Quantum quenches, linear response and superfluidity out of equilibrium

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Dipartimento di Fisica, Università di Pisa

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Outlook

Long-time behavior of isolated quantum many-body systems

Can asymptotic behavior of certain observables be predicted according to quantum statistical mechanics ?

Are there underlying dynamical rules admitting this possibility ?

Thermalization

A long-standing and fascinating problem (since Von Neumann, 1929)

Yet mostly an academic question, up to the 21st century…

Outlook

Experimental breakthroughs

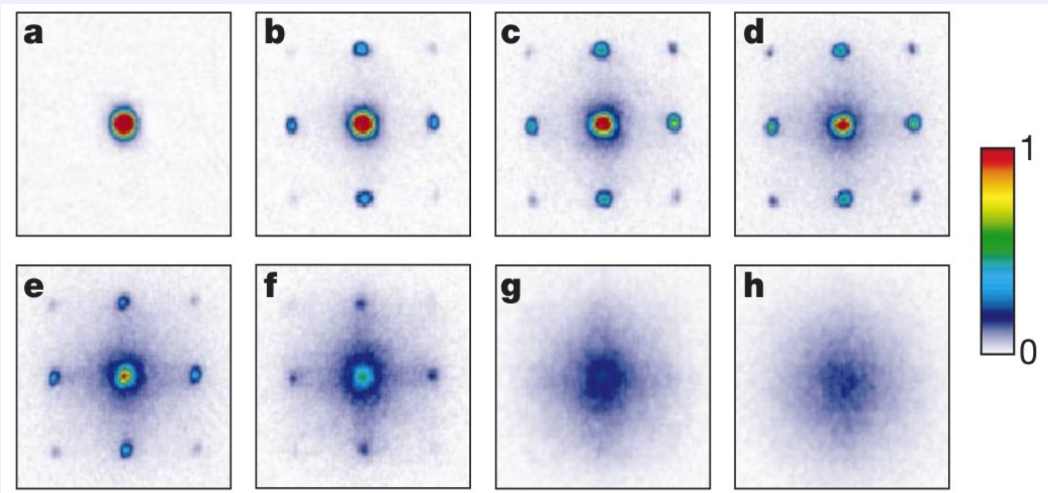
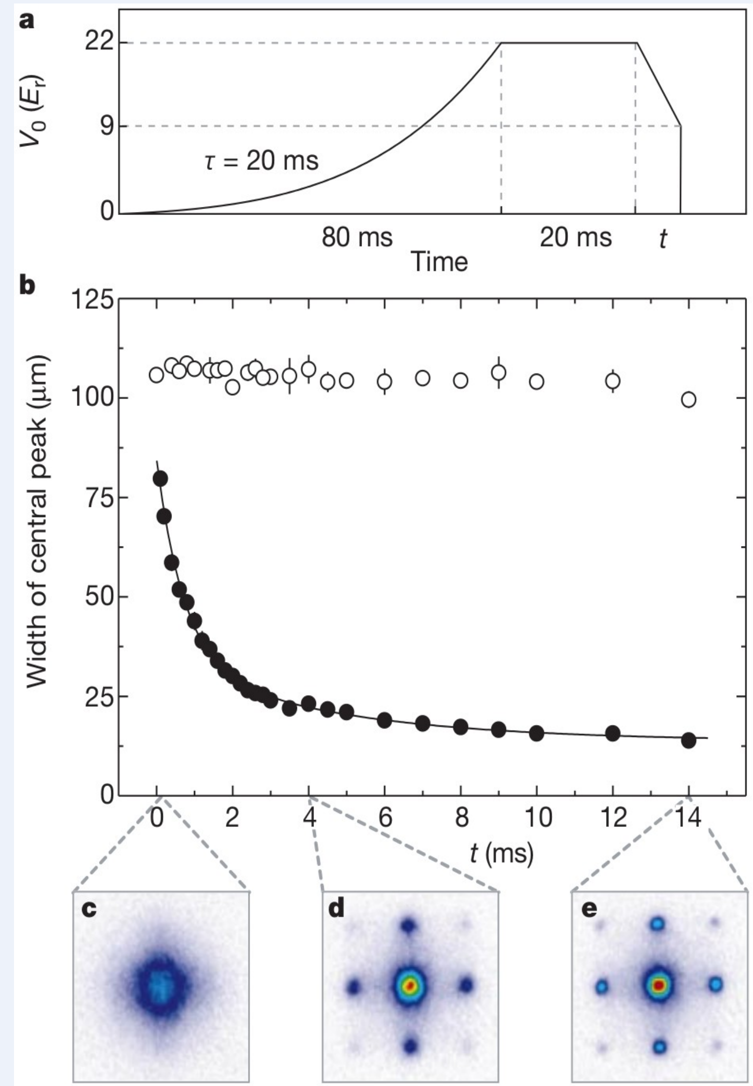


Figure 2 Absorption images of multiple matter wave interference patterns. These were obtained after suddenly releasing the atoms from an optical lattice potential with different potential depths V_0 after a time of flight of 15 ms. Values of V_0 were: **a**, 0 E_r; **b**, 3 E_r; **c**, 7 E_r; **d**, 10 E_r; **e**, 13 E_r; **f**, 14 E_r; **g**, 16 E_r; and **h**, 20 E_r.

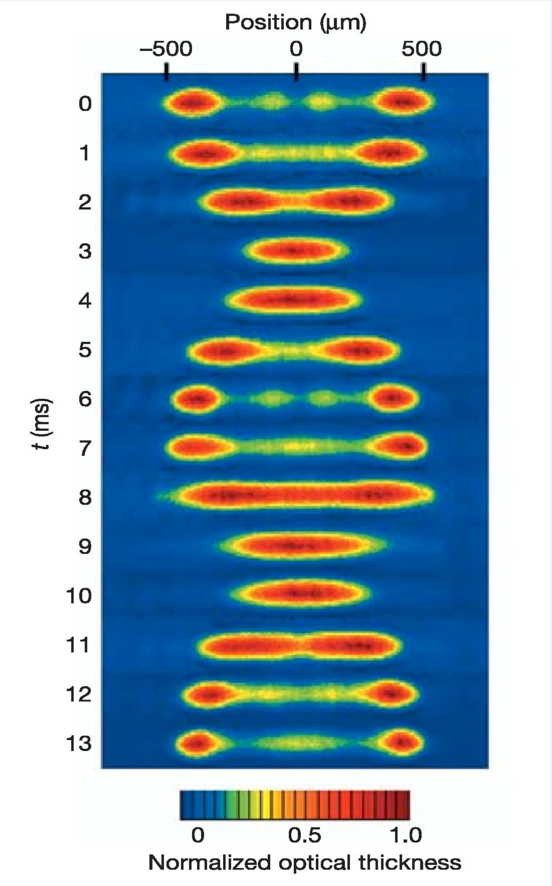


Greiner, Mandel, Esslinger, Hänsch, Bloch (Nature 2002)

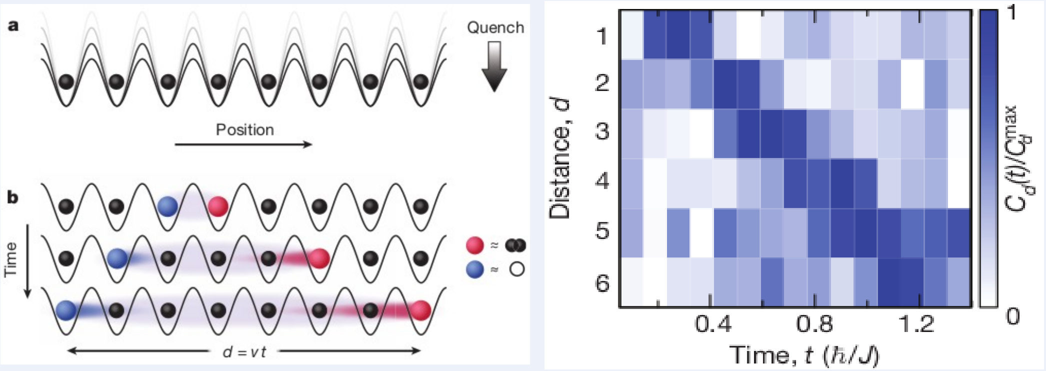
Bloch, Dalibard, Zwerger, Rev. Mod. Phys. **80**, 885 (2008)

Outlook

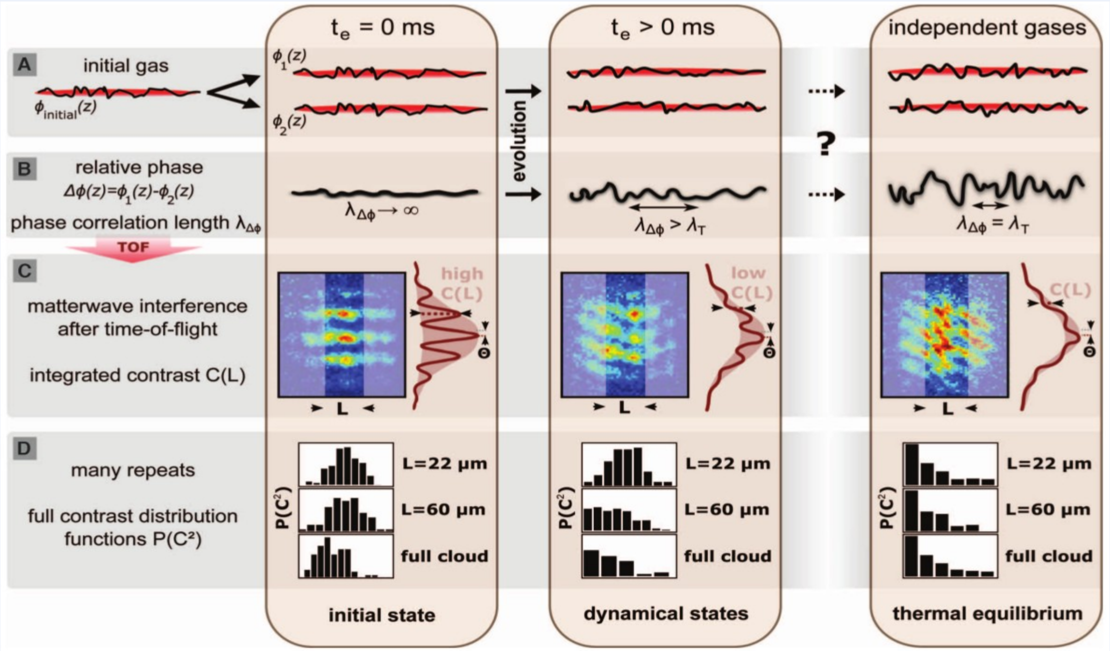
Experimental breakthroughs



Kinoshita, Wenger, Weiss
(Nature, 2006)



Cheneau et al. [*I. Bloch group*] (Nature, 2012)



Gring et al. [*J. Schmiedmayer group*] (Science, 2012)

Long-time behavior of isolated quantum many-body systems

Out-of-equilibrium dynamics:
(quantum quench protocol)

$$|\psi(t)\rangle = e^{-i\hat{\mathcal{H}}(V_f)\,t}\,|\psi_0\rangle$$

$|\psi_0\rangle$ ground state of Hamiltonian $\hat{\mathcal{H}}(V_i)$

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- Non-integrable systems: usually thermalize (ETH)
- Quasi-integrable systems (pre-thermalization)
- Integrable systems: usually do not thermalize (GGE)

Polkovnikov, Sengupta, Silva, Vengalattore, RMP **83**, 863 (2011)

Dziarmaga, Adv. Phys. **59**, 1063 (2010)

Lamacraft, Moore, arXiv:1106.3567 in "Ultracold bosonic and Fermionic gases", Elsevier (2012)

$$\hat{\rho}_{\mathrm{therm}} = \frac{e^{-\beta^\star \hat{\mathcal{H}}}}{\mathcal{Z}}$$

canonical ensemble (thermal)

Deutsch (1991), *Srednicki* (1994)
Rigol, Dunjko, Olshanii (2008)

non-integrable systems

vs.

$$\hat{\rho}_{\mathrm{GGE}} = \frac{e^{\sum_j \lambda_j \hat{I}_j}}{\mathcal{Z}}$$

generalized Gibbs ensemble

Rigol, Dunjko, Yurovsky, Olshanii (2007)

integrable systems

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integrable systems

Scenario widely scrutinized:

- Stationary states
- Local observables

$$\overline{\langle\psi(t)|\,\hat{A}\,|\psi(t)\rangle} \approx \sum_n |\langle\varphi_n|\psi_0\rangle|^2 \cdot \langle\varphi_n|\,\hat{A}\,|\varphi_n\rangle$$

p_n

diagonal ensemble

$$\hat{\rho} = \sum_n p_n\,|\varphi_n\rangle\,\langle\varphi_n|$$

In frequent cases **thermal ≈ GGE**

See, e.g.,
Rossini, Silva, Mussardo, Santoro (PRL, 2009) – *Calabrese, Essler, Fagotti* (PRL, 2011)

BUT a different point of view can be adopted:

→ Linear response out of equilibrium

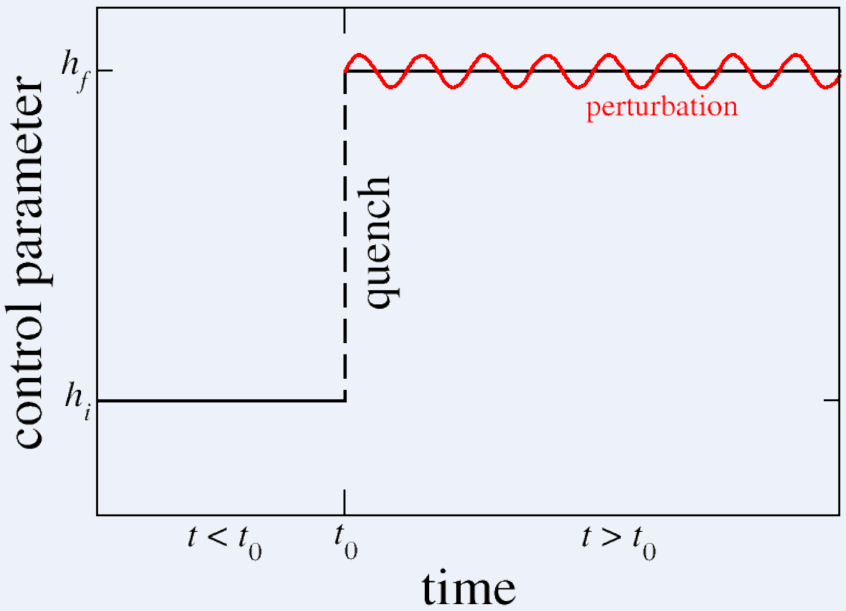
Physical characterization of steady states after relaxation

→ Application to superfluidity

Generalization of ***phase stiffness*** and ***helicity modulus*** out of equilibrium

Linear response out of equilibrium

$$\hat{\mathcal{H}}_i \to \hat{\mathcal{H}}_f + \hat{V}(t)$$



Linear response out of equilibrium

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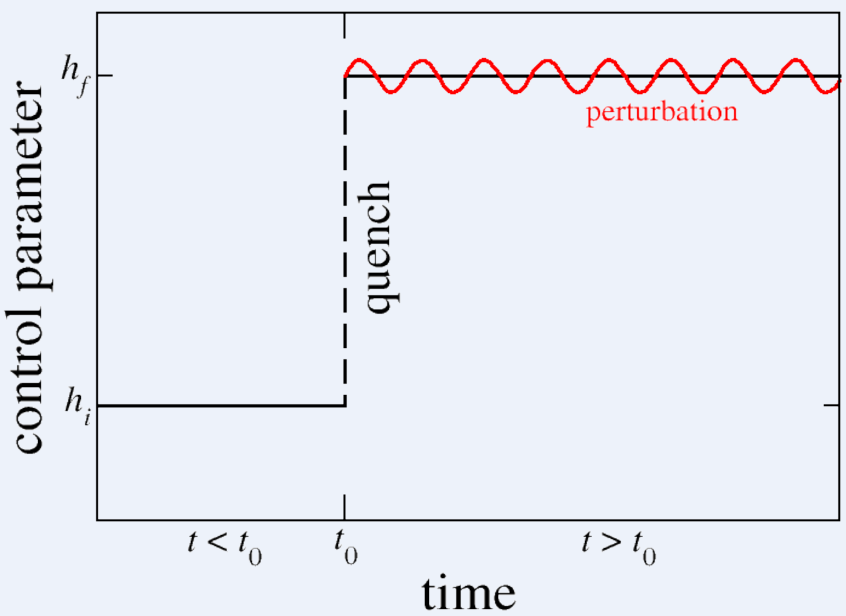


$$\langle \hat{A}(t) \rangle = \mathrm{Tr}[\hat{\rho}(t)\hat{A}]$$

$$i\partial_t \hat{\rho}(t) = [\hat{\mathcal{H}}_f, \hat{\rho}(t)] + [\hat{V}(t), \hat{\rho}(t)]$$

equation of motion

usual quench evolution (stationary states)

effect of perturbation

$$\hat{\rho}(t) = \hat{\rho}_0(t) + \delta\hat{\rho}(t)$$

$$\hat{\rho}_0(t) = e^{-i\hat{\mathcal{H}}_f t}\hat{\rho}_0(0)e^{i\hat{\mathcal{H}}_f t}$$

Linear response out of equilibrium

$$\hat{V}(t) = h(t)\,\hat{B}$$

response function

$$\langle \hat{A}(t) \rangle = \langle \hat{A}(t) \rangle_0 + \int_0^t \mathrm{d}t'\,\chi(t,t')\,h(t')$$

$$\boxed{\chi(t,t') = -i\,\theta(t-t')\,\big\langle [\hat{A}(t), \hat{B}(t')] \big\rangle_0}$$

$\langle\,\cdot\,\rangle_0 = \mathrm{Tr}[\hat{\rho}(0)\,\cdot\,]$ unperturbed expectation value

Linear response out of equilibrium

… usual linear response: $\chi(t,t') \equiv \chi(t-t') \longrightarrow \chi(\omega)$ spectrum (peaks, singularities...)

… here, in non-equilibrium: $\chi(t,t')$ depends on two separate times

so use Wigner coordinates $T = (t+t')/2, \quad \tau = t - t'$

and average over T: $\overline{\chi(T,\tau)} = \lim_{T\to\infty} \frac{1}{T}\int_0^T \chi(t,t')\mathrm{d}t$

$$\overline{\chi(\tau)} = -i\,\theta(\tau)\sum_n p_n \langle\psi_n|\left[\hat{A}(\tau/2), \hat{B}(-\tau/2)\right]|\psi_n\rangle$$

average over the diagonal ensemble

$\overline{\sigma(\omega)}$

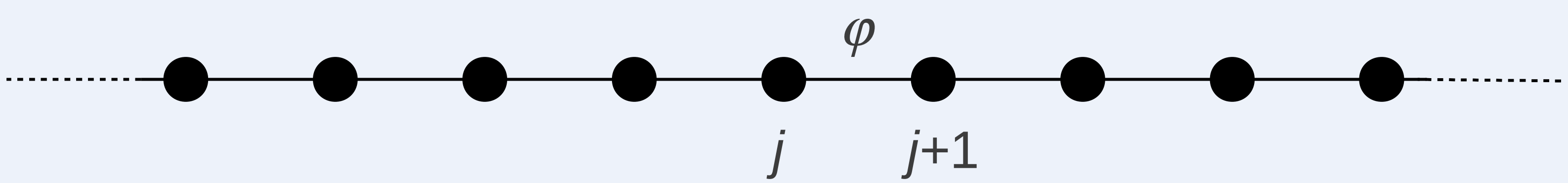
Relaxation towards stationary state

A specific example:

Out-of-equilibrium superfluidity

Model: 1D lattice bosonic Hamiltonian

$$\hat{\mathcal{H}}(\varphi) = -\sum_j \left(e^{i\varphi}\hat{b}^\dagger_{j+1}\hat{b}_j + \mathrm{H.c.}\right) + \hat{\mathcal{V}}_{\mathrm{int}}$$



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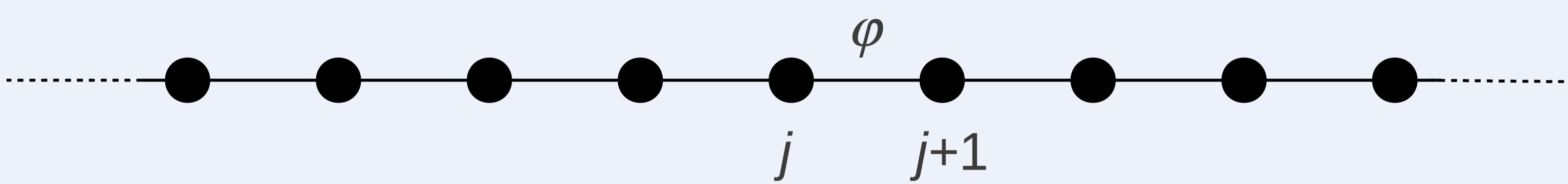

$$\hat{\mathcal{J}} = -\partial_\varphi \hat{\mathcal{H}} = i\sum_j \left(e^{i\varphi}\hat{b}^\dagger_{j+1}\hat{b}_j - \mathrm{H.c.}\right)$$ *current operator*

Now perform linear response:

$$\hat{\mathcal{H}}_i \;\downarrow\; \hat{\mathcal{H}}_f(\varphi) \simeq \hat{\mathcal{H}}_f - \varphi\hat{\mathcal{J}}$$

$$\langle\hat{\mathcal{J}}\rangle(t) = \int_0^{+\infty} \chi(t,t')\Phi(t')\mathrm{d}t'$$

$\chi(t,t') \rightarrow \sigma(t,t')$

$\Phi(t') \rightarrow \Phi(t) \sim A(t) = -\frac{i}{\omega}E(t)$

- Compute the conductivity $\sigma(t,t')$ [current-current correlator]
- Perform the time average $\overline{\sigma(\tau)}$
- Take the Fourier transform and get $\sigma_{avg}(\omega)$

$$\mathcal{D} = \tfrac{1}{2}[\omega\, \sigma''(\omega)]_{\omega\to 0}$$

Kohn (1964)

Drude peak, *i.e.*, stiffness: $\mathcal{D} = \frac{1}{L}\sum_n p_n \partial_\varphi^2 \varepsilon_{n|_{\varphi=0}}$

At equilibrium: $p_n \propto e^{-\beta\varepsilon_n}$ (Boltzmann weights)

After a quench: $p_n = |\langle\varphi_n|\psi_0\rangle|^2$ (diagonal ensemble)

Model I

$$\hat{\mathcal{V}}_{\text{int}} = V \sum_j (-1)^j \hat{b}_j^\dagger \hat{b}_j$$

$V = 0$ superfluid

$V \neq 0$ insulator

Free-fermionic problem,
exactly solvable

Klich, Lannert, Refael (PRL, 2007)

Model I $\hat{\mathcal{V}}_{\rm int} = V \sum_j (-1)^j \hat{b}_j^\dagger \hat{b}_j$

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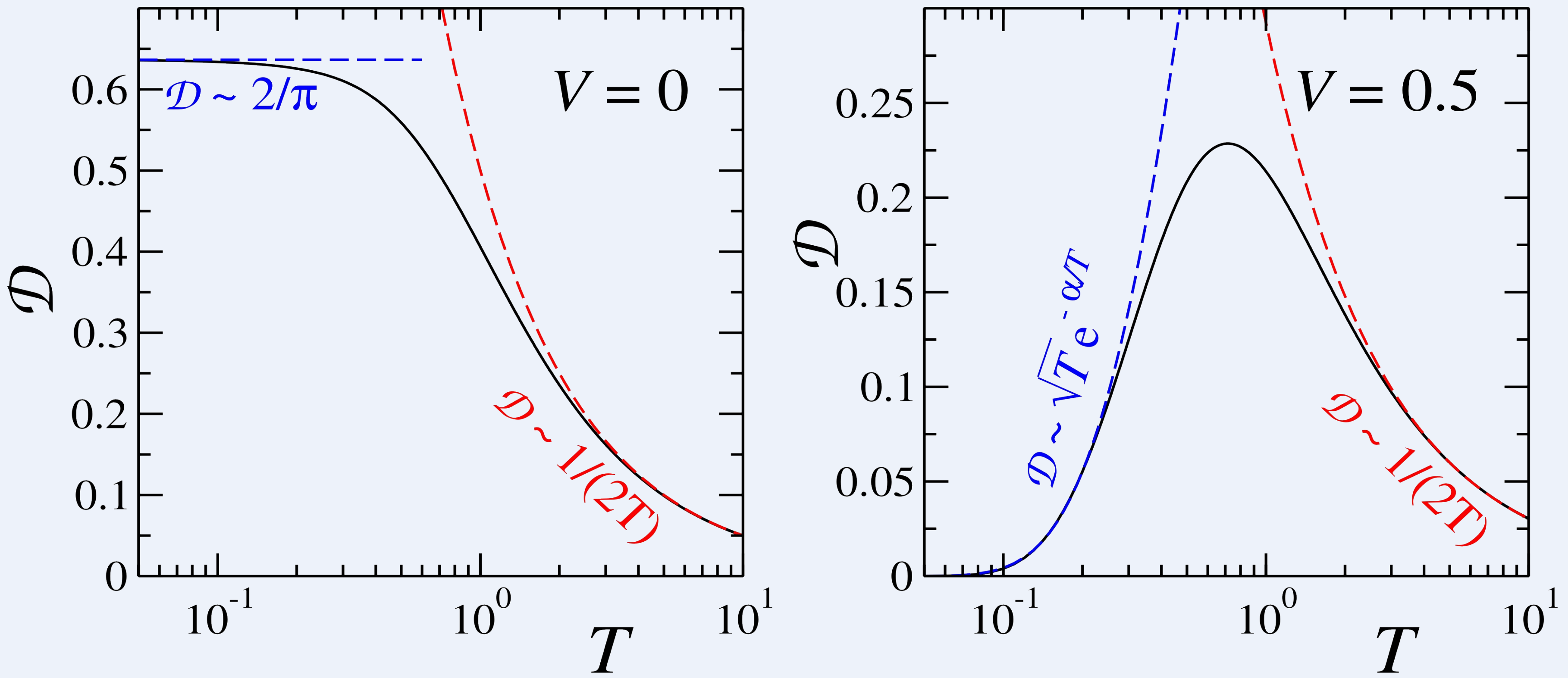
Equilibrium:



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$V = 0$ superfluid

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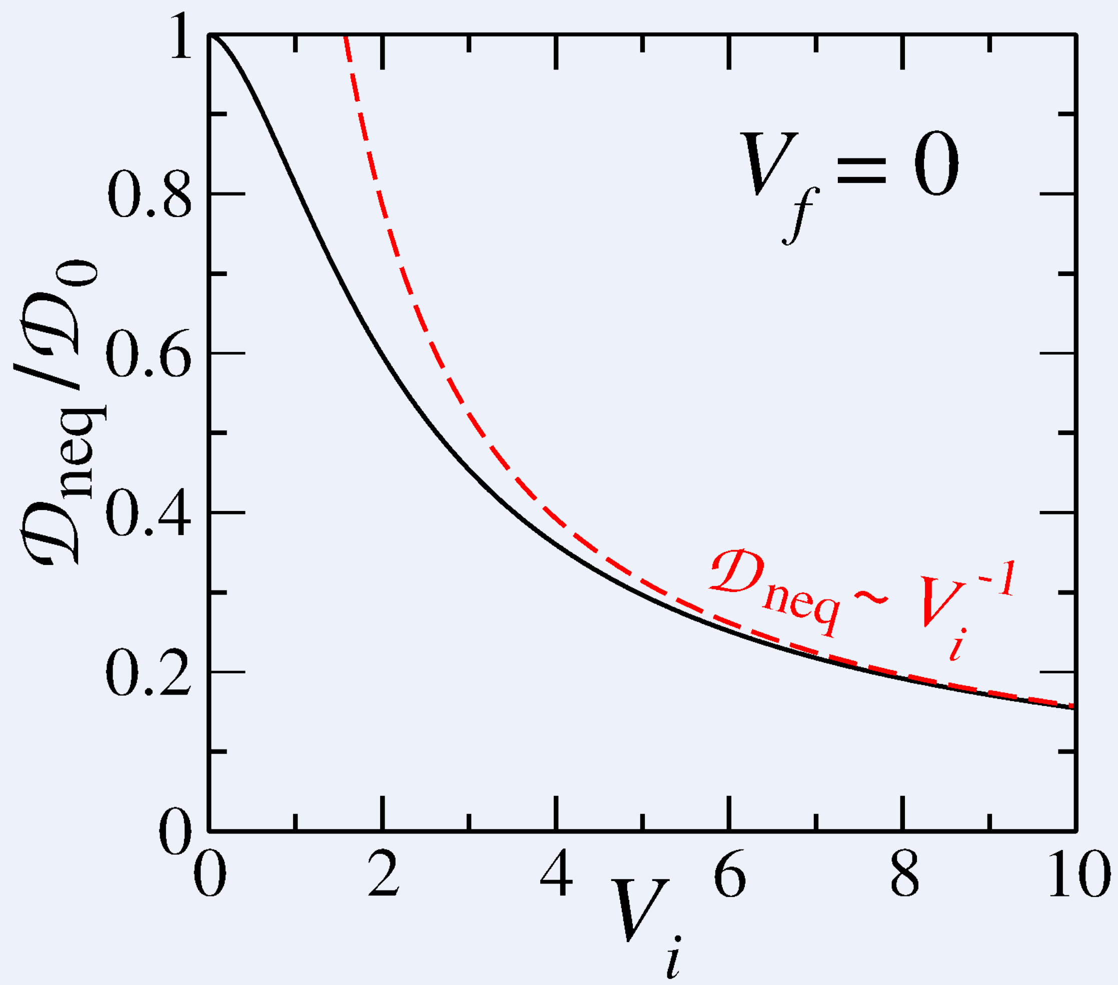
Quench:

Insulator → Superfluid

- Smooth approach to the equilibrium value



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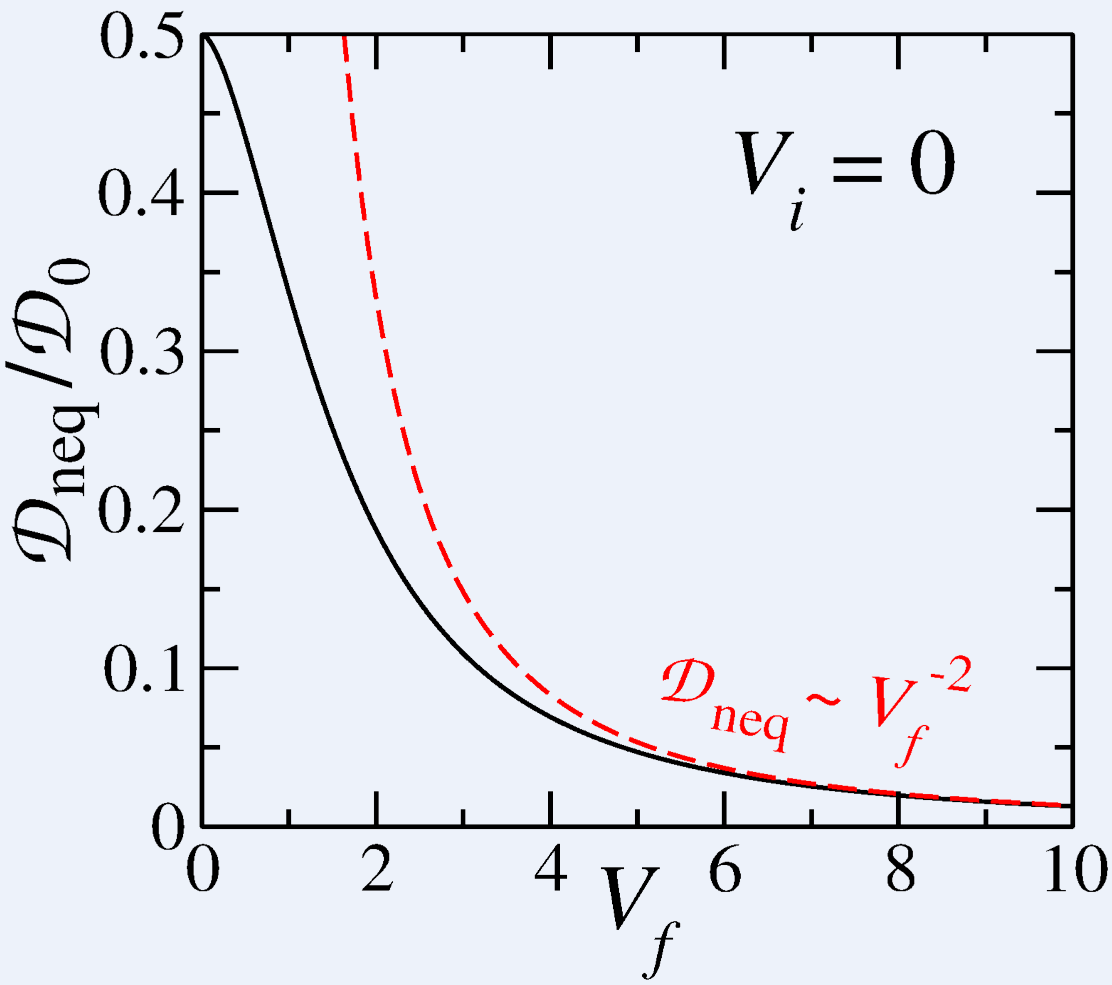
Quench:

Superfluid → Insulator

- The stiffness is NOT zero! (excitations created after the quench do not relax)



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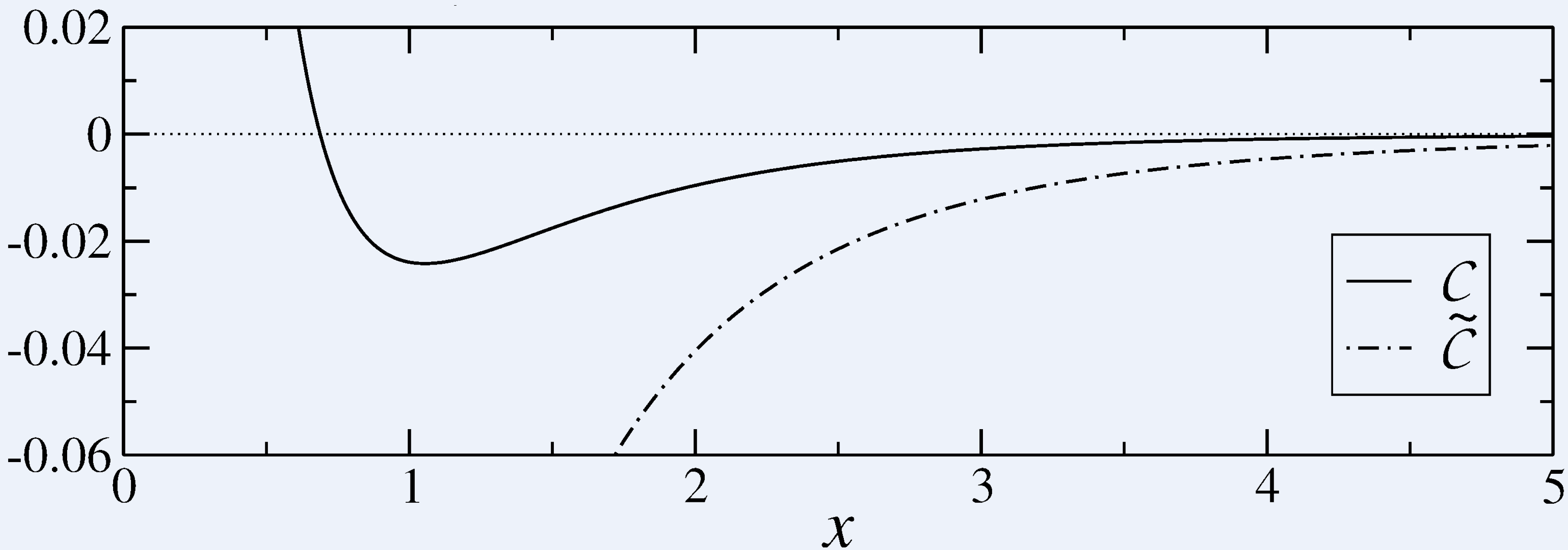
Quench: ***Insulator → Insulator***

For small quenches:

$$\mathcal{D}_{\rm neq} \approx \mathcal{D}_0 \, \delta V^2 \, \mathcal{C}(V_f)$$

- The stiffness is NOT zero!
- The stiffness can be negative!



Model II

$$\hat{\mathcal{V}}_{\text{int}} = V \sum_j \hat{n}_j \hat{n}_{j+1}$$

Interacting bosons,
Bethe-Ansatz solvable

XXZ spin-1/2 Heisenberg chain

$V \le 2$ superfluid

$V > 2$ insulator

Model II $\hat{\mathcal{V}}_{\rm int} = V \sum_j \hat{n}_j \hat{n}_{j+1}$

Interacting bosons,
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$V \leq 2$ superfluid

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Equilibrium: $\mathcal{D} = \pi \frac{\sin(\mu)}{\mu(\pi - \mu)}, \quad \mu = \cos^{-1} V$ (at $T = 0$)

Shastry, Sutherland (1990)

at finite-T – in general, broadening of the Drude peak

Castella, Zotos, Prelovsek (1995)
Prosen (2011)
Karrasch, Bardarson, Moore (2012)
Karrasch, Hauschild, Langer, Heidrich-Meisner (2013)
…

Model II $\hat{\mathcal{V}}_{\text{int}} = V \sum_j \hat{n}_j \hat{n}_{j+1}$

Interacting bosons,
Bethe-Ansatz solvable

XXZ spin-1/2 Heisenberg chain

$V \leq 2$ superfluid

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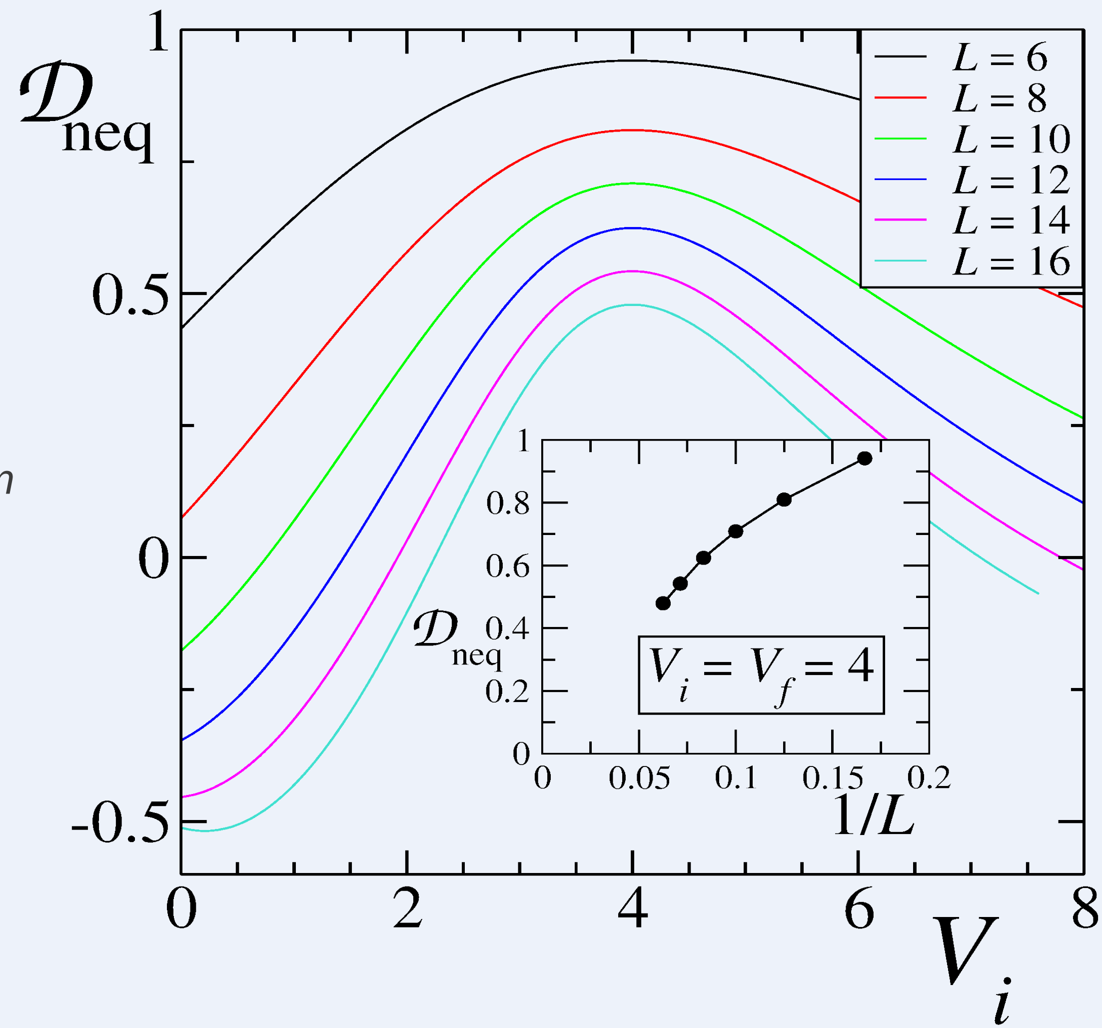
Quench: $(V_f = 4)$

Towards the insulator

- Finite-size scaling *at equilibrium* predicts *zero* stiffness
- The stiffness can be negative!



Model II $\hat{\mathcal{V}}_{\text{int}} = V \sum_j \hat{n}_j \hat{n}_{j+1}$

Interacting bosons,
Bethe-Ansatz solvable

XXZ spin-1/2 Heisenberg chain

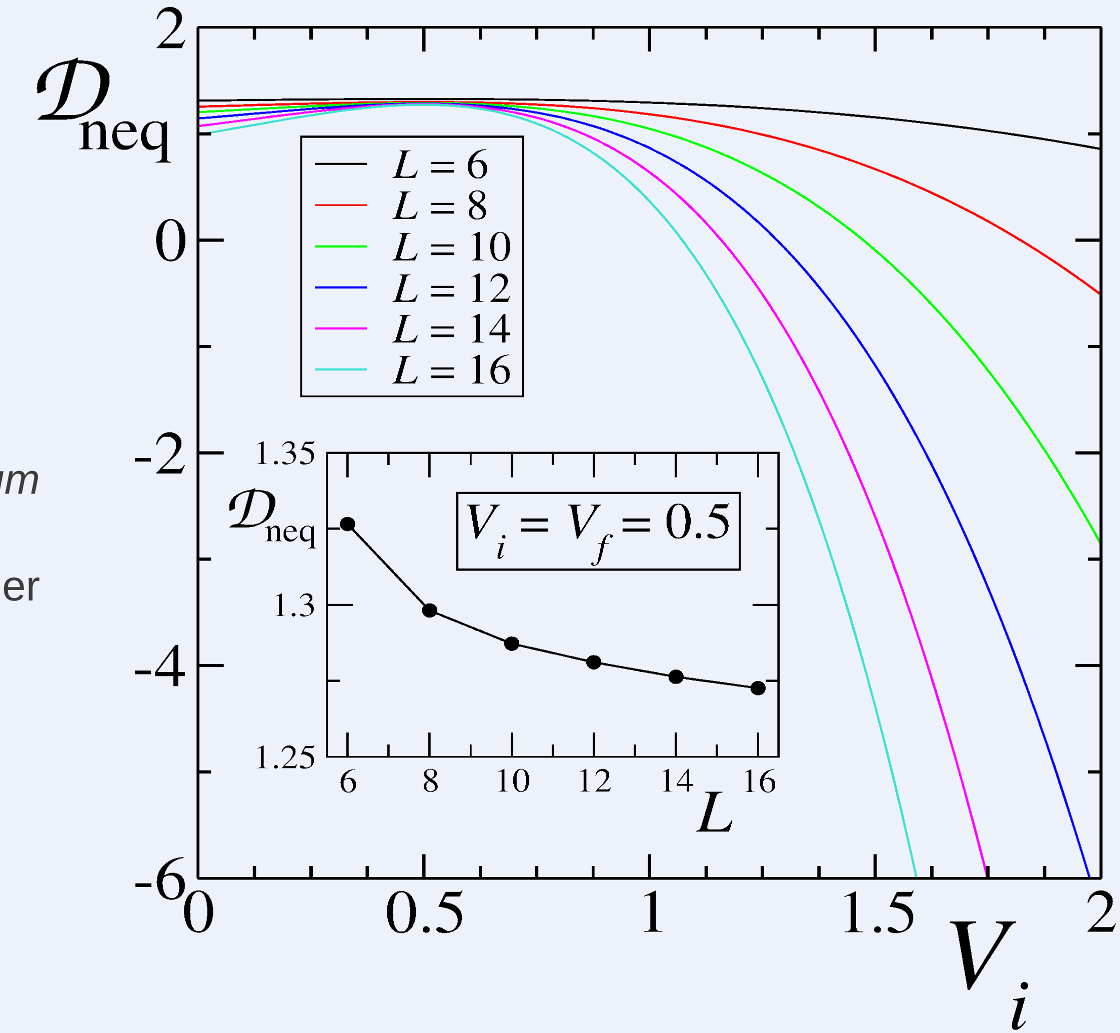
$V \leq 2$ superfluid

$V > 2$ insulator

Quench: $(V_f = 0.5)$

Towards the superfluid

- Finite-size scaling *at equilibrium* predicts *finite* stiffness
- The superfluid presents stronger susceptibility to phase twists



Model II $\hat{\mathcal{V}}_{\text{int}} = V \sum_j \hat{n}_j \hat{n}_{j+1}$

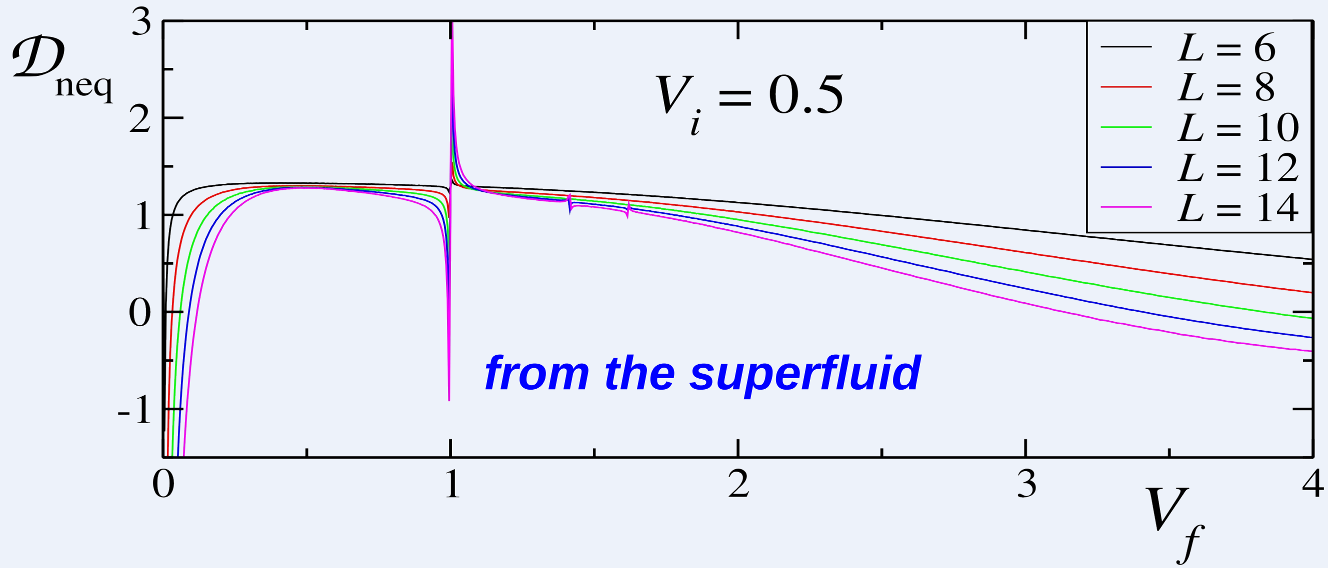
Interacting bosons,
Bethe-Ansatz solvable

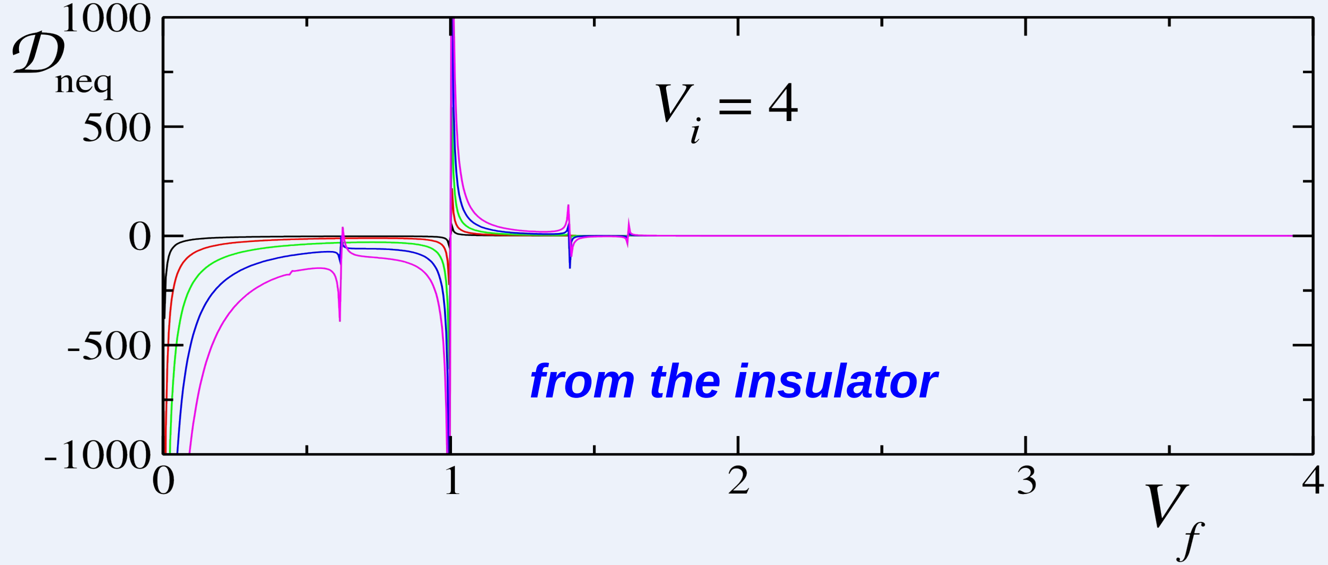
XXZ spin-1/2 Heisenberg chain

$V \leq 2$ superfluid

$V > 2$ insulator

- QPT signaled by a discontinuity in the stiffness
- Appearance of peaks with divergent first derivative, signaling the emergence of additional symmetries





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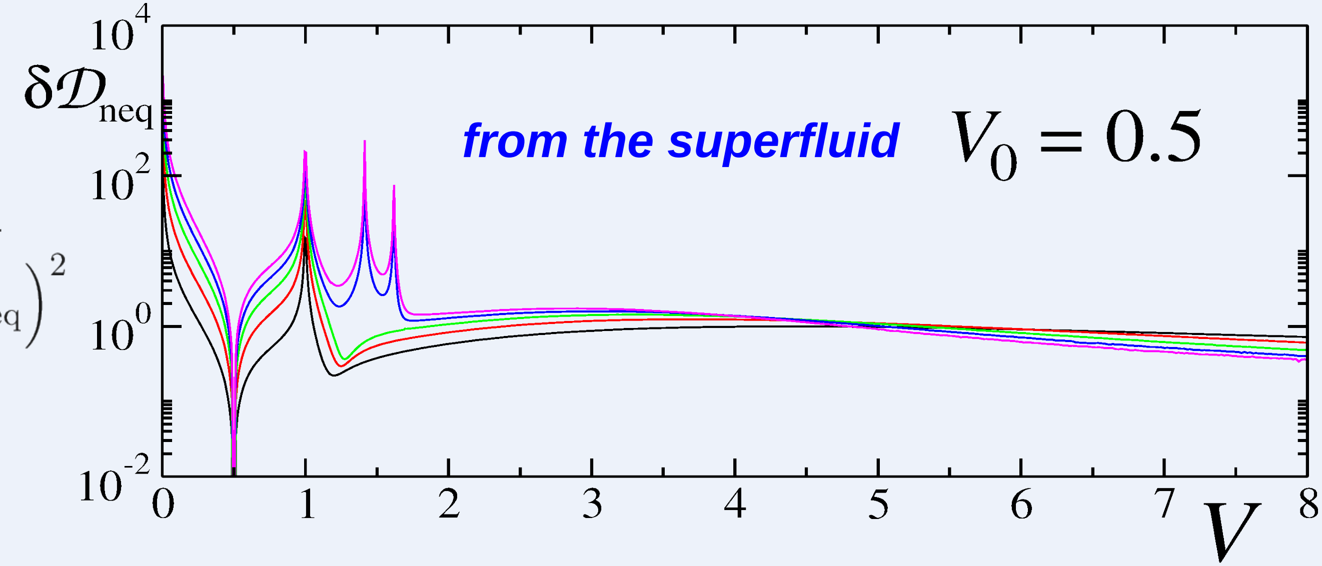
Fluctuations of the stiffness:

$$\delta\mathcal{D}^2_{\text{neq}} = \sum_n p_n \left(\frac{1}{L} \partial^2_\varphi \varepsilon_{n|_{\varphi=0}} - \mathcal{D}_{\text{neq}} \right)^2$$

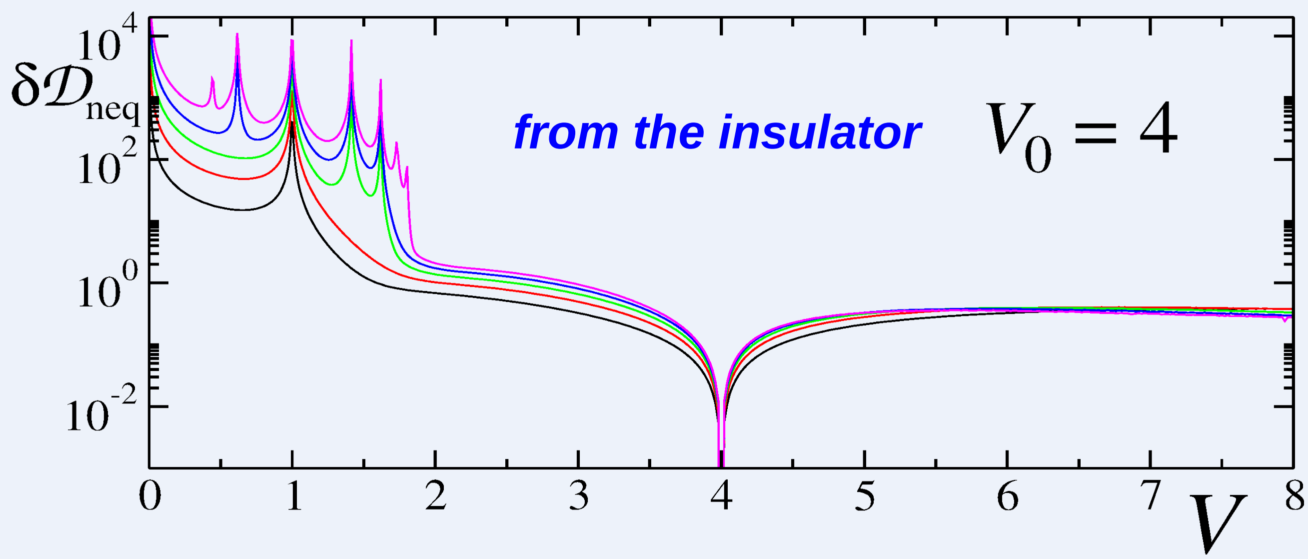


- QPT signaled by a sudden jump of the fluctuations
- Appearance of divergent peaks signaling additional sl_2 loop-algebra symmetry

$$V^\star = 2\cos(k\pi/K)$$

$$k = 0, 1, \ldots, K-1$$

Deguchi, Fabricius, McCoy (2001)



Back to the concept of superfluidity … How to probe it ?

Helicity modulus: coefficient connecting the current density to a *static* perturbation induced by a change in the flux density

$$j_{|\varphi \to 0} = -\mathcal{Y} \cdot \varphi$$

Fisher, Barber, Jasnow (1973)

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$$j_{|\varphi\to 0} = -\mathcal{Y}\cdot\varphi$$

Fisher, Barber, Jasnow (1973)

$$\overline{j_\varphi} = \frac{1}{L}\lim_{T\to\infty}\frac{1}{T}\int_0^T \mathrm{d}t\,\langle\psi(t)|\,\hat{\mathcal{J}}_\varphi\,|\psi(t)\rangle$$

$\hat{\mathcal{J}}_\varphi = -\partial_\varphi\hat{\mathcal{H}}$ current operator

after averaging over *T* and using diagonal ensemble: $\overline{j_\varphi} = -L^{-1}\sum_n p_n(\varphi)\,\partial_\varphi\varepsilon_n(\varphi)$

$$\mathcal{Y} = -\partial_\varphi\overline{j_\varphi}_{|\varphi=0} = \mathcal{D} + \frac{1}{L}\sum_n(\partial_\varphi p_n)\cdot(\partial_\varphi\varepsilon_n)_{|\varphi=0}$$

At equilibrium (*T* = 0): $\mathcal{Y} = \mathcal{D}$ At *T* > 0 or after a quench there are corrections

Model I

A) Insulator → Superfluid $\mathcal{Y}_{\mathrm{neq}} = \mathcal{D}_{\mathrm{neq}}$

B) Superfluid → Insulator $\mathcal{Y}_{\mathrm{neq}} = 0$

C) Insulator → Insulator $\mathcal{Y}_{\mathrm{neq}} = \mathcal{D}_{\mathrm{neq}} + \mathcal{D}_0 \, \Delta V \, \tilde{\mathcal{C}}(V_f)$

Model II

The helicity modulus coincides with the stiffness apart when $V_f = V^\star$

Summary

- Linear response out of equilibrium

- Application to 1D superfluidity after a sudden quench
  – two examples of hard-core bosons on a ring

- Non-equilibrium steady states can be very different from thermal states
  → Non-zero stiffness even in the insulator…
  → Negative stiffness in some cases…

Preprint available on arXiv:1310.4757