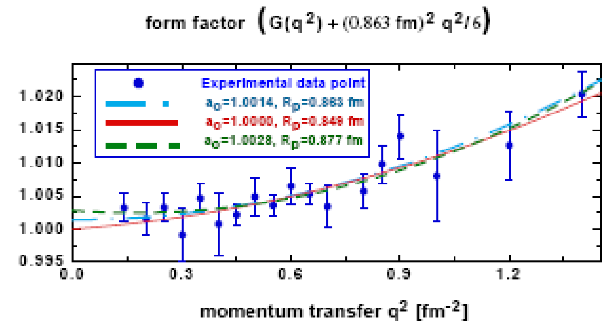


Fig. 15. Fitting the electric form factor of the proton from the Mainz experimental data [80]. Since details of fitting by Wong [83] and a particular result on the coefficient a_2 are not available, we present here similar fits from [36].

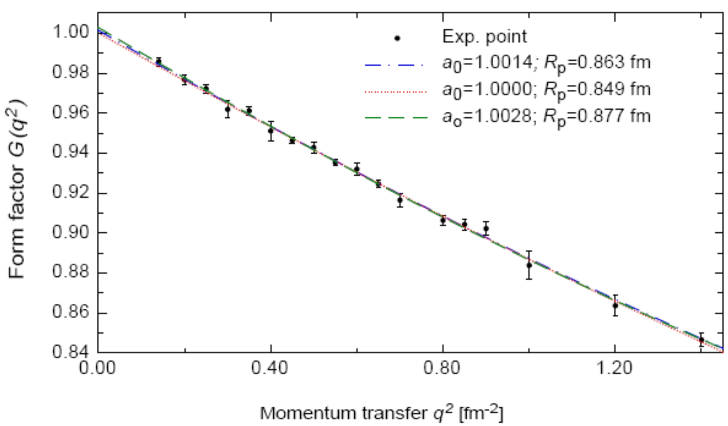


Figure 4: The Wong fits [58] of the Mainz-1980 data [2]

Normalization problem: a value denoted as $G(q^2)$ is a `true' form factor as long as systematic errors are introduced.

$$G(q^2) = a_0\,(1 + a_1\,q^2 + a_2\,q^4)$$

Electron-proton scattering: new Mainz experiment

High-precision determination of the electric and magnetic form factors of the proton

J. C. Bernauer,[1,*] P. Achenbach,[1] C. Ayerbe Gayoso,[1] R. Böhm,[1] D. Bosnar,[2] L. Debenjak,[3] M. O. Distler,[1,†] L. Doria,[1] A. Esser,[1] H. Fonvieille,[4] J. M. Friedrich,[5] J. Friedrich,[1] M. Gómez Rodríguez de la Paz,[1] M. Makek,[2] H. Merkel,[1] D. G. Middleton,[1] U. Müller,[1] L. Nungesser,[1] J. Pochodzalla,[1] M. Potokar,[3] S. Sánchez Majos,[1] B. S. Schlimme,[1] S. Širca,[6,3] Th. Walcher,[1] and M. Weinriefer[1]

$$\langle r_E^2 \rangle^{\frac{1}{2}} = 0.879(5)_{\mathrm{stat.}}(4)_{\mathrm{syst.}}(2)_{\mathrm{model}}(4)_{\mathrm{group}}\,\mathrm{fm},$$

$$\langle r_M^2 \rangle^{\frac{1}{2}} = 0.777(13)_{\mathrm{stat.}}(9)_{\mathrm{syst.}}(5)_{\mathrm{model}}(2)_{\mathrm{group}}\,\mathrm{fm}.$$

Electron-proton scattering: evaluations of `the World data'

- Mainz:

$$\langle r_E^2\rangle^{\frac{1}{2}} = 0.879(5)_{\text{stat.}}(4)_{\text{syst.}}(2)_{\text{model}}(4)_{\text{group}}\,\text{fm},$$
$$\langle r_M^2\rangle^{\frac{1}{2}} = 0.777(13)_{\text{stat.}}(9)_{\text{syst.}}(5)_{\text{model}}(2)_{\text{group}}\,\text{fm}.$$

- JLab (similar results also from Ingo Sick)

$$\langle r_E^2\rangle^{1/2} = 0.875 \pm 0.008_{\text{exp}} \pm 0.006_{\text{fit}}\ \text{fm} \qquad (3)$$
$$\langle r_M^2\rangle^{1/2} = 0.867 \pm 0.009_{\text{exp}} \pm 0.018_{\text{fit}}\ \text{fm}, \qquad (4)$$

- Charge radius:

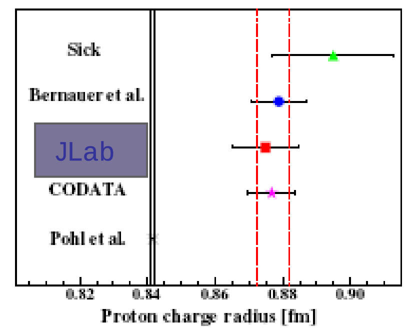


FIG. 3: (Color online) The proton RMS charge radius from previous ep scattering analysis (Sick [40]), Mainz low Q^2 measurement (Bernauer *et al.* [37]) and this work compared to the CODATA [41] and muonic hydrogen spectroscopy (Pohl *et al.* [42]). The red dashed lines show the combined results from CODATA, Bernauer *et al.* and this work, while the black dotted lines show the Pohl *et al.* uncertainty.

High Precision Measurement of the Proton Elastic Form Factor Ratio $\mu_p G_E/G_M$ at Low Q^2

X. Zhan,[1,2] K. Allada,[3] D. S. Armstrong,[4] J. Arrington,[2] W. Bertozzi,[1] W. Boeglin,[5] J.-P. Chen,[6] K. Chirapatpimol,[7] S. Choi,[8] E. Chudakov,[6] E. Cisbani,[9,10] P. Decowski,[11] C. Dutta,[12] S. Frullani,[9] E. Fuchey,[13] F. Garibaldi,[9] S. Gilad,[1] R. Gilman,[6,14] J. Glister,[15,16] K. Hafidi,[2] B. Hahn,[4] J.-O. Hansen,[6] D. W. Higinbotham,[6] T. Holmstrom,[17] R. J. Holt,[2] J. Huang,[1] G. M. Huber,[18] F. Itard,[13] C. W. de Jager,[6] X. Jiang,[14] J. Johnson,[19] J. Katich,[4] R. de Leo,[20] J. J. LeRose,[6] R. Lindgren,[7] E. Long,[21] D. J. Margaziotis,[22] S. May-Tal Beck,[23] D. Meekins,[6] R. Michaels,[6] B. Moffit,[1,6] B. E. Norum,[7] M. Olson,[24] E. Piasetzky,[25] I. Pomerantz,[25] D. Protopopescu,[26] X. Qian,[27] Y. Qiang,[27,6] A. Rakhman,[28] R. D. Ransome,[14] P. E. Reimer,[2] J. Reinhold,[29] S. Riordan,[7] G. Ron,[25,30] A. Saha,[6] A. J. Sarty,[31] B. Sawatzky,[6,32] E. C. Schulte,[14] M. Shabestari,[7] A. Shahinyan,[33] S. Širca,[34,35] P. Solvignon,[2,6] N. F. Sparveris,[1,32] S. Strauch,[36] R. Subedi,[7] V. Sulkosky,[1,6] I. Vilardi,[20] Y. Wang,[37] B. Wojtsekhowski,[6] Z. Ye,[38] and Y. Zhang[39]
(Jefferson Lab Hall A Collaboration)

Magnetic radius does not agree!

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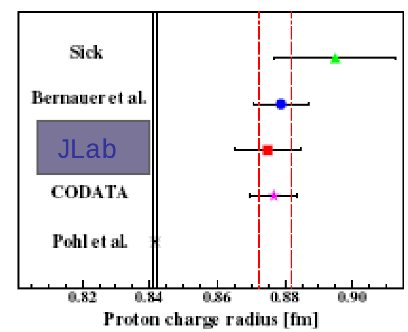

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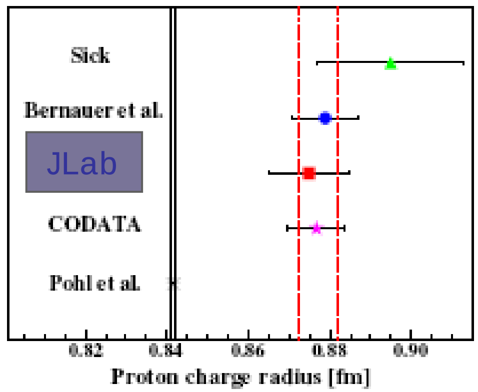
(Jefferson Lab Hall A Collaboration)

Different methods to determine the proton charge radius

- *spectroscopy of hydrogen (and deuterium)*
- *the Lamb shift in muonic hydrogen*
- *electron-proton scattering*

- Comparison:



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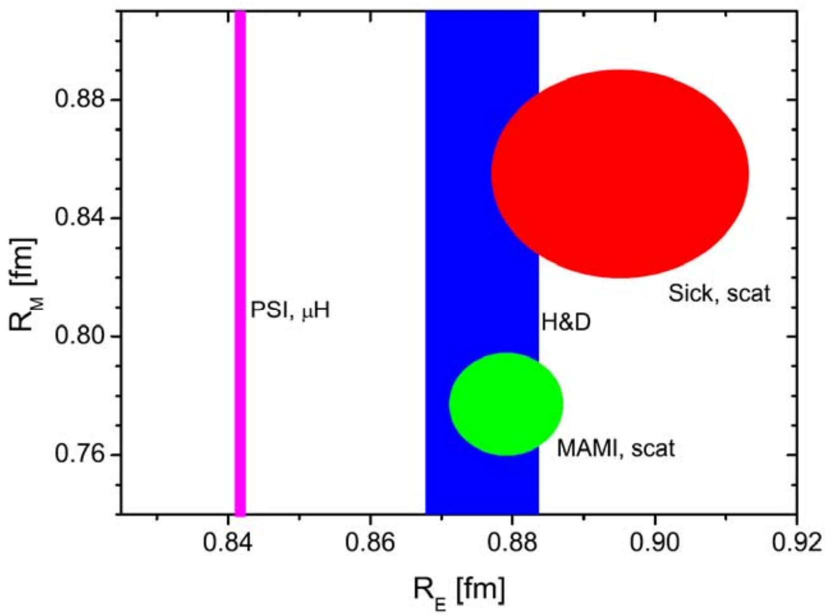
Present status of proton radius: three convincing results

charge radius and the Rydberg constant: a strong discrepancy.

- If I would bet:
  - *systematic effects in hydrogen and deuterium spectroscopy*
  - *error or underestimation of uncalculated terms in 1s Lamb shift theory*
- Uncertainty and model-independence of scattering results.

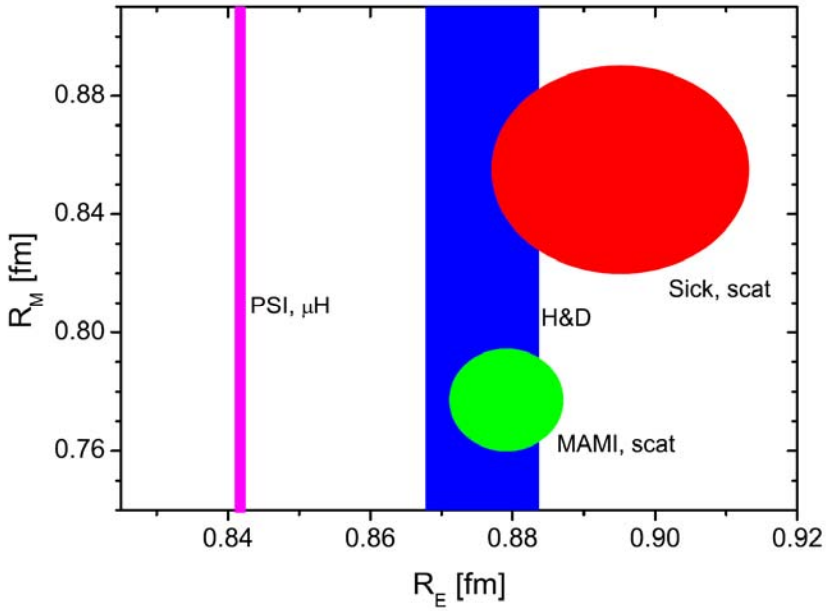
magnetic radius:

a strong discrepancy between different evaluation of the data and maybe between the data



Proton radius determination as a probe of the Coulomb law

hydrogen e-p	q ~ 1 – 4 keV
muonic hydrogen μ-p	q ~ 0.35 MeV
scattering e-p	q from few MeV to 1 GeV



What is next?

- *new evaluations of scattering data (old and new)*
- *new spectroscopic experiments on hydrogen and deuterium*
- *evaluation of data on the Lamb shift in muonic deuterium (from PSI) and new value of the Rydberg constant*
- *systematic check on muonic hydrogen and deuterium theory*

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