## Lectures & objectives

ISAPP 2014 (Belgirate) 21-30 July 2014

## Transport of cosmic rays in the Galaxy and in the heliosphere (~4h30)

- What is GCR (Galactic Cosmic Ray) physics and transport
- Relevant time scales:  $\neq$  species have  $\neq$  phenomenology
- Main modelling ingredients: key parameters and uncertainties
- Tools to solve the transport equation

### Charged signals: electrons/positrons, antibaryons (~1h30)

- What is astroparticle physics and DM (Dark Matter) indirect detection
- What are the astrophysical backgrounds + uncertainties [nuclear]
- Phenomenology of DM signals + uncertainties [transport and dark matter]
- Pros and Cons of DM indirect detection with charged GCRs





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## Transport of cosmic rays (CR) in the Galaxy

#### I. Introduction; Galactic Cosmic Rays

- 1. Early history of CRs: discovery and disputes
- 2. GCR journey (from source to detector)
- 3. Timeline
- 4. Observables and questions
- II. Processes, ingredients, characteristic times
  - 1. Definitions
  - 2. Diffusion (space and momentum)
  - 3. Convection and adiabatic losses
  - 4. Energy losses
  - 5. Catastrophic losses
  - 6. All together

#### III. Solving the equations: GCR phenomenology

- 1. The full transport equation
- 2. Source terms: primary and secondary contributions
- 3. A matrix of transport equations
- 4 (Semi-)Analytical, numerical, & MC solutions
- 5. Stable species: degeneracy K<sub>0</sub> /L
- 6. Radioactive species and local ISM
- 7. Leptons and local sources

### GCRs-II.pdf

# Transport of cosmic rays (CR) in the Galaxy

## II. Processes, ingredients, characteristic times

## 1. Definitions

- 2. Diffusion (space and momentum)
- 3. Convection and adiabatic losses
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• "Energy" units

 $\begin{cases} c = \hbar = e \equiv 1 \\ m_e = 511 \text{ keV} \\ m_p = 0.938 \text{ GeV} \end{cases}$ 

$$\begin{cases} E_k(=T) = E - m \\ \beta \equiv \frac{v}{c} = \frac{p}{1E} \\ \gamma \equiv \frac{1}{\sqrt{1 - \beta^2}} = \frac{E}{m} \end{cases}$$

**II.1 Definitions** 

3	Expression	Unit	Natural for	$c = \hbar = e \equiv 1$
Rigidity	$R = \frac{pc}{Ze} = \frac{p}{Z} = r_l B$	[GV]	Acceleration, diffusion	$\begin{cases} m_e = 511 \text{ keV} \\ m_p = 0.938 \text{ GeV} \end{cases}$ $\begin{cases} E_k(=T) = E - m \\ \beta \equiv \frac{v}{c} = \frac{p}{1E} \end{cases}$
				$\int \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} = \frac{B}{m}$

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Total Energy	$E^2 = p^2 + m^2$	[GeV]	Calorimeter	$\begin{cases} E_k(=T) = E - m \\ \beta \equiv \frac{v}{c} = \frac{p}{1E} \\ \gamma \equiv \frac{\sqrt{1 - \beta^2}}{\sqrt{1 - \beta^2}} = \frac{E}{m} \end{cases}$

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Kinetic E per nucleon	$E_{k/n}(=T) = \frac{E_k}{A}$	[GeV/n]	CR fragmentation on ISM	$\int \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{m}$

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### • CR intensity

Intensity: 
$$I = \# \text{ particles } \text{m}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$$
  
Differential intensity:  $\frac{dI}{d\mathcal{E}} = \# \text{ particles } \text{m}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \mathcal{E}^{-1}$   
Differential density:  $N = \frac{dN}{d\mathcal{E}} = \frac{4\pi}{v} \frac{dI}{dE}$  (# particules  $\text{m}^{-3} \mathcal{E}^{-1}$ )

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- 1. Suppose IS flux is  $dI/dR = I_0 R^{-\gamma}$ : express dI/dR in terms of  $dI/dE_{k/n}$
- 2. If  $R_{max}$  maximum rigidity of a CR source, at which E is there a cut-off in EAS?



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### • Diffusion (or Heat) equation

$$\begin{bmatrix} \text{continuity} \end{bmatrix} \begin{cases} \frac{\partial N(r,t)}{\partial t} + \vec{\nabla} \cdot \vec{j_d} = 0 \\ \vec{j_d} = -D\vec{\nabla}N(r,t) \end{cases} \qquad \frac{\partial N(r,t)}{\partial t} - \vec{\nabla} \cdot [D\vec{\nabla}N(r,t)] = 0 \end{cases}$$

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#### Comments

- Dimensionality
- Geometry
- D(r) spatial dependenceBoundary conditions

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- 1. Write equation in 1D
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N.B.: boundary conditions are

$$N(z,t) = \frac{N_0}{(4\pi Dt)^{1/2}} \exp\left(\frac{-z^2}{4Dt}\right)$$
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#### • Mean distance

Calculate 
$$\langle z(t) \rangle$$
 and  $\langle z^2(t) \rangle$ , using  $\int_{-\infty}^{-\infty} e^{-x^2/A} dx = \sqrt{A\pi}$ 

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N.B.: solution (behaviour) depends on

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- Analog to spatial diffusion (+ drift): 
$$D_{EE} \equiv \frac{1}{2} \left\langle \frac{(\Delta E)^2}{\Delta t} \right\rangle - d_{\text{diff}} = \sqrt{2Dt}$$

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 $\frac{\partial N}{\partial t} = \frac{\partial}{\partial E} \left( - \left\langle \frac{\Delta E}{\Delta t} \right\rangle N \right) + \frac{\partial}{\partial E} \left( D_{EE} \frac{\partial N}{\partial E} \right)$ 

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- Natural mechanisms for reacceleration: Fermi 2<sup>nd</sup> order
  - CR (v) collides with magnetic scatterer (V):  $\Delta E = \frac{2Vvcos\theta}{c^2} + 2\left(\frac{V}{v}\right)^2$

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$$\mathbf{Fermi 2} \\ \beta = V/c \sim 10^{-4} \\ \Delta E/E \propto \beta^2$$
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II 2 Diffusion

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[Thornbury & Drury, MNRAS 442 (2014) 3010]

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Order of magnitude for VA  

$$<\Delta E >^2 = 2D_{EE}t \approx \frac{2}{9} \frac{p^2}{DV_A^2} t$$

$$\begin{cases} <\Delta E > ~1 \text{ GeV} \\ T_{esc} \sim 50 \text{ Myr} \\ D_0 \simeq 0.05 \text{ kpc}^2 \text{ Myr}^{-1} \end{cases} \rightarrow \text{VA} \sim 10 \text{ km/s} \\ 1 \text{ km s}^{-1} = 10^{-3} \text{kpc Myr}^{-1} \end{cases}$$

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$$\rightarrow \text{Energy gain } @ \text{ GeV/n} \\ \rightarrow \text{Strength mediated by V}_{A}$$

# Transport of cosmic rays (CR) in the Galaxy

## II. Processes, ingredients, characteristic times

- 1. Definitions
- 2. Diffusion (space and momentum)
- 3. Convection and adiabatic losses
- 4. Energy losses
- 5. Catastrophic losses
- 6. All together

• Advection/diffusion or drift/diffusion (or Smoluchowski equation)

 $\begin{bmatrix} \text{continuity} \end{bmatrix} \begin{cases} \frac{\partial N(r,t)}{\partial t} + \vec{\nabla}.\vec{j_d} = 0 \\ \vec{j_d} = -D\vec{\nabla}N(r,t) & \frac{\partial N}{\partial t} - \vec{\nabla}.[D\vec{\nabla}N + \vec{V_c}N] = 0 \\ \vec{j_c} = \vec{V_c}N(r,t) & \frac{\partial N}{\partial t} - \vec{\nabla}.[D\vec{\nabla}N + \vec{V_c}N] = 0 \end{cases}$ 

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 $\begin{array}{l} \mbox{[continuity]} \\ \mbox{[Fick's law]} \\ \mbox{[drift]} \end{array} \left\{ \begin{array}{l} \displaystyle \frac{\partial N(r,t)}{\partial t} + \vec{\nabla}.\vec{j_d} = 0 \\ \\ \displaystyle \vec{j_d} = -D\vec{\nabla}N(r,t) \\ \\ \displaystyle \vec{j_c} = \vec{V_c}N(r,t) \end{array} \right. \quad \begin{array}{l} \displaystyle \frac{\partial N}{\partial t} - \vec{\nabla}.\left[D\vec{\nabla}N + \vec{V_c}N\right] = 0 \\ \\ \displaystyle \vec{j_c} = \vec{V_c}N(r,t) \end{array} \right.$ 

#### Comments

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#### • Characteristic time scales: diffusion vs convection

- Competition D vs V

- N.B.:  $D(R) = \beta D_0 R^{\delta}$ 

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#### • Characteristic time scales: diffusion vs convection

- Competition D vs V  $\rightarrow t_{diff} \text{ and } t_{conv}$ ? - N.B.: D(R) =  $\beta D_0 R^{\delta}$ 



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#### Numerical application (Galaxy)

$$\begin{cases} L \simeq 10 \text{ kpc} & \text{Halo half-size} \\ D_0 \simeq 0.05 \text{ kpc}^2 \text{ Myr}^{-1} & \text{Diffusion} \\ \delta \simeq 0.5 & \text{Diffusion slope} \\ V_c \simeq 10 \text{ km s}^{-1} & \text{Convection} \end{cases}$$

$$1 \text{ km s}^{-1} = 10^{-3} \text{kpc Myr}^{-3}$$
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### • Characteristic time scales: diffusion vs convection

- Competition D vs V  $t_{\text{diff}} \simeq 300 \left(\frac{L}{10 \text{ kpc}}\right)^2 \left(\frac{D_0}{0.05 \text{ kpc}^2 \text{ Myr}^{-1}}\right)^{-1} \left(\frac{R}{1 \text{ GV}}\right)^{-\delta} \text{Myr}$ - N.B.: D(R) =  $\beta D_0 R^{\delta}$  $t_{\text{conv}} \simeq 10^3 \left(\frac{L}{10 \text{ kpc}}\right) \left(\frac{V_c}{10 \text{ km s}^{-1}}\right)^{-1} \text{Myr}$ 

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### Numerical application (Galaxy)

$$\begin{cases} L \simeq 10 \text{ kpc} & \text{Halo half-size} \\ D_0 \simeq 0.05 \text{ kpc}^2 \text{ Myr}^{-1} & \text{Diffusion} \\ \delta \simeq 0.5 & \text{Diffusion slope} \\ V_c \simeq 10 \text{ km s}^{-1} & \text{Convection} \end{cases}$$

$$1 \text{ km s}^{-1} = 10^{-3} \text{kpc Myr}^{-3}$$

### • Advection/diffusion or drift/diffusion (or Smoluchowski equation)

 $\begin{bmatrix} \text{continuity} \end{bmatrix} \begin{cases} \frac{\partial N(r,t)}{\partial t} + \vec{\nabla}.\vec{j_d} = 0 \\ \vec{j_d} = -D\vec{\nabla}N(r,t) \\ \vec{j_c} = \vec{V_c}N(r,t) \end{cases} \quad \frac{\partial N}{\partial t} - \vec{\nabla}.[D\vec{\nabla}N + \vec{V_c}N] = 0 \\ \vec{j_c} = \vec{V_c}N(r,t) \end{cases}$ 

### • Characteristic time scales: diffusion vs convection

- Competition D vs V - N.B.: D(R) =  $\beta D_0 R^{\delta}$   $t_{diff} \simeq 300 \left(\frac{L}{10 \text{ kpc}}\right)^2 \left(\frac{D_0}{0.05 \text{ kpc}^2 \text{ Myr}^{-1}}\right)^{-1} \left(\frac{R}{1 \text{ GV}}\right)^{-\delta} \text{Myr}$   $t_{conv} \simeq 10^3 \left(\frac{L}{10 \text{ kpc}}\right) \left(\frac{V_c}{10 \text{ km s}^{-1}}\right)^{-1} \text{Myr}$ 



### Comments

- N.B.: solution (behaviour) depends on
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### Numerical application (Galaxy)

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• Adiabatic losses (in expanding plasma)

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### • Adiabatic losses (in expanding plasma)

V: volume N (# particles) = nV U (internal energy) = NE E (average energy/particle) Non-relativistic E = 3/2 kT P = NkT/VP=2/3 nE

### Comments

- N.B.: solution (behaviour) depends on
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V: volume N (# particles) = nV U (internal energy) = NE E (average energy/particle) Non-relativistic E = 3/2 kT P = NkT/VP=2/3 nE Relativistic U=3NkT P = 1/3 U

### Comments

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### Numerical application (Galaxy)

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**II.3** Convection

### • Advection/diffusion or drift/diffusion (or Smoluchowski equation)

 $\begin{bmatrix} \text{continuity} \end{bmatrix} \begin{cases} \frac{\partial N(r,t)}{\partial t} + \vec{\nabla}.\vec{j_d} = 0 \\ \vec{j_d} = -D\vec{\nabla}N(r,t) \\ \vec{j_c} = \vec{V_c}N(r,t) \end{cases} \quad \frac{\partial N}{\partial t} - \vec{\nabla}.\left[D\vec{\nabla}N + \vec{V_c}N\right] = 0 \\ \vec{j_c} = \vec{V_c}N(r,t) \end{cases}$ 

### • Characteristic time scales: diffusion vs convection

- Competition D vs V  $t_{\text{diff}} \approx 300 \left(\frac{L}{10 \text{ kpc}}\right)^2 \left(\frac{D_0}{0.05 \text{ kpc}^2 \text{ Myr}^{-1}}\right)^{-1} \left(\frac{R}{1 \text{ GV}}\right)^{-\delta} \text{Myr}$ - N.B.: D(R) =  $\beta D_0 R^{\delta}$  $t_{\text{conv}} \approx 10^3 \left(\frac{L}{10 \text{ kpc}}\right) \left(\frac{V_c}{10 \text{ km s}^{-1}}\right)^{-1} \text{Myr}$ 

### • Adiabatic losses (in expanding plasma)

V: volumeNon-relativisticRelativisticN (# particles) = nVE = 3/2 kTU=3NkTU (internal energy) = NEP = NkT/VP = 1/3 UE (average energy/particle)P=2/3 nEP=2/3 nEAdiabatic: dU = -PdV (work done by gas)  $\rightarrow$  NdE = -2/3 nE dV

### Comments

- N.B.: solution (behaviour) depends on
  - Dimensionality
  - Geometry
  - D(r) spatial dependence
  - V spatial dependence and direction
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### Numerical application (Galaxy)

$$\begin{cases} L \simeq 10 \text{ kpc} & \text{Halo half-size} \\ D_0 \simeq 0.05 \text{ kpc}^2 \text{ Myr}^{-1} & \text{Diffusion} \\ \delta \simeq 0.5 & \text{Diffusion slope} \\ V_c \simeq 10 \text{ km s}^{-1} & \text{Convection} \end{cases}$$

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 $\begin{bmatrix} \text{continuity} \end{bmatrix} \begin{cases} \frac{\partial N(r,t)}{\partial t} + \vec{\nabla}.\vec{j_d} = 0 \\ \vec{j_d} = -D\vec{\nabla}N(r,t) \\ \vec{j_c} = \vec{V_c}N(r,t) \end{cases} \quad \frac{\partial N}{\partial t} - \vec{\nabla}.[D\vec{\nabla}N + \vec{V_c}N] = 0 \\ \vec{j_c} = \vec{V_c}N(r,t) \end{cases}$ 

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V: volume N (# particles) = nV U (internal energy) = NE E (average energy/particle) Adiabatic: dU = -PdV (work done by gas)  $\rightarrow$  NdE = -2/3 nE dV  $\frac{dE}{dt} = -\frac{2}{3}\frac{nE}{N}\frac{dV}{dt}$ 

### Comments

- N.B.: solution (behaviour) depends on
  - Dimensionality
  - Geometry
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### Numerical application (Galaxy)

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 $\begin{bmatrix} \text{continuity} \end{bmatrix} \begin{cases} \frac{\partial N(r,t)}{\partial t} + \vec{\nabla}.\vec{j_d} = 0 \\ \vec{j_d} = -D\vec{\nabla}N(r,t) & \frac{\partial N}{\partial t} - \vec{\nabla}.\left[D\vec{\nabla}N + \vec{V_c}N\right] = 0 \\ \vec{j_c} = \vec{V_c}N(r,t) & \frac{\partial N}{\partial t} - \vec{\nabla}.\left[D\vec{\nabla}N + \vec{V_c}N\right] = 0 \end{cases}$ 

### Characteristic time scales: diffusion vs convection

- Competition D vs V  $t_{\rm diff} \simeq 300 \left(\frac{L}{10 \text{ kpc}}\right)^2 \left(\frac{D_0}{0.05 \text{ kpc}^2 \text{ Myr}^{-1}}\right)^{-1} \left(\frac{R}{1 \text{ GV}}\right)^{-\delta} \text{Myr}$  $t_{\rm conv} \simeq 10^3 \left(\frac{L}{10 \text{ kpc}}\right) \left(\frac{V_c}{10 \text{ km s}^{-1}}\right)^{-1} \text{Myr}$ - N.B.:  $D(R) = \beta D_0 R^{\delta}$ 

### • Adiabatic losses (in expanding plasma)

V: volume Non-relativistic N (# particles) = nV E = 3/2 kTU (internal energy) = NE P = NkT/VE (average energy/particle) P=2/3 nEAdiabatic: dU = -PdV (work done by gas)  $\rightarrow NdE = -2/3$  nE dV dE2 nE dV $\overline{dt} = -\overline{3} \overline{N} \overline{dt}$ 

### **Comments**

- *N.B.: solution (behaviour) depends on* 
  - Dimensionality
  - Geometry

Relativistic

U=3NkT

P = 1/3 U

- D(**r**) spatial dependence
- V spatial dependence and direction
- Boundary conditions

### *Numerical application (Galaxy)*

 $L \simeq 10 \text{ kpc}$  Halo half  $D_0 \simeq 0.05 \text{ kpc}^2 \text{ Myr}^{-1}$  Diffusion Halo half-size  $\begin{array}{ll} \delta\simeq 0.5 \\ V_c\simeq 10 \ {\rm km \ s^{-1}} \end{array} \qquad \begin{array}{ll} {\rm Diffusion \ slope} \\ {\rm Convection} \end{array}$ 

$$1 \text{ km s}^{-1} = 10^{-3} \text{kpc Myr}^{-1}$$

1. Rate of expansion in velocity field 
$$\mathbf{v}(\mathbf{r})$$
  

$$\frac{dV}{dt} = (v_{x+dx} - v_x)dydz + \cdots$$

$$= \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)dxdydz = \left(\vec{\nabla} \cdot \vec{v}(\vec{r})\right)V$$

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\end{cases}$ Non-relativistic E = 3/2 kT U = 3NkT P = NkT/V P = 1/3 U P=2/3 nE Adiabatic: dU = -PdV (work done by gas)  $\rightarrow$  NdE = -2/3 nE dV  $\begin{cases}
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### Comments

- N.B.: solution (behaviour) depends on
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### Numerical application (Galaxy)

 $\begin{cases} L \simeq 10 \text{ kpc} & \text{Halo half-size} \\ D_0 \simeq 0.05 \text{ kpc}^2 \text{ Myr}^{-1} & \text{Diffusion} \\ \delta \simeq 0.5 & \text{Diffusion slope} \\ V_c \simeq 10 \text{ km s}^{-1} & \text{Convection} \end{cases}$ 

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1. Rate of expansion in velocity field 
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### Comments

- N.B.: solution (behaviour) depends on
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- 1. Rate of expansion in velocity field  $\mathbf{v}(\mathbf{r})$  $\frac{dV}{dt} = (v_{x+dx} - v_x)dydz + \cdots$   $= \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)dxdydz = \left(\vec{\nabla} \cdot \vec{v}(\vec{r})\right)V$
- 2. Application: spherical case,  $v(r) = v_0$

• Use 
$$(\vec{\nabla} \cdot \vec{v})_r = \frac{1}{r^2} \frac{\partial (r^2 v_r)}{\partial r}$$

• Calculate E=f(r)



### **II.3** Convection

### • Advection/diffusion or drift/diffusion (or Smoluchowski equation)

 $\begin{bmatrix} \text{continuity} \end{bmatrix} \begin{cases} \frac{\partial N(r,t)}{\partial t} + \vec{\nabla}.\vec{j_d} = 0 \\ \vec{j_d} = -D\vec{\nabla}N(r,t) \\ \vec{j_c} = \vec{V_c}N(r,t) \end{cases} \quad \frac{\partial N}{\partial t} - \vec{\nabla}.[D\vec{\nabla}N + \vec{V_c}N] = 0 \\ \vec{j_c} = \vec{V_c}N(r,t) \end{cases}$ 

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V: volume N (# particles) = nV U (internal energy) = NE E (average energy/particle) Adiabatic: dU = -PdV (work done by gas)  $\rightarrow$  NdE = -2/3 nE dV  $\begin{cases}
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### Comments

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### Numerical application (Galaxy)

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P. Application: spherical case, 
$$v(r) = v_0$$
  
• Use  $(\vec{\nabla} \cdot \vec{v})_r = \frac{1}{r^2} \frac{\partial (r^2 v_r)}{\partial r}$   
• Calculate E=f(r)  $E = E_0 \left(\frac{r_0}{r}\right)^{4/3}$ 

**II.3** Convection

### • Advection/diffusion or drift/diffusion (or Smoluchowski equation)

 $\begin{bmatrix} \text{continuity} \end{bmatrix} \begin{cases} \frac{\partial N(r,t)}{\partial t} + \vec{\nabla}.\vec{j_d} = 0 \\ \vec{j_d} = -D\vec{\nabla}N(r,t) \\ \vec{j_c} = \vec{V_c}N(r,t) \end{cases} \quad \frac{\partial N}{\partial t} - \vec{\nabla}.[D\vec{\nabla}N + \vec{V_c}N] = 0 \\ \vec{j_c} = \vec{V_c}N(r,t) \end{cases}$ 

### • Characteristic time scales: diffusion vs convection

- Competition D vs V  $t_{\text{diff}} \simeq 300 \left(\frac{L}{10 \text{ kpc}}\right)^2 \left(\frac{D_0}{0.05 \text{ kpc}^2 \text{ Myr}^{-1}}\right)^{-1} \left(\frac{R}{1 \text{ GV}}\right)^{-\delta} \text{ Myr}$   $- \text{ N.B.: } D(R) = \beta D_0 R^{\delta}$   $t_{\text{conv}} \simeq 10^3 \left(\frac{L}{10 \text{ kpc}}\right) \left(\frac{V_c}{10 \text{ km s}^{-1}}\right)^{-1} \text{ Myr}$ 

### • Adiabatic losses (in expanding plasma)

V: volume N (# particles) = nV U (internal energy) = NE E (average energy/particle) Adiabatic: dU = -PdV (work done by gas)  $\rightarrow$  NdE = -2/3 nE dV  $\begin{cases} \frac{dE}{dt} = -\frac{2}{3}\frac{nE}{N}\frac{dV}{dt} \\ \frac{dV}{dt} = (\vec{\nabla} \cdot \vec{v}(\vec{r}))V \end{cases}$   $\frac{dE}{dt} = -\frac{2}{3}(\vec{\nabla} \cdot \vec{v}(\vec{r}))E \qquad -\frac{1}{3}(\vec{\nabla} \cdot \vec{v}(\vec{r}))E \end{cases}$ 

### Comments

- N.B.: solution (behaviour) depends on
  - Dimensionality
  - Geometry
  - D(r) spatial dependence
  - V spatial dependence and direction
  - Boundary conditions

### Numerical application (Galaxy)

 $\begin{cases} L \simeq 10 \text{ kpc} & \text{Halo half-size} \\ D_0 \simeq 0.05 \text{ kpc}^2 \text{ Myr}^{-1} & \text{Diffusion} \\ \delta \simeq 0.5 & \text{Diffusion slope} \\ V_c \simeq 10 \text{ km s}^{-1} & \text{Convection} \end{cases}$ 

 $1 \text{ km s}^{-1} = 10^{-3} \text{kpc Myr}^{-1}$ 

1. Rate of expansion in velocity field  $\mathbf{v}(\mathbf{r})$  $\frac{dV}{dt} = (v_{x+dx} - v_x)dydz + \cdots$   $= \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)dxdydz = \left(\vec{\nabla} \cdot \vec{v}(\vec{r})\right)V$ 

2. Application: spherical case, 
$$v(r) = v_0^{-1}$$
  
• Use  $(\vec{\nabla} \cdot \vec{v})_r = \frac{1}{r^2} \frac{\partial (r^2 v_r)}{\partial r}^{-1}$   
• Calculate E=f(r)  $E = E_0 \left(\frac{r_0}{r}\right)^{4/3}$   
 $\rightarrow$  Solar modulation: CRs loose  
energy in expanding Solar wind  
II.3 Convection

## Transport of cosmic rays (CR) in the Galaxy

### II. Processes, ingredients, characteristic times

- 1. Definitions
- 2. Diffusion (space and momentum)
- 3. Convection and adiabatic losses
- 4. Energy losses
- 5. Catastrophic losses
- 6. All together

### Synchrotron



### **Inverse Compton**









### Synchrotron

- Power emitted || and H to B (polarised emission)
- If (E)<sup>-s</sup>  $\rightarrow$   $(\varepsilon_{\gamma})^{(p-1)/2}$
- $v(P_{max}) \sim 2\gamma^2 (B/1\mu G) Hz \sim 300 \text{ MHz} \text{ (for 100 MeV e}^-)$



### **Inverse Compton**









### Synchrotron

408 MHz

- Power emitted || and H to B (polarised emission)
   If (E)<sup>-s</sup> → (ε<sub>γ</sub>)<sup>(p-1)/2</sup>
- $v(P_{max}) \sim 2\gamma^2 (B/1\mu G) \text{ Hz} \sim 300 \text{ MHz} \text{ (for 100 MeV e}^-)$





### **Inverse Compton**









### Synchrotron

408 MHz

- Power emitted || and H to B (polarised emission)
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- $v(P_{max}) \sim 2\gamma^2 (B/1\mu G) Hz \sim 300 MHz (for 100 MeV e)$

## electron B radio waves

### Ingredients

### **B** tracers

- Faraday rotation: free e (ionised regions)
- Synchrotron emission: CR e
- Zeeman splitting: lines (neutral regions)
- Dust thermal emission, starlight polar.

### **Inverse Compton**









### Synchrotron

408 MHz

- Power emitted || and H to B (polarised emission)
- If (E)<sup>-s</sup>  $\rightarrow (\epsilon_{\gamma})^{(p-1)/2}$
- $v(P_{max}) \sim 2\gamma^2 (B/1\mu G) Hz \sim 300 \text{ MHz} \text{ (for 100 MeV e}^-\text{)}$



### Ingredients

### **B** tracers

- Faraday rotation: free e<sup>-</sup> (ionised regions)
- Synchrotron emission: CR e<sup>-</sup>
- Zeeman splitting: lines (neutral regions)
- Dust thermal emission, starlight polar.

### Uncertainties $[2 \ \mu G < B_{sync} < 6 \ \mu G]$

- Geometry (z dependence)
- Arm-interarm strenght
- Regular vs irregular component

### **Inverse Compton**

ev'

### **Bremsstrahlung (or free-free)**

**Ionisation and Coulomb** 





### Synchrotron

- Power emitted || and H to B (polarised emission)
- If (E)<sup>-s</sup>  $\rightarrow$   $(\varepsilon_{\gamma})^{(p-1)/2}$
- $v(P_{max}) \sim 2\gamma^2 (B/1\mu G) Hz \sim 300 \text{ MHz} (\text{for } 100 \text{ MeV e}^-)$

 $-\frac{dE}{dt}_{\rm sync} \propto \sigma_{\rm T} B_{\perp}^2 \gamma$ 

408 MHz

### **Inverse Compton**

**Ionisation and Coulomb** 

**Bremsstrahlung (or free-free)** 



### Ingredients

### **B** tracers

- Faraday rotation: free e<sup>-</sup> (ionised regions)
- Synchrotron emission: CR e<sup>-</sup>
- Zeeman splitting: lines (neutral regions)
- Dust thermal emission, starlight polar.

### Uncertainties $[2 \ \mu G < B_{sync} < 6 \ \mu G]$

- Geometry (z dependence)
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- Regular vs irregular component







### Synchrotron

- Power emitted || and H to B (polarised emission)
- If (E)<sup>-s</sup>  $\rightarrow (\varepsilon_{\gamma})^{(p-1)/2}$ •  $\nu(P_{max}) \sim 2\gamma^2 (B/1\mu G) Hz \sim 300 \text{ MHz} (\text{for 100 MeV e}^-)$

$$-\frac{dE}{dt}_{
m sync} \propto \sigma_{
m T} B_{\perp}^2 \gamma$$

408 MHz 🔹

$$_{\rm c} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{B_{\perp}}{3 \,\mu\text{G}}\right)^{-1} \text{ Myr}$$

### **Inverse Compton**



### **B** tracers

- Faraday rotation: free e<sup>-</sup> (ionised regions)
- Synchrotron emission: CR e<sup>-</sup>
- Zeeman splitting: lines (neutral regions)
- Dust thermal emission, starlight polar.

### Uncertainties $[2 \ \mu G < B_{sync} < 6 \ \mu G]$

- Geometry (z dependence)
- Arm-interarm strenght
- Regular vs irregular component



electron

radio

waves

### **Bremsstrahlung (or free-free)**

**Ionisation and Coulomb** 





### Synchrotron

- Power emitted || and H to B (polarised emission)
- If (E)<sup>-s</sup>  $\rightarrow (\epsilon_{\gamma})^{(p-1)/2}$ •  $v(P_{max}) \sim 2\gamma^2 (B/1\mu G) Hz \sim 300 \text{ MHz} (\text{for } 100 \text{ MeV e}^-)$

## $-\frac{dE}{dt}_{ m sync} \propto \sigma_{ m T} B_{\perp}^2 \gamma^2$

408 MHz

 $t_{\rm sync} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{B_{\perp}}{3 \,\mu\text{G}}\right)^{-1} \text{ Myr}$ 

Inverse Compton (2 regimes) Thomson

Klein-Nishina



electron

radio

waves

**Bremsstrahlung (or free-free)** 



e-



### Ingredients

### **B** tracers

- Faraday rotation: free e<sup>-</sup> (ionised regions)
- Synchrotron emission: CR e
- Zeeman splitting: lines (neutral regions)
- Dust thermal emission, starlight polar.

### Uncertainties $[2 \mu G < B_{sync} < 6 \mu G]$

- Geometry (z dependence)
- Arm-interarm strenght
- Regular vs irregular component

hina

### Synchrotron

- Power emitted || and H to B (polarised emission)
- If  $(E)^{-s} \rightarrow (\epsilon_{\gamma})^{(p-1)/2}$ •  $\nu(P_{max}) \sim 2\gamma^2 (B/1\mu G) \text{ Hz} \sim 300 \text{ MHz} \text{ (for 100 MeV e})$

## $-rac{dE}{dt}_{ m sync} \propto \sigma_{ m T} B_{\perp}^2$

• Scattered  $\varepsilon^{IC}$ 

408 MHz

$$300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{B_{\perp}}{3 \,\mu\text{G}}\right)^{-1} \text{ Myr}$$

Inverse Compton (2 regimes) Thomso

$$n$$
 Klein-Nisk  
 $\gamma^2$   $\sim 4 \epsilon^0 \gamma$ 

• Power: fold cross-section to density of photons

 $t_{\rm sync} \simeq$ 

• Energy losses  $\propto \gamma^2 \qquad \propto \ln(\gamma)$ 

 $[\varepsilon_{v}^{0} - 4\varepsilon_{v}^{0}]$ 



### Ingredients

### **B** tracers

- Faraday rotation: free e<sup>-</sup> (ionised regions)
- Synchrotron emission: CR e
- Zeeman splitting: lines (neutral regions)
- Dust thermal emission, starlight polar.

### Uncertainties $[2 \ \mu G < B_{sync} < 6 \ \mu G]$

- Geometry (z dependence)
- Arm-interarm strenght
- Regular vs irregular component

### **Bremsstrahlung (or free-free)**

**Ionisation and Coulomb** 





### Synchrotron

408 MHz

- Power emitted || and H to B (polarised emission)
- If (E)<sup>-s</sup>  $\rightarrow (\epsilon_{\gamma})^{(p-1)/2}$ •  $\nu(P_{max}) \sim 2\gamma^2 (B/1\mu G) \text{ Hz} \sim 300 \text{ MHz} (\text{for 100 MeV e}^-)$

## $-rac{dE}{dt}_{ m sync} \propto \sigma_{ m T} B_{\perp}^2$

• Scattered  $\varepsilon^{IC}$ 

$$= 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{B_{\perp}}{3 \,\mu\text{G}}\right)^{-1} \text{ Myr}$$

### **Inverse Compton (2 regimes)**

- $\begin{array}{ll} \textit{Thomson} & \textit{Klein-Nishina} \\ [\epsilon_{\gamma}^{\ 0} 4\epsilon_{\gamma}^{\ 0}\gamma_{e}^{\ 2}] & \sim 4\epsilon_{\gamma}^{\ 0}\gamma_{e}^{\ 2} \end{array}$
- Power: fold cross-section to density of photons

l<sub>sync</sub>

• Energy losses  $\propto \gamma^2 \qquad \propto \ln(\gamma)$ 

γ αι

### **Bremsstrahlung (or free-free)**

**Ionisation and Coulomb** 



### Ingredients

### **B** tracers

- Faraday rotation: free e<sup>-</sup> (ionised regions)
- Synchrotron emission: CR e<sup>-</sup>
- Zeeman splitting: lines (neutral regions)
- Dust thermal emission, starlight polar.

### Uncertainties $[2 \ \mu G < B_{sync} < 6 \ \mu G]$

- Geometry (z dependence)
- Arm-interarm strenght
- Regular vs irregular component







### Synchrotron

- Power emitted || and H to B (polarised emission)
- If  $(E)^{-s} \rightarrow (\epsilon)^{(p-1)/2}$  $v(P_{max}) \sim 2\gamma^2 (B/1\mu G) Hz \sim 300 \text{ MHz} \text{ (for 100 MeV e}^{-})$

### 408 MHz

dt syne

dF

$$_{\rm nc} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{B_{\perp}}{3 \,\mu\text{G}}\right)^{-1} \text{ Myr}$$

### **Inverse Compton (2 regimes)**

$$\begin{bmatrix} 1 \text{ nomson} & K \\ E^0 - 4E^0 \sqrt{2} \end{bmatrix}$$

$$A = c^0 \alpha^2$$

 $\propto \ln(\gamma)$ 

 $U_{\rm rad}$ 

ľ,

Mvr

Power: fold cross-section to density of photons

Ellergy losses	•	Energy	losses
----------------	---	--------	--------

• Scattered  $\varepsilon^{IC}$ 

 $\propto \sigma_{
m T} B_\perp^2$ 

$$t_{\rm IC} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{U_{\rm rad}}{0.3 \text{ GeV m}}\right)^{-1}$$

$$\frac{1}{dt} \frac{\alpha \sigma_{\rm T} U_{\rm rad} \gamma^2}{t_{\rm IC}} = t_{\rm I}$$

### **Bremsstrahlung (or free-free)**





### Ingredients

### **B** tracers

electron

radio

waves

- Faraday rotation: free e<sup>-</sup> (ionised regions)
- Synchrotron emission: CR e<sup>-</sup>
- Zeeman splitting: lines (neutral regions)
- Dust thermal emission, starlight polar. •

### Uncertainties $[2 \ \mu G < B_{sync} < 6 \ \mu G]$

- Geometry (z dependence)
- Arm-interarm strenght
- Regular vs irregular component

### Uncertainties [Porter et al., ApJ 682 (2008) 400]



### **Ionisation and Coulomb**

### Synchrotron

- Power emitted || and H to B (polarised emission)
- If (E)<sup>-s</sup>  $\rightarrow$  ( $\epsilon$ )<sup>(p-1)/2</sup> •  $v(P_{max}) \sim 2\gamma^2 (B/1\mu G) \text{ Hz} \sim 300 \text{ MHz} \text{ (for 100 MeV e}^-)$

### 408 MHz

$$\frac{dE}{dt}_{
m sync} \propto \sigma_{
m T} B_{\perp}^2$$

$$4 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{B_{\perp}}{3 \,\mu\text{G}}\right)^{-1} \text{ Myr}$$

**Inverse Compton (2 regimes)** 

ThomsonKlein-Ni
$$[\varepsilon_{\gamma}^{0} - 4\varepsilon_{\gamma}^{0}\gamma_{e}^{2}]$$
 $\sim 4\varepsilon_{\gamma}^{0}$ 

$$\sim 4 \epsilon^0 \gamma^2$$

 $\propto \ln(\gamma)$ 

Mvr

- Power: fold cross-section to density of photons
- Energy los

• Scattered  $\varepsilon^{IC}$ 

• Energy losses 
$$\frac{dE}{dt} \propto \sigma_{\rm T} U_{\rm rad} \gamma^2$$
  $t_{\rm IC} \simeq 300 \left(\frac{H}{1000}\right)$ 

<sup>l</sup>sync

$$_{\rm IC} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{U_{\rm rad}}{0.3 \text{ GeV m}^{-1}}\right)^{-1}$$

Thoms

### **Bremsstrahlung (or free-free)**

- Loss in plasma or atomic hydrogen within a factor of two
- In the ISM: H (neutral and molecular) and He dominant •
- $\varepsilon_{\gamma} \sim E/2$ , and  $(E)^{-s} \rightarrow (\varepsilon_{\gamma})^{-s}$

### **Ionisation and Coulomb**



### Ingredients

### **B** tracers

- Faraday rotation: free e<sup>-</sup> (ionised regions)
- Synchrotron emission: CR e<sup>-</sup>
- Zeeman splitting: lines (neutral regions)
- Dust thermal emission, starlight polar.

### Uncertainties [2 $\mu G < B_{sync} < 6 \mu G$ ]

- Geometry (z dependence)
- Arm-interarm strenght
- Regular vs irregular component







### Synchrotron

- Power emitted || and H to B (polarised emission)
- If (E)<sup>-s</sup>  $\rightarrow$  ( $\epsilon$ )<sup>(p-1)/2</sup>  $v(P_{max}) \sim 2\gamma^2 (B/1\mu G) \text{ Hz} \sim 300 \text{ MHz} \text{ (for 100 MeV e}^-)$

### 408 MHz

$$-\frac{dE}{dt}_{
m sync} \propto \sigma_{
m T} B_{\perp}^2$$

$$(4.300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{B_{\perp}}{3 \,\mu\text{G}}\right)^{-1} \text{ Myr}$$

**Inverse Compton (2 regimes)** 

$$n$$
 Klein-Nis

$$\gamma^2$$
] ~  $2 \epsilon_0^0 \gamma^2$ 

 $\propto \ln(\gamma)$ 

Myr

Power: fold cross-section to density of photons

<sup>l</sup>sync

Energy los

• Scattered  $\varepsilon^{IC}$ 

Energy losses 
$$\propto \tau$$
  
 $\propto \sigma_{\rm T} U_{\rm rad} \gamma^2$   $t_{\rm IC} \simeq 300 \left(\frac{E}{1 \ {\rm GeV}}\right)$ 

$$_{\rm IC} \simeq 300 \, \left(\frac{E}{1 \, {\rm GeV}}\right)^{-1} \left(\frac{U_{\rm rad}}{0.3 \, {\rm GeV} \, {\rm m}^{-3}}\right)^{-1}$$

Thoms

 $\left[\epsilon^{0} - 4\epsilon\right]$ 

### **Bremsstrahlung (or free-free)**

- Loss in plasma or atomic hydrogen within a factor of two
- In the ISM: H (neutral and molecular) and He dominant

• 
$$\varepsilon_{\gamma} \sim E/2$$
, and  $(E)^{-s} \rightarrow (\varepsilon_{\gamma})^{-s}$ 

$$-\frac{dE}{dt} \propto \sigma_{\rm T} n_{\rm ISM} \gamma$$

### **Ionisation and Coulomb**



### Ingredients

### **B** tracers

- Faraday rotation: free e<sup>-</sup> (ionised regions)
- Synchrotron emission: CR e<sup>-</sup>
- Zeeman splitting: lines (neutral regions)

### Dust thermal emission, starlight polar. •

### Uncertainties $[2 \ \mu G < B_{sync} < 6 \ \mu G]$

- Geometry (z dependence)
- Arm-interarm strenght
- Regular vs irregular component







### Synchrotron

- Power emitted || and H to B (polarised emission)
- If  $(E)^{-s} \rightarrow (\epsilon)^{(p-1)/2}$  $v(P_{max}) \sim 2\gamma^2 (B/1\mu G) \text{ Hz} \sim 300 \text{ MHz} \text{ (for 100 MeV e}^-\text{)}$

dt sync

dE

$$_{\rm T}B_{\perp}^2\gamma^2$$
  $t_{\rm sync}\simeq 300~\left(\frac{1}{100}\right)$ 

$$800 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{B_{\perp}}{3 \,\mu\text{G}}\right)^{-1} \text{ Myr}$$

**Inverse Compton (2 regimes)** 

ThomsonKlein-Nishina
$$[\varepsilon_{1}^{0} - 4\varepsilon_{1}^{0}\gamma^{2}]$$
 $\sim 4\varepsilon_{1}^{0}\gamma^{2}$ 

$$\gamma^2$$
] ~  $2 \epsilon^0 \gamma^2$ 

 $\propto \ln(\gamma)$ 

Mvr

- Power: fold cross-section to density of photons
- **Energy** losses

 $\propto \sigma_{\rm T} U_{\rm rad} \gamma$ 

• Scattered  $\varepsilon^{IC}$ 

$$\propto \gamma^2 \qquad \propto \tau^2$$
$$t_{\rm IC} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{U_{\rm rad}}{0.3 \text{ GeV m}}\right)^{-1}$$

Thoms

### **Bremsstrahlung (or free-free)**

- Loss in plasma or atomic hydrogen within a factor of two
- In the ISM: H (neutral and molecular) and He dominant

• 
$$\varepsilon_{\gamma} \sim E/2$$
, and  $(E)^{-s} \rightarrow (\varepsilon_{\gamma})^{-s}$ 

$$-\frac{dE}{dt} \propto \sigma_{\rm T} n_{\rm ISM} \gamma$$

### **Ionisation and Coulomb**

- Ionisation: interaction in neutral matter •
- Coulomb: scattering off free electrons



### Ingredients

### **B** tracers

- Faraday rotation: free e<sup>-</sup> (ionised regions)
- Synchrotron emission: CR e<sup>-</sup>
- Zeeman splitting: lines (neutral regions)

### • Dust thermal emission, starlight polar.

### Uncertainties $[2 \ \mu G < B_{sync} < 6 \ \mu G]$

- Geometry (z dependence)
- Arm-interarm strenght
- Regular vs irregular component







### Synchrotron

- Power emitted || and H to B (polarised emission)
- If  $(E)^{-s} \rightarrow (\epsilon_{s})^{(p-1)/2}$ •  $v(P_{max}) \sim 2\gamma^2 (B/1\mu G) Hz \sim 300 \text{ MHz} \text{ (for 100 MeV e}^-\text{)}$

$$-\frac{dE}{dt}_{
m sync} \propto \sigma_{
m T} B_{\perp}^2 \gamma$$

$$\simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{B_{\perp}}{3 \,\mu\text{G}}\right)^{-1} \text{ Myr}$$

**Inverse Compton (2 regimes)** 

ThomsonKlein-Ni.
$$[\varepsilon_{\gamma}^{0} - 4\varepsilon_{\gamma}^{0}\gamma_{e}^{2}]$$
~ 4  $\varepsilon_{\gamma}^{0}$ 

$$\sim 4 \epsilon^0 \gamma^2$$

 $\propto \ln(\gamma)$ 

Myr

Power: fold cross-section to density of photons

l<sub>svnc</sub>

Energy losse

 $_{_{
m T}} \propto \sigma_{
m T} U_{
m rad} \gamma$ 

• Scattered  $\varepsilon^{IC}$ 

$$\frac{\alpha \gamma^2}{2} + \frac{\alpha \gamma^2}{(E - 1)^{-1}}$$

$$e \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{U_{\text{rad}}}{0.3 \text{ GeV m}^{-3}}\right)^{-1}$$

Thoms

### **Bremsstrahlung (or free-free)**

- Loss in plasma or atomic hydrogen within a factor of two
- In the ISM: H (neutral and molecular) and He dominant

• 
$$\varepsilon_{\gamma} \sim E/2$$
, and  $(E)^{-s} \rightarrow (\varepsilon_{\gamma})^{-s}$ 

$$-\frac{dE}{dt} \propto \sigma_{\rm T} n_{\rm ISM} \gamma$$

### **Ionisation and Coulomb**

- Ionisation: interaction in neutral matter •
- Coulomb: scattering off free electrons 0





### Ingredients

### **B** tracers

- Faraday rotation: free e<sup>-</sup> (ionised regions)
- Synchrotron emission: CR e<sup>-</sup>
- Zeeman splitting: lines (neutral regions)
- Dust thermal emission, starlight polar.

### Uncertainties $[2 \ \mu G < B_{sync} < 6 \ \mu G]$

- Geometry (z dependence)
- Arm-interarm strenght
- Regular vs irregular component







### Synchrotron

- Power emitted || and H to B (polarised emission)
- If  $(E)^{-s} \rightarrow (\epsilon_{s})^{(p-1)/2}$ •  $v(P_{max}) \sim 2\gamma^2 (B/1\mu G) Hz \sim 300 \text{ MHz} \text{ (for 100 MeV e}^-\text{)}$

dE

$$_{\rm ync} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{B_{\perp}}{3 \,\mu\text{G}}\right)^{-1} N_{\rm s}$$

**Inverse Compton (2 regimes)** 

 $\propto \sigma_{
m T} B_{\perp}^2$ 

ThomsonKlein-Ni
$$[\varepsilon_{0}^{0} - 4\varepsilon_{0}^{0}\gamma^{2}]$$
~ 4  $\varepsilon_{0}^{0}\gamma^{2}$ 

$$\gamma_{a}^{2}$$
] ~ ~4  $\varepsilon_{\gamma}^{0} \gamma_{a}^{2}$ 

 $\propto \ln(\gamma)$ 

Myr

Power: fold cross-section to density of photons

 $-rac{dE}{dt}_{IC} \propto \sigma_{
m T} U_{
m rad} \gamma^2$ 

• Scattered  $\varepsilon^{IC}$ 

$$t_{\rm IC} \simeq 300 \left(\frac{E}{1 \, {\rm GeV}}\right)^{-1} \left(\frac{U_{\rm rad}}{0.2 \, {\rm GeV}}\right)^{-1}$$

Thoms

### **Bremsstrahlung (or free-free)**

- Loss in plasma or atomic hydrogen within a factor of two
- In the ISM: H (neutral and molecular) and He dominant

• 
$$\varepsilon_{\gamma} \sim E/2$$
, and  $(E)^{-s} \rightarrow (\varepsilon_{\gamma})^{-s}$ 

$$-\frac{dE}{dt} \propto \sigma_{\rm T} n_{\rm ISM} \gamma$$

### **Ionisation and Coulomb**

- Ionisation: interaction in neutral matter •
- Coulomb: scattering off free electrons 0





### Ingredients

### **B** tracers

- Faraday rotation: free e<sup>-</sup> (ionised regions)
- Synchrotron emission: CR e<sup>-</sup>
- Zeeman splitting: lines (neutral regions)

### • Dust thermal emission, starlight polar.

### Uncertainties $[2 \ \mu G < B_{sync} < 6 \ \mu G]$

- Geometry (z dependence)
- Arm-interarm strenght
- Regular vs irregular component







### Synchrotron

- Power emitted || and H to B (polarised emission)
- If (E)<sup>-s</sup>  $\rightarrow$  ( $\epsilon_{\gamma}$ )<sup>(p-1)/2</sup> •  $\nu(P_{max}) \sim 2\gamma^2$  (B/1 $\mu$ G) Hz  $\sim$  300 MHz (for 100 MeV e<sup>-</sup>)

dt syne

dE

$$_{\rm nc} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{B_{\perp}}{3 \,\mu\text{G}}\right)^{-1} \text{ My}$$

Inverse Compton (2 regimes)

 $\propto \sigma_{
m T} B_\perp^2$ 

$$n$$
 Klein-Nis

$$\gamma_a^2$$
] ~ ~ 4  $\epsilon_{\gamma}^0 \gamma_a^2$ 

 $\propto \ln(\gamma)$ 

Mvi

• Power: fold cross-section to density of photons

• Energy	losses
----------	--------

 $\overline{dt}_{IC} \propto \sigma_{
m T} U_{
m rad} \gamma^2$ 

• Scattered  $\epsilon^{IC}$ 

$$\frac{\alpha \gamma}{t_{\rm IC} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{U_{\rm rad}}{0.3 \text{ GeV m}}\right)^{-1}}$$

Thomse

 $\left[\epsilon_{...}^{0} - 4\epsilon_{...}^{0}\right]$ 

### Bremsstrahlung (or free-free)

- Loss in plasma or atomic hydrogen within a factor of two
- In the ISM: H (neutral and molecular) and He dominant

• 
$$\varepsilon_{\gamma} \sim E/2$$
, and  $(E)^{-s} \rightarrow (\varepsilon_{\gamma})^{-s}$ 

$$-\frac{dE}{dt} \propto \sigma_{\rm T} n_{\rm ISM} \gamma$$

### Ionisation and Coulomb

- Ionisation: interaction in neutral matter
- Coulomb: scattering off free electrons





### Ingredients

### **B** tracers

- Faraday rotation: free e<sup>-</sup> (ionised regions)
- Synchrotron emission: CR e
- Zeeman splitting: lines (neutral regions)
- Dust thermal emission, starlight polar.

### Uncertainties [2 $\mu$ G < B<sub>sync</sub> < 6 $\mu$ G]

- Geometry (z dependence)
- Arm-interarm strenght
- Regular vs irregular component

### Uncertainties [Porter et al., ApJ 682 (2008) 400]



### Uncertainties $[n_{disc} \sim 1 - 2 \text{ cm}^{-3}]$



- Distribution of HI, HII, H2, He...
- Geometry: radial and z-dependence
  - Arm-interarm contrast



Plasma (Coulomb)

Atomic matter (ionisation)

### Synchrotron

- Power emitted || and H to B (polarised emission)
- If  $(E)^{-s} \rightarrow (\epsilon_{s})^{(p-1)/2}$ •  $v(P_{max}) \sim 2\gamma^2 (B/1\mu G) Hz \sim 300 \text{ MHz} (\text{for } 100 \text{ MeV e}^-)$

dE

 $\overline{dt}$  sync

$$_{\rm c} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{B_{\perp}}{3 \,\mu\text{G}}\right)^{-1} \text{ Myr}$$

**Inverse Compton (2 regimes)** 

ThomsonKlein-Nise
$$[\boldsymbol{\varepsilon}_{y}^{0} - 4\boldsymbol{\varepsilon}_{y}^{0}\boldsymbol{\gamma}_{e}^{2}]$$
 $\sim 4 \boldsymbol{\varepsilon}_{y}^{0} \boldsymbol{\gamma}_{e}^{3}$ 

1(Y)

Myr

e-

Power: fold cross-section to density of photons

 $\frac{dT}{dt}_{IC} \propto \sigma_{\rm T} U_{\rm rad} \gamma^2$ 

• Scattered  $\epsilon^{IC}$ 

 $\propto \sigma_{
m T} B_\perp^2$ 

$$t_{\rm IC} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{U_{\rm rad}}{0.3 \text{ GeV m}^{-3}}\right)^{-3}$$

Thomso

### **Bremsstrahlung (or free-free)**

- Loss in plasma or atomic hydrogen within a factor of two
- In the ISM: H (neutral and molecular) and He dominant

• 
$$\varepsilon_{\gamma} \sim E/2$$
, and  $(E)^{-s} \rightarrow (\varepsilon_{\gamma})^{-s}$   
 $-\frac{dE}{dt}_{\text{brem}} \propto \sigma_{\text{T}} n_{\text{ISM}} \gamma$ 

$$t_{\rm brem} \simeq 300 \left(\frac{n_{\rm ISM}}{1 \,{\rm cm}^{-3}}\right)^{-1} \,{\rm Myr}$$

### **Ionisation and Coulomb**

- Ionisation: interaction in neutral matter •
- Coulomb: scattering off free electrons

$$\frac{dE^{\text{ion}}}{dt} \propto \sigma_{\rm T} n_{\rm plasma}^{\rm ISM} \qquad t_{\rm ion} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right) \left(\frac{n_{\rm ISM}}{1 \text{ cm}^{-3}}\right)^{-1} \text{ M}$$



K-rav

<u>e-</u>

 $(\mathbf{f})$ 

Plasma (Coulomb) Atomic matter (ionisation)

### Ingredients

### **B** tracers

- Faraday rotation: free e<sup>-</sup> (ionised regions)
- Synchrotron emission: CR e<sup>-</sup>
- Zeeman splitting: lines (neutral regions)
- Dust thermal emission, starlight polar.

### Uncertainties $[2 \ \mu G < B_{sync} < 6 \ \mu G]$

- Geometry (z dependence)
- Arm-interarm strenght
- Regular vs irregular component

### Uncertainties [Porter et al., ApJ 682 (2008) 400]



### Uncertainties $[n_{disc} \sim 1 - 2 \text{ cm}^{-3}]$



- Distribution of HI. HII, H2, He...
- Geometry: radial and z-dependence
  - Arm-interarm contrast

hina

Myr

e-

n(Y)

### Synchrotron

- Power emitted || and H to B (polarised emission)
- If (E)<sup>-s</sup>  $\rightarrow (\epsilon_{\gamma})^{(p-1)/2}$ •  $\nu(P_{max}) \sim 2\gamma^2 (B/1\mu G) Hz \sim 300 \text{ MHz} (\text{for 100 MeV e}^-)$

dt syne

dE

$$v_{\rm nc} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{B_{\perp}}{3 \,\mu \text{G}}\right)^{-1} \text{ M}$$

**Inverse Compton (2 regimes)** 

 $\propto \sigma_{
m T} B_\perp^2 \gamma$ 

$$[\epsilon_{\gamma}^{0} - 4\epsilon_{\gamma}^{0}\gamma_{e}^{2}] \sim$$

• Power: fold cross-section to density of photons

•	Energy losses	
dE	$\sim \sigma U \sigma^2$	
dt	$C \propto \sigma_{\rm T} \sigma_{\rm rad}$	

• Scattered  $\varepsilon^{IC}$ 

$$t_{\rm IC} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{U_{\rm rad}}{0.3 \text{ GeV m}^{-3}}\right)^{-1}$$

Thoms

### **Bremsstrahlung (or free-free)**

- Loss in plasma or atomic hydrogen within a factor of two
- In the ISM: H (neutral and molecular) and He dominant

• 
$$\varepsilon_{\gamma} \sim E/2$$
, and  $(E)^{-S} \rightarrow (\varepsilon_{\gamma})$   
 $-\frac{dE}{dt}_{\text{brem}} \propto \sigma_{\text{T}} n_{\text{ISM}} \gamma$ 

$$t_{\text{brem}} \simeq 300 \left(\frac{n_{\text{ISM}}}{1 \text{ cm}^{-3}}\right)^{-1} \text{ Myr}$$

### Ionisation and Coulomb

- Ionisation: interaction in neutral matter
- Coulomb: scattering off free electrons

$$-\frac{dE^{\text{ion}}}{dt_{\text{Coulomb}}} \propto \sigma_{\text{T}} n_{\text{plasma}}^{\text{ISM}} \qquad t_{\text{ion}} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right) \left(\frac{n_{\text{ISM}}}{1 \text{ cm}^{-3}}\right)^{-1} \text{ My}$$



K-rav

<u>e-</u>

 $(\mathbf{f})$ 

Plasma (Coulomb)+× Atomic matter (ionisation)

### Ingredients

### **B** tracers

- Faraday rotation: free e<sup>-</sup> (ionised regions)
- Synchrotron emission: CR e
- Zeeman splitting: lines (neutral regions)
- Dust thermal emission, starlight polar.

### Uncertainties $[2 \mu G < B_{sync} < 6 \mu G]$

- Geometry (z dependence)
- Arm-interarm strenght
- Regular vs irregular component

### Uncertainties [Porter et al., ApJ 682 (2008) 400]



### Uncertainties $[n_{disc} \sim 1 - 2 \text{ cm}^{-3}]$



- Distribution of HI, HII, H2, He...
- Geometry: radial and z-dependence
  - Arm-interarm contrast

 $\rightarrow$  Crucial for  $\gamma$ -ray emissions

II.4 E losses

### Synchrotron [disc + halo]

$$-rac{dE}{dt}_{
m sync} \propto \sigma_{
m T} B_{\perp}^2 \gamma^2$$

$$_{\rm sync} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{B_{\perp}}{3 \,\mu\text{G}}\right)^{-1} \text{ Myr}$$





## Are the formulae the same for electrons and nuclei?

### Inverse Compton [disc + CMB halo]

$$-\frac{dE}{dt}_{IC} \propto \sigma_{\rm T} U_{\rm rad} \gamma^2 \qquad t_{\rm IC} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{U_{\rm rad}}{0.3 \text{ GeV m}^{-3}}\right)^{-1} \text{ Myr}$$

### Bremsstrahlung [disc]

$$-\frac{dE}{dt}_{\rm brem} \propto \sigma_{\rm T} n_{\rm ISM} \gamma$$

$$_{\rm em} \simeq 300 \left(\frac{n_{\rm ISM}}{1 \,{\rm cm}^{-3}}\right)^{-1} \,{\rm Myr}$$

### Ionisation and Coulomb [disc]

$$-\frac{dE^{\rm ion}}{dt_{\rm Coulomb}} \propto \sigma_{\rm T} n_{\rm plasma}^{\rm ISM} \quad t_{\rm ion} \simeq 300$$



Mvr



$$-rac{dE}{dt}_{
m sync} \propto \sigma_{
m T} B_{\perp}^2 \gamma^2$$

$$_{\rm ync} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{B_{\perp}}{3 \,\mu\text{G}}\right)^{-1} \text{ Myr}$$



Inverse Compton [disc + CMB halo]

$$-\frac{dE}{dt_{IC}} \propto \sigma_{\rm T} U_{\rm rad} \gamma^2 \qquad t_{\rm IC} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{U_{\rm rad}}{0.3 \text{ GeV m}^{-3}}\right)^{-1} \text{ Myr}$$



- Lepton X-section  $\rightarrow$  nucleus X-section

$$\sigma_{\rm T} = 8\pi r_e^2/3$$
 
$$r_e \propto e^2/m_e$$



Relate  $\sigma_{_N}$  to  $\sigma_{_T}$ 

# Bremsstrahlung [disc] $-\frac{dE}{dt} \propto \sigma_{\rm T} n_{\rm ISM} \gamma \qquad t_{\rm brem} \simeq 300 \left(\frac{n_{\rm ISM}}{1 \,{\rm cm}^{-3}}\right)^{-1} \,{\rm Myr}$ Ionisation and Coulomb [disc]

$$-\frac{dE^{\rm ion}}{dt_{\rm Coulomb}} \propto \sigma_{\rm T} n_{\rm plasma}^{\rm ISM} \qquad t_{\rm ion} \simeq 30$$



Mvr

🗴-ray

### Synchrotron [disc + halo]

$$-\frac{dE}{dt}_{
m sync} \propto \sigma_{
m T} B_{\perp}^2 \gamma^2$$

$$_{\rm ync} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{B_{\perp}}{3 \,\mu\text{G}}\right)^{-1} \text{ Myr}$$



- Changes in formulae
  - Lepton X-section  $\rightarrow$  nucleus X-section

$$\sigma_{\rm T} = 8\pi r_e^2/3$$
$$r_e \propto e^2/m_e$$

$$\sigma_{\rm N} = \frac{Z^4}{A^2} \frac{m_e^2}{m_p^2} \sigma_{\rm T} \simeq Z^2 \cdot 10^{-7} \sigma_{\rm T}$$

Inverse Compton [disc + CMB halo]

$$-\frac{dE}{dt_{IC}} \propto \sigma_{\rm T} U_{\rm rad} \gamma^2 \qquad t_{\rm IC} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{U_{\rm rad}}{0.3 \text{ GeV m}^{-3}}\right)^{-1} \text{ Myr}$$



e-

**X**-ray

$$-\frac{dE}{dt}_{\rm brem} \propto \sigma_{\rm T} n_{\rm ISM}$$

$$t_{\rm brem} \simeq 300 \left(\frac{n_{\rm ISM}}{1\,{\rm cm}^{-3}}\right)^{-1} {\rm Myr}$$

 $n_{\rm ISM}$ 

1cm

Mvr

### Ionisation and Coulomb [disc]

$$-\frac{dE^{\rm ion}}{dt_{\rm Coulomb}} \propto \sigma_{\rm T} n_{\rm plasma}^{\rm ISM} \qquad t_{\rm ion} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right) ($$



(+)

### Synchrotron [disc + halo]

$$-rac{dE}{dt}_{
m sync} \propto \sigma_{
m T} B_{\perp}^2 \gamma^2$$

$$_{\rm rnc} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{B_{\perp}}{3 \,\mu\text{G}}\right)^{-1} \text{ Myr}$$



### • Changes in formulae

- Lepton X-section  $\rightarrow$  nucleus X-section

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- At 1 GeV, 
$$\gamma_{e} \neq \gamma_{I}$$

$$\frac{\gamma_N}{\gamma_e} (E^e = E^N_{k/n}) = \frac{m_e}{m_N} \simeq 5 \cdot 10^{-4}$$

Inverse Compton [disc + CMB halo]

$$-\frac{dE}{dt_{IC}} \propto \sigma_{\rm T} U_{\rm rad} \gamma^2 \qquad t_{\rm IC} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{U_{\rm rad}}{0.3 \text{ GeV m}^{-3}}\right)^{-1} \text{ Myr}$$



Bremsstrahlung [disc]

 $-\frac{dE}{dt}_{\rm brem} \propto \sigma_{\rm T} n_{\rm ISM} \gamma$ 

$$t_{\rm brem} \simeq 300 \left(\frac{n_{\rm ISM}}{1\,{\rm cm}^{-3}}\right)^{-1} {\rm Myr}$$

 $n_{\rm ISM}$ 

1cm

Mvr

Ionisation and Coulomb [disc]

$$\frac{dE^{\rm ion}}{dt_{\rm Coulomb}} \propto \sigma_{\rm T} n_{\rm plasma}^{\rm ISM} \quad t_{\rm ion} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right) \left(\frac{E}{1 \text{ GeV}}\right)$$




$$-rac{dE}{dt}_{
m sync} \propto \sigma_{
m T} B_{\perp}^2 \gamma^2 \qquad t_{
m sync}$$

$$_{\rm nc} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{B_{\perp}}{3 \,\mu\text{G}}\right)^{-1} \text{ Myr}$$

electron B radio waves

### **Inverse Compton [disc + CMB halo]**

$$-\frac{dE}{dt}_{IC} \propto \sigma_{\rm T} U_{\rm rad} \gamma^2 \qquad t_{\rm IC} \simeq 300 \left(\frac{E}{1 \text{ GeV}}\right)^{-1} \left(\frac{U_{\rm rad}}{0.3 \text{ GeV m}^{-3}}\right)^{-1} \text{ Myr}$$



 $(\mathbf{+})$ 

Plasma (Coulomb)+ Atomic matter (ionisation)

e-

X-rav

e-

### • Changes in formulae

- Lepton X-section  $\rightarrow$  nucleus X-section

$$\sigma_{\rm T} = 8\pi r_e^2/3$$
$$r_e \propto e^2/m_e$$

$$\sigma_{\rm N} = \frac{Z^4}{A^2} \frac{m_e^2}{m_p^2} \sigma_{\rm T} \simeq Z^2 \cdot 10^{-7} \sigma_{\rm T}$$

- At 1 GeV,  $\gamma_e \neq \gamma_p$  $\frac{\gamma_N}{\gamma_e} (E^e = E_{k/n}^N) = \frac{m_e}{m_N} \simeq 5 \cdot 10^{-4}$
- Suppression factors in dE/dt

  all effects: Z<sup>2</sup> 10<sup>-7</sup>
  each time a γ is in dE/dt: 5 10<sup>-4</sup>



Which losses can be neglected?

 $-rac{dE}{dt}_{
m brem} \propto \sigma_{
m T} n_{
m ISM} \gamma$ 

$$t_{\rm brem} \simeq 300 \left(\frac{n_{\rm ISM}}{1\,{\rm cm}^{-3}}\right)^{-1} {\rm Myr}$$

#### Ionisation and Coulomb [disc]

$$-\frac{dE^{\rm ion}}{dt}_{\rm Coulomb} \propto \sigma_{\rm T} n_{\rm plasma}^{\rm ISM}$$



- Changes in formulae
- Lepton X-section  $\rightarrow$  nucleus X-section

$$\sigma_{\rm T} = 8\pi r_e^2/3$$
  
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$$\frac{\gamma_N}{\gamma_e} (E^e = E^N_{k/n}) = \frac{m_e}{m_N} \simeq 5 \cdot 10^{-4}$$

- Suppression factors in dE/dt - all effects: Z<sup>2</sup> 10<sup>-7</sup>
  - each time a  $\gamma$  is in dE/dt: 5 10<sup>-4</sup>



- Changes in formulae
- Lepton X-section  $\rightarrow$  nucleus X-section

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- Suppression factors in dE/dt

  all effects: Z<sup>2</sup> 10<sup>-7</sup>
  each time a γ is in dE/dt: 5 10<sup>-4</sup>



Why do we keep ionisation and Coulomb?



### • Changes in formulae

- Lepton X-section  $\rightarrow$  nucleus X-section

$$\sigma_{\rm T} = 8\pi r_e^2/3$$
  
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- Suppression factors in dE/dt - all effects: Z<sup>2</sup> 10<sup>-7</sup>
  - each time a  $\gamma$  is in dE/dt: 5 10<sup>-4</sup>
  - → Coulomb/ion amplitude redeemed [non-relativistic nucleus vs relativisic leptons for same kinetic energy]

N.B.: ionisation and heating of the ISM!

### Transport of cosmic rays (CR) in the Galaxy

### II. Processes, ingredients, characteristic times

- 1. Definitions
- 2. Diffusion (space and momentum)
- 3. Convection and adiabatic losses
- 4. Energy losses
- 5. Catastrophic losses
- 6. All together

#### **Nuclear interactions**

- Elastic:  $N + ISM \rightarrow N + ISM$
- Inelastic:  $N + ISM \rightarrow X + ... (X \neq N)$



#### **Nuclear interactions**

- Elastic:  $N + ISM \rightarrow N + ISM$ •
- Inelastic: N + ISM  $\rightarrow$ X + ... (X $\neq$ N) •



#### **Comments/accuracy**

- Destruction  $\rightarrow$  "source" for fragments

- Semi-empirical models fit on data  $\rightarrow \sigma_{inel} \propto A^{2/3}$ 
  - Bradt & Peters (1950) •
    - Letaw et al. (1970-2000)
    - $\Delta\sigma/\sigma \sim 2-5\%$ Tripathi et al. (1998-2003)

#### **Nuclear interactions**

- Elastic:  $N + ISM \rightarrow N + ISM$
- Inelastic:  $N + ISM \rightarrow X + ... (X \neq N)$
- $\rightarrow$  Interaction rate  $\Gamma_{inel} = n v \sigma$



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- Destruction  $\rightarrow$  "source" for fragments
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- $\rightarrow$  Interaction rate  $\Gamma_{inel} = n v \sigma$





Destruction time for p and Fe (in Myr)?



#### **Nuclear interactions**

- Elastic:  $N + ISM \rightarrow N + ISM$
- Inelastic:  $N + ISM \rightarrow X + ... (X \neq N)$
- $\rightarrow$  Interaction rate  $\Gamma_{inel} = n v \sigma$

$$t_{\text{inel}} \simeq 10^3 \left(\frac{n_{\text{ISM}}}{1 \text{ cm}^{-3}}\right)^{-1} \left(\frac{\sigma_{\text{inel}}}{1 \text{ mb}}\right)^{-1} \text{Myr}$$



#### Comments/accuracy - Destruction $\rightarrow$ "source" for fragments - Semi-empirical models fit on data - Bradt & Peters (1950) $\rightarrow \sigma_{inel} \propto A^{2/3}$ - Letaw *et al.* (1970-2000) $\rightarrow \sigma_{inel} \propto A^{2/3}$ - Tripathi *et al.* (1998-2003) $\Delta \sigma / \sigma \sim 2-5\%$ $\sigma_{inel}(\mathbf{p}, \mathbf{C}, \mathbf{Fe}) \sim 40, 250, 750 \text{ mb}$ $n_{ISM} = 1 \text{ cm}^{-3}$ Disc density $c \simeq 3 \cdot 10^{10} \text{ cm s}^{-1}$ Speed of light 1 mb = $10^{-27} \text{ cm}^2$ 1 s $\simeq 3 \cdot 10^{-14}$ Myr

#### Nuclear interactions

- Elastic:  $N + ISM \rightarrow N + ISM$
- Inelastic:  $N + ISM \rightarrow X + ... (X \neq N)$



$$t_{\text{inel}} \simeq 10^3 \left(\frac{n_{\text{ISM}}}{1 \text{ cm}^{-3}}\right)^{-1} \left(\frac{\sigma_{\text{inel}}}{1 \text{ mb}}\right)^{-1} \text{Myr}$$



#### **Comments/accuracy**

Destruction $\rightarrow$ "sou	rce" for fragments
Semi-empirical mod	lels fit on data
• Bradt & Peters (1950	$\rightarrow \sigma_{\rm c} \propto A^{2/3}$
• Letaw <i>et al.</i> (1970-20	$\frac{1000}{1000} \qquad \frac{1000}{1000} \qquad \frac{1000}{1000$
• Tripathi <i>et al.</i> (1998-2	2003) <b>Δ0/0</b> ~ <b>2-3</b> %
$\sigma_{inel}(p, C, Fe) \sim 40$	0, 250, 750 mb
$n_{ISM} = 1 \text{ cm}^{-3}$	Disc density
$c \simeq 3 \cdot 10^{10} \mathrm{~cm~s^{-1}}$	Speed of light
$1 \text{ mb} = 10^{-27} \text{ cm}^2$	$1 \mathrm{s} \simeq 3 \cdot 10^{-14} \mathrm{Myr}$

### Spontaneous $\beta$ decay

#### **Nuclear interactions**

- Elastic:  $N + ISM \rightarrow N + ISM$
- Inelastic:  $N + ISM \rightarrow X + ... (X \neq N)$



$$t_{\text{inel}} \simeq 10^3 \left(\frac{n_{\text{ISM}}}{1 \text{ cm}^{-3}}\right)^{-1} \left(\frac{\sigma_{\text{inel}}}{1 \text{ mb}}\right)^{-1} \text{Myr}$$



#### **Comments/accuracy**

- Destruction  $\rightarrow$  "source" for fragments - Semi-empirical models fit on data • Bradt & Peters (1950)  $\rightarrow \sigma_{inel} \propto A^{2/3}$ • Letaw *et al.* (1970-2000) • Tripathi *et al.* (1998-2003)  $\Delta\sigma/\sigma \sim 2-5\%$   $\sigma_{inel}(\mathbf{p}, \mathbf{C}, \mathbf{Fe}) \sim 40, 250, 750 \text{ mb}$   $n_{ISM} = 1 \text{ cm}^{-3}$  Disc density  $c \simeq 3 \cdot 10^{10} \text{ cm s}^{-1}$  Speed of light  $1 \text{ mb} = 10^{-27} \text{ cm}^2$   $1 \text{ s} \simeq 3 \cdot 10^{-14} \text{ Myr}$ 

### Spontaneous $\beta$ decay

• Mean lifetime t

• Half-life 
$$t_{1/2} = \tau \ln(2)$$

#### **Nuclear interactions**

- Elastic:  $N + ISM \rightarrow N + ISM$ •
- Inelastic: N + ISM  $\rightarrow$ X + ... (X $\neq$ N)  $\bullet$



$$t_{\text{inel}} \simeq 10^3 \left(\frac{n_{\text{ISM}}}{1 \text{ cm}^{-3}}\right)^{-1} \left(\frac{\sigma_{\text{inel}}}{1 \text{ mb}}\right)^{-1} \text{Myr}$$

### Spontaneous $\beta$ decay

- Mean lifetime t
- $t_{1/2} = \tau \ln(2)$ Half-life  $t_{1/2}$ •



$$\rightarrow \mathbf{Decay \ rate} \quad \Gamma_{rad} = \frac{\ln(2)}{\gamma t_{1/2}}$$
$$t_{\beta} \simeq E_{k/n} \left( \frac{t_{1/2}}{1.51 \text{ Myr}} \right) \text{ Myr}$$

#### **Comments/accuracy** - Destruction $\rightarrow$ "source" for fragments - Semi-empirical models fit on data Bradt & Peters (1950) $\rightarrow \sigma_{inel} \propto A^{2/3}$ Letaw et al. (1970-2000) $\Delta\sigma/\sigma \sim 2-5\%$ Tripathi *et al.* (1998-2003) $\sigma_{inel}(p, C, Fe) \sim 40, 250, 750 \text{ mb}$ $n_{ISM} = 1 \text{ cm}^{-3}$ Disc density $c \simeq 3 \cdot 10^{10} \text{ cm s}^{-1}$ Speed of light $1 \text{ mb} = 10^{-27} \text{ cm}^2$ $1 \text{ s} \simeq 3 \cdot 10^{-14} \text{ Myr}$

#### **Nuclear interactions**

- Elastic:  $N + ISM \rightarrow N + ISM$ •
- Inelastic: N + ISM  $\rightarrow$ X + ... (X $\neq$ N) •



$$t_{\text{inel}} \simeq 10^3 \left(\frac{n_{\text{ISM}}}{1 \text{ cm}^{-3}}\right)^{-1} \left(\frac{\sigma_{\text{inel}}}{1 \text{ mb}}\right)^{-1} \text{Myr}$$

### Spontaneous **B** decay

- Mean lifetime t •
- $t_{1/2} = \tau \ln(2)$ Half-life t<sub>1/2</sub> •



$$\rightarrow \mathbf{Decay \ rate} \quad \Gamma_{rad} = \frac{\ln(2)}{\gamma t_{1/2}}$$
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#### **Comments/accuracy**

Dest Sem	ruction $\rightarrow$ "sour i-empirical mod	rce" f lels fit	or fragme t on data	ents
• ]	Bradt & Peters (1950) tetaw et al. (1970-20)	) 00)	$\rightarrow \sigma_{_{inel}} \propto$	$A^{2/3}$
•	Tripathi <i>et al.</i> (1970-20	2003)	$\Delta\sigma/\sigma\sim 2$	-5%
	$\sigma_{inel}(p, C, Fe) \sim 40$	, 250,	750 mb	
$n_I$	$v_{SM} = 1 \text{ cm}^{-3}$	Disc	density	
c :	$\simeq 3 \cdot 10^{10} \mathrm{~cm~s^{-1}}$	Spee	d of light	
	1			

#### - Measurements

Z	Nucleus	Daughter	$t_{1/2}^{\text{unit.}}(\text{error})$
4	$^{10}_{4}\mathrm{Be}$	${}_{5}^{10}B$	$1.51^{Myr}$ (0.06)
6	$^{14}_{6}C$	$^{14}_{7}N$	$5.73^{kyr}$ (0.04)
13	$^{26}_{13}Al$	$^{26}_{12}{ m Mg}$	$0.91^{Myr}$ (0.04)
17	$^{36}_{17}Cl$	$^{36}_{18}{ m Ar}$	$0.307^{Myr}$ (0.002)
26	$^{60}_{26}{ m Fe}$	<sup>60</sup> <sub>28</sub> Ni	$1.5^{Myr}$ (0.3)

#### **Nuclear interactions**

- Elastic:  $N + ISM \rightarrow N + ISM$ •
- Inelastic: N + ISM  $\rightarrow$ X + ... (X $\neq$ N)





- Mean lifetime t
- $\overline{t_{1/2}} = \tau \ln(2)$ Half-life t<sub>1/2</sub>

 $\rightarrow$  **Decay rate**  $\Gamma_{rad} = \frac{\ln(2)}{\gamma t_{1/2}}$  $t_{\beta} \simeq E_{k/n}$ Myr

#### **Comments/accuracy**

- Destruction  $\rightarrow$  "source" for fragments - Semi-empirical models fit on data Bradt & Peters (1950)  $\rightarrow \sigma_{\rm inel} \propto A^{2/3}$ Letaw et al. (1970-2000)  $\Delta\sigma/\sigma \sim 2-5\%$ Tripathi *et al.* (1998-2003)  $\sigma_{inel}^{}(p, C, Fe) \sim 40, 250, 750 \text{ mb}$  $n_{ISM} = 1 \text{ cm}^{-3}$ Disc density  $c \simeq 3 \cdot 10^{10} \text{ cm s}^{-1}$  Speed of light.  $1 \text{ mb} = 10^{-27} \text{ cm}^2$  $1 \text{ s} \simeq 3 \cdot 10^{-14} \text{ Myr}$ 

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#### Electronic capture with a K-shell electron

#### **Nuclear interactions**

- Elastic:  $N + ISM \rightarrow N + ISM$
- Inelastic: N + ISM  $\rightarrow$ X + ... (X $\neq$ N) •

### $\rightarrow$ Interaction rate $\Gamma_{inel} = n v \sigma$

$$t_{\text{inel}} \simeq 10^3 \left(\frac{n_{\text{ISM}}}{1 \text{ cm}^{-3}}\right)^{-1} \left(\frac{\sigma_{\text{inel}}}{1 \text{ mb}}\right)^{-1} \text{Myr}$$



- Mean lifetime t •
- $\overline{t_{1/2}} = \tau \ln(2)$ Half-life t<sub>1/2</sub> •

#### Electronic capture with a K-shell electron

- Attachment
- Stripping



 $\sigma_{inel} = \sigma_{tot} - \sigma_{el}$ 

$$r_{\beta} \simeq E_{k/n} \left( \frac{t_{1/2}}{1.51 \text{ Myr}} \right) \text{ Myr}$$



#### **Comments/accuracy**

Des Sen	struction → "sour mi-empirical mod Bradt & Peters (1950) Letaw <i>et al.</i> (1970-20) Tripathi <i>et al.</i> (1998-2)	rce" f lels fi ) 00) 2003)	For fragment on data $\rightarrow \sigma_{inel} \circ \Delta \sigma / \sigma \sim \sigma$	nents a < A <sup>2/3</sup> 2-5%
	$\sigma_{inel}(p, C, Fe) \sim 40$	, 250,	750 mb	
1	$n_{ISM} = 1 \ \mathrm{cm}^{-3}$	Disc	density	
C	$c \simeq 3 \cdot 10^{10} \mathrm{~cm~s^{-1}}$	Spee	ed of ligh	t
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#### - Measurements

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t<sub>strip</sub>≪ t

attach

 $\rightarrow GCRs \ are \ ions!$ 

- Attachment 0
- Stripping



Attachment

Stripping

Ec[GeV/n]

10

10

10

10

10

10

10 10 10 25

 $= \sigma_{tot} - \sigma_{el}$ 

 $\sigma_{inel}$ 

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Destruction $\rightarrow$ "sou	rce" for fragments
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 $t_{\rm EC} \gtrsim E_{k/n} \times 50 \,\,{\rm Myr}$ 



Decay rate 
$$\Gamma_{rad} = \frac{4\pi (2)}{\gamma t_{1/2}}$$
  
 $t_{\beta} \simeq E_{k/n} \left( \frac{t_{1/2}}{1.51 \text{ Myr}} \right) \text{Myr}$ 

Z=5-10-15-20-2

t [Myr]

10<sup>5</sup>

10

10

10

10

10 10

10

10

10

$$\Rightarrow \textbf{Decay rate} \quad \Gamma_{rad} = \frac{\ln(2)}{\gamma t_{1/2}}$$
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Attachment

Stripping

Ec[GeV/n]

struction 
$$\rightarrow$$
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Ec[GeV/n]

10

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- EC decay:
  - Very few candidates for Z<30
  - Number of EC-unstable isotopes is a nightmare for UHCR!

II.5 Other losses

### Transport of cosmic rays (CR) in the Galaxy

### II. Processes, ingredients, characteristic times

- 1. Definitions
- 2. Diffusion (space and momentum)
- 3. Convection and adiabatic losses
- 4. Energy losses (continuous)
- 5. Catastrophic losses
- 6. All together



### Discussion: what do you conclude?

- For nuclei?
- For leptons?



 $\rightarrow$  Numbers depend on MW model parameters (halo size, diffusion coefficient...)

 $\rightarrow$  Time scale for effects in the disc overestimated: CRs see density  $n_{ISM} \ge \langle n \rangle \ge (h/H) n_{ISM}$ 



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- Leptons loose their energy

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- 1. Dominant effects
  - Nuclei escape from the Galaxy
  - Leptons loose their energy
- 2. Local origin
  - Low energy radioactive nuclei
  - High energy electrons and positrons

### Is the Galaxy an efficient "calorimeter"?

 $\rightarrow$  GALPROP run (exact numbers depend on the model used)



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### **Energy budget of the Galaxy**

- Energy density of GCRs  $w_{CR} = \int E_k N(E) dE = \int \frac{4\pi}{v} E_k I(E) dE$   $w_{CR} \simeq 0.9 \text{ eV cm}^3 \simeq 4 \cdot 10^{52} \text{ erg kpc}^{-3}$ 

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Supernovae as source of GCRs?

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- $W_{SN} = 10^{51} \text{ erg}$
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- $U_{_{SN}} = W_{_{SN}} / f_{_{SN}} \approx 10^{41} \text{ erg/s}$

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→ Requires acceleration efficiency ~ 10-50%

II.6 Time scales

### Conclusions and summary

### II. Processes, ingredients, characteristic times

- $\rightarrow$  In any (astro-)physics problem, always
  - check the time scales;
  - check the energetics.
- $\rightarrow$  For GCRs:
  - nuclei: plenty of competing effects @ GeV, escape at HE;
  - electrons: energy losses dominate at LE and HE.