Introduction to particle physics: Standard Model and beyond

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Lecture 1

CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

ASTROPARTICLES
Astroparticles and High Energy Physics Group

VNIVERSITAT DE VALÈNCIA

http://astroparticles.es/

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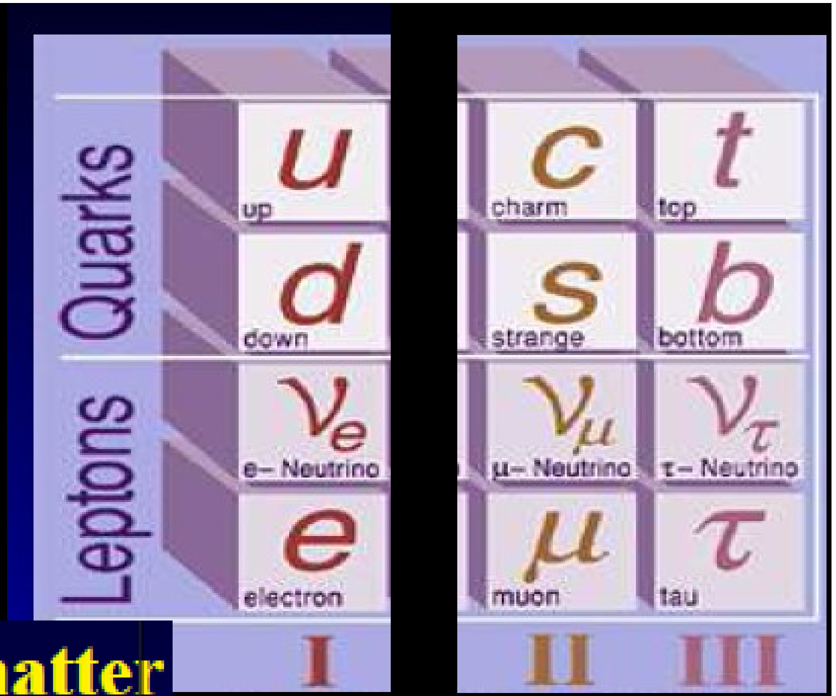
basic building blocks of matter

- fundamental forces:

electromagnetism — photon
strong interaction — gluons
weak interaction — $Z, W^{\pm}$



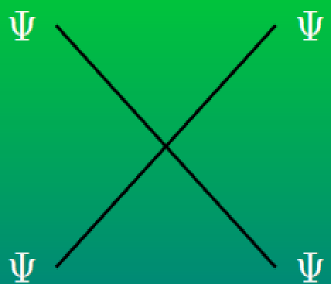
first generation makes up everyday matter

others are produced only at high energies

Each has its neutrino

Towards a weak interaction theory

- short range $\Rightarrow$ contact 4-fermion interaction,



$\mu \to e\nu\bar{\nu}$ and $n \to pe\bar{\nu}$ **works well ...**

but theory is not renormalizable nor unitary.

- To develop the weak interaction as a gauge theory we need to incorporate gauge boson masses by combining
  - φ^4 scalar field theory
  - unbroken Yang-Mills gauge theory

- Spontaneous symmetry breaking is the key ingredient

The Standard Model

non-abelian gauge theory

focus on $SU(2) \otimes U(1)$

- result of detailed interplay of different fields over 30 years
- we adopt anti-historical approach, present SM and derive implications
- product gauge group, each factor with its gauge coupling

$$G = SU(3) \otimes SU(2)_L \otimes U(1) \qquad \text{with} \qquad g_3 \qquad g_2 \qquad g_1$$

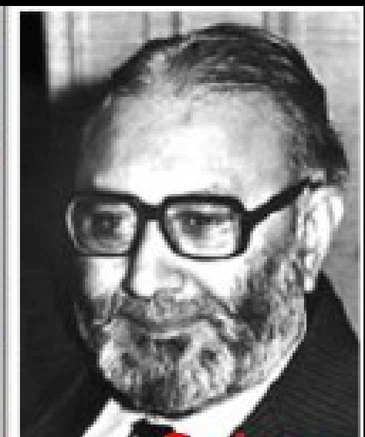
- chiral fermion multiplets: Q's & L's
- scalar multiplet

$SU(2) \otimes U(1)$ gauge fields

$$\left\{ A_\mu = A_\mu^a \frac{\tau^a}{2} = \frac{1}{2}\begin{pmatrix} A_3 & A_1 - iA_2 \\ \text{h.c.} & -A_3 \end{pmatrix}, B_\mu \right\} \qquad \Phi \rightarrow \begin{pmatrix} \varphi_+ \\ \varphi_0 \end{pmatrix}$$

the full SM Lagrangean





$$\mathcal{L} = \mathcal{L}_{YM}^{A} + \mathcal{L}_{S}^{\varphi, A} + \mathcal{L}_{matter}^{\psi, \Phi, A}$$

scalar-gauge Lagrangean

$$\mathcal{L}_{\mathbf{YM}} = -\frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \frac{1}{2}\mathbf{Tr}(F_{\mu\nu}F_{\mu\nu})$$

$$F_{\mu\nu} = \frac{\tau^a}{2}F^a_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\nu, A_\mu]$$

$$\mathcal{L}_S = -D_\mu\varphi^\dagger D_\mu\varphi - V(\varphi)$$

$$D_\mu\varphi = \partial_\mu\varphi - igA_\mu\varphi + \frac{ig'}{2}B_\mu\varphi$$

choose the scalar potential as

$$V(\varphi) \;=\; a(\varphi^\dagger\varphi - v^2/2)^2, \quad a, v^2 > 0$$

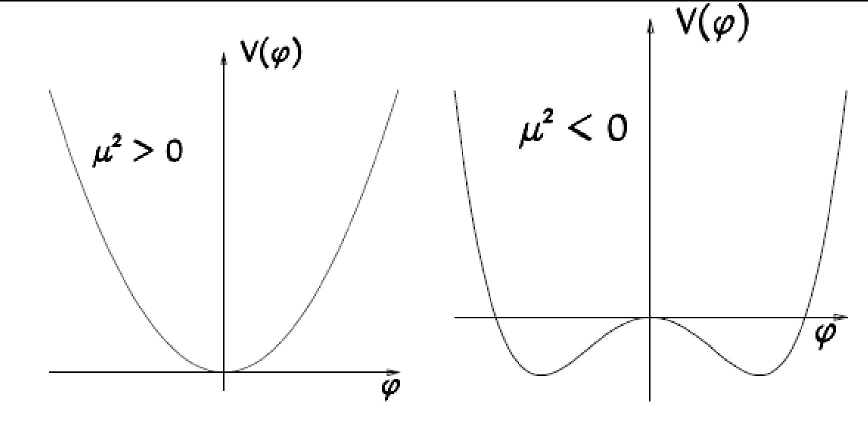
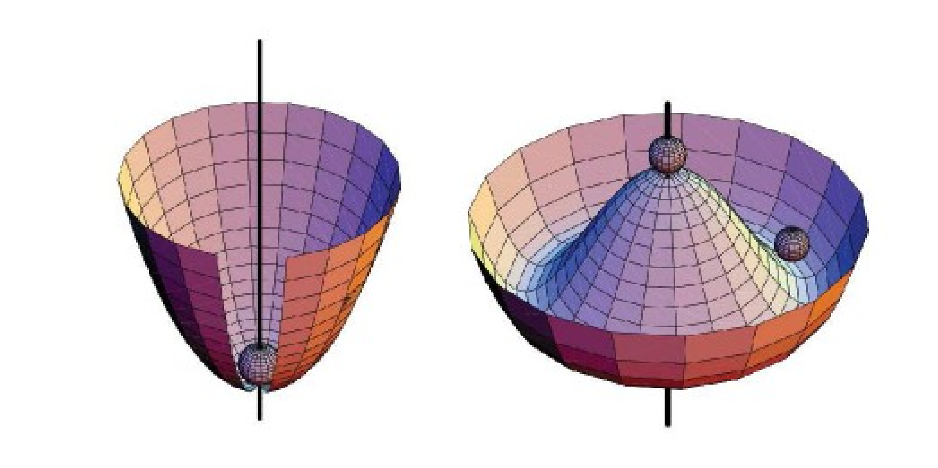


Figure 2.1 Classical potential V of the scalar field for different signs of μ^2.



HIGGS MECHANISM
and the origin of mass

$$V = a\left(\varphi^\dagger\varphi - \frac{v^2}{2}\right)^2 \equiv \underbrace{-av^2}_{\mu^2<0}\varphi^\dagger\varphi + a(\varphi^\dagger\varphi)^2 + \text{const}$$

under a gauge transformation it is always possible to bring

$$\varphi \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ (\tilde{\varphi}+v)/\sqrt{2} \end{pmatrix}$$

so that

$$\begin{aligned} D_\mu\varphi &= \begin{pmatrix} 0 \\ \partial_\mu\tilde{\varphi}/\sqrt{2} \end{pmatrix} - \frac{ig}{2}\begin{pmatrix} A^3_\mu & \sqrt{2}W^+_\mu \\ \sqrt{2}W^+_\mu & -A^3_\mu \end{pmatrix}\begin{pmatrix} 0 \\ (\tilde{\varphi}+v)/\sqrt{2} \end{pmatrix} \\ &\quad -\frac{ig'}{2}\begin{pmatrix} 0 \\ (\tilde{\varphi}+v)/\sqrt{2} \end{pmatrix} B_\mu \end{aligned}$$

$$= \begin{pmatrix} \frac{-ig}{2}\frac{\tilde{\varphi}+v}{\sqrt{2}}\sqrt{2}\, W^+_\mu \\ \frac{\partial_\mu\tilde{\varphi}}{\sqrt{2}} + i\frac{\tilde{\varphi}+v}{2\sqrt{2}}(gA^3_\mu + g'B_\mu) \end{pmatrix}$$

where

$$\sqrt{2}W^\pm = (A^1_\mu \mp A^2_\mu),$$

HIGGS MECHANISM

$$-(D_\mu\varphi)^\dagger D_\mu\varphi = -\frac{g^2}{4}(\tilde{\varphi}+v)^2 W_\mu^+ W_\mu^- - \frac{1}{2}(\partial_\mu\tilde{\varphi})^2$$

$$- \frac{(\tilde{\varphi}+v)^2}{8} \underbrace{\left(\frac{gA_\mu^3 + g' B_\mu}{\sqrt{g^2+g'^2}}\right)^2}_{Z_\mu^2} \underbrace{\frac{(g^2+g'^2)}{g^2}}_{\frac{1}{\cos^2\theta_W}} g^2$$

$$\frac{g^2 v^2}{4} = m_W^2 \qquad \frac{(g^2+g'^2)v^2}{4} = m_Z^2$$

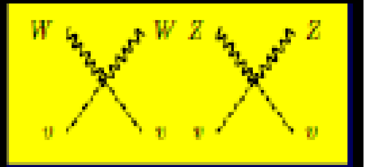
The vacuum is invariant under gauge transformations generated by the electric charge $Q = I_3 + \frac{Y}{2}$ so the photon is massless,

$$\mathcal{L}_{\text{mass}}^{\text{gauge}} = -m_W^2 W_\mu^+ W_\mu^- - \frac{m_Z^2}{2} Z_\mu^2 + 0\, A_\mu^2$$

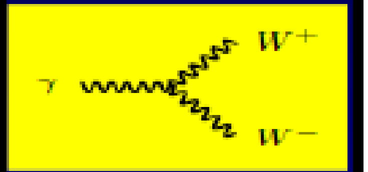
$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} A_\mu^3 \\ B_\mu \end{pmatrix}$$

gauge part of Standard Model Lagrangean

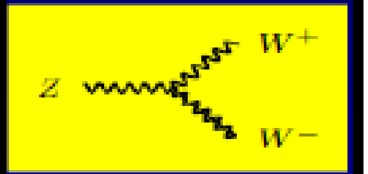
$$-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) - \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)^2$$



$$-m_W^2 W_\mu^+ W_\mu^- - \frac{m_Z^2}{2} Z_\mu^2$$

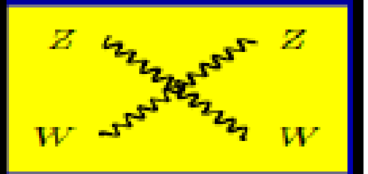


$$ie[A_\mu(W_\nu^+ \overleftrightarrow{\partial}_\mu W_\nu^- + W_\nu^+ \overleftrightarrow{\partial}_\mu W_\mu^- - \partial_\nu W_\mu^+ W_\nu^-) + (\partial_\mu A_\nu - \partial_\nu A_\mu) W_\mu^+ W_\nu^-]$$

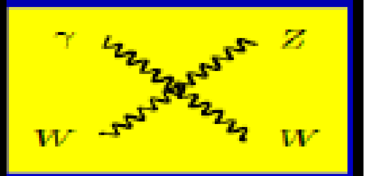


$$igc_W\,[Z_\mu(W_\nu^+ \overleftrightarrow{\partial}_\mu W_\nu^- + W_\nu^+ \overleftrightarrow{\partial}_\mu W_\mu^- - \partial_\nu W_\mu^+ W_\nu^+) + (\partial_\mu Z_\nu - \partial_\nu Z_\mu) W_\mu^+ W_\nu^-]$$

$$-\frac{g^2}{2}(W_\mu^+ W_\mu^-)^2 + \frac{g^2}{2}(W_\mu^+ W_\nu^-)^2 + e^2(A_\mu A_\nu W_\mu^+ W_\nu^- - A_\mu A_\mu W_\nu^+ W_\mu^-)$$



$$g^2 c_W^2 (Z_\mu Z_\nu W_\mu^+ W_\nu^- - Z_\mu^2 W_\nu^+ W_\nu^-)$$



$$egc_W\,[A_\mu Z_\nu(W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - 2A_\mu Z_\nu W_\nu^+ W_\nu^-]$$

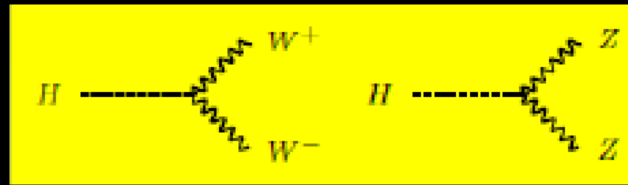
$m_W = m_Z \cos\theta_W$ | $\frac{\sin\theta_W}{\cos\theta_W} = \frac{g'}{g} = \tan\theta_W$ | $e = g \sin\theta_W = g' \cos\theta_W$

THE STANDARD MODEL HIGGS SECTOR

Higgs interactions with e-w gauge bosons

$$\mathcal{L}_{\tilde{\varphi}-\mathrm{gauge}} = \mathcal{L}^{(3)}_{\tilde{\varphi}-\mathrm{gauge}} + \mathcal{L}^{(4)}_{\tilde{\varphi}-\mathrm{gauge}}$$

$$\mathcal{L}^{(3)}_{\tilde{\varphi}-\mathrm{gauge}} = -\frac{g m_W}{2}\tilde{\varphi}\left(W^+_\mu W^-_\nu + \frac{1}{2\cos^2\theta_W}Z^2_\mu\right)$$

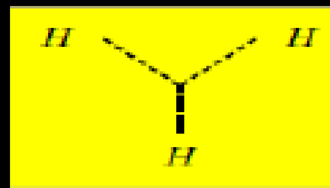


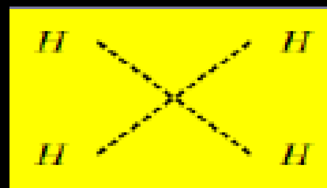
$$\mathcal{L}^{(4)}_{\tilde{\varphi}-\mathrm{gauge}} = -\frac{g^2}{4}\tilde{\varphi}^2\left(W^+_\mu W^-_\nu + \frac{1}{2\cos^2\theta_W}Z^2_\mu\right)$$

$$\mathcal{L}_{\tilde{\varphi}} = -\frac{1}{2}(\partial_\mu\tilde{\varphi})^2 - \frac{1}{2}m_H^2\tilde{\varphi}^2 + \mathcal{L}^{(3)}_{\tilde{\varphi}} + \mathcal{L}^{(4)}_{\tilde{\varphi}}$$

$$\mathcal{L}^{(3)}_{\tilde{\varphi}} = -\frac{g}{4}\frac{m_H^2}{m_W}\tilde{\varphi}^3$$

$$\mathcal{L}^{(4)}_{\tilde{\varphi}} = -\frac{g^2}{32}\frac{m_H^2}{m_W^2}\tilde{\varphi}^4$$





$m_H^2 = 8av^2$ **undetermined**

if heavy, Higgs may become strongly coupled

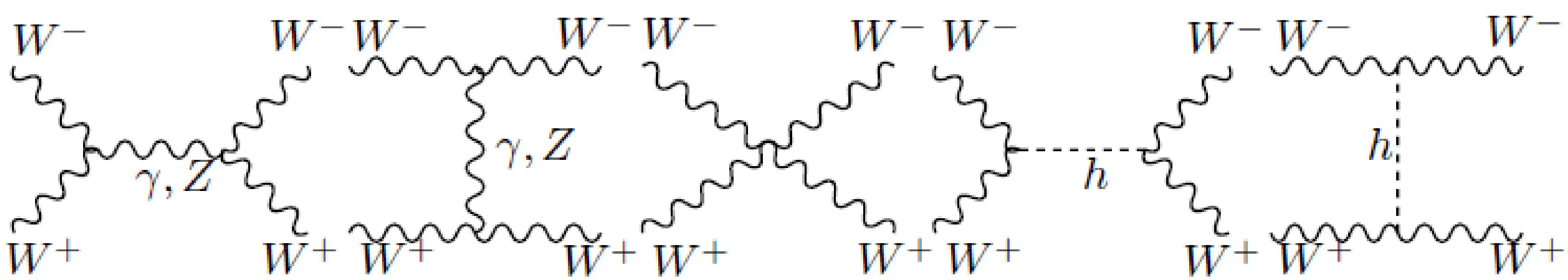
Higgs boson and unitarity in the standard model

The most important indication for physics beyond the standard model at the 1 TeV scale is the need to unitarize the cross sections for some processes. The most serious problem is the scattering of longitudinal gauge bosons, that only respects the unitarity limit if there is one Higgs boson, either elementary or composite but having the same effective coupling as in the standard model. Let us show here how these cancellations occur for the Higgs boson of the SM.

We will discuss now one particular process where the Higgs boson is crucial to unitarize the amplitudes. The process we consider is the scattering of longitudinal $W_L^\pm$,

$$W_L^-(p_1) + W_L^+(p_2) \rightarrow W_L^-(q_1) + W_L^+(q_2)\,, \tag{2.73}$$

where the momenta are as indicated and the subscript L means that the gauge bosons $W^\pm$ are longitudinally polarized. In the standard model this process has seven tree-level diagrams shown in Fig. 2.3.



Higgs and unitarity in the Standard model

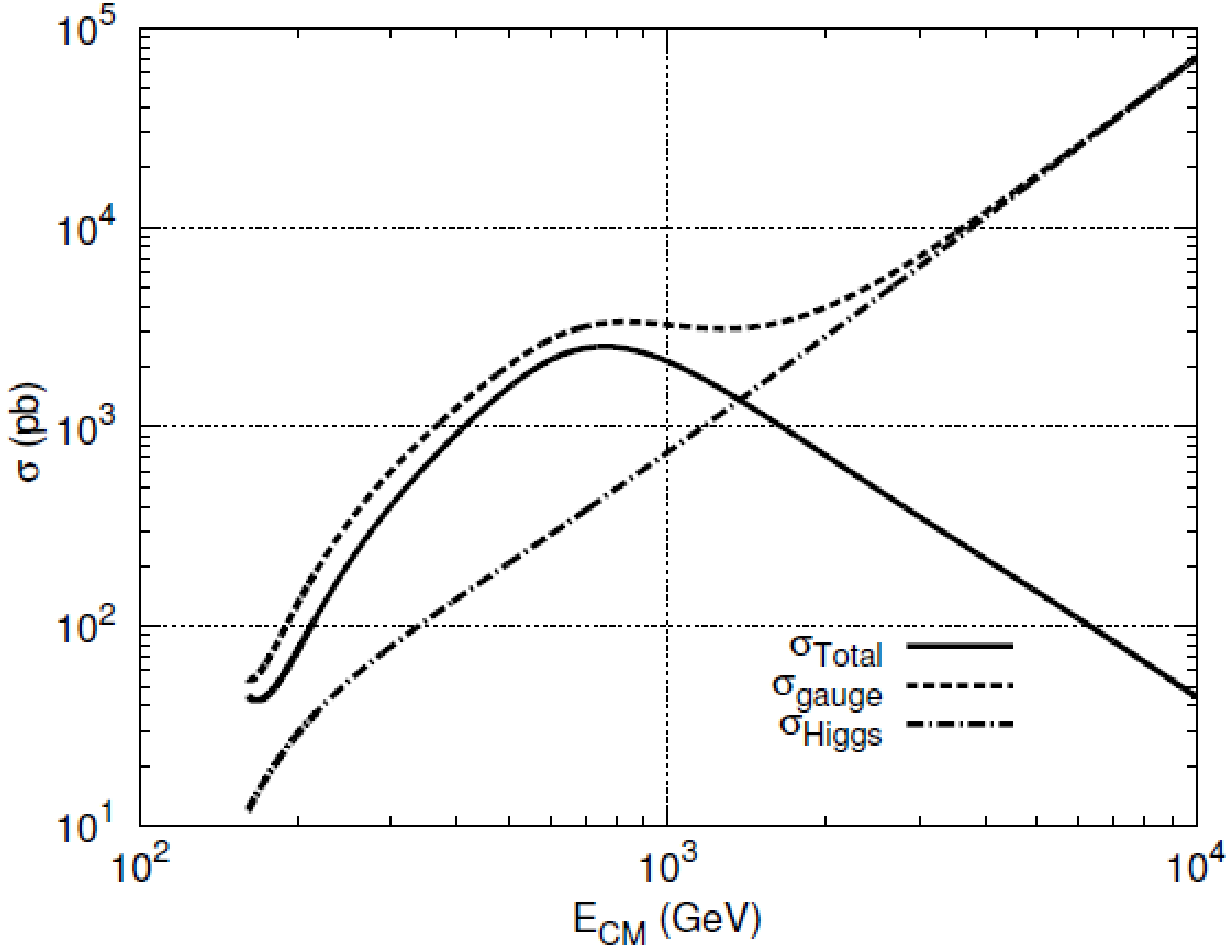


Figure 2.4 Cross section for $W_L^- + W_L^+ \rightarrow W_L^- + W_L^+$. Shown are the contribution of the gauge diagrams (dashed), the contribution from the Higgs (dot-dashed) and the total cross section (solid line). The sum of the amplitudes from the gauge part have the opposite sign from those from the Higgs (not visible in the figure because we are plotting cross sections) forcing the cross section to decrease.

SM Fermion assignments

- 3 left-handed, color singlet, $SU(2)\otimes U(1)$ doublet leptons

$$\chi_{AL} = \begin{pmatrix} \chi_{0A} \\ \chi_{-A} \end{pmatrix}_L = \frac{1+\gamma_5}{2}\chi_A \qquad A = 1,2,3$$

- 3 right-handed, color singlet, $SU(2)\otimes U(1)$ singlet leptons

$$\chi_{AR} = \frac{1+\gamma_5}{2}\chi_{AR}$$

- 3 left-handed, SU(3)-color triplet, $SU(2)\otimes U(1)$ doublet quarks

$$\psi_{AL} = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}_{AL} = \frac{1+\gamma_5}{2}\psi_A$$

- 6 right-handed, SU(3)-color triplet, $SU(2)\otimes U(1)$ singlet quarks

$$\psi^u_{AR} = \frac{1-\gamma_5}{2}\psi^u_R$$
$$\psi^d_{AR} = \frac{1-\gamma_5}{2}\psi^d_R$$

$$L_L = \begin{bmatrix} \nu_e \\ e^- \end{bmatrix}_L, \ Q_L = \begin{bmatrix} u \\ d \end{bmatrix}_L, \ e^-_R, u_R, d_R,$$

Particle	ν_{eL}	e_L	u_L	d_L	e_R	u_R	d_R
T_3	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0
Y	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$	-1	$\frac{2}{3}$	$-\frac{1}{3}$

Still need to specify U(1) assignments to define gauge transf & cov derivatives…

$$\Psi_L \to \Psi'_L = e^{i\alpha^a \frac{\sigma^a}{2}} e^{i\alpha_Y Y}\,\Psi_L\,,$$
$$\psi_R \to \psi'_R = e^{i\alpha_Y Y}\,\psi_R\,.$$

$$\mathcal{D}_\mu \Psi_L = \left(\partial_\mu + ig\frac{\sigma_a}{2}W^a_\mu + ig' Y B_\mu\right)\Psi_L\,,$$
$$\mathcal{D}_\mu \psi_R = \left(\partial_\mu + ig' Y B_\mu\right)\psi_R\,.$$

Show that

$$\mathcal{D}_\mu \Psi_L \to \mathcal{D}_\mu \Psi'_L = e^{i\alpha^a \frac{\sigma^a}{2}} e^{i\alpha_Y Y}\,\mathcal{D}_\mu \Psi_L\,,$$
$$\mathcal{D}_\mu \psi_R \to \mathcal{D}_\mu \psi'_R = e^{i\alpha_Y Y}\,\mathcal{D}_\mu \psi_R\,.$$

Standard Model recap

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L$$

$$e_R \quad \mu_R \quad \tau_R$$

$$\begin{pmatrix}u\\ d\end{pmatrix}_L \quad \begin{pmatrix}c\\ s\end{pmatrix}_L \quad \begin{pmatrix}t\\ b\end{pmatrix}_L$$

$$u_R \quad c_R \quad t_R$$

$$d_R \quad s_R \quad b_R$$

$$\varphi = \begin{pmatrix}\varphi_+\\ \varphi_0\end{pmatrix}$$

χ_L	2	-1	1	gauge-scalar sector			
χ_R	1	-2	1	W_μ, Z_μ	3	0	1
ψ_L	2	1/3	3	A_μ	1	0	1
ψ_R^u	1	4/3	3	g_μ	1	0	8
ψ_R^d	1	-2/3	3	φ	2	1	1

$$Y = 2(Q - T_3)$$

$$m_W = m_Z \cos\,\theta_W$$

$$\frac{\sin\,\theta_W}{\cos\,\theta_W} = \frac{g'}{g} = \tan\,\theta_W$$

$$e = g\,\sin\,\theta_W = g'\,\cos\,\theta_W$$

Check above assignments of all SM multiplets

Yukawa couplings

couplings of fermions to the scalar doublet, allowed by gauge invariance and renormalizability

$$\mathcal{L}_{\textbf{Yukawa}} = -\sum_{AB} h^e_{AB} \bar{\chi}^A_R \varphi^\dagger \chi^B_L + \textbf{h.c.}$$

$$-\sum_{AB} \left[h^d_{AB} \bar{\psi}^A_{dR} \varphi^\dagger \psi^B_L + h^u_{AB} \bar{\psi}^A_L (i\tau_2 \varphi^*) \psi^B_{uR} \right] + \text{h.c.}$$

under SU(2) $i\tau_2\varphi^* \sim \varphi$

These Yukawa terms introduce many new arbitrary parameters

λ	g	g'	v	|	$h^{d,u,e}_{AB}$
θ_W	m_H	m_W	m_Z	|	

some are spurious, others directly given by experiment

general matrix diagonalization theorem

any arbitrary square matrix M may be decomposed in polar form as

$$\underbrace{M}_{2n^2} = \underbrace{UK}_{n^2+n^2} \quad \textbf{where } K = K^\dagger \quad \textbf{and} \quad U^\dagger U = UU^\dagger = 1$$

one can always fi nd a unitary matrix V so that

$$K = VDV^\dagger \qquad D \textbf{ real diag} > 0$$

$$V^\dagger V = VV^\dagger = 1$$

$$M = UVDV^\dagger$$

$$\Rightarrow \boxed{W^\dagger MV = D}; \quad \textbf{where} \quad W = UV$$

thus any square matrix can be bi-diagonalized $V = W$ **when** $M = M^\dagger$

diagonalizing matrices may be found by solving

$$\begin{aligned} MM^\dagger &= WDV^\dagger VDW^\dagger = WD^2W^\dagger \\ M^\dagger M &= VDW^\dagger WDV^\dagger = VD^2V^\dagger \end{aligned}$$

FERMION MASSES

rewrite Yukawa in matrix form

$$\mathcal{L}_{\textbf{Yukawa}} = -\bar{\chi}_R \varphi^\dagger h^e \chi_L - \bar{\psi}_R^d \varphi^\dagger h^d \psi_L - \bar{\psi}_R^u (i\tau_2 \varphi^*) h^u \psi_L + \textbf{h.c.}$$

where h^e, h^u, h^d are arbitrary, non-hermitian matrices.

In the U-gauge we have $\varphi^\dagger = \left(0, \frac{v+H}{\sqrt{2}}\right)$

thus the lepton piece becomes

$$\bar{\chi}_R \left(0, \frac{v+H}{\sqrt{2}}\right) h^e \begin{pmatrix} \chi_L^0 \\ \chi_L^- \end{pmatrix}$$

$$= \bar{\chi}_R^- h^e \chi_L^- \frac{(v+H)}{\sqrt{2}} + \textbf{h.c.} = \bar{E}_R \, \Omega_R^\dagger h^e \Omega_L \, E_L \frac{(v+H)}{\sqrt{2}} + \textbf{h.c.}$$

After SSB these Yukawa terms will generate

$$\frac{v}{\sqrt{2}} \Omega_R^{e\dagger} h^e \Omega_L^e = m(e) \text{ real diagonal}$$

$$\frac{v}{\sqrt{2}} \Omega_R^{u\dagger} h^u \Omega_L^u = m(u) \text{ real diagonal}$$

$$\frac{v}{\sqrt{2}} \Omega_R^{d\dagger} h^d \Omega_L^d = m(d) \text{ real diagonal}$$

Fermion masses ... fitted parameters

Higgs boson couplings

- prop to mass, all Yukawa terms proportional to 1+H/v
- Higgs boson couplings diagonal in mass eigenstate basis thus no Higgs-mediated FCNC

Indeed it is seen that the Higgs couplings correlate to mass ...