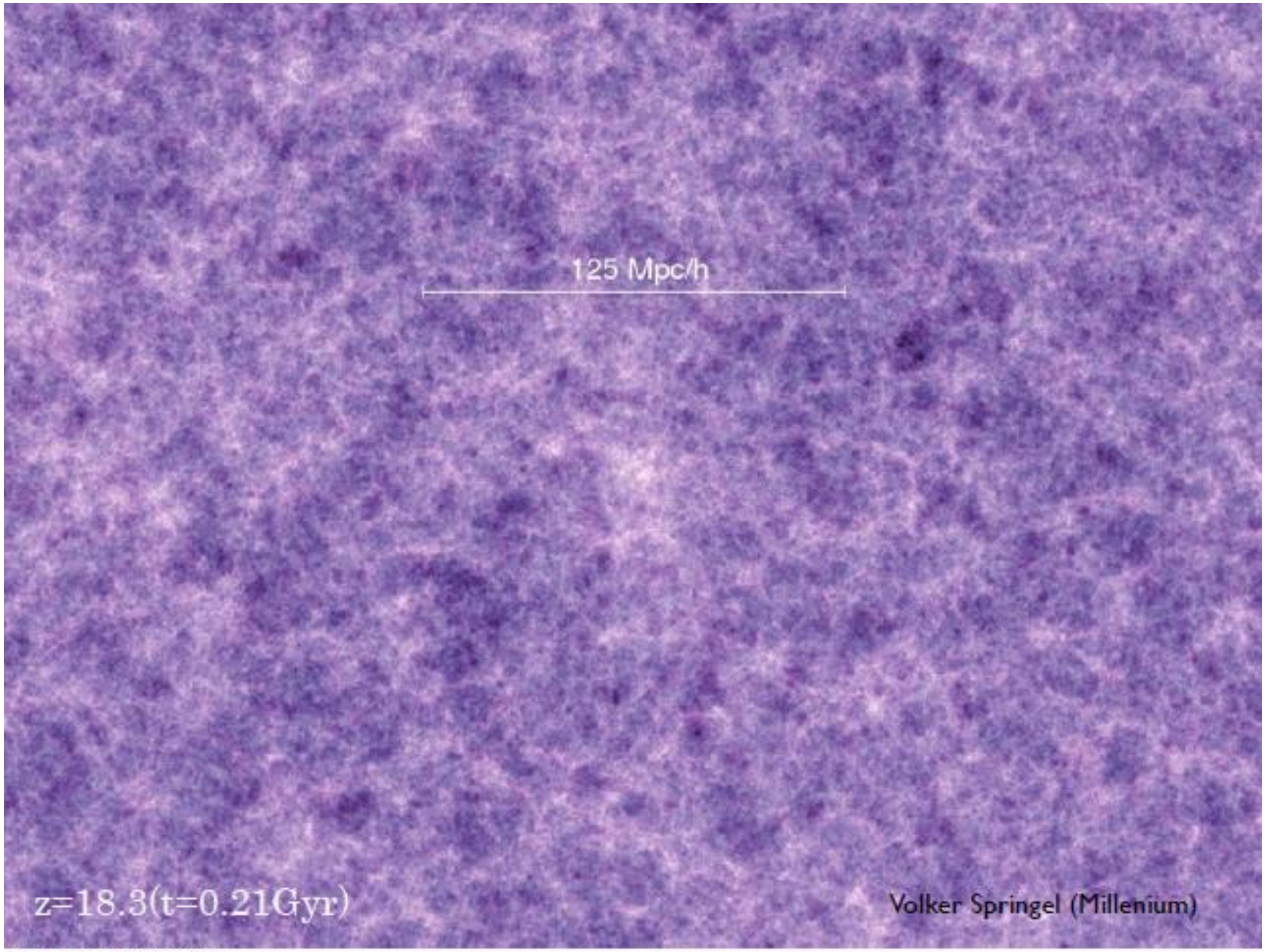
Dark Matter and Structure Formation

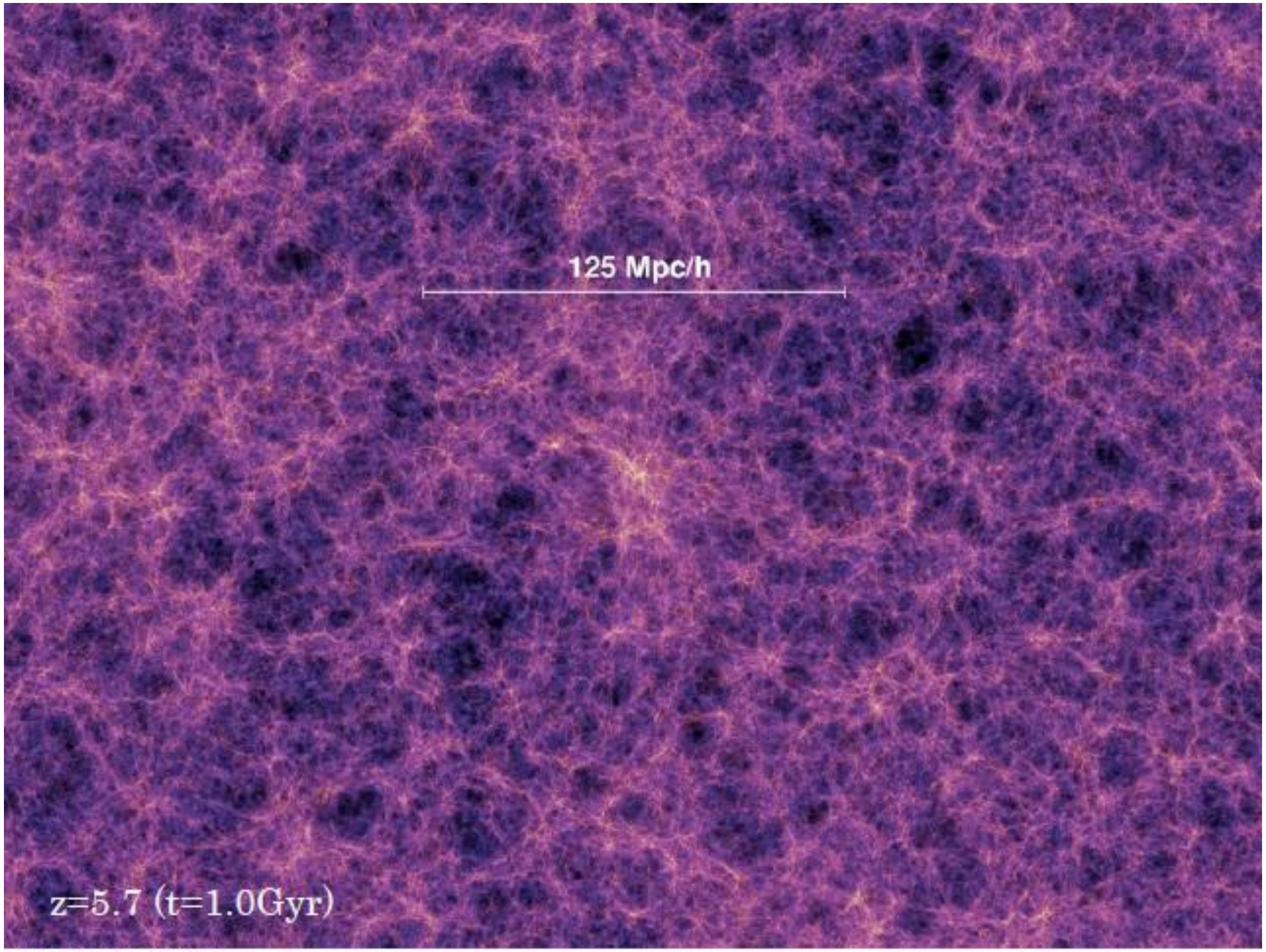
The transfer function

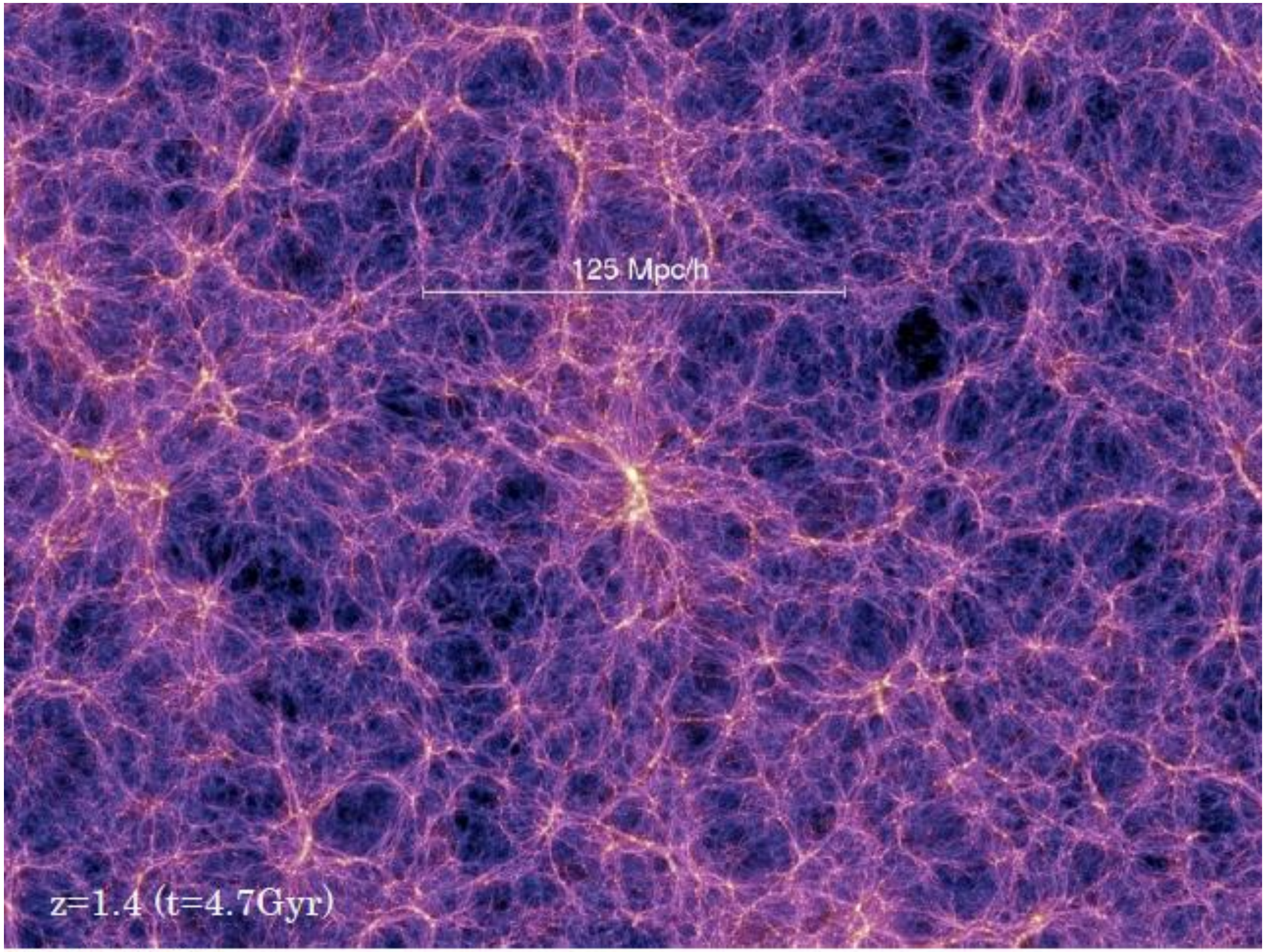
Linear theory

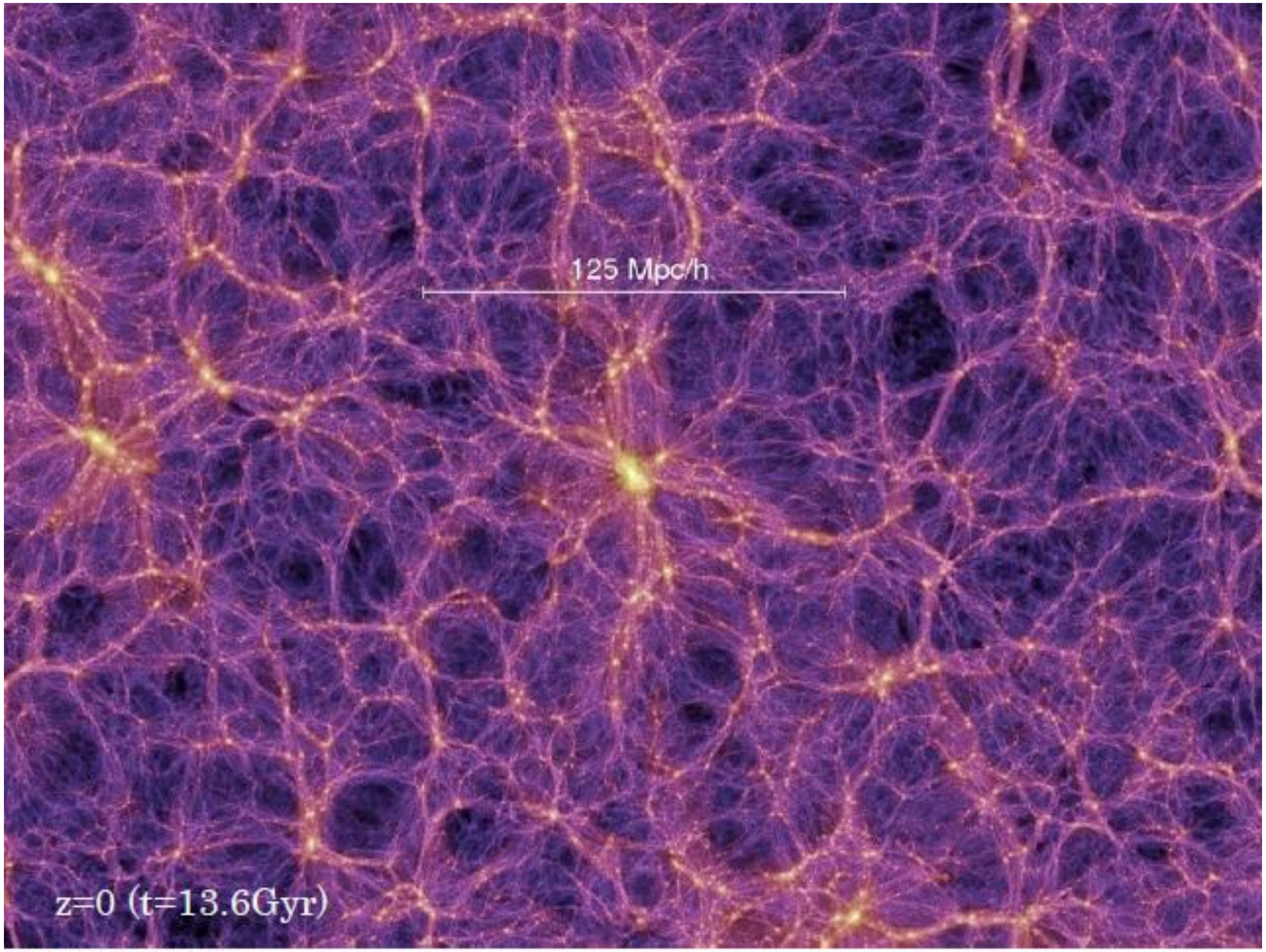
Nonlinear scalings

Baryon Acoustic Oscillations, Clusters

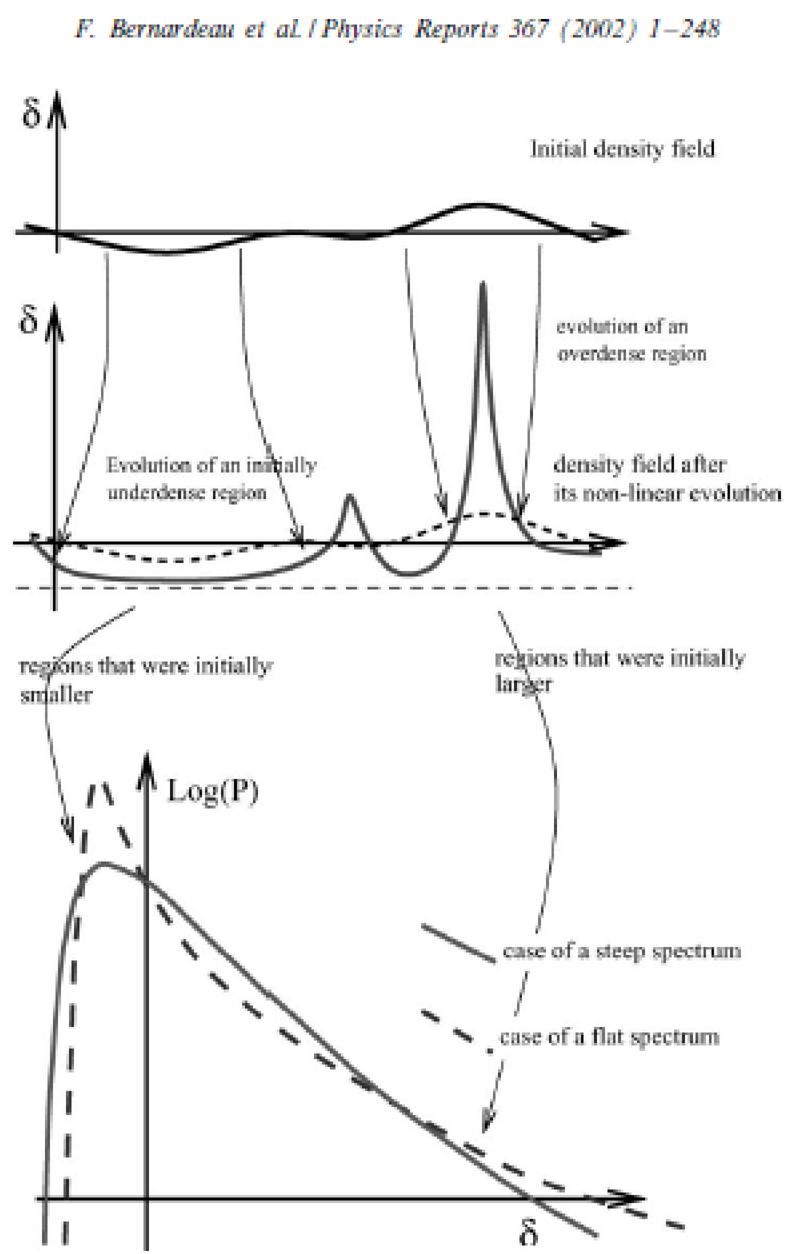

125 Mpc/h
z=18.3(t=0.21Gyr)
Volker Springel (Millenium)


125 Mpc/h
z=5.7 (t=1.0Gyr)

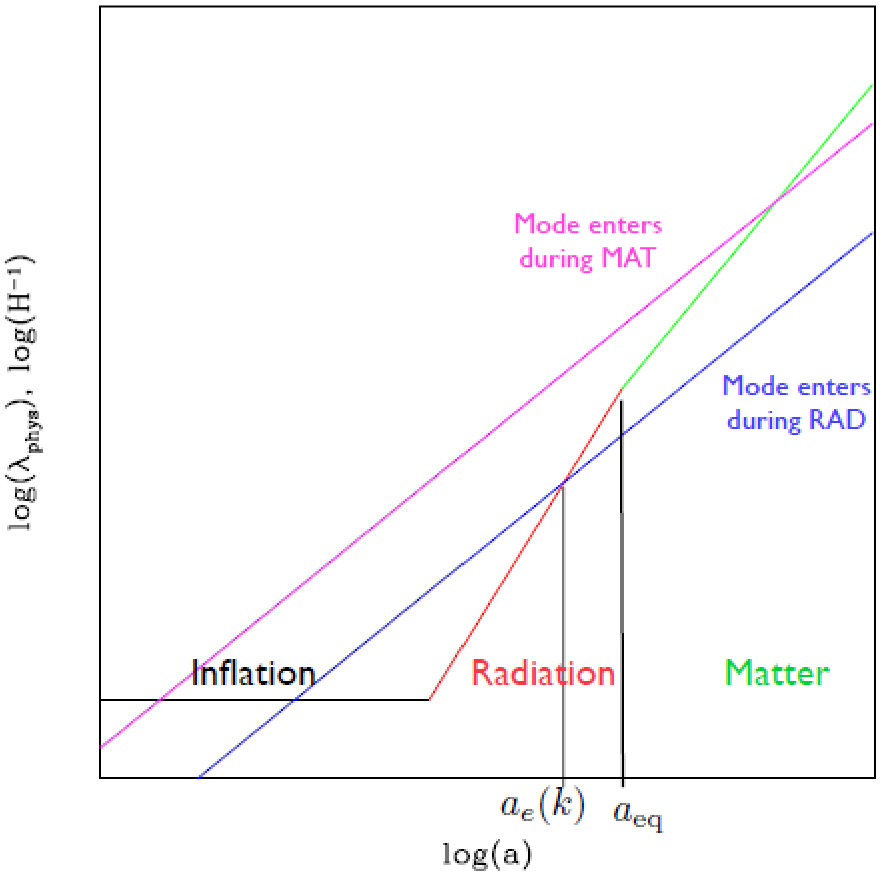

125 Mpc/h
z=1.4 (t=4.7Gyr)


125 Mpc/h
z=0 (t=13.6Gyr)

Initially Gaussian fluctuation field becomes very non-Gaussian



Different wavelengths enter horizon at different times



Sub-horizon: Linear theory

- Newtonian analysis:

  $d^2R/dt^2 = - GM/R^2(t) = - (4\pi/3)\, G\rho(t)R(t)\, [1+\delta(t)]$
- M constant means $R^3 \propto \rho^{-1} [1+\delta]^{-1} \propto a^3 [1+\delta]^{-1}$
- So $dR/dt \propto HR - d\delta/dt\, R\, [1+\delta]^{-1}/3$ and when $|\delta| << 1$

  $(d^2R/dt^2)/R = (d^2a/dt^2)/a - (d^2\delta/dt^2)/3 – (2/3)H\, (d\delta/dt)$
- So $(d^2a/dt^2)/a - (d^2\delta/dt^2)/3 – (2/3)H\, (d\delta/dt)$

  $= - (4\pi/3)\, G\rho(t)\, [1+\delta(t)]$

  $(d^2\delta/dt^2) + 2H\, (d\delta/dt) = 4\pi\, G\rho(t)\, \delta(t) = (3/2)\, \Omega_m H^2\, \delta(t)$

Linear theory (contd.)

- When radiation dominated ($H = 1/2t$):

  $(d^2\delta/dt^2) + 2H\,(d\delta/dt) = (d^2\delta/dt^2) + (d\delta/dt)/t = 0$

  $\delta(t) = C_1 + C_2 \ln(t)$ (weak growth)

- In distant future (H = constant):

  $(d^2\delta/dt^2) + 2H_\Lambda(d\delta/dt) = 0$

  $\delta(t) = C_1 + C_2 \exp(-2H_\Lambda t)$

- If flat matter dominated ($H = 2/3t$):

  $\delta(t) = D_+ t^{2/3} + D_- t^{-1} \propto a(t)$ at late times

- Because linear growth just multiplicative factor, it cannot explain non-Gaussianity at late times

Super-horizon growth

- Start with Friedmann equation when $\kappa=0$:

$$H^2 = (8\pi G/3)\,\rho$$

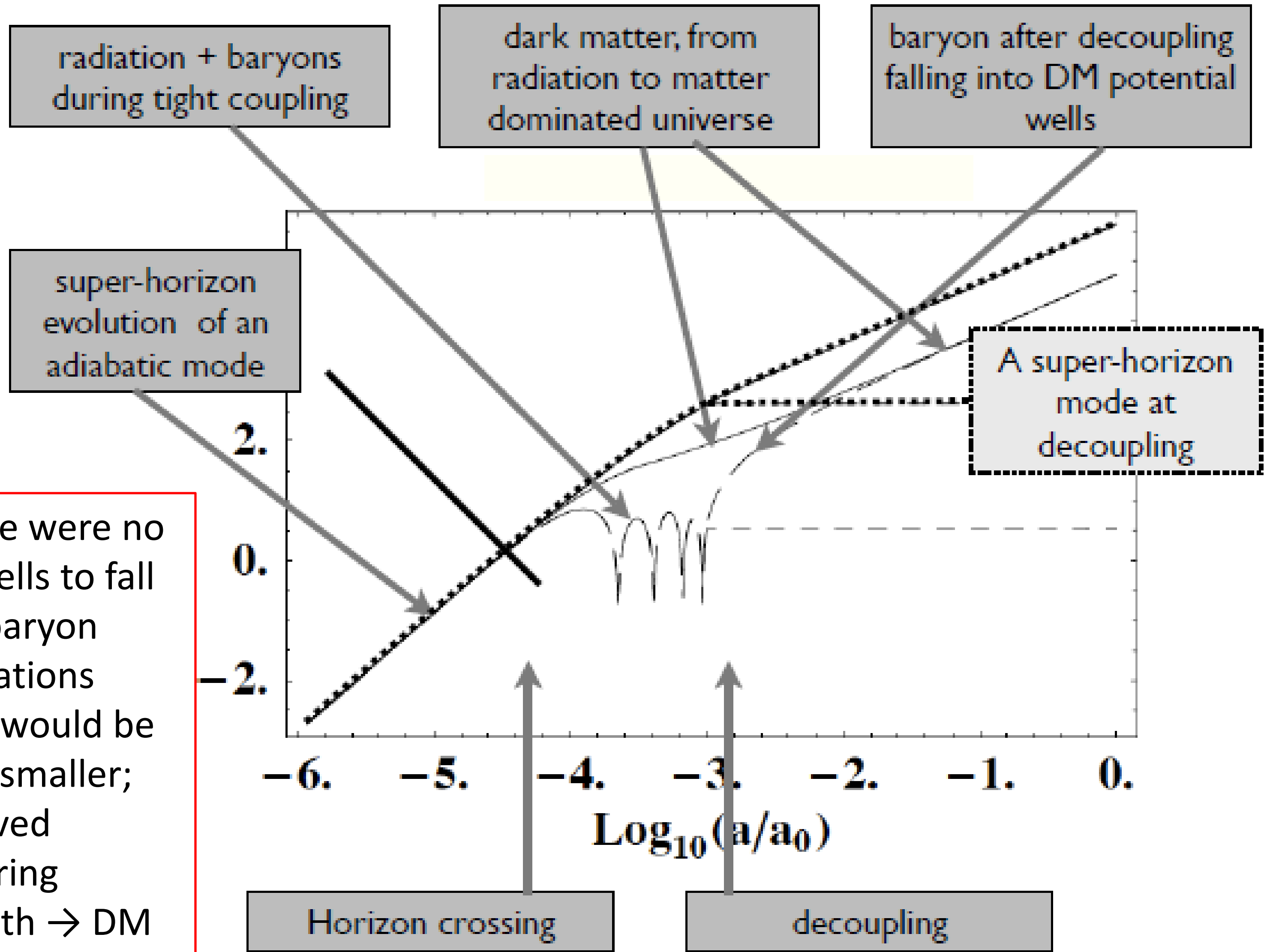
- Now consider a model with same H but slightly higher ρ (so it is a closed universe):

$$H^2 = 8\pi G\rho_1/3 - \kappa/a^2$$

- Then $\delta = (\rho_1 - \rho)/\rho = (\kappa/a^2)/(8\pi G\rho/3)$
- For small δ we have $\delta \propto a$ (matter dominated) but $\delta \propto a^2$ (radiation dominated)

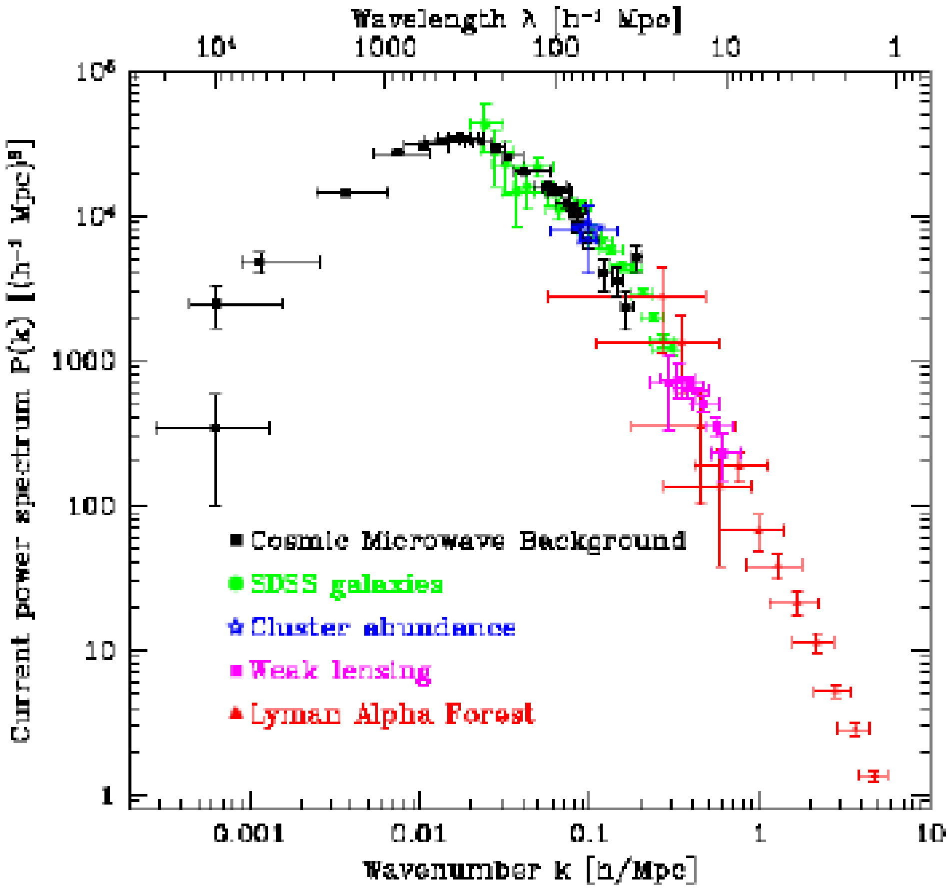
Putting it together

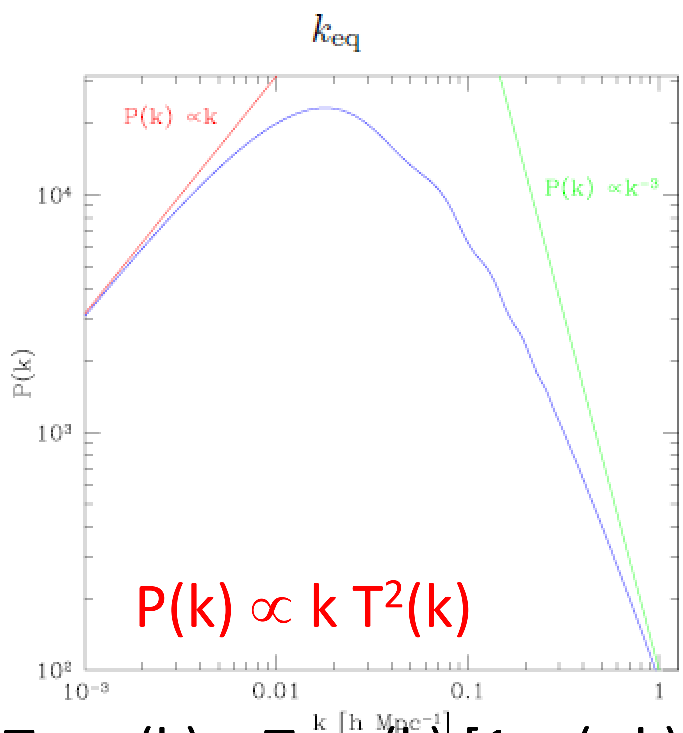
- Consider two modes, λ_1 and $\lambda_2 < \lambda_1$, which entered at $a_1/a_2 = \lambda_1/\lambda_2$ while radiation dominated
- Their amplitudes will be $(a_1/a_2)^2 = (k_2/k_1)^2$ so expect suppression of power $\propto k^{-2}$ at $k>k_{eq}$ (i.e. for the short wavelength modes which entered earlier)
- After entering horizon, dark matter grows only logarithmically until matter domination, after which it grows $\propto a$
- Baryons oscillate (i.e. don't grow) until decoupling, after which they fall into the deeper wells defined by the dark matter



If there were no DM wells to fall into, baryon fluctuations today would be much smaller; observed clustering strength → DM must exist!

Transfer function: $T(k) \propto 1/(1+k^2)$



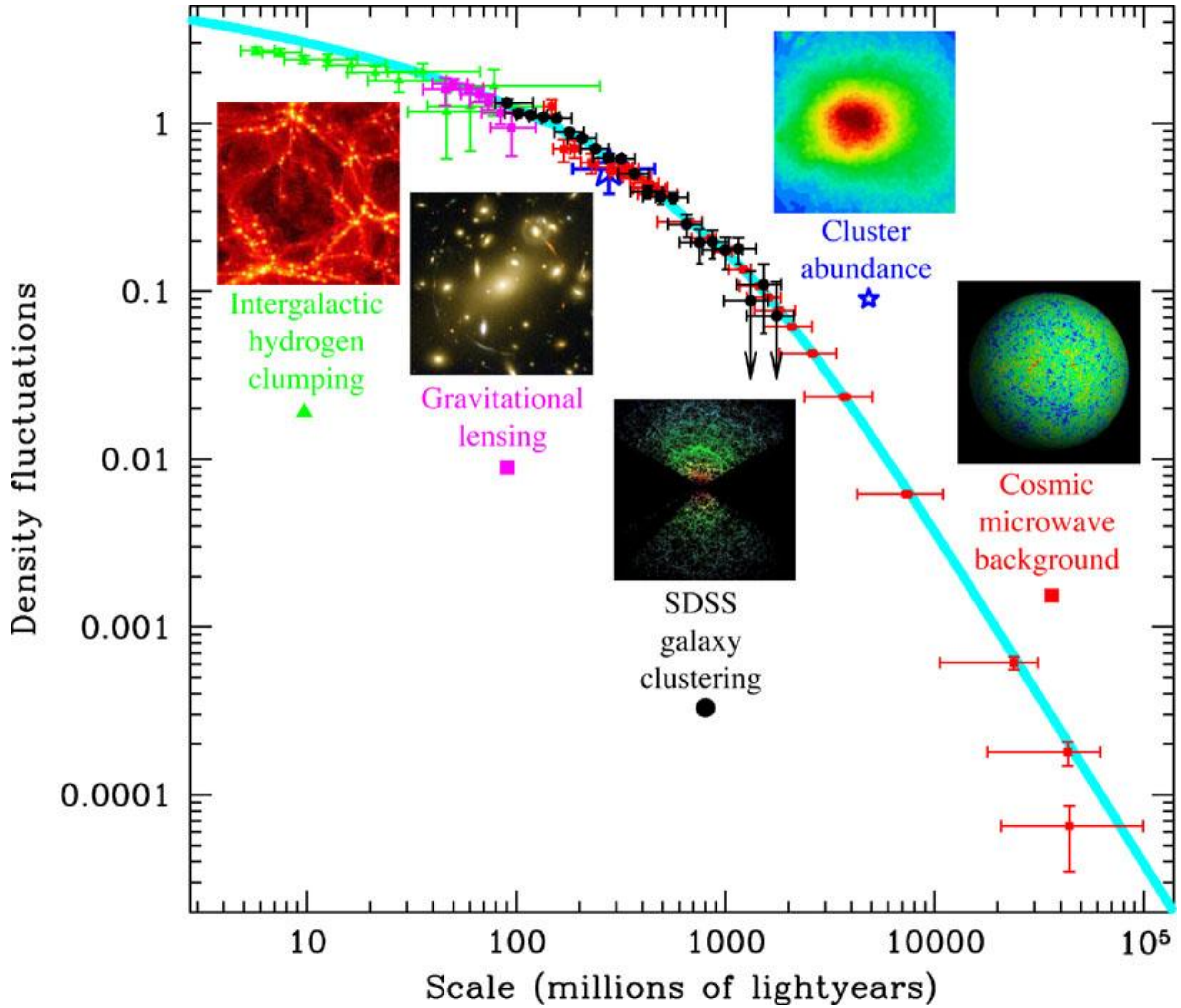


$P(k) \propto k\,T^2(k)$

$T_{WDM}(k) \approx T_{CDM}(k)\,[1 + (\alpha k)^2]^{-5}$

$$\alpha \equiv 0.05 \left(\frac{\Omega_m}{0.4}\right)^{0.15} \left(\frac{h}{0.65}\right)^{1.3} \left(\frac{m_{\rm dm}}{1\,{\rm keV}}\right)^{-1.15} h^{-1}{\rm Mpc}$$

Same, but position- (rather than k-) space



$$\sigma^2(r) = (2\pi)^{-3} \int dk\, 4\pi k^2\, P(k)\, W^2(kr) \qquad W(x) \sim (3/x)\, j_1(x)$$

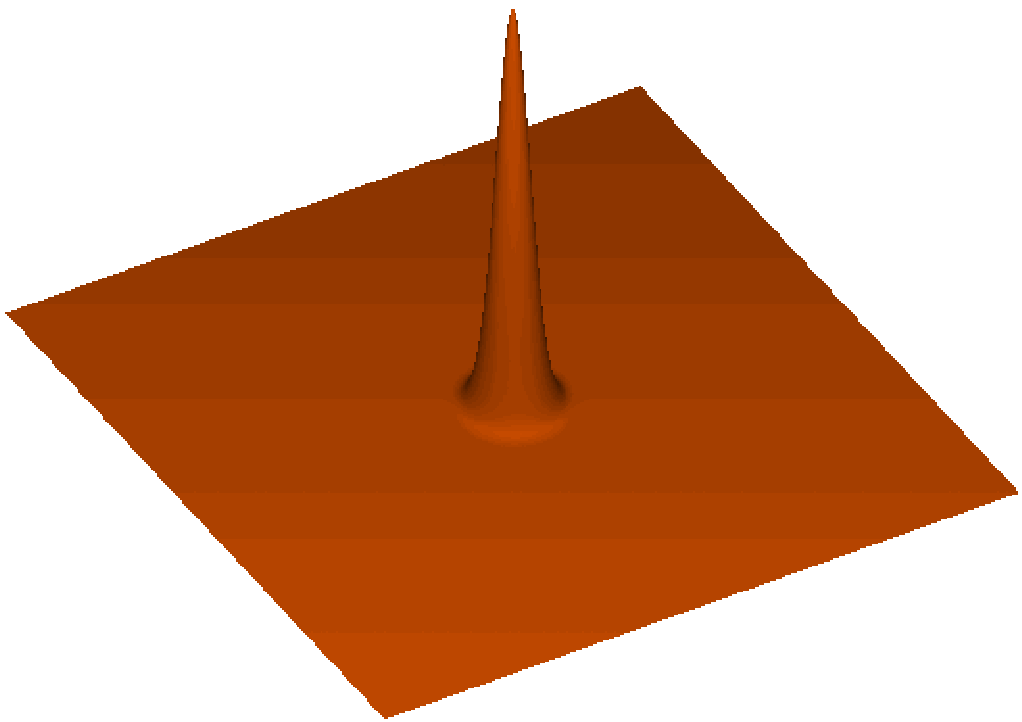
Cosmology from the same physics imprinted in the galaxy distribution at different redshifts:

Baryon Acoustic Oscillations

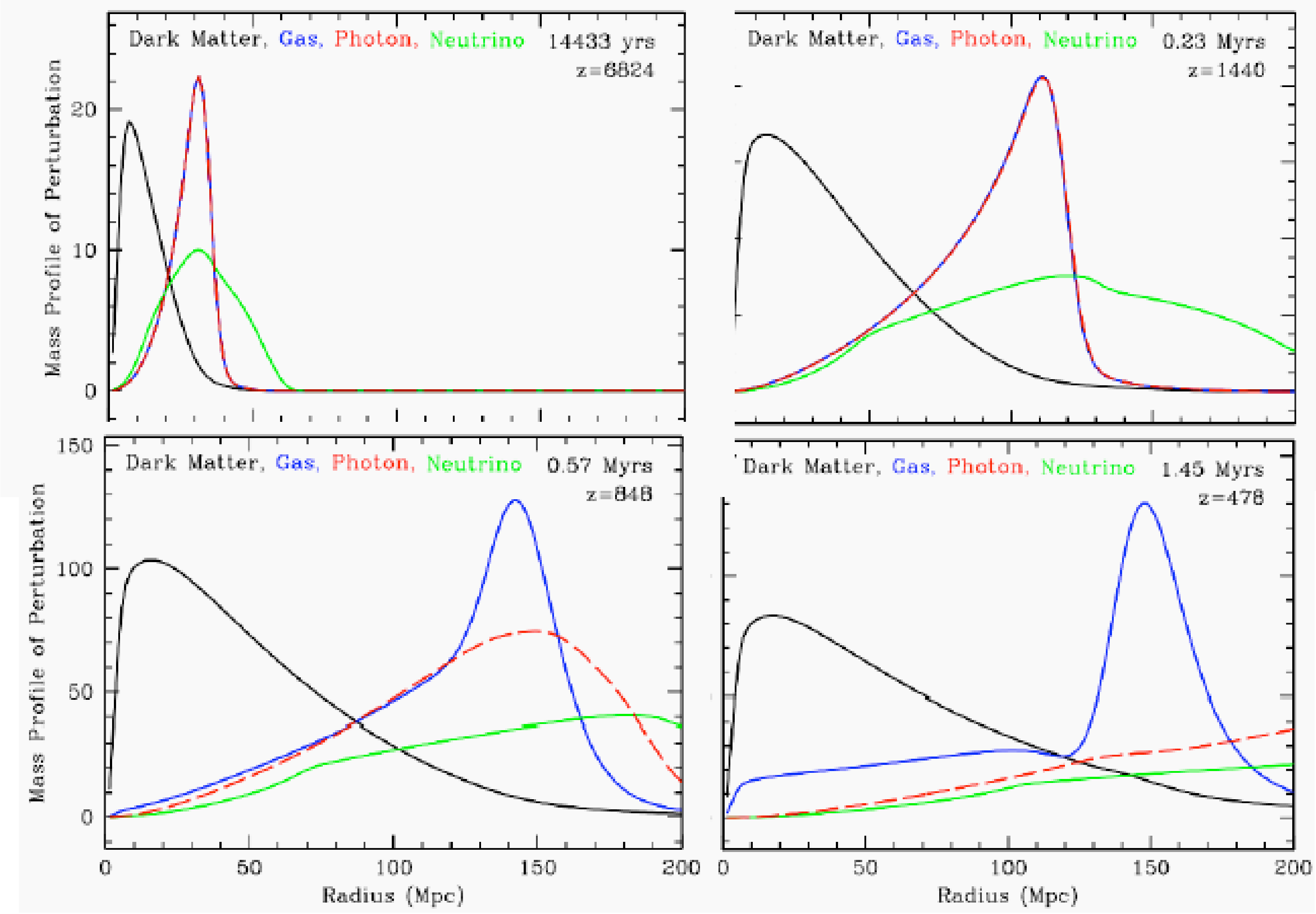
CMB from interaction between photons and baryons when Universe was 3,000 degrees (about 300,000 years old)

- Do galaxies which formed much later carry a memory of this epoch of last scattering?

Photons ‘drag’ baryons for 300,000 years…
300,000 light years ~ 100,000 pc ~ 100 kpc

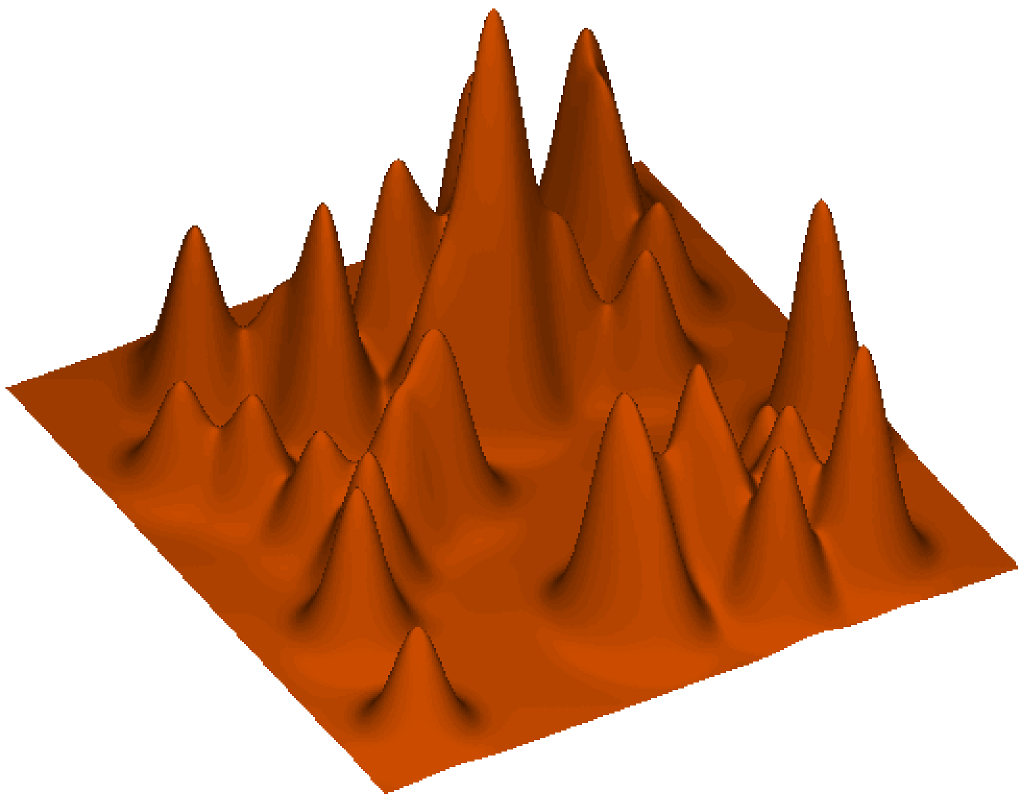


Expansion of Universe since then stretches this to (3000/2.725) ×100 kpc ~ 100 Mpc



Dark Matter, Gas, Photon, Neutrino
14433 yrs
z=6824
0.23 Myrs
z=1440
0.57 Myrs
z=848
1.45 Myrs
z=478
Mass Profile of Perturbation
Radius (Mpc)
0
10
20
50
100
150
200

Expect to see a feature in the Baryon distribution on scales of 100 Mpc today

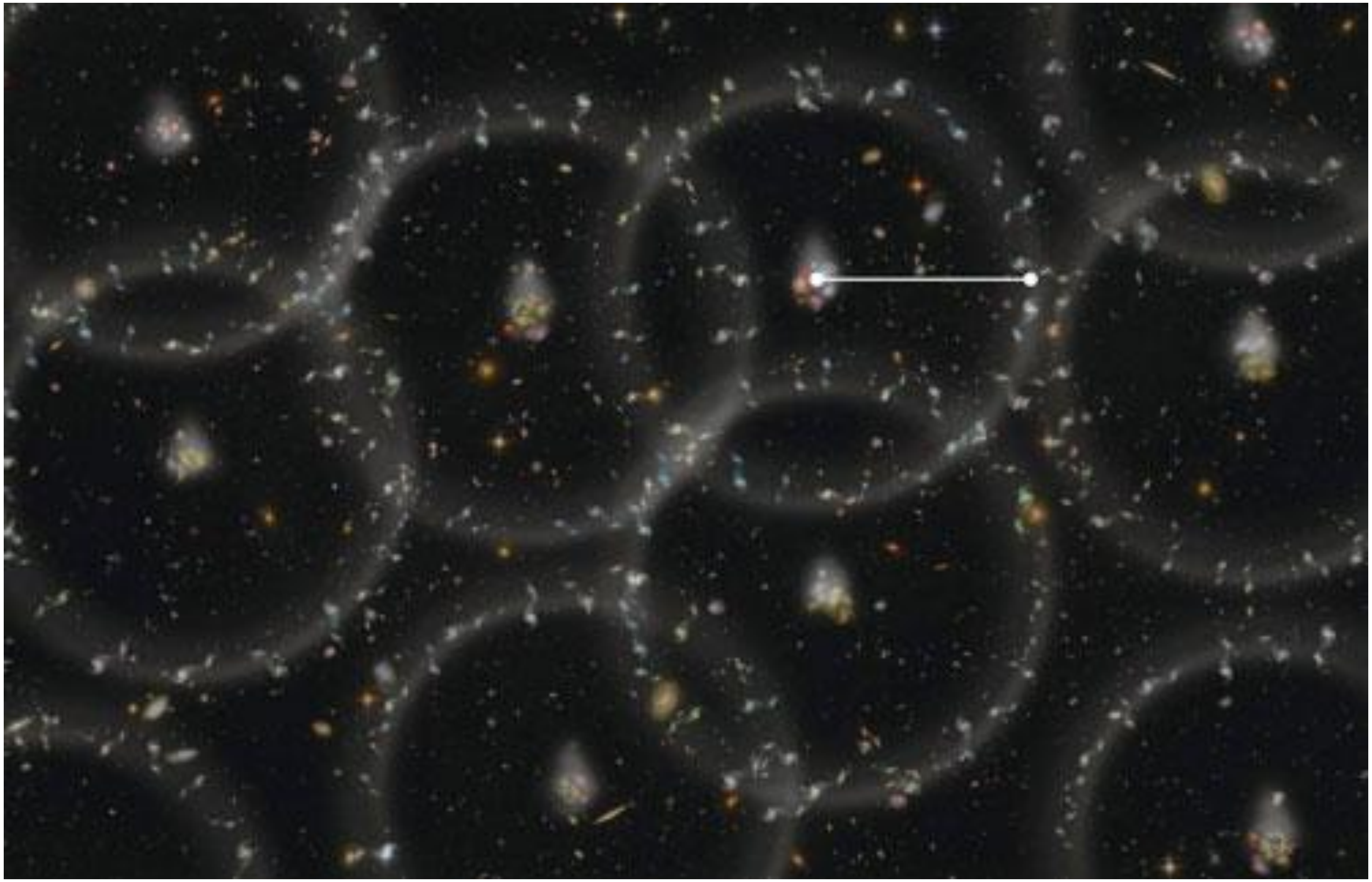


But this feature is like a standard rod:

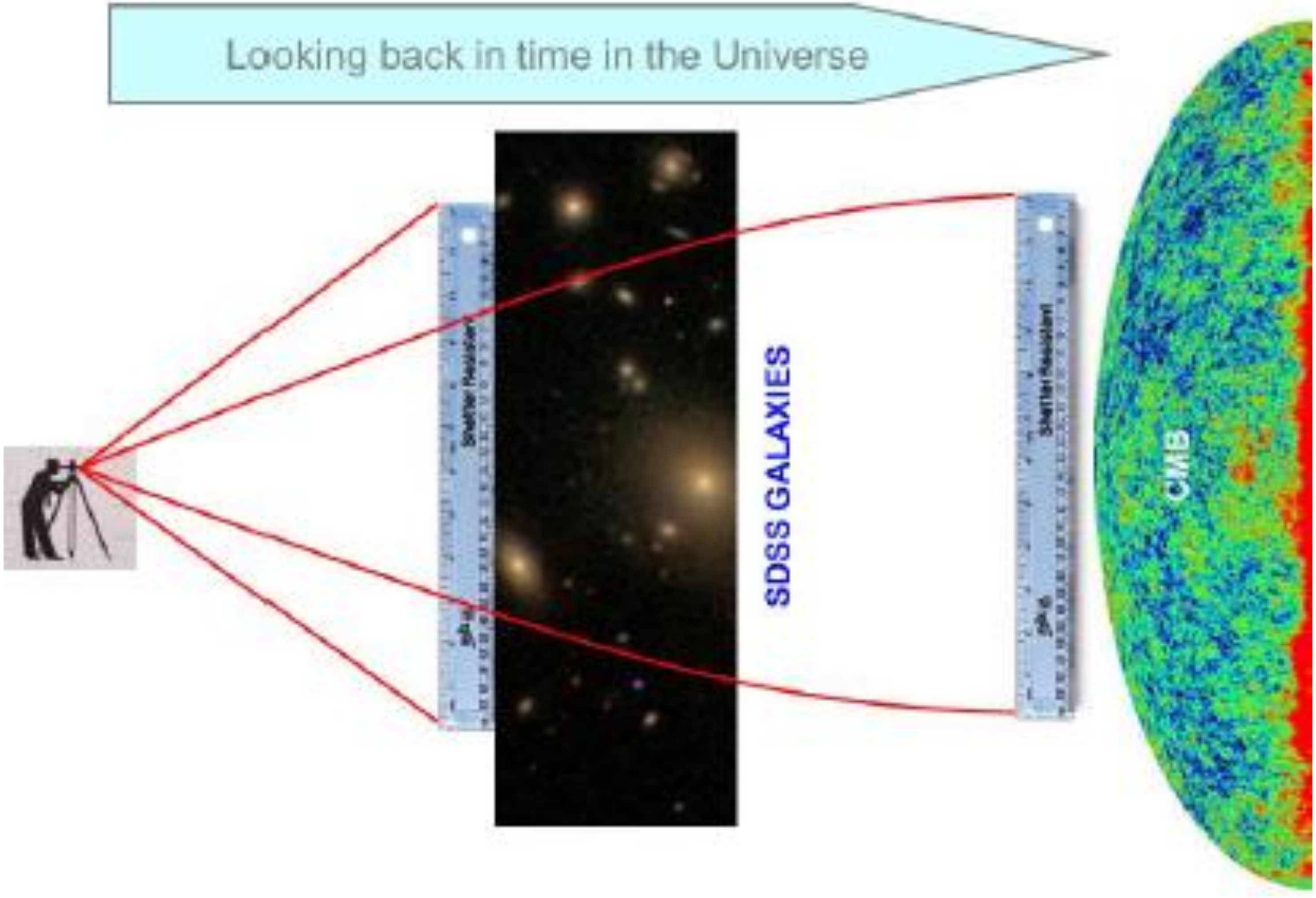
We see it in the CMB itself at z~1000

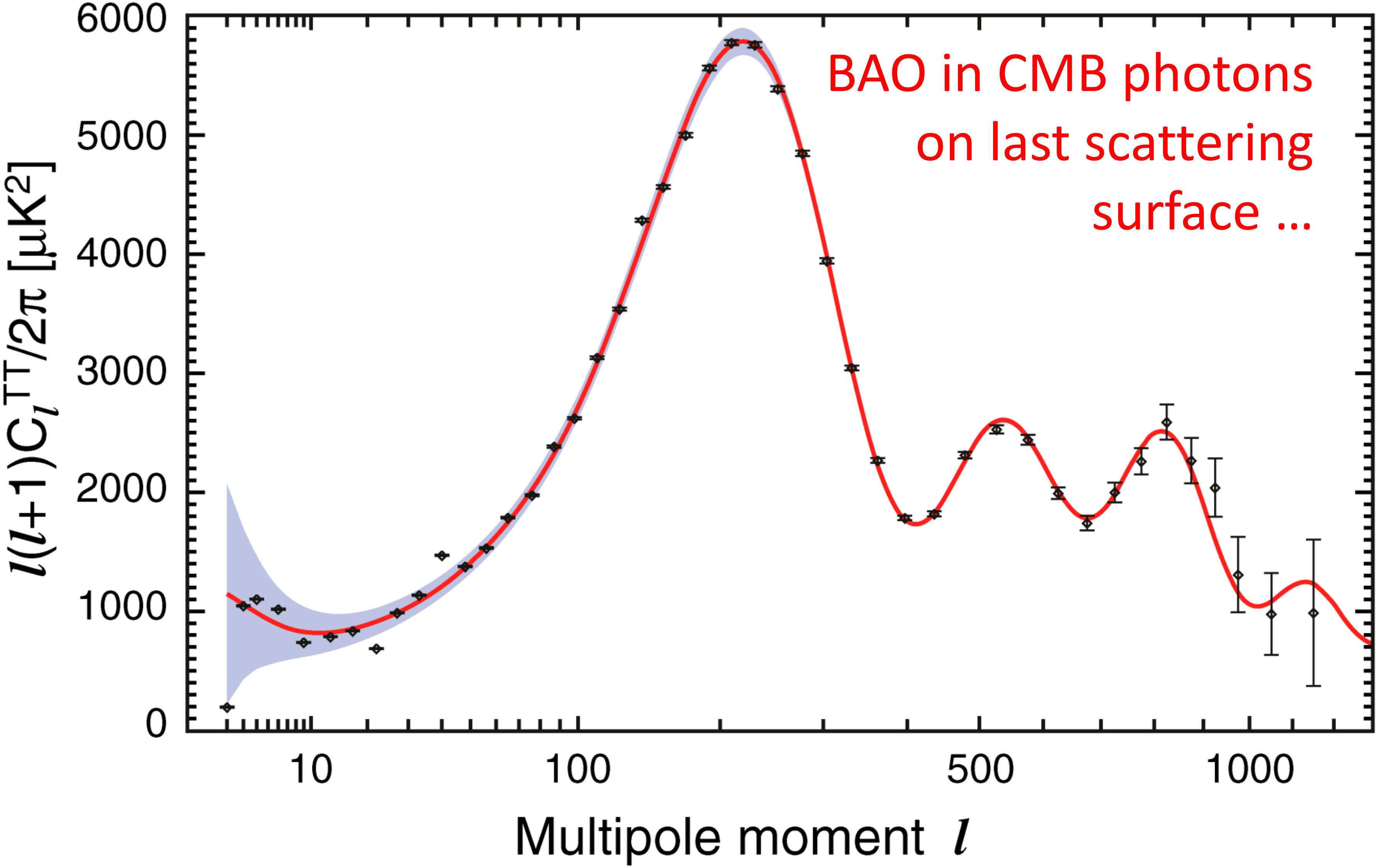
Should see it in the galaxy distribution at other z

Cartoon of expected effect

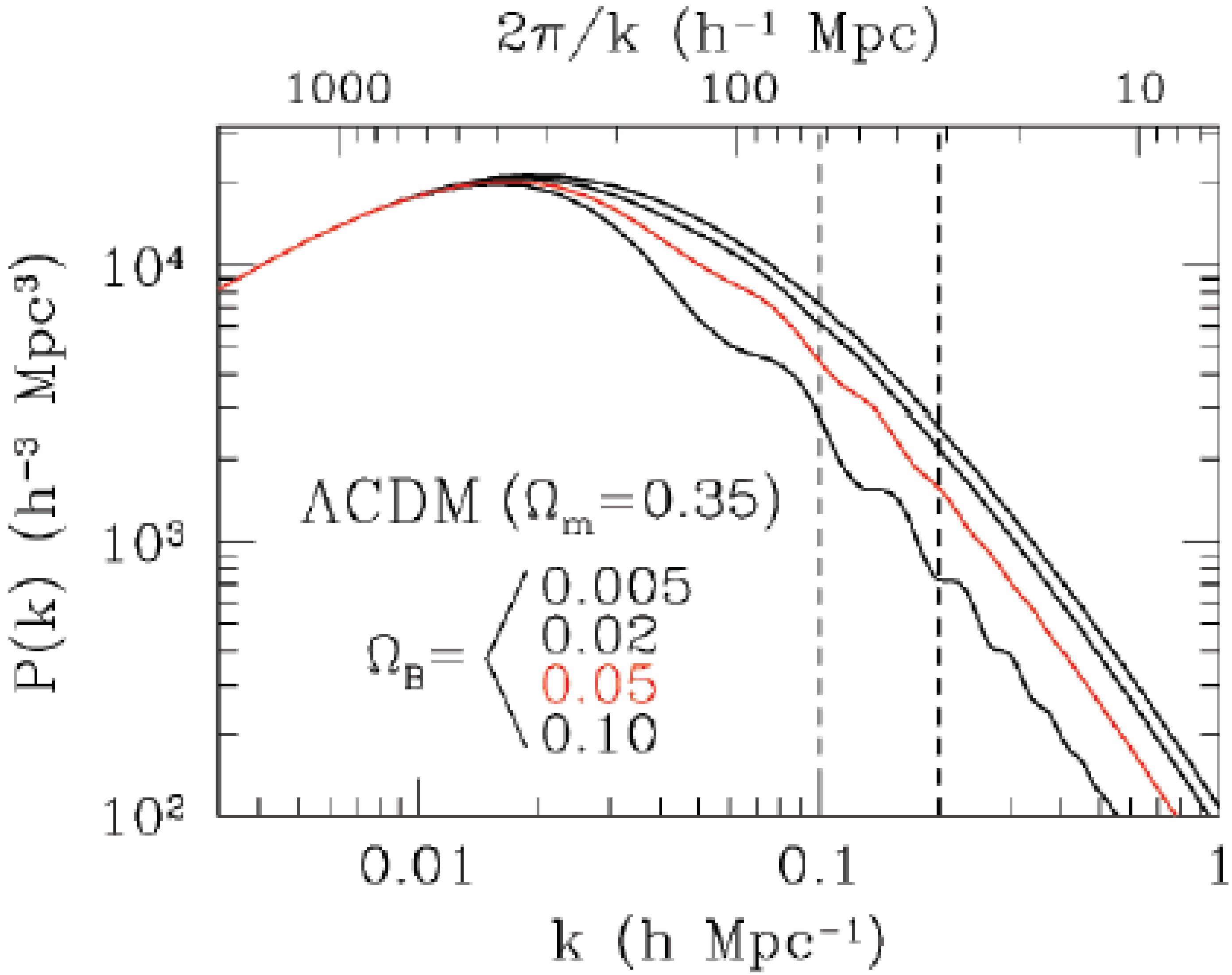


Baryon Oscillations in the Galaxy Distribution





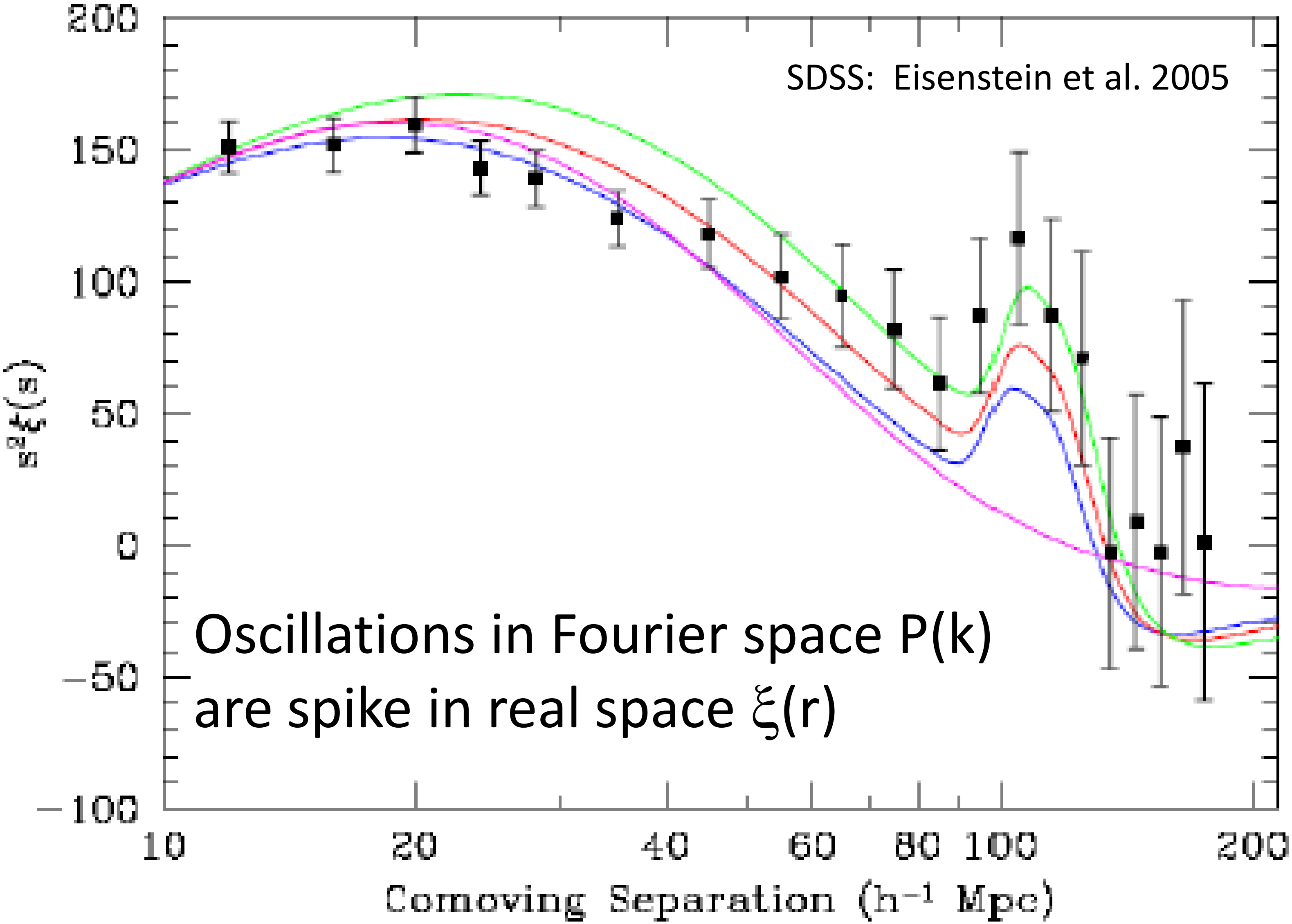
6000
5000
4000
3000
2000
1000
0
$l(l+1)C_l^{TT}/2\pi$ [μK²]
10
100
500
1000
Multipole moment l
BAO in CMB photons
on last scattering
surface ...

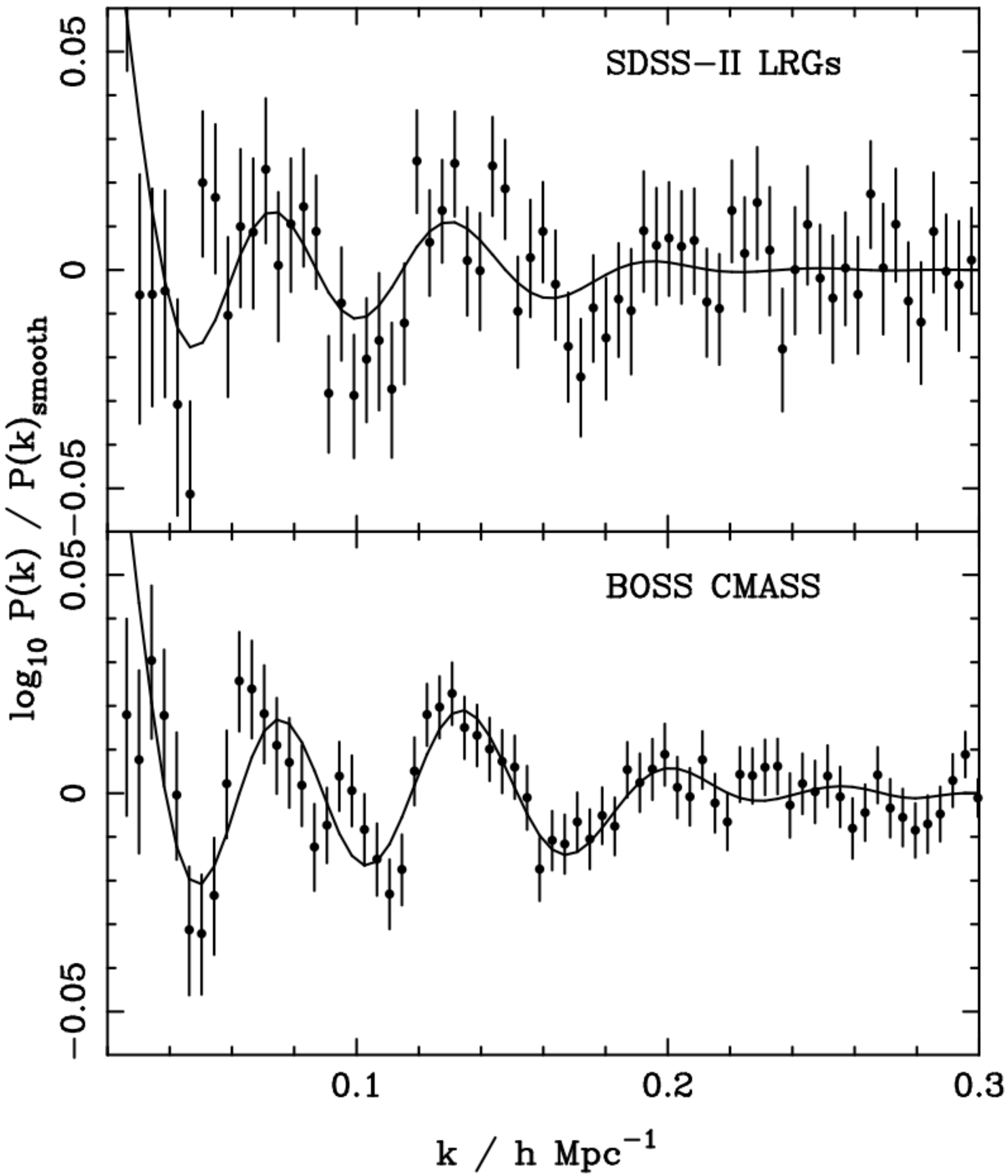


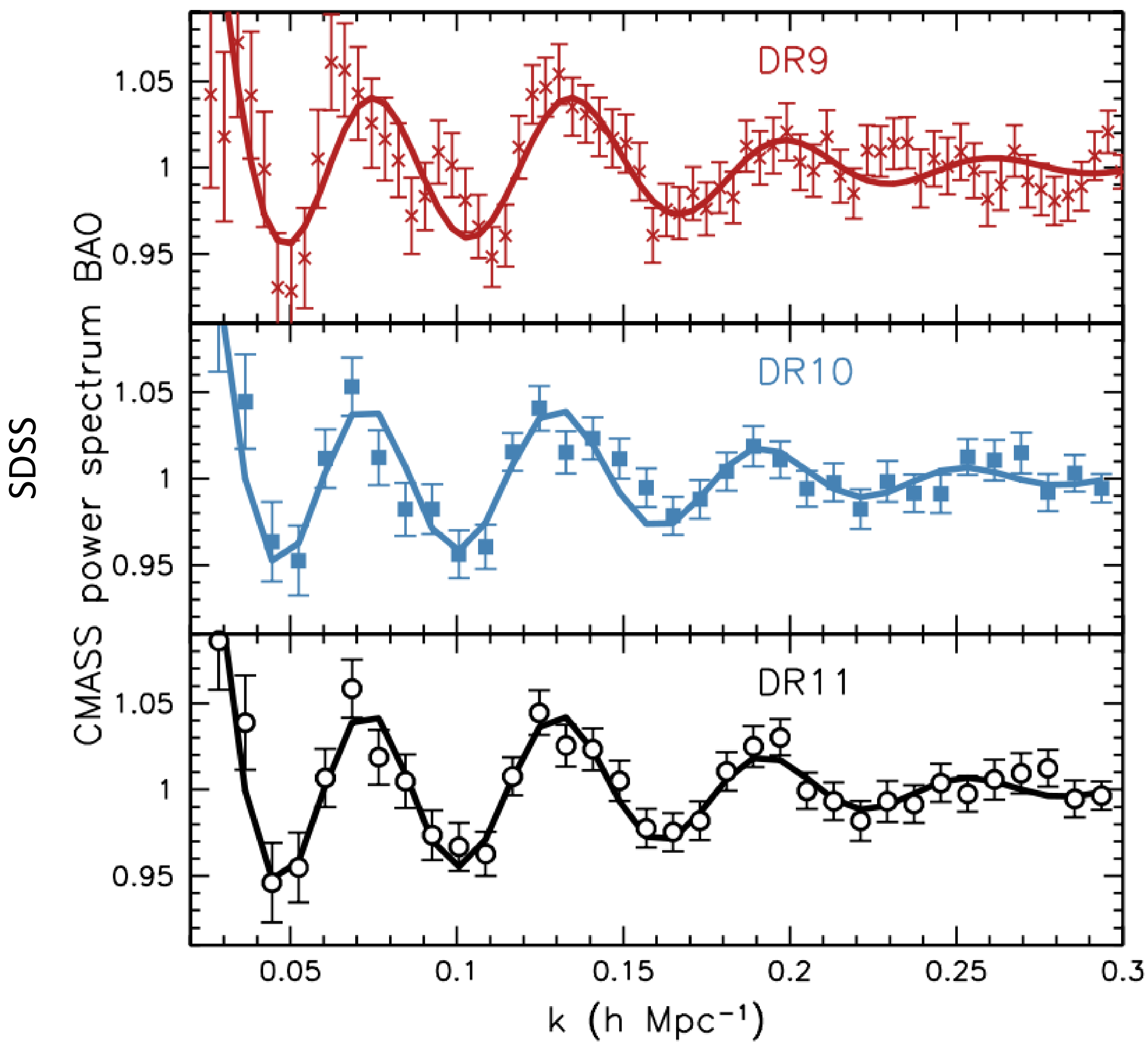
… should still be seen in matter distribution at later times

…we need a tracer of the baryons

- Luminous Red Galaxies
  - Luminous, so visible out to large distances
  - Red, presumably because they are old, so probably single burst population, so evolution relatively simple
  - Large luminosity suggests large mass, so probably strongly clustered, so signal easier to measure
  - Linear bias on large scales, so *length of rod* not affected by galaxy tracer!


SDSS: Eisenstein et al. 2005
Oscillations in Fourier space P(k)
are spike in real space ξ(r)
s²ξ(s)
200
150
100
50
0
−50
−100
10
20
40
60
80
100
200
Comoving Separation (h⁻¹ Mpc)


SDSS−II LRGs
BOSS CMASS
0.05
0
−0.05
0.1
0.2
0.3
$\log_{10}$ P(k) / P(k)$_{smooth}$
k / h Mpc^{-1}

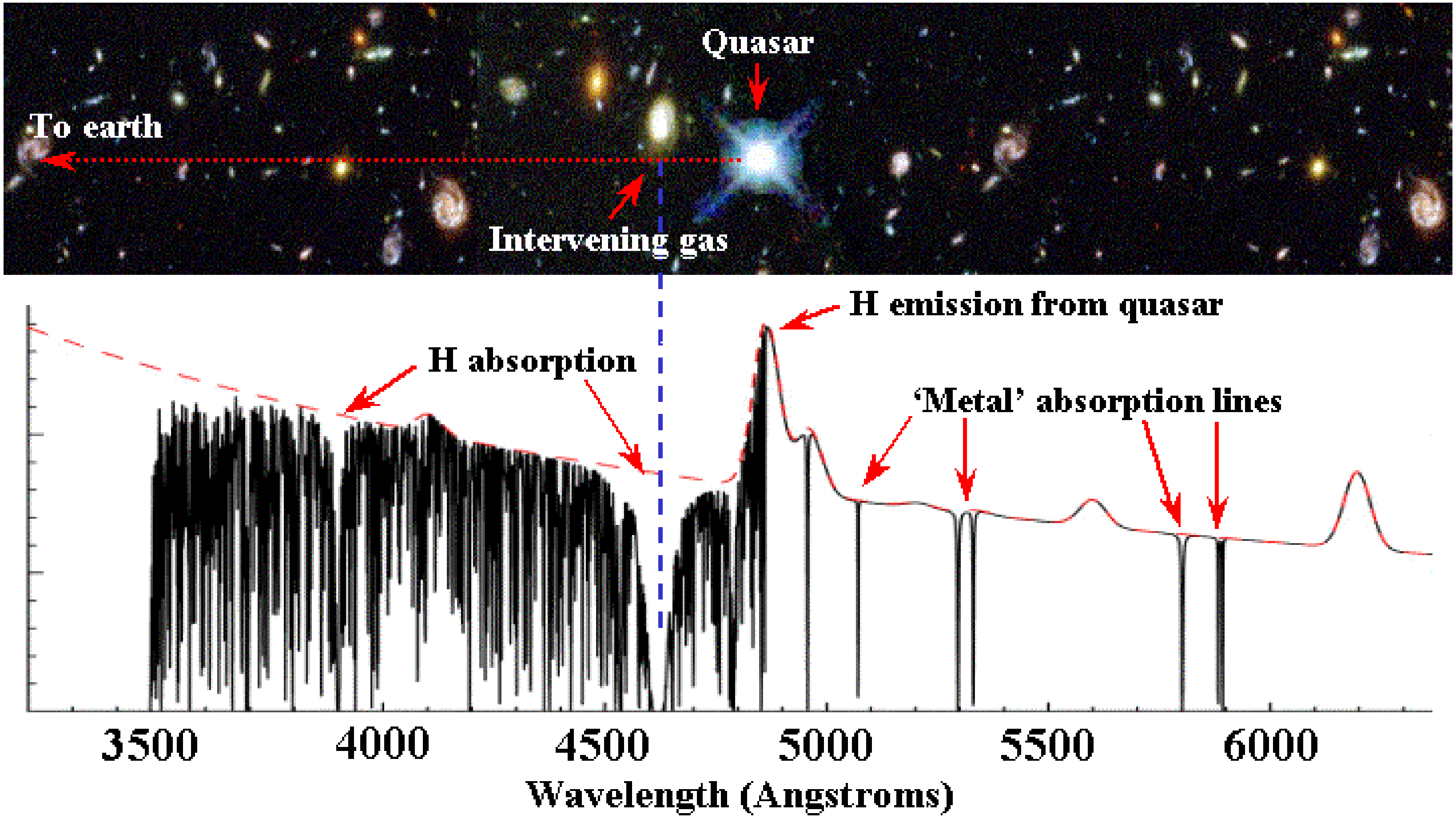


DR9
DR10
DR11
1.05
1
0.95
0.05
0.1
0.15
0.2
0.25
0.3
k (h Mpc⁻¹)
SDSS
CMASS power spectrum BAO

- The baryon distribution today ‘remembers’ the time of decoupling/last scattering; can use this to build a ‘standard rod’

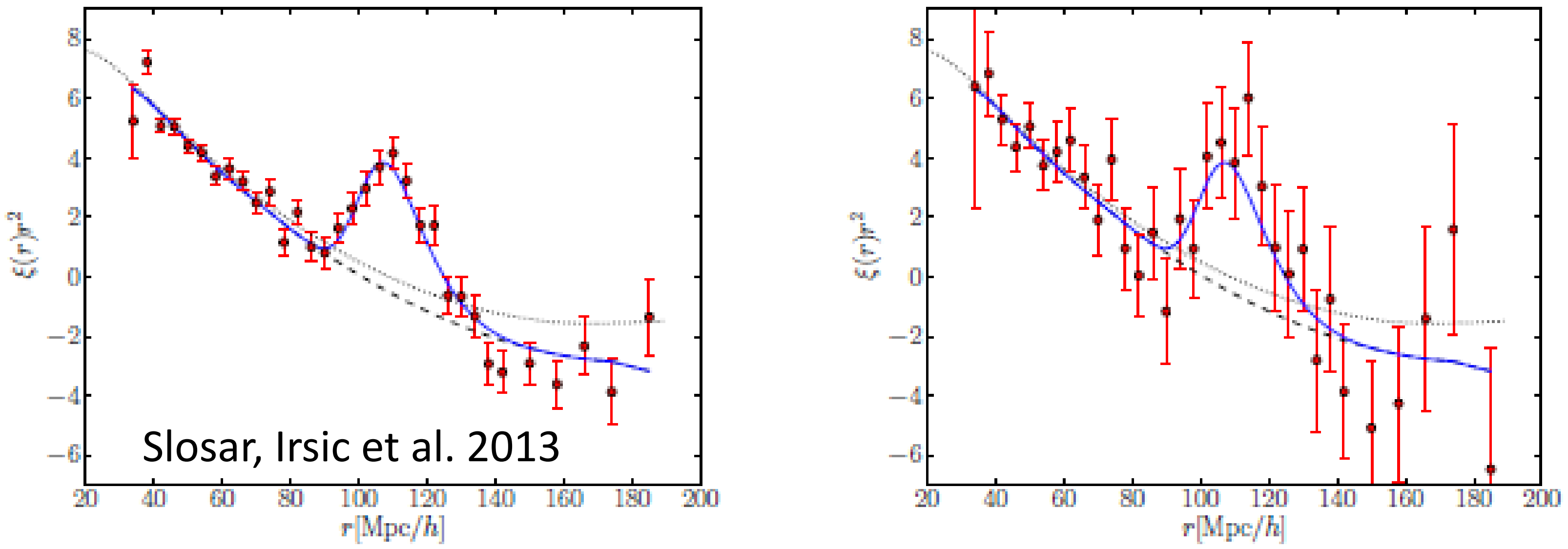
- Next decade will bring observations of this standard rod out to redshifts $z \sim 1$. Constraints on model parameters from 10% to 1%

Can see baryons that are not in stars …



High redshift structures constrain neutrino mass

BAO in Ly-α forest at z~2.4



- Signal from cross-correlating different lines of sight

Nonlinear scale

- $\langle\delta^2(t)\rangle = \int dk/k \; 4\pi \; k^3 \; P(k,t) \; W^2(kR)$
- If $P(k) = Ak^n$ then $\langle\delta^2(t)\rangle \sim R^{-(3+n)} \sim M^{-(3+n)/3}$ converges only for n>-3.
- Convergence of potential fluctuations only if n=1.
- Note: $P(k,t) = D_+^2(t) \; P(k)$, so $\langle\delta^2(t)\rangle \sim 1$ means nonlinear structure on scales smaller than $R_{nl} \sim D_+^{2/(3+n)} \sim t^{(4/3)/(3+n)}$

Hierarchical structure formation for -3<n<1

N-body simulations of

gravitational clustering

in an expanding universe

It’s a capitalist’s life…

- Most of the action is in the big cities
- Newcomers to the city are rapidly stripped of (almost!) all they have
- Encounters generally too high-speed to lead to long-lasting mergers
- Repeated ‘harassment’ can lead to change
- Real interactions take place in the outskirts
- A network exists to channel resources from the fields to feed the cities

Nonlinear evolution

Assume a spherical cow ....

Spherical evolution model

$$d^2R/dt^2 = -\,GM/R^2 + \Lambda\, R$$
$$= -\,\rho\,(4\pi G/3H^2)\, H^2 R + \Lambda\, R$$
$$= -\,½\, \Omega(t) H(t)^2\, R \quad + \Lambda\, R$$

- Note: currently fashionable to modify gravity. Should we care that only $1/R^2$ or R give stable circular orbits?

Spherical evolution model

- Initially, $E_i = -GM/R_i + (H_iR_i)^2/2$
- Shells remain concentric as object evolves; if denser than background, object pulls itself together as background expands around it
- At 'turnaround': $E = -GM/r_{\rm max} = E_i$
- So $-GM/r_{\rm max} = -GM/R_i + (H_iR_i)^2/2$
- Hence $(R_i/r) = 1 - H_i^2R_i^3/2GM$

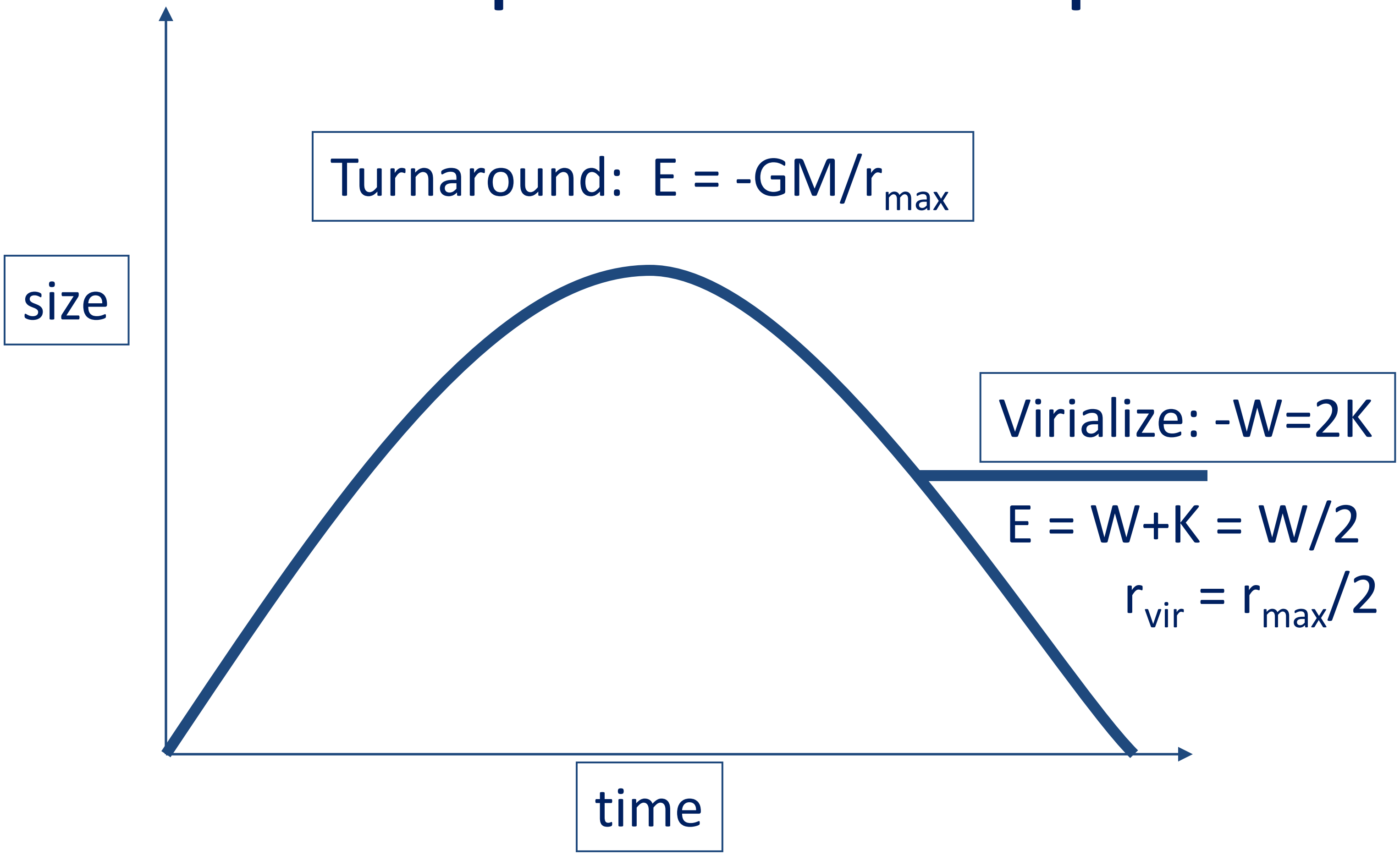
$$= 1 - (3H_i^2/8\pi G)\,(4\pi R_i^3/3)/M$$

$$= 1 - 1/(1+\Delta_i) = \Delta_i/(1+\Delta_i) \approx \Delta_i$$

Virialization

- Final object virializes: $-W = 2K$
- $E_{vir} = W+K = W/2 = -GM/2r_{vir} = -GM/r_{max}$
  - so $r_{vir} = r_{max}/2$:
- Ratio of initial to final size = (density)$^{1/3}$
  - final density determined by initial overdensity
- To form an object at present time, must have had a critical over-density initially
- Critical density same for all objects!
- To form objects at high redshift, must have been even more over-dense initially

Spherical collapse



Modify gravity → modify collapse

Exact Parametric Solution

(R_i/R) vs. θ and (t/t_i) vs. θ

very well approximated by…

$$(R_{\rm initial}/R)^3$$

$$= \text{Mass}/(\rho_{\rm com}\text{Volume})$$

$$= 1 + \delta \approx (1 - D_{\rm Linear}(t)\ \delta_i/\delta_{sc})^{-\delta sc}$$

Dependence on cosmology from $\delta_{sc}(\Omega,\Lambda)$, but this is rather weak

$$1 + \delta \approx (1 - \delta_{Linear}/\delta_{sc})^{-\delta sc}$$

- As $\delta_{Linear} \rightarrow \delta_{sc}$, $\delta \rightarrow$ infinity
  - This is virialization limit
- As $\delta_{Linear} \rightarrow 0$, $\delta \approx \delta_{Linear}$
- If $\delta_{Linear} = 0$ then $\delta = 0$
  - This does not happen in modified gravity models where $D(t) \rightarrow D(k,t)$
  - Related to loss of Birkhoff's theorem when r^{-2} lost?

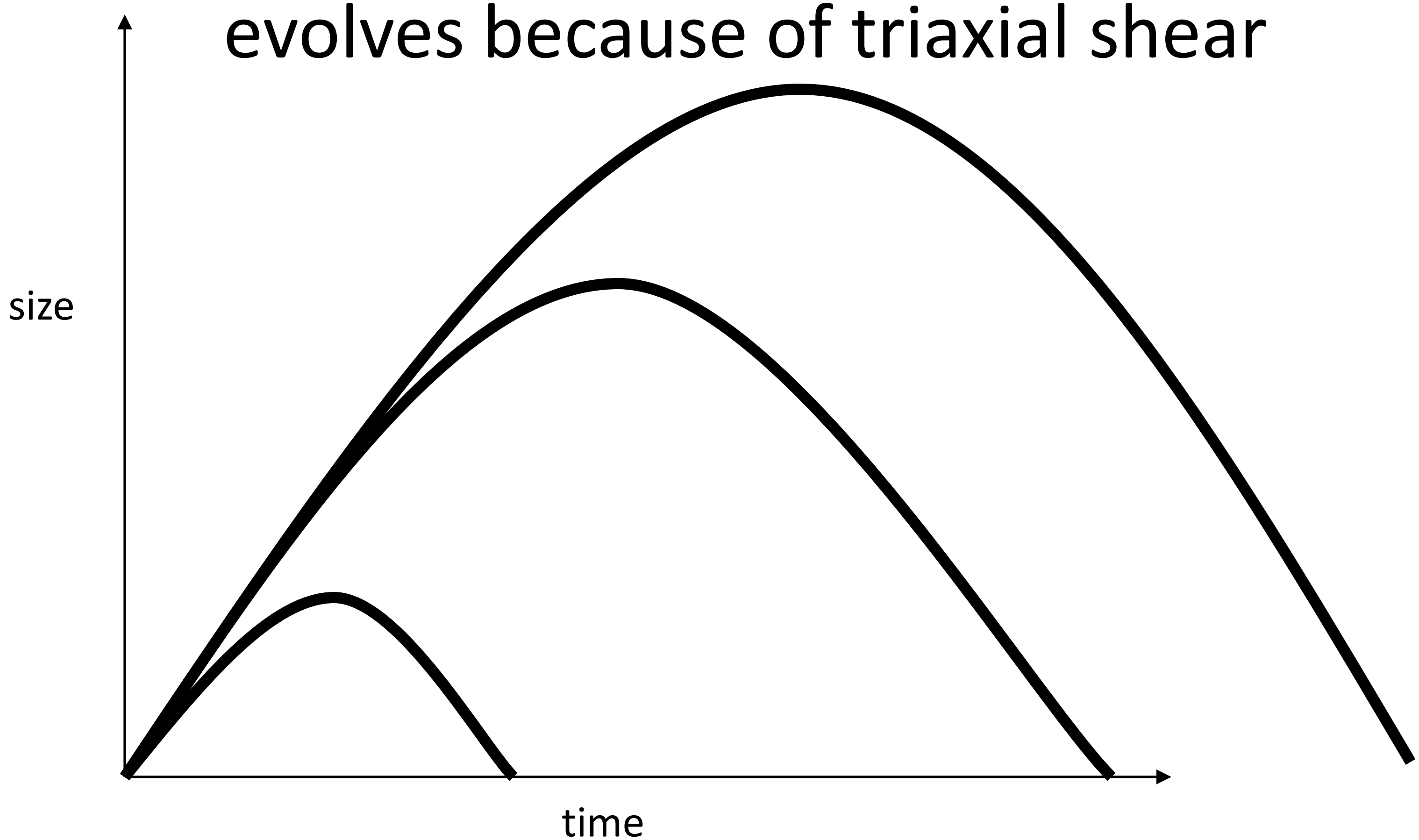
Virial Motions

- $(R_i/r_{\rm vir}) \sim f(\Delta_i)$: ratio of initial and final sizes depends on initial overdensity
- Mass $M \sim R_i^3$ (since initial overdensity « 1)
- So final virial density $\sim M/r_{\rm vir}^3 \sim (R_i/r_{\rm vir})^3 \sim$ function of critical density: Hence, all virialized objects have the same density, $\Delta_{\rm vir}\,\rho_{\rm crit}(z)$, whatever their mass
- $V^2 \sim GM/r_{\rm vir} \sim (Hr_{\rm vir})^2\Delta_{\rm vir} \sim (HGM/V^2)^2\,\Delta_{\rm vir} \sim (HM)^{2/3}$: massive objects have larger internal velocities or temperatures; *H* decreases with time, so, for a given mass, virial motions (or temperature) higher at high *z*

Only very fat cows are spherical….

(Lin, Mestel & Shu 1963; Icke 1973; White & Silk 1978; Bond & Myers 1996; Sheth, Mo & Tormen 2001; Ludlow, Boryazinski, Porciani 2014)

Triaxial collapse: initial sphere evolves because of triaxial shear



Evolution of 2nd axis very similar to spherical model of same initial density

Collapse of 1st axis sooner than in spherical model; collapse of all 3 axes takes longer

Tri-axial (ellipsoidal) collapse

- Evolution determined by properties of initial deformation field, described by 3×3 matrix at each point (Doroshkevich 1970)
- Tri-axial because 3 eigenvalues/invariants; Trace = initial density $\delta_{\rm in}$ = quantity which determines spherical model; other two (e,p) describe anisotropic evolution of patch
- Critical density for collapse no longer constant: On average, $\delta_{\rm ec}(\delta_{\rm in},e,p)$ larger for smaller patches → low mass objects

Convenient Approximations

- Zeldovich Approximation (1970):

$$(1+\delta)_{Zel} = \Pi_{i=1}^{3} (1 - D(t)\lambda_i)^{-1}$$

- Zeldovich Sphere ($\lambda_1 = \lambda_2 = \lambda_3 = \delta_{Linear}/3$):

$$(1+\delta)_{ZelSph} = (1 - \delta_{Linear}/3)^{-3}$$

$$(1+\delta)_{EllColl} \approx (1+\delta)_{Zel}/(1+\delta)_{ZelSph}$$

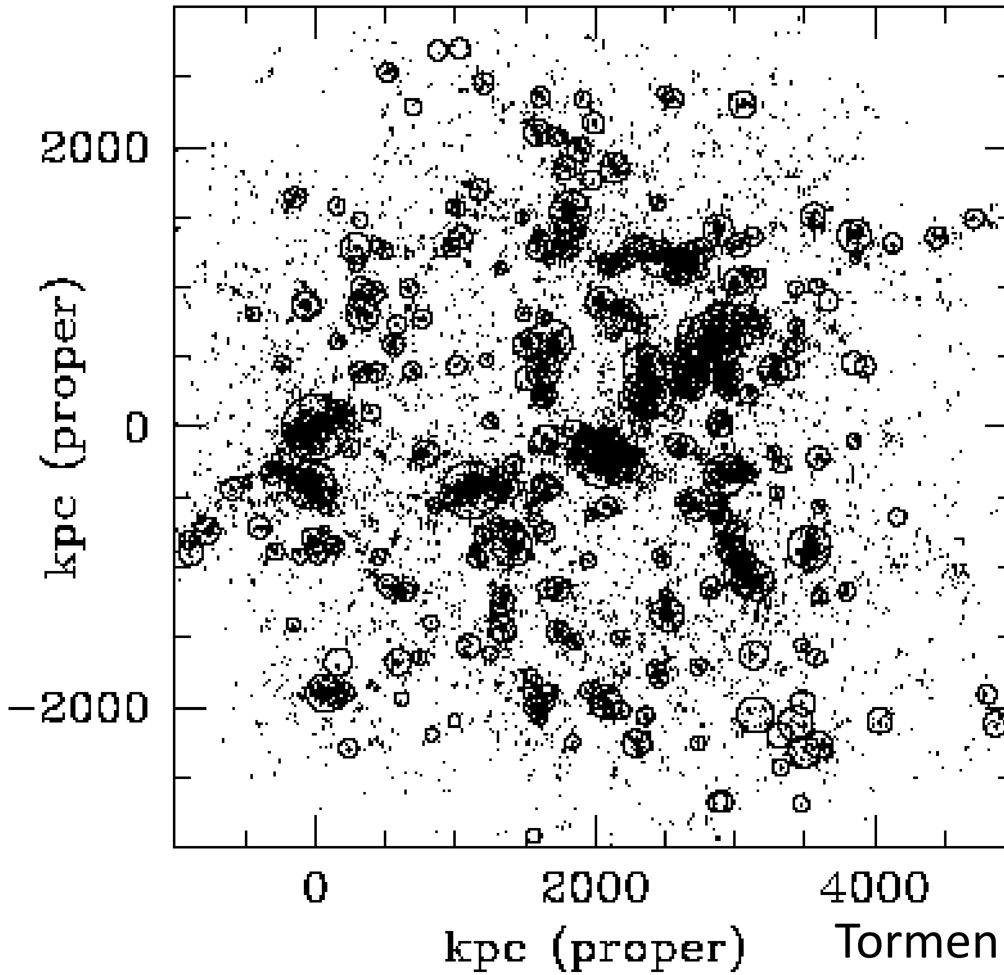
Open questions

- Virial density scales with background or critical density?
  - In ΛCDM, critical seems more reasonable
  - Can address by running simulations beyond present epoch!
- Tri-axial collapse from initially spherical or tri-axial patches?
  - How best to incorporate tidal effects? Simulations suggest longest axis initially aligned with direction of largest compression (correlation is reversed by the final time)
  - What is equivalent of virial size?
  - Predicting final axial ratios is tough problem (generically predict larger halos rounder; this is true in initial conditions, but not at final time)

Spherical collapse with DM + DE + νs!

Spherical evolution model

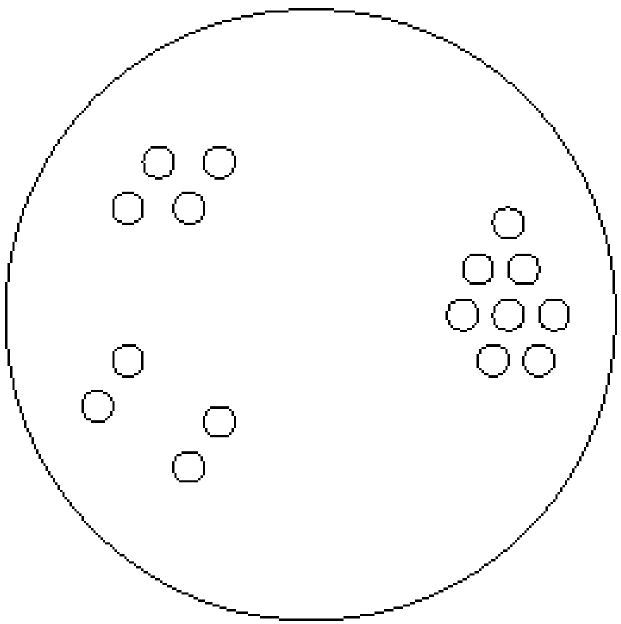
- 'Collapse' depends on initial over-density Δ_i; same for all initial sizes
- Critical density depends on cosmology
- Final objects all have same density, whatever their initial sizes
- Collapsed objects called halos are ~ 200× denser than critical (background?!), whatever their mass

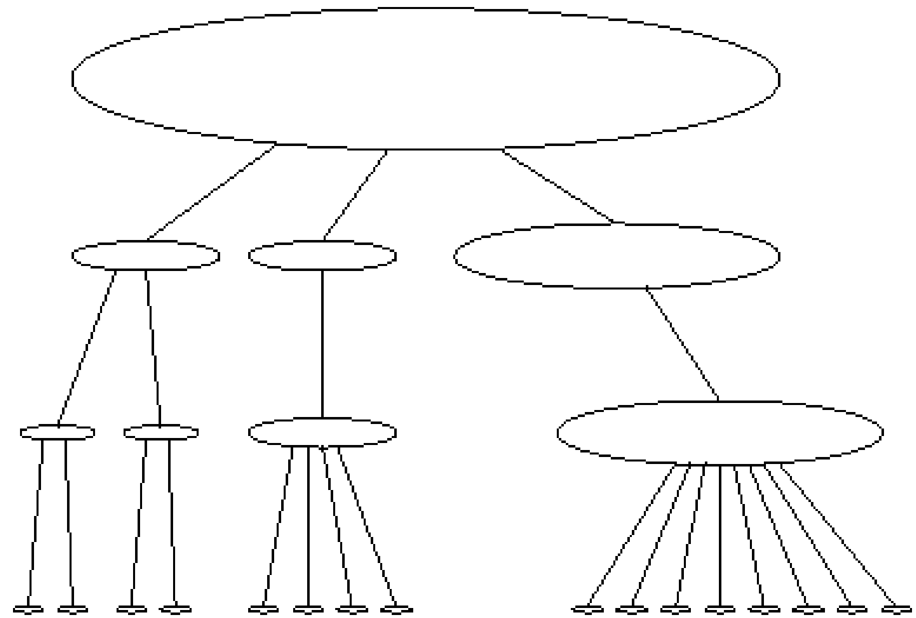


Tormen 1997

Assume a spherical herd of spherical cows…

Initial spatial distribution within patch (at z~1000)…





…stochastic (initial conditions Gaussian random field); study ‘forest’ of merger history ‘trees’.

…encodes information about subsequent ‘merger history’ of object

(Mo & White 1996; Sheth 1996)

Models of halo abundances and clustering: Gravity in an expanding universe

Use knowledge of initial conditions (CMB) to make inferences about late-time, nonlinear structures

The phenomenology of large scale structure

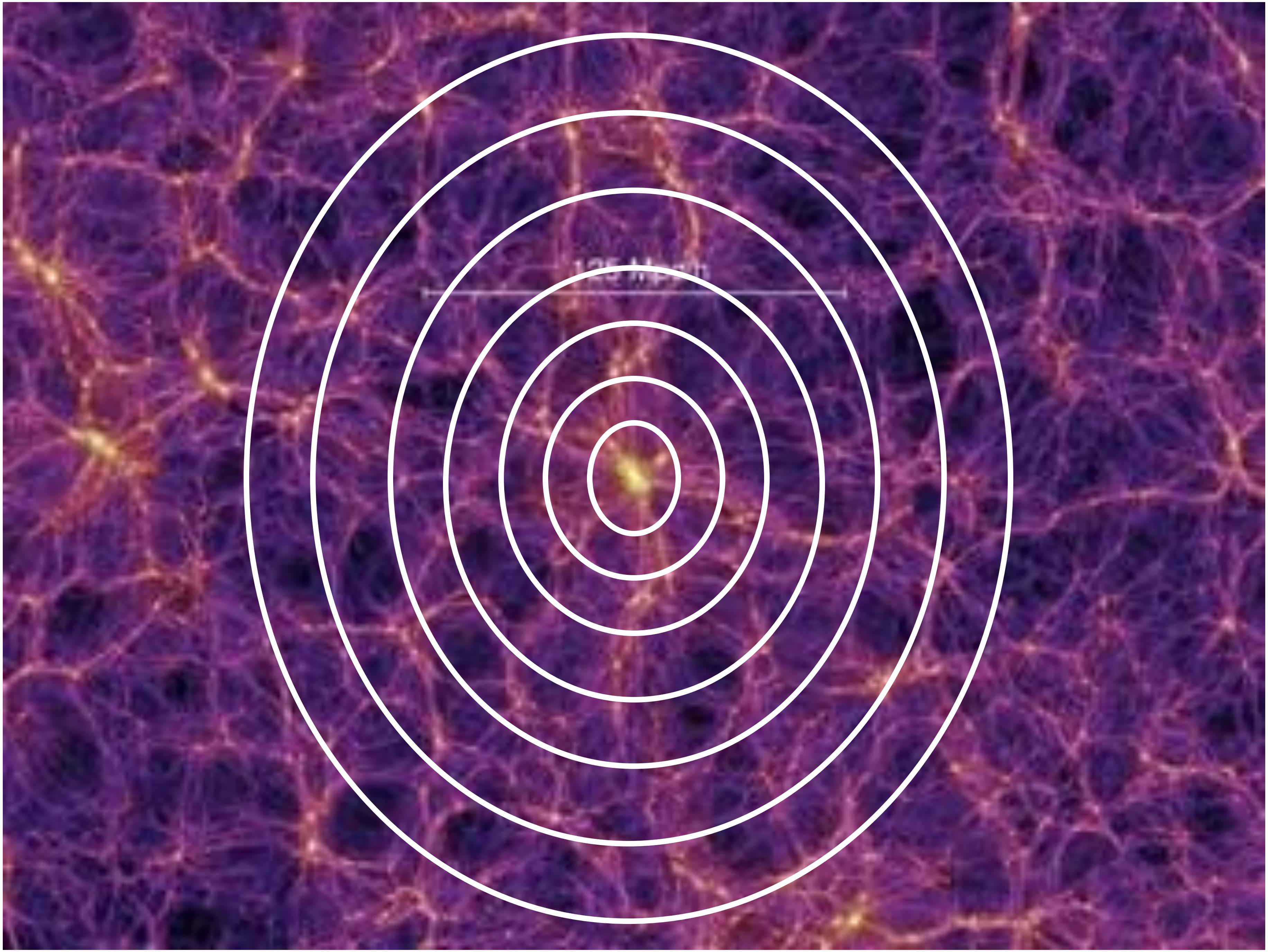
- Halo abundances and clustering
- Halo profiles
- The halo model

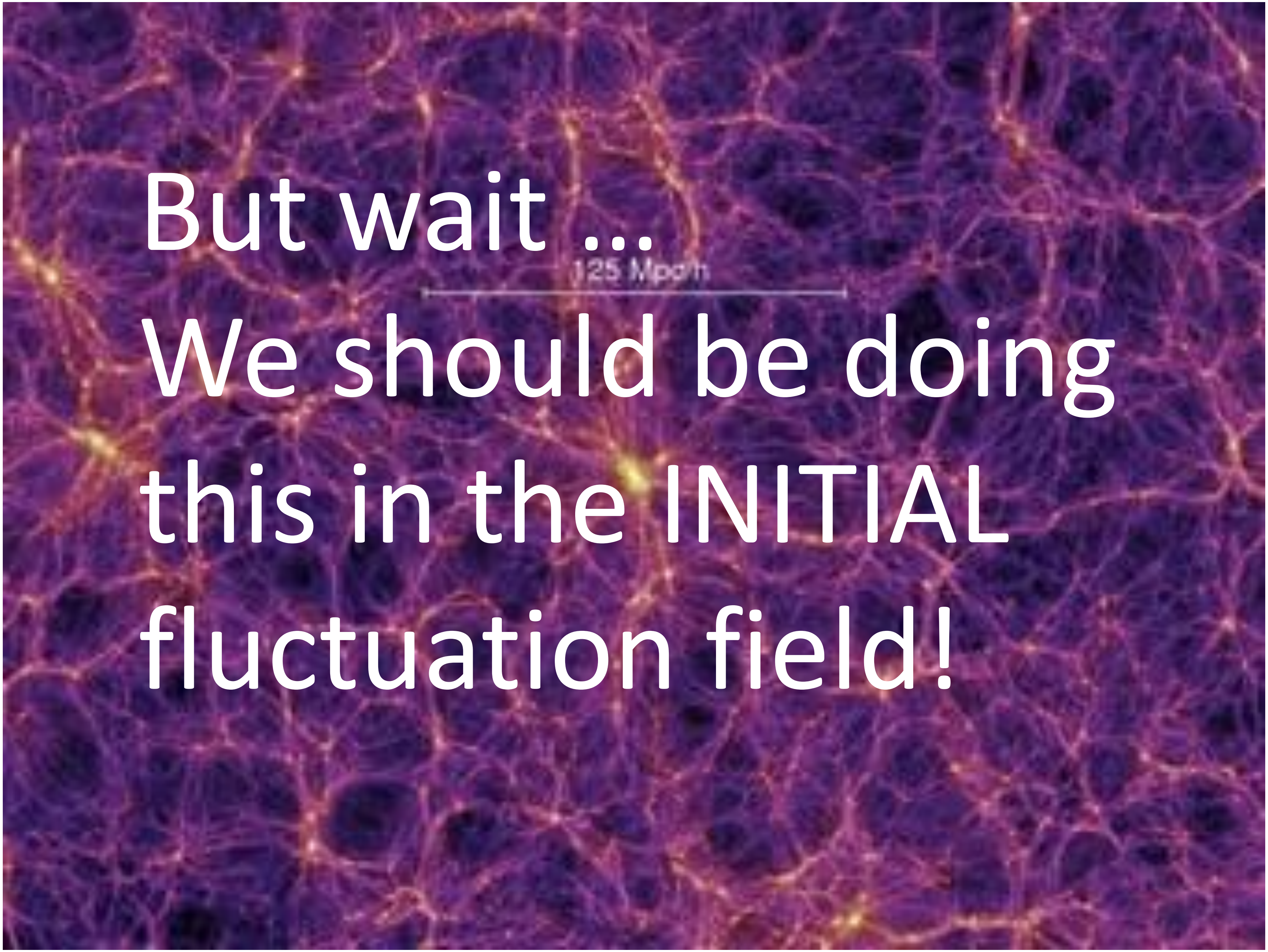
Why study halos?

- Cluster counts contain information about volume and about how gravity won/lost compared to expansion
- Probe geometry and expansion history of Universe, and nature of gravity

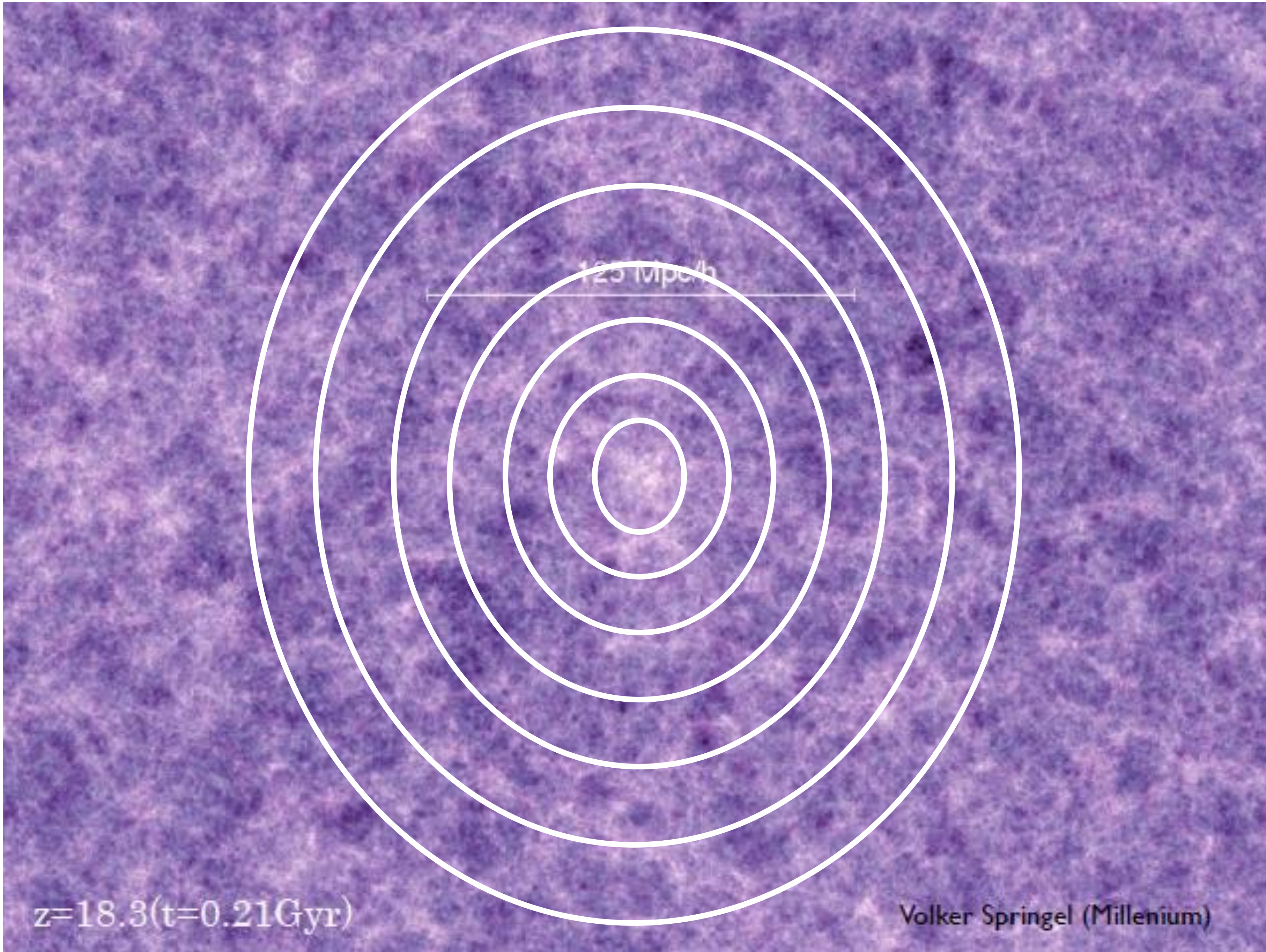
Massive halo = Galaxy cluster

(Simpler than studying galaxies? Less gastrophysics?)

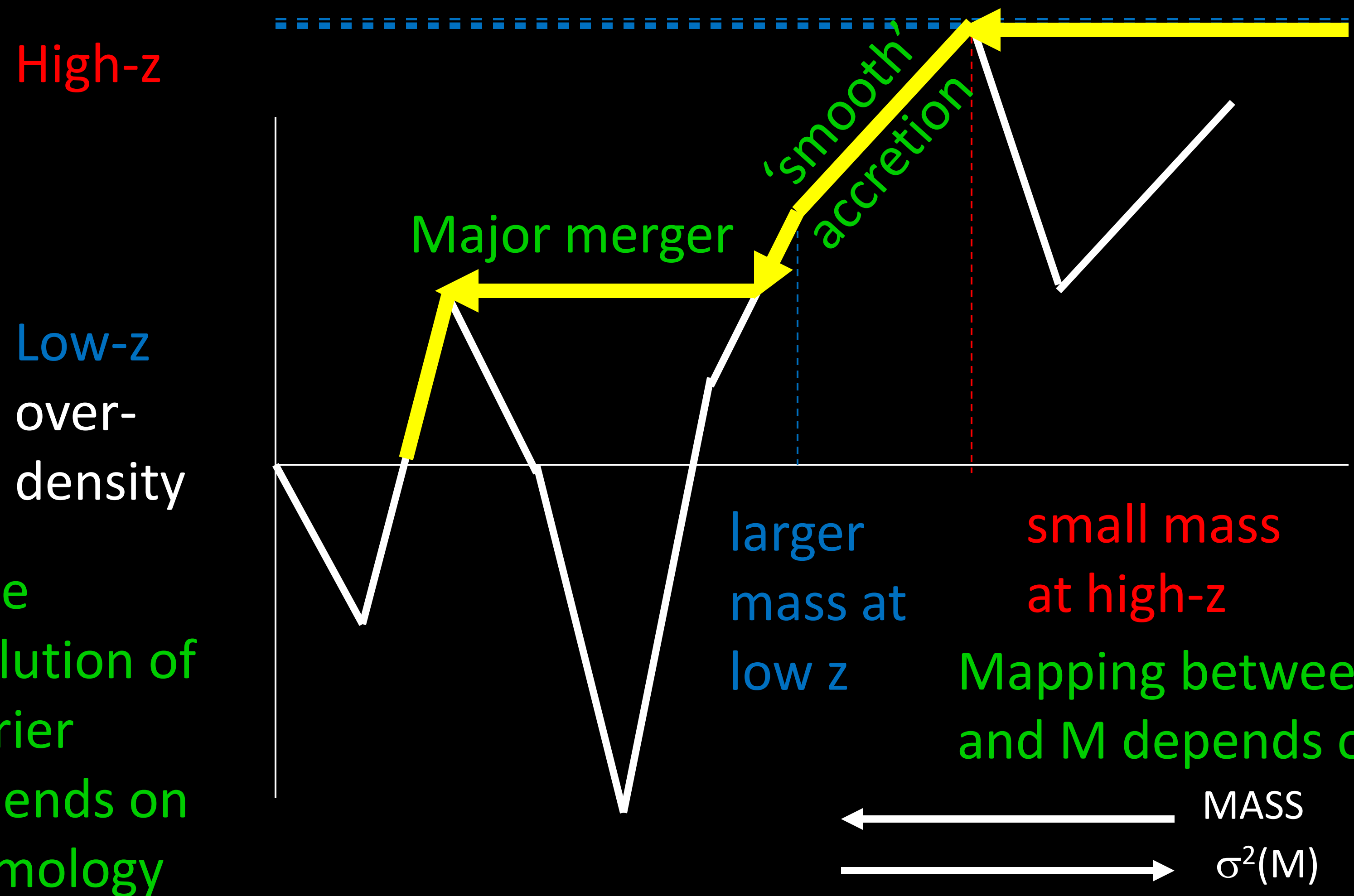

125 Mpc/h



But wait …
We should be doing this in the INITIAL fluctuation field!


125 Mpc/h
z=18.3(t=0.21Gyr)
Volker Springel (Millenium)

The excursion set approach



Time evolution of barrier depends on cosmology

Mapping between σ^2 and M depends on P(k)

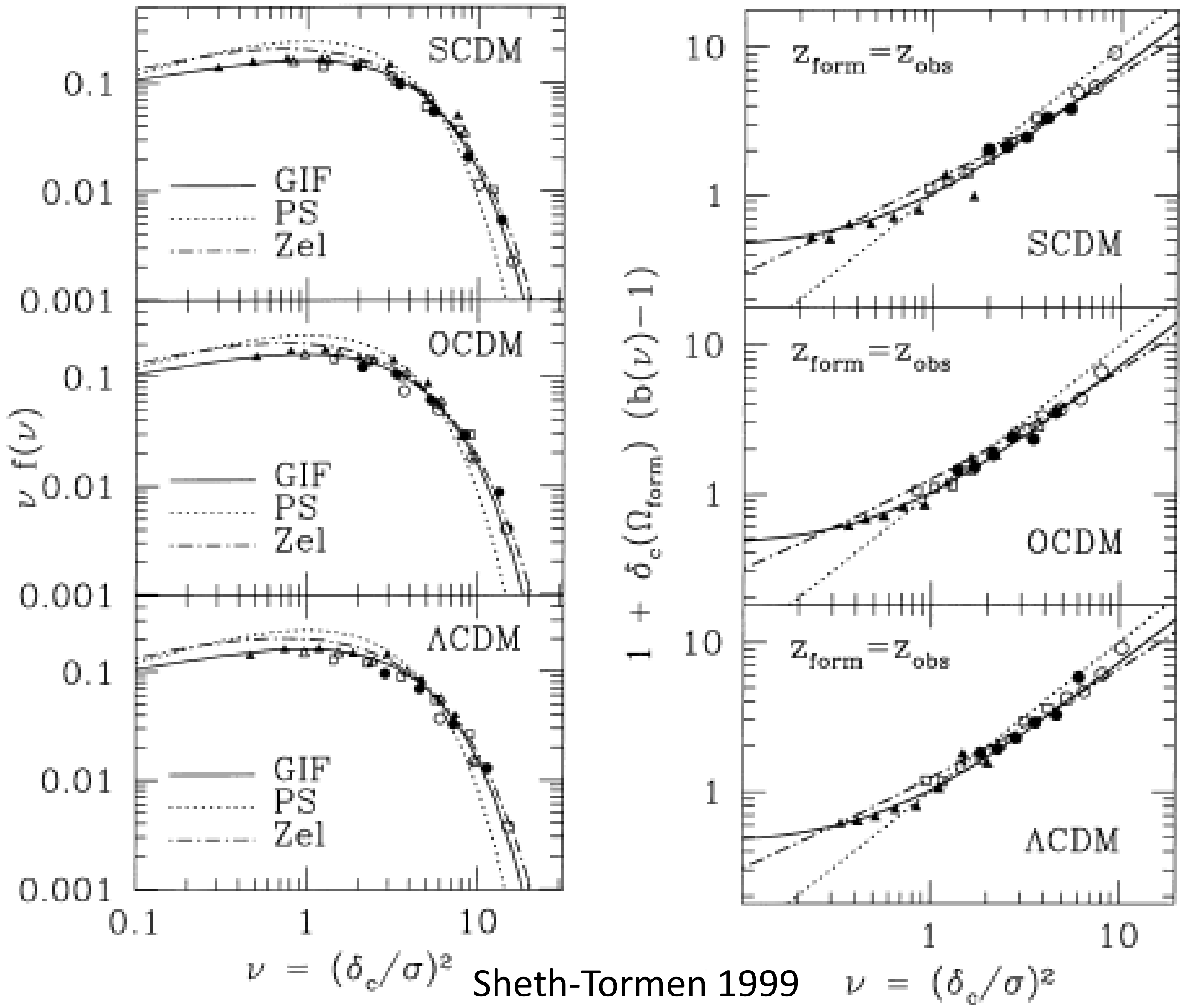
Simplification because…

- Everything local
- Evolution determined by cosmology (competition between gravity and expansion)
- Statistics determined by initial fluctuation field: for Gaussian, specified by initial power-spectrum $P(k)$
- Nearly universal in scaled units: $\delta_c(z)/\sigma(m)$ where $\sigma^2(m) = \langle\delta_m^2\rangle = \int dk/k \; k^3 P(k)/2\pi^2 \; W^2(kR_m)$ $m \propto R_m^3$
- Fact that only very fat cows are spherical is a detail (*crucial* for precision cosmology); in excursion set approach, mass-dependent barrier height increases with distance along walk

(Almost) universal mass function and halo bias

See Paranjape et al (2013) for recent progress in modeling this from first principles

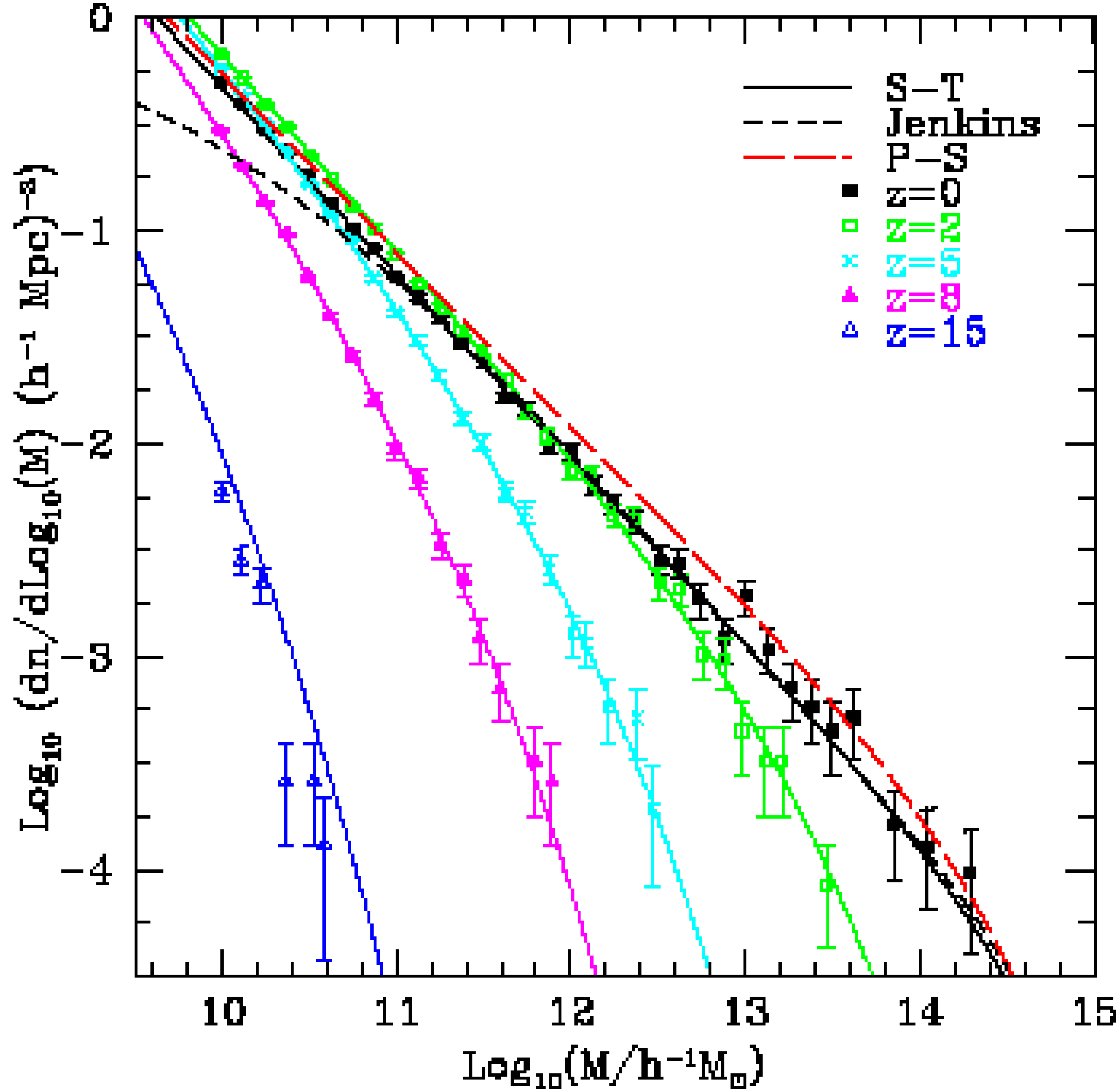
See Castorina et al. (2014) for ν's



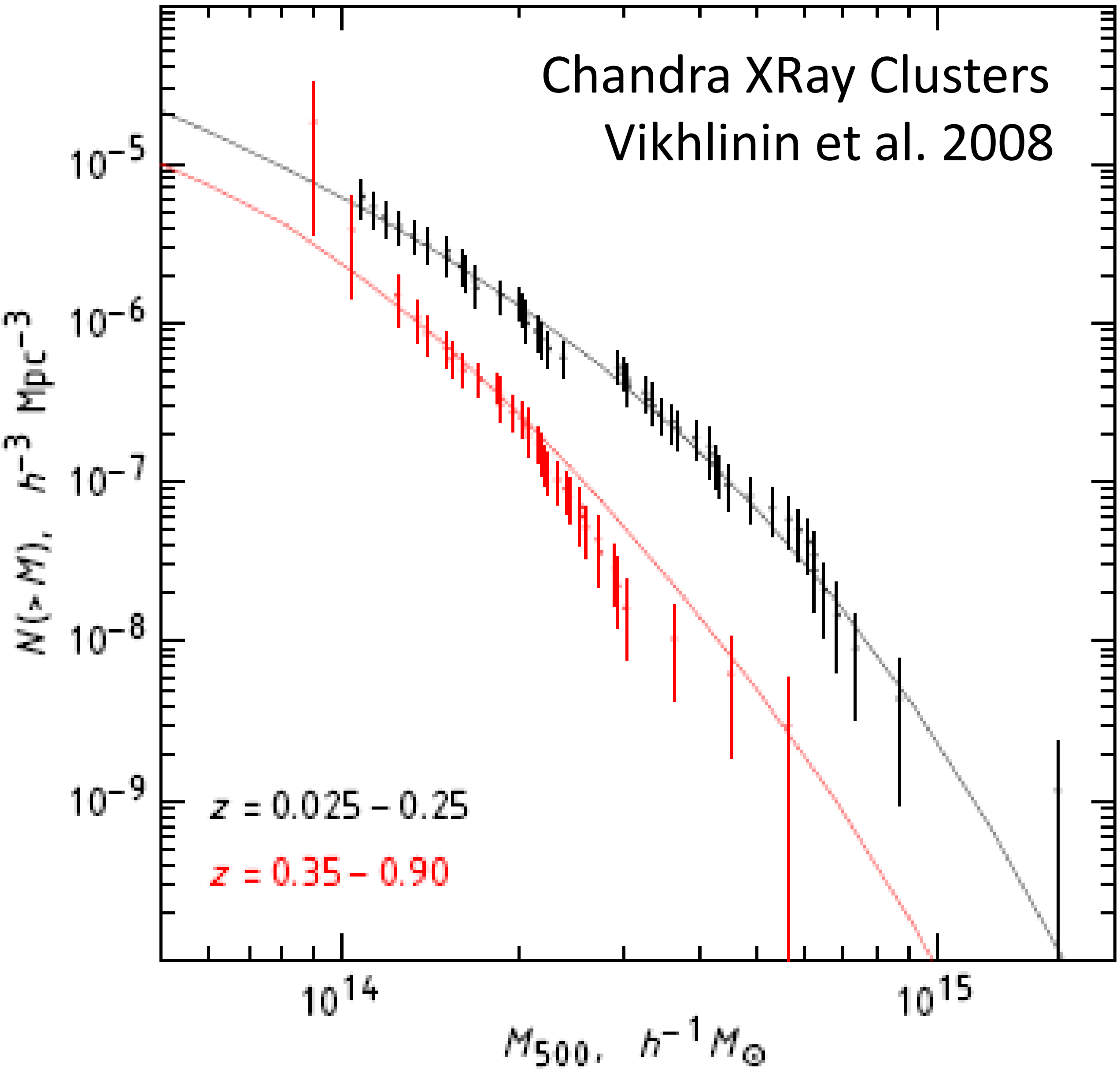
Sheth-Tormen 1999

The Halo Mass Function

(Reed et al. 2003)

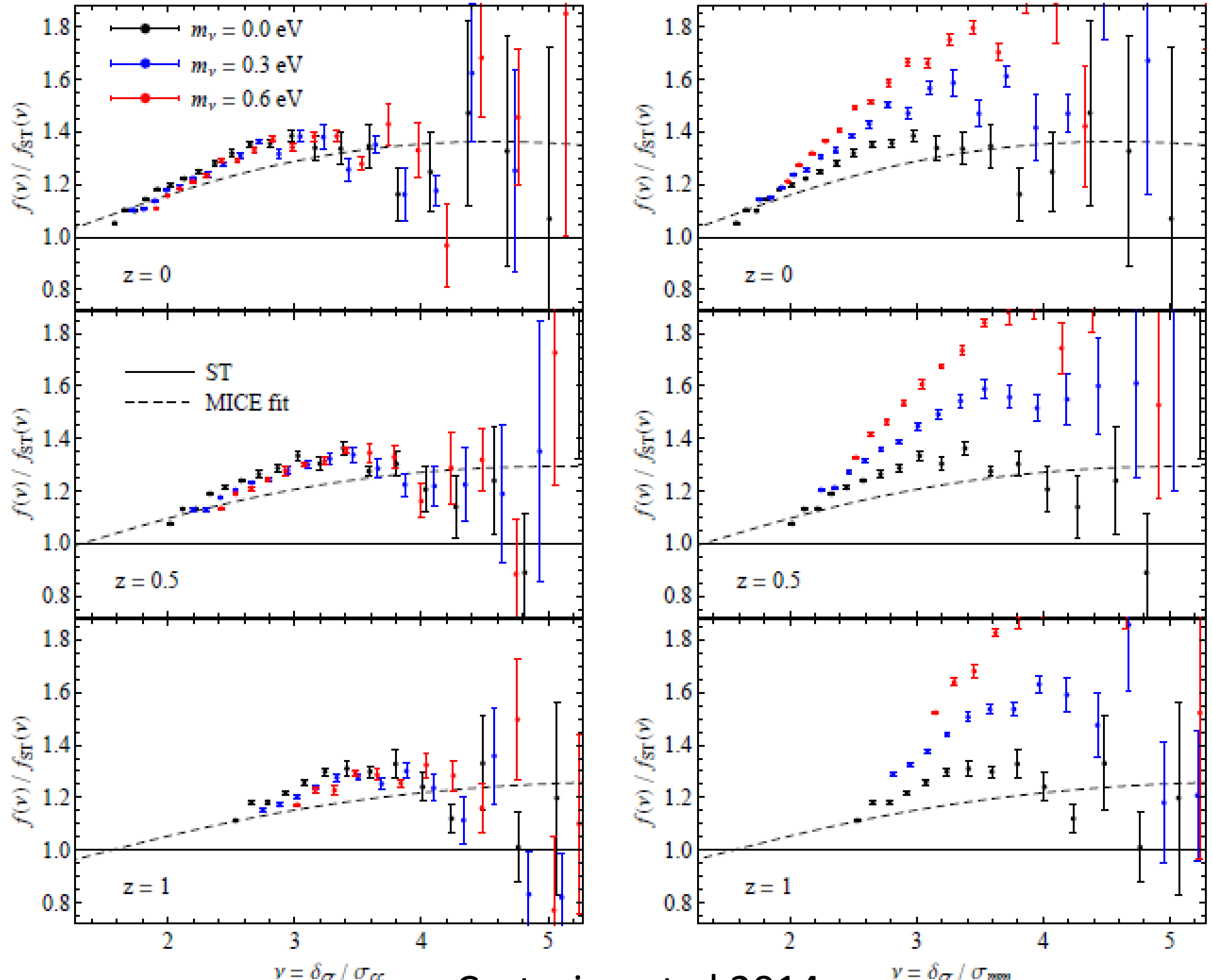


- Small halos collapse/virialize first
- Can also model halo spatial distribution
- Massive halos more strongly clustered



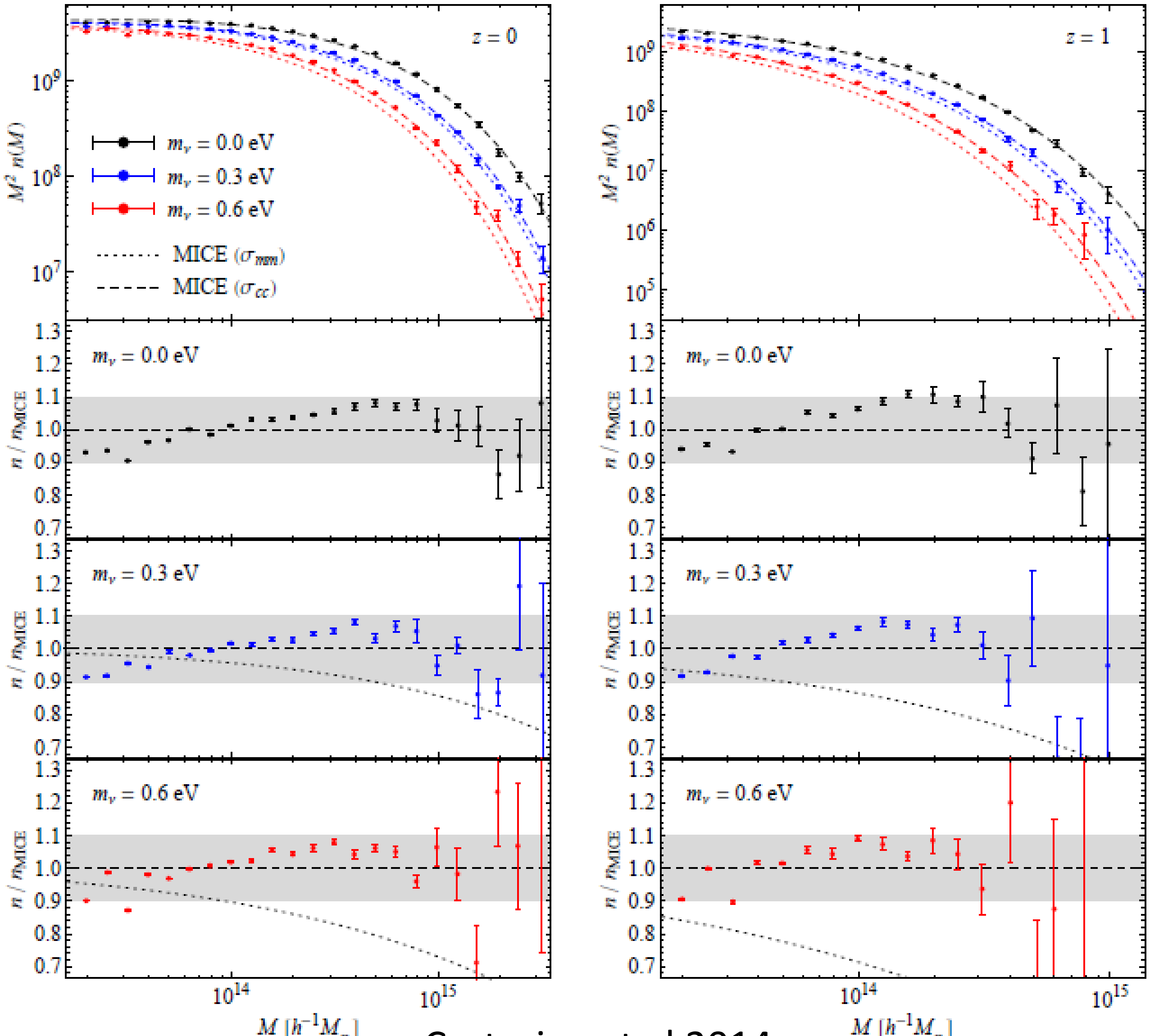
Chandra XRay Clusters
Vikhlinin et al. 2008
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
$N(>M)$, h^{-3} Mpc^{-3}
$z = 0.025 - 0.25$
$z = 0.35 - 0.90$
10^{14}
10^{15}
M_{500}, $h^{-1} M_{\odot}$

0.3eV νs act as effective background cosmology, only fluct'ns in CDM component matter: so relevant quantity is $P_{cc}(k)$



Castorina et al 2014

Universality more evident for $\sigma_{cc}(m)$ than for $\sigma_{mm}(m)$



Castorina et al 2014

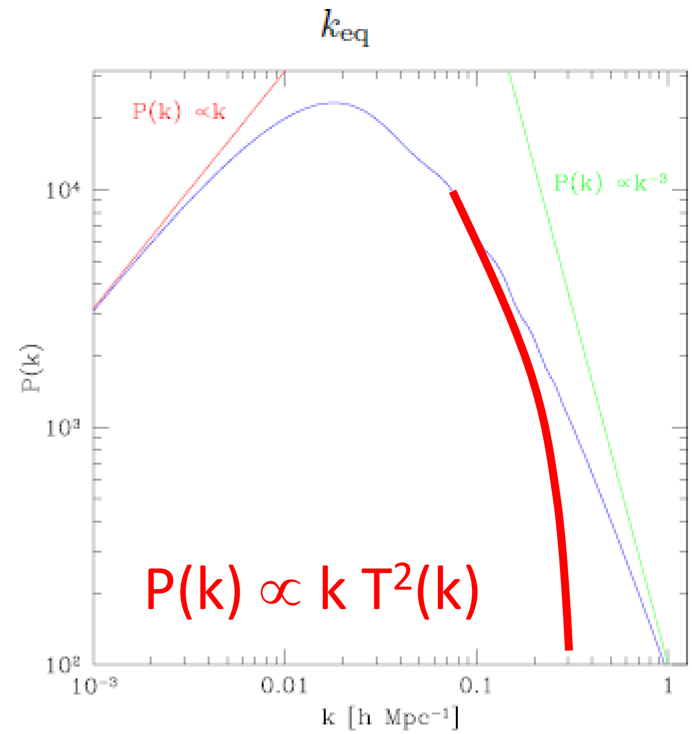
For keV WDM:

$T_{WDM}(k) = T_{CDM}(k)/[1+(\alpha k)^2]^5$

$T_{CDM}(k) \propto [1 + k^2]^{-1}$

Think of α as a free streaming scale; smaller scale fluctuations are erased; associated mass $\propto \alpha^3$

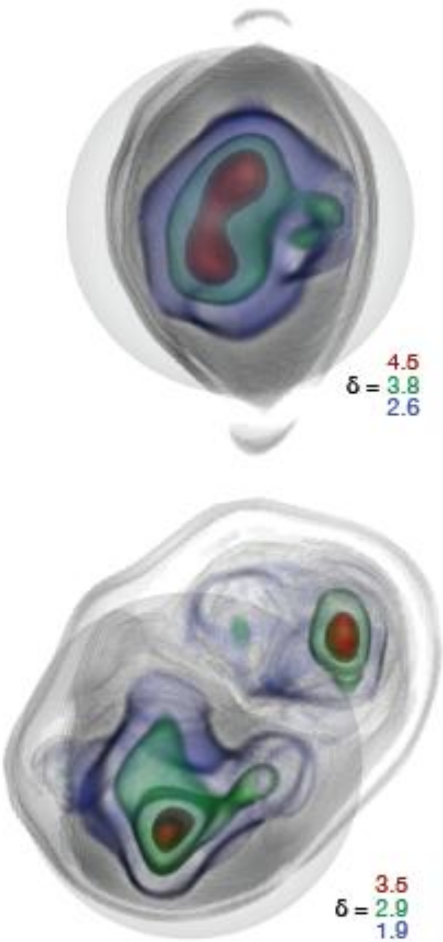
For m_{dm} = 0.25 keV expect no structures smaller than 7×10^8 $h^{-1}M_{sun}$

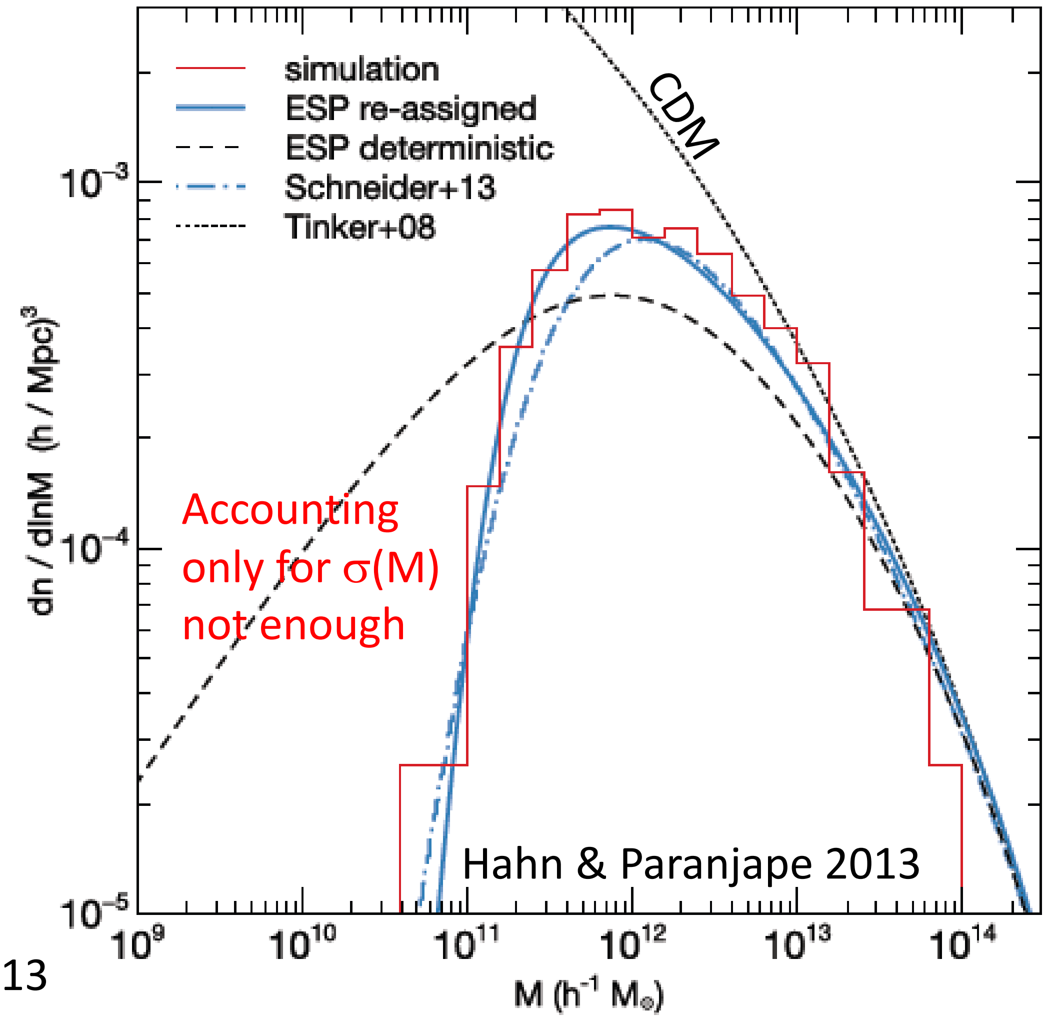


$$\alpha \equiv 0.05 \left(\frac{\Omega_m}{0.4}\right)^{0.15} \left(\frac{h}{0.65}\right)^{1.3} \left(\frac{m_{\rm dm}}{1\,{\rm keV}}\right)^{-1.15} h^{-1}{\rm Mpc}$$

Sterile neutrino similar: $m_{\nu_s} = 4.43{\rm keV}\left(\frac{m_{\rm WDM}}{1{\rm keV}}\right)^{4/3}\left(\frac{\Omega_{\rm WDM}}{0.1225}\right)^{-1/3}$

For keV WDM





Steep P(k) is difficult to simulate:
recent progress Angulo, Hahn, Abel 2013

Study of random walks with correlated steps

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Cosmological constraints from large scale structures