Dipartimento di Matematica e Fisica
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The Standard Model in a weak gravitational background

Dilatons and Conformal Anomalies

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Outline

Quantum Field Theory in a gravitational background

Gravitational interactions with matter

$$R^{\mu\nu} - g^{\mu\nu}\frac{R}{2} + \Lambda g^{\mu\nu} = \frac{8\pi G}{c^4}T^{\mu\nu}$$

Analysis of the conformal symmetry and its anomalous breaking $J_D^{\mu} = x_\nu T^{\mu\nu}$

Holographic applications in cosmology

Constraints on CFT

Scale-invariant extensions of the Standard Model

The conformal anomaly 1.

At the classical level the trace of the energy-momentum tensor (EMT) takes into account the explicit conformal symmetry breaking terms

$$g_{\mu\nu} T^{\mu\nu}_{cl} \sim m_\phi^2 \, \phi^2 + m_\psi \, \bar{\psi}\psi + m_A^2 \, A_\mu A^\mu$$

Even if we start with a classical conformally invariant theory the renormalization procedure radiatively generates a non-zero trace in the EMT (the renormalization scale is introduced by quantum corrections)

We end up with a well defined conformal anomaly $\langle g_{\mu\nu} T^{\mu\nu} \rangle_{m=0} \neq 0$

The conformal anomaly 2.

The trace anomaly in 4 space-time dimensions generated by quantum effects in a classical gravitational and gauge background is given by

$$T^{\mu}_{\mu\, anom} = -\frac{1}{8}\left[2bC^2 + 2b'\left(E - \frac{2}{3}\Box R\right) + 2cF^2\right]$$

gravitational contribution

gauge contribution

Weyl and Euler tensors

$$C^2 = C_{\lambda\mu\nu\rho}C^{\lambda\mu\nu\rho} = R_{\lambda\mu\nu\rho}R^{\lambda\mu\nu\rho} - 2R_{\mu\nu}R^{\mu\nu} + \frac{R^2}{3}$$

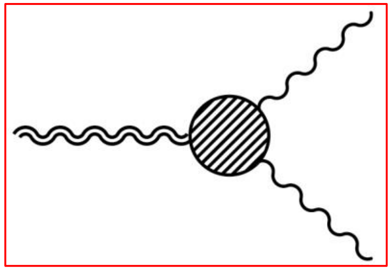
$$E = {}^*R_{\lambda\mu\nu\rho}\,{}^*R^{\lambda\mu\nu\rho} = R_{\lambda\mu\nu\rho}R^{\lambda\mu\nu\rho} - 4R_{\mu\nu}R^{\mu\nu} + R^2.$$

The coefficients b,b' and c depend on the field content of the theory, in particular c is related to the β function

In this talk I will focus on the gauge contribution to the conformal anomaly analyzed in the framework of perturbation theory

The perturbative expansion

The one-loop corrections to the <TVV> correlator are the first order contributions responsible for the appearance of the conformal anomaly



Main goals of the analysis:

- *This class of correlators describes the interaction of a weak gravitational field with gauge fields*
- *Implications of the conformal anomaly*

the anomalous breaking of conformal symmetry → Existence of effective degrees of freedom

This is true in every gauge-invariant sector of the Standard Model and it is described by a pole singularity in the correlation functions

Conformal anomalies and QFT

Quantum field theories examined in the context of conformal anomalies:

- *Abelian gauge field theory (QED)*
  Giannotti, Mottola, Phys.Rev.D79 (2009) 045014,
  Armillis, Corianò, L. D. R., Phys.Rev.D81 (2010) 085001

- *Fermion with an axial-vector coupling to a background vector field (axial QED)*
  Armillis, Corianò, L. D. R., Manni, Int.J.Mod.Phys.A26 (2011)

- *Exact non abelian gauge field theory (QCD)*
  Armillis, Corianò, L. D. R., Phys.Rev.D82 (2010) 064023

- *Electroweak sector of the Standard Model (SM)*
  Corianò, L. D. R., Quintavalle, Serino, Phys.Lett.B700 (2011) 29-38
  Corianò, L. D. R., Serino, Phys.Rev.D83 (2011) 125028

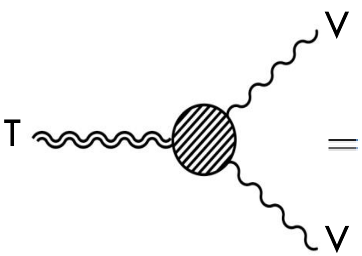
- *N=1 Supersymmetric Yang-Mills*
  work in progress …

The computation

- All diagrams are computed at one loop order by means of tensor reduction in dimensional regularization
- The result is expressed in terms of scalar master integrals
- The results are expanded onto a suitable tensor basis defined by symmetry constraints
- The traceful and traceless parts of the correlators are explicitly separeted
- The conformal anomaly is identified

Explicit results: the QED case 1.

Consider the simplest case, QED, in order to avoid all the complications of the non abelian gauge theories



$$= \sum_{i=1}^{13} F_i(s; s_1, s_2, m^2)\; t_i^{\mu\nu\alpha\beta}(p,q)$$

i	$t_i^{\mu\nu\alpha\beta}(p,q)$
1	$\left(k^2 g^{\mu\nu} - k^\mu k^\nu\right) u^{\alpha\beta}(p.q)$
2	$\left(k^2 g^{\mu\nu} - k^\mu k^\nu\right) w^{\alpha\beta}(p.q)$
3	$\left(p^2 g^{\mu\nu} - 4p^\mu p^\nu\right) u^{\alpha\beta}(p.q)$
4	$\left(p^2 g^{\mu\nu} - 4p^\mu p^\nu\right) w^{\alpha\beta}(p.q)$
5	$\left(q^2 g^{\mu\nu} - 4q^\mu q^\nu\right) u^{\alpha\beta}(p.q)$
6	$\left(q^2 g^{\mu\nu} - 4q^\mu q^\nu\right) w^{\alpha\beta}(p.q)$
7	$\left[p\cdot q\, g^{\mu\nu} - 2(q^\mu p^\nu + p^\mu q^\nu)\right] u^{\alpha\beta}(p.q)$
8	$\left[p\cdot q\, g^{\mu\nu} - 2(q^\mu p^\nu + p^\mu q^\nu)\right] w^{\alpha\beta}(p.q)$
9	$\left(p\cdot q\, p^\alpha - p^2 q^\alpha\right)\left[p^\beta\left(q^\mu p^\nu + p^\mu q^\nu\right) - p\cdot q\left(g^{\beta\nu} p^\mu + g^{\beta\mu} p^\nu\right)\right]$
10	$\left(p\cdot q\, q^\beta - q^2 p^\beta\right)\left[q^\alpha\left(q^\mu p^\nu + p^\mu q^\nu\right) - p\cdot q\left(g^{\alpha\nu} q^\mu + g^{\alpha\mu} q^\nu\right)\right]$
11	$\left(p\cdot q\, p^\alpha - p^2 q^\alpha\right)\left[2\, q^\beta q^\mu q^\nu - q^2(g^{\beta\nu} q^\mu + g^{\beta\mu} q^\nu)\right]$
12	$\left(p\cdot q\, q^\beta - q^2 p^\beta\right)\left[2\, p^\alpha p^\mu p^\nu - p^2(g^{\alpha\nu} p^\mu + g^{\alpha\mu} p^\nu)\right]$
13	$(p^\mu q^\nu + p^\nu q^\mu) g^{\alpha\beta} + p\cdot q\left(g^{\alpha\nu} g^{\beta\mu} + g^{\alpha\mu} g^{\beta\nu}\right) - g^{\mu\nu} u^{\alpha\beta}$ $-(g^{\beta\nu} p^\mu + g^{\beta\mu} p^\nu) q^\alpha - (g^{\alpha\nu} q^\mu + g^{\alpha\mu} q^\nu) p^\beta$

This basis is chosen so that:

- t_1 and t_2 are traceful
- t_1 accounts for the anomaly
- $g_{\mu\nu} t_i^{\mu\nu\alpha\beta}(p,q) = 0\,, i = 3\ldots 13$

$$u^{\alpha\beta}(p,q) \equiv (p\cdot q)\, g^{\alpha\beta} - q^\alpha\, p^\beta\,,$$

$$w^{\alpha\beta}(p,q) \equiv p^2\, q^2\, g^{\alpha\beta} + (p\cdot q)\, p^\alpha\, q^\beta - q^2\, p^\alpha\, p^\beta - p^2\, q^\alpha\, q^\beta$$

Giannotti, Mottola, Phys.Rev.D79 (2009) 045014,
Armillis, Corianò, L. D. R., Phys.Rev.D81 (2010) 085001

Explicit results: the QED case 2.

We present the first two (traceful) form factors

Armillis, Corianò, L. D. R., Phys.Rev.D81 (2010) 085001

The off-shell massive form factors F_i, with

- $s \neq 0 \quad s_1 \neq 0 \quad s_2 \neq 0 \quad m \neq 0$ $\qquad s = k^2 = (p+q)^2, \quad s_1 = p^2, \quad s_2 = q^2$

and with $\gamma \equiv s - s_1 - s_2$, $\sigma \equiv s^2 - 2(s_1+s_2)\,s + (s_1-s_2)^2$ are given by [2]

$$\begin{aligned}
\mathbf{F_1(s;\, s_1,\, s_2,\, m^2)} &= \frac{e^2\gamma m^2}{3\pi^2 s\sigma} + \frac{e^2\,\mathcal{D}_2(s,s_2,m^2)\,s_2\left[s^2+4s_1 s-2s_2 s-5s_1^2+s_2^2+4s_1 s_2\right]m^2}{3\pi^2 s\sigma^2} \\
&- \frac{e^2}{18\pi^2 s} - \frac{e^2\,\mathcal{D}_1(s,s_1,m^2)\,s_1\left[-(s-s_1)^2+5s_2^2-4(s+s_1)\,s_2\right]m^2}{3\pi^2 s\sigma^2} \\
&- e^2\,\mathcal{C}_0(s,s_1,s_2,m^2)\left[\frac{m^2\gamma\left[(s-s_1)^3-s_2^3+(3s+s_1)\,s_2^2+\left(-3s^2-10s_1 s+s_1^2\right)s_2\right]}{6\pi^2 s\sigma^2} - \frac{2m^4\gamma}{3\pi^2 s\sigma}\right],
\end{aligned}$$

Anomalous form factor with an anomaly pole

$$\begin{aligned}
\mathbf{F_2(s;\, s_1,\, s_2,\, m^2)} &= -\frac{2e^2 m^2}{3\pi^2 s\sigma} - \frac{2\,e^2\,\mathcal{D}_2(s,s_2,m^2)\,\left[(s-s_1)^2-2s_2^2+(s+s_1)\,s_2\right]m^2}{3\pi^2\,s\,\sigma^2} \\
&- \frac{2e^2\,\mathcal{D}_1(s,s_1,m^2)\,m^2}{3\pi^2 s\sigma^2}\left[s^2+(s_1-2s_2)\,s-2s_1^2+s_2^2+s_1 s_2\right] \\
&- e^2\,\mathcal{C}_0(s,s_1,s_2,m^2)\left[\frac{4m^4}{3\pi^2 s\sigma} + \frac{m^2}{3\pi^2 s\sigma^2}\left[s^3-(s_1+s_2)\,s^2-\left(s_1^2-6s_2 s_1+s_2^2\right)s\right.\right. \\
&\left.\left. + (s_1-s_2)^2(s_1+s_2)\right]\right],
\end{aligned}$$

Non anomalous form factor: $F_2 = 0$ for $m = 0$

D_1, D_2 and C_0 are scalar integrals with bubble and triangle topologies

Explicit results: the QED case 3.

We present the first two (traceful) form factors

Armillis, Corianò, L. D. R., Phys.Rev.D81 (2010) 085001

The off-shell massive form factors F_i, with

- $s \neq 0 \qquad s_1 \neq 0 \qquad s_2 \neq 0$

and with $\gamma \equiv s - s_1 - s_2$, $\sigma \equiv s^2 - 2(s_1 +$

$$\mathbf{F_1(s;\, s_1,\, s_2,\, m^2)} = \frac{e^2 \gamma m^2}{3\pi^2 s \sigma} + \frac{e^2 \mathcal{D}}{} \ldots$$
$$- \frac{e^2}{18\pi^2 s} - \frac{e^2 \mathcal{D}_1(s, s_1, m^2)\, s_1 \left[- \ldots\right.}{}$$
$$- e^2\, \mathcal{C}_0(s, s_1, s_2, m^2) \left[\frac{m^2 \gamma \left[(s - \ldots\right.}{} \right.$$

$$\mathbf{F_2(s;\, s_1,\, s_2,\, m^2)} = -\frac{2e^2 m^2}{3\pi^2 s \sigma} - \frac{2\, e}{} \ldots$$
$$- \frac{2e^2\, \mathcal{D}_1(s, s_1, m^2)\, m^2}{3\pi^2 s \sigma^2} \left[s^2 + (s_1 \ldots\right.$$
$$- e^2\, \mathcal{C}_0(s, s_1, s_2, m^2) \left[\frac{4m^4}{3\pi^2 s \sigma} + \frac{}{3} \ldots \right.$$

Conformal anomaly pole

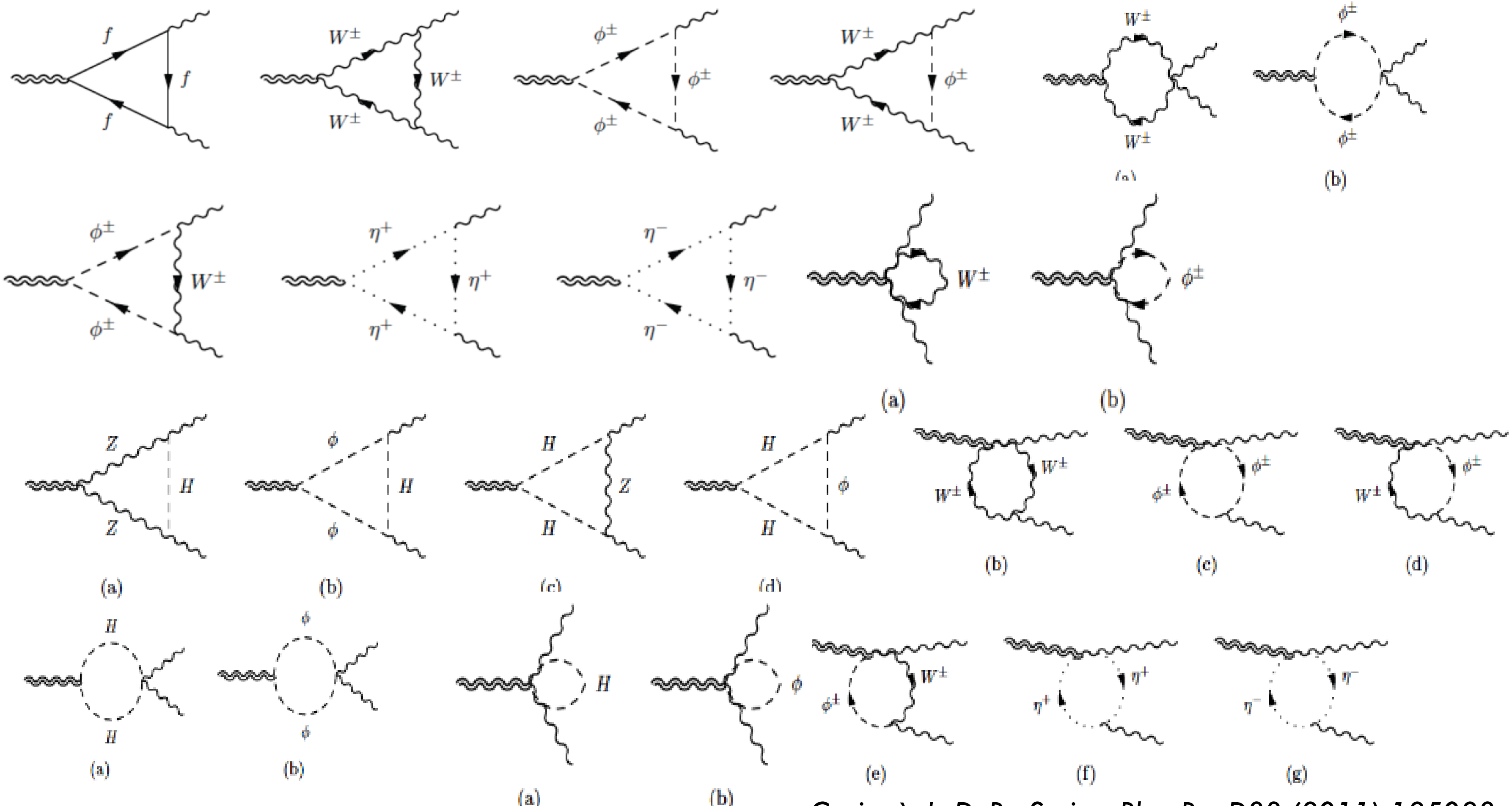
This is the only contribution which breaks the naïve trace identity

$$g_{\mu\nu} \Gamma^{\mu\nu\alpha\beta}(p,q) = -\frac{2\beta(e)}{e} u^{\alpha\beta}(p,q)$$

remember the anomaly equation in the gauge sector $T^{\mu}_{\mu} \sim F^2$

$$u^{\alpha\beta}(p,q) = \mathcal{F.T.}\left\{ -\frac{1}{4} \frac{\delta^2 [F_{\rho\sigma} F^{\rho\sigma}]}{\delta A_\alpha \delta A_\beta} \right\}$$

Computation in the Standard Model



Corianò, L. D. R., Serino, Phys.Rev.D83 (2011) 125028

Conformal anomaly poles in the SM

As in the QED case, the traceful part of the <TVV'> amplitudes contains conformal anomaly poles in every gauge invariant sector of the SM, for instance:

<Tgg>

$$\Phi_{1,pole} = \Phi^{(F)}_{1,pole} + \Phi^{(g)}_{1,pole} = \frac{i\,\kappa}{3}\frac{\beta_g}{g}\frac{1}{k^2}$$

<TAA>

$$\Phi_{1,pole} = \Phi^{(F)}_{1,pole} + \Phi^{(W)}_{1,pole} + \Phi^{(I)}_{1,pole} = \frac{i\,\kappa}{3}\frac{\beta_e}{e}\frac{1}{k^2}$$

<TZZ>

$$\Phi_{1,pole} = \Phi^{(F)}_{1,pole} + \Phi^{(W)}_{1,pole} + \Phi^{(Z,H)}_{1,pole} + \Phi^{(I)}_{1,pole} = \frac{i\,\kappa}{3}\left[s_w^2\frac{\beta_1}{g_1} + c_w^2\frac{\beta_2}{g_2}\right]\frac{1}{k^2}$$

The 1/k^2 singularity characterises the anomalous part of all the TVVs

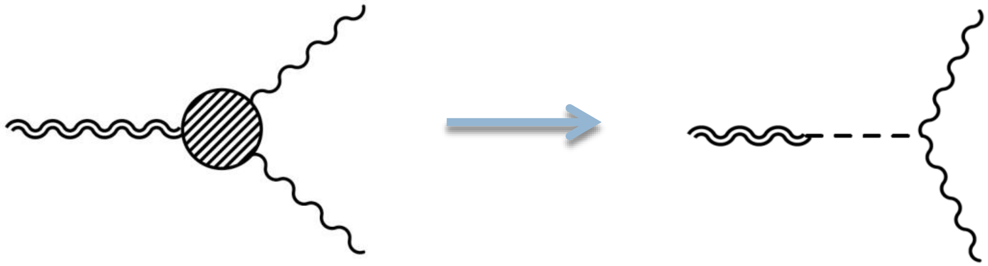
The coefficients of the poles depend on the β functions of $SU(3)xSU(2)_W xU(1)_Y$

Polology*: a correlation function can develop a pole in one of the momenta of the external lines, or in a combination of them, through the coupling of a particle (elementary or composite) in the physical spectrum of the theory*

Conformal anomaly poles in the SM

Perturbative computations show the appearance of effective degrees of freedom.

This is a consequence of the anomalous breaking of the conformal symmetry

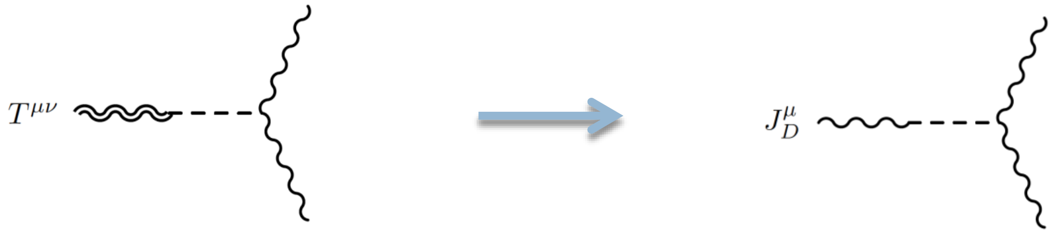


***Polology**: a correlation function can develop a pole in one of the momenta of the external lines, or in a combination of them, through the coupling of a particle (elementary or composite) in the physical spectrum of the theory*

The pole of the dilatation current

Intepretation of the results:

1) Notice that the correlation functions with the dilation current J_D inherit the same pole behavior of those with the EMT $J_D^\mu = x_\nu T^{\mu\nu}$



$$\langle T^{\mu\nu} V^\alpha V^\beta \rangle \sim \frac{\beta}{g} \frac{1}{k^2} (k^2 \eta^{\mu\nu} - k^\mu k^\nu) u^{\alpha\beta}(p,q) \qquad \langle J_D^\mu V^\alpha V^\beta \rangle \sim \frac{\beta}{g} \frac{k^\mu}{k^2} u^{\alpha\beta}(p,q)$$

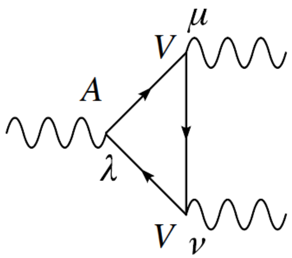
Corianò, L.D.R., Quintavalle, Serino, JHEP 1306 (2013) 077

2) Use the anology with the pion case and the axial anomaly in the AVV diagram …

The analogy with the pion case

Indeed correlation functions with the axial-vector current A are characterized by a massless pole singularity

Dolgov, Zakharov. Nucl. Phys. B27, 525 (1971)



$$\Delta^{\lambda\mu\nu} = a_n \frac{k^\lambda}{k^2} \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta}$$

The pole structure clearly describes the pseudoscalar pion π, the Nambu-Goldstone boson of the spontaneous breaking of the chiral symmetry, which interpolates between the axial and the two vector currents

Therefore

The anomaly pole in <J_DVV> can be interpreted as an effective scalar, the Nambu-Goldstone boson of the breaking of conformal symmetry, in analogy with the pion in the chiral theory of the strong interactions

The analogy with the pion case

Nature has already chosen this mechanism!

- ✓ The axial anomaly pole in the <AVV> diagram → *the pseudoscalar pion* ✓
- ✓ The conformal anomaly pole in the $<J_D VV>$ diagram → *the scalar dilaton (a composite state ?)* ?

Pertubation theory provides a hint on the existence of new states

We can speculate on the existence of a dilaton in the spectrum of the SM

Corianò, L.D.R., Quintavalle, Serino, JHEP 1306 (2013) 077

SM corrections to fermion interactions in a gravitational background

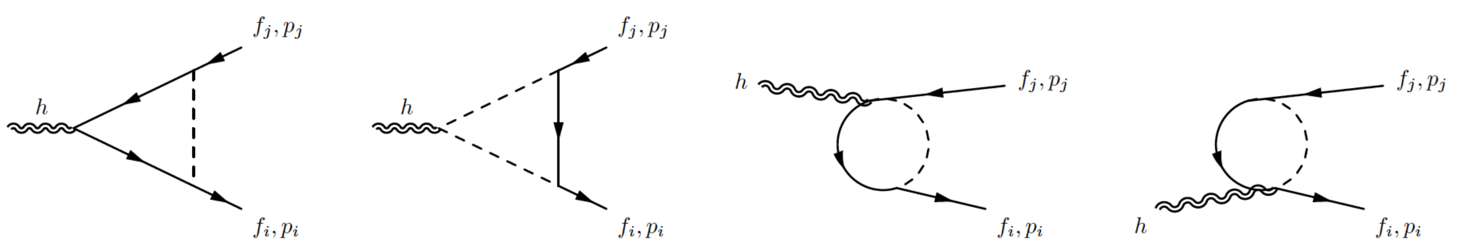
One-loop Standard Model corrections to $\langle T f_i f_j \rangle$ in the *flavor diagonal* and *off-diagonal* sector

- Extend the SM effective action in a gravitational background to the fermion sector
- Inspection of the electroweak corrections to the Newton's potential
- Analysis of the gravitational interactions of neutrinos (differences between Dirac and Majorana neutrinos?)

Degrassi, Gabrielli, Trentadue, Phys.Rev.D79 (2009)

Corianò, L.D.R., Gabrielli, Trentadue, Phys.Rev.D87 (2013)

Corianò, L.D.R., Gabrielli, Trentadue, Phys.Rev.D88 (2013)



Dashed lines can be gluons, photons, Higgs, Z and W bosons

Scale invariant extensions of the SM

We have to find a model (with a bottom-up construction) to accomodate the results obtained from perturbation theory:

a scalar state coupling to the dilatation current

We consider a scale invariant extension of the Standard Model

Corianò, L.D.R., Quintavalle, Serino, JHEP 1306 (2013) 077
Goldberger, Grinstein, Skiba, PRL 100 (2008) 111802
Abe, Kitano, Konishi, Oda, Sato, Sugiyama, PRD 86 (2012) 115016
Fan, Goldberger, Ross, Skiba, PRD 79 (2009) 035017

Scale invariant extensions of the SM

We have to find a model (with a bottom-up construction) to accomodate the results obtained from perturbation theory:

a scalar state coupling to the dilatation current

We consider a scale invariant extension of the Standard Model

The only source of (classical) scale breaking in the Standard Model is in the Higgs potential

$$V = \lambda \left(H^\dagger H - \frac{\mu^2}{2\lambda} \right)^2$$

$$\mu \to \mu \frac{\Sigma}{\Lambda}$$

Classical scale invariance can be recovered introducing a new dynamical field, the dilaton Σ

$$V = \lambda \left(H^\dagger H - \frac{\mu^2}{2\lambda} \frac{\Sigma^2}{\Lambda^2} \right)^2$$

Note also that the invariance of the potential under the addition of constant terms is lifted once we require the dilatation symmetry. This may affect the vacuum energy.

Scale invariant extensions of the SM

Analysis of the scalar potential

- Spontaneous electroweak symmetry breaking $$H = \frac{v+h}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
- Spontaneous conformal symmetry breaking $$\Sigma = \Lambda + \rho$$

Diagonalization of the mass matrix:

$$\begin{pmatrix} \rho_0 \\ h_0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \rho \\ h \end{pmatrix}$$

$$\cos\alpha = \frac{1}{\sqrt{1+v^2/\Lambda^2}}$$
$$\sin\alpha = \frac{1}{\sqrt{1+\Lambda^2/v^2}}$$

Mass eigenstates:

$$m_{h_0}^2 = 2\lambda v^2 \left(1 + \frac{v^2}{\Lambda^2}\right)$$ *Higgs*

$$m_{\rho_0}^2 = 0$$ ***Dilaton, a Nambu-Goldstone boson***

Explicit (small) symmetry breaking terms → $m_{\rho_0} \ll \Lambda$ Small Dilaton mass

Dilaton interactions with SM states

The dilaton interactions with fermions and gauge bosons can be obtained from the SM Higgs Lagrangian after diagonalization

$$\mathcal{L}_h = \left(2\frac{h}{v} + \frac{h^2}{v^2}\right)\left(m_W^2 W_\mu^+ W^{-\mu} + \frac{m_Z^2}{2} Z_\mu Z^\mu\right) - \frac{h}{v} m \bar{\psi}\psi$$

$$h = \sin\alpha\, \rho_0 + \cos\alpha\, h_0$$

Anomalous terms must be introduced by hand

The identification of the pole structure in the <J_DVV> correlator with a dilaton exchange provides its anomalous coupling with massless gauge bosons

$$\mathcal{L} = \frac{\alpha_s}{8\pi} b_g \frac{\rho}{\Lambda} (F^a_{g\,\mu\nu})^2 + \frac{\alpha_{em}}{8\pi} b_{em} \frac{\rho}{\Lambda} (F_{\gamma\,\mu\nu})^2 \qquad \beta = \frac{g^3}{16\pi^2} b$$

A phenomenological analysis (*work in progress*) could be helpful to distinguish the dilaton from other scalar states appearing in ED models, MSSM, NMSSM, inflation…

Superconformal anomalies

In a supersymmetric field theory the energy-momentum tensor belongs to a *chiral multiplet*, together with the ***R-current*** and the ***supersymmetric current***.

The Ferrara-Zumino multiplet: $\mathcal{J}^\mu = (R^\mu, T^{\mu\nu}, K^\mu)$

For a SU(N) gauge theory with a matter chiral multiplet:

vector (gauge) supermultiplet: $V = \left(A^a_\mu, \lambda^a, D^a\right)$

chiral (matter) supermultiplet: $\Phi = (\phi_i, \chi_i, F_i)$

we define

$$R^\mu = \bar{\lambda}^a \bar{\sigma}^\mu \lambda^a + \frac{1}{3}\Big[-\bar{\chi}_i \bar{\sigma}^\mu \chi_i + 2i\phi_i^\dagger \mathcal{D}^\mu_{ij}\phi_j - 2i\left(\mathcal{D}^\mu_{ij}\phi_j\right)^\dagger \phi_i \Big]$$

$$K^\mu = -\sqrt{2}\left(\sigma_\nu \bar{\sigma}^\mu \chi_i\right)\left(\mathcal{D}^\nu_{ij}\phi_j\right)^\dagger - \frac{1}{2}\left(\sigma^\nu \bar{\sigma}^\rho \sigma^\mu \bar{\lambda}^a\right) F^a_{\nu\rho} - ig\left(\phi_i^\dagger T^a_{ij}\phi_j\right)\sigma^\mu \bar{\lambda}^a$$

Anomalies in $\mathcal{N}=1$ Super-YM

The Ferrara-Zumino supercurrent satisfies the anomaly equation:

$$\bar{D}^{\dot{A}} \mathcal{J}_{A\dot{A}} = a_n \, D_A W^2$$

$$a_n = -\frac{2}{3} g^2 \frac{3T_G - \sum_f T(R_f)}{16\pi^2}$$

The component anomaly equations:

$$\partial_\mu R^\mu = -\frac{a_n}{2} F^a_{\mu\nu} \tilde{F}^{a\,\mu\nu}$$

$$T^\mu_\mu = \frac{3}{4} a_n \, F^a_{\mu\nu} F^{a\,\mu\nu}$$

$$\bar{\sigma}_\mu K^\mu = 3i \, a_n \, \bar{\lambda}^a \bar{\sigma}^{\mu\nu} F^a_{\mu\nu}$$

The supersymmetric current is characterized by an anomaly pole and this suggests the existence of an effective spin ½ state (*dilatino*)

We can build a new chiral supermultiplet with a *dilaton*, an *axion* and a *dilatino*

$$\mathcal{S} = (\varphi, \psi, F)$$

$$\varphi = \rho + i\, a$$

Conclusions and perspectives

- ✓ One-loop effective action of the Standard Model in a gravitational background
- ✓ The conformal anomaly implies the existence of massless effective degrees of freedom
- ✓ Dilaton as the Nambu-Goldstone boson of the conformal symmetry breaking
- ✓ Phenomenological applications in cosmology, low gravity scale models and scale-invariant extensions of the SM