Dark Universe and Marvels of Gravity

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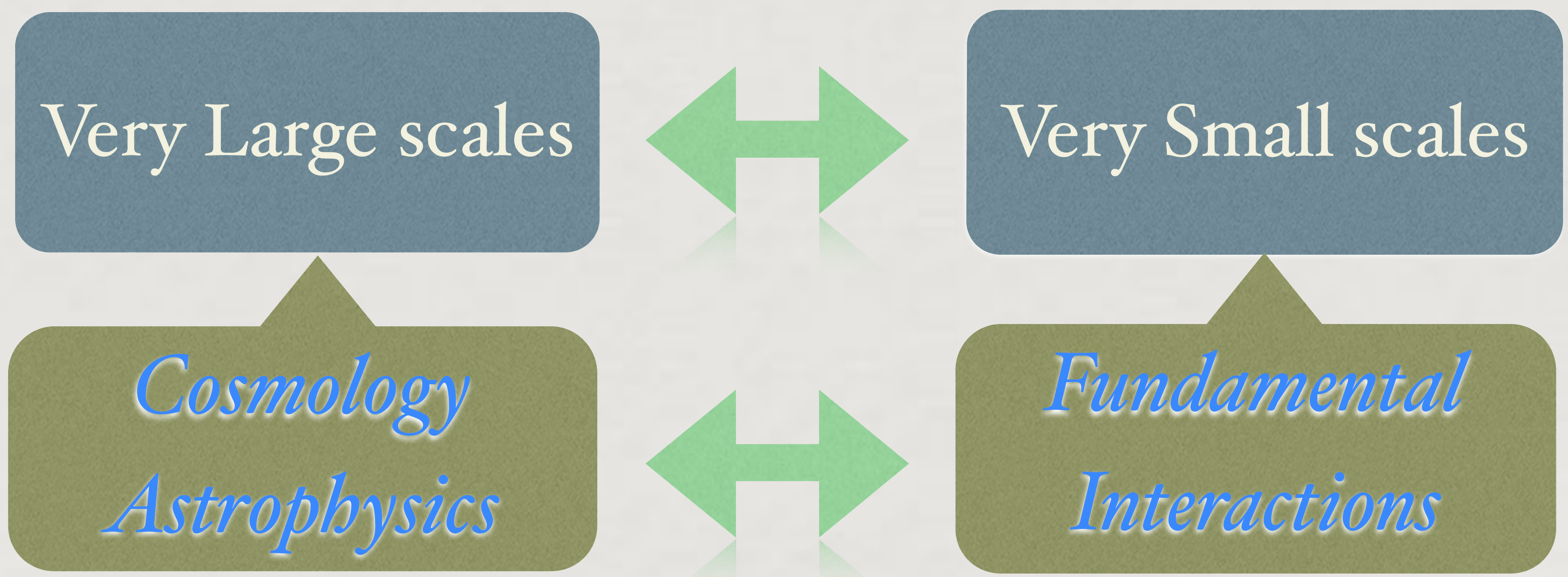
Outline

- *Dark Universe: what we (don't) know*
- *Alternative Gravity and Dark Phenomenology*
- *Some methods to test the idea Geometric Dark Energy*
    - *Dynamical Systems Approach*
    - *Covariant approaches*
- *Perspectives*

Microscopic and Macroscopic

The Universe can be considered our best laboratory for the understanding of the fundamental interactions.



So one can learn about fundamental physics via the study of the large scale Universe.

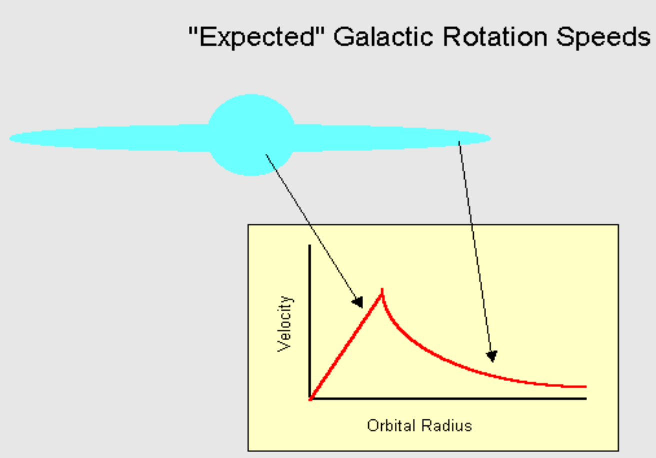
Dark Mysteries (I)

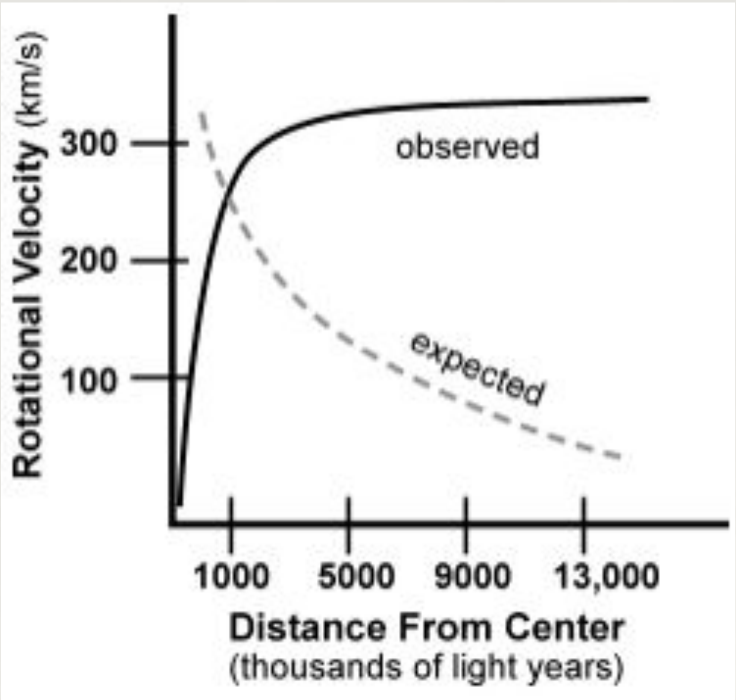
Astrophysical objects do not appear to move according to Newton law of gravitation.

Evidence of this behavior have been found on:

- galactic scale (flattening of rotation curves),
- local cluster scale (motion of the galaxies),
- supercluster scale (motion of the clusters).

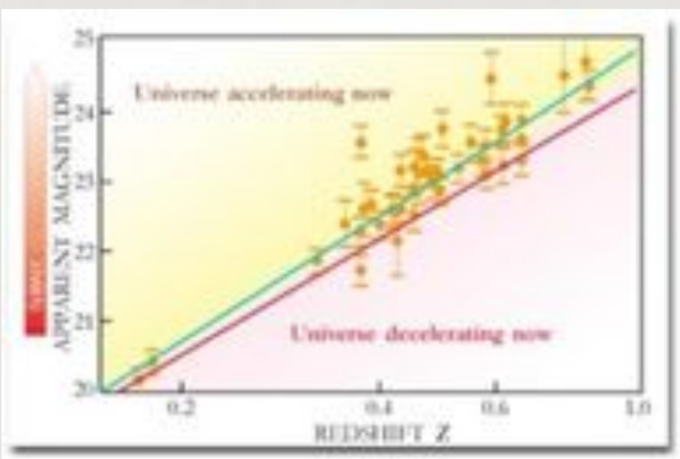
The missing mass has been called Dark Matter.





Dark Mysteries (II)

The expansion rate of the Universe does not decrease as GR predicts.



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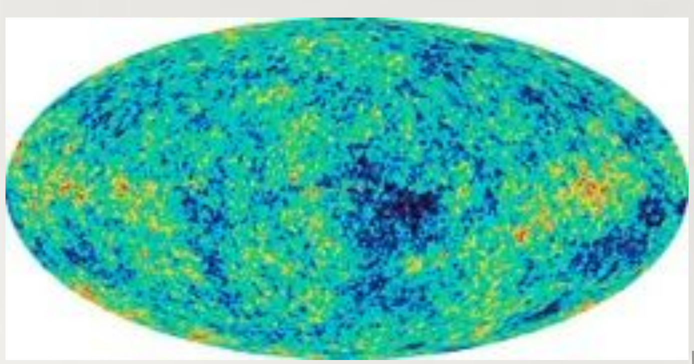
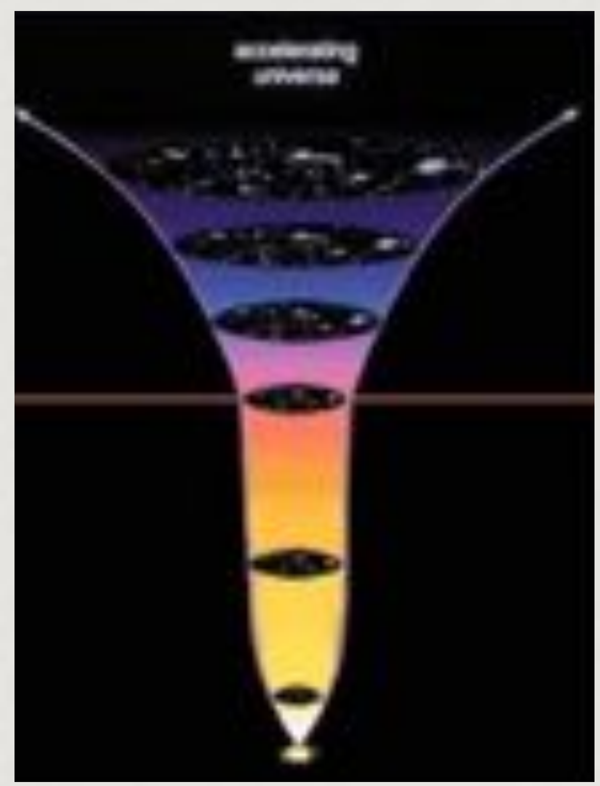
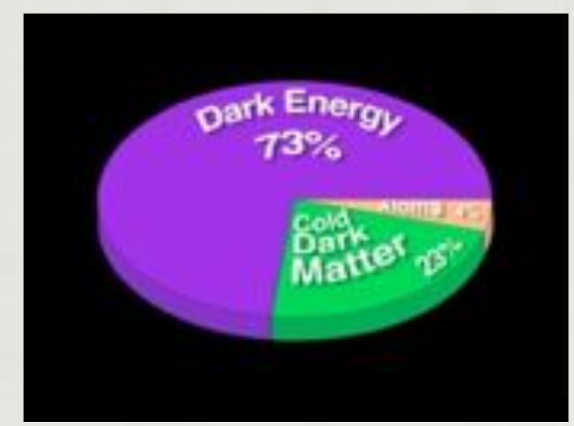
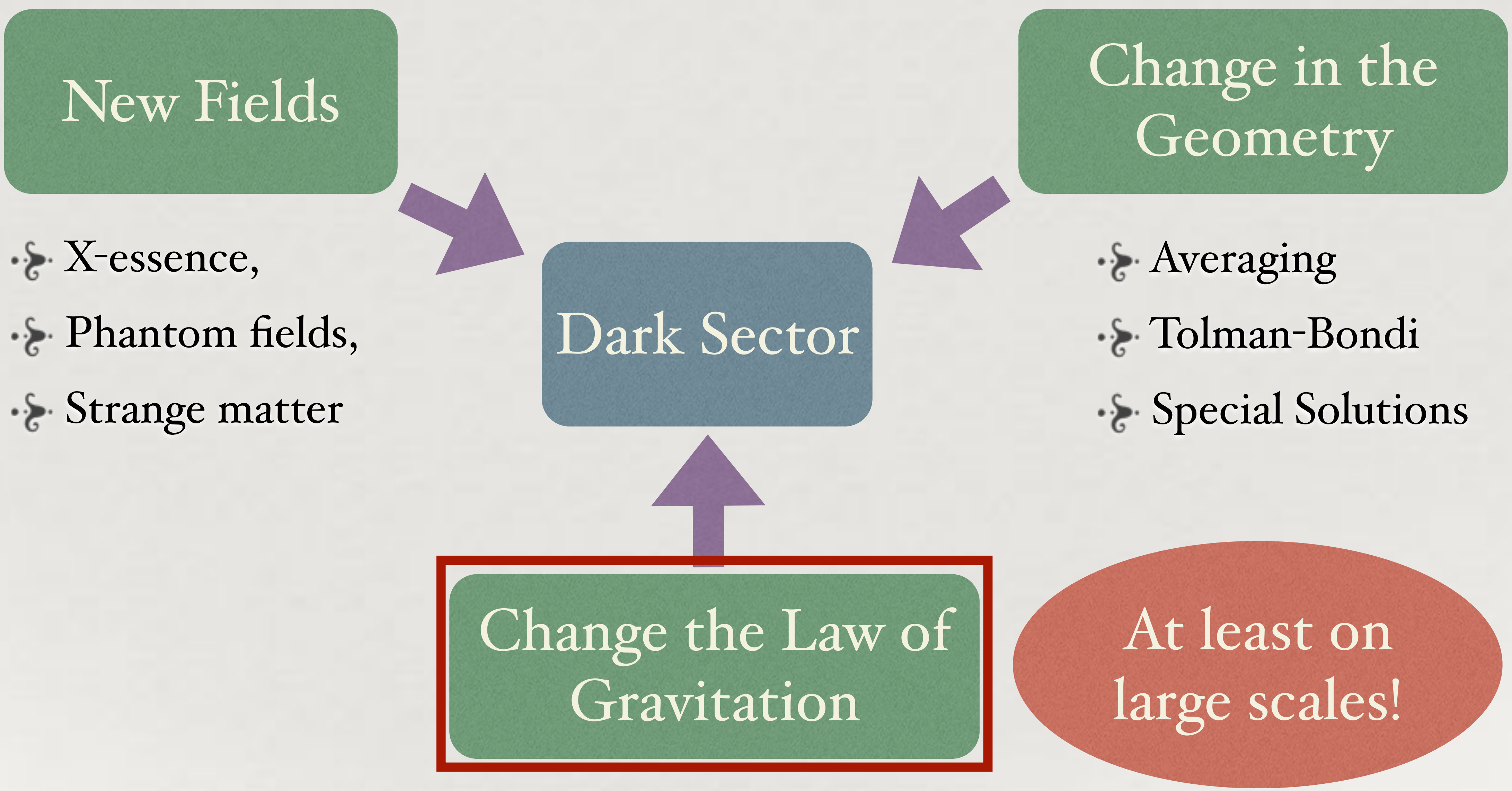






Can we deal with the Dark Menace?



HOW to change gravity?

There are many different ways to modify Einstein's theory. Some popular models are:

- Scalar Tensor Gravity $\mathcal{A} = \frac{1}{2}\int d^4x\sqrt{-g}\left[F(\phi)R + \frac{1}{2}\nabla_a\phi\nabla^a\phi - V(\phi)\right]$
- Higher Order Gravity $\mathcal{A} = \int d^4x\sqrt{-g}\left[f(R, R_{\mu\nu}R^{\mu\nu}, \Box R...)\right]$
- Hořava-Lifschits $\mathcal{A} = \int N\ d^3x\ dt\sqrt{g}\left\{\alpha(K^{ij}K_{ij} - \lambda K^2) - V\right\}$
- Others (TeVeS, MOND, etc.)

WHY do we study these models?

- They are often recovered from very fundamental schemes (e.g. M-theory, Supergravity, Renormalized theories of gravitation);
- They are known to reproduce in cosmology cosmic acceleration;
- They are known to alter the dynamics of galaxies and clusters.

Complications...

A problem in the study of the astrophysics and cosmology of these theories is that they often present serious technical problems. For example:

- their equations are often highly non linear;
- their equations can be of order higher than two;

Therefore a major part of the research in this sector is devoted to the development of techniques able to overcome these problems.

...and Clever Solutions

In the past 10 years I have been developing tools to deal with these theories. Two of them have been particularly successful:

- *The dynamical systems approach*
- *The covariant approaches (1+3 and 1+1+2)*

These methods are based on a simple idea: Translation.

stop to go shopping ⟷ *fermarsi per andare a fare la spesa*

Dynamical Systems Approach

Cosmological Equations $\rightarrow$ Autonomous system of first order differential equations

So that:

- Fixed Points (FP) $\longrightarrow$ Particular exact solutions;
- Stability of the FP $\longrightarrow$ Relation of the solutions with general solutions;
- General orbits $\longrightarrow$ Features of the general solution.

Dynamical Systems Approach

The method consists of three basic steps:

1. Define suitable dimensionless variables including a time variable (logarithmic time);
2. Write the cosmological equations above as an autonomous system of first order differential equations;
3. Use the standard dynamical system theory to achieve a semi-quantitative description of the evolution of these models (fixed points, their stability, phase space).

Covariant Approaches

In the two covariant approaches:

Gravitational Field Equations $\rightarrow$ Simpler set of Propagation and Constraint equations

In particular:

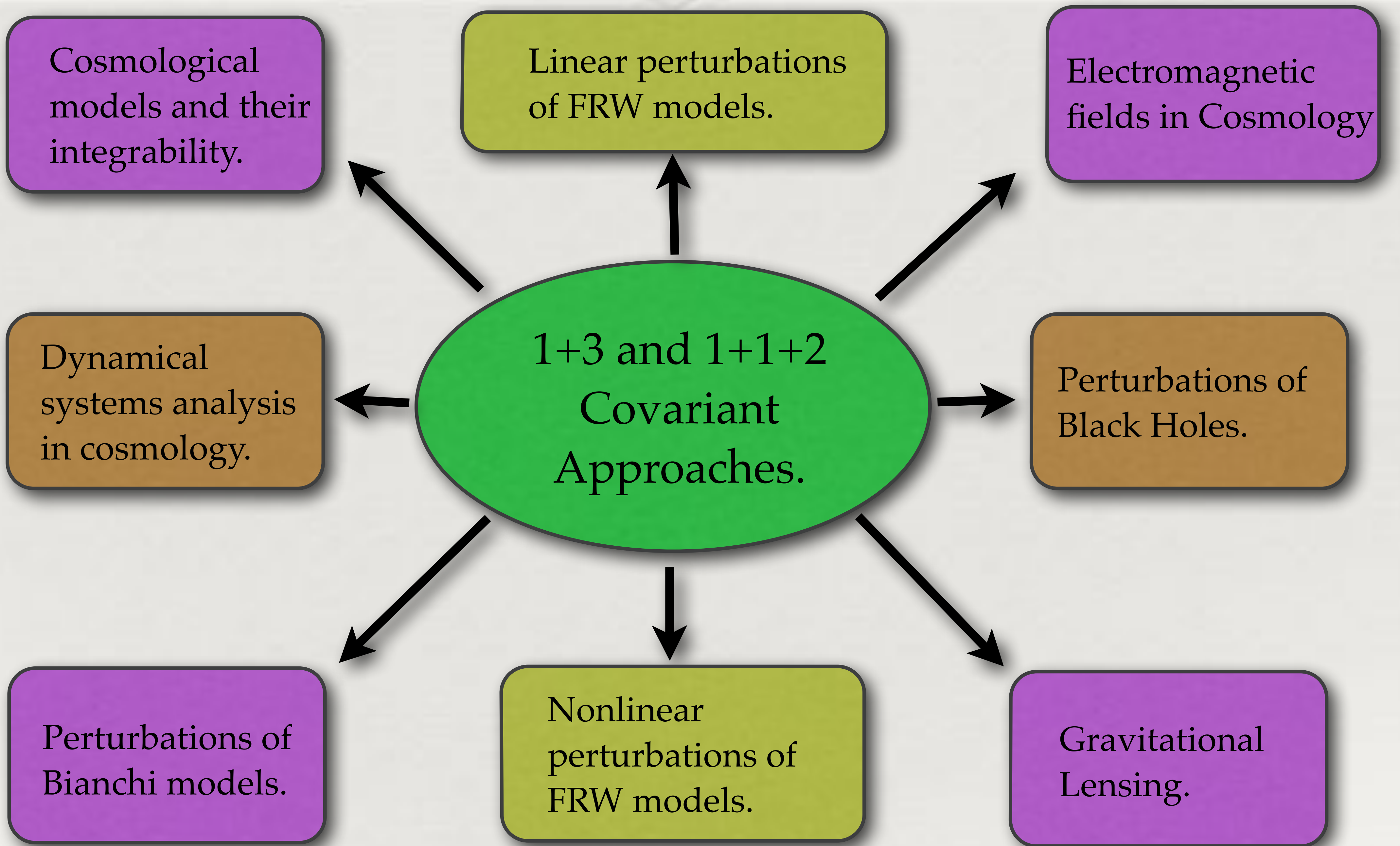
- 1+3 approach they are optimized for cosmology;
- 1+1+2 approach they are optimized for astrophysics.

Covariant Approaches

The use of both these formalisms has many advantages:

- The variables are covariant and with easy physical interpretation;
- Can be used to investigate both the background and the perturbations;
- Can be adapted to alternative gravity.

Covariant Approaches



Dynamical Systems Approach

FLRW F(R) cosmology

Let us consider, as an example, f(R)-gravity

$$\mathcal{A} = \int d^4x\sqrt{-g}\,[f(R) + \mathcal{L}_m] \ ,$$

In homogeneous and isotropic cosmologies the field equations reduce to the system:

$$2\dot{H} + H^2 + \frac{k}{a^2} = -\frac{1}{f'}\left\{\frac{1}{2}\left[f'R - f\right] + \ddot{f}' - 3H\dot{f}' + p_m\right\} \ ,$$

$$H^2 + \frac{k}{a^2} = \frac{1}{3f'}\left\{\frac{1}{2}\left[f'R - f\right] - 3H\dot{f}' + \mu_m\right\} \ ,$$

$$\dot{\mu}_m + 3H\left(\mu_m + p_m\right) = 0 \, .$$

We will use the Dynamical System Approach to analyze these equations.

DSA for a general FOG

1. We define the general dimensionless variables (single fluid case):

$$x = \frac{\dot{f}'}{f'H}, \quad y = \frac{R}{6H^2}, \quad z = \frac{f}{6f'H^2}, \quad \Omega = \frac{\mu}{3f'H^2}, \quad K = \frac{k}{a^2H^2} \ ,$$

and the logarithmic time

$$\mathcal{N} = |\ln a| \ .$$

In the multifluid case we have a different variable Ω for every different fluid.

The dimension of the phase space is reduced when

$$\frac{f}{f'} = \alpha R \Rightarrow f(R) = R^\alpha$$

DSA for a general FOG

2. The cosmological equations are equivalent to

$$\frac{dx}{d\mathcal{N}} = \varepsilon(2 - x^2 + 2K + 2z) + x\varepsilon(K + y + 1) + \Omega\varepsilon(-3\omega - 1),$$
$$\frac{dy}{d\mathcal{N}} = y\varepsilon(2y + 2K + x\mathfrak{q} + 4),$$
$$\frac{dz}{d\mathcal{N}} = z\varepsilon(2K - x + 2y + 4) + \varepsilon xy\mathfrak{q},$$
$$\frac{d\Omega}{d\mathcal{N}} = \Omega\varepsilon(2K - x + 2y - 3\omega + 1),$$
$$\frac{dK}{d\mathcal{N}} = K\varepsilon(2K + 2y + 2),$$

with the constraint

$$1 = -K - x - y + z + \Omega\,.$$

Here the function "ε" ensures the positivity of our time coordinate.

DSA for a general FOG

An important part of the system above is the function

$$\mathfrak{q} \equiv \left(\frac{d\log f'}{d\log R}\right)^{-1} = \frac{f'}{Rf''}\,.$$

In order to obtain a closed system on has to express this function in terms of the dynamical variables....

...but this is far from a trivial task!

In fact:

- The operations involved to obtain this relation might be non trivial e.g. a transcendental form of f(R).
- This transformation might have a non trivial dominion and this would imply constraints on the phase space

DSA for a general FOG

3. Once the system is closed the standard dynamical system theory allows to:

- find fixed points i.e. particular exact solutions:

$$\dot{H} = \alpha H^2 \,, \qquad \alpha = -1 - \Omega_i + x_i - z_i \,,$$
$$\dot{\mu}_m = -\frac{3(1+w)}{\alpha\, t}\mu_m \,,$$

- find their stability i.e. the relation of the above solutions with the general integral

- have an idea of the global properties of the phase space i.e. the behavior of the general integral

Let us now consider an example.....

An Example

Let us consider, for example

$$\mathcal{A} = \int d^4x\sqrt{-g}\left[\chi R^n + L_M\right] \ ,$$

Using the variables above we obtain

$$\frac{dx}{d\mathcal{N}} = \varepsilon\left(-2x^2 + \frac{yx}{n} + \Omega x - 2x + \frac{4y}{n} - 2y - 3w\Omega + \Omega\right) \ ,$$

$$\frac{dy}{d\mathcal{N}} = y\varepsilon\left[\left(\frac{1}{n-1} - 2\right)x + 2\left(\frac{y}{n} + 1\right) + 2\Omega\right] \ ,$$

$$\frac{d\Omega}{d\mathcal{N}} = -\varepsilon\Omega\left(-3w - 3x + \frac{2y}{n} + 2\Omega - 1\right) \ ,$$

$$1 + x + y + K - \Omega = 0 \ ,$$

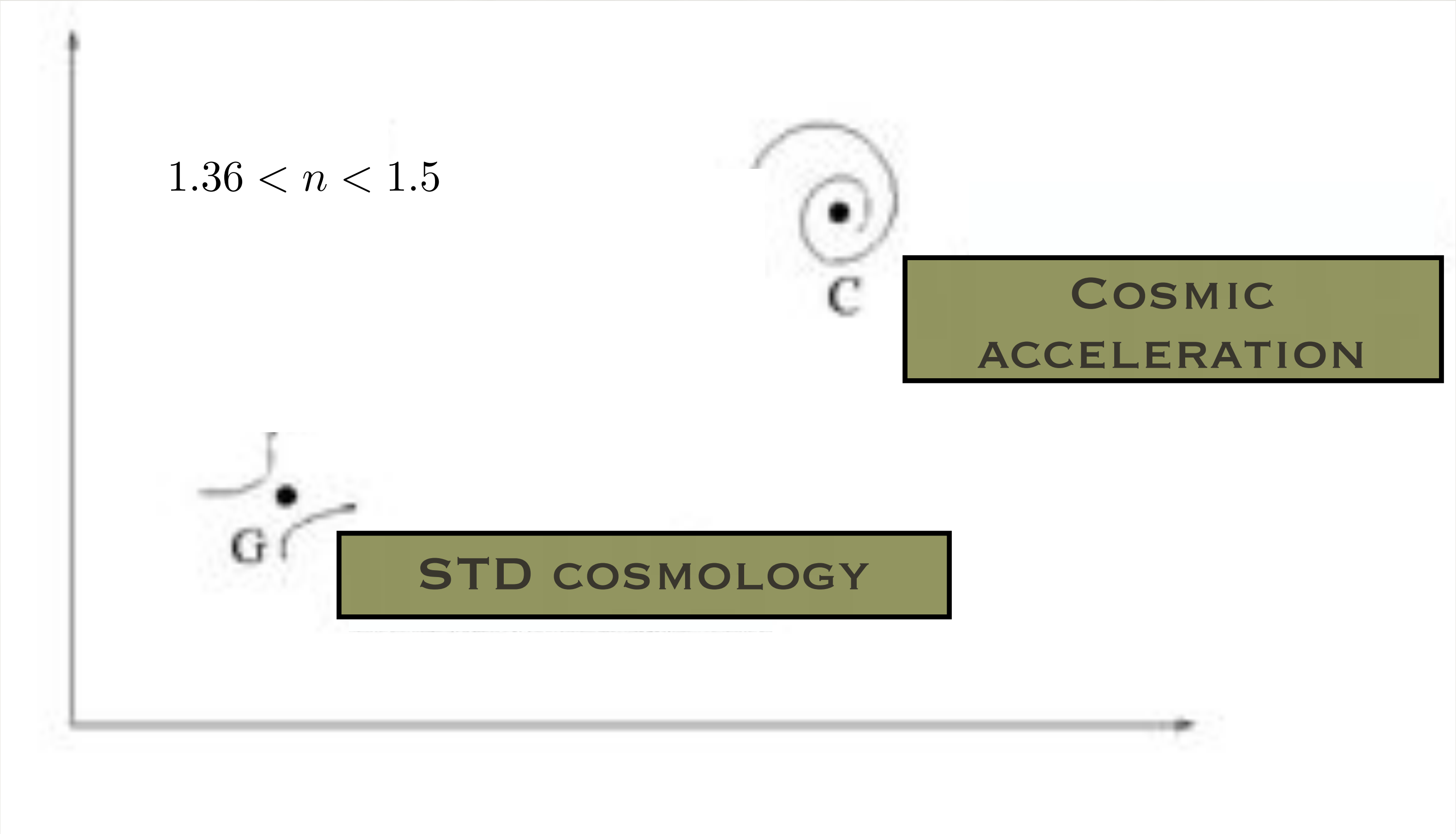
note that this case is degenerate.

An Example

Point	Coordinates (x, y, z)	Scale Factor
$\mathcal{A}$	$[0, 0, 0]$	$a = a_0(t - t_0)$
$\mathcal{B}$	$[-1, 0, 0]$	$a = a_0(t - t_0)^{1/2}$ (only for $n = 3/2$)
$\mathcal{C}$	$\left[\frac{2(n-2)}{2n-1}, \frac{4n-5}{2n-1}, 0\right]$	$a = a_0\, t^{\frac{(1-n)(2n-1)}{n-2}}$
$\mathcal{D}$	$[2(1-n), 2(n-1)^2, 0]$	$\begin{cases} a = \frac{kt}{2n^2-2n-1} & \text{if} \quad k \neq 0 \\ a = a_0 t & \text{if} \quad k = 0 \end{cases}$
$\mathcal{E}$	$[-1-3\omega, 0, -1-3\omega]$	$a = a_0(t - t_0)$
$\mathcal{F}$	$[1-3\omega, 0, 2-3\omega]$	$a = a_0(t - t_0)^{1/2}$ (only for $n = 3/2$)
$\mathcal{G}$	$\left[-\frac{3(n-1)(1+\omega)}{n}, \frac{(n-1)[4n-3(\omega+1)]}{2n^2}, \frac{n(13+9\omega)-2n^2(4+3\omega)-3(1+\omega)}{2n^2}\right]$	$a = a_0\, t^{\frac{2n}{3(1+\omega)}}$

An Example

In terms of the phase space one schematically has



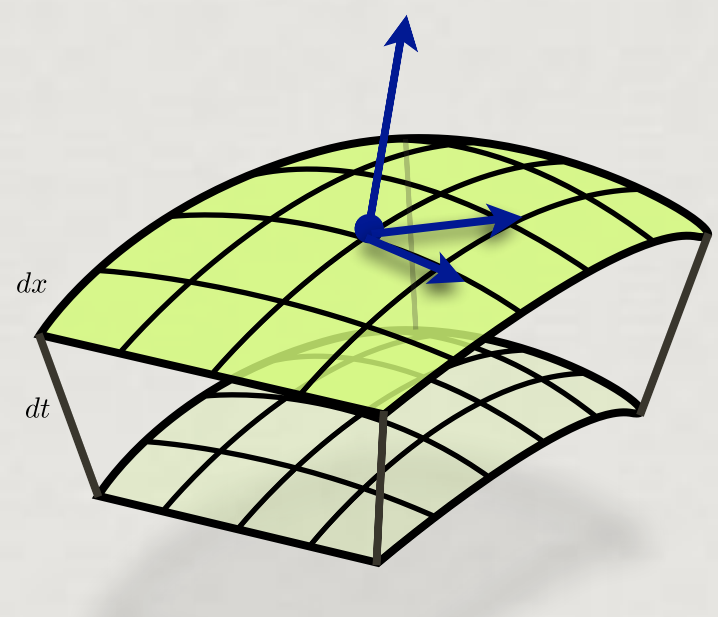
i.e. one has a smooth transition between Friedmannian cosmologies and dark energy era!

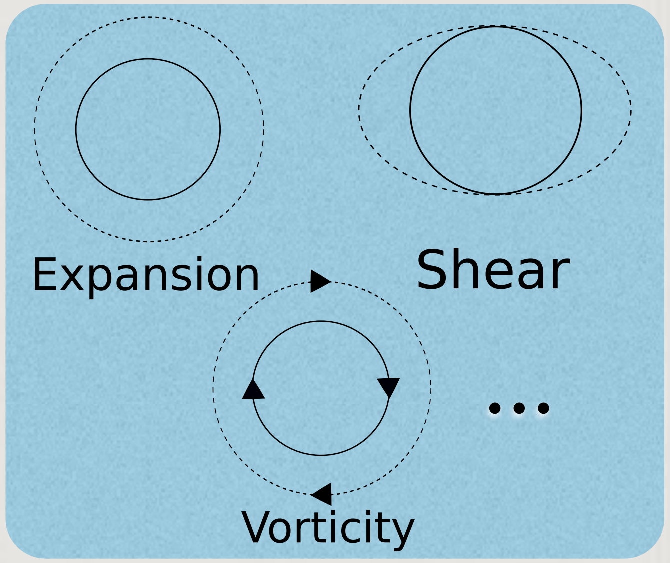
Covariant Approaches

(1+3)

1+3 Covariant Approach

Given the path of an observer



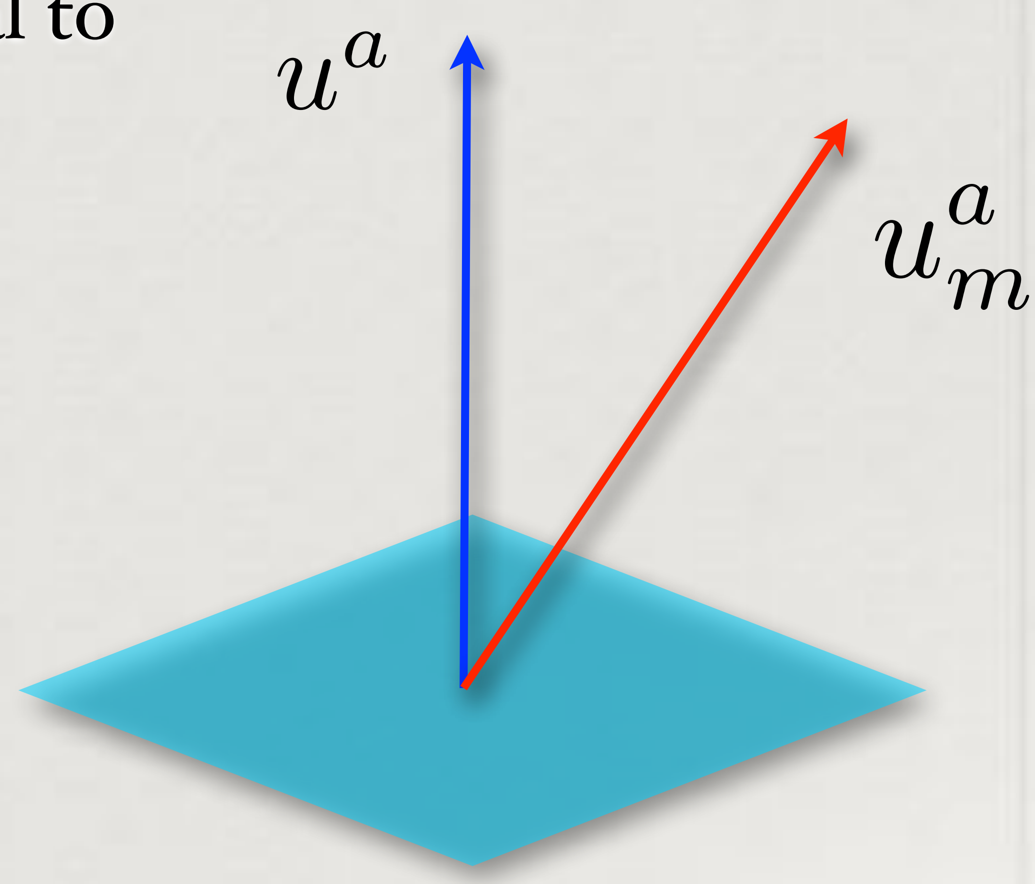


Thermodynamics

1+3 Covariant Approach

- From the time-like flow u^a we construct the projection onto surfaces orthogonal to the flow: $h_{ab} = g_{ab} + u_a u_b$.
- Three-volume form: $\eta_{abc} = u^d \eta_{dabc}$.
- Covariant convective derivative and projected derivative on scalar:

$$\dot{f} = u^a \nabla_a f \qquad \tilde{\nabla}_a f = h^b{}_a \nabla_b f$$



1+3 Covariant Approach

- Kinematics of u^a gives geometry of congruence of flow lines:

$$\nabla_a u_b = \overbrace{-u_a \dot{u}_b}^{\text{acceleration}} + \overbrace{\tfrac{1}{3}\Theta h_{ab}}^{\text{expansion}} + \overbrace{\sigma_{ab}}^{\text{shear}} + \overbrace{\omega_{ab}}^{\text{vorticity}}$$

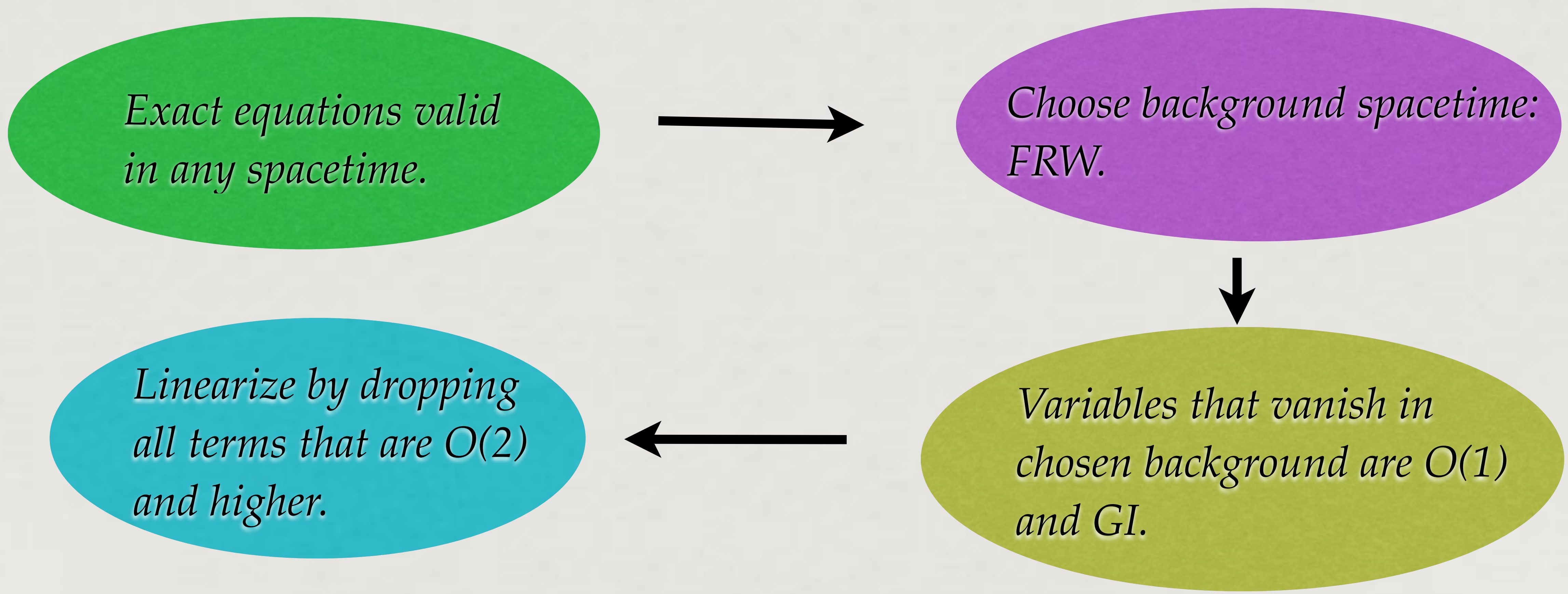
- Other relevant quantities can be defined using the Weyl tensor.
- The total energy-momentum tensor can be decomposed relative to u^a giving:

$$T^{tot}_{ab} = \mu^{tot} u_a u_b + p^{tot} h_{ab} + 2q^{tot}_{(a} u_{b)} + \pi^{tot}_{ab}$$

- We can then give a set of propagation and constraint equations for these quantities. Which are very complicated...
- We can now treat any space times with these equations.

Linearization

We will focus , however, their linearized version...



$$\dot{\Theta} + \frac{1}{3}\Theta^2 + \sigma_{ab}\sigma^{ab} - 2\omega_a\omega^a - \tilde{\nabla}^a\dot{u}_a + \dot{u}_a\dot{u}^a + \frac{1}{2}(\mu^{tot} + 3p^{tot}) = 0$$

$$\dot{\Theta} + \frac{1}{3}\Theta^2 - \tilde{\nabla}^a\dot{u}_a + \frac{1}{2}(\mu^{tot} + 3p^{tot}) = 0$$

1+3 Covariant Equations

The FLRW-linearized propagation equations are

$$\dot{\Theta} + \frac{1}{3}\Theta^2 - \tilde{\nabla}^a \dot{u}_a + \frac{1}{2}(\tilde{\mu}^m + 3\tilde{p}^m) = -\frac{1}{2}(\mu + 3p) \ ,$$

$$\dot{\omega}_{\langle a \rangle} + \frac{2}{3}\Theta\omega_a + \frac{1}{2}\mathrm{curl}\, \dot{u}_a = 0 \ ,$$

$$\dot{\sigma}_{\langle ab \rangle} + \frac{2}{3}\Theta\sigma_{ab} + E_{ab} - \tilde{\nabla}_{\langle a}\dot{u}_{b\rangle} = \frac{1}{2}\pi_{ab} \ ,$$

$$\begin{aligned} \dot{E}_{\langle ab \rangle} + \Theta E_{ab} - \mathrm{curl} H_{ab} + \frac{1}{2}(\tilde{\mu}^m + \tilde{p}^m)\sigma_{ab} \\ = -\frac{1}{2}(\mu + p)\sigma_{ab} - \frac{1}{2}\dot{\pi}_{\langle ab \rangle} - \frac{1}{2}\tilde{\nabla}_{\langle a} q_{b\rangle} - \frac{1}{6}\Theta\pi_{ab} \ , \end{aligned}$$

$$\dot{H}_{\langle ab \rangle} + \Theta H_{ab} + \mathrm{curl} E_{ab} = \frac{1}{2}\mathrm{curl}\pi_{ab} \ .$$

1+3 Covariant Equations

The FLRW-linearized constraint equations are

$$\tilde{\nabla}^a \omega_a = 0 \ ,$$

$$\tilde{\nabla}^b \sigma_{ab} - \mathrm{curl}\omega_a - \frac{2}{3}\tilde{\nabla}_a \Theta = -q_a \ ,$$

$$\mathrm{curl}\sigma_{ab} + \tilde{\nabla}_{\langle a}\omega_{b\rangle} - H_{ab} = 0 \ ,$$

$$\tilde{\nabla}^b E_{ab} - \frac{1}{3}\tilde{\nabla}_a \tilde{\mu}^m - [\sigma, H]_a = -\frac{1}{2}\tilde{\nabla}^b \pi_{ab} + \frac{1}{3}\tilde{\nabla}_a \mu - \frac{1}{3}\Theta q_a \ ,$$

$$\tilde{\nabla}^b H_{ab} - (\tilde{\mu}^m + \tilde{p}^m)\omega_a = -\frac{1}{2}\mathrm{curl}\, q_a + (\mu + p)\omega_a \ .$$

1+3 Covariant Equations

The matter conservation equations are

$$\dot{\mu} + \tilde{\nabla}^a q_a = -\,\Theta\,(\mu + p)\ ,$$
$$\dot{q}_{\langle a \rangle} + \tilde{\nabla}_a p + \tilde{\nabla}^b \pi_{ab} = -\,\frac{4}{3}\,\Theta\, q_a - (\mu + p)\,\dot{u}_a\ .$$

These equations are equivalent to the linearized gravitational field equations, but are expressed in terms of quantities which have a clear physical meaning.

CoGI Perturbations

1+3 Covariant Equations $\rightarrow$ Covariant Gauge Invariant theory of perturbations

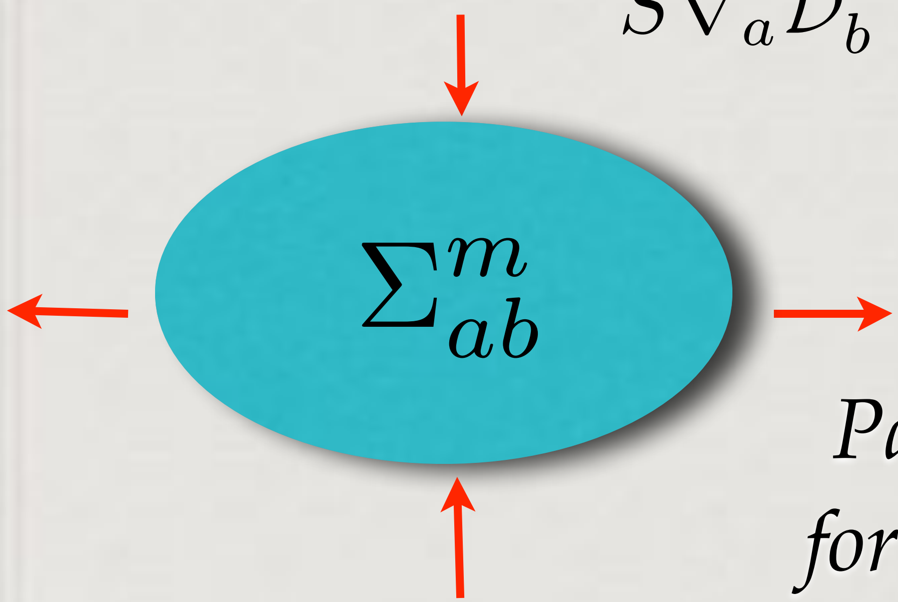
The natural set of inhomogeneity variables are:

$$\mathcal{D}^m_a = \frac{S}{\mu^m}\tilde{\nabla}_a \mu^m\,, \qquad Z_a = S\tilde{\nabla}_a\Theta\,, \qquad C_a = S\tilde{\nabla}_a\tilde{R}\,,$$

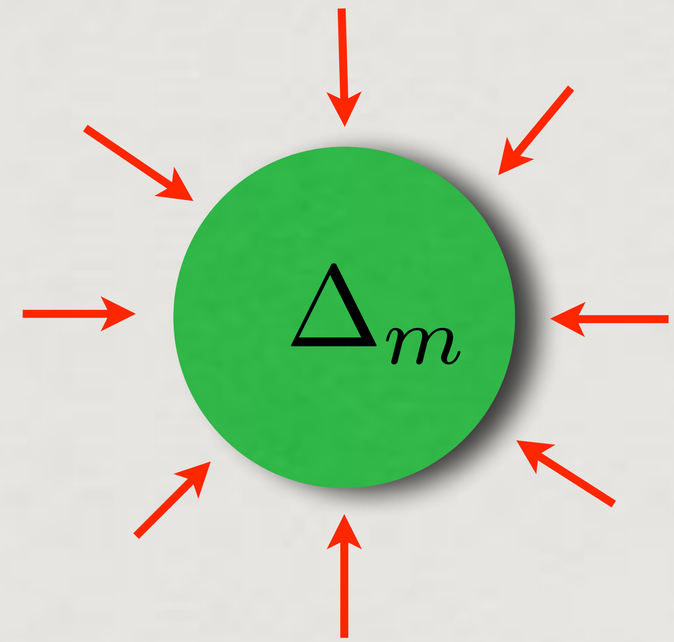
together with other variables able to capture the additional degrees of freedom of the theory we are analyzing.

Extracting the scalar modes

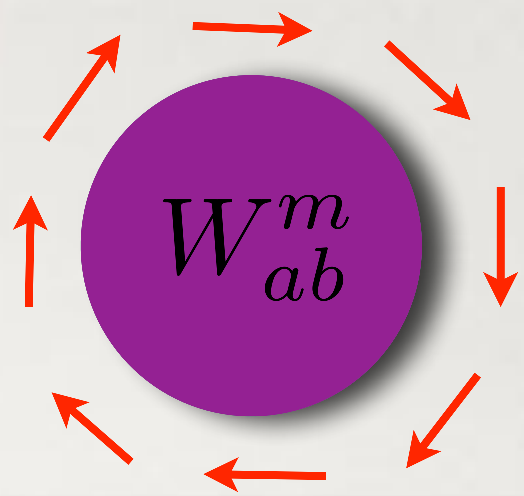
$$S\tilde{\nabla}_a \mathcal{D}^m_b = \Sigma^m_{ab} + W^m_{ab} + \frac{1}{3}\Delta_m h_{ab}$$



Pancake formation



Spherical clumping



Vortices formation

$$\Delta_m = S\tilde{\nabla}^a \mathcal{D}^m_a\,,$$

$$C = S\tilde{\nabla}^a C_a\,,\ Z = S\tilde{\nabla}^a Z_a$$

Harmonic Decomposition

To simplify the resolution of the perturbations equations it is common to exploit an harmonic decomposition.

Using the covariant harmonics defined by

$$\tilde{\nabla}^2 Q = -\frac{k^2}{S^2} Q \ ,$$

where $k^2 = 1/\lambda$ is the wavenumber and $\dot{Q} = 0$, we can expand every first order quantity as

$$X = \sum X^{(k)}(t)\, Q_{(k)}$$

CoGI f(R) Perturbations

Let us consider the case of f(R)-gravity

$$\mathcal{A} = \int d^4x\sqrt{-g}\left[f(R) + \mathcal{L}_m\right]\ ,$$

The natural set of inhomogeneity variables are:

$$\mathcal{D}^m_a = \frac{S}{\mu^m}\tilde{\nabla}_a\mu^m\,, \qquad Z_a = S\tilde{\nabla}_a\Theta\,, \qquad C_a = S\tilde{\nabla}_a\tilde{R}\ ,$$

$$\mathcal{R}_a = S\tilde{\nabla}_a R\,, \qquad \Re_a = S\tilde{\nabla}_a\dot{R}\ .$$

Let us see what the perturbation equations look like...

CoGI f(R) Perturbations

The full first order equations are too long to be given in full, but it is useful to look at the second order ones in their harmonically developed form:

$$\ddot{\Delta}_m^{(k)} + \left[\left(\frac{2}{3} - w\right)\Theta - \frac{\dot{R}f''}{f'}\right]\dot{\Delta}_m^{(k)} - \left[w\frac{k^2}{S^2} - w(3p^R + \mu^R) - \frac{2w\dot{R}\Theta f''}{f'} - \frac{\left(3w^2 - 1\right)\mu}{f'}\right]\Delta_m^{(k)} =$$

$$= \frac{1}{2}(w+1)\left[2\frac{k^2}{S^2}f'' - 1 + \left(f - 2\mu + 2\dot{R}\Theta f''\right)\frac{f''}{f'^2} - 2\dot{R}\Theta\frac{f^{(3)}}{f'}\right]\mathcal{R}^{(k)} - \frac{(w+1)\Theta f''}{f'}\dot{\mathcal{R}}^{(k)},$$

$$f''\ddot{\mathcal{R}}^{(k)} + \left(\Theta f'' + 2\dot{R}f^{(3)}\right)\dot{\mathcal{R}}^{(k)} - \left[\frac{k^2}{S^2}f'' + 2\frac{K}{S^2}f'' + \frac{2}{9}\Theta^2 f'' - (w+1)\frac{\mu}{2f'}f'' - \frac{1}{6}(\mu^R + 3p^R)f'' + \right.$$

$$\left. -\frac{f'}{3} + \frac{f}{6f'}f'' + \dot{R}\Theta\frac{f''^2}{6f'} - \ddot{R}f^{(3)} - \Theta f^{(3)}\dot{R} - f^{(4)}\dot{R}^2\right]\mathcal{R}^{(k)} =$$

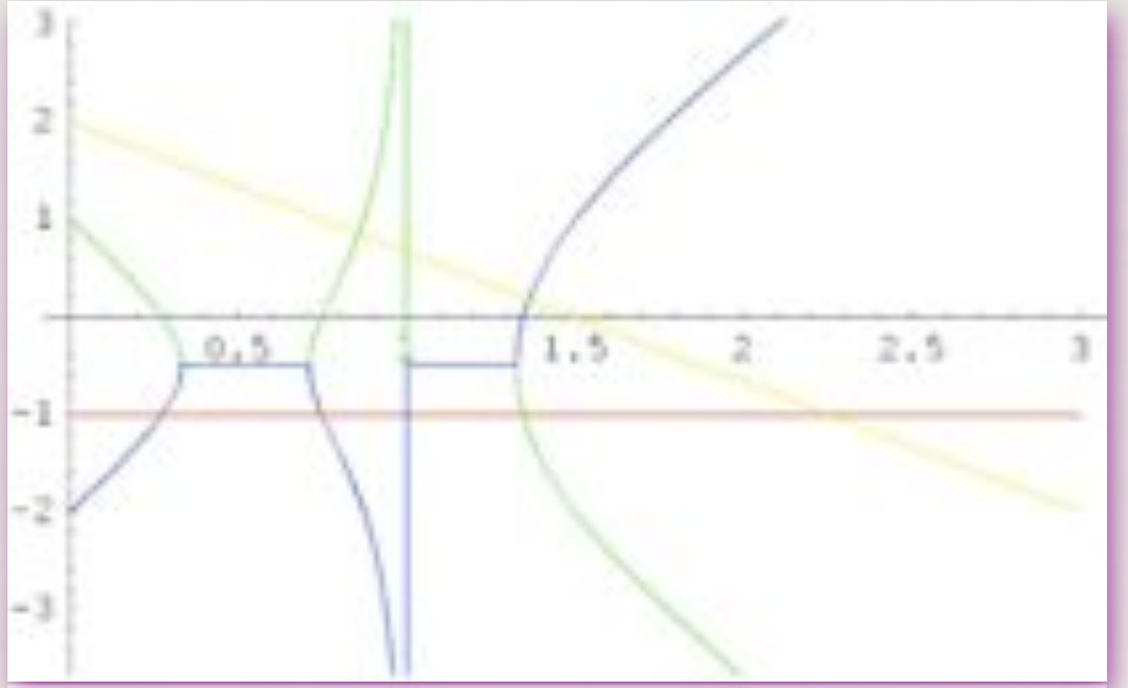
$$-\left[\frac{1}{3}(3w-1)\mu + \frac{w}{1+w}\left(f^{(3)}\dot{R}^2 + (p^R + \mu^R)f' + \frac{7}{3}\dot{R}\Theta f'' + \ddot{R}f''\right)\right]\Delta_m^{(k)} - \frac{(w-1)\dot{R}f''}{w+1}\dot{\Delta}_m^{(k)}.$$

R^n gravity Perturbations

Let us apply these equation R^n gravity. In the long wavelength limit the equations in the quasi-friedmann phase admit an exact solution

$$S = S_0 t^{\frac{2n}{3}}$$

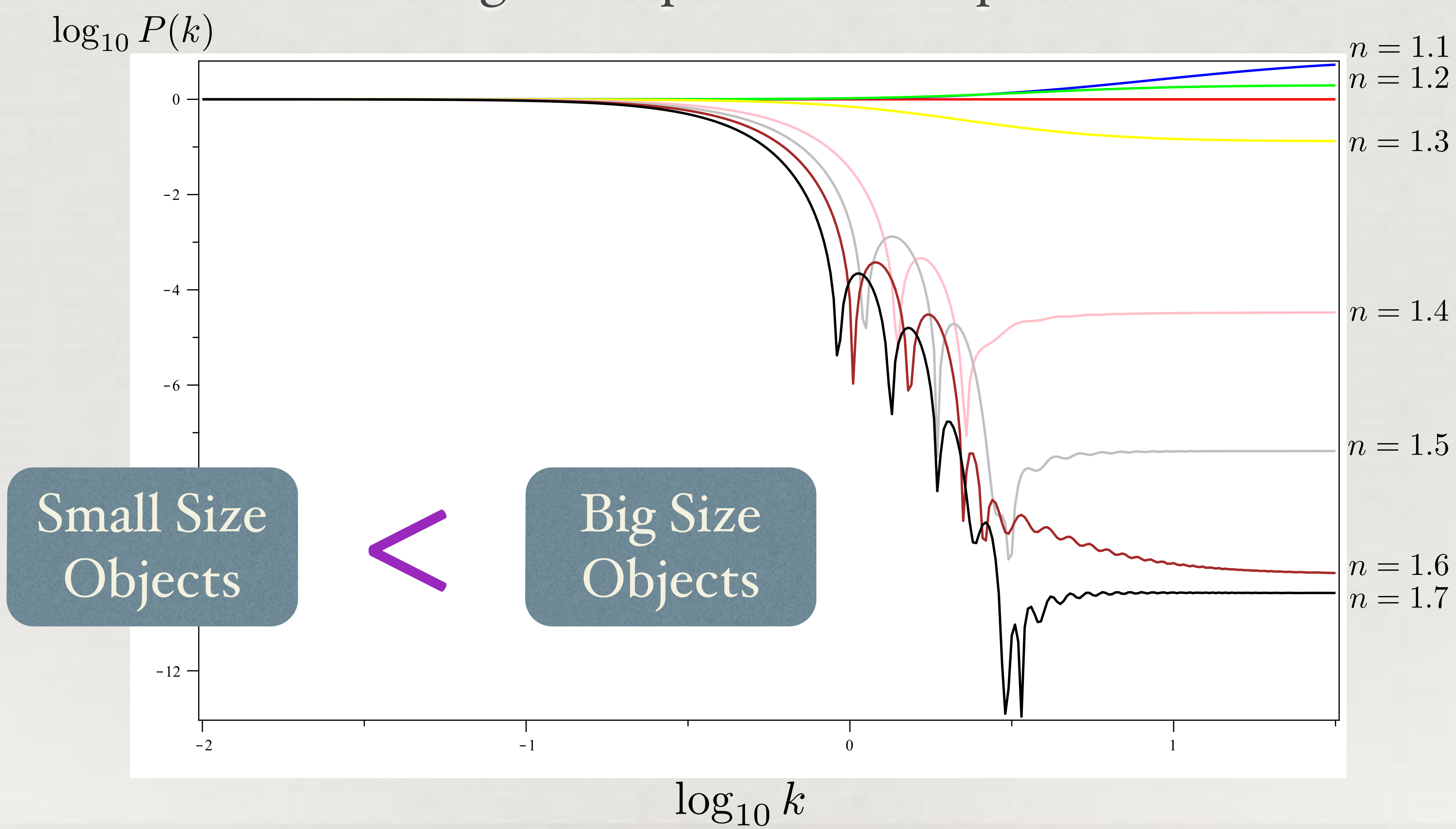
$$\Delta_m = K_1 t^{-1} + K_2 t^{\alpha_+|_{w=0}} + $$
$$+ K_3 t^{\alpha_-|_{w=0}} - K_4 \frac{\mathcal{C}_0}{S_0^2} t^{2-\frac{4n}{3}}$$



The perturbations can grow even in a accelerating expanding backgrounds!!

R^n gravity Perturbations

Looking at the perturbation spectrum...

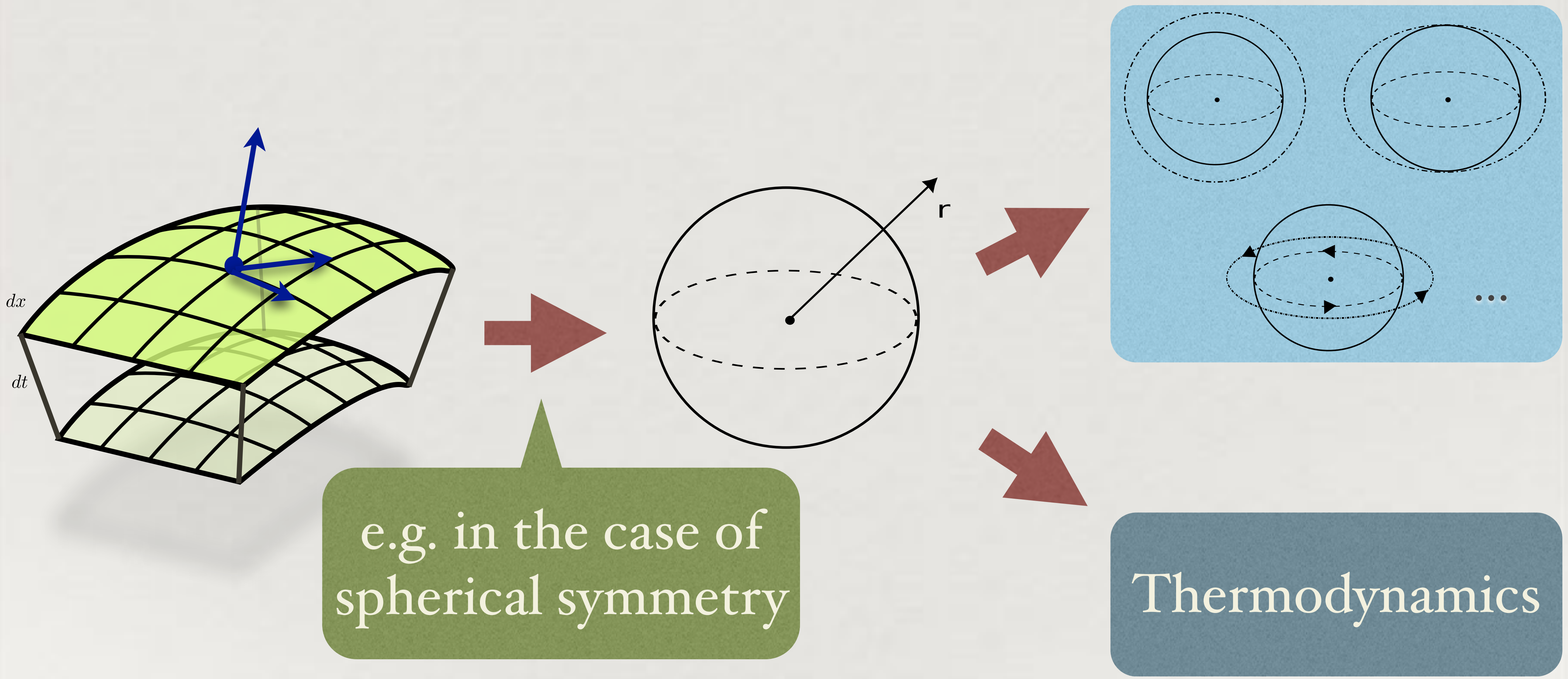


Covariant Approaches

(1+1+2)

1+1+2 Covariant Approach

This formalism is a further specialization of the 1+3 approach:



1+1+2 Covariant Approach

* From the spacelike surfaces we single out a special direction e_a.

$$g_{ab} = -u_a u_b + e_a e_b + N_{ab}$$

* The projected derivative is split accordingly:

$$\hat{\psi}_{a..b}{}^{c..d} \equiv e^f \tilde{\nabla}_f \psi_{a..b}{}^{c..d} \qquad \delta_f \psi_{a..b}{}^{c..d} \equiv N_a{}^f ... N_b{}^g N_h{}^c .. N_i{}^d N_f{}^j \tilde{\nabla}_j \psi_{f..g}{}^{i..j} \ .$$

* Kinematics of e_a gives geometry of the spatial 2-hypersurfaces:

$$\tilde{\nabla}_a e_b = e_a a_b + \frac{1}{2}\phi N_{ab} + \xi \varepsilon_{ab} + \zeta_{ab} \ ,$$

* All the 1+3 quantities can be further split in terms of the projectors above

1+1+2 Covariant Approach

- Considering spherically symmetric metrics the non trivial variables reduce to

$$\mathcal{A}, \Theta, \phi, \xi, \Sigma, \Omega, \mathcal{E}, \mathcal{H}, \mu, p, \Pi, Q$$

Tensor Equations → Scalar Equations

1+1+2 formalism

1+1+2 Covariant Approach

The general equations for static spherically symmetric metrics read

$$\begin{aligned}
\hat{\phi} &= - \frac{1}{2}\phi^2 - \frac{2}{3}\mu - \frac{1}{2}\Pi - \mathcal{E}\ , \\
\hat{\mathcal{E}} - \frac{1}{3}\hat{\mu} + \frac{1}{2}\hat{\Pi} &= - \frac{3}{2}\phi\left(\mathcal{E} + \frac{1}{2}\Pi\right)\ , \\
\hat{p} + \hat{\Pi} &= - \left(\frac{3}{2}\phi + \mathcal{A}\right)\Pi - (\mu + p)\,\mathcal{A}\ , \\
\hat{\mathcal{A}} &= - (\mathcal{A} + \phi)\,\mathcal{A} + \frac{1}{2}(\mu + 3p)\ .
\end{aligned}$$

with the constraint

$$0 = -\mathcal{A}\phi + \frac{1}{3}(\mu + 3p) - \mathcal{E} + \frac{1}{2}\Pi$$

1+1+2 f(R)-gravity

In the case of f(R) gravity

$$\mathcal{A} = \int d^4x\sqrt{-g}\,[f(R) + \mathcal{L}_m] \ ,$$

the above equations, in vacuum, become:

$$f'\left[\hat{\phi} + \phi\left(\frac{1}{2}\phi - \mathcal{A}\right)\right] = \frac{1}{3}Rf' - \frac{2}{3}f$$
$$+f''X\left(\phi + 2\mathcal{A}\right) \ ,$$
$$f'\left[\hat{\mathcal{A}} + \mathcal{A}(\mathcal{A} + \phi)\right] = \frac{1}{6}f - \frac{1}{3}Rf' - f''X\mathcal{A} \ ,$$
$$\hat{R} = X \ ,$$
$$f''\hat{X} = -\frac{1}{3}Rf' + \frac{2}{3}f - f'''X^2 - X(\phi + \mathcal{A})f'' \ .$$

1+1+2 f(R)-gravity

These equations can be used to find exact solutions, but also to understand better general properties of these theories.

For example the structure of the equations reveals that

Birkhoff Theorem $\Leftrightarrow$ $\begin{cases} f(0) = 0\,, \\ f'(0) \neq 0\,. \end{cases}$

Which means that cosmological constant and a linear term are crucial to avoid e.g. gravitational monopole radiation.

Perspectives

There are many possible future developments of these research lines:

- Dynamical Systems Approach can be further refined and potentiated;
- Study of Structure Formation and Cosmic Microwave Background with 1+3 approach;
- Study of Relativistic Astrophysics with 1+1+2 approach;
- These tools can be applied to any other modification of Einstein theory might be proposed in the future.