Lectures & objectives

ISAPP 2014
(Belgirate)
21-30 July 2014

Transport of cosmic rays in the Galaxy and in the heliosphere (~ 4h30)
- What is GCR (Galactic Cosmic Ray) physics and transport
- Relevant time scales: ≠ species have ≠ phenomenology
- Main modelling ingredients: key parameters and uncertainties
- Tools to solve the transport equation

Charged signals: electrons/positrons, antibaryons (~1h30)
- What is astroparticle physics and DM (Dark Matter) indirect detection
- What are the astrophysical backgrounds + uncertainties [nuclear]
- Phenomenology of DM signals + uncertainties [transport and dark matter]
- Pros and Cons of DM indirect detection with charged GCRs

David Maurin (LPSC)
dmaurin@lpsc.in2p3.fr

Transport of cosmic rays (CR) in the Galaxy

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II. Processes, ingredients, characteristic times

1. Definitions
2. Diffusion (space and momentum)
3. Convection and adiabatic losses
4. Energy losses
5. Catastrophic losses
6. All together

Useful units/definitions

- **“Energy” units**

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\begin{cases}
c = \hbar = e \equiv 1 \\
m_e = 511 \text{ keV} \\
m_p = 0.938 \text{ GeV}
\end{cases}
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E_k (= T) = E - m \\
\beta \equiv \dfrac{v}{c} = \dfrac{p}{E} \\
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$\mathcal{E}$	Expression	Unit	*Natural for*
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- **CR intensity**

Intensity: $I=\#\text{ particles m}^{-2}\text{ s}^{-1}\text{ sr}^{-1}$

Differential intensity: $\dfrac{dI}{d\mathcal{E}}=\#\text{ particles m}^{-2}\text{ s}^{-1}\text{ sr}^{-1}\,\mathcal{E}^{-1}$

Differential density: $\boxed{N\equiv\dfrac{dN}{d\mathcal{E}}=\dfrac{4\pi}{v}\dfrac{dI}{dE}\quad(\#\ particules\text{ m}^{-3}\,\mathcal{E}^{-1})}$

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- **Warm up...**

1. Suppose IS flux is $dI/dR = I_0\, R^{-\gamma}$: express dI/dR in terms of $dI/dE_{k/n}$
2. If R_{max} maximum rigidity of a CR source, at which E is there a cut-off in EAS?

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$$\boxed{\frac{dI}{dE_{k/n}}=\frac{A}{\beta Z}\frac{dI}{dR}}$$

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Diffusion equation: reminder

- **Diffusion (or Heat) equation**

$$\begin{array}{l} \text{[continuity]} \\ \\ \text{[Fick's law]} \end{array} \begin{cases} \dfrac{\partial N(r,t)}{\partial t} + \vec{\nabla}.\vec{j}_d = 0 \\ \vec{j}_d = -D\vec{\nabla}N(r,t) \end{cases} \qquad \frac{\partial N(r,t)}{\partial t} - \vec{\nabla}.[D\vec{\nabla}N(r,t)] = 0$$

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Comments

N.B.: solution (behaviour) depends on

- Dimensionality
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- **1D geometry, constant D**

1. Write equation in 1D

2. Check that the solution is $\boxed{N(z,t) = \frac{N_0}{(4\pi Dt)^{1/2}} \exp\left(\frac{-z^2}{4Dt}\right)}$

N.B.: boundary conditions are $\begin{cases} N(t=0, z=0) = N_0 \\ N(t, z=\pm\infty) = 0 \end{cases}$

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1. Fourier Transform
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Calculate $\langle z(t)\rangle$ and $\langle z^2(t)\rangle$, using $\displaystyle\int_{-\infty}^{-\infty} e^{-x^2/A}dx = \sqrt{A\pi}$

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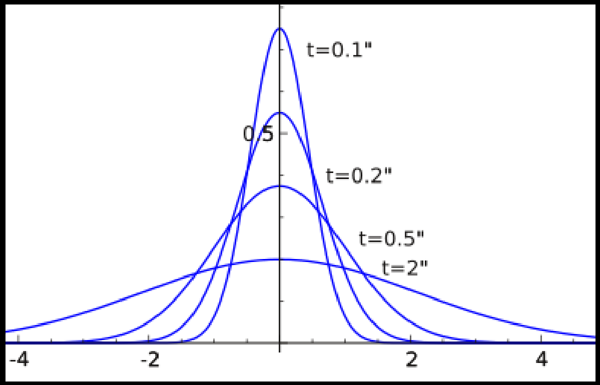
N.B.: broadening of the distribution to decrease density gradients



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- Analog to spatial diffusion (+ drift): $D_{EE} \equiv \dfrac{1}{2}\left\langle \dfrac{(\Delta E)^2}{\Delta t} \right\rangle \longleftarrow \boxed{d_{\rm diff} = \sqrt{2Dt}}$

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- Natural mechanisms for reacceleration: Fermi 2nd order

- CR (v) collides with magnetic scatterer (V): $\quad \Delta E = \dfrac{2Vv\cos\theta}{c^2} + 2\left(\dfrac{V}{v}\right)^2$

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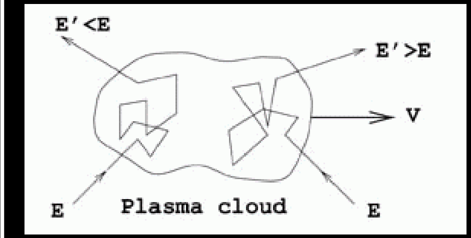

Diffusion equation: spatial and momentum space

- **Diffusion (or Heat) equation**

$$\begin{cases} \frac{\partial N(r,t)}{\partial t} + \vec{\nabla}.\vec{j}_d = 0 & \text{[continuity]} \\ \vec{j}_d = -D\vec{\nabla}N(r,t) & \text{[Fick's law]} \end{cases} \qquad \frac{\partial N(r,t)}{\partial t} - \vec{\nabla}.[D\vec{\nabla}N(r,t)] = 0$$

- **1D geometry, constant D**

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- **Diffusion in momentum space (reacceleration)**

- Analog to spatial diffusion (+ drift): $D_{EE} \equiv \frac{1}{2}\left\langle \frac{(\Delta E)^2}{\Delta t} \right\rangle \longleftarrow \boxed{d_{\rm diff} = \sqrt{2Dt}}$

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$$\left\langle \frac{\Delta E}{E} \right\rangle = 4\left(\frac{V}{c}\right)^2 \qquad \langle(\Delta E)^2\rangle = 4E^2\left(\frac{V}{c}\right)^2$$

Comments

N.B.: solution (behaviour) depends on

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- Geometry
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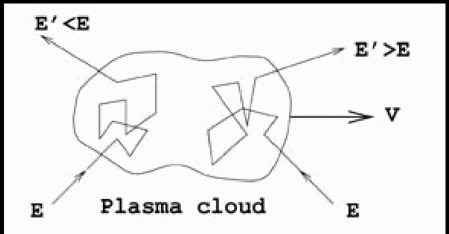

Fermi 2
β=V/c ~ 10^{-4}
ΔE/ E ∝ β^2

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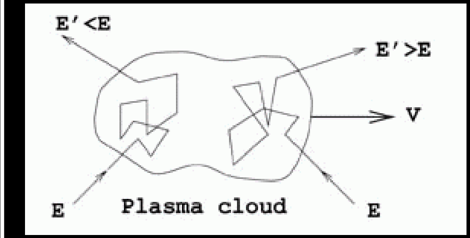

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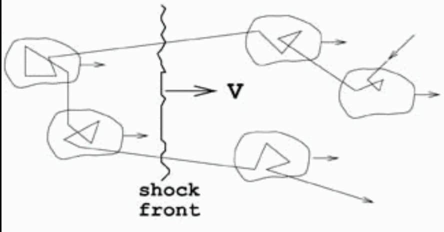

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$$V_A \approx \sqrt{<B>^2/\rho} \simeq 20 \text{ km s}^{-1}$$
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[Thornbury & Drury, MNRAS 442 (2014) 3010]

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- Order of magnitude for V_A

$$<\Delta E>^2 = 2D_{EE}t \approx \frac{2}{9}\frac{p^2}{DV_A^2}t$$

$<\Delta E>\sim 1$ GeV
$T_{esc} \sim 50$ Myr
$D_0 \simeq 0.05$ kpc^2 Myr^{-1}

$\rightarrow V_A \sim 10$ km/s

$1 \text{ km s}^{-1} = 10^{-3} \text{kpc Myr}^{-1}$

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→ Energy gain @ GeV/n
→ Strength mediated by V_A

Transport of cosmic rays (CR) in the Galaxy

II. Processes, ingredients, characteristic times

1. Definitions
2. Diffusion (space and momentum)
3. Convection and adiabatic losses
4. Energy losses
5. Catastrophic losses
6. All together

Diffusion + convection equation: adiabatic losses

- **Advection/diffusion or drift/diffusion (or Smoluchowski equation)**

$$
\begin{array}{ll}
\text{[continuity]} & \\
\text{[Fick's law]} & \\
\text{[drift]} &
\end{array}
\left\{
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• **Characteristic time scales: diffusion vs convection**

- Competition D vs V

- N.B.: $D(R) = \beta D_0 R^\delta$

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    $\rightarrow$ t_{diff} and t_{conv} ?

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Numerical application (Galaxy)

$$\begin{cases} L \simeq 10 \text{ kpc} & \text{Halo half-size} \\ D_0 \simeq 0.05 \text{ kpc}^2 \text{ Myr}^{-1} & \text{Diffusion} \\ \delta \simeq 0.5 & \text{Diffusion slope} \\ V_c \simeq 10 \text{ km s}^{-1} & \text{Convection} \end{cases}$$

$$1 \text{ km s}^{-1} = 10^{-3}\text{kpc Myr}^{-1}$$

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$$\begin{cases} \dfrac{\partial N(r,t)}{\partial t} + \vec{\nabla}.\vec{j}_d = 0 & \text{[continuity]} \\ \vec{j}_d = -D\vec{\nabla}N(r,t) & \text{[Fick's law]} \\ \vec{j}_c = \vec{V}_c N(r,t) & \text{[drift]} \end{cases}$$

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- **Characteristic time scales: diffusion vs convection**
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  - N.B.: $D(R) = \beta D_0 R^\delta$

$$\boxed{t_{\rm diff} \simeq 300 \left(\frac{L}{10\ \rm kpc}\right)^2 \left(\frac{D_0}{0.05\ \rm kpc^2\ Myr^{-1}}\right)^{-1} \left(\frac{R}{1\ \rm GV}\right)^{-\delta} \rm Myr}$$

$$\boxed{t_{\rm conv} \simeq 10^3 \left(\frac{L}{10\ \rm kpc}\right) \left(\frac{V_c}{10\ \rm km\ s^{-1}}\right)^{-1} \rm Myr}$$

Comments

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- D(**r**) spatial dependence
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Numerical application (Galaxy)

$$\begin{cases} L \simeq 10\ \rm kpc & \text{Halo half-size} \\ D_0 \simeq 0.05\ \rm kpc^2\ Myr^{-1} & \text{Diffusion} \\ \delta \simeq 0.5 & \text{Diffusion slope} \\ V_c \simeq 10\ \rm km\ s^{-1} & \text{Convection} \end{cases}$$

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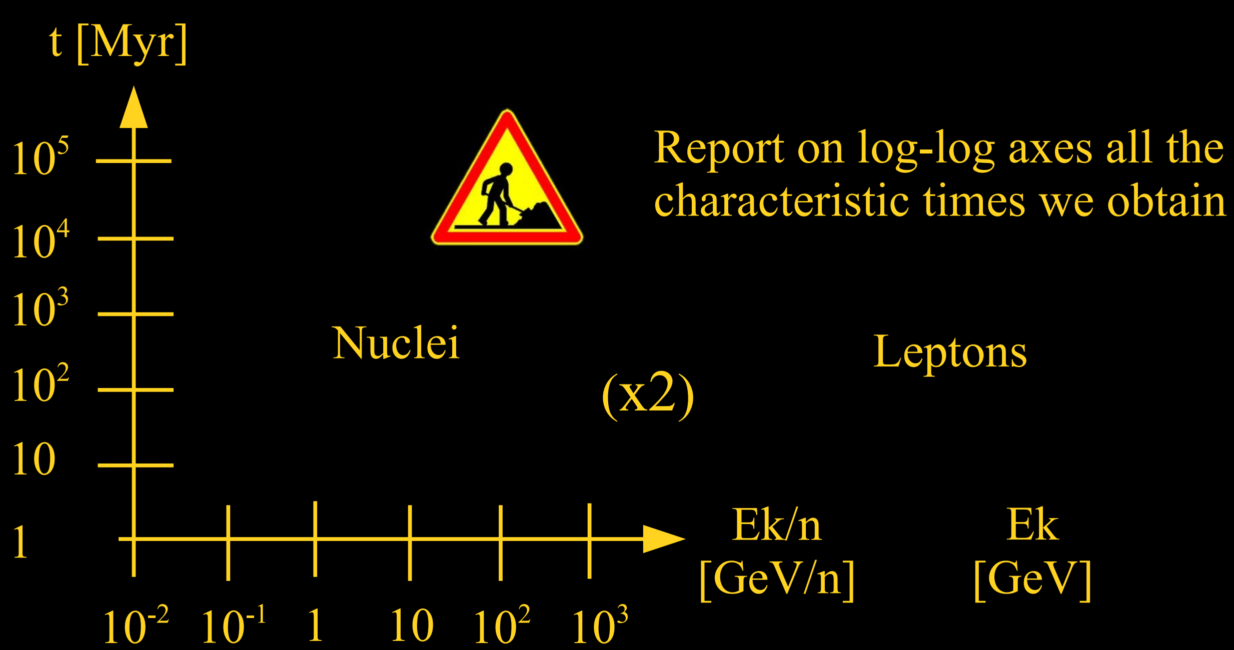

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2. Application: spherical case, $v(r) = v_0$

- Use $(\vec{\nabla}\cdot\vec{v})_r = \dfrac{1}{r^2}\dfrac{\partial(r^2 v_r)}{\partial r}$
- Calculate E=f(r)

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$$= \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) dxdydz = \left(\vec{\nabla}\cdot\vec{v}(\vec{r})\right) V$$

2. Application: spherical case, $v(r) = v_0$

- Use $(\vec{\nabla}\cdot\vec{v})_r = \dfrac{1}{r^2}\dfrac{\partial(r^2 v_r)}{\partial r}$
- Calculate E=f(r) $\qquad E = E_0 \left(\dfrac{r_0}{r}\right)^{4/3}$

Diffusion + convection equation: adiabatic losses

- **Advection/diffusion or drift/diffusion (or Smoluchowski equation)**

$$\begin{cases} \frac{\partial N(r,t)}{\partial t} + \vec{\nabla}.\vec{j}_d = 0 & \text{[continuity]} \\ \vec{j}_d = -D\vec{\nabla}N(r,t) & \text{[Fick's law]} \\ \vec{j}_c = \vec{V}_c N(r,t) & \text{[drift]} \end{cases}$$

$$\frac{\partial N}{\partial t} - \vec{\nabla}.[D\vec{\nabla}N + \vec{V}_c N] = 0$$

- **Characteristic time scales: diffusion vs convection**

- Competition D vs V

$$\boxed{t_{\rm diff} \simeq 300 \left(\frac{L}{10\text{ kpc}}\right)^2 \left(\frac{D_0}{0.05\text{ kpc}^2\text{ Myr}^{-1}}\right)^{-1} \left(\frac{R}{1\text{ GV}}\right)^{-\delta} \text{Myr}}$$

- N.B.: D(R) = β D_0 R^δ

$$\boxed{t_{\rm conv} \simeq 10^3 \left(\frac{L}{10\text{ kpc}}\right) \left(\frac{V_c}{10\text{ km s}^{-1}}\right)^{-1} \text{Myr}}$$

- **Adiabatic losses (in expanding plasma)**

V: volume
N (# particles) = nV
U (internal energy) = NE
E (average energy/particle)

Non-relativistic
E = 3/2 kT
$\underbrace{\text{P = NkT/V}}_{\text{P=2/3 nE}}$

Relativistic
U=3NkT
P = 1/3 U

Adiabatic: dU = -PdV (work done by gas) → NdE = -2/3 nE dV

$$\begin{cases} \frac{dE}{dt} = -\frac{2}{3}\frac{nE}{N}\frac{dV}{dt} \\ \frac{dV}{dt} = \left(\vec{\nabla}\cdot\vec{v}(\vec{r})\right)V \end{cases} \qquad \frac{dE}{dt} = -\frac{2}{3}\left(\vec{\nabla}\cdot\vec{v}(\vec{r})\right)E \qquad \boxed{-\frac{1}{3}\left(\vec{\nabla}\cdot\vec{v}(\vec{r})\right)E}$$

Comments

N.B.: solution (behaviour) depends on

- Dimensionality
- Geometry
- D(**r**) spatial dependence
- V_c spatial dependence and direction
- Boundary conditions

Numerical application (Galaxy)

$$\begin{cases} L \simeq 10\text{ kpc} & \text{Halo half-size} \\ D_0 \simeq 0.05\text{ kpc}^2\text{ Myr}^{-1} & \text{Diffusion} \\ \delta \simeq 0.5 & \text{Diffusion slope} \\ V_c \simeq 10\text{ km s}^{-1} & \text{Convection} \end{cases}$$

$$1\text{ km s}^{-1} = 10^{-3}\text{kpc Myr}^{-1}$$

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$\boxed{\text{→ Solar modulation: CRs loose energy in expanding Solar wind}}$

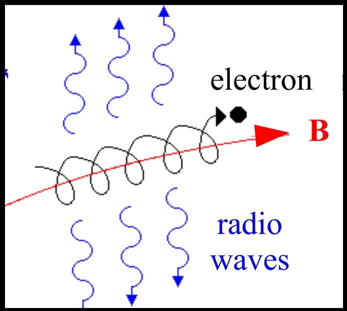
Transport of cosmic rays (CR) in the Galaxy

II. Processes, ingredients, characteristic times

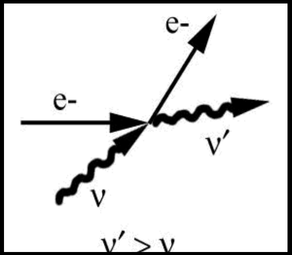
1. Definitions
2. Diffusion (space and momentum)
3. Convection and adiabatic losses
4. Energy losses
5. Catastrophic losses
6. All together

Other energy losses: electrons and positrons

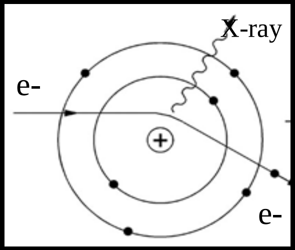
Synchrotron



Inverse Compton



Bremsstrahlung (or free-free)



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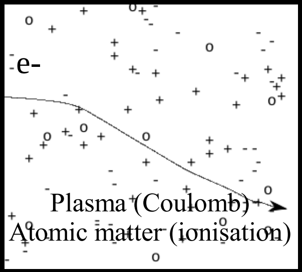

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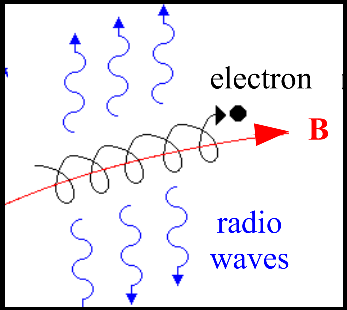

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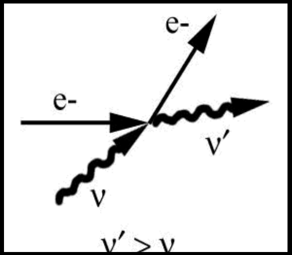

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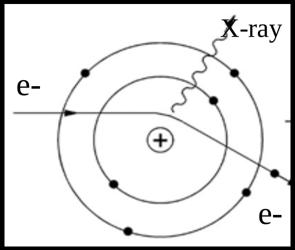

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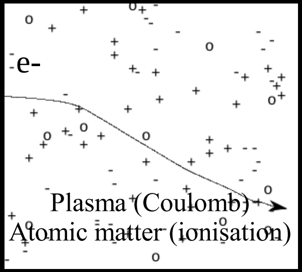

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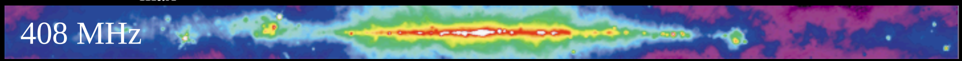

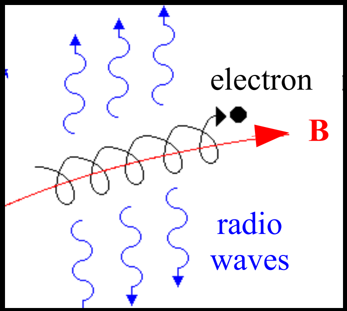

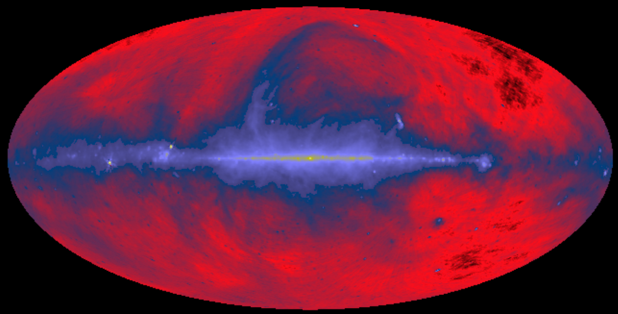

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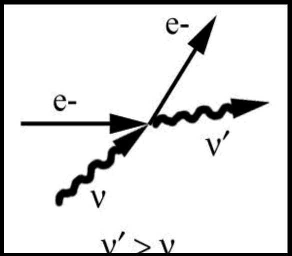

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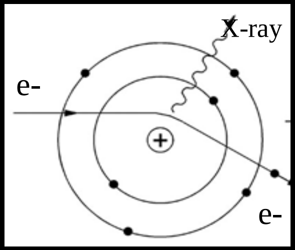

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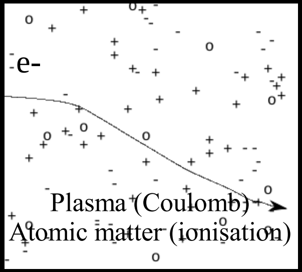

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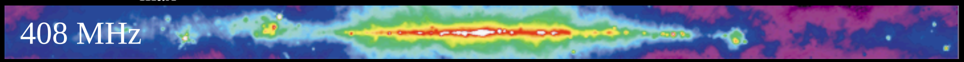

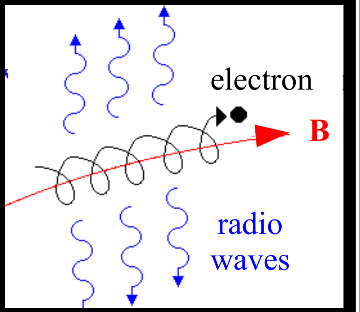

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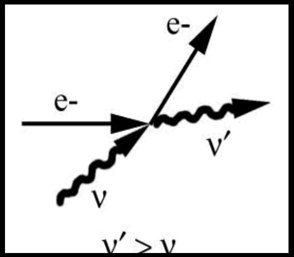

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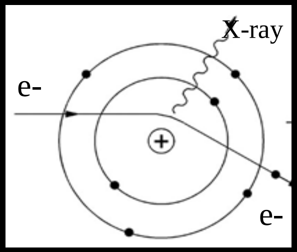

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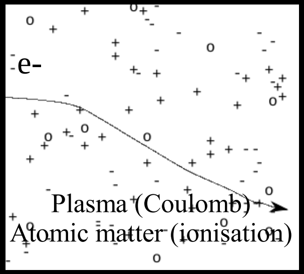

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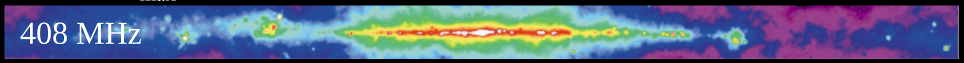

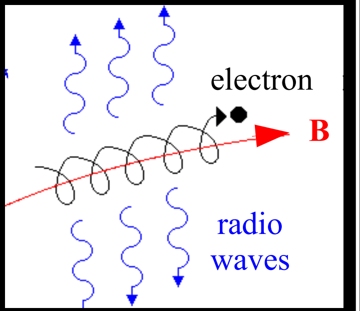

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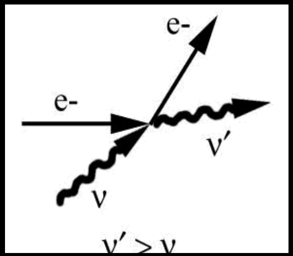

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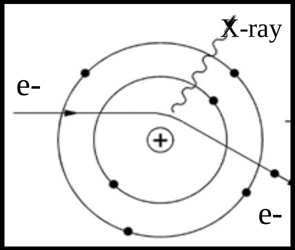

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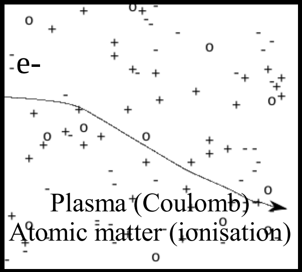

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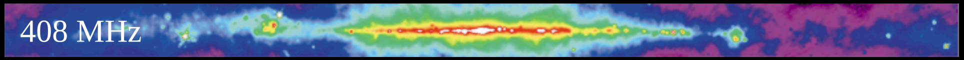

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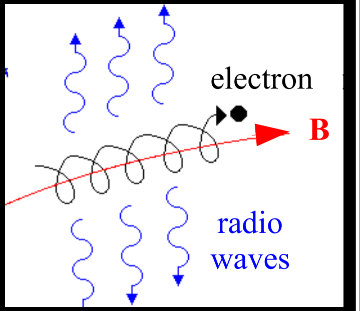

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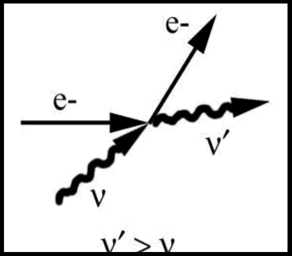

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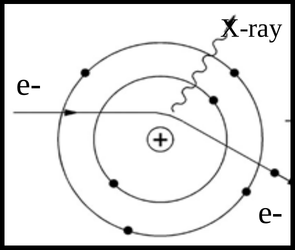

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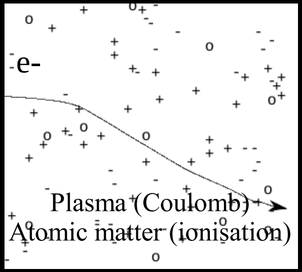

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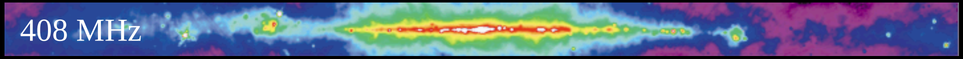

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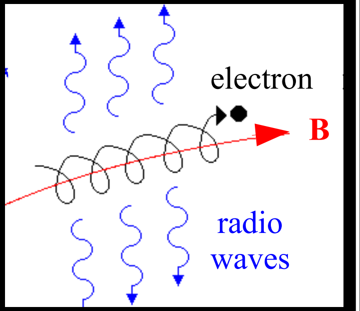
$$t_{\rm sync} \simeq 300 \left(\frac{E}{1\ {\rm GeV}}\right)^{-1} \left(\frac{B_\perp}{3\ \mu{\rm G}}\right)^{-1}\ {\rm Myr}$$



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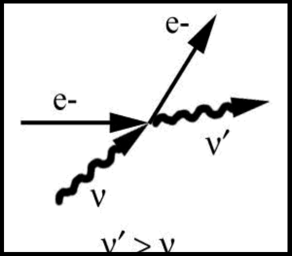

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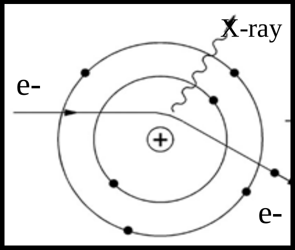

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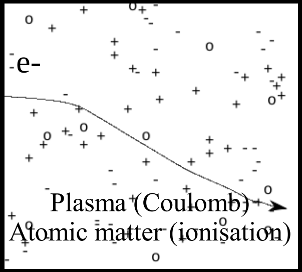

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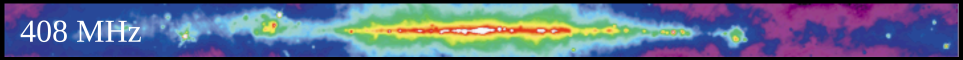

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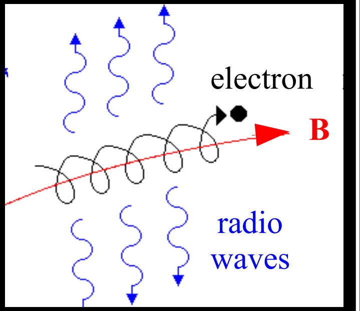
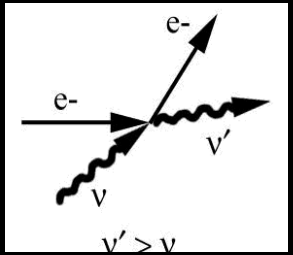


Inverse Compton (2 regimes) *Thomson* *Klein-Nishina*



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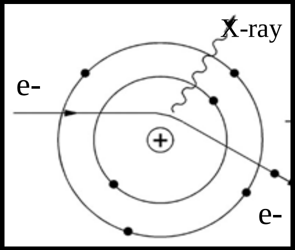

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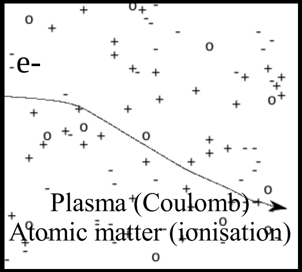

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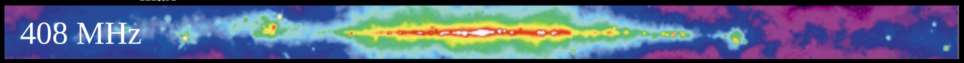

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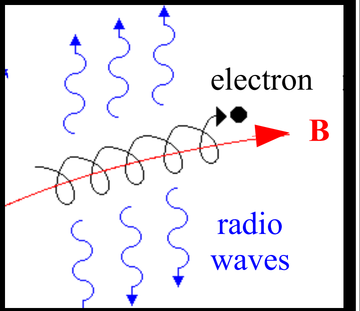

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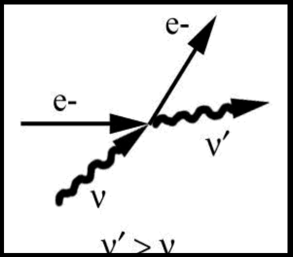
- Scattered ε_γ^{IC} $[\varepsilon_\gamma^0 - 4\varepsilon_\gamma^0 \gamma_e^2]$ $\sim 4\,\varepsilon_\gamma^0 \gamma_e^2$
- Power: fold cross-section to density of photons
- Energy losses **$\propto \gamma^2$** **$\propto \ln(\gamma)$**



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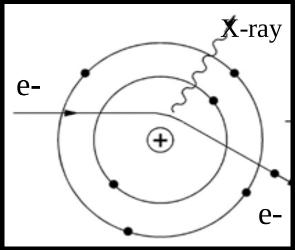

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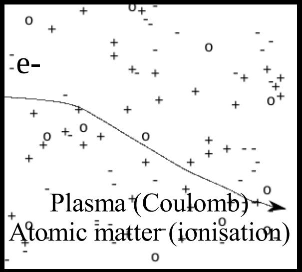

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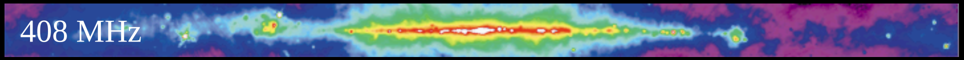

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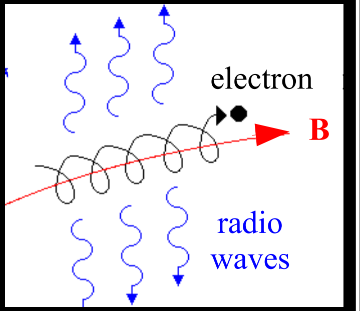

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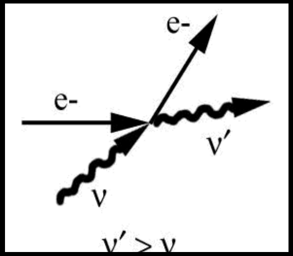

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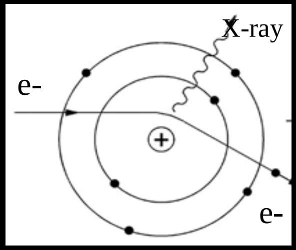

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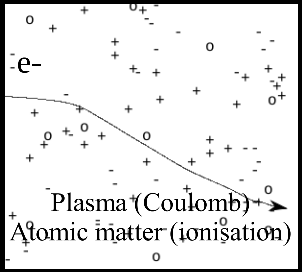

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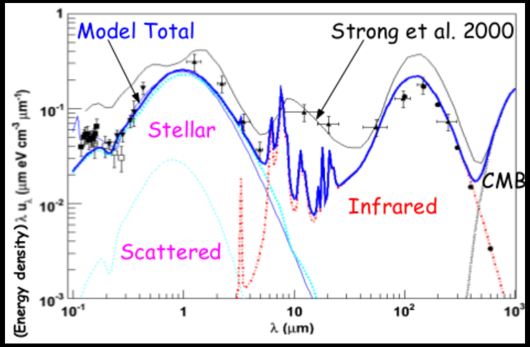

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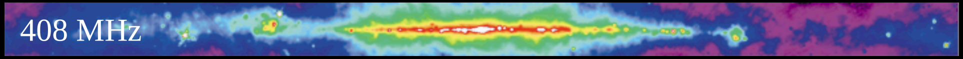

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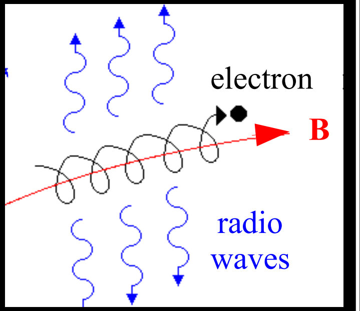

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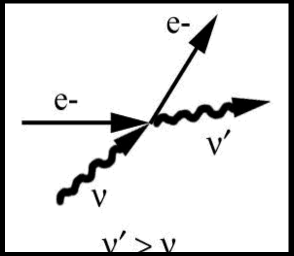

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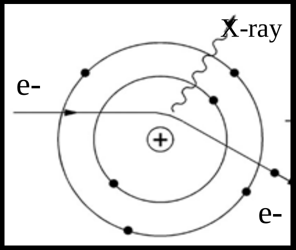

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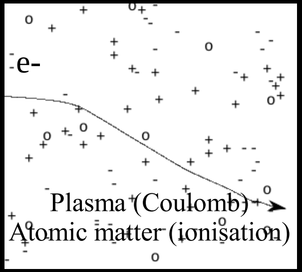

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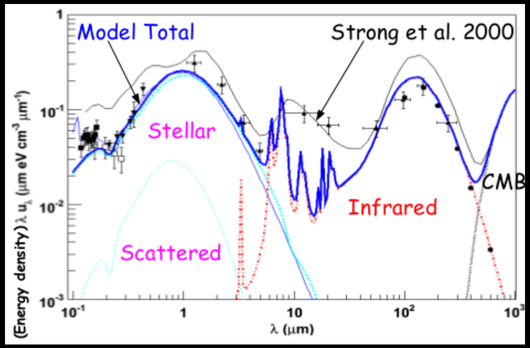

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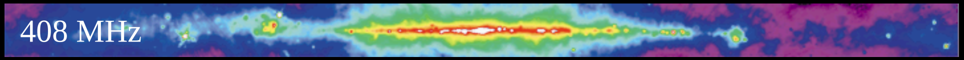

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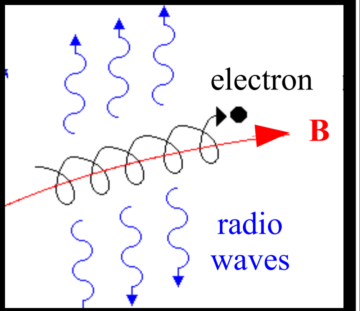

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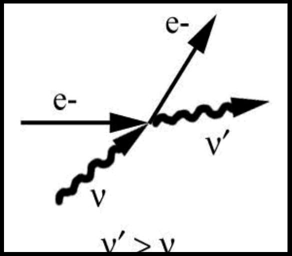

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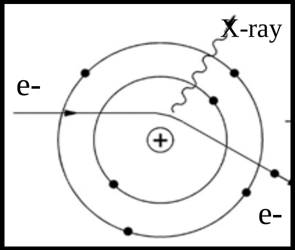

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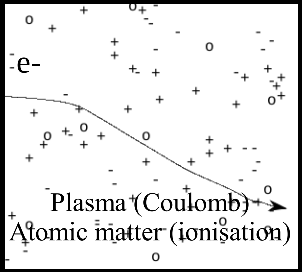

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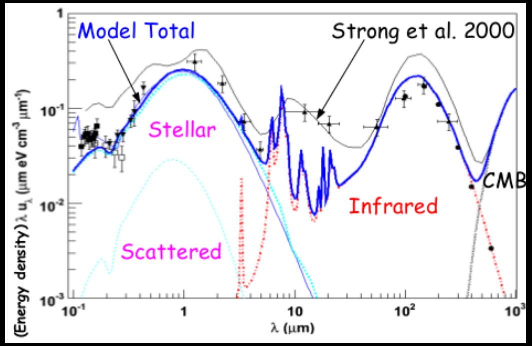

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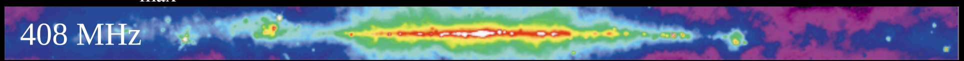

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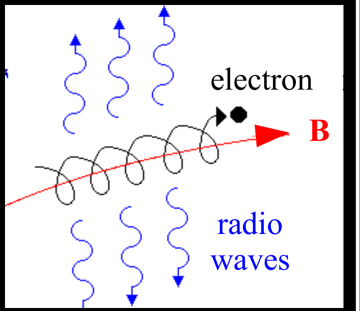

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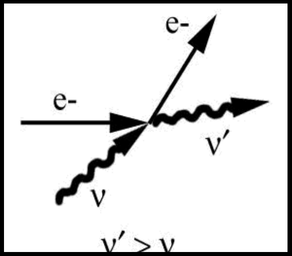

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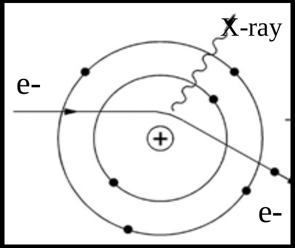

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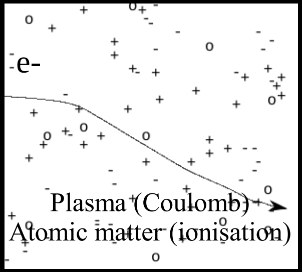

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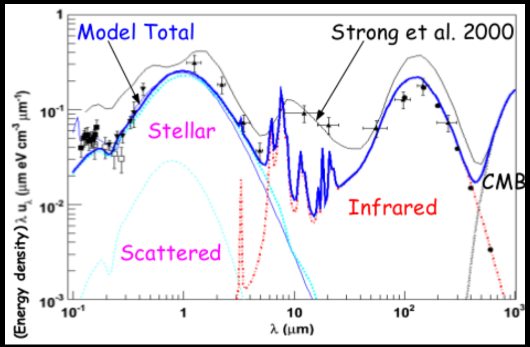

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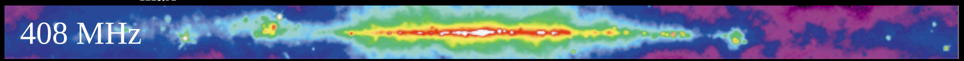

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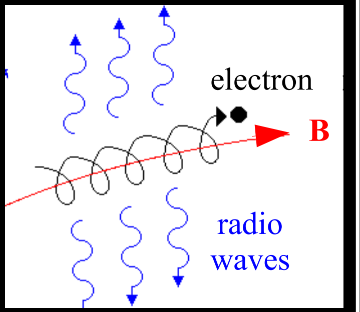

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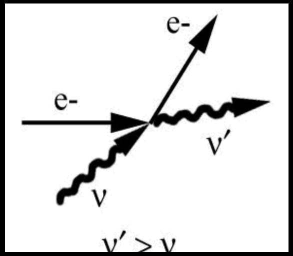

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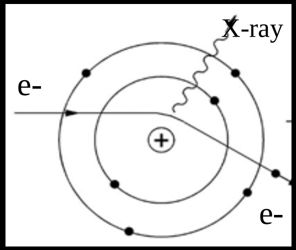

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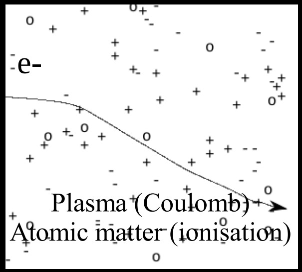

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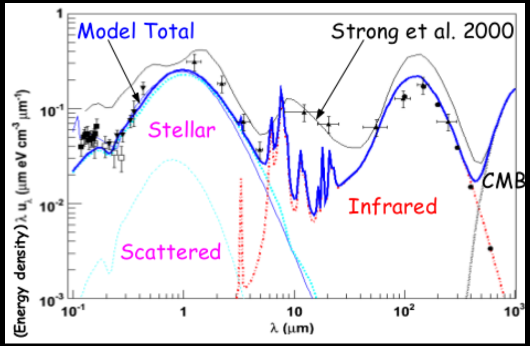

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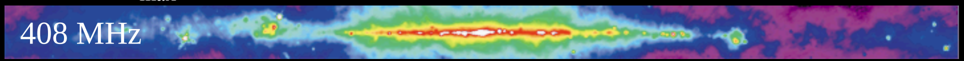

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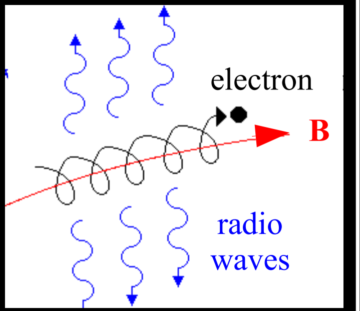

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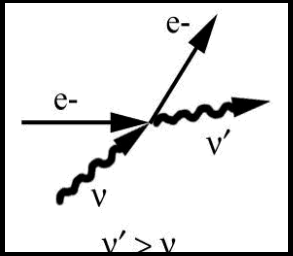

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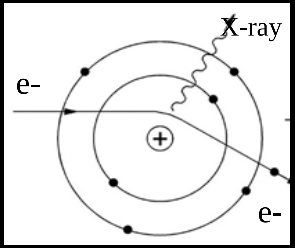

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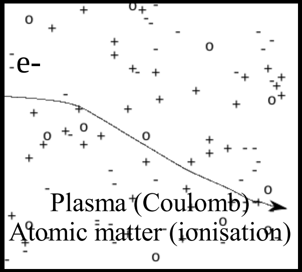

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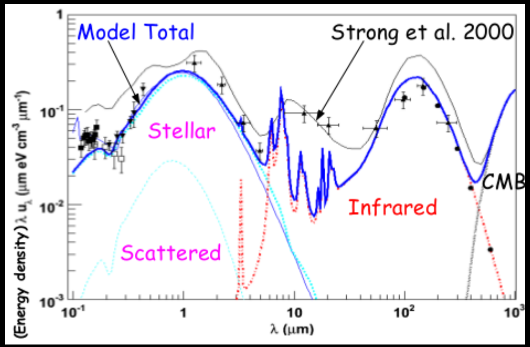

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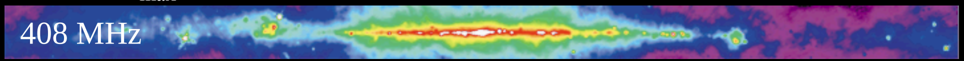

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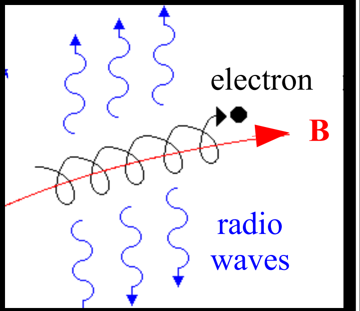

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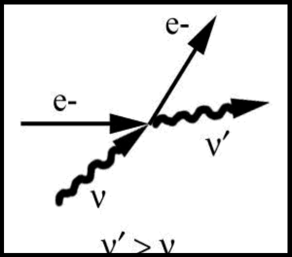

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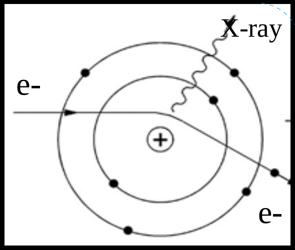

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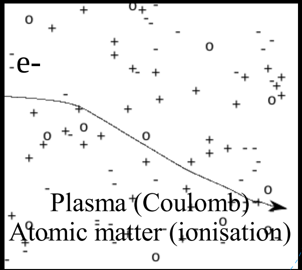

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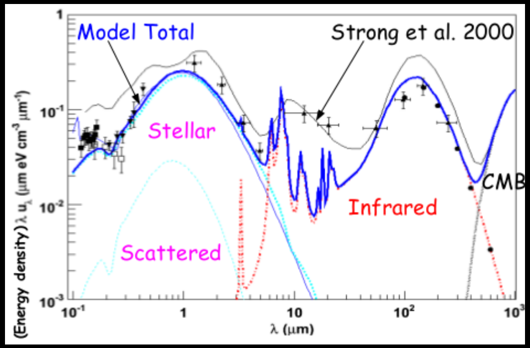
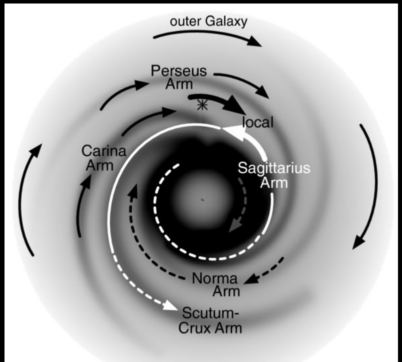
Uncertainties [Porter et al., ApJ 682 (2008) 400]





Van Eck et al. ApJ 728 (2011) 97

Other energy losses: electrons and positrons $\sigma_T = 8\pi r_e^2/3$

Synchrotron
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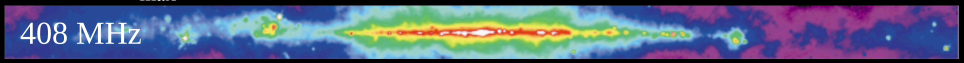

$$-\frac{dE}{dt}_{\rm sync} \propto \sigma_T B_\perp^2 \gamma^2$$

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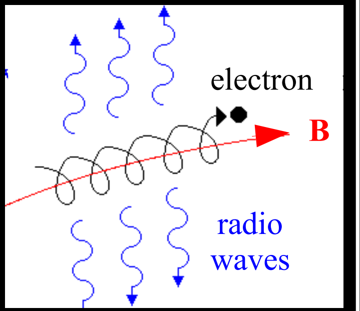

Inverse Compton (2 regimes) *Thomson* *Klein-Nishina*
- Scattered ε_γ^{IC} — Thomson: $[\varepsilon_\gamma^0 - 4\varepsilon_\gamma^0 \gamma_e^2]$ — Klein-Nishina: $\sim 4\,\varepsilon_\gamma^0\,\gamma_e^2$
- Power: fold cross-section to density of photons
- Energy losses — Thomson: $\propto \gamma^2$ — Klein-Nishina: $\propto \ln(\gamma)$

$$-\frac{dE}{dt}_{IC} \propto \sigma_T U_{\rm rad} \gamma^2$$

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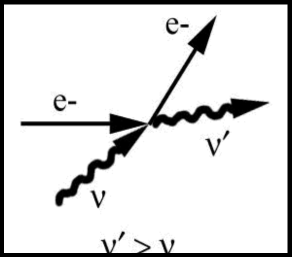

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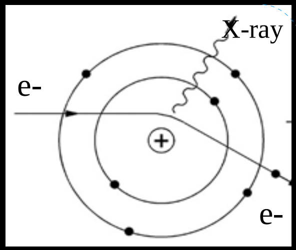

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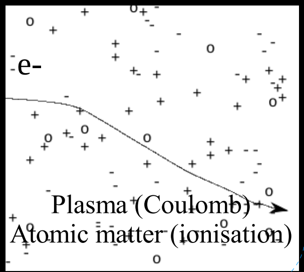

Ingredients

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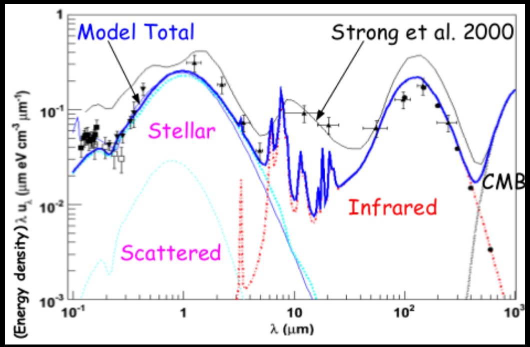

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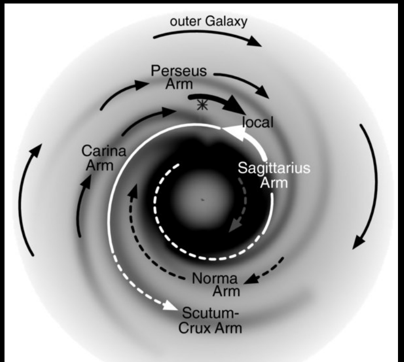

Van Eck et al. ApJ 728 (2011) 97

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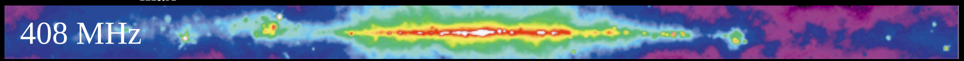

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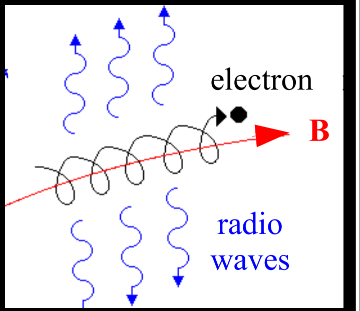

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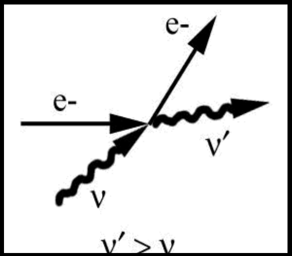

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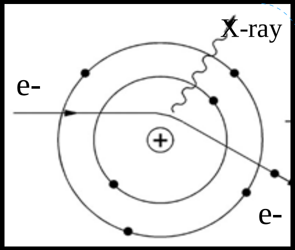

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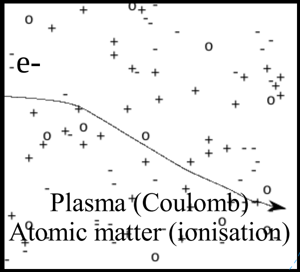

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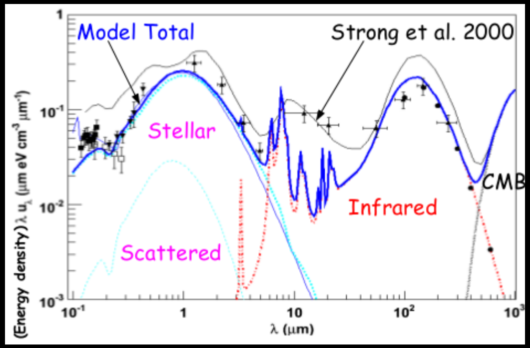

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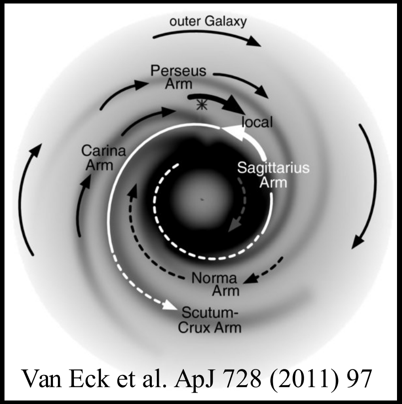

Van Eck et al. ApJ 728 (2011) 97

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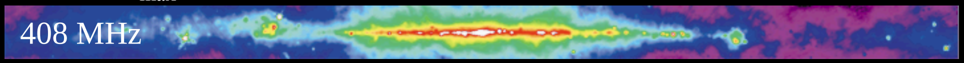

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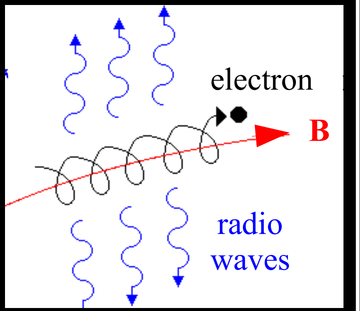

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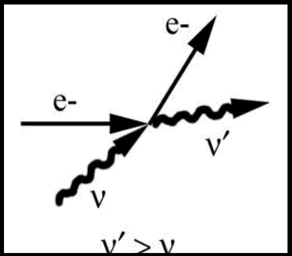

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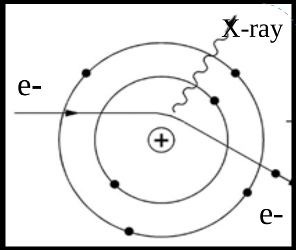

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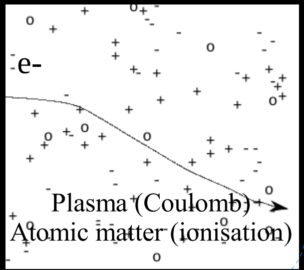

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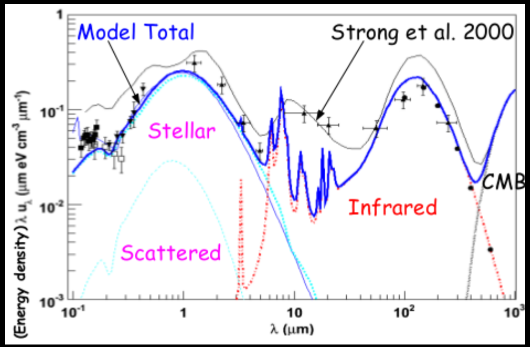

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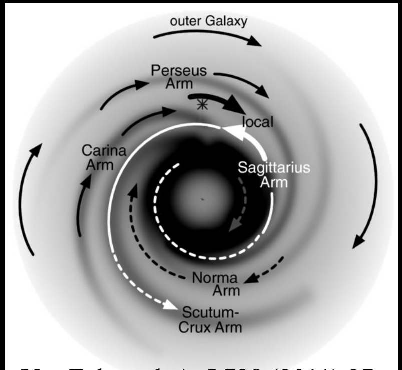

Van Eck et al. ApJ 728 (2011) 97

- Distribution of HI, HII, H2, He...
- Geometry: radial and z-dependence
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→ Crucial for γ-ray emissions

Other energy losses: nuclei

Synchrotron [disc + halo]

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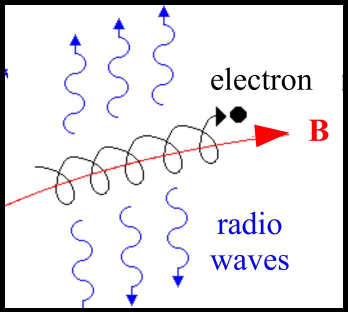

Are the formulae the same for electrons and nuclei?

Inverse Compton [disc + CMB halo]

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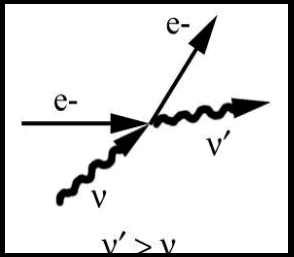

Bremsstrahlung [disc]

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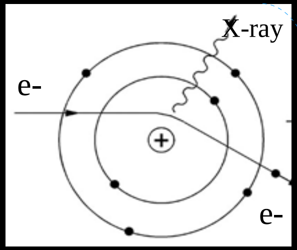

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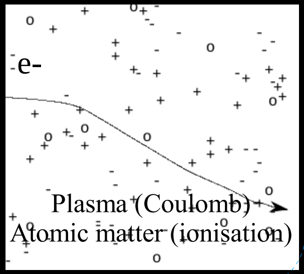

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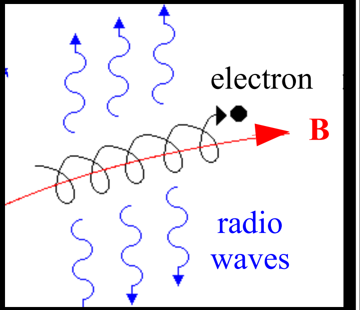

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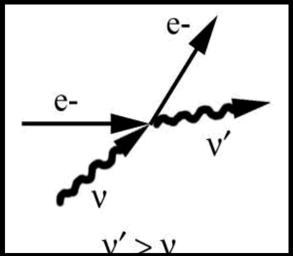

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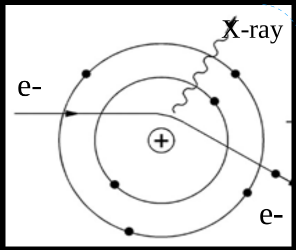

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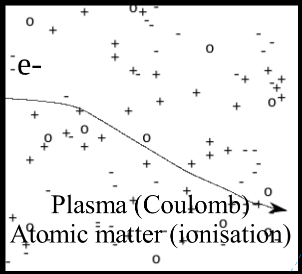

- **Changes in formulae**
  - Lepton X-section → nucleus X-section

$$\sigma_{\rm T} = 8\pi r_e^2/3$$
$$r_e \propto e^2/m_e$$

Relate σ_N to σ_T

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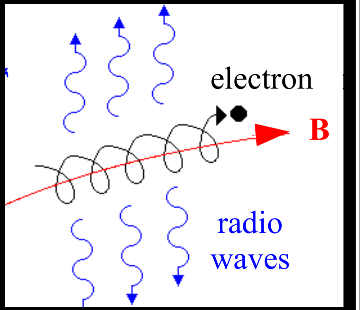

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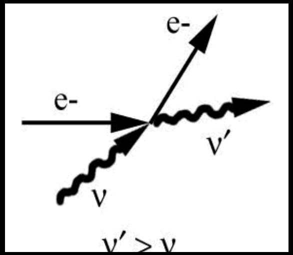

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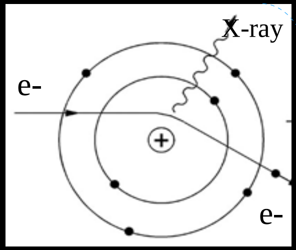

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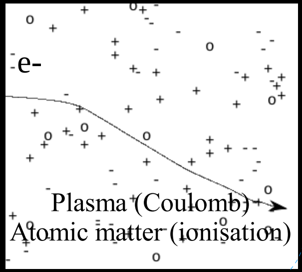

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Synchrotron [disc + halo]

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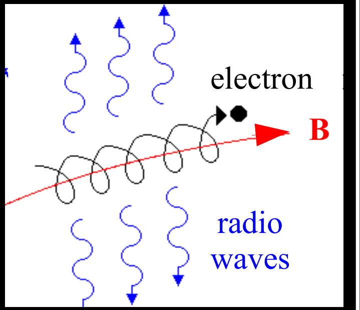

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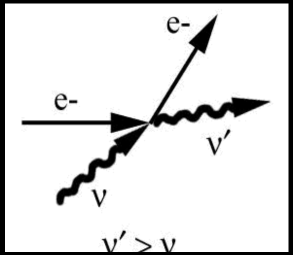

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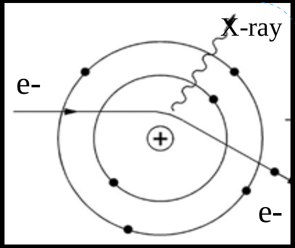

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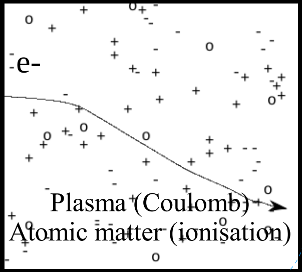

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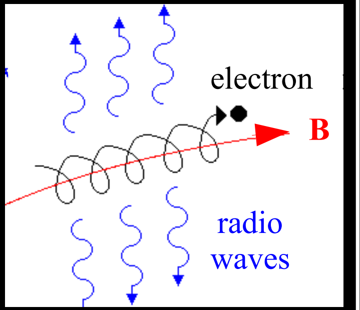
Other energy losses: nuclei

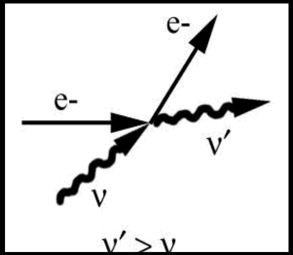
Synchrotron [disc + halo]

$$-\frac{dE}{dt}_{\rm sync} \propto \sigma_{\rm T} B_\perp^2 \gamma^2 \qquad t_{\rm sync} \simeq 300 \left(\frac{E}{1\ {\rm GeV}}\right)^{-1} \left(\frac{B_\perp}{3\ \mu{\rm G}}\right)^{-1} {\rm Myr}$$



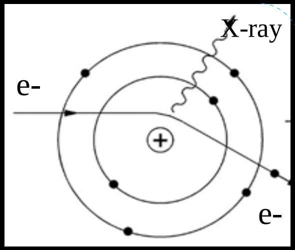
Inverse Compton [disc + CMB halo]

$$-\frac{dE}{dt}_{IC} \propto \sigma_{\rm T} U_{\rm rad} \gamma^2 \qquad t_{\rm IC} \simeq 300 \left(\frac{E}{1\ {\rm GeV}}\right)^{-1} \left(\frac{U_{\rm rad}}{0.3\ {\rm GeV\ m^{-3}}}\right)^{-1} {\rm Myr}$$



Bremsstrahlung [disc]

$$-\frac{dE}{dt}_{\rm brem} \propto \sigma_{\rm T} n_{\rm ISM} \gamma \qquad t_{\rm brem} \simeq 300 \left(\frac{n_{\rm ISM}}{1{\rm cm}^{-3}}\right)^{-1} {\rm Myr}$$



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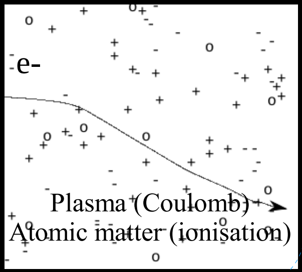

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- **Suppression factors in dE/dt**
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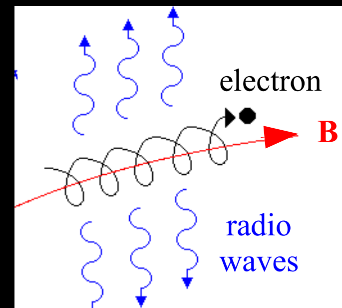
Which losses can be neglected?

Other energy losses: nuclei

Synchrotron [disc + halo]

$$-\frac{dE}{dt}_{\rm sync} \propto \sigma_{\rm T} B_\perp^2 \gamma^2 \qquad t_{\rm sync} \simeq 300 \left(\frac{E}{1\ {\rm GeV}}\right)^{-1} \left(\frac{B_\perp}{3\ \mu{\rm G}}\right)^{-1} {\rm Myr}$$



Inverse Compton [disc + CMB halo]

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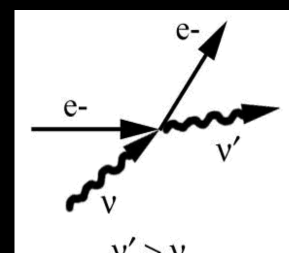
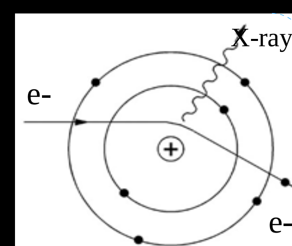


Bremsstrahlung [disc]

$$-\frac{dE}{dt}_{\rm brem} \propto \sigma_{\rm T} n_{\rm ISM} \gamma \qquad t_{\rm brem} \simeq 300 \left(\frac{n_{\rm ISM}}{1{\rm cm}^{-3}}\right)^{-1} {\rm Myr}$$



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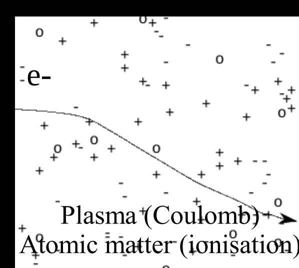

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Other energy losses: nuclei

Synchrotron [disc + halo]

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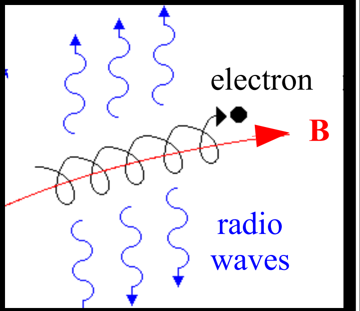

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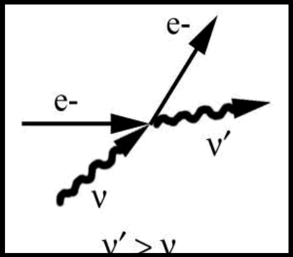
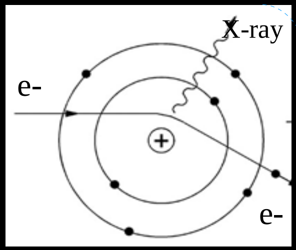


Bremsstrahlung [disc]

$$\cancel{-\frac{dE}{dt}_{\rm brem} \propto \sigma_{\rm T} n_{\rm ISM} \gamma \qquad t_{\rm brem} \simeq 300 \left(\frac{n_{\rm ISM}}{1{\rm cm^{-3}}}\right)^{-1} {\rm Myr}}$$



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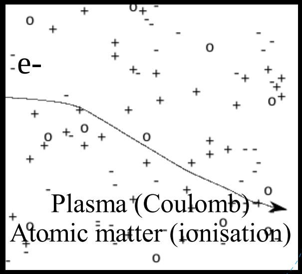

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Why do we keep ionisation and Coulomb?

Other energy losses: nuclei

Synchrotron [disc + halo]

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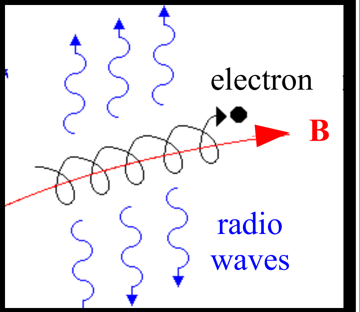

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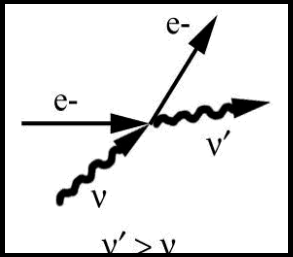

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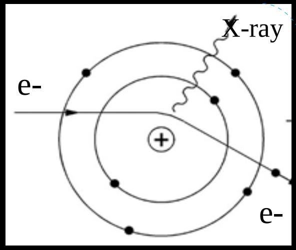

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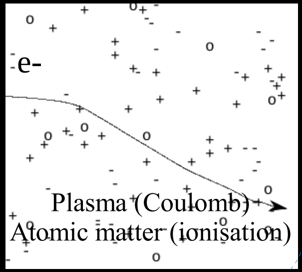

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→ Coulomb/ion amplitude redeemed
[non-relativistic nucleus vs relativisic leptons for same kinetic energy]

N.B.: ionisation and heating of the ISM!

Transport of cosmic rays (CR) in the Galaxy

II. Processes, ingredients, characteristic times

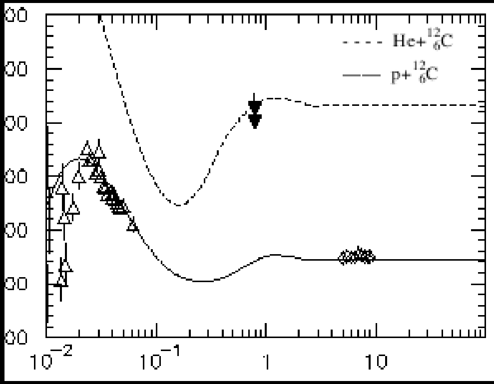
1. Definitions
2. Diffusion (space and momentum)
3. Convection and adiabatic losses
4. Energy losses
5. Catastrophic losses
6. All together

Other losses: catastrophic losses [nuclei]

Nuclear interactions

- Elastic: N + ISM →N + ISM
- Inelastic: N + ISM →X + ... (X≠N)

$$\sigma_{inel} = \sigma_{tot} - \sigma_{el}$$



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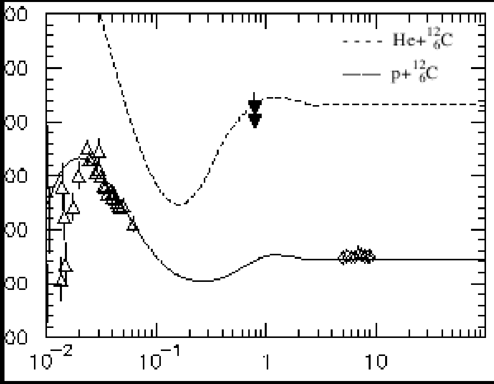

Comments/accuracy

- Destruction → "source" for fragments
- Semi-empirical models fit on data
  - Bradt & Peters (1950)
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→ $\sigma_{inel} \propto A^{2/3}$

$\Delta\sigma/\sigma \sim$ 2-5%

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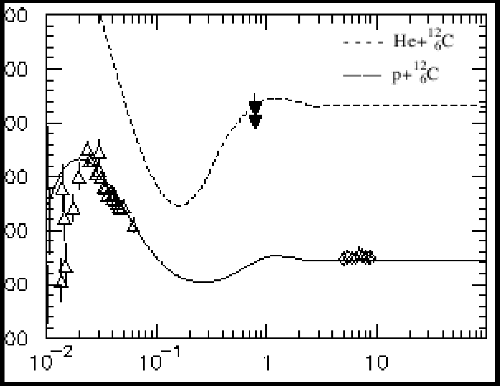

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Destruction time for p and Fe (in Myr) ?

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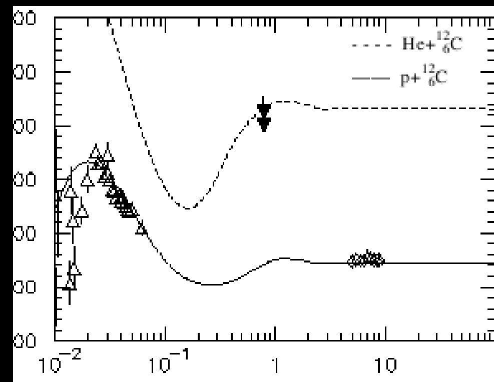

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$\Delta\sigma/\sigma$ ~ 2-5%

σ_{inel}(p, C, Fe) ~ 40, 250, 750 mb

$n_{ISM} = 1\ \mathrm{cm}^{-3}$	Disc density
$c \simeq 3 \cdot 10^{10}\ \mathrm{cm\ s}^{-1}$	Speed of light
$1\ \mathrm{mb} = 10^{-27}\ \mathrm{cm}^2$	$1\ \mathrm{s} \simeq 3 \cdot 10^{-14}\ \mathrm{Myr}$

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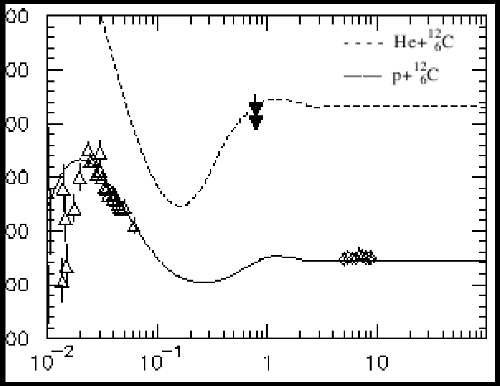
$$\sigma_{inel} = \sigma_{tot} - \sigma_{el}$$



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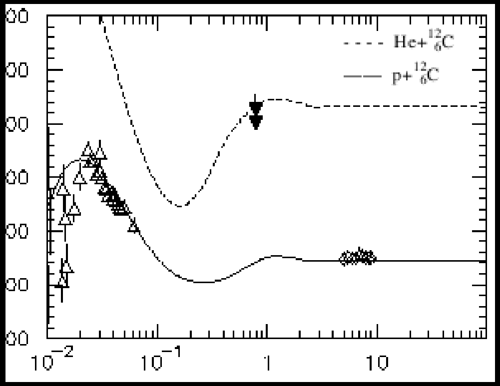

Spontaneous β decay

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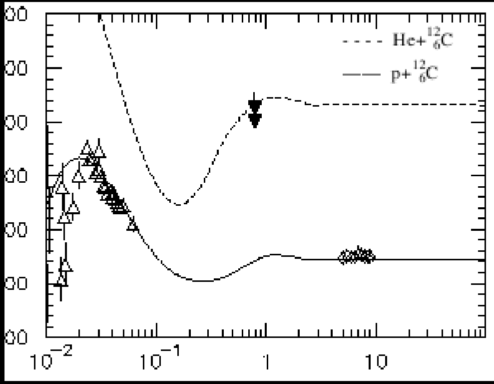

Spontaneous β decay

- Mean lifetime t
- Half-life $t_{1/2}$

$$t_{1/2} = \tau \ln(2)$$

Comments/accuracy

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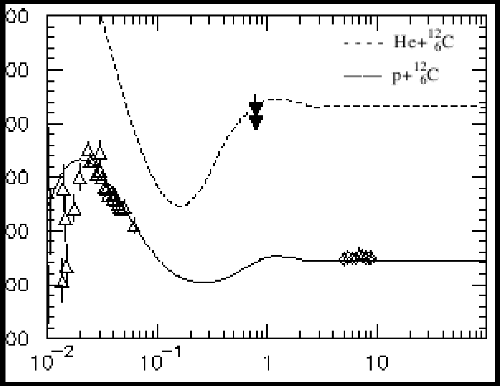

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→ **Decay rate** $\Gamma_{rad} = \dfrac{\ln(2)}{\gamma\, t_{1/2}}$

$$t_\beta \simeq E_{k/n} \left(\frac{t_{1/2}}{1.51\ {\rm Myr}}\right) {\rm Myr}$$

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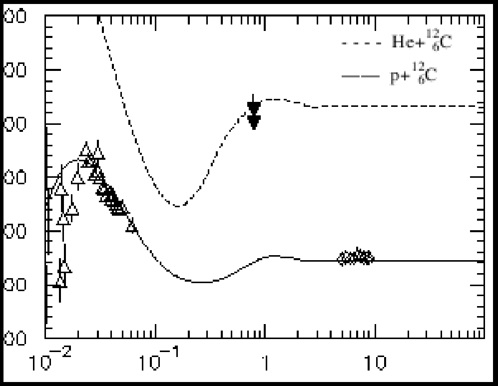

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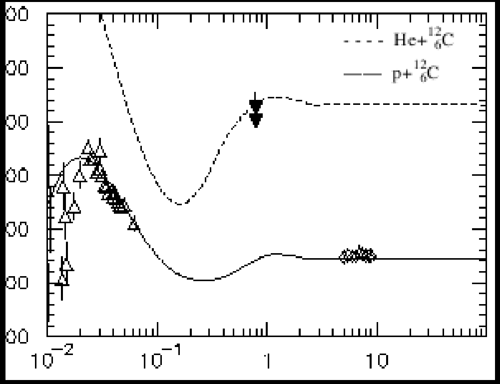

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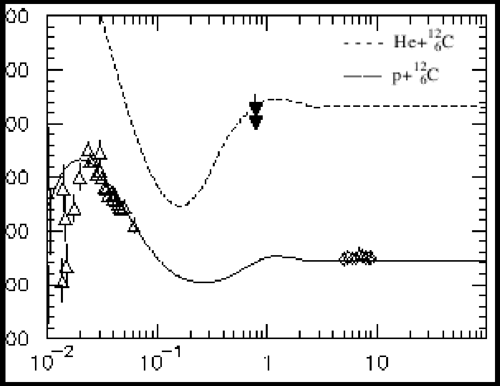

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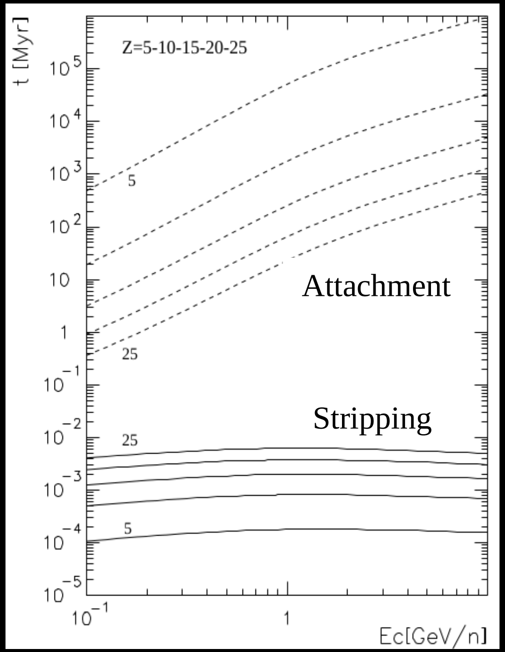

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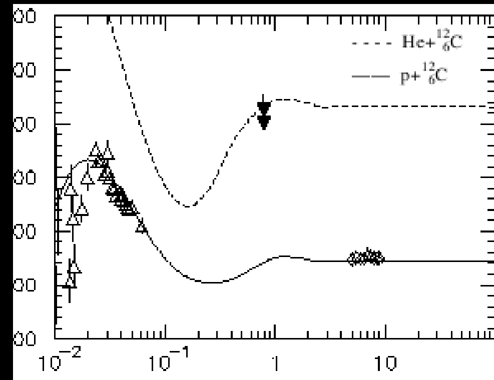

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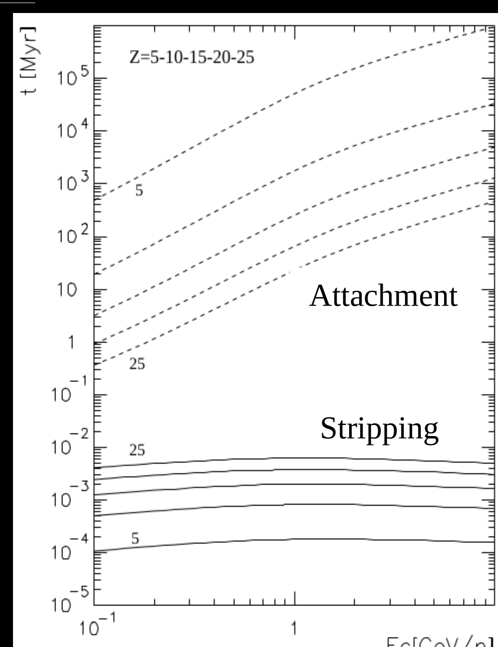

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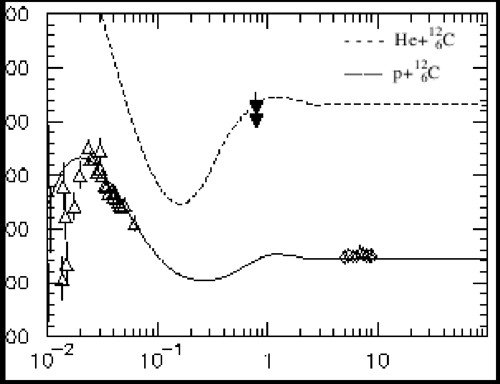

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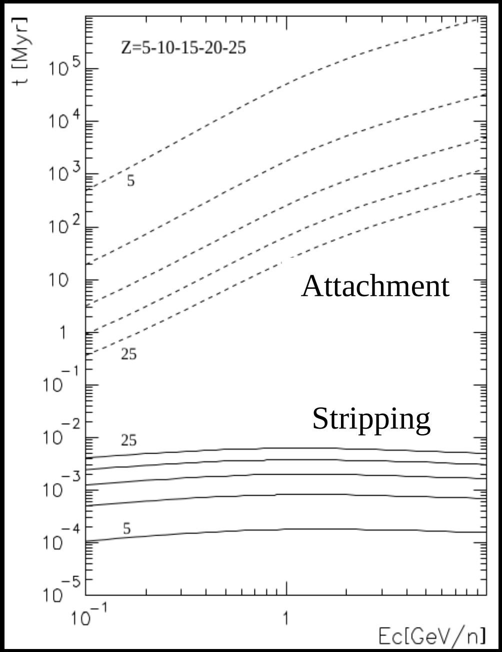

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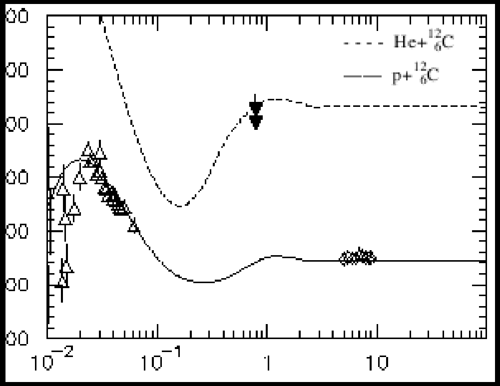

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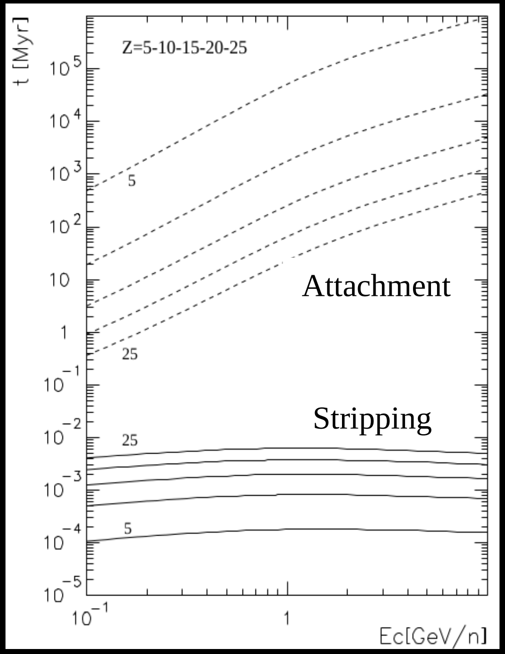

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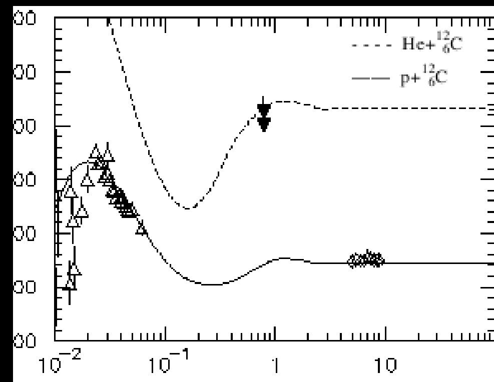

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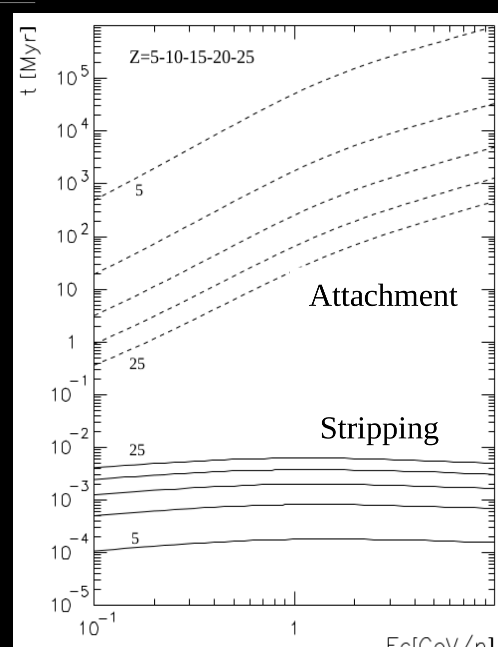

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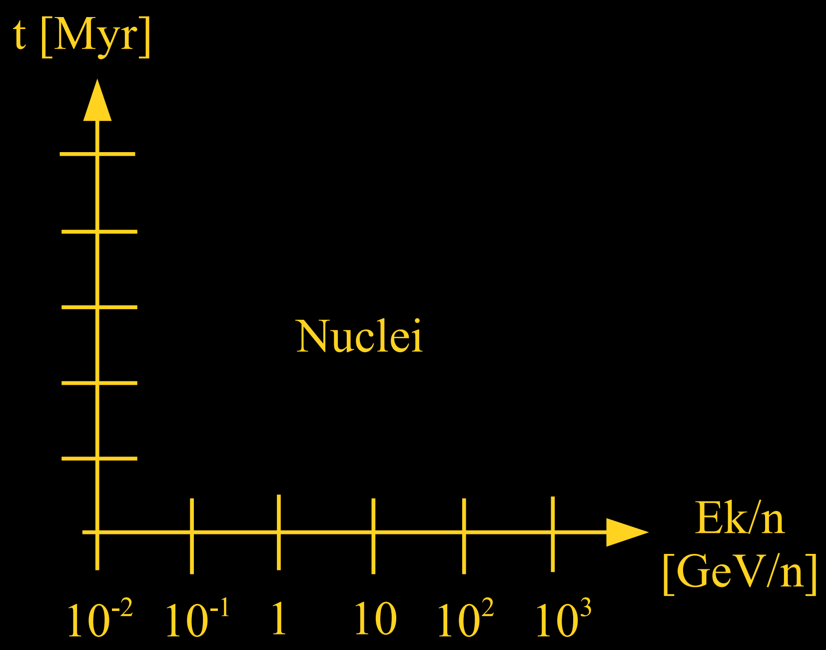
II. Processes, ingredients, characteristic times

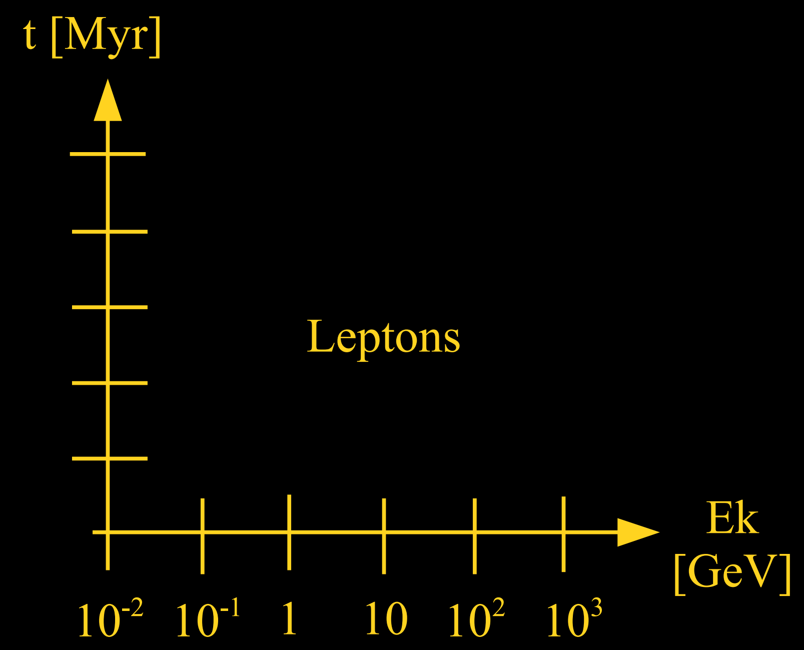
1. Definitions
2. Diffusion (space and momentum)
3. Convection and adiabatic losses
4. Energy losses (continuous)
5. Catastrophic losses
6. All together

Time scales: all together

Discussion: what do you conclude?

- For nuclei?
- For leptons?





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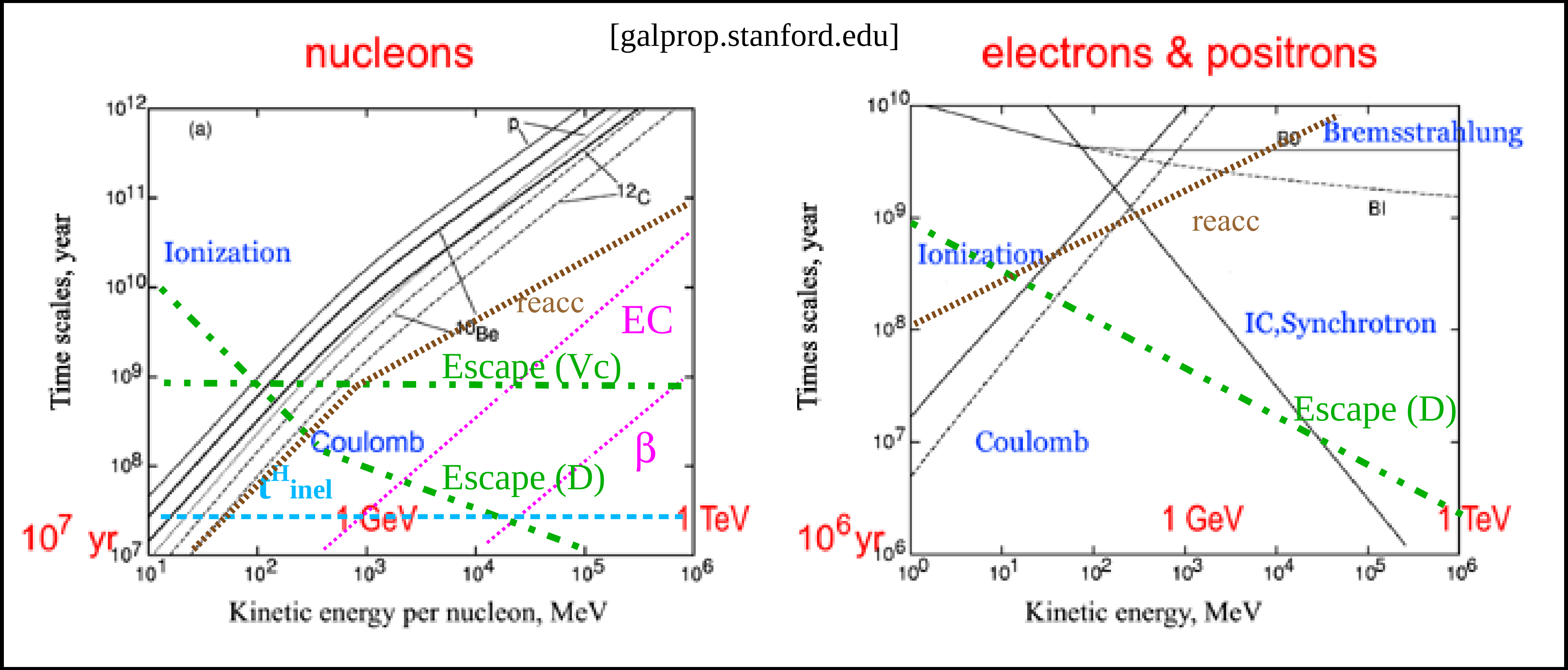

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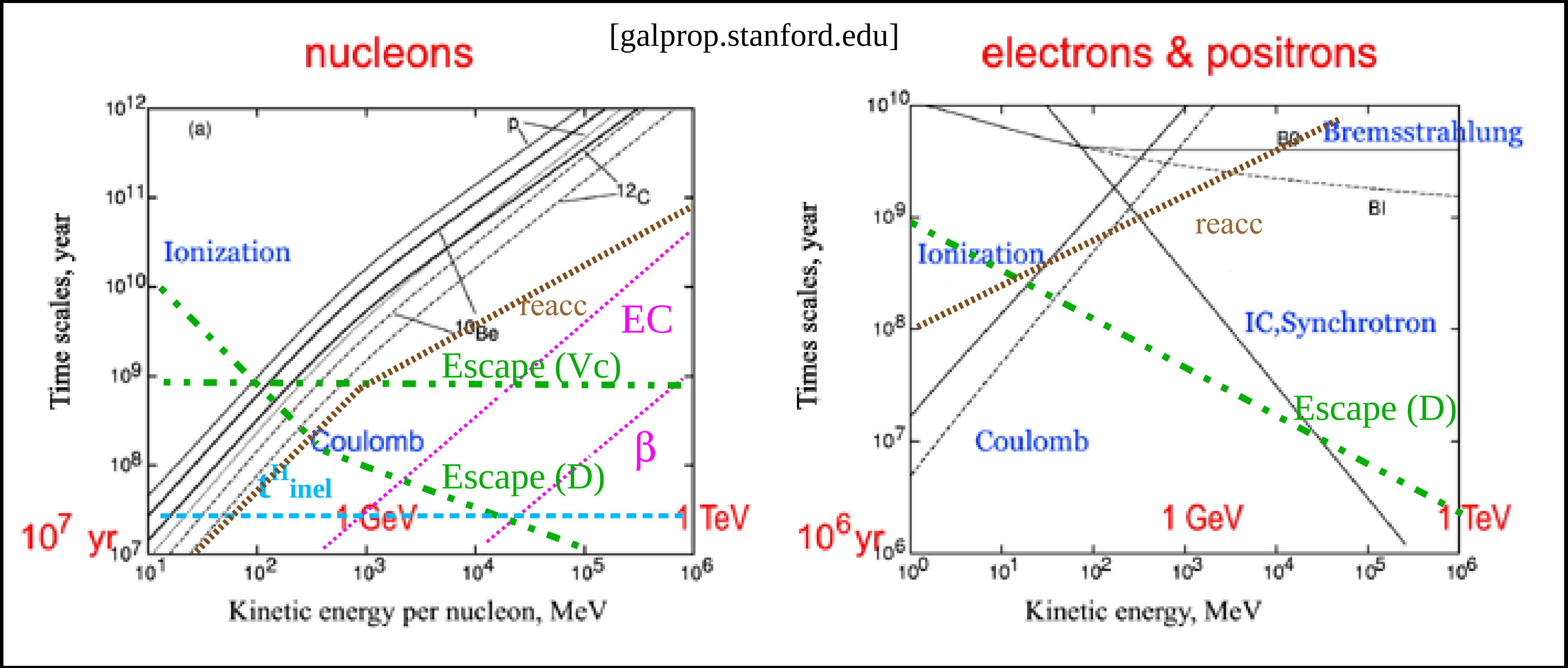

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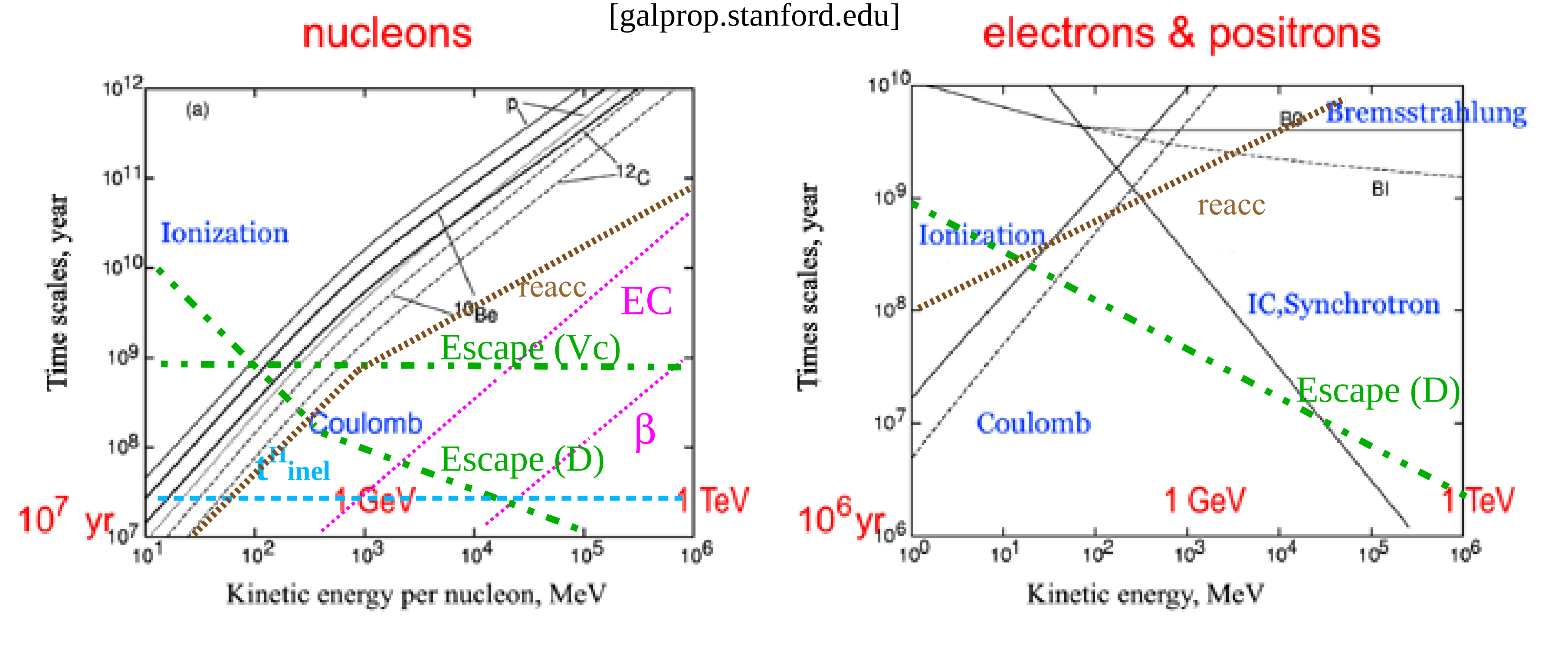
[galprop.stanford.edu]



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2. Local origin
   - Low energy radioactive nuclei
   - High energy electrons and positrons

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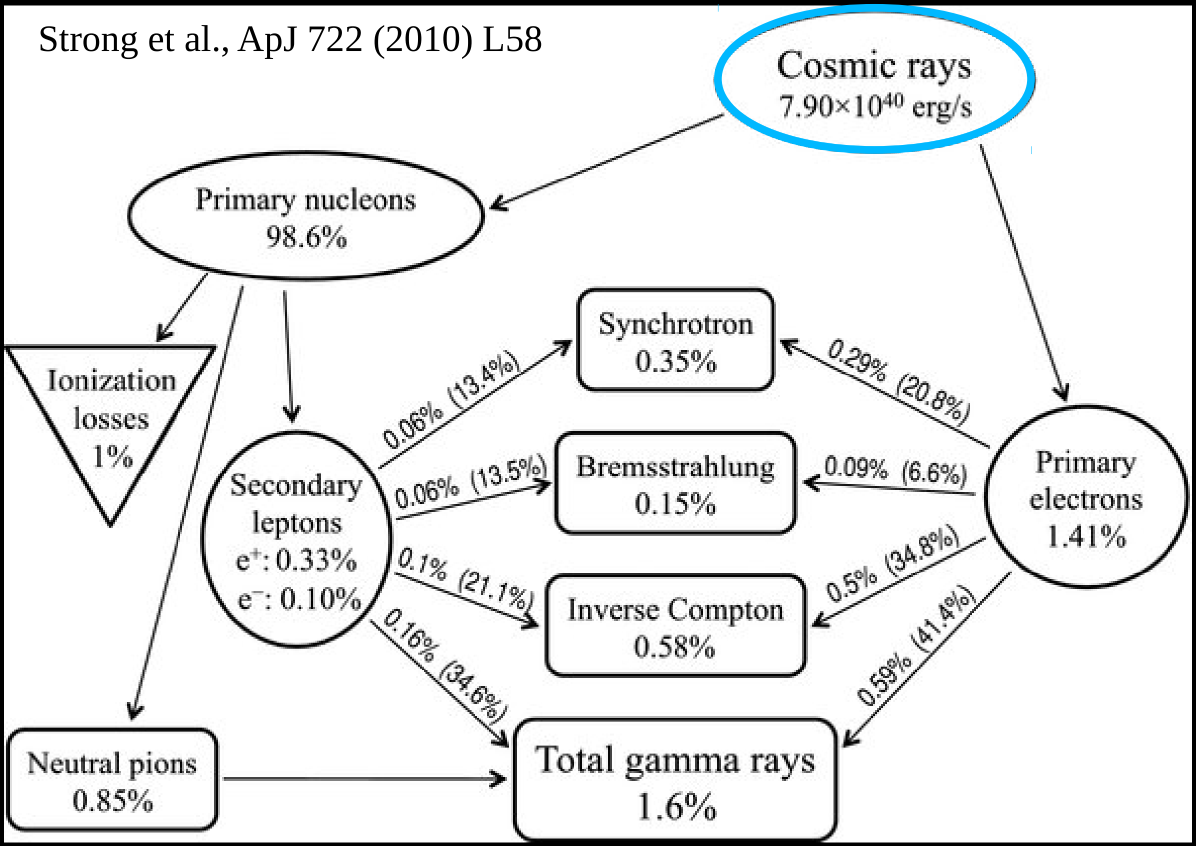
→ GALPROP run (exact numbers depend on the model used)



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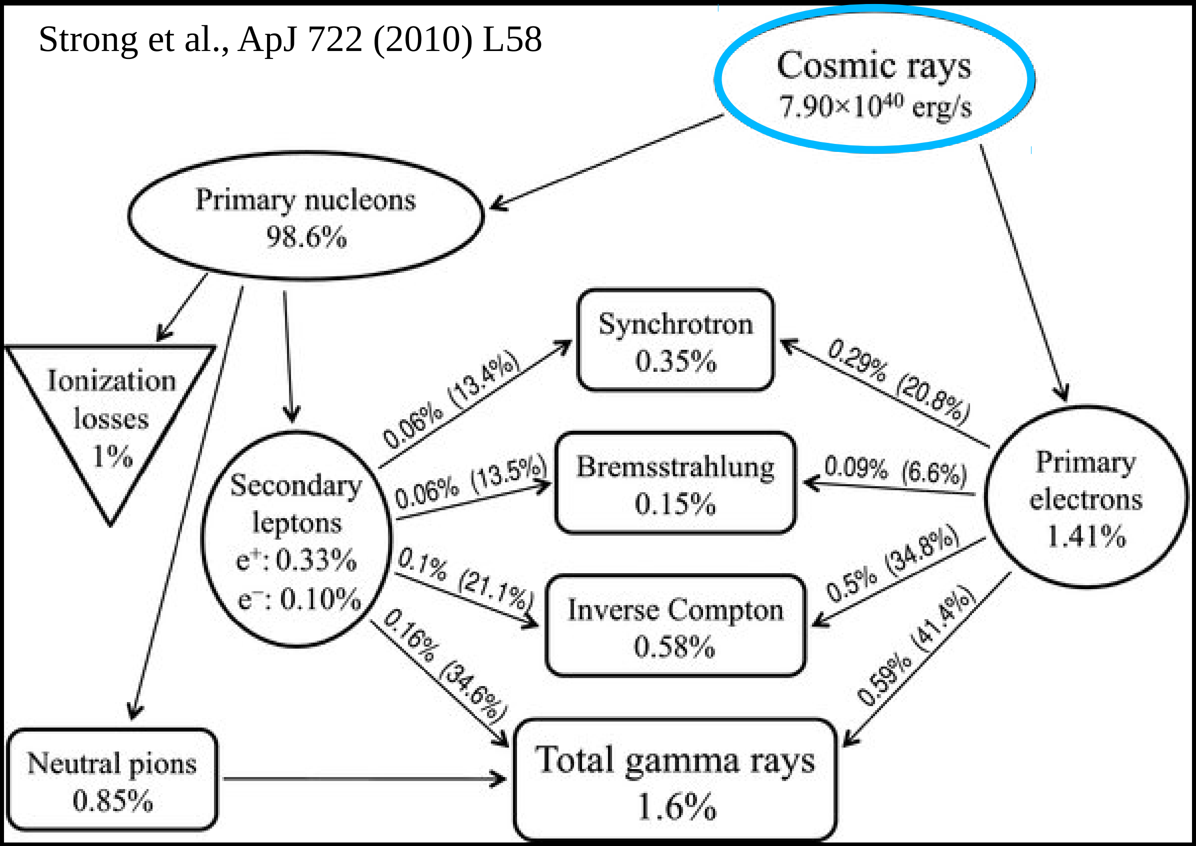

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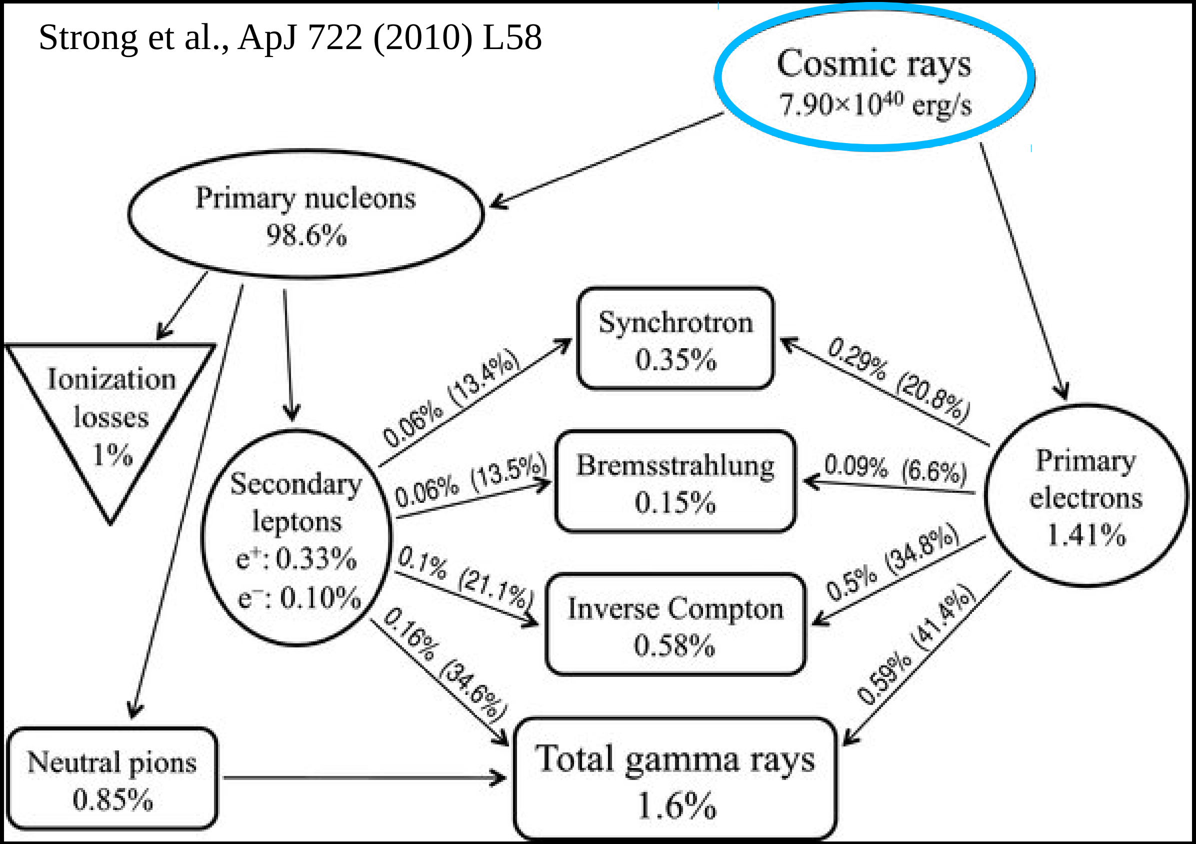

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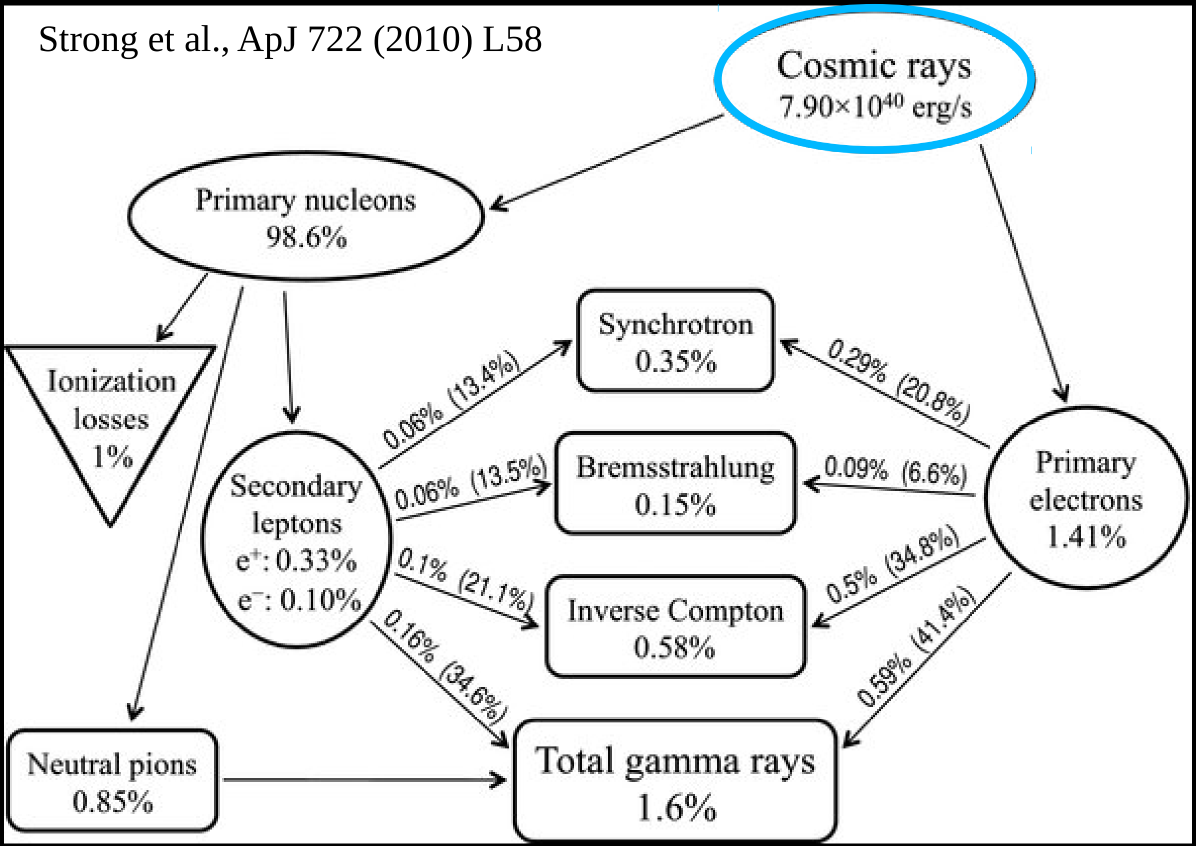

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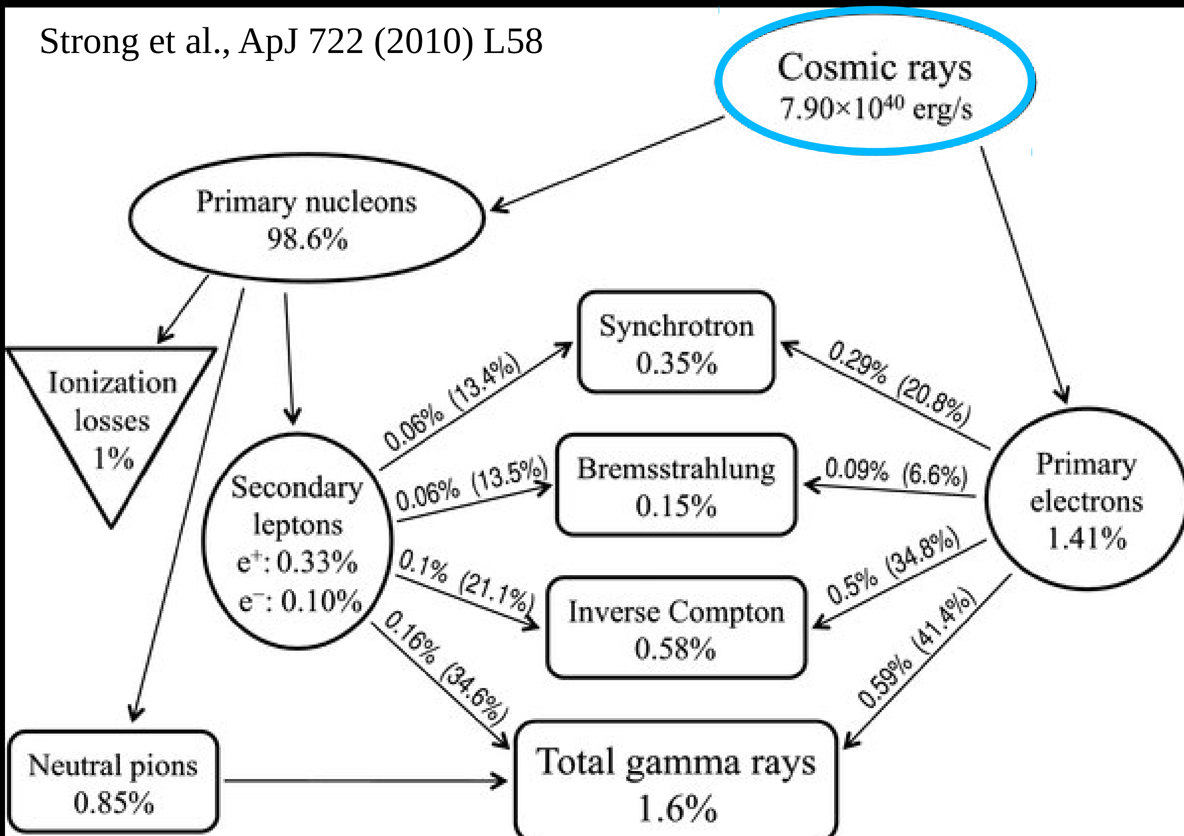

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- Total energy in Milky Way

$$V_{\rm MW} = 4\pi R^2 \times 2L \simeq 10^5 \text{ kpc}^3$$

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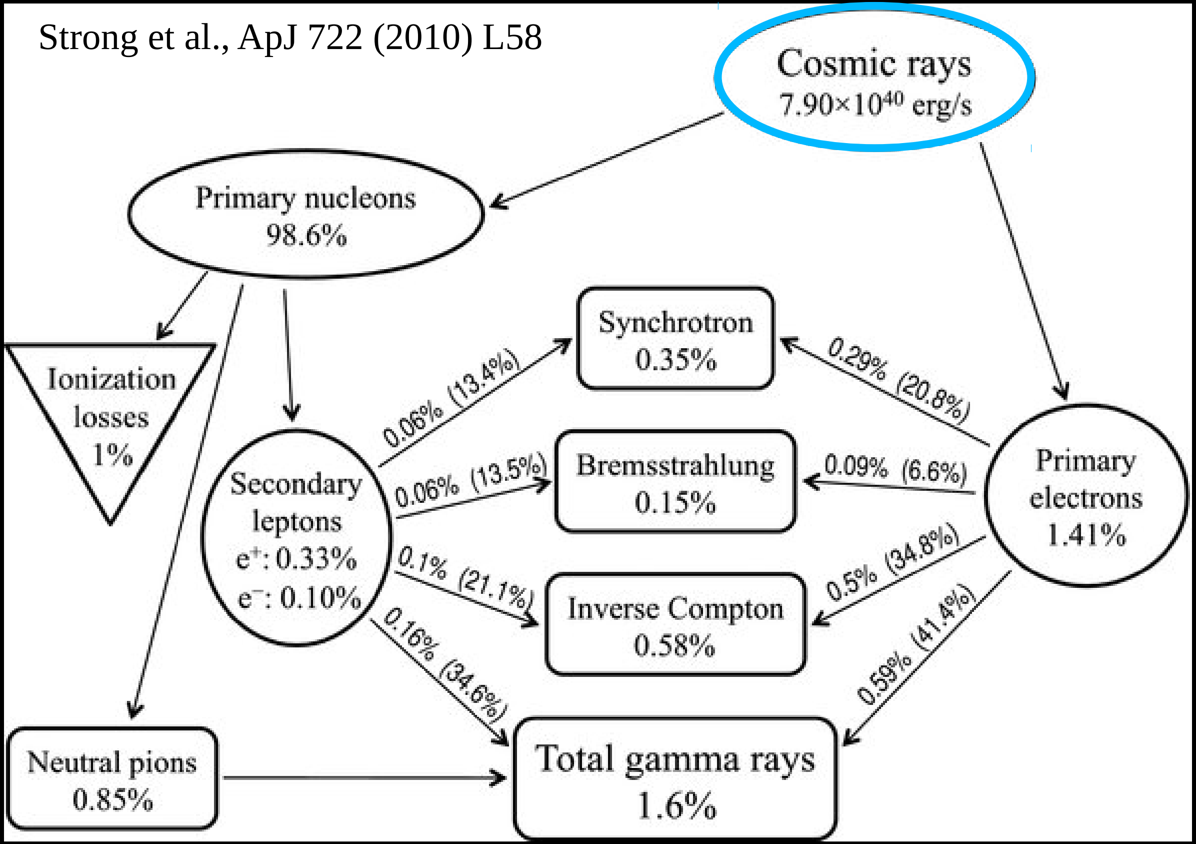

→ Very inefficient for protons (escape)
→ Very efficient for electrons (convert e^- to radiation)

Energy budget of the Galaxy

- Energy density of GCRs

$$w_{\rm CR} = \int E_k N(E) dE = \int \frac{4\pi}{v} E_k I(E) dE$$

$$w_{\rm CR} \simeq 0.9 \text{ eV cm}^3 \simeq 4 \cdot 10^{52} \text{ erg kpc}^{-3}$$

- Total energy in Milky Way

$$V_{\rm MW} = 4\pi R^2 \times 2L \simeq 10^5 \text{ kpc}^3$$

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- Power to sustain this energy

$$U_{\rm CR} \simeq \frac{W_{\rm CR}}{T_{\rm esc}} \qquad T_{\rm esc} = 20 \text{ Myr} \simeq 5 \cdot 10^{17} \text{ s}$$

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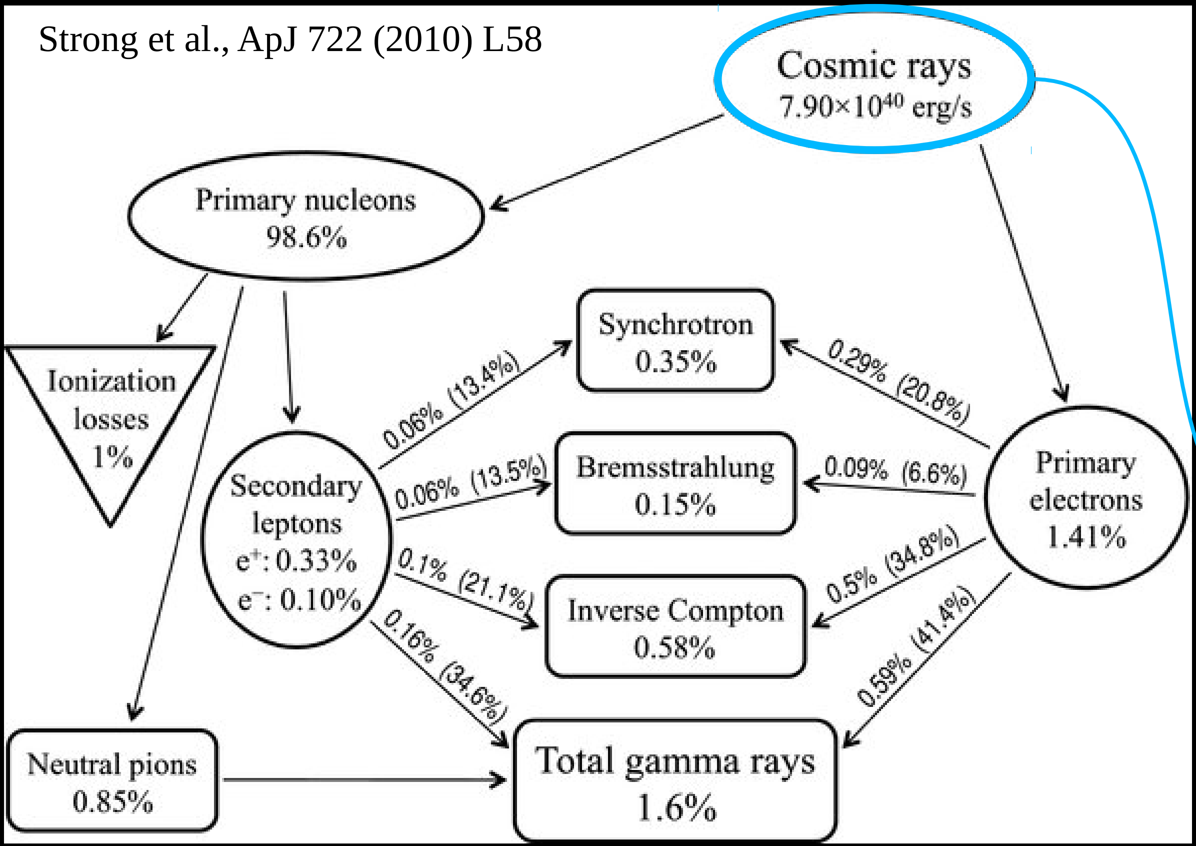

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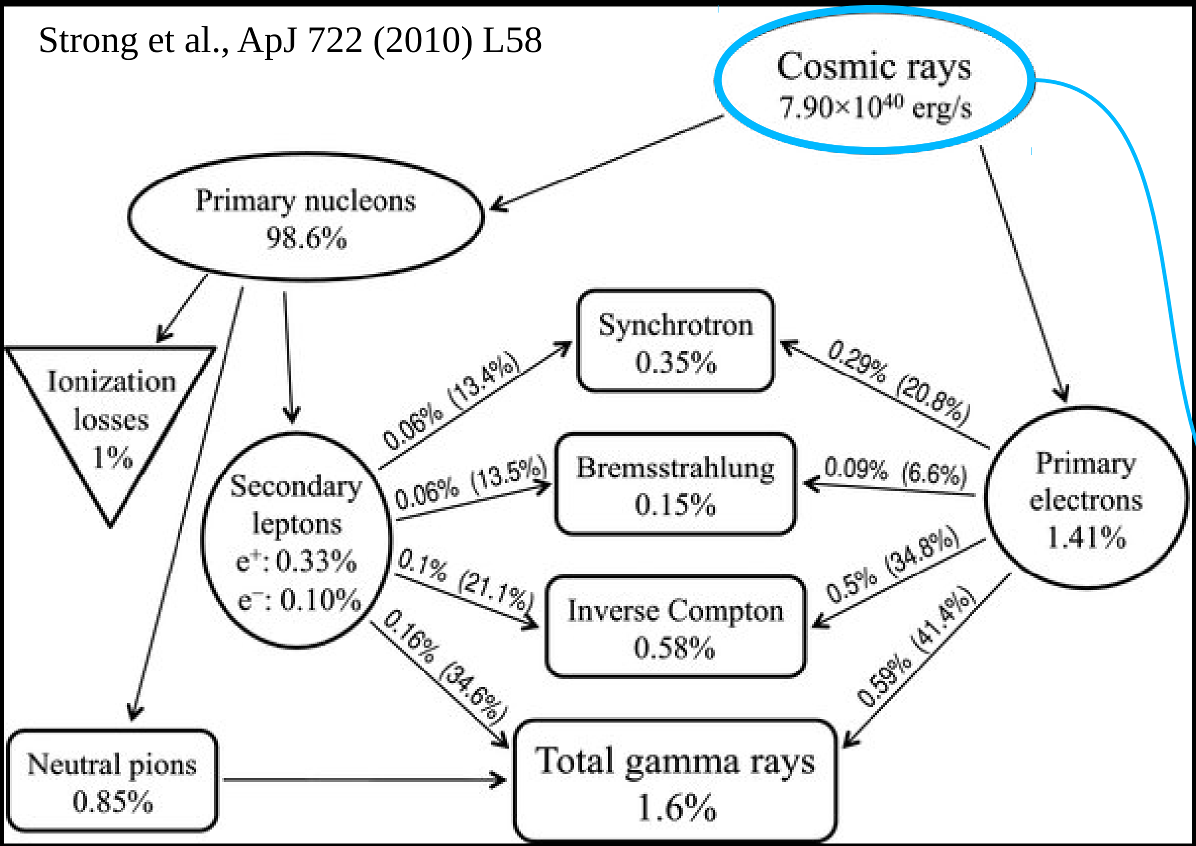

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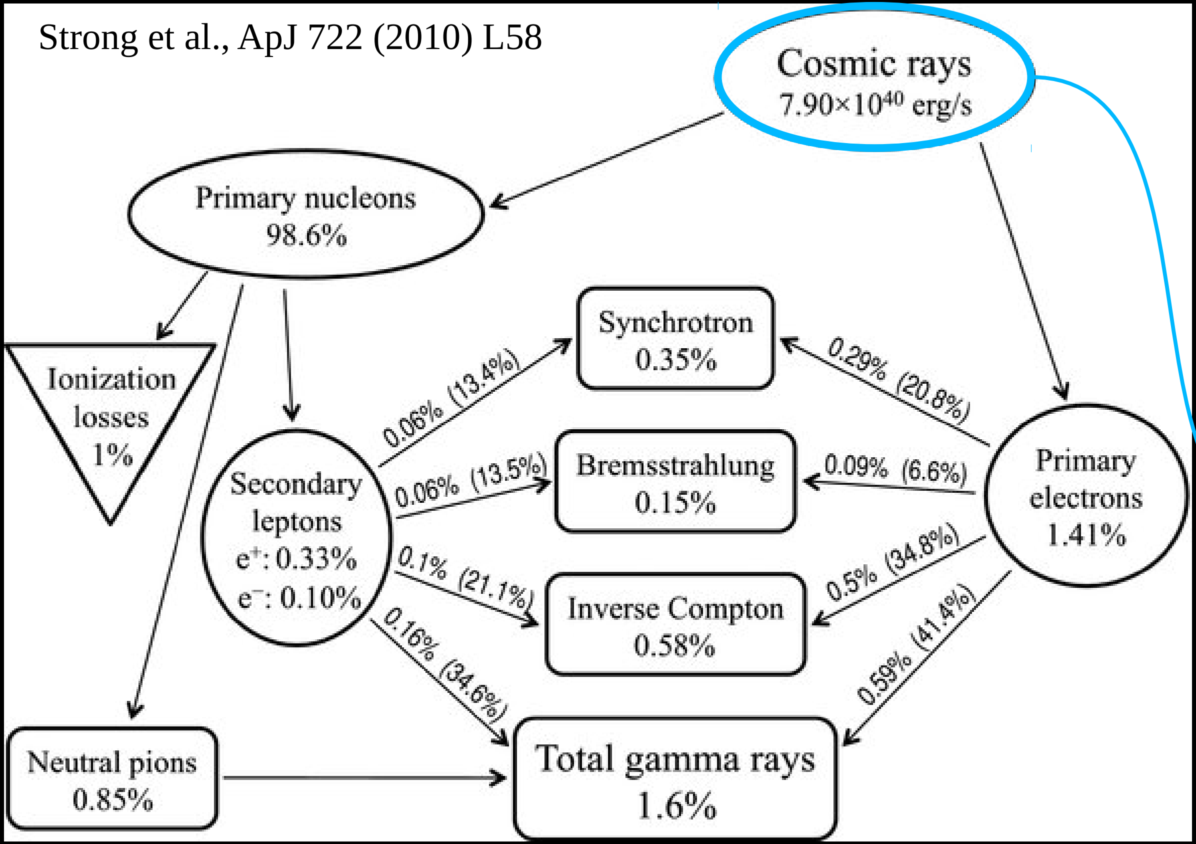

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Supernovae as source of GCRs?

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- $f_{SN} = 50$ yr $\approx 1.6\ 10^9$ s (~2 per century)
- $U_{SN} = W_{SN} / f_{SN} \approx 10^{41}$ erg/s

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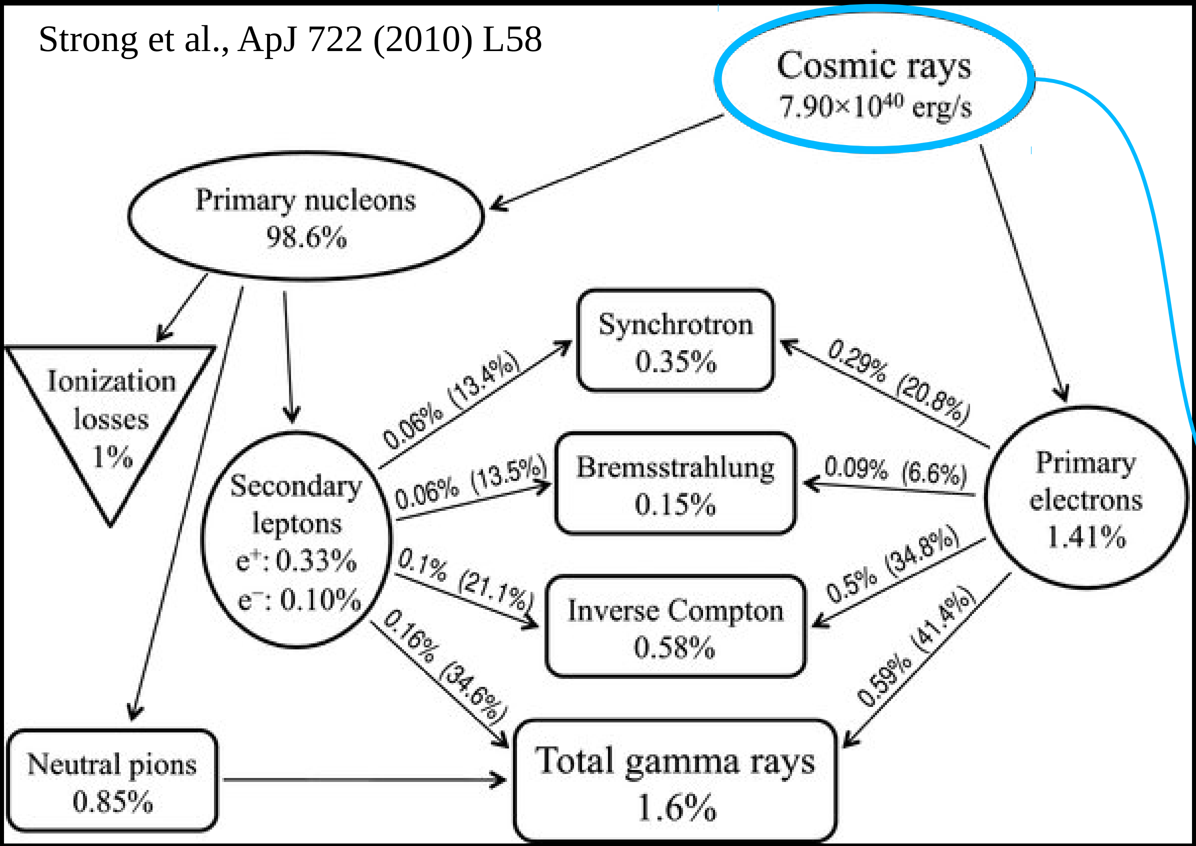

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→ Requires acceleration efficiency ~ 10-50%

Conclusions and summary

II. Processes, ingredients, characteristic times

→ In any (astro-)physics problem, always

- check the time scales;
- check the energetics.

→ For GCRs:

- nuclei: plenty of competing effects @ GeV, escape at HE;
- electrons: energy losses dominate at LE and HE.