Introduction to particle physics: Standard Model and beyond

José W F Valle

IFIC
INSTITUT DE FÍSICA CORPUSCULAR

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CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

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Some history

- Since its proposal as a fundamental ingredient in the description of β decay by Pauli and Fermi, we have learned a lot about the neutrino. First we now know for certain that there are at least 3 types ν_e (1956), ν_μ (1962) and ν_τ (2000).
- Although the possibility of neutrinos having mass was present since the birth of the neutrino idea, it was abandoned in the 50's because of the great success of the V-A hypothesis of Lee and Yang, in accounting for the observed parity violation in the weak interaction (Wu et al). This was then taken as an indication that neutrinos are massless.
- In the 80's, however, there was a tremendous amount of activity in neutrino physics, devoted mainly to the issue of nonzero masses and other novel properties of neutrinos, absent in the standard model, partly due to theoretical ideas such as grand unification.
- Gauge theoretic formulation of neutrino mass and oscillations with the characterization of the charged and neutral current weak interactions in gauge theories of massive neutrinos.
- Other important landmarks were the birth of extra solar system neutrino astronomy with the detection of neutrinos from SN87a and the formulation of the MSW effect
- The ultimate confirmation of the neutrino mass hypothesis only came recently, with the conclusive atmospheric neutrino data and the subsequent resolution of the solar neutrino problem which came by combining these data with reactor data

Accelerators and reactors give crucial confirmation of oscillation hypothesis ...

DIRAC & MAJORANA

arXiv:0710.0554 **CP Violation and Neutrino Oscillations.** Hiroshi Nunokawa, Stephen Parke, Jose W. F. Valle.

- massive Majorana fermion is **more basic** than the more familiar *Dirac fermion* since it has just half the number of degrees of freedom (lowest rep of the Lorentz group) [**only for electrically neutral fermions**]
- **basic Lagrangean (in terms of a 2-component spinor ρ)**

$$\mathcal{L}_M = -i\rho^\dagger \sigma_\mu \partial_\mu \rho - \frac{m}{2}\rho^T \sigma_2 \rho + H.C.$$

- **Under a Lorentz transformation,** $x \to \Lambda x$**, the spinor ρ transforms as** $\rho \to S(\Lambda)\rho(\Lambda^{-1}x)$ **where S obeys**

$$S^\dagger \sigma_\mu S = \Lambda_{\mu\nu}\sigma_\nu$$

- **kinetic term clearly invariant. Also, the mass term is invariant, due to unimodular property $det\ S = 1$. However, not invariant under a phase transformation** $\rho \to e^{i\alpha}\rho$

The resulting equation of motion is

$$-i\sigma_\mu \partial_\mu \rho = m\sigma_2 \rho^*$$

conjugation and Clifford properties of the σ-matrices, implies that each component of the spinor ρ obeys the Klein-Gordon wave-equation.

- **In order to display clearly the relationship between our theory, and the usual theory of a massive spin $1/2$ Dirac fermion, defi ned by the familiar Lagrangean**

$$L_D = -\bar{\Psi}\gamma_\mu \partial_\mu \Psi - m\,\bar{\Psi}\Psi,$$

let us construct the solutions to the Majorana equation in terms of those of the Dirac eqn, which are well known.

- **can use any representation of the Dirac algebra $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\,\delta_{\mu\nu}$.**
- **To develop the weak interaction theory, however, the chiral representation, in which γ_5 is diagonal, is the better**

$$\gamma_i = \begin{pmatrix} 0 & -i\sigma_i \\ i\sigma_i & 0 \end{pmatrix} \quad \gamma_4 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

In this representation the charge conjugation matrix C obeying

$$C^T = -C$$
$$C^\dagger = C^{-1}$$
$$C^{-1}\,\gamma_\mu\,C = -\,\gamma_\mu^T$$

is simply given by

$$C = \begin{pmatrix} -\sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix}$$

A Dirac spinor may then be written as

$$\Psi_D = \begin{pmatrix} \chi \\ \sigma_2\,\phi^* \end{pmatrix}$$

so that the corresponding charge-conjugate spinor $\Psi_D^c = C\,\bar{\Psi}_D^T$ is the same as Ψ_D but exchanging ϕ and χ, *i.e.*,

$$\Psi_D^c = \begin{pmatrix} \phi \\ \sigma_2\,\chi^* \end{pmatrix}$$

A 4-component spinor is said to be Majorana or self-conjugate if $\Psi = C\bar{\Psi}^T$ which amounts, to setting $\chi = \phi$. We can rewrite the D-Lagragean as follows

$$L_D = -i\sum_{\alpha=1}^{2} \rho_\alpha^\dagger \sigma_\mu \partial_\mu \rho_\alpha - \frac{m}{2}\sum_{\alpha=1}^{2} \rho_\alpha^T \sigma_2 \rho_\alpha + H.C.$$

$$\chi = \frac{1}{\sqrt{2}}(\rho_2 + i\rho_1)$$

$$\phi = \frac{1}{\sqrt{2}}(\rho_2 - i\rho_1)$$

are the left handed components of Ψ_D and of the charge-conjugate fi eld Ψ_D^c, respectively. Thus

Dirac fermion equivalent to two Majoranas of equal mass

fermion number conservation in the Dirac theory, i.e. the phase symmetry

$$\Psi_D \to e^{i\alpha}\Psi_D$$

corresponds to invariance under a continuous rotation symmetry between ρ_1 and ρ_2

$$\rho_1 \to c\rho_1 + s\rho_2$$

$$\rho_2 \to -s\rho_1 + c\rho_2$$

QUANTUM THEORY OF MAJORANAS

to obtain solutions of the Maj-theory we start from the usual Fourier expansion for the Dirac spinor,

$$\Psi_D = (2\pi)^{-3/2} \int d^3k \sum_{r=1}^{2} (\frac{m}{E})^{1/2} [e^{ik.x} a_r(k)\ u_r(k) + e^{-ik.x} b_r^\dagger(k)\ v_r(k)]$$

where $u = C\,\bar{v}^T$ and $E(k) = (\vec{k}^2 + m^2)^{1/2}$ One can show that [PRD22 (1980) 2227]

$$\Psi_M = (2\pi)^{-3/2} \int d^3k \sum_{r=1}^{2} (\frac{m}{E})^{1/2} [e^{ik.x} A_r(k)\ u_{Lr}(k) + e^{-ik.x} A_r^\dagger(k)\ v_{Lr}(k)]$$

is the correct Maj expansion, with a single set of Fock operators A for each value of r and k obeying canonical anticommutation rules. This gives a

consistent Fock-space interpretation of massive 2-component Majorana theory

- massive Majorana fi eld operator given in terms of the chiral projections of the **ordinary** massive Dirac wave functions u and v.
- **same** creation and annihilation operators appear in Ψ_M showing that "Majorana is half of Dirac"

- **2 types of propagators**

$$\langle 0 \mid \rho(x)\, \rho^*(y) \mid 0\rangle = i\sigma_\mu \partial_\mu \Delta_F(x-y;m)$$

$$\langle 0 \mid \rho(x)\, \rho(y) \mid 0\rangle = m\, \sigma_2\, \Delta_F(x-y;m)$$

where $\Delta_F(x-y;m)$ is the usual Feynman function.

The first is the "normal" propagator that intervenes in total lepton number conserving ($\mid \Delta L \mid = 0$) processes, while the second describes $\mid \Delta L \mid = 2$ processes such as neutrinoless double-beta decay.

what is the massless limit of Majorana?

to answer this we define helicity eigenstate wavefunctions by

$$\vec{\sigma}\cdot\vec{k}\, u_L^{\pm}(k) = \pm \mid \vec{k} \mid u_L^{\pm}(k)$$
$$\vec{\sigma}\cdot\vec{k}\, v_L^{\pm}(k) = \mp \mid \vec{k} \mid v_L^{\pm}(k)$$

and note that, out of the 4 linearly independent wave functions $u_L^{\mp}(k)$ and $v_L^{\mp}(k)$, only two survive as the mass approaches zero, namely, $u_L^{-}(k)$ and $v_L^{+}(k)$

This is just the old 2-component massless neutrino theory of Lee and Yang

several Majoranas

the most general (free) Lagrangean allowed by Lorentz invariance is

$$L_M = -i \sum_{\alpha=1}^{n} \rho_\alpha^\dagger \sigma_\mu \partial_\mu \rho_\alpha - \frac{1}{2} \sum_{\alpha,\beta=1}^{n} M_{\alpha\beta} \rho_\alpha^T \sigma_2 \rho_\beta + H.C.$$

where the sum runs over α and β. By Fermi statistics the mass coeffi cients $M_{\alpha\beta}$ must form a symmetric matrix, in general complex. This matrix can always be diagonalized by a complex $n \times n$ unitary matrix U

$$M_{diag} = U^T M U \; .$$

When M is taken to be real (CP conserving) its diagonalizing matrix U may be chosen to be orthogonal and, in general, the mass eigenvalues can have different signs. These may be assembled as a signature matrix

$$\eta = diag(+, +, ..., -, -, ..)$$

- It should be apparent from the above analysis that there is no reason, in general, to expect a conserved fermion number symmetry to arise in a gauge theory where the basic building blocks are 2-component massive electrically neutral fermions, such as neutrinos or the supersymmetric *inos*.
- D-M confusion theorem
  Majorana and Dirac neutrinos can only be distinguished in the standard $SU(2) \otimes U(1)$ electroweak theory, to the extent that neutrinos are massive

LEPTONIC CKM MATRIX – DIRAC CASE

Neutrinos may be given a mass by coupling new fermions beyond those in table, such as right handed neutrinos, in such a way that total lepton number is preserved. The new leptons will be denoted by ν_i^c, $1 \leq i \leq m$. From the point of view of the standard model one can add any number m of isosinglets since, being completely inert under the gauge group, they do not upset the renormalizability of the theory. There is, in this case a new gauge invariant Yukawa interaction, similar to that of the up quarks,

$$h_{Dia}\nu_i^c \ell_a \tau_2 \phi^* .$$

assuming $n = m$ this generates a $Dirac$ mass for the neutrinos.

in this case we find that the structure of lepton mixing is identical with that describing quarks, so the same parametrization applies. It can be given as

$$K = O_{23}\Gamma_\delta O_{13}\Gamma_\delta^\dagger O_{12}$$

where O_{ij} is the orthogonal rotation matrix in the ij-plane which depends on the mixing angle θ_{ij}, and $\Gamma_\delta = \mathrm{diag}(1, 1, \mathrm{e}^{\mathrm{i}\delta_{\mathrm{CP}}})$, δ_{CP} being the Dirac-type CP-violating phase.