1. Particle Cosmology

basic notions, evidence & properties of dark matter

Pasquale D. Serpico

Camille Flammarion (Paris, 1888) "L'Atmosphere: Météorologie Populaire"

ISAPP - Belgirate 23 July 2014

A disclaimer

Particle cosmology (topics at interface of particle physics and cosmology) is way too broad, it includes for instance:

- *microscopic models of inflation*
- *microscopic models of dark energy*
- *baryogenesis mechanisms*
- *neutrino cosmology*
- *signatures of phase transitions in the early universe*
- *dark matter (DM)*

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Choice here: follow the Dark (Matter) Path

Focus on DM (its "physical properties" & production mechanisms) narrow enough to cover sufficiently well in ~4 h, broad enough to touch either directly or indirectly several ingredients of other particle cosmo applications

"reasonable excuse" to illustrate a few items

Outline

more qualitative, slide-based

more quantitative (slides to accompany bb derivation)

Lecture one

- ***basic notions of cosmology for "particle astrophysics"***
- ***The evidence for Dark Matter and the limited number of particle physics properties/constraints from astro/cosmo observations***

Lecture two

- ***freeze-out (hot, cold), "WIMPs & their relatives", freeze-in***
- ***sterile neutrinos***
- ***...***

Lecture three

- ***Misalignment mechanism (+ some peculiarities of axion DM?)***
- ***Gravitational production***
- ***...***

Some references

General references

- The Early Universe", E. W. Kolb & M. S. Turner
- "Physical Foundations of Cosmology", V. Mukhanov

...

Specific monographs

- "Kinetic Theory in the expanding Universe", J. Bernstein
- "Neutrino Cosmology", J. Lesgourgues, G. Mangano, G. Miele, Pastor
- "Particle Dark Matter" Edited by Gianfranco Bertone
(chapters on different particle physics candidates and probes)
- "Introduction to Quantum Fields in Classical Backgrounds"
V. Mukhanov, S. Winitzki (accessible intro to gravitational production...)

...

Yet another disclaimer...

- *Differently e.g. from a course on the Standard Model or Cosmology, the difficulty is that here the goal is... to define the object of our study:“What’s Dark Matter?”*

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✦ *First Goal (this lecture):*
to understand "what's out there that needs to be explained"
(otherwise meaningless to build a theory-to do what?-not to speak of testing it!)

✦ *Preliminary to that, some MINIMAL notions in cosmology. Sorry for possible partial overlap with Ravi Sheth's lectures (only in lecture I!) but "Repetita iuvant".*

Basic Notions of smooth cosmology*

**Minimum you need to know to follow the rest of the lectures. Cannot replace a proper knowledge in cosmology needed to work on this subject!*

Pillars of the Standard Cosmological Model

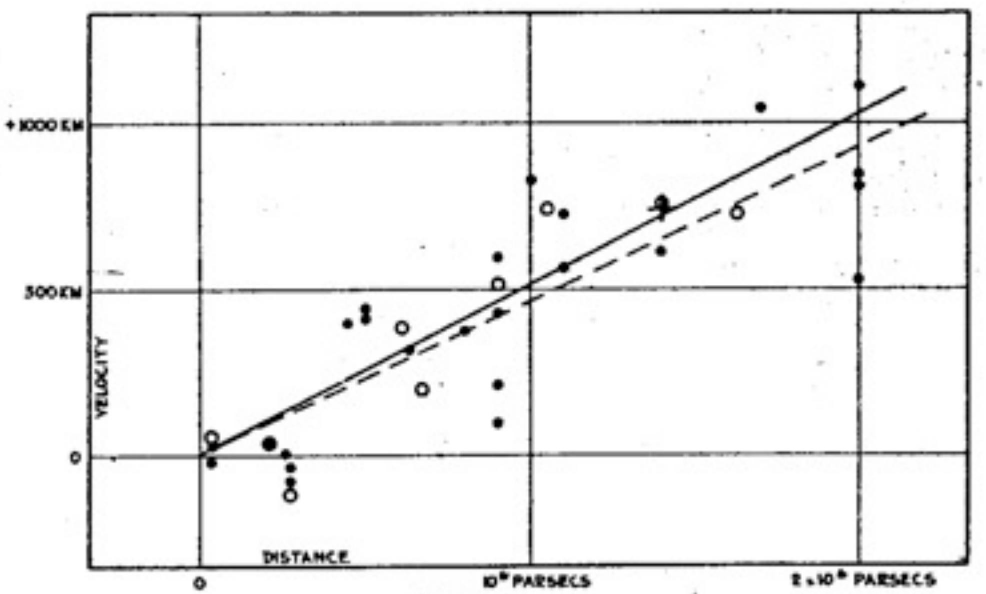
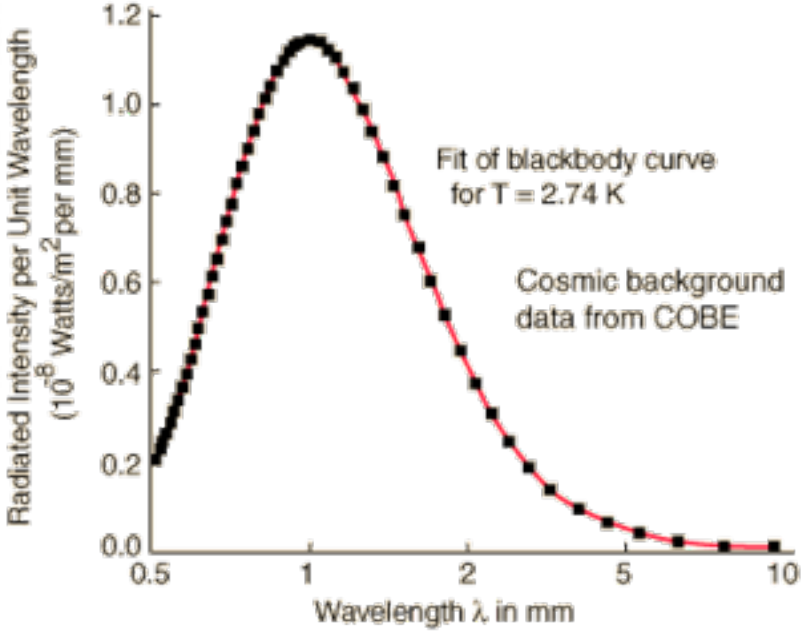


FIGURE 1



- Galaxies sufficiently far away from us recede with $v=Hd$ (Hubble law)
- The Universe is permeated by an almost perfect blackbody radiation, with T~2.73 K (Cosmic Microwave Background)
- Yields of light elements (notably Deuterium and Helium) way larger than what expected from “stellar” phenomena.

Standard Cosmological Model

Based on:

- General Relativity (GR): metric theory of gravitation
- Cosmological Principle (spatial homogeneity & isotropy on large scales)
- "Standard Physics", in particular Kinetic Theory of Fluids, Particle & Nuclear Physics, Plasma Physics, Atomic Physics.

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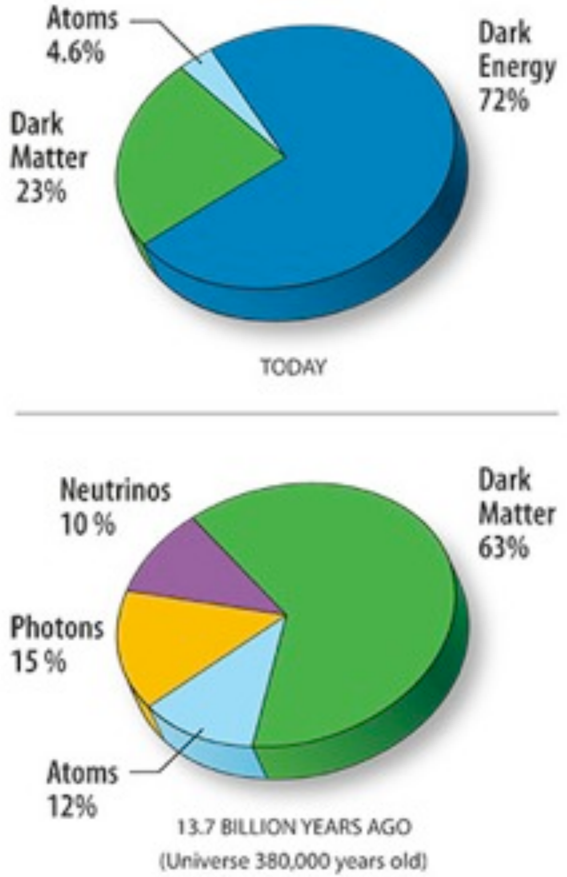
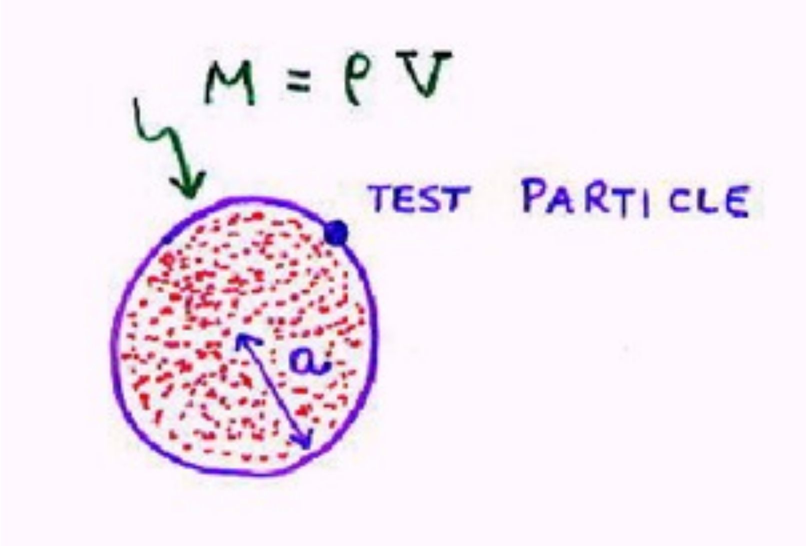
Evolving the expanding universe backwards in time leads to the picture of a hot Early Universe, made of a plasma which has been cooling while expanding.

One basic (not the sole!) task of cosmology is to understand what the universe is made of, now and in the past (the "mixture" can and does evolve with time…)



$$\text{Natural units}: c = \hbar = k_B = 1$$

Friedmann Equations for dummies



Consider the Newtonian toy model of a sphere of dust. The acceleration is

$$\ddot{a} = -\frac{G_N M}{a^2} \qquad M = \frac{4\pi}{3}\rho a^3$$

by integration

$$\frac{\dot{a}^2}{2} = \frac{G_N M}{a} - \frac{k}{2}$$

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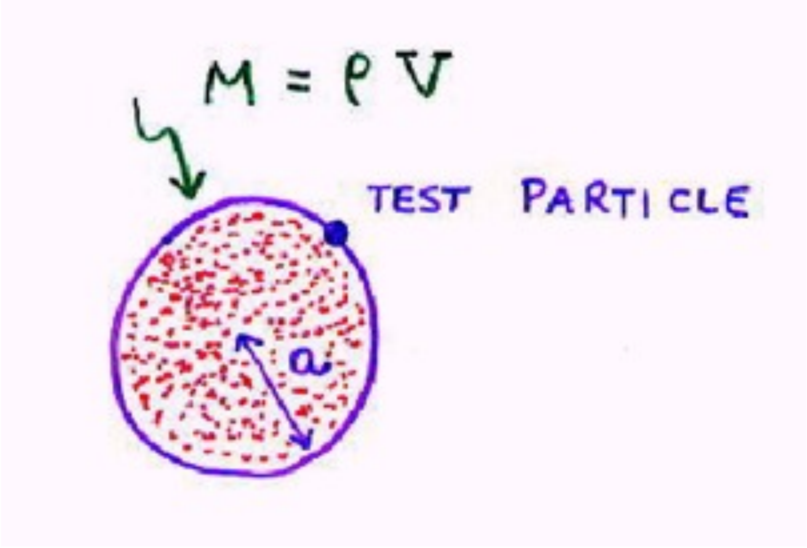

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$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}$$

This naïve model reproduces correctly one of the 2 independent GR equations in the FLRW metric=(implementing the Cosm. Pr.)

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0$$

The additional independent equation implements “energy conservation” and contains a peculiar GR term

closed system if an Equation Of State P=P(ρ) is provided

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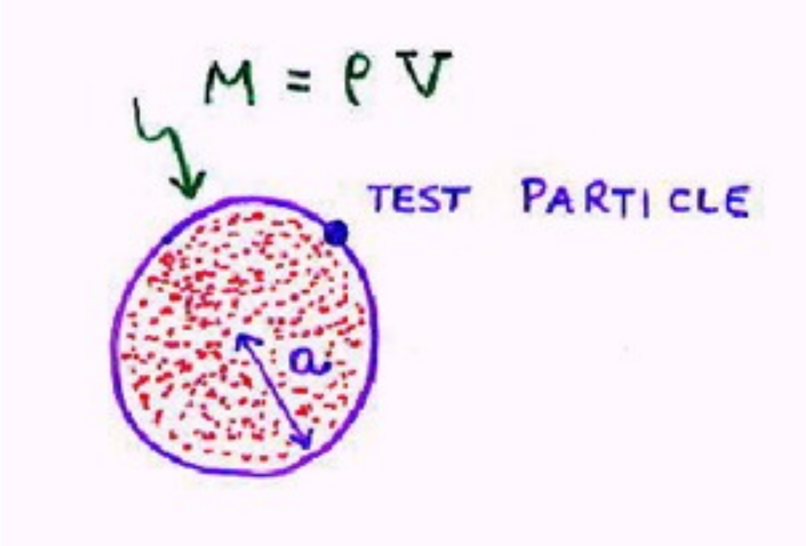

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Compositions usually expressed in **Ω_i's**, ratios of density of i-species to "critical density"

$$\rho_c = \frac{3}{8\pi G_N} H_0^2$$

Some Generic Solutions (k=0)

	Equation of State	Behaviour of ρ	Scale Factor
Matter	$P \simeq 0$ $(T \ll m)$	$\rho \propto a^{-3}$	$a \propto t^{2/3}$
Radiation	$P = \rho/3$	$\rho \propto a^{-4}$	$a \propto t^{1/2}$
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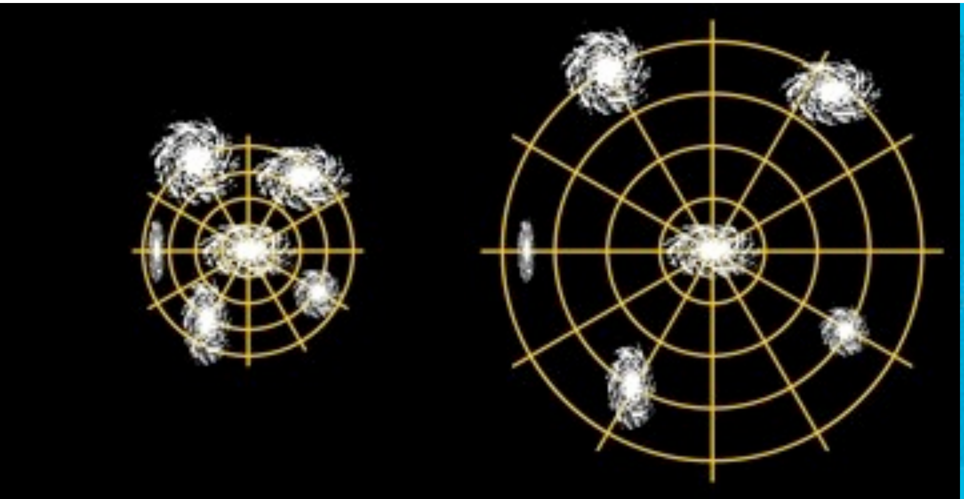
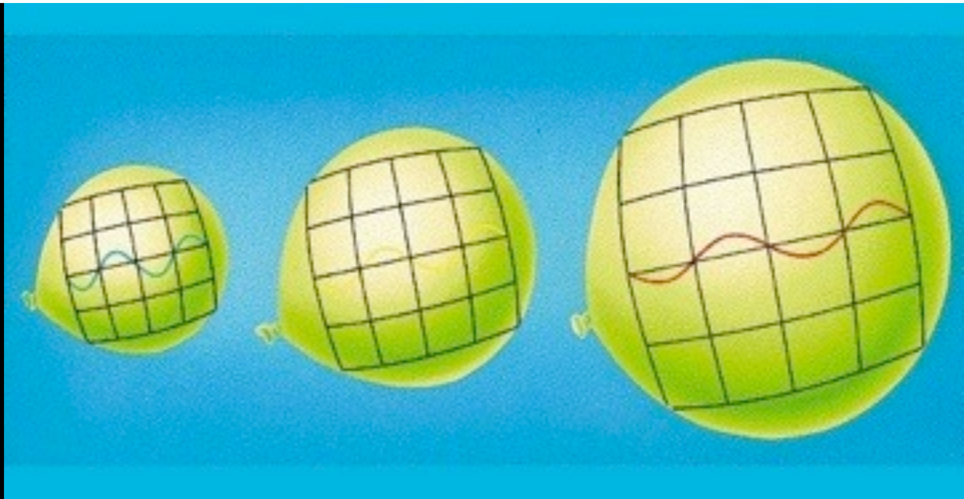
conservation of particles per comoving volume
For radiation, further a-factor due to wavelength stretching, also called "redshift"

$$1 + z = \frac{\lambda_{\text{today}}}{\lambda_{\text{then}}} = \frac{a_{\text{today}}}{a_{\text{then}}}$$





"Thermodynamics"

Let's introduce the phase space density f describing the occupation number of microstates of different energies.

The Universe is not a system in equilibrium with an external bath, need nonequilibrium system tools.

However, for sufficiently fast processes *(wrt expansion rate)* exchanging both energy & particles, locally the entropy gets maximized & "local equilibrium conditions" hold

$$f(E) = \frac{1}{\exp[(E-\mu)/T] \pm 1}$$

T and μ: parameters maximizing the entropy under a given constraints on the energy and number of particles present per unit volume, respectively.

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- If energy is exchanged rapidly, different species share the the same T
- Similarly, if particle changing reactions of the type $A + B \leftrightarrow C + D$ are fast enough

a conservation rule holds $\mu_A + \mu_B = \mu_C + \mu_D$

⇒ chemical potential μ vanishes for particles that can be freely created/ annihilated, like photons, and that particles and antiparticles have opposite μ

Useful recipe

To know if LTE holds, compare

Rate of process of interest Γ vs. H Hubble expansion rate

Most of the interesting cosmological processes happen when those quantities become comparable ("freeze-out"): departures from equilibria!

- T~ 1 eV (@ t~10^{13} s)

$$e + p \leftrightarrow \gamma + H$$

freezes-out: recombination, photons nowadays forming CMB decouple

- T~ 0.1 MeV (@ t~10^{2} s)

$$p + n \leftrightarrow \gamma + D$$

freezes-out: the "nuclear statistical equilibrium" ends, BBN takes place

"Thermodynamics" in the expanding universe

If f is the phase space distribution function, homogeneity and isotropy imply that it can only depend on t and $|\mathbf{p}|=p$

"Kinetic theory" demands a dynamical equation for f (Boltzmann Eq.)
However, in most applications the whole energy spectrum is not needed and one can work with moments of f (and corresponding equations)

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current density of particles

internal (spin) dof

due to isotropy, only $n^0 \neq 0$

$$n^\mu = g \int f \frac{p^\mu}{p^0} \frac{d\vec{p}}{(2\pi)^3} \Rightarrow n = \int f \frac{d\vec{p}}{(2\pi)^3}$$

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can be proven that the covariant conservation of particle number

$$\nabla_\mu n^\mu = 0 \Rightarrow \nabla_\mu n^\mu = \frac{1}{a^3}\frac{\partial}{\partial t}(a^3 n) = 0$$

OK with physical intuition of previous cartoon $n \propto a^{-3} \propto V^{-1}$

Second moment

In GR, the Einstein tensor depends on second moments

Stress-energy Tensor

$$T^{\mu\nu} = g \int f \frac{p^\mu p^\nu}{p^0} \frac{d\vec{p}}{(2\pi)^3}$$

Energy density

$$\rho = T^{00} = g \int f \, p^0 \frac{d\vec{p}}{(2\pi)^3}$$

Pressure

(note the isotropy assumption) →

$$-P\delta^{ij} = T^{ij} = -\delta^{ij} g \int f \frac{|\vec{p}|^2}{3} \frac{d\vec{p}}{(2\pi)^3}$$

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Bianchi identities (1 ind. eq.), "energy conservation"

$$\nabla_\mu T^{\mu\nu} = 0 \quad \longrightarrow \quad \frac{d\rho}{dt} = -3H(\rho + P)$$

We recover the second Friedmann equation!

If we express f in terms of "temperature", this equation provides a time-temperature relation!

Explicit equilibrium expressions for μ=0...

Relativistic species

$$n = g\frac{\zeta(3)}{\pi^2}T^3 \times \left\{1(-), \frac{3}{4}(+)\right\}$$

$$\rho = g\frac{\pi^2}{30}T^4 \times \left\{1(-), \frac{7}{8}(+)\right\} \qquad P = \rho/3$$

applying comoving particle number conservation law we obtain a simple *t(T)*

$$a^3T^3 = \text{const.} \rightarrow T \propto a^{-1}$$

we can use e.g. CMB photon "temperature" as "clock variable" for the epoch of the universe, at least after recombination when the # of photons does not change...

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Non-relativistic species at LTE

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2}\exp\left(-\frac{m}{T}\right) \qquad \rho = m\,n \qquad P = nT \ll \rho$$

Entropy

Remember Boltzmann's formula? It naturally suggests the following formula for the entropy density/current (classical limit)

$$s^{\mu} = -g \int f(\ln f - 1)\frac{p^{\mu}}{p^{0}}\frac{d\vec{p}}{(2\pi)^3} \Rightarrow s^{0} = -g \int f(\ln f - 1)\frac{d\vec{p}}{(2\pi)^3}$$

Exercise: using *f~exp[(μ-E)/T in the parenthesis*, check that @ equilibrium & for a perfect fluid, this gives

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For relativistic species (the entropy is dominated by relativistic species)

$$s \propto T^3 \qquad s \simeq \frac{4}{3}\frac{\rho}{T} \qquad s = \frac{2\pi^2}{45}h_{\rm eff}(T)T^3$$

$$h_{\rm eff}(T) = \sum_{i=\rm rel.bos.} g_i\left(\frac{T_i}{T}\right)^3 + \frac{7}{8}\sum_{j=\rm rel.ferm.} g_j\left(\frac{T_j}{T}\right)^3$$

Energy & Entropy density in relativistic era

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entering

$$\rho_{\rm tot} = \frac{\pi^2}{30} g_{\rm eff}(T) T^4$$

&

$$H^2 = \frac{8\pi}{3\,M_P^2}\rho_{\rm tot} = \frac{4\pi^3}{45\,M_P^2} g_{\rm eff} T^4$$

$$s = \frac{2\pi^2}{45} h_{\rm eff}(T) T^3$$

they vary when species annihilate!

for reference, currently-accounting for photons and neutrinos-one has **$h_{\rm eff}$** ~ 2+3*2(4/11)*7/8~**3.91, T~2.73 K**

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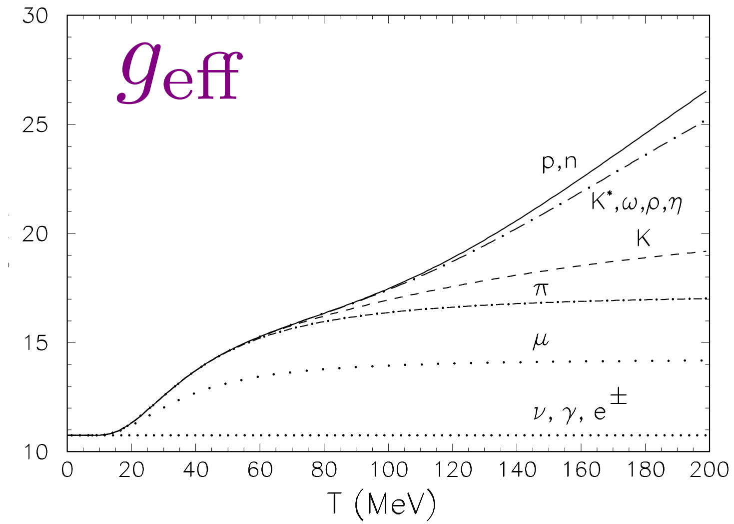

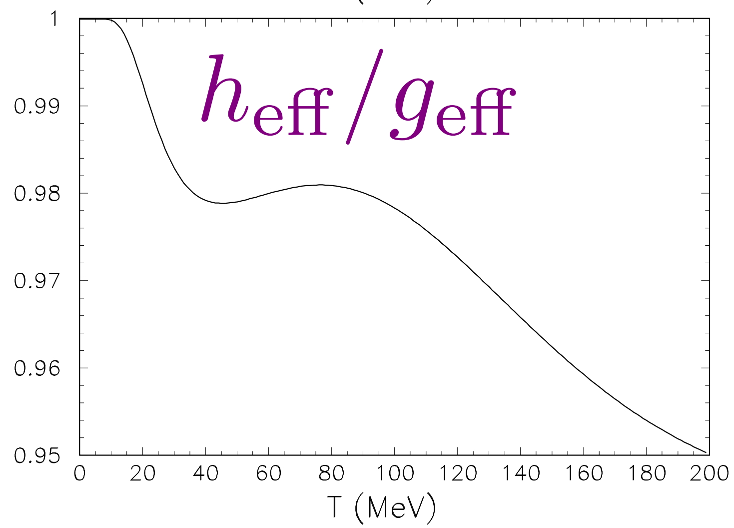

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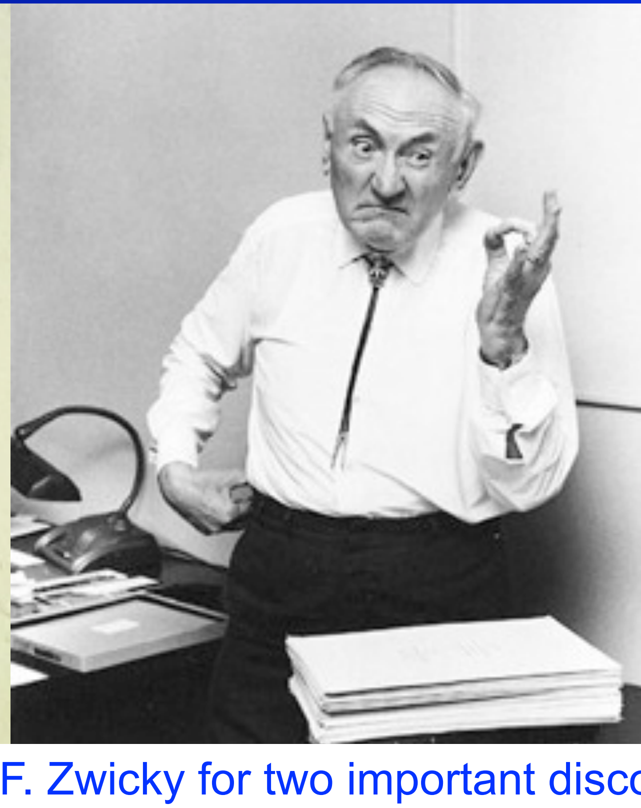
Dark Matter enters the scene

Dark Matter Discovery in Coma cluster: 1933

Varna, Bulgaria



~10[3]galaxies in ~1 Mpc radius region

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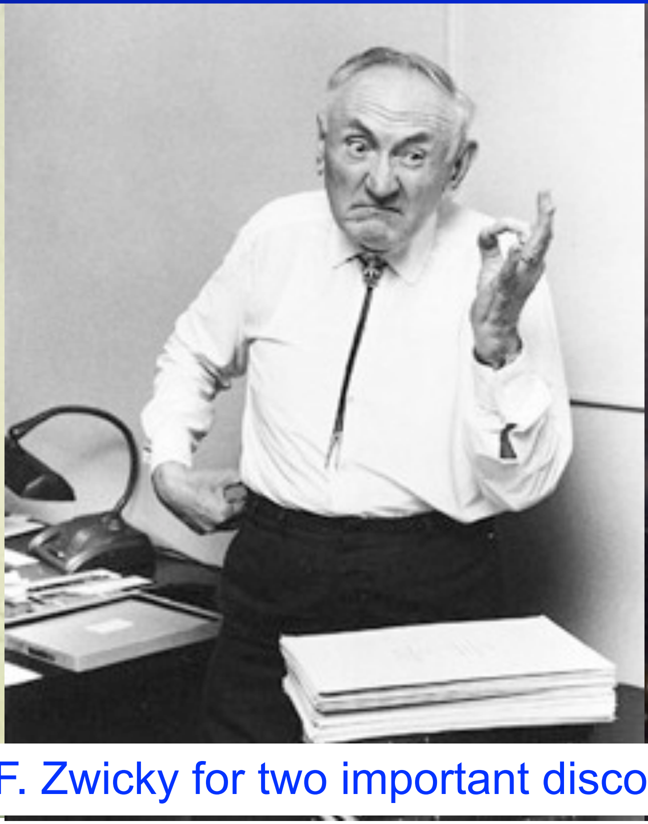

~10^3galaxies in ~1 Mpc radius region

- We recall here F. Zwicky for two important discoveries:

- “Astronomers are spherical bastards. No matter how you look at them they are just bastards.”
- Inferred the mass of the Coma cluster from the proper motion of the Galaxies, finding that the required mass is much larger than what could be accounted for

Die Rotverschiebung von extragalaktischen Nebeln", Helvetica Physica Acta (1933)* ***6****, 110–127.*
"On the Masses of Nebulae and of Clusters of Nebulae", Astrophysical Journal (1937)* ***86****, 217*
*Nebula=Early XXth century name for what we call now galaxy

I. No “BSM” implications (yet)
II. How did he do it? Clever & original application of Virial Theorem

Sketch of the method

Expression of time average of total kinetic energy T of N particles bounded by conservative forces F

$$2\langle T\rangle = -\sum_{k=1}^{N}\langle \mathbf{r}_k \cdot \mathbf{F}_k\rangle$$

Average total potential energy <U>

$$U(r) = A\,r^n \Longrightarrow -\sum_{k=1}^{N}\langle \mathbf{r}_k \cdot \mathbf{F}_k\rangle = n\langle U_{tot}\rangle$$

For Gravity, U~ r $^{-1}$

$$2\langle T\rangle + \langle U_{tot}\rangle = 0$$

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$$\langle T\rangle = N\frac{\langle m\,v^2\rangle}{2}$$

$N^2/2$ pairs of Galaxies

$$\langle U_{tot}\rangle \simeq -\frac{N^2}{2}G_N\frac{\langle m^2\rangle}{\langle r\rangle}$$

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For Gravity, $U \sim r^{-1}$

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$N^2/2$ pairs of Galaxies

$$\langle U_{tot}\rangle \simeq -\frac{N^2}{2}G_N\frac{\langle m^2\rangle}{\langle r\rangle}$$

doppler shifts in galactic spectra ← $\langle v^2\rangle$

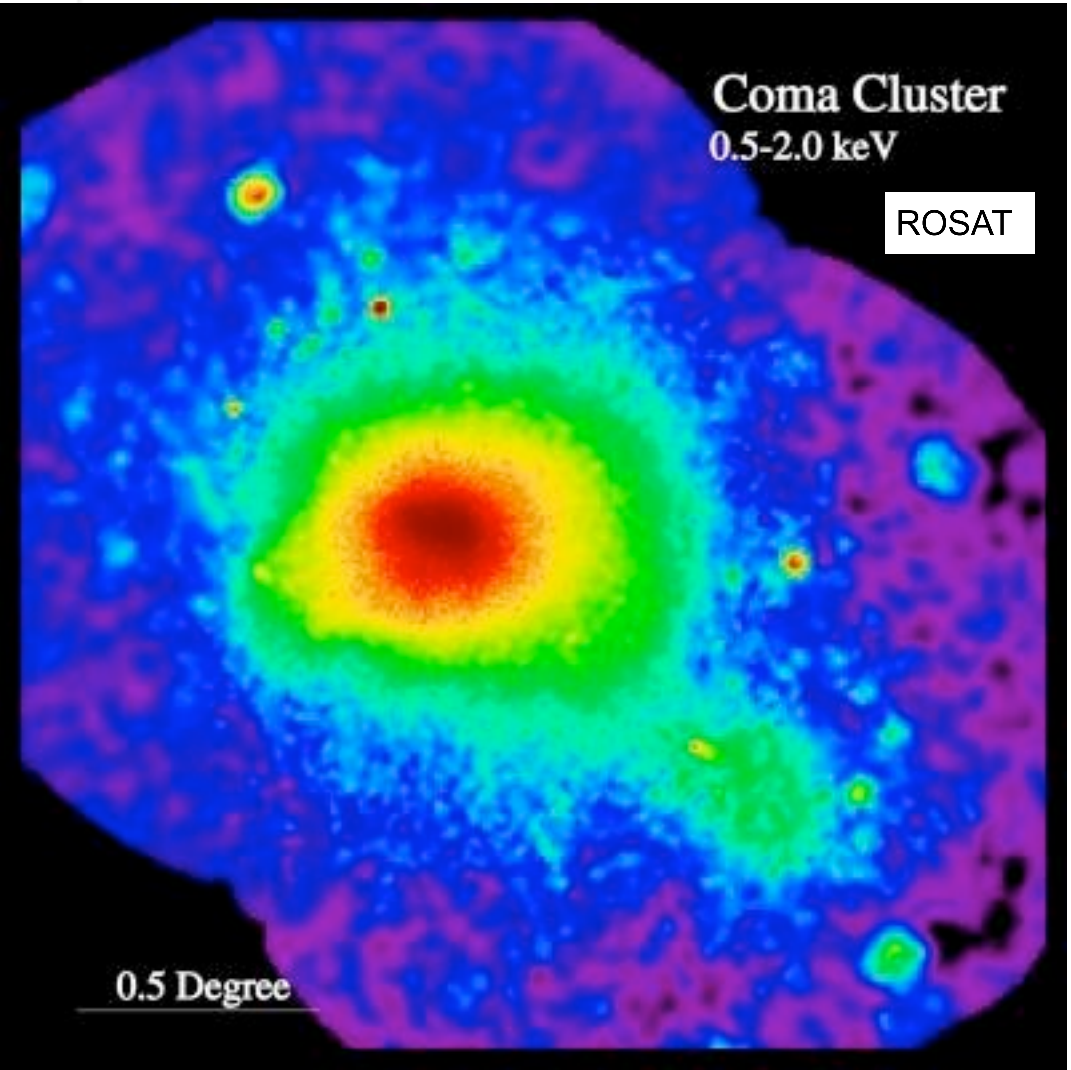
$$M_{tot} \simeq N\langle m\rangle \simeq -\frac{2\langle v^2\rangle\,\langle r\rangle}{G_N}$$

$\langle r\rangle$ → inferred geometrically

found a factor ~400 larger mass than the one from converting luminosity into mass!

Modern “proofs” from Clusters: X-rays

We know today that most of the mass in clusters (not true for galaxies!) is in the form of hot, intergalactic gas, which can be traced via X rays: bolometric X-luminosity can be eventually converted into gas density maps, spectral info into pressure information (or potential depth)

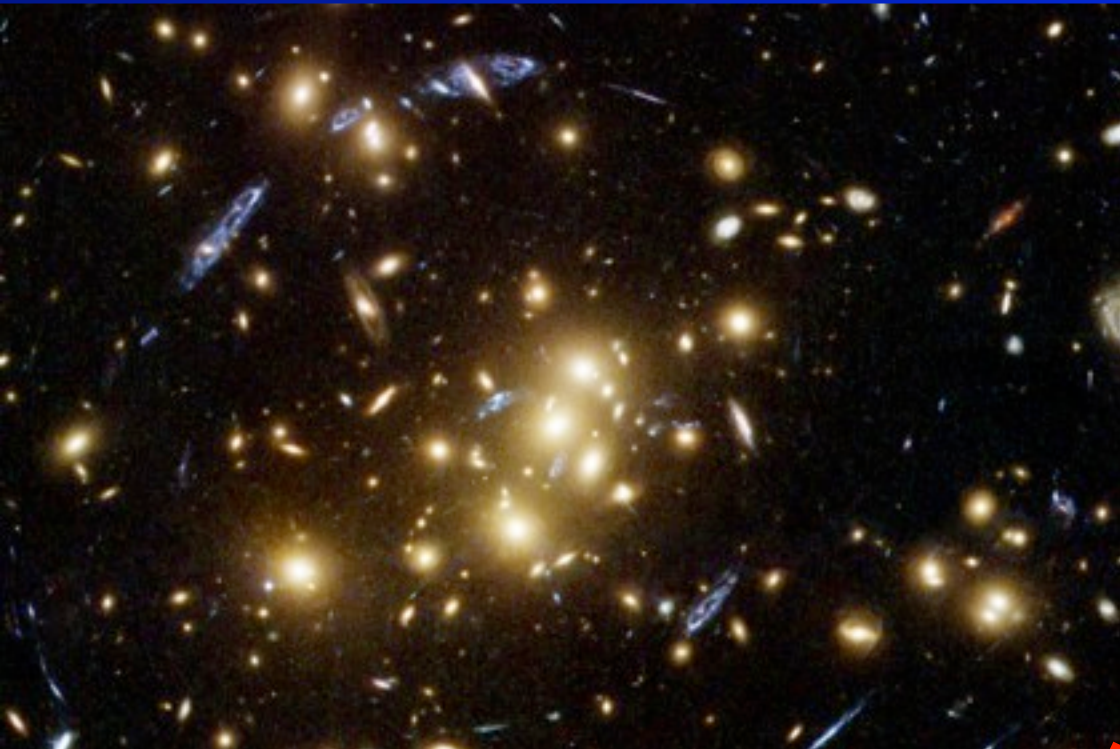


$$\frac{dP_{gas}}{dr} = G_N \frac{M(<r)\rho_{gas}}{r^2}$$

See for example
Lewis, Buote, and Stocke, ApJ (2003), 586, 135

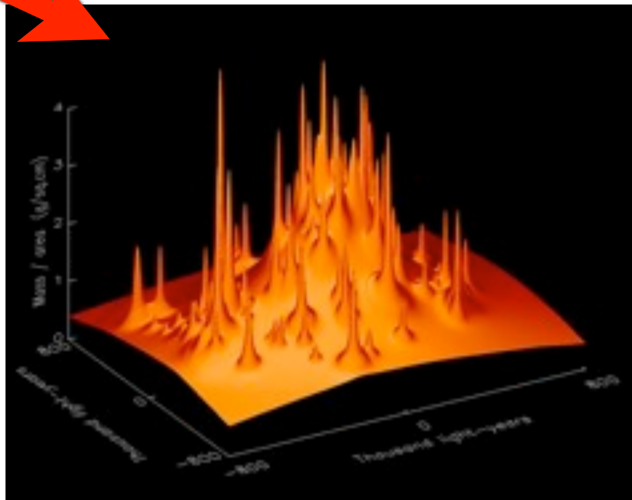
Again, a factor ~7 more mass than those in gas form is inferred (also its profile can be traced...)

Modern "proofs" from Clusters: lensing



CL0024+1654,
Hubble space telescope

its gravitating mass distribution
inferred from lensing tomography

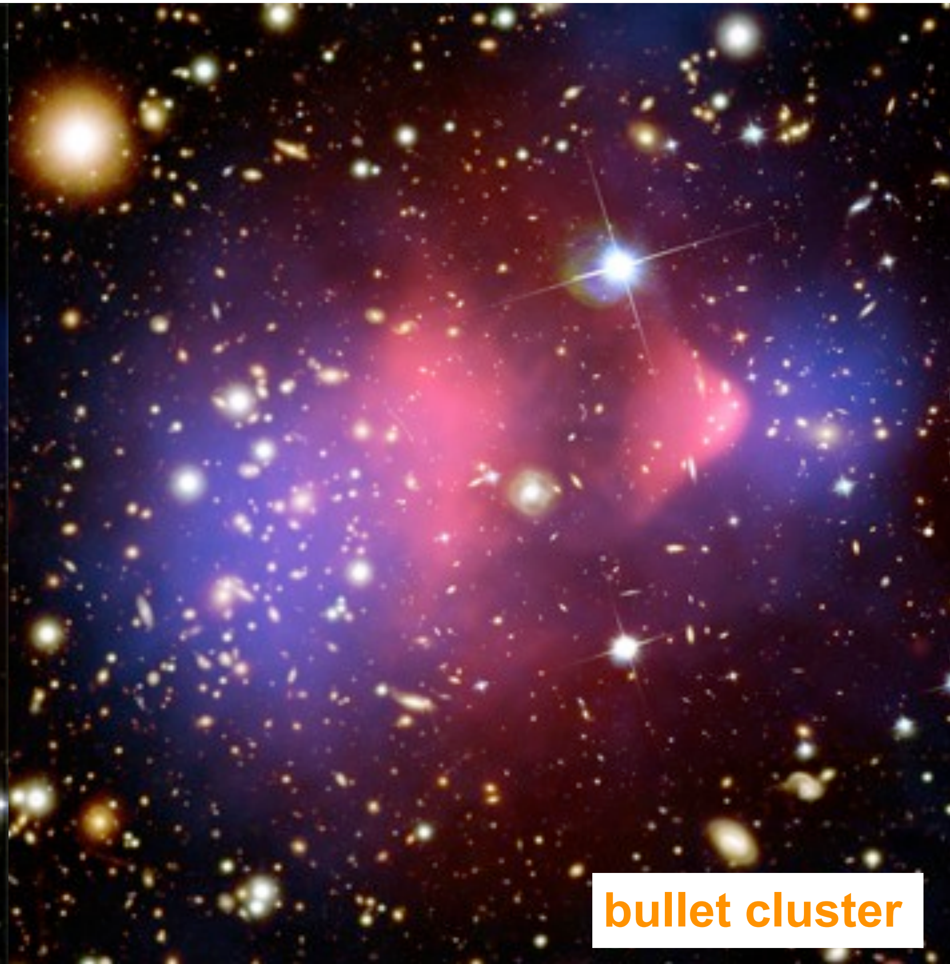


Consistent inference done from clusters of Galaxies: Presence of Dark Matter smoothly distributed in-between galaxies is required (and actually must dominate total potential)

Even more spectacular: segregation in colliding clusters

Baryonic gas gets “shocked” in the collision and stays behind. The mass causing lensing (as well as the subdominant galaxies) pass trough each other (non-collisional)

(most of the) Mass is not in the collisional gas



Flat galaxy rotations curves

- observed (equate centripetal acc. & Newton's law)

$$v_{rot}^2 = \frac{G\,M(R)}{R} \simeq const. \qquad M(R) = \int_0^R 4\pi\,r^2\,\rho(r)\,d\,r$$

- predicted based on visible light

$$v_{rot}^2 \propto \frac{1}{R}$$

Flat galaxy rotations curves

- observed (equate centripetal acc. & Newton's law)

$$v_{rot}^2 = \frac{G\,M(R)}{R} \simeq const. \qquad M(R) = \int_0^R 4\pi\, r^2\, \rho(r)\, d\,r$$

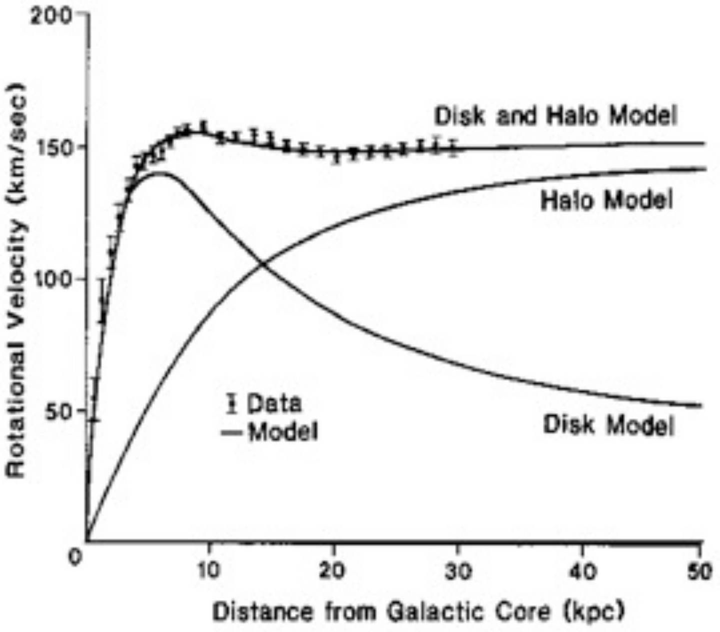
- predicted based on visible light

$$v_{rot}^2 \propto \frac{1}{R}$$

Data are well described by an additional component extending to distance >> visible mass scale, with a profile

$\rho(r) \propto r^{-2}$ (clearly not valid at asymptotically large r!)

Historically, only after these studies (in the ~'70-'80) people started to take the dark matter problem seriously



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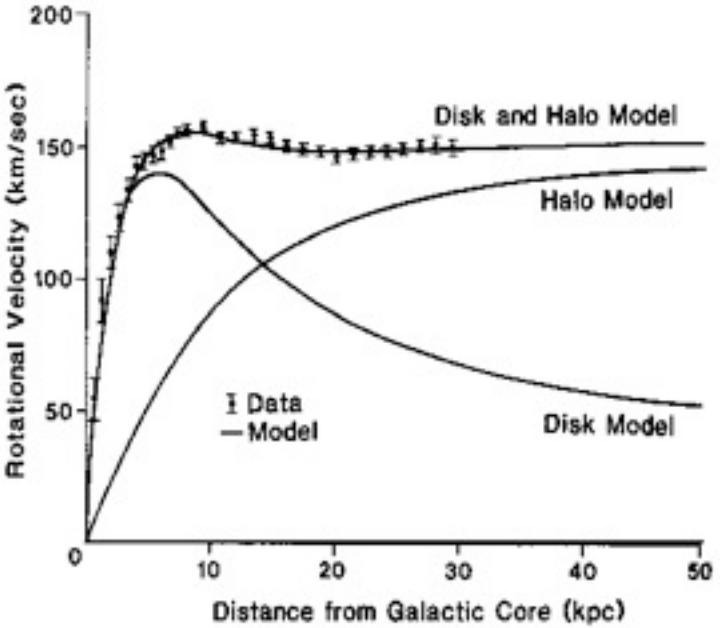

The determination of "local" (Galactic) DM properties require a multi-parameter fit including parameterizations for stellar disk, gas, bulge...

$$\rho_\odot \simeq 0.4\, \mathrm{GeV/cm^3}$$

Important for direct and indirect searches of DM, not so important/robust to infer its existence and properties

Growth of structures: Jeans Equation

What determines if perturbations grow or not? For collisional fluids, it's who wins the struggle between gravity and pressure!

The combination of continuity, Euler equation & Poisson Equation (conservation of mass/energy and momentum and gravity law) linearized in small perturbation around a "smooth" solution leads to the evolution eq. for the density perturbation of the form

$$\delta'' + \frac{a'}{a}\delta' + (k^2 - k_J^2)c_s^2\delta = 0$$

(derivative with respect to conformal time $d\tau = dt/a$)

$$c_s^2 = \left(\frac{\partial P}{\partial \rho}\right)_s$$

sound speed

$$k_J^2 = \frac{4\pi G_{\rm N}\bar{\rho}(t)a(t)^2}{c_s(t)^2}$$

Jeans wavenumber

For details see e.g. R. Sheth's lectures

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Jeans wavenumber

In absence of expansion, very simple solutions:

- Modes with $k \gg k_J$ oscillate with $\omega = kc_s$
- Modes with $k \ll k_J$ grow exponentially with typical time $(k_J c_s)^{-1} \sim (4\pi G_N \rho\, a^2)^{-1/2}$
- For pressureless fluids (uncoupled to photons), linear growth.

For details see e.g. R. Sheth's lectures

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Jeans wavenumber

In presence of expansion (=friction) qualitative modification of growths:

- Exponential growth → power-law
- linear growth → logarithmic

Jeans Equation in cosmology

- In the radiation-dominated era, the sound speed is large and ~constant, k_J is thus very small (all k oscillate) $c_s \approx c/\sqrt{3}$

$$k_J^2 = \frac{4\pi G_N \bar{\rho}(t) a(t)^2}{c_s(t)^2}$$

$$\bar{\rho} \sim a^{-4} \rightarrow (k_J\, a)^2 \sim \text{const.}$$

Hence, the Jeans length is comparable with the horizon and stays constant in time: no sub-horizon growth of perturbations!

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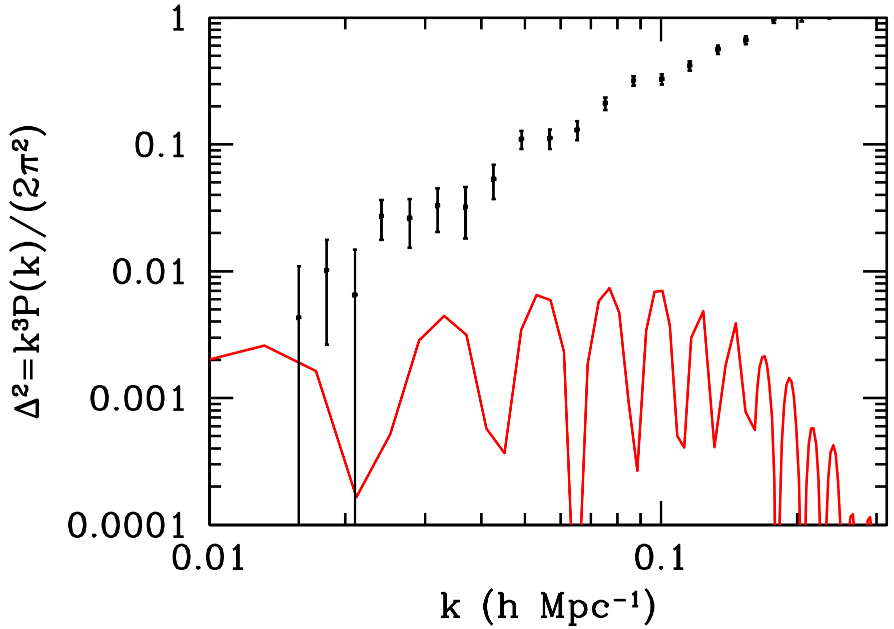
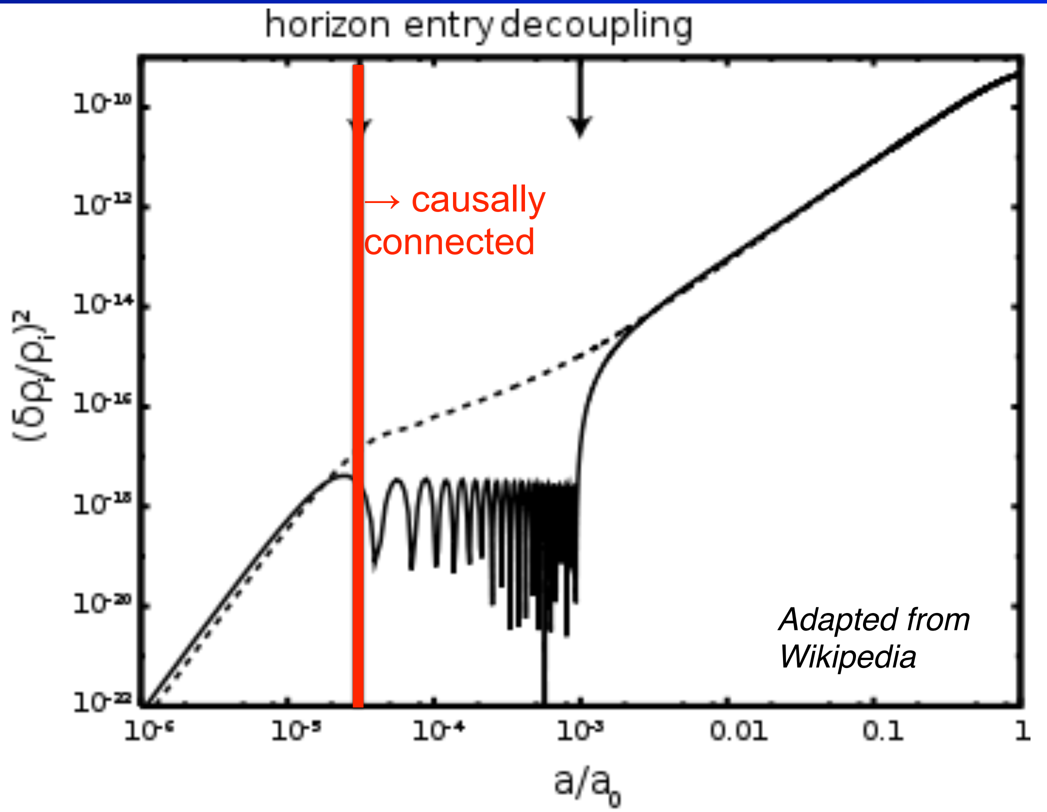
But is there enough time for the ~10^{-5} level perturbations we see in the CMB to grow, by now?

NO, by orders of magnitude, and even the k-shape does not match



Dark Matter to the rescue



CDM mode (dashed) and Baryonic (solid) mode growth

Ignore evolution before entering the (Hubble) horizon (gauge dependent).

Upon horizon entry (in radiation era) the baryonic mode is coupled to the baryon-radiation fluid, and oscillates as pressure prevents overdensities from collapsing below the Jeans Mass

The (pressureless) CDM mode grows logarithmically during radiation domination (by some orders of magnitude).

At matter-radiation equality the CDM mode can grow (enormous drop in the Jeans mass for baryons), but it receives a quick boost since it “falls” in the already much deeper gravitational potentials established by the CDM (from now until non-linear scale the two are identical)

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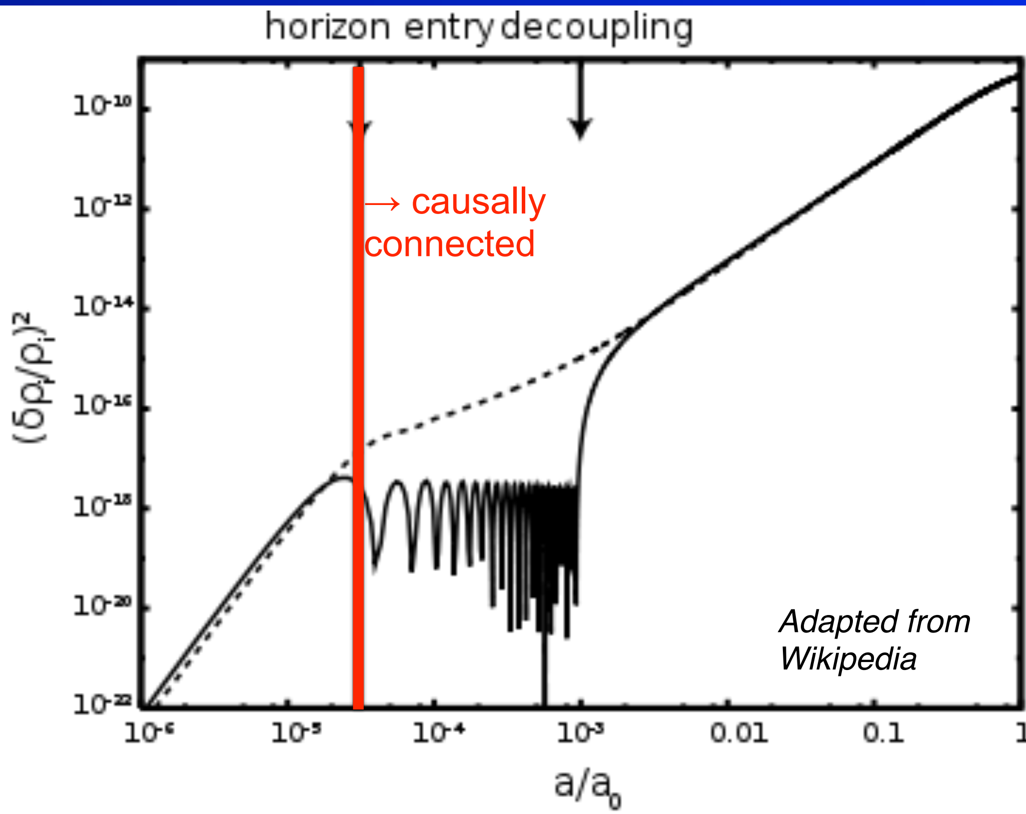

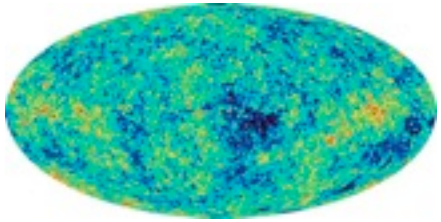

→ The smallness of fluctuations in the CMB tells that Dark Matter must be there!

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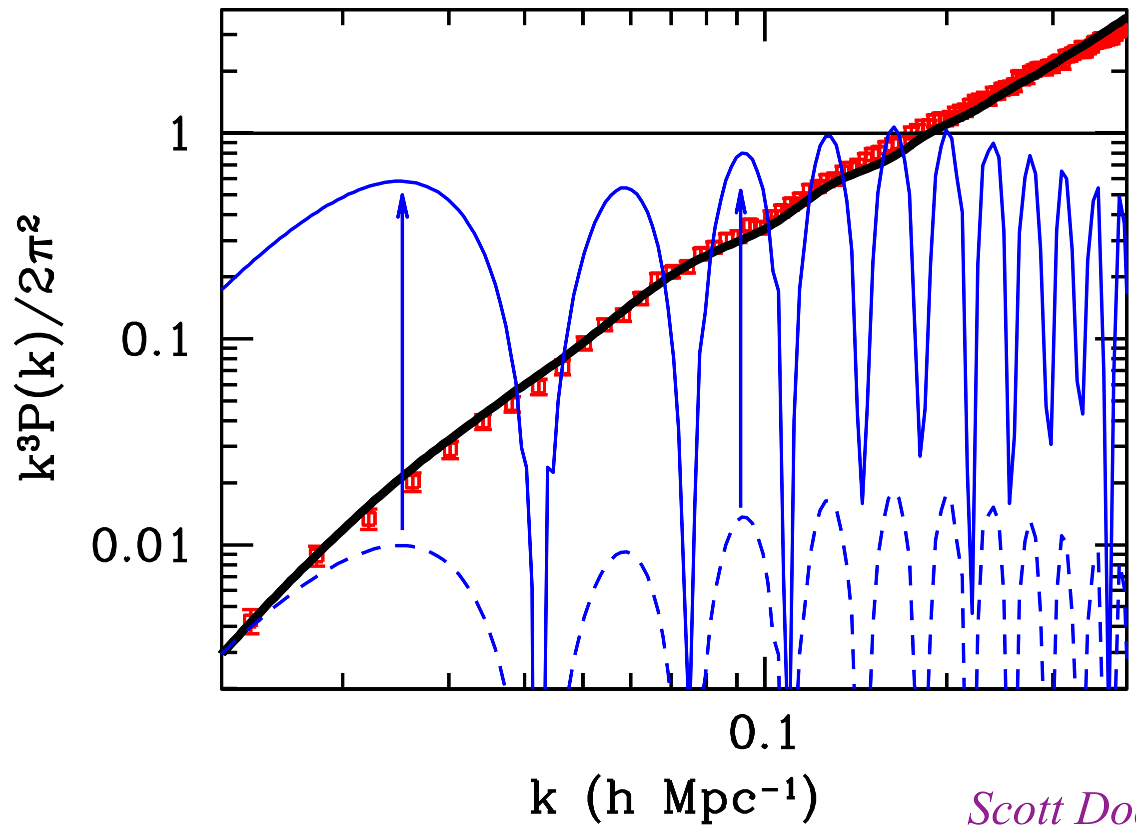
Dark Matter vs Baryons

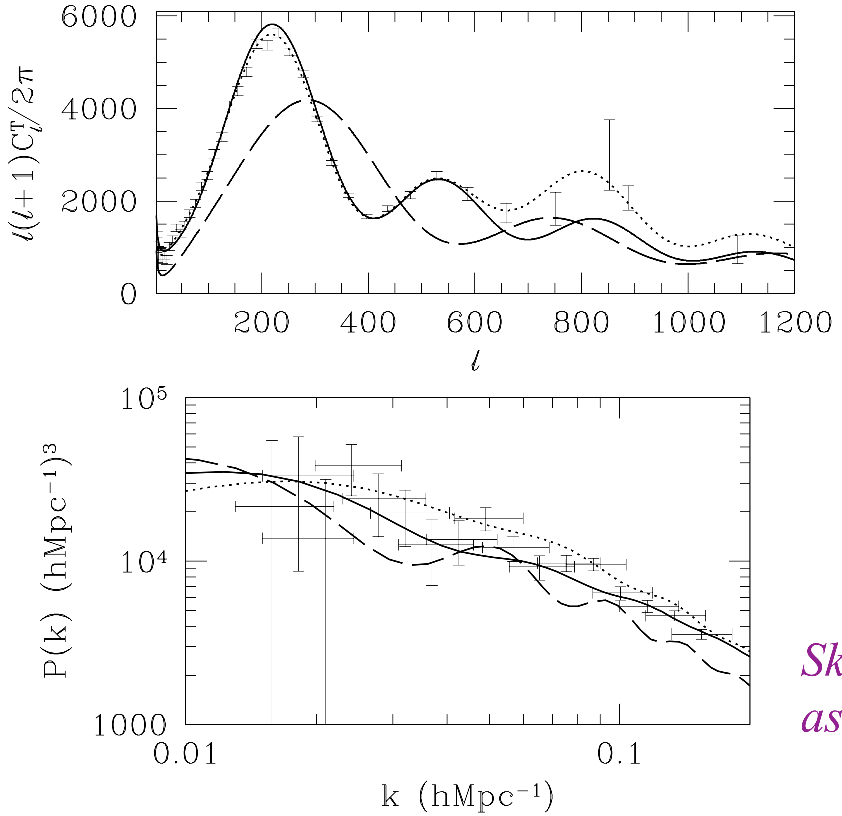


Scott Dodelson, arXiv:1112.1320

The power spectrum of matter. Red points with error bars are the data from the Sloan Digital Sky Survey; heavy **black curve is the ΛCDM model**, which assumes standard general relativity and contains **6 times more dark matter than ordinary baryons**. The dashed blue curve is a "No Dark Matter" model in which all matter consists of baryons (with density equal to 20% of the critical density), and the baryons and a cosmological constant combine to form a flat Universe with the critical density. This model predicts that inhomogenities on all scales are less than unity (horizontal black line), so the Universe never went nonlinear, and no structure could have formed. **TeVeS (solid blue curve) solves the no structure problem by modifying gravity to enhance the perturbations (amplitude enhancement shown by arrows).** While the amplitude can now exceed unity, the spectrum has pronounced Baryon Acoustic Oscillations, in violent disagreement with the data.

CMB also...

A few years ago, modified gravity models could still accommodate data (with large Ω_ν)



Skordis et al.
astro-ph/0505519

MOND universe (with $a_0 \simeq 4.2\times10^{-8}$cm/s^2) with $\Omega_\Lambda = 0.78$ and $\Omega_\nu = 0.17$ and $\Omega_b = 0.05$ (solid line), for a MOND universe $\Omega_\Lambda = 0.95$ and $\Omega_b = 0.05$ (dashed line) and for the ΛCDM model (dotted line).

CMB also...

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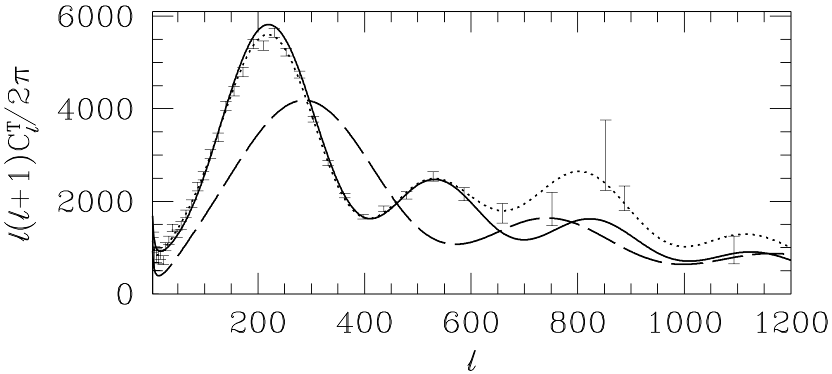
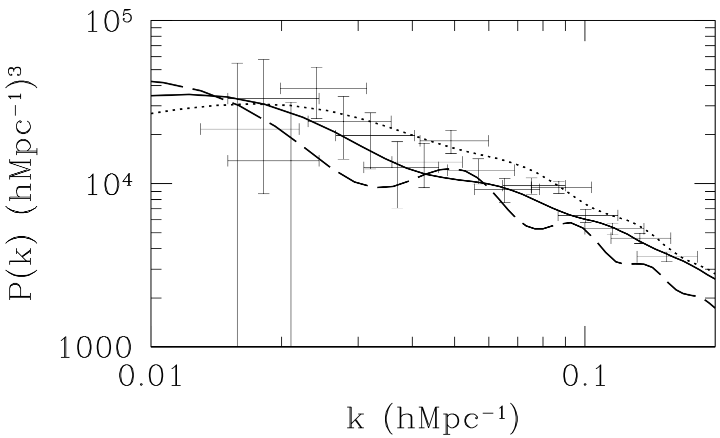
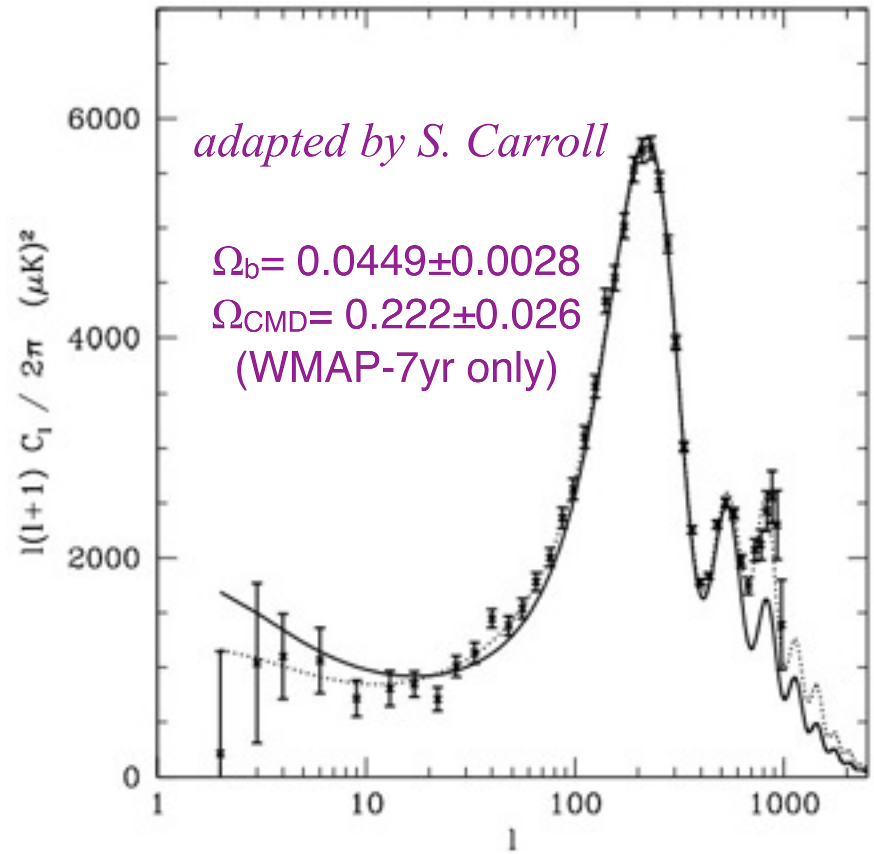
recent data inconsistent with these “old” proposals: e.g. CMB 3rd peak, baryon acoustic oscillations...







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Ω_b^{CMB} (from atomic physics) is also in agreement with Ω_b^{BBN}, sensitive to total number of nucleons in the plasma at T~0.01-1 MeV (nuclear physics)

Great success of cosmology!

Why cosmological evidence for DM is important

***I.* It is essentially based on exact solutions or linear perturbation theory applied to simple physical systems (gravity, atomic physics...): credible and robust!**

***II.* It suggests additional species, rather than a modification of gravity.**

***III.* Because it tells us that the largest fraction of required dark matter is non-baryonic, rather than brown dwarf stars, planets, etc.**

Only (even more radical) way out: modify cosmology to allow "collapsed" objects at very early times (e.g. primordial Black Holes, But very constrained or completely excluded, see F. Capela, M. Pshirkov and P. Tinyakov,arXiv:1209.6021 and refs. therein)

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But neutrinos (at least known ones) do not work!

This implies that Dark Matter requires "new physics", beyond the theories of the SM and/or gravity known today. Only a handful of similar indications, explains the interest of particle physicists!

Neutrinos as Dark Matter?

Condition 1. Must be massive (which is already a departure from SM...)

Fulfilled! Oscillations established, at least 2 massive states, measured splitting implies at least one state heavier than 0.05 eV

$$\Delta m^2_{\rm atm} \simeq 2.4 \times 10^{-3}\,{\rm eV}^2$$

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Condition 2. Must match cosmological abundance

Failed! Direct mass limits combined with splittings from oscillation experiments impose upper limit of about 7 eV to the sum (After KATRIN, potentially improved to ~0.7 eV)

$$\Omega_\nu = \frac{\rho_\nu}{\rho_c} \simeq \frac{\sum_i m_i}{45\,{\rm eV}}$$

$\Omega_{DM} \approx 0.3$(WMAP)$\Rightarrow \Sigma m_i \approx 15$ eV

we will perform this computation in lecture 2.

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Condition 3. Must allow for structure formation (of the right kind)

Failed! We will see shortly why it is so... which applies to more general classes of candidates.

Sometimes one hears: DM is a theory for a number

Current determination (Planck 2013, 68% CL)

$\Omega_c h^2 = 0.120 \pm 0.003$, i.e. $\Omega_c \sim 0.27$

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Since $$\rho_{X,0} = M_X\, n_{X,0} = M_X\, s_0\, Y_0$$

$$\rho_c = \frac{3H_0^2}{8\pi G_N} = 1.054 \times 10^{-5} h^2 \mathrm{GeV\,cm}^{-3}$$

$$s_0 = 2889 \left(\frac{T_{\gamma,0}}{2.725}\right)^3 \mathrm{cm}^{-3}$$

where $h_{eff} \sim 2+3\times2(4/11)\times7/8 \sim$ **3.91** comes from accounting for γ's & ν's

$$\Omega_X h^2 = 2.74 \times 10^8 \left(\frac{M_X}{\mathrm{GeV}}\right) Y_0$$

[Main] Goal: *compute value of number to entropy density ratio, Y_0*

In reality, must be sure that your DM candidate...

...also fulfills some basic requirements from astro/cosmo

- ***Dark matter... is dark, and dissipationless***
- ***Dark matter is collisionless (or not very collisional)***
- ***Dark matter is smoothly distributed (at astrophysical scales)***
- ***Dark matter behaves as a classical fluid at astrophysical scales***
- ***Dark matter is not "hot" (non-relativistic velocity distribution)***

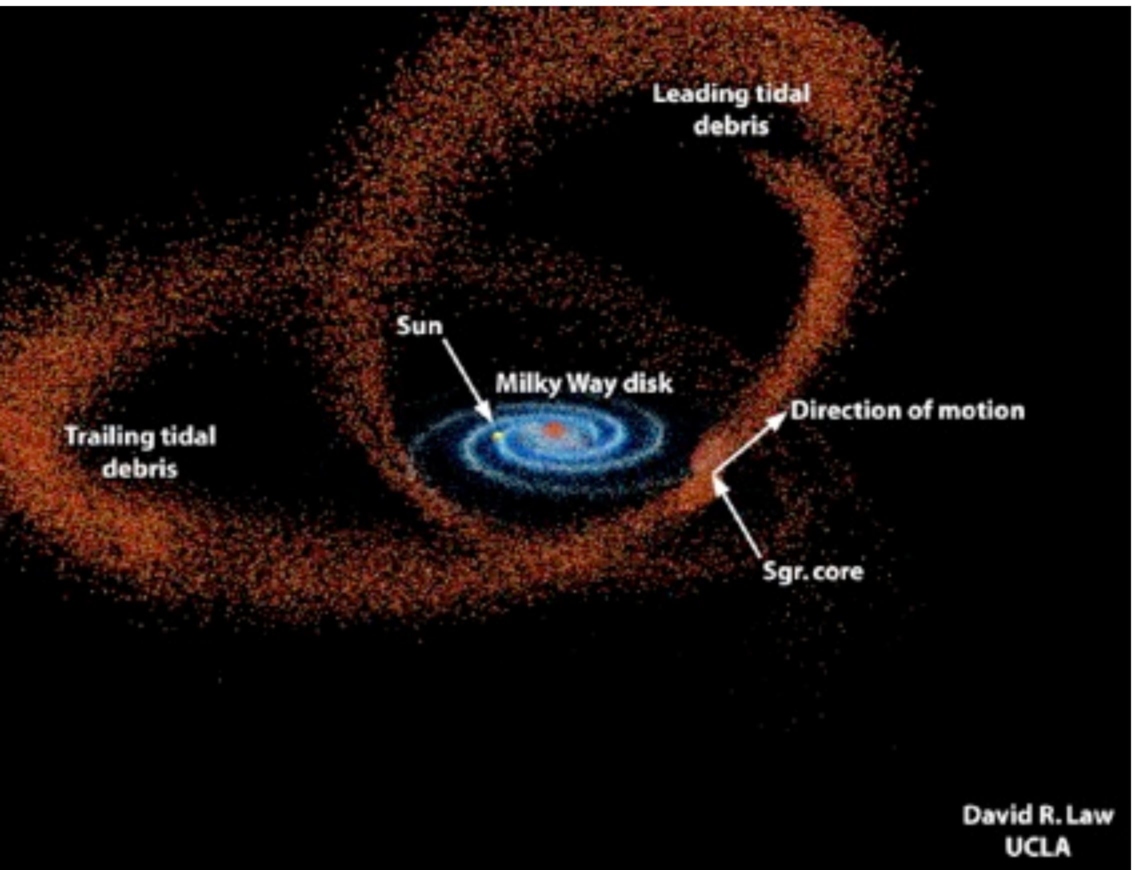
Let's detail them one by one: they have more or less stringent particle physics implications

Observationally-inferred properties of DM. I

Dark matter... is dark, and dissipationless

- DM must not couple "much" to photons (perturbation shape & amplitude argument, invisibility in e.m. channels...)

- *<u>DM forms extended, triaxial halos</u>*, while baryons "sink" in inner halo parts, form disk, etc. since they can dissipate energy by e.m. emission. At Galactic scale, evidence from tidal streams of satellite galaxies



D. R. Law, S. R. Majewski, K. V. Johnston, "Evidence for a Triaxial Milky Way Dark Matter Halo from the Sagittarius Stellar Tidal Stream"
Astrophys. J. 703, L67 (2009)

Observationally-inferred properties of DM. II

Dark matter is collisionless (or not very collisional)

- if DM-DM interaction too strong, spherical structures would be obtained rather than triaxial. From actual clusters, one can derive σ/m<0.02 cm²/g

*Jordi Miralda-Escudé ApJ **564** 60 (2002)*

- From Bullet cluster, σ/m<0.7-1.3 cm²/g,

S. W. Randall et al. ApJ 679, 1173 (2008)

- similar bounds from different arguments, for a compilation see e.g.

System	v_0 [km/s]	σ/m_χ [cm^2/g]	References
Bullet Cluster	1000	1.25	[41, 43]
Galactic Evaporation	1000	0.3	[45]
Elliptic Cluster	1000	0.02	[46]
Dwarf Evaporation	100	$0.1^\star$	[45]
Black Hole	100	$0.02^\star$	[59]
Mean Free Path	44 – 2400	0.01 – 0.6	[57]
Dwarf Galaxies	10	0.1	[56]

From M. R. Buckley and P. J. Fox, Phys. Rev. D 81, 083522 (2010) (=v-dependent)*

- Very loose from particle physics standard (barn level!), but much less than atomic or molecular cross sections characteristic of gas.

$$\frac{\text{cm}^2}{\text{g}} = 1.78\,\frac{\text{barn}}{\text{GeV}}$$

Observationally-inferred properties of DM. III

At least at astrophysical scales, dark matter has a "continuum" (fluid limit), rather than having discrete/granular structure.

✤ Granular distribution would provide time-dependent gravitational potentials, distrupting bound systems of different sizes (function of "grain mass")

- thickness of disks: $M_X < 10^6\ M_{sun}$
  satellites, globular clusters: $M_X < 10^3\ M_{sun}$

H-W.Rix and G. Lake, astro-ph/9308022 & refs. therein

- Halo-wide binaries: $M_X < 43\ M_{sun}$

J. Yoo, J. Chaname and A. Gould, Astrophys. J. 601, 311 (2004)

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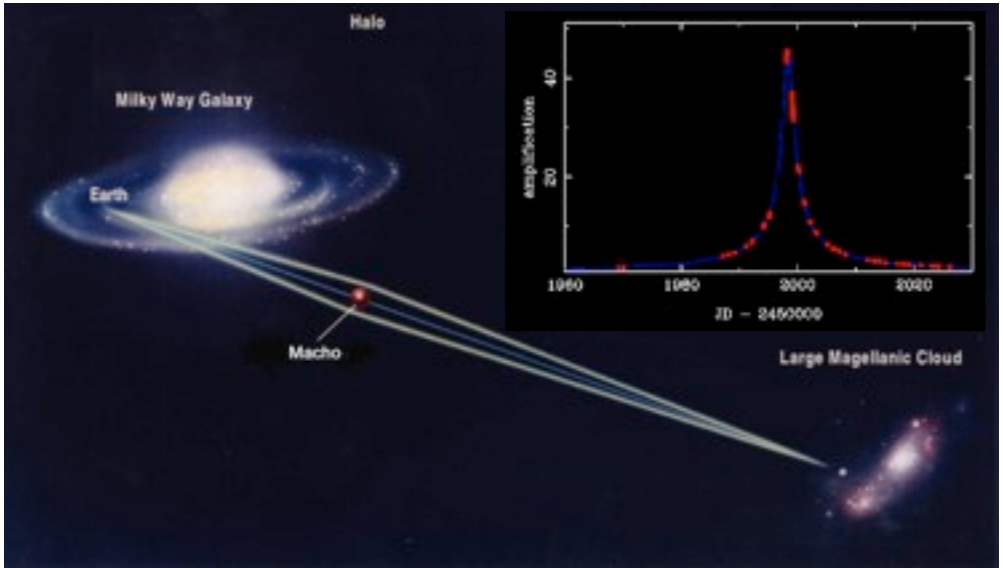
✤Several searches (EROS, OGLE...) for μlensing events towards Magellanic Cloud exclude dominant MACHOs component as halo DM for 10^{-7} to 10 M_{sun}

e.g. L. Wyrzykowski et al., arXiv:1106.2925 & refs. therein



idea: constrain the frequency of a peculiar magnification pattern

$$A(u) = \frac{u^2+2}{u\sqrt{u^2+4}}$$

$u = \theta/\theta_E$ ang. distance source-lens

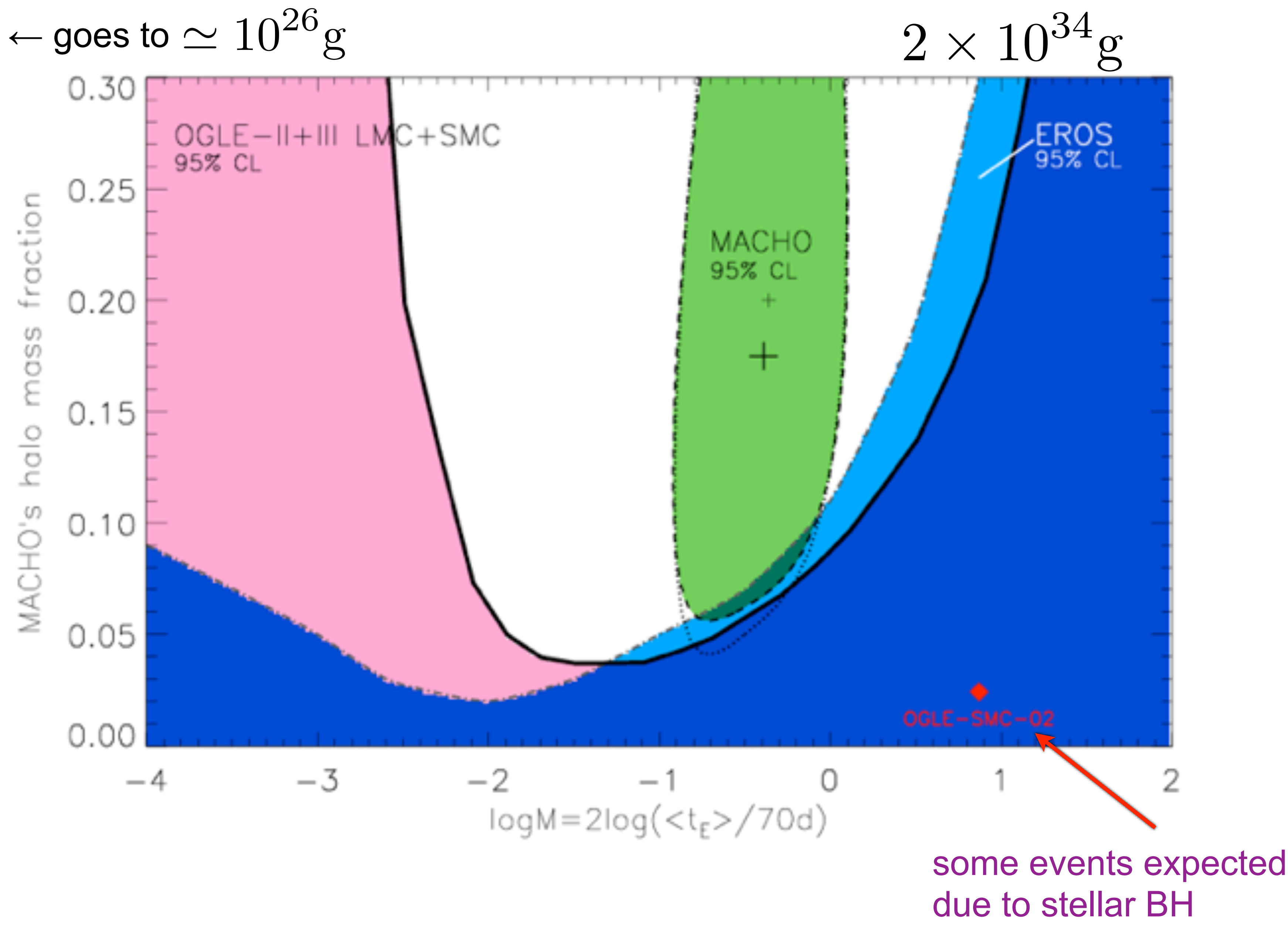
$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{d_S - d_L}{d_S d_L}}$$

depends on lens mass and Geometry

$$u(t) = \sqrt{u_{min}^2 + \left(\frac{t-t_0}{t_E}\right)^2}$$

t_E =time to cross einstein angular size

Microlensing Constraints



Observationally-inferred properties of DM. IV

dark matter is confined/detected at least at astrophysical scales, hence must be "localized" and behave classically there.

$$\lambda_{De\,Broglie} = \frac{h}{m\,v} \lesssim \mathrm{kpc} \Longrightarrow m \gtrsim 10^{-22}\,\mathrm{eV}\;(v \simeq 100\,\mathrm{km/s})$$

Observationally-inferred properties of DM. IV

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$$\lambda_{De\,Broglie} = \frac{h}{m\,v} \lesssim \mathrm{kpc} \Longrightarrow m \gtrsim 10^{-22}\,\mathrm{eV}\;(v \simeq 100\,\mathrm{km/s})$$

For *fermions* a much stronger bound holds, due to the fact that their quantum nature emerges more easily, so to speak, thanks to Pauli principle/Fermi-Dirac statistics

$$f \leq \frac{g}{h^3}$$

From the conservation of phase space density of a non-interacting fluid (Liouville Eq.) and from the condition that any observable, coarse grained p.s. density must be lower than the real one, in turn lower than the above maximum, one derives

$$m > \mathcal{O}(10 - 100)\,\mathrm{eV}$$

S. Tremaine and J. E. Gunn, Phys. Rev. Lett. 42, 407 (1979)

updated lower limit around ~400 eV

A. Boyarsky, O. Ruchayskiy and D. Iakubovskyi, JCAP 0903, 005 (2009)

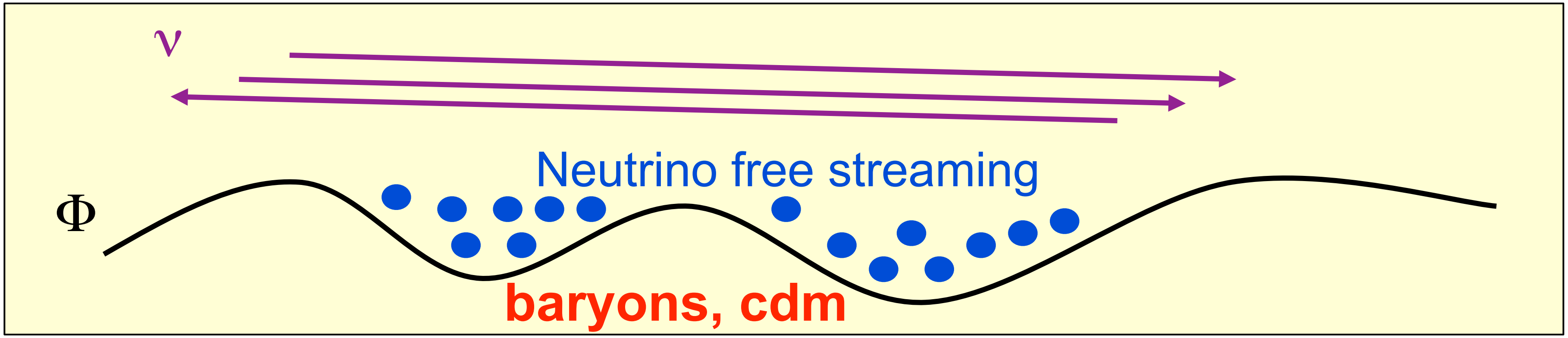
Observationally-inferred properties of DM. V

dark matter is not "hot": cannot have a relativistic velocity distribution (at least from matter-radiation equality for perturbation to grow)

Observationally-inferred properties of DM. V

dark matter is not "hot": cannot have a relativistic velocity distribution (at least from matter-radiation equality for perturbation to grow)

This is the more profound reason why neutrinos would not work as DM, even if they had the correct mass: they were born with relativistic velocity distribution which prevents structures below O(100 Mpc) to grow till late!

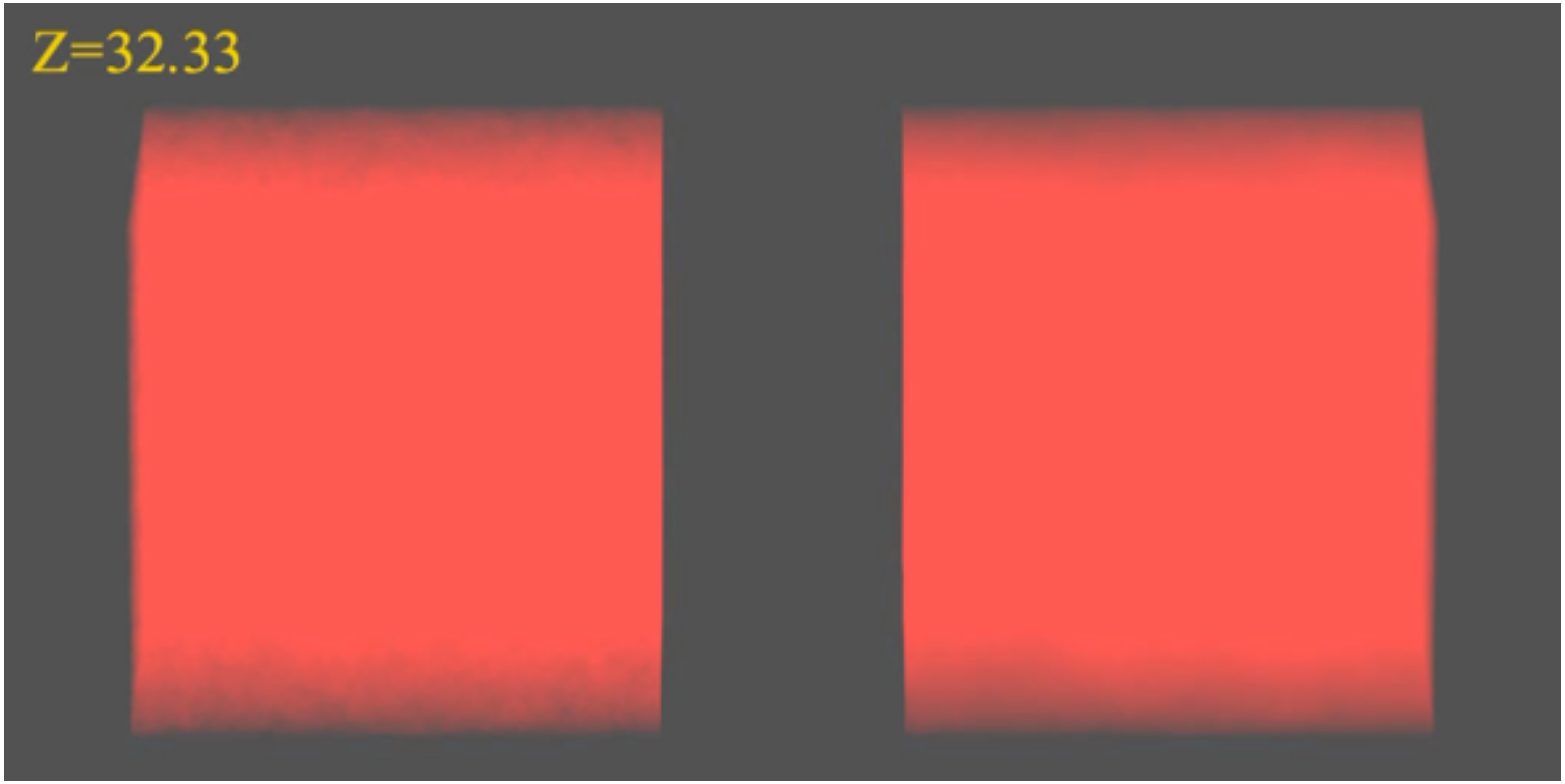


Cartoon Picture:

ν's "do not settle" in potential wells that they can overcome by their typical velocity: compared with CDM, they suppress power at small-scales

Observationally-inferred properties of DM. V

ΛCDM run vs. cosmology including neutrinos (total mass of 6.9 eV)



simulation by Troels Haugbølle, see

http://users-phys.au.dk/haugboel/projects.shtml

Summary of what we learned

✤ Apart for unavoidable simplifications, that's about **all nature tells us** of "generic" about Dark Matter.

✤ On one side, it's a lot: **we need new physics**, with some specific properties. Justifies the enormous amount of attention particle physicists devote to it!

✤ On the other side, *<u>it does not tell us what kind of physics it is</u>*. Notice that I never mention TeV or electroweak scale, nor "WIMPs": these aspects are theoretical creativity... but also prejudice.

✤ We shall see a glimpse of how widely different scales and production mechanisms can be envisaged.

✤ Of course, no matter how wild our speculation is, at the end we must compare with nature again for validating it (or, at very least, to constrain model-dependent free parameters). Mostly outside of my lectures, though...