Tesi di laurea magistrale in Fisica.
Anno 2012-2013

Search for a jet-jet resonance associated with a W/Z, decaying leptonically, with the ATLAS detector: application of a multiresolution analysis.

INTRODUCTION

- The problem of the detection of weak signals is of high importance in high energy physics. The method used should have high sensitivity and reliability.

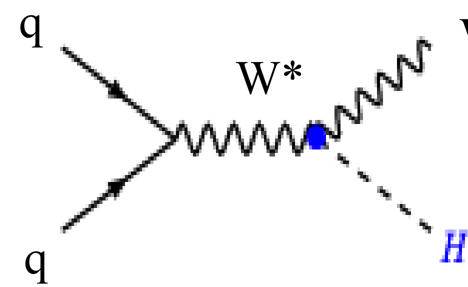
- In this thesis, a multiresolution analysis method has been applied to the search of resonances in invariant mass spectra.
  - The method we have investigated (*wavelet analysis*) has never been used in high energy physics, although it has been applied for analysis in various fields.
  - This is an explorative work still evolving.

- We analyzed the invariant mass of jet pairs produced in association with a leptonically decaying *W* (from *p-p* collisions in ATLAS at $\sqrt{s}$=7 TeV).
  - This channel is sensitive to Standard Model signals of interest (*W/Z* and Higgs boson decays) and also to hypothetical particles from unconventional theories.

STANDARD MODEL EXPECTED SIGNALS

- ***WW/WZ*** (diboson).
  - Measured in this channel by the ATLAS collaboration.

$$\sigma_{WW/WZ} = 72 \pm 9 \text{ (stat.) } \pm 15 \text{ (syst.) } \pm 13 \text{ (MC stat.)}$$



- Higgs boson contributes via specific channels of production and decay:
  - Produced via *WH* associated production.
  - *W* decays leptonically and *H* decays into two jets (mostly $H \to b\bar{b}$).
  - $H \to b\bar{b}$ was recently observed by CMS, no signal has yet been observed by ATLAS.

Cross section at √s=7 TeV

$$\sigma_{HW} = 0.558^{+3.7\%}_{-4.3\%} \text{ pb}$$

Higgs total width: $\Gamma = 4.18 \cdot 10^{-3}$ GeV

Higgs decay	BR (Γ_i/Γ)
$H \to b\bar{b}$	$5.61 \cdot 10^{-1}{}^{+3.3\%}_{-3.4\%}$

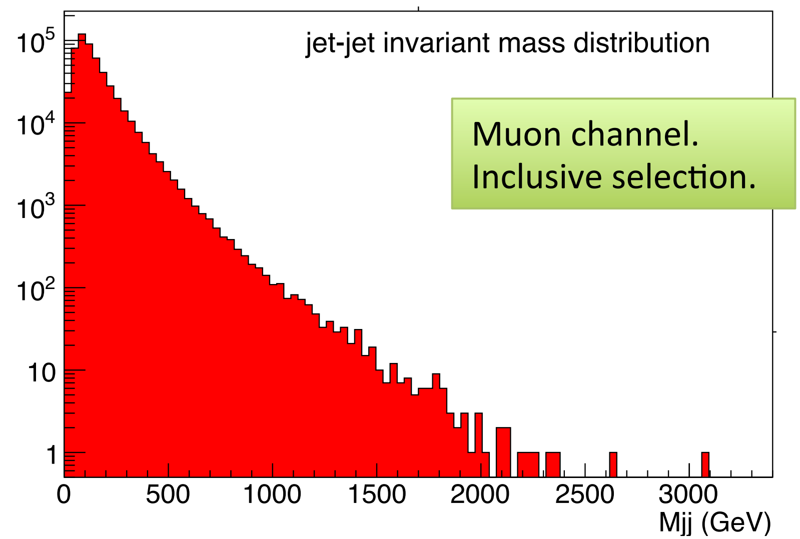
W^+/W^- decay modes	BR (Γ_i/Γ)
$e^+\nu/e^-\overline{\nu}$	$(10.75 \pm 0.13)\%$
$\mu^+\nu/\mu^-\overline{\nu}$	$(10.57 \pm 0.15)\%$

- The analized dataset has an integrated luminosity of *L*=4702 pb^{-1}. Considering the *W* decay rates, the number of produced events in this channel is expected to be:

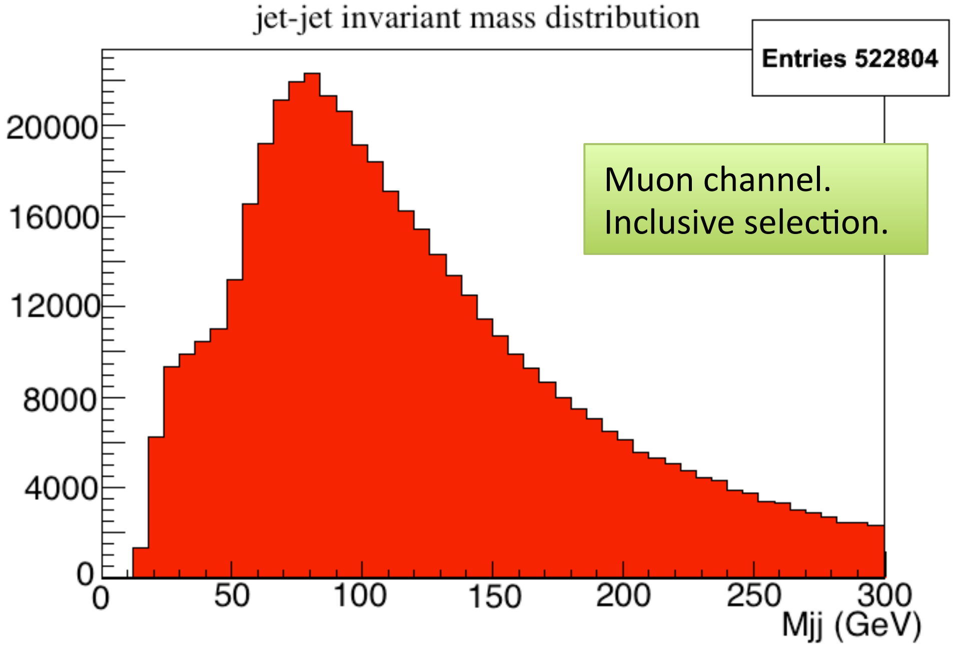
$$N_{WH} \sim 200 \text{ events}$$

EVENT SELECTION

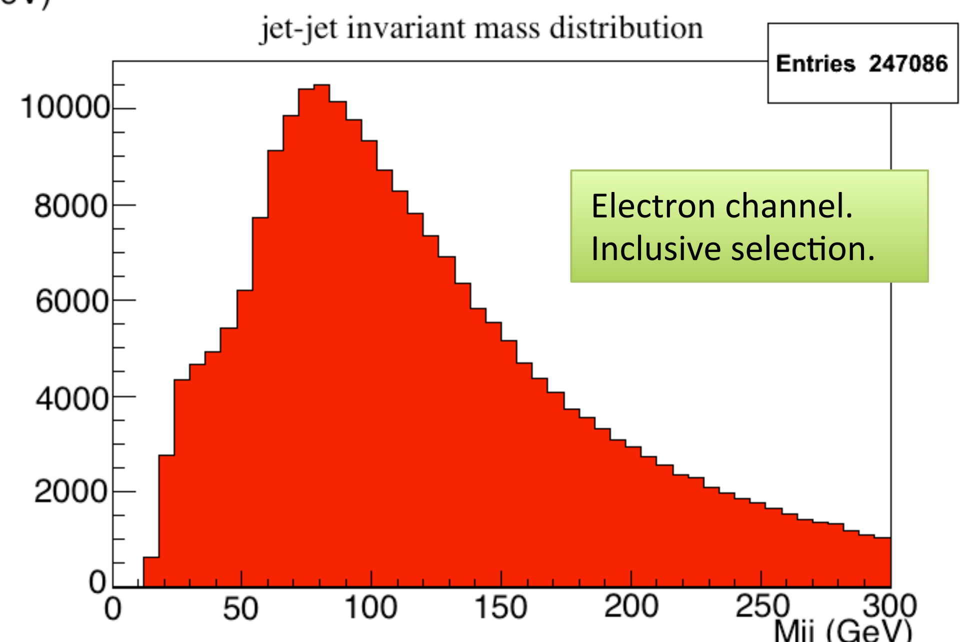
- The analysis is performed on data acquired by the ATLAS experiment in 2011: $\sqrt{s}=7$ TeV and integrated luminosity $L=4.702\ fb^{-1}$.
- Dijets events are selected by requiring a W ⟶ lν decay.
  - Select one single charged lepton (muon or electron) passing the <u>lepton selection</u>:
    - Lepton trigger + $p_T > 25\ GeV$
    - $|\eta| < 2.4$
    - Cut on impact parameter with respect to primary vertex.
    - Track and calorimeter isolation.
  - Events must have a neutrino: $E_t^{miss} > 25$ GeV
  - Select *W* events: $M_T > 40$ GeV
- <u>Jet selection</u> to reduce background:
    - $p_T > 25$ GeV
    - $|\eta| < 2.8$
    - Jet Vertex Fraction > 0.75 (to reject pile-up)
    - $\Delta R(j,l) > 0.5$
  - The two jets of highest p_T are used to build the invariant mass spectrum.



INVARIANT MASS SPECTRUM



- The mass spectrum has a maximum at around 80 GeV.
- At higher mass it decreases exponentially
- It reaches values of Mjj > 2 TeV in both channels.



WAVELET ANALYSIS: AN INTRODUCTION

- A multiresolution method allows to separate structures of different dimensions in mass.

- The *wavelet analysis* is a multiscale method based on *wavelet transform.*
  - It was developed for the detection of local structures in time series.
  - It can be applied to the analysis of any random variable *m* of density *f(m)*.

- Wavelet transform (continuous case):
  - Here, ψ is the *Mexican Hat* (DoG) function.
  - It can be any local function with zero mean.

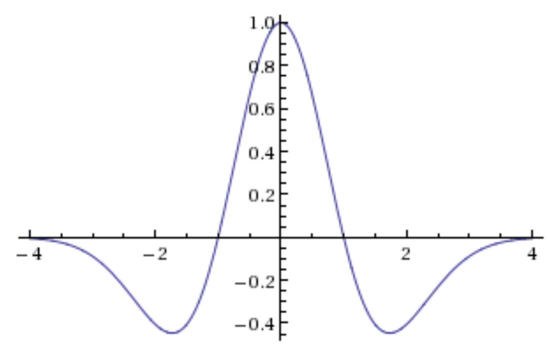
$$W(m,s) \;=\; \int f(m') \cdot \psi^* \left(\frac{m'-m}{s} \right) \, dm'$$

- Varying *m* and the *scale s*, *W(m,s)* gives a global picture of *f(m)* features.

$$s_j \;=\; s_0 2^{j\delta j}, \qquad j \;=\; 0, 1, ..., J$$

- In practice, *f(m)* is substituted by the mass histogram.

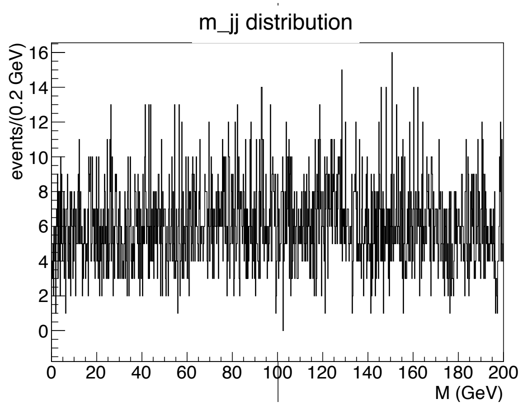
$$W(m,s) \;=\; \sum_{n'=0}^{N-1} x_{n'} \cdot \psi^* \left(\frac{(n'-n)\delta m}{s} \right)$$

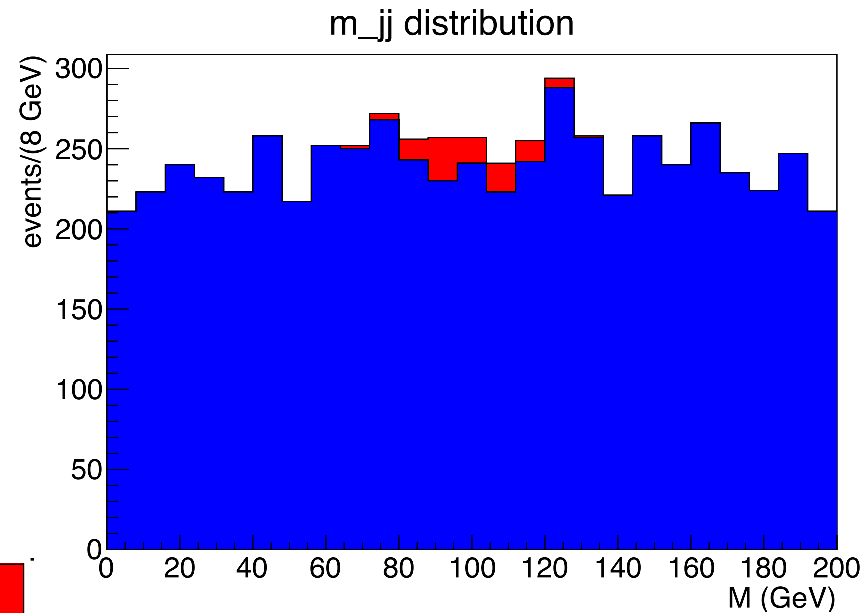


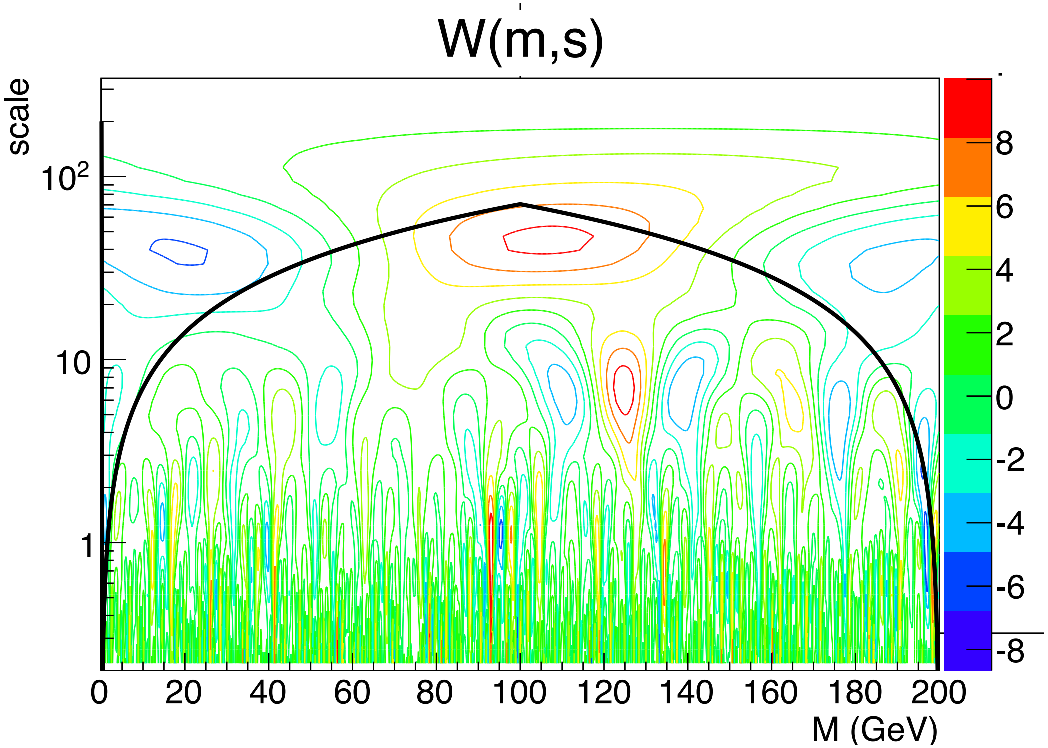
WAVELET ANALYSIS: AN EXAMPLE

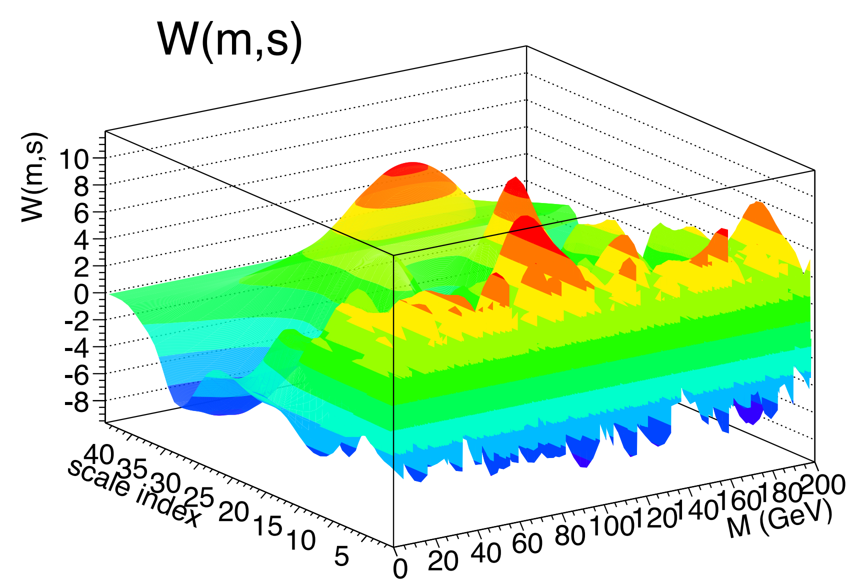
Flat background:
6000 events.

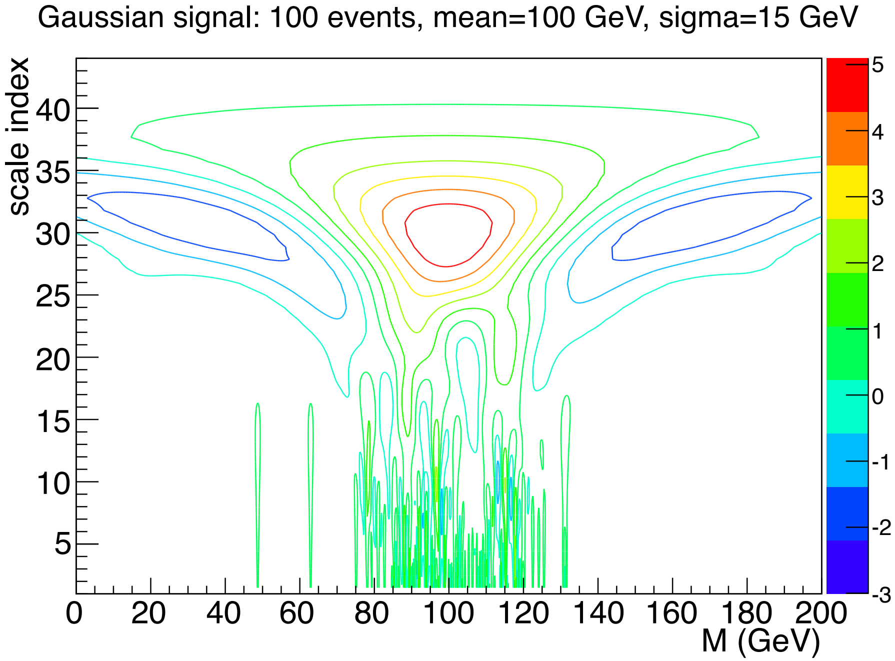
Gaussian signal:
100 events,
mean μ=100 GeV,
width σ=15 GeV.

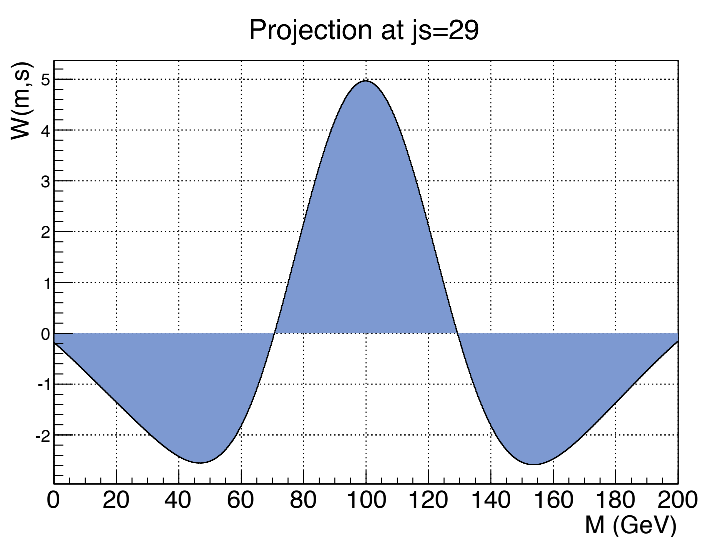












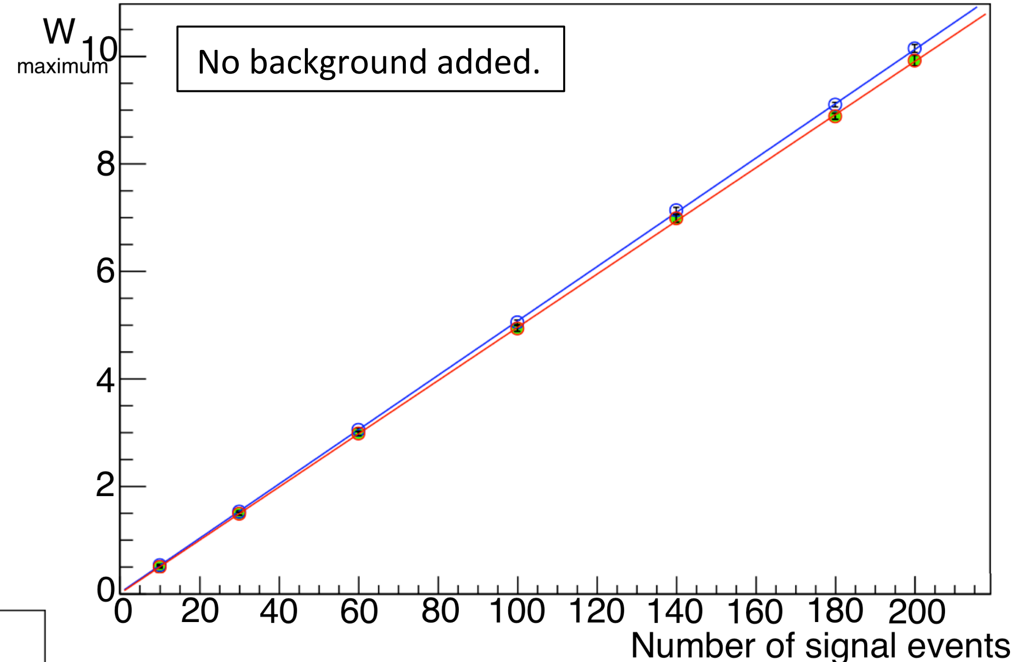
<u>Wavelet transform of a gaussian signal.</u>

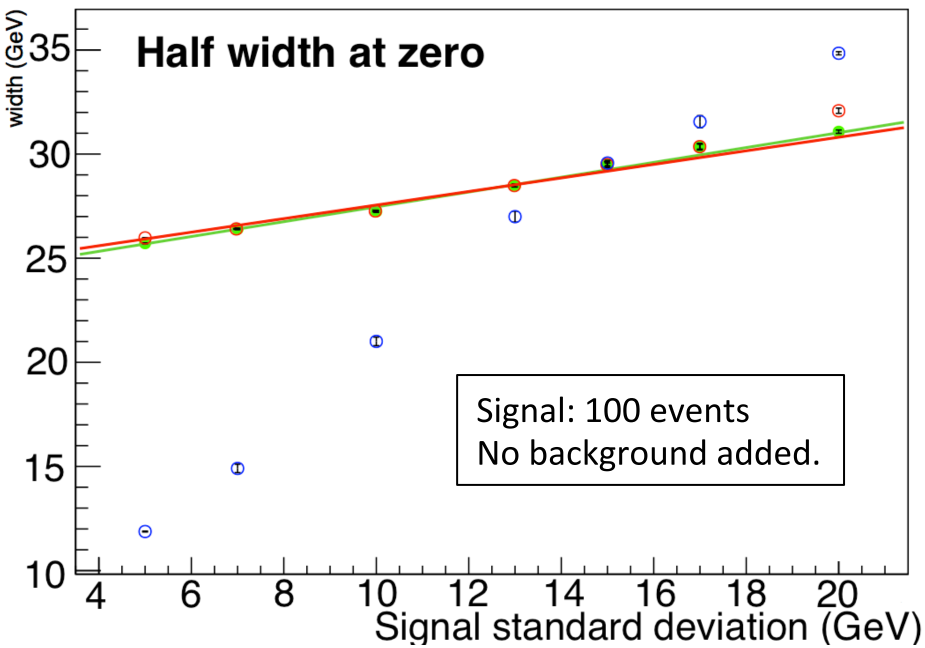
$$W(m,s) \;=\; A(s,\sigma)\cdot\delta m\cdot N_{ev}\cdot\left(1-\frac{(n\delta m-\mu)^2}{\sigma^2+s^2}\right)e^{-\frac{(n\delta m-\mu)^2}{2(\sigma^2+s^2)}}$$

- *W(m,s)* has a DoG-like shape, with mean corresponding to the signal mean.
- *W(m,s)* depends linearly on the number of events.
- *W(m,s)* depends also on the signal standard deviation.
  - It is not expected to be highly sensitive to signal width, due to the DoG shape.

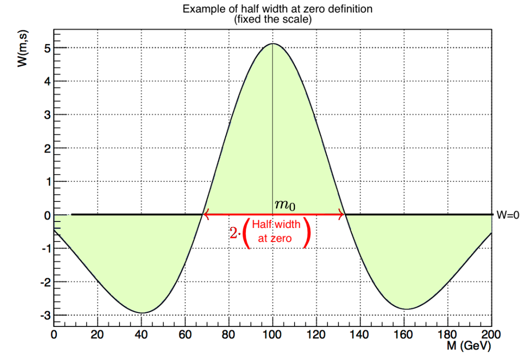
DEPENDENCE ON SIGNAL PARAMETERS: CHECK WITH TOY MONTECARLO

- We use maximum of *W(m,s)* to estimate the number of signal events.
  - The maximum has been defined both at fixed scale (red, green) and variable scale (blue).
  - All linear in N_{ev} with zero intercept.



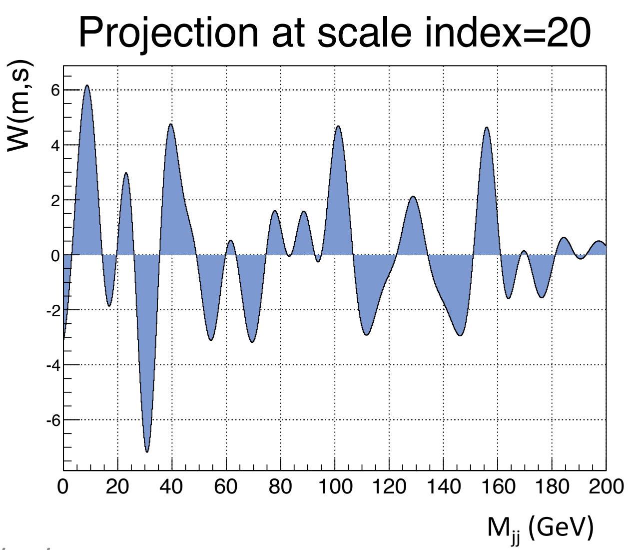


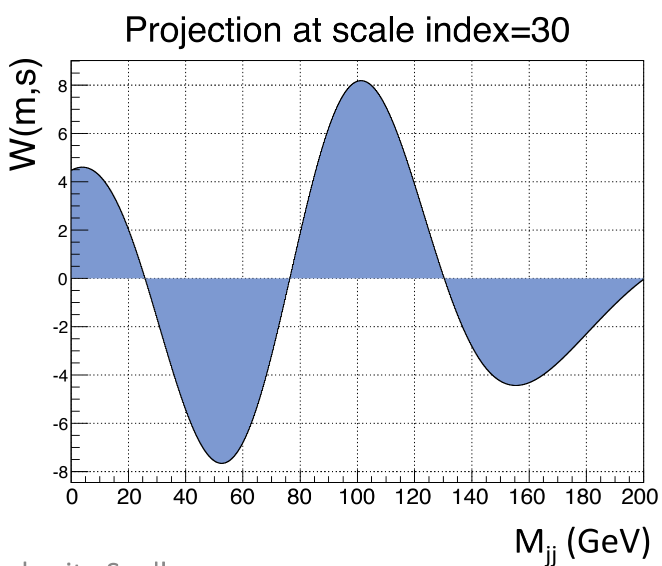
- We use the half width at zero as a function of signal width.
  - Signal σ will not be evaluated from wavelet analysis.



- Flat background is the condition in which wavelet analysis applied in most of literature.
  - *W(m,s)* is computed with respect to arithmetic mean of the data.

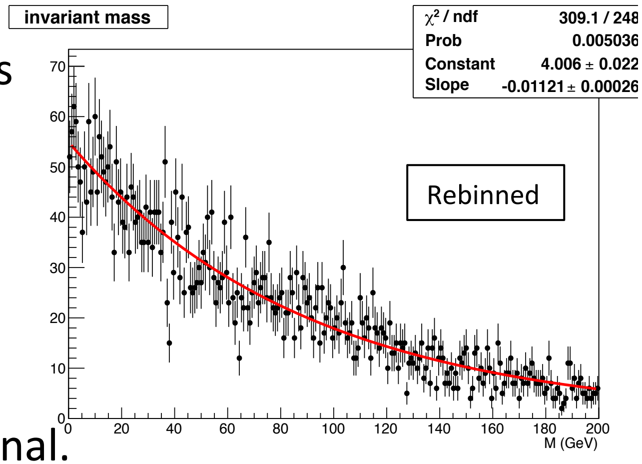
- At lower scale the wavelet transform is dominated by statistical fluctuation: only the scale region js ≥ 25 is used for the analysis.

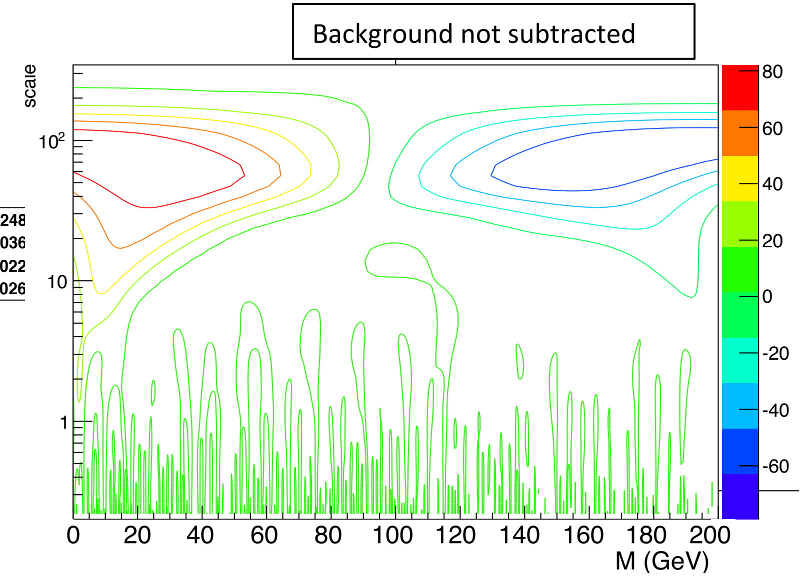




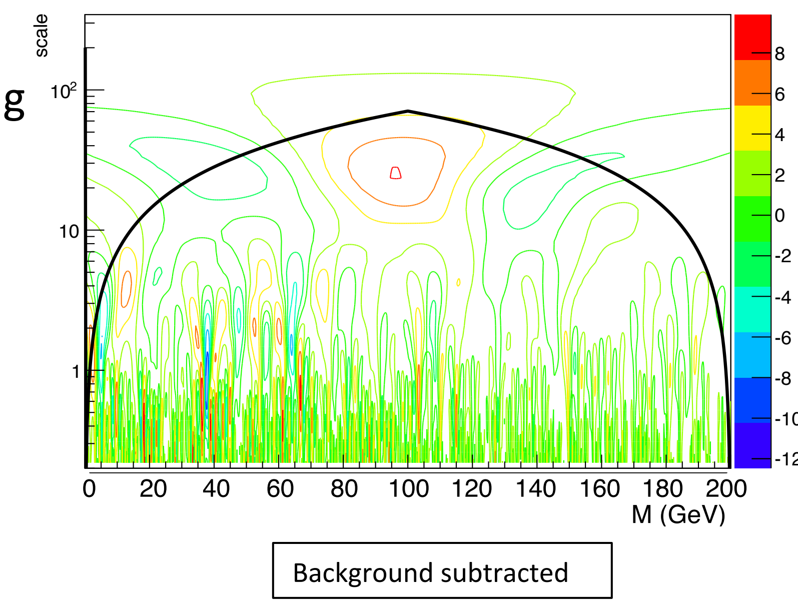
EXPONENTIAL BACKGROUND

- Exponential background affects *W(m,s)*.
- The adopted strategy is background subtraction.
  - Background shape is fitted to data.
  - Fit problems may generate fakes.
  - Proper subtraction retains a good sensitivity to the signal.

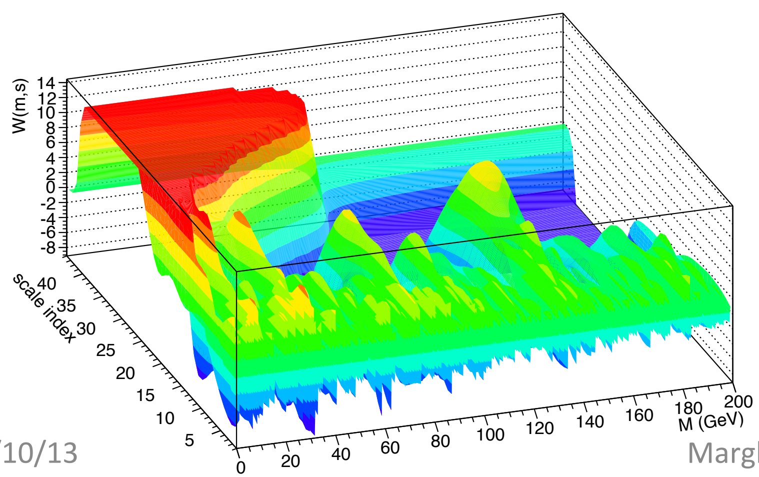




Wavelet transform



- A qualitative check on a signal can be done performing wavelet transform without background subtraction.



- Maximum of *W(m,s)*:
  - Nonzero intercept, due to the inclusion of background events in the wavelet convolution.

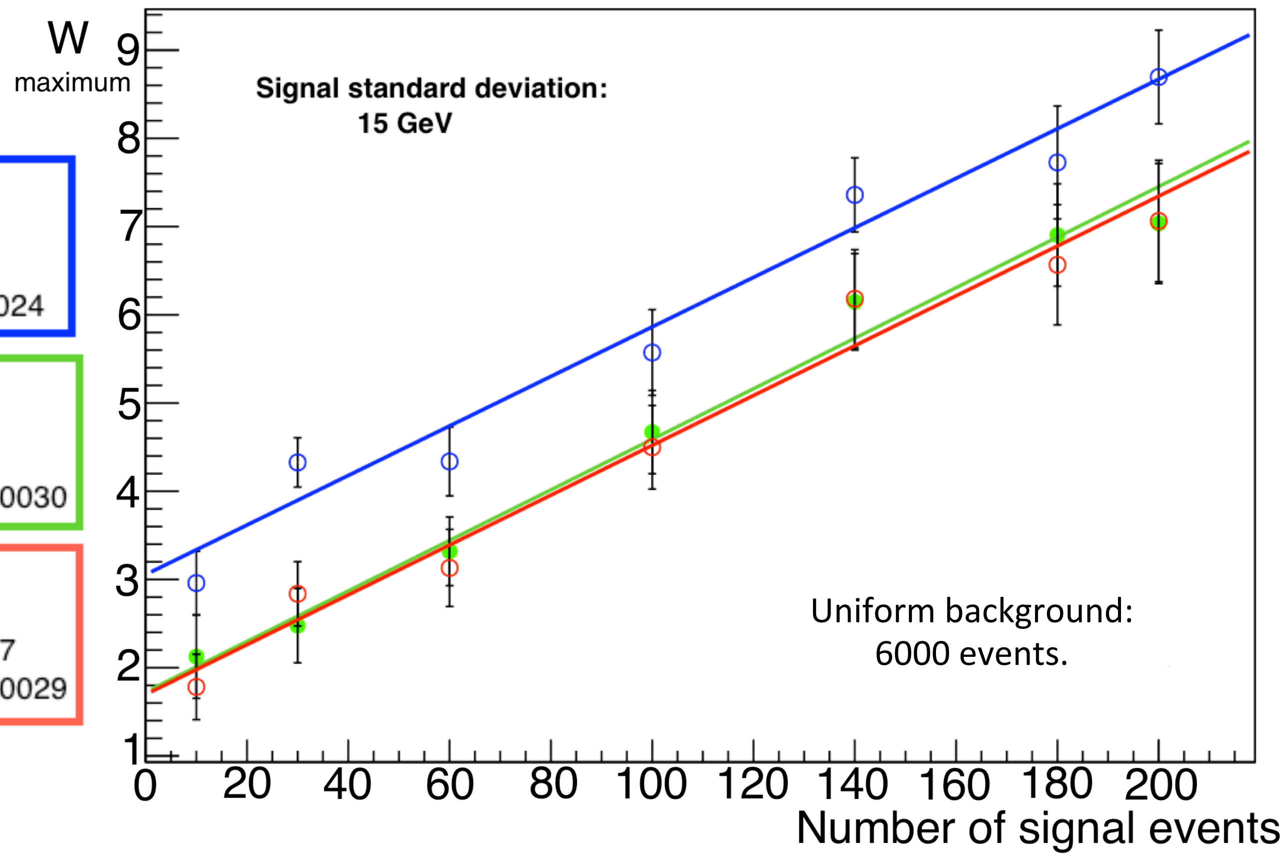
Blue: $W_{max}(N)$ at variable scale

Green and red: $W_{max}(N)$ at fixed scale

Chi2	=	6.02423
NDf	=	5
p0	=	3.1 +/- 0.24
p1	=	0.028 +/- 0.0024

Chi2	=	1.20486
NDf	=	5
p0	=	1.7 +/- 0.31
p1	=	0.029 +/- 0.0030

Chi2	=	2.46798
NDf	=	5
p0	=	1.70 +/- 0.27
p1	=	0.028 +/- 0.0029



EXPONENTIAL BACKGROUND: DEPENDENCE ON NUMBER OF SIGNAL EVENTS

- Dependence on number of signal events after background subtraction.
  - Conditions are similar to that with flat background.

$W_{max}(N)$
Variable scale

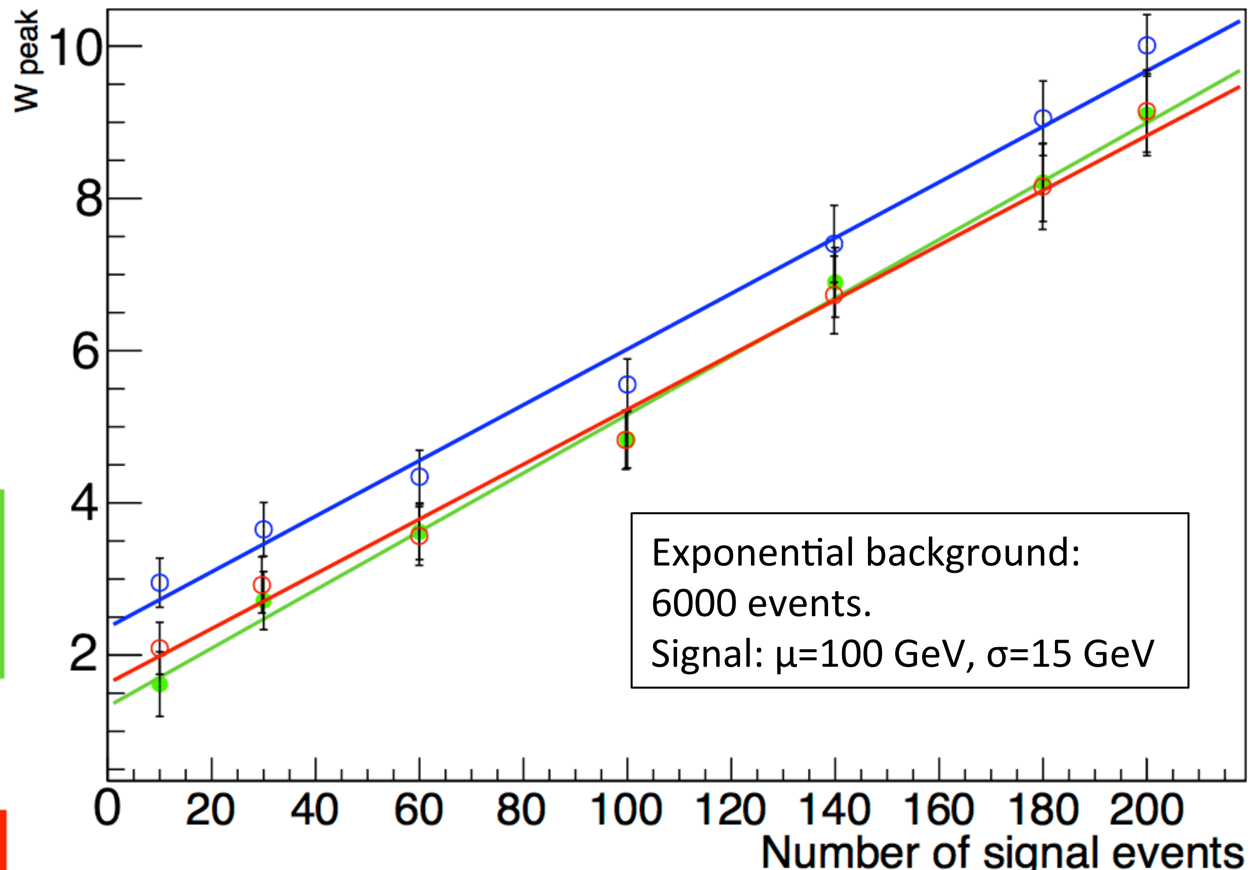
Chi2	=	3.82179
NDf	=	5
p0	=	2.4 +/- 0.23
p1	=	0.037 +/- 0.0021

$W_{max}(N)$
Fixed scale

Chi2	=	1.47943
NDf	=	5
p0	=	1.3 +/- 0.28
p1	=	0.038 +/- 0.0026

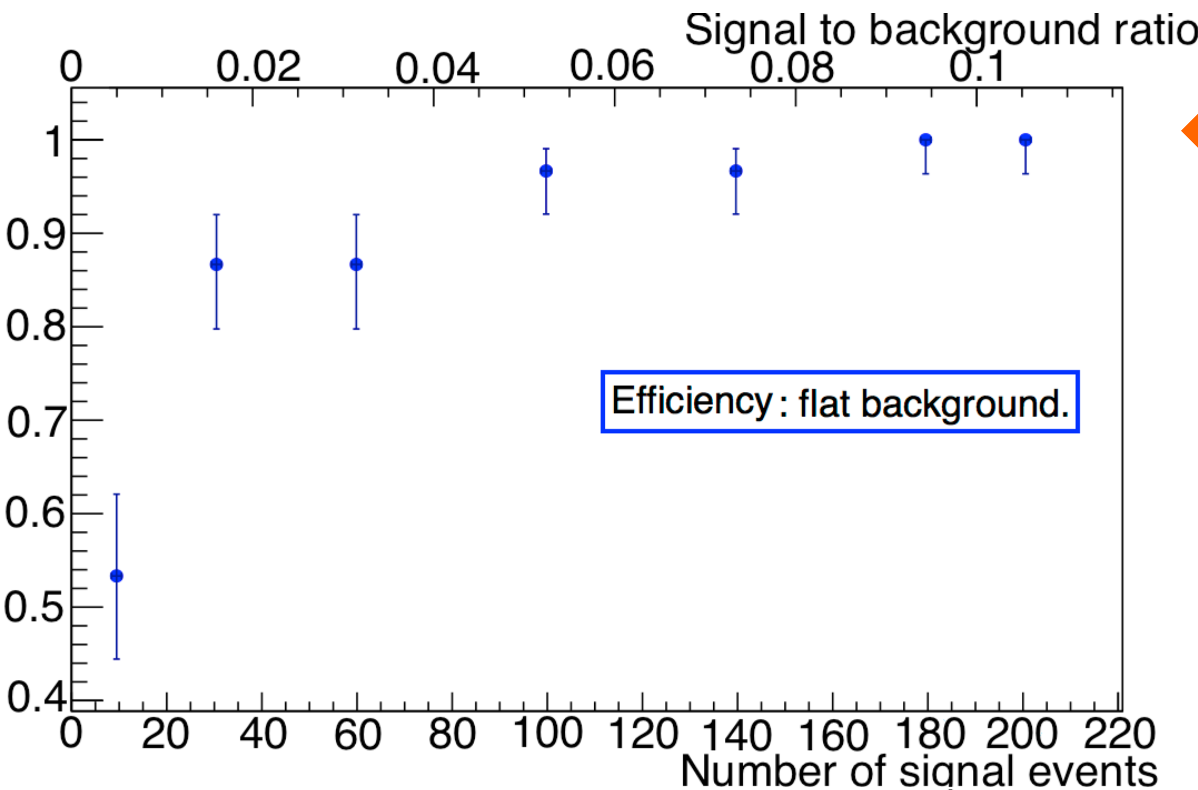
$W_{max}(N)$
Fixed scale

Chi2	=	2.18783
NDf	=	5
p0	=	1.6 +/- 0.25
p1	=	0.036 +/- 0.0025



- For each toy MonteCarlo sample, we looked for a *W(m,s)* local maximum:
  - within the region $j_s \geq 25$
  - compatible with the inserted signal.

- The efficiency is defined as the fraction of cases in which a compatible *W(m,s)* peak is found.
  - It is large even for very small signals.

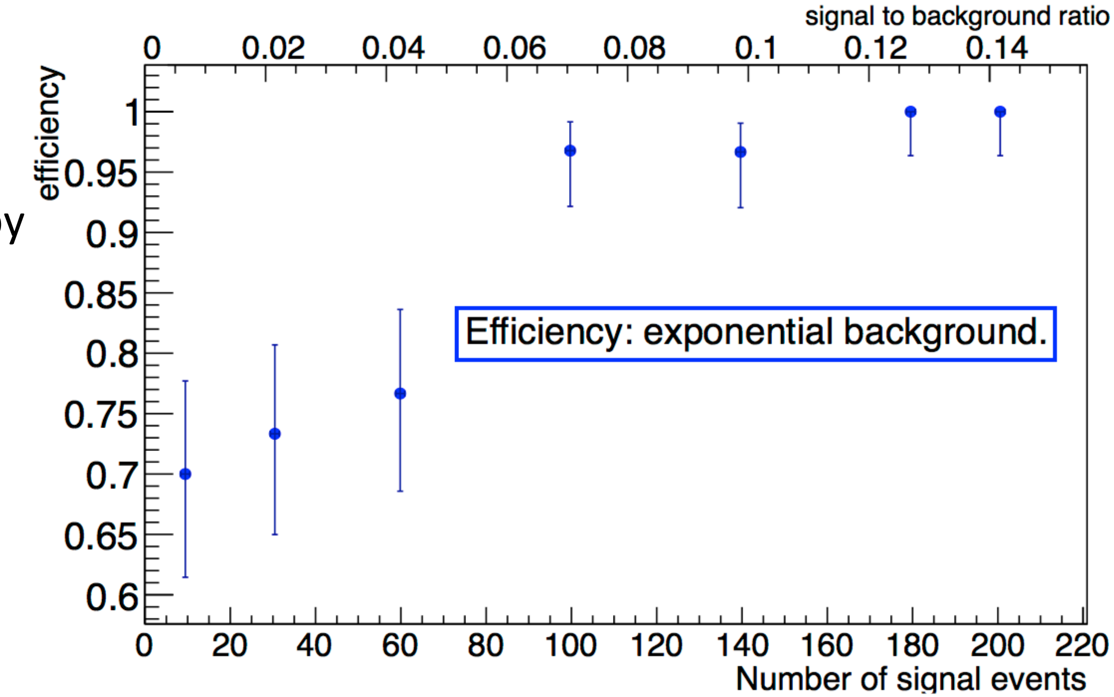


- Efficiency has been compared to what found with simple fit methods (gaussian + constant). This method is definitely more efficient.

Flat background:
6000 events.
Signal:
μ=100 GeV, σ=15 GeV

EXPONENTIAL BACKGROUND: EFFICIENCY AND FAKE RATE

- The efficiency is similar to what computed for flat background.
  - A bit smaller and affected by greater fluctuations.



- The fake rate is possibly increased by fit problems:
  - evaluated applying the efficiency algorithm to background-only MonteCarlo samples (6000 events).

$$R_{fakes} = 0.567 \pm 0.0082$$

- A confidence level must be defined to evaluate the significance of found peaks.
  - We use the *W(m,s)* maximum height as statistic.

CONFIDENCE LEVEL

- The significance level is computed locally, evaluating *W(m,s)* distribution, fixed *m,s*.
  - x_n are Poisson variables: we assume gaussian approximation to be valid

$$W(m,s) = \sum_{n'=0}^{N-1} x_{n'} \cdot \psi^* \left(\frac{(n'-n)\delta m}{s} \right) \quad \Rightarrow W(m,s) \sim N(0,\sigma_{(m,s)})$$

$$\sigma^2_{(m,s)} = Var(W(m,s)) = \sum_{n'=0}^{N-1} x_{n'} \cdot |c_{n'}(m,s)|^2$$

  - The α confidence level should be compared to $W(m,s)/\sigma_{(m,s)}$.

$$\alpha = \int_{x_{CL}}^{\infty} N(0,1)dx = 5\%$$

- We evaluated the mean number of peaks exceeding the 95% confidence level via toy MonteCarlo.

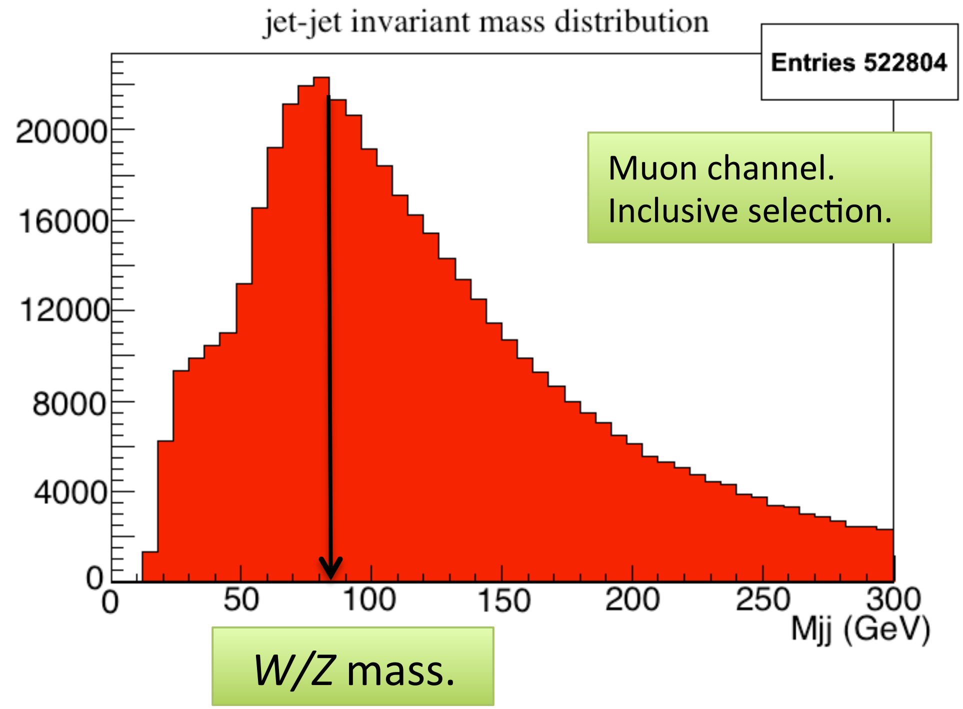
$$R_{fakes}^{overCL} = 0.46$$

- The fraction of false positive is large since in *W(m,s)* plot the number of independent channels is large and difficult to quantify because high and not uniform correlation between *m×s* bins.
  - To reduce the fake rate, a global confidence level should be defined.
  - The definition is made difficult by correlation effects.

WAVELET ANALYSIS OF JET-JET MASS SPECTRUM

W/Z SIGNAL

- The detection of *W/Z* boson is complicated because the background peaks at about 80 GeV.
  - The wavelet transform detects a huge peak, but it is impossible to correctly separate signal and background effects.
- The problem could be fixed by refining the sample selection.



[100,200] GeV MASS REGION

- The decreasing background have been fitted with an exponential and subtracted.
  - Fit quality appeared to be satisfactory
- Wavelet transform has then been computed.

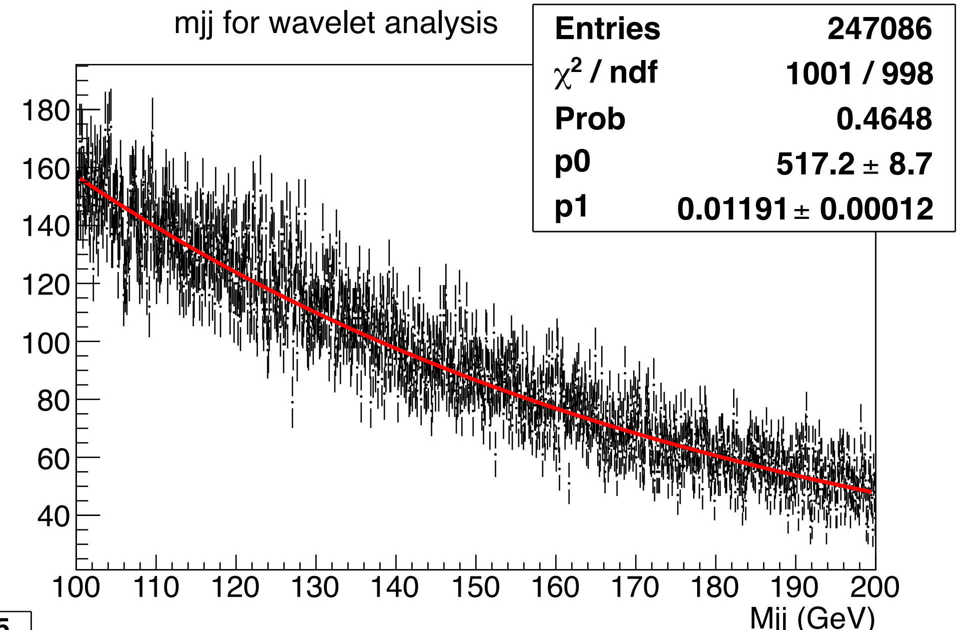
FIT QUALITY: EXAMPLE OF ELECTRON CHANNEL

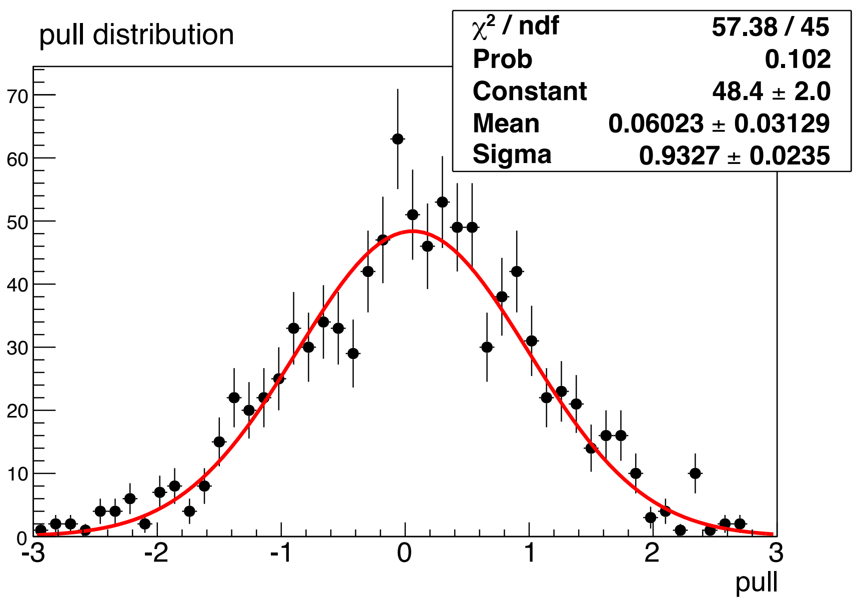
pulls

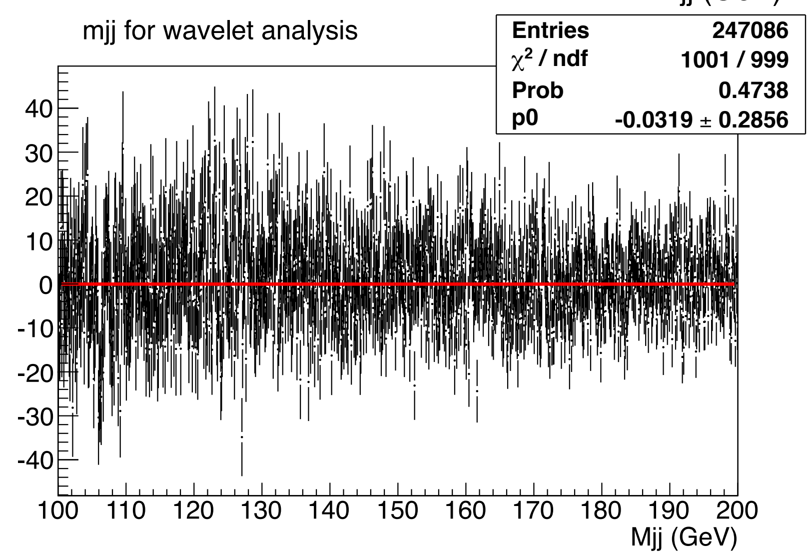
$$\epsilon_n = \frac{x_n - f(m_n)}{\sigma_n}$$

Expected to be a standard normal distribution.

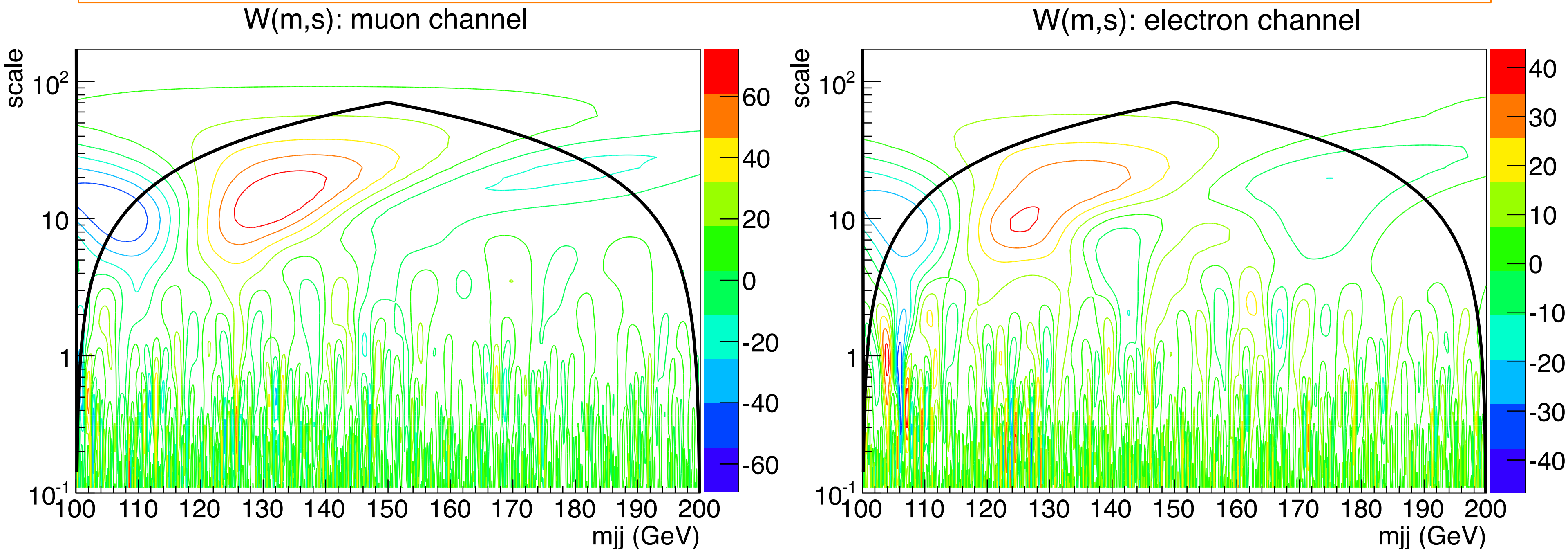
Mean	**0.06023 ± 0.03129**
Sigma	**0.9327 ± 0.0235**







RESULTS IN [100,200] GeV MASS REGION



- *W(m,s)* shows a signal at a mass compatible to Higgs mass in both e and mu channels

- The peak mass have been computed, its uncertainty is given by the scale at which the maximum have been found.

Detected peak mass (GeV)	
Muon channel	131 ± 14
Electron channel	125 ± 10

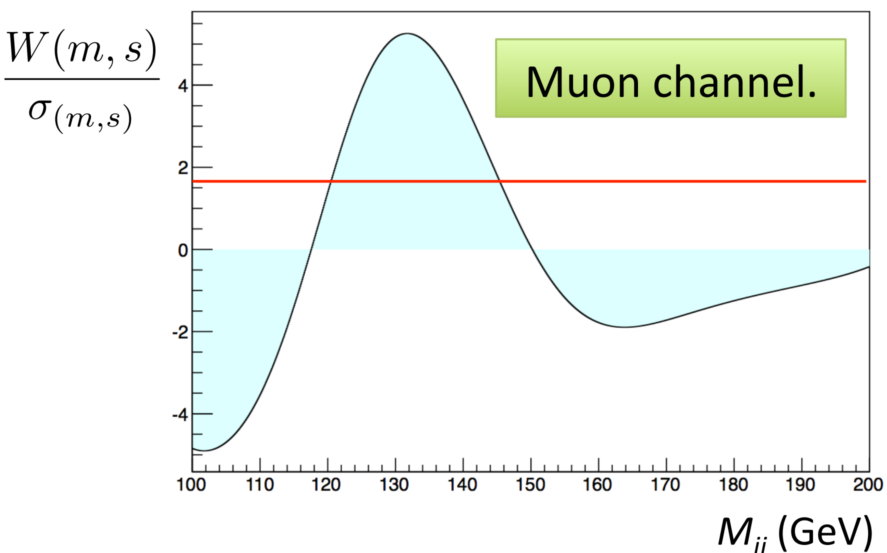
Higgs' boson mass from ATLAS:

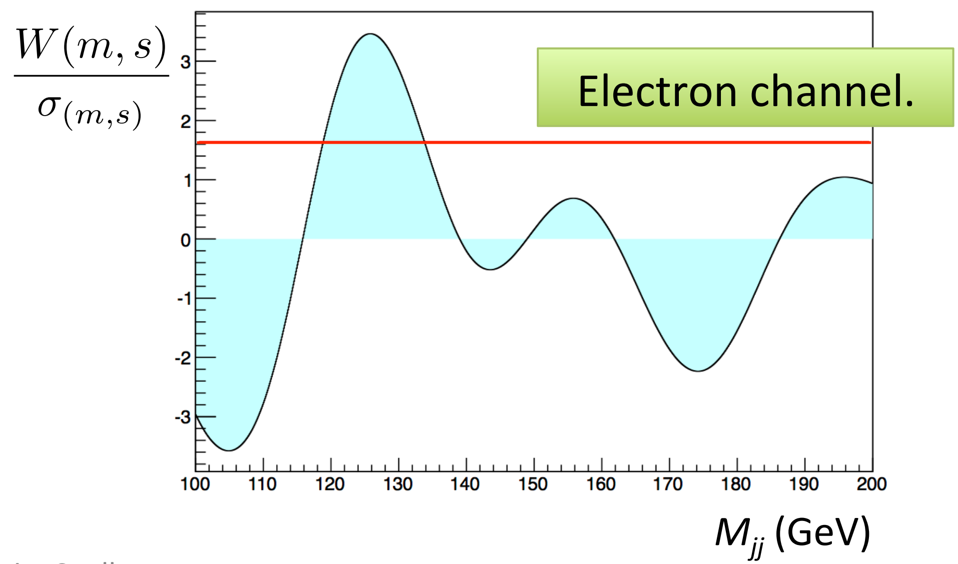
$$m_H = 126 \pm 0.4 \text{ (stat)} \pm 0.4 \text{(sys) GeV}$$

RESULTS IN [100,200] GeV MASS REGION: CONFIDENCE LEVEL

- The 95% local confidence level computed for a standard normal distribution has been compared to $W(m,s)/\sigma_{(m,s)}$ of the peak.
- Both peaks in muon and electron channel resulted to be significative.
- Better considerations could be done via a global confidence level
  - Due to the difficulties in defining a global confidence level for *W(m,s)*, this topic has not been developed in this thesis.

Slice at the scale $s = s_{max}$ where the peak maximum have been found.





RESULTS IN [100,200] GeV MASS REGION: QUALITATIVE CHECKS

- Check 1: the analysis was repeated moving the mass interval of ±10 GeV.

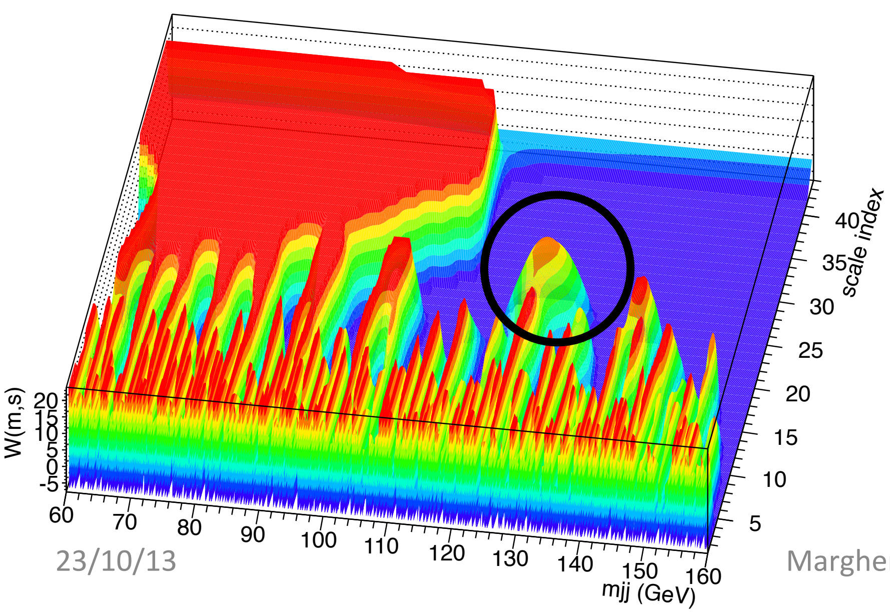
mass range (GeV)	Detected peak mass (GeV)	
	Muon channel	Electron channel
[90, 190]	129 ± 17	129 ± 21
[110, 210]	132 ± 10	126 ± 7.5

- Check 2: the muon sample was divided in two subsample and the analysis repeated for each one.

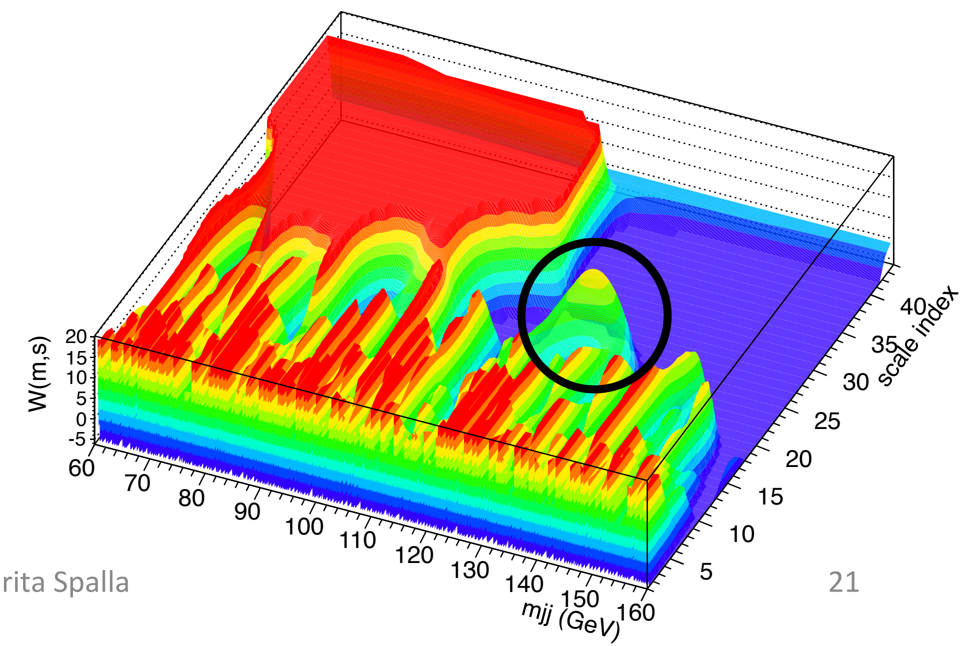
Detected peak mass (GeV): muon channel	
Subsample A	138 ± 17
Subsample B	128 ± 12

- Check 3, only qualitative: *W(m,s)* has been computed without background subtraction. The bump at 126 GeV is still visible

W(m,s): muon channel



W(m,s): electron channel



DETERMINATION OF SIGNAL INTENSITY: CALIBRATION

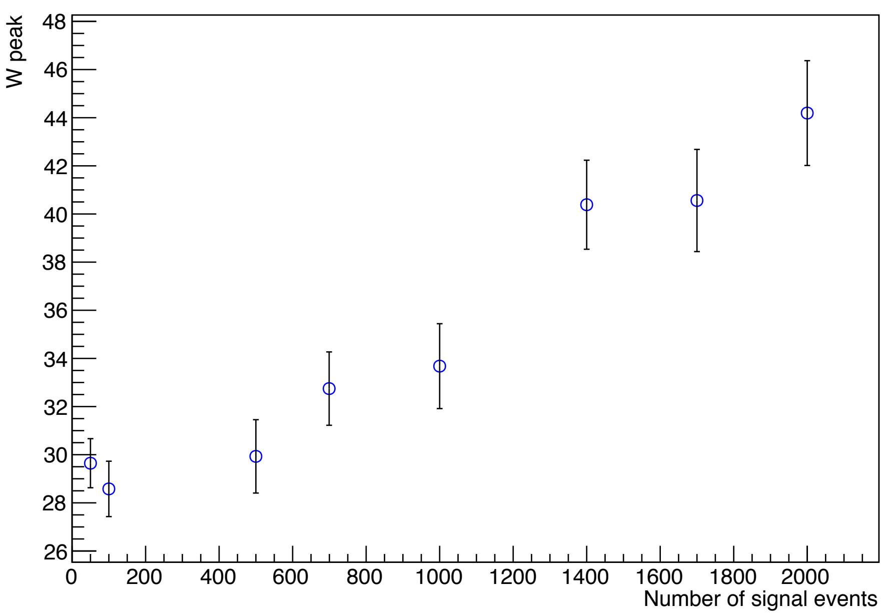
- The number of background events is much larger in real data than in sample used for calibration.
  - After subtraction, residual background has larger fluctuations.

<u>Muon channel</u>:
522804 events
<u>Electron channel</u>:
247086 events

- The direct determination of the signal intensity via the maximum of W(m,s) becomes badly conditioned by the strong fluctuation of the background.

- The *W(m,s)* maximum, as a function of the number of signal events, has a large constant term and a small slope.
  - In this way we cannot provide an adequate calibration.
  - The precision in background modelling and the search algorithm should be refined

W peak height vs number of signal events: exponential background fitted to real data.



DETERMINATION OF SIGNAL INTENSITY: AN ALTERNATIVE SOLUTION

- Since background variations are difficult to control, fix the background distribution and vary only the signal intensity.
  - A simple way: subtract a gaussian signal (μ = 126 GeV, σ = 15 GeV) from the data.
  - When the wavelet transform is not able to detect a peak any more, the number of subtracted events is an estimation of the number of signal events.

- The index used for this evaluation is the *Shannon entropy* (*H(W)*).
  - It quantifies the unevenness of a probability distribution.

$$H(W) = -\frac{1}{\ln 2}\sum_{m,s} p_W(m,s)\,\ln(p_W(m,s))$$

$$p_W(m,s) = \begin{cases} W(m,s)/I_+ & \text{if } W(m,s) > 0 \\ 0 & \text{if } W(m,s) \leqslant 0 \end{cases}$$

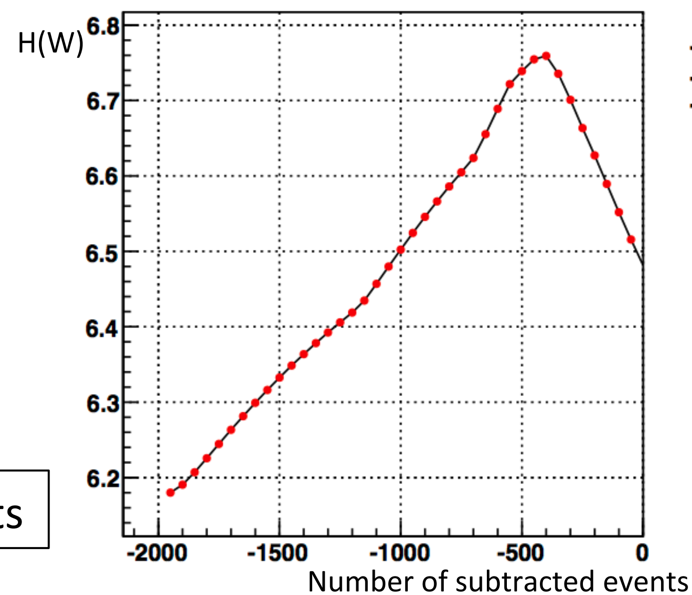
(I_+ is the integral of *W(m,s)*, computed by summing up the positive values of *W(m,s)*.)

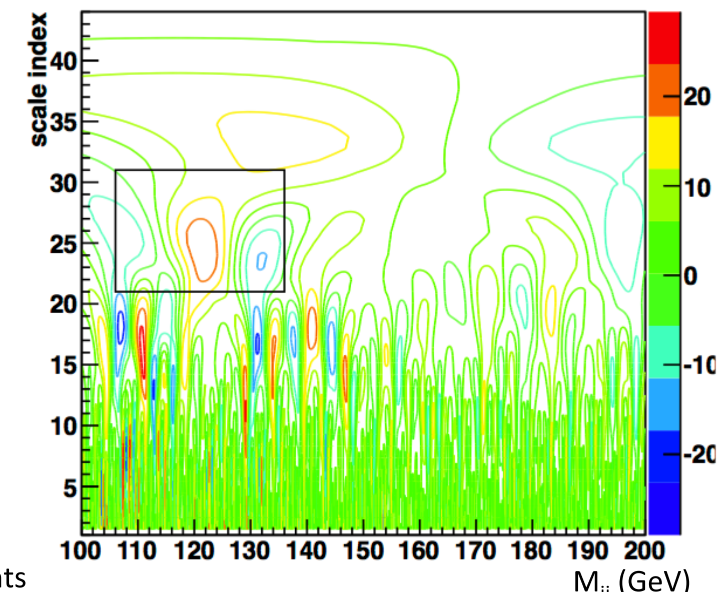
SIGNAL SUBTRACTION: FIRST CHECKS

- The method has to be validated using toy MonteCarlo.
  - A systematic uncertainty of 200 events has been added to the statistical uncertainties.
- The subtraction method has been preliminary defined to fix the problem of calibration.
  - It seems to provide consistent results, but needs a more accurate optimization.
- If the distribution was not perfectly exponential, eventual background structures could be included in the signal peak.
  - The number of events could be overestimated.

Number of signal inserted events	Average number of events measured with subtraction method
500	522 ± 40
1000	1025 ± 40
1500	1490 ± 30

Example with 500 events





SIGNAL SUBTRACTION: RESULTS

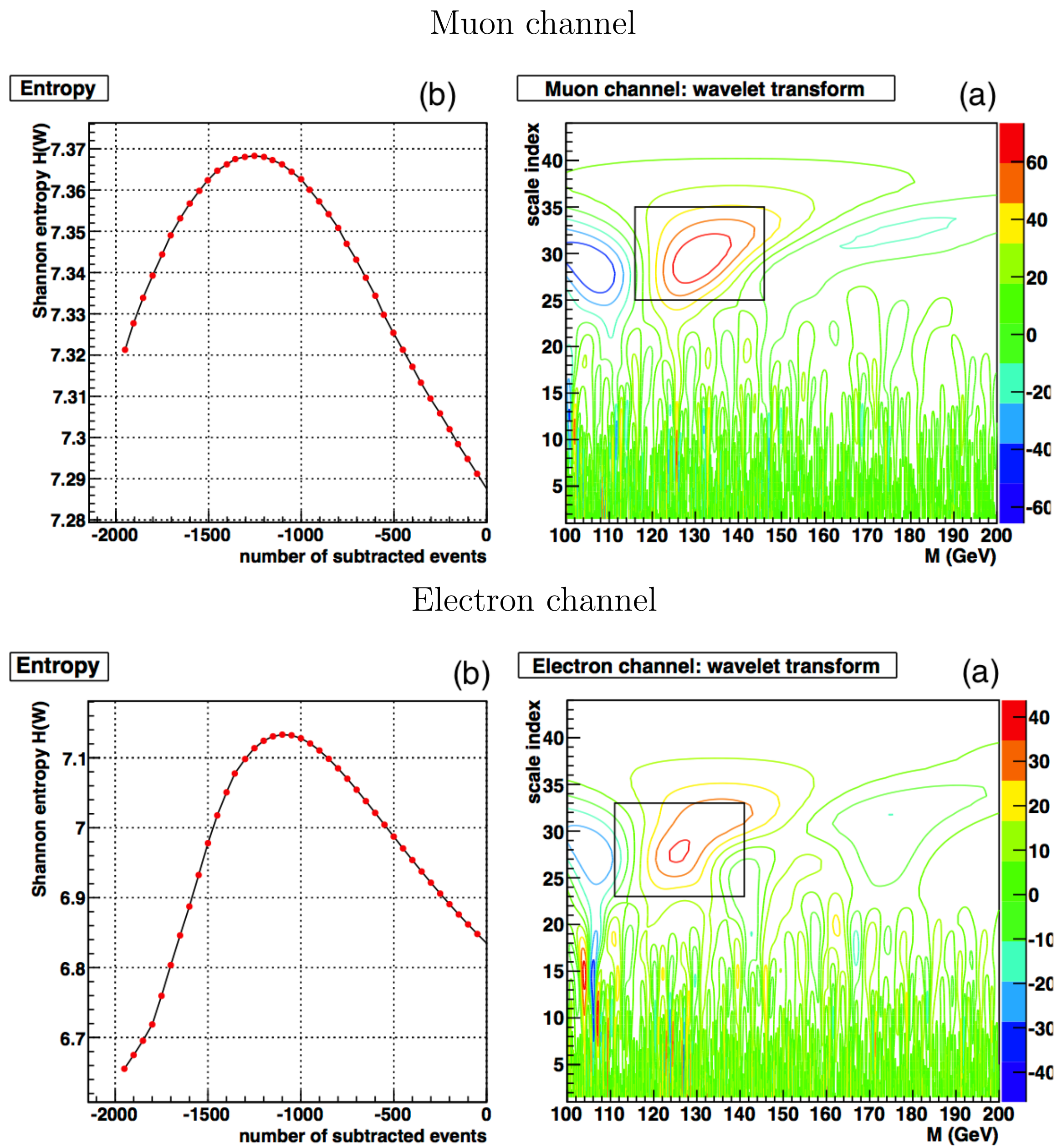
Number of signal events measured with the signal subtraction method	
Muon channel	1250 ± 200
Electron channel	1100 ± 200

- These results are not in agreement with what expected from Standard Model.
  - The number of produced events was:

$$N_{WH} \sim 200 \text{ events}$$

- *W(m,s)* peak could include eventual underlying background structures.
  - Signal subtraction may overestimate the number of events.

- Further work is needed to have a better separation of signal and background effects.

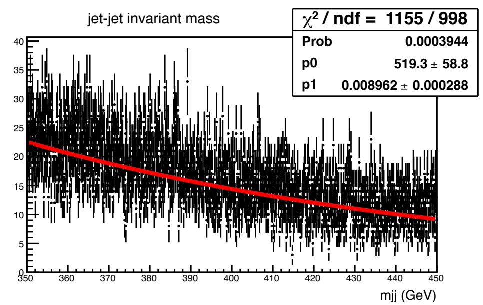


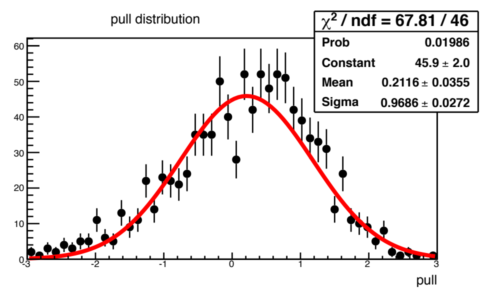
WAVELET ANALYSIS: RESULTS IN THE HIGH MASS REGION

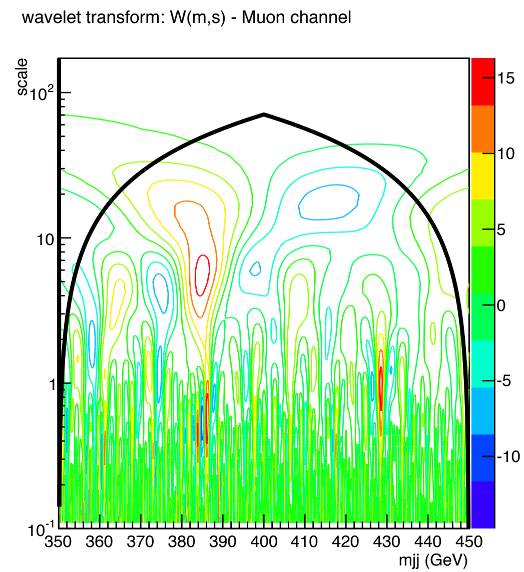
- Wavelet transform has been computed in [150,500] GeV mass region.
  - At higher masses the fit quality is not sufficiently good to obtain reliable results.

- The wavelet transform is computed in a mass range of 100 GeV, which has been moved upwards in steps of 50 GeV.
  - This avoids that part of the mass range is analyzed only in the edges of mass intervals.
  - Only structures appearing at compatible masses in overlapping mass intervals have been considered.

> - Muon channel:
>   - m_{peak}= 385 GeV
> - Electron channel:
>   - m_{peak} = 360 GeV
>   - m_{peak} = 424 GeV







- The nature of these peaks still has to be investigated:
  - The fit quality is poor, fit needs to be improved.

- Further work is needed: we then avoid any further comments.

CONCLUSIONS

- From tests with toy MonteCarlo, the wavelet analysis resulted to be able to detect small signals, invisible to simple observation.

- The quantitative treatment (significance and determination of signal intensity) needs further work to be refined.
  - It is influenced by background, especially if the background is very large.

- By applying wavelet analysis to real data, a signal evidence has been found at $m_{jj} \approx 126$ GeV.
  - It is above the 95% local confidence level.
  - It it confirmed by two independent channels.
  - Its intensity could not be completely estimated.

<u>SOME POSSIBLE DEVELOPMENTS</u>

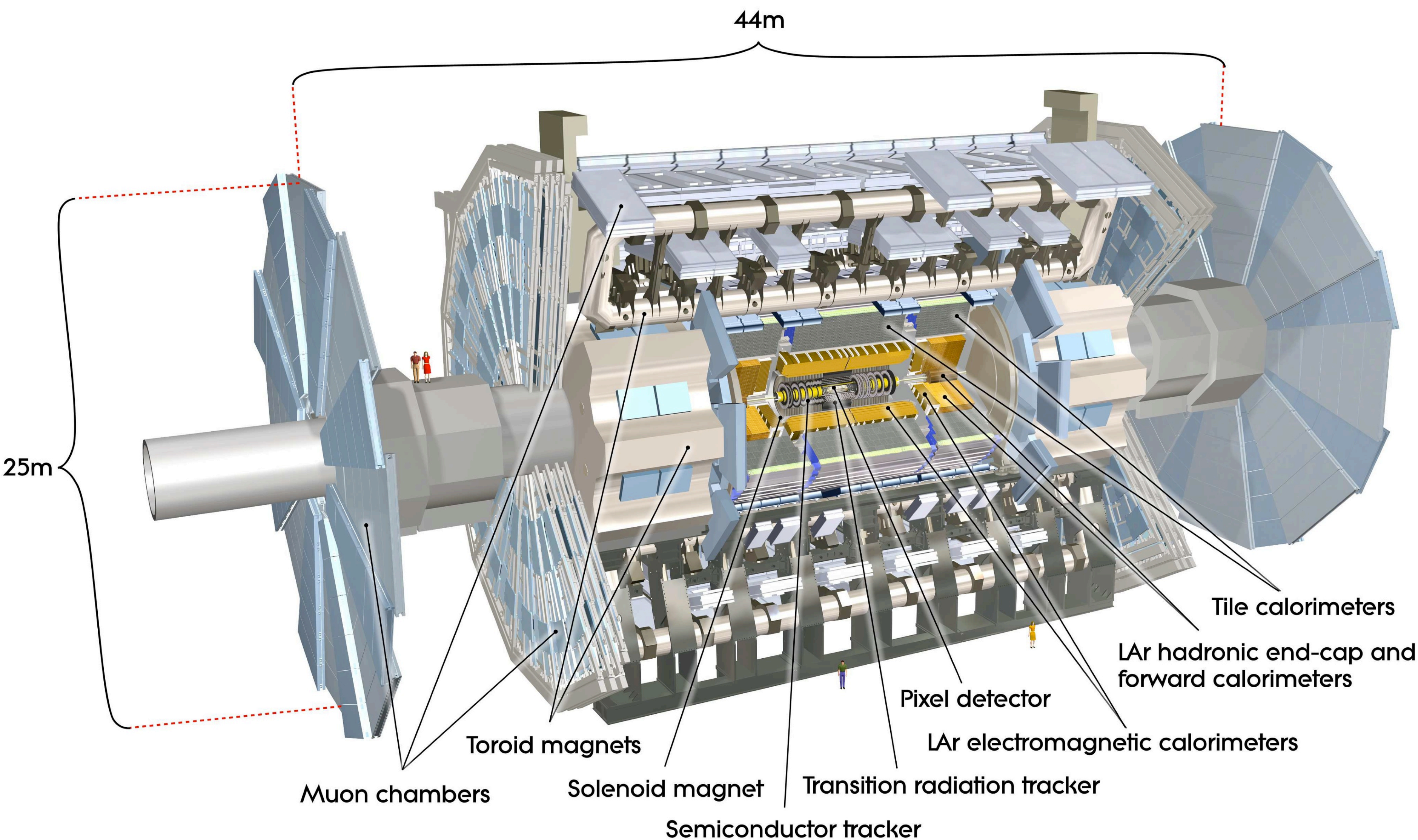
- Refine the peak search algorithm in wavelet analysis and the calibration method: other variables could be used instead of $W(m,s)$ maximum.

- Define a quantitative treatment of wavelet analysis performed without background subtraction.

- Try to use other wavelet functions instead of the DoG.

BACKUP

THE ATLAS DETECTOR



DATA PREPARATION: PHYSICAL OBJECTS RECONSTRUCTION

Reconstruction of physical objects in ATLAS is performed via different algorithms depending on the particular object.

In this thesis, we used muons, electrons, jets and E_t^{miss}.

Muons

- We used *combined muons*: muon tracks are reconstructed independently in the muon spectrometer (MS) and inner detector (ID), the (MS) and (ID) tracks are then matched.

Electrons

- The reconstruction starts from a *seed cluster* (an η-ϕ window of predefined dimension) in electromagnetic calorimeter with E_T>2.5 GeV. *Seed clusters* matching an ID track are taken as electron candidates.
- Electron candidates are then identified to reject photons and hadrons. Three levels are provided: *loose, medium, tight.* We used the tighter identification level.

Jets

- Jets are reconstructed from calorimeters: neighboring cells with significant signal-to-noise ratio are collected in *topoclusters*, *topoclusters* are processed with the *Anti-kt* algorithm to form jets.
- The four-momentum must be corrected for energy losses in uninstrumented material or calorimeter non-compensation: a calibration scale factor has been applied before the analysis.

E_T^{miss}

- It is defined as the sum of the measured energy of all physics objects changed by sign.
- Due to jets momentum correction, it has been rebuilt at the beginning of the selection.

SELECTION APPLIED TO DATA: OBJECT SELECTION

Objects passing the selection are defined as *good* objects.

MUON SELECTION.

- *Combined muons* are used.
- Trigger: EF_mu18_MG, EF_mu18_MG_medium. p_T>25 GeV is required to restrict to the trigger efficiency plateau.
- Track quality cuts.
- $|\eta| < 2.4$
- Impact parameter: $|d_0/\sqrt{\sigma(d_0)}| < 3$ and z_0<1 mm.
- Isolation.

  Track: $\Sigma(p_T^{track})/p_T < 0.15$ in a cone of radius R=0.3

  Calorimeter: $\Sigma(E_T^{corr})/p_T < 0.14$ in a cone of radius R=0.3

ELECTRON SELECTION.

- Candidates satisfying the *tight++* identification criteria.
- Trigger: EF_e20_medium, EF_e22_medium, EF_e22vh_medium1. p_T>25 GeV is required to restrict to the trigger efficiency plateau.
- $|\eta| < 2.47$, excluding $1.37 < |\eta| < 1.52$.
- Impact parameter: $|d_0/\sqrt{\sigma(d_0)}| < 10$ and z_0<1 mm.
- Isolation.

  Track: $\Sigma(p_T^{track})/p_T < 0.14$ in a cone of R=0.3

  Calorimeter: $\Sigma(E_T^{corr})/p_T < 0.13$ in a cone of R=0.3

JET SELECTION.

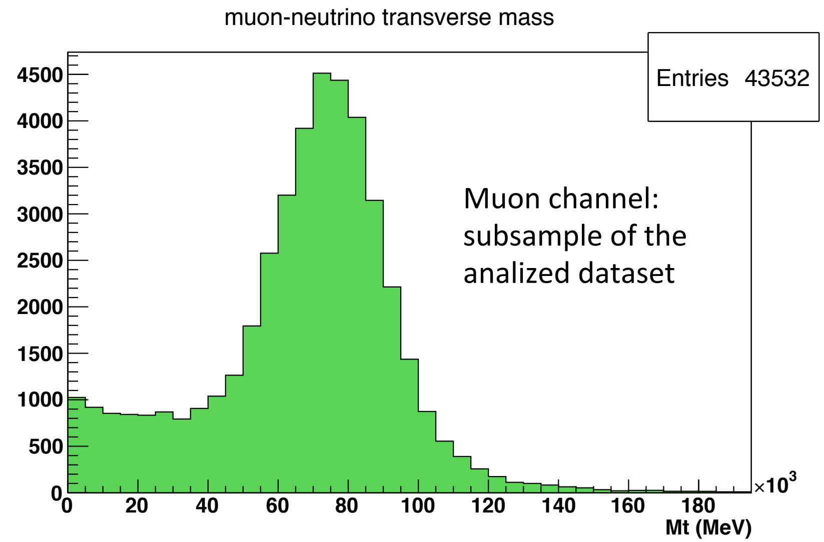
- Jets reconstructed with *Anti-kt* algorithm, passing *looser* quality criteria.
- $p_T > 25$ GeV
- $|\eta| < 2.8$
- Jet Vertex Fraction > 0.75 to reject jets from pile-up interactions.
- $\Delta R(j,l) > 0.5$, *l* is the selected lepton. This to remove overlap between jets and energy deposits due to leptons.

EVENT SELECTION

Dijets events are triggered by requiring a $W \longrightarrow l\nu$ decay.

- Events are firstly pre-selected applying cuts on event quality:
  - Stable beam conditions, absence of large noise bursts or data integrity errors in the LAr, no jets of p_T>20 GeV pointing to the Lar non-sensitive area (*Lar hole*).
  - A reconstructed primary vertex with at least three associated tracks of p_T>0.5 GeV

- Events with one charged lepton passing the object selection.
  - Events are discarded if a second lepton passes the object selection.
  - *Trigger-matching*: a check to verify that the selected lepton is the one that fired the trigger in the event.



- Events containing also a neutrino: E_t^{miss} >25 GeV
  - Cleaning cuts are applied to the jets before E_T^{miss} cut to avoid non-physical E_T^{miss} due to jet reconstruction errors.

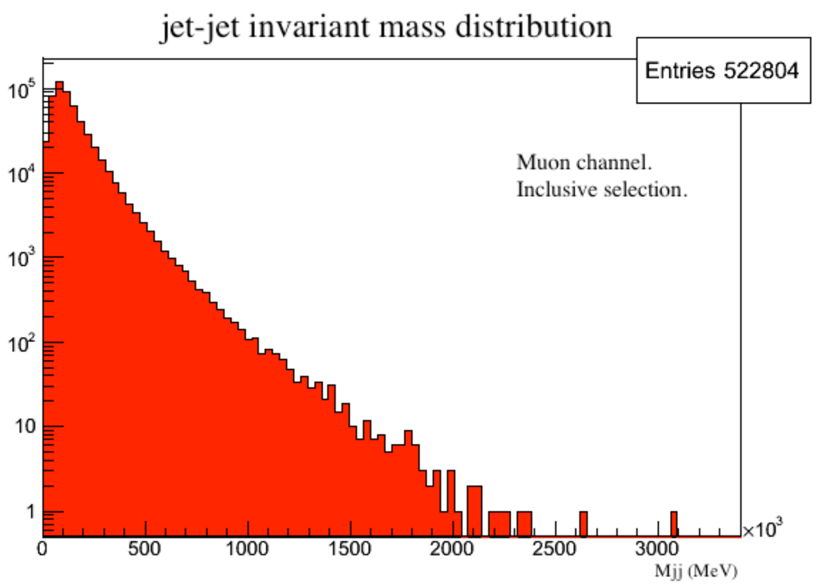
- Cut on the lepton-neutrino transverse mass: M_T > 40 GeV

Once $W \longrightarrow l\nu$ events are selected, further cuts are applied to jets.

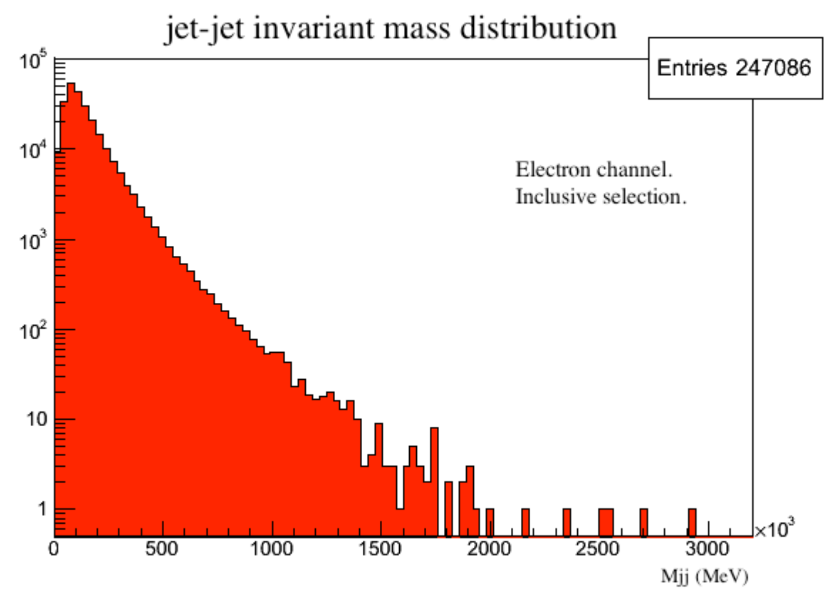
→ with respect to the selection used in Standard Model diboson measurement, fewer cuts are applied to apply wavelet analysis at a more inclusive level.

- At least two jets passing the object selection
- $\Delta\phi(E_t^{miss}, j_1) > 0.8$. Where j_1 is the jet of highest p_T
- The dijet invariant mass is built using the two selected jets of highest p_T

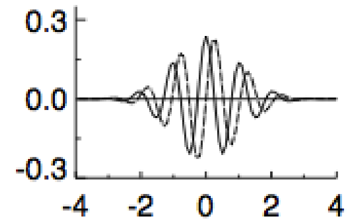
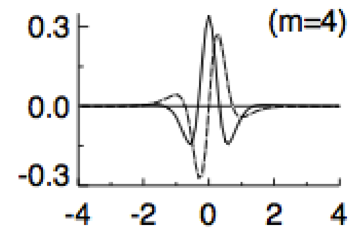
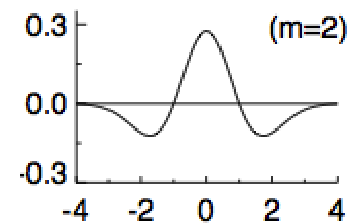
Jet-Jet invariant mass (logarithmic scale), obtained with $L_{int} = 4702\ \mathrm{pb}^{-1}$.



(a)



(b)

Name	$\psi_0(\eta)$	$\hat{\psi}_0(s\omega)$	$\psi_0(\eta)$ (graphic)
Morlet (ω_0 = frequency)	$\pi^{-1/4}e^{i\omega_0\eta}e^{-\eta^2/2}$	$\pi^{-1/4}H(\omega)e^{-(s\omega-\omega_0)^2/2}$	
Paul (m = order)	$\frac{2^m i^m m!}{\sqrt{\pi(2m)!}}(1-i\eta)^{-(m+1)}$	$\frac{2^m}{\sqrt{m(2m-1)!}}H(\omega)(s\omega)^m e^{-s\omega}$	
DOG (m = derivative)	$\frac{(-1)^{m+1}}{\sqrt{\Gamma\left(m+\frac{1}{2}\right)}}\frac{d^m}{d\eta^m}\left(e^{-\eta^2/2}\right)$	$\frac{-i^m}{\sqrt{\Gamma\left(m+\frac{1}{2}\right)}}(s\omega)^m e^{-(s\omega)^2/2}$	

$H(\omega)$ = Heaviside step function, $H(\omega) = 1$ if $\omega > 0$, $H(\omega) = 0$ otherwise.
DOG = derivative of a Gaussian; $m = 2$ is the Marr or Mexican hat wavelet.

Three wavelet mother functions and their Fourier transform. Constant factors for ψ_0 and $\hat{\psi}_0$ are for normalisation. The plots on the right give the real part (solid) and imaginery part (dashed) for the wavelets as functions of the parameter η.

Reference:

C. Torrence and G. P. Compo, "A practical guide to wavelet analysis," *Bulletin of the American Meteorological society*, vol. 79, no. 1, pp. 61–78, 1998.

DETAILS ON WAVELET TRANSFORM CALCULATION

- It is considerably faster to compute the wavelet transform in Fourier space.
  - The discrete Fourier transform of x_n is: $\hat{x}_k = \frac{1}{N}\sum_{n=0}^{N-1} x_n e^{-i2\pi kn/N}$
  - $\hat{\psi}(s\omega_k)$ is the Fourier transform of a (continuous) function ψ(*m/s*).

$$W(m,s) = \sum_{k=0}^{N-1} \hat{x}_k \hat{\psi}^*(s\omega_k) e^{i\omega_k n\delta m}$$

$$\omega_k = \begin{cases} \frac{2\pi k}{N\delta m} & \text{if } k \leqslant \frac{N}{2} \\ -\frac{2\pi k}{N\delta m} & \text{if } k > \frac{N}{2} \end{cases}$$

- *W(m,s)*, as a continuous function of *s*, can be approximated by computing the wavelet transform for a set of scales.
  - s_0 is the smallest resolvable scale: $s_0 = \delta m$
  - δj sets the smallest wavelet resolution: $\delta j = 0.25$
  - J sets the value of the largest scale: $J = 44$

$$s_j = s_0 2^{j\delta j}, \qquad j = 0, 1, \ldots, J$$

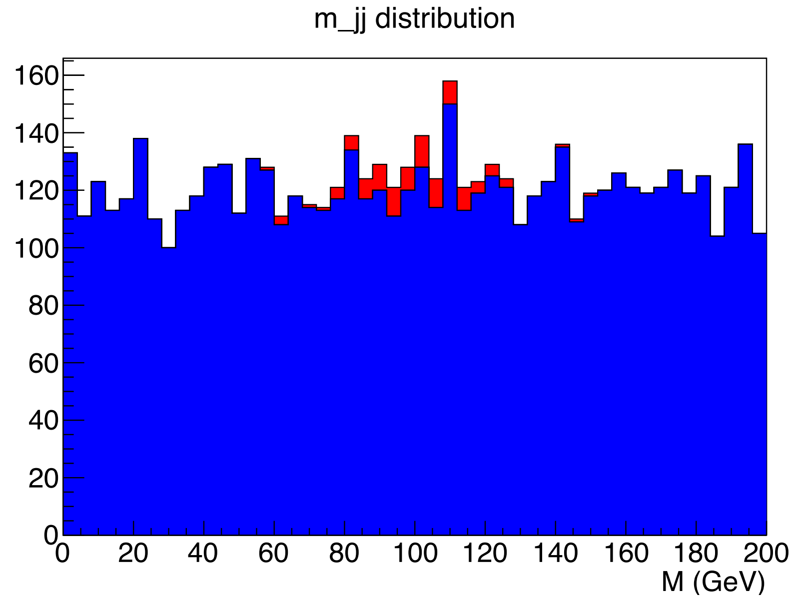
- Normalization: *W(m,s)* at different scales must be directly compared, therefore it is necessary that they all have the same normalization.
  - The normalization is fixed for the Fourier transform of the *mother* wavelet function: it is normalized to have unit energy.

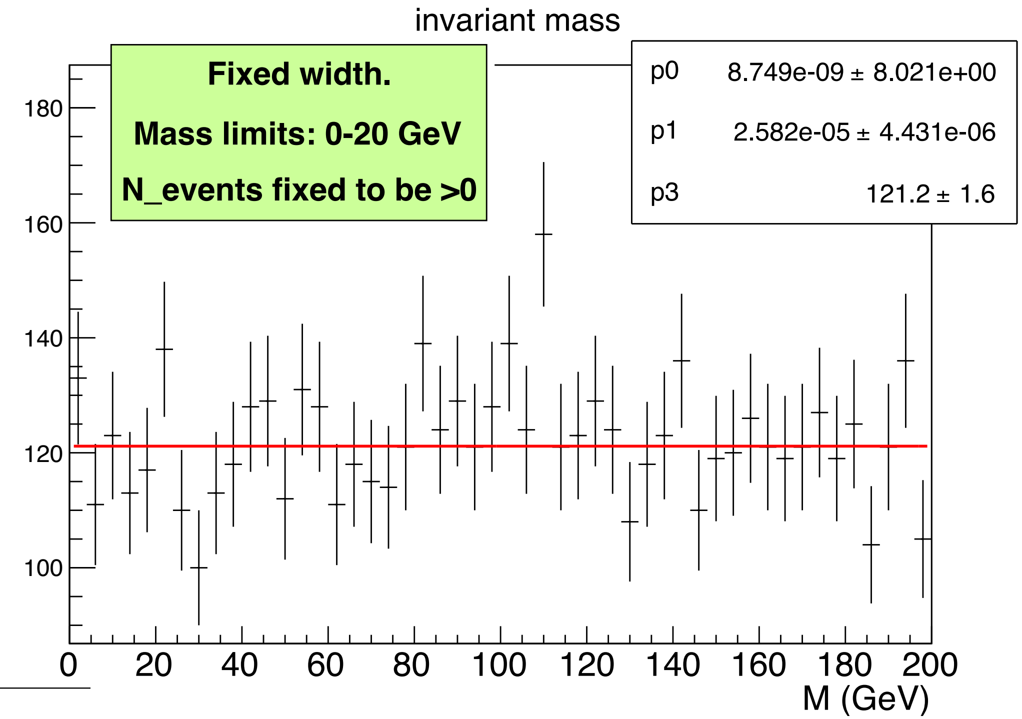
$$\int_{-\infty}^{+\infty} |\hat{\psi}_0(s\omega)|^2 d\omega = 1$$

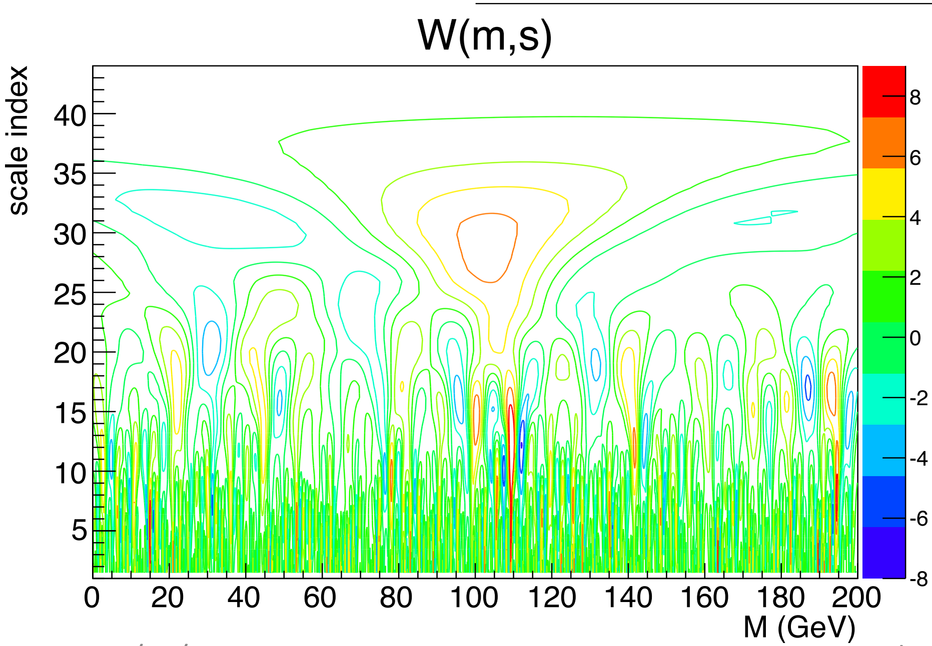
  - The wavelet *daughter* are normalized in the same way adding a normalization constant to their Fourier transform.

$$\hat{\psi}(s\omega_k) = \left(\frac{2\pi s}{\delta m}\right)^{1/2} \hat{\psi}_0(s\omega_k)$$

- Fourier transform is computed padding with zeroes the end of the mass range: this influence *W(m,s)* in the region close to the edges.
  - ➢ The *Cone of Influence* (COI) is the region in *m* × *s* plane where edge effects are important. Discontinuities at the edges decrease exponentially: at each scale, COI is defined by the 'characteristic length' of this decrease.







CHECK WITH ALTERNATIVE METHODS.

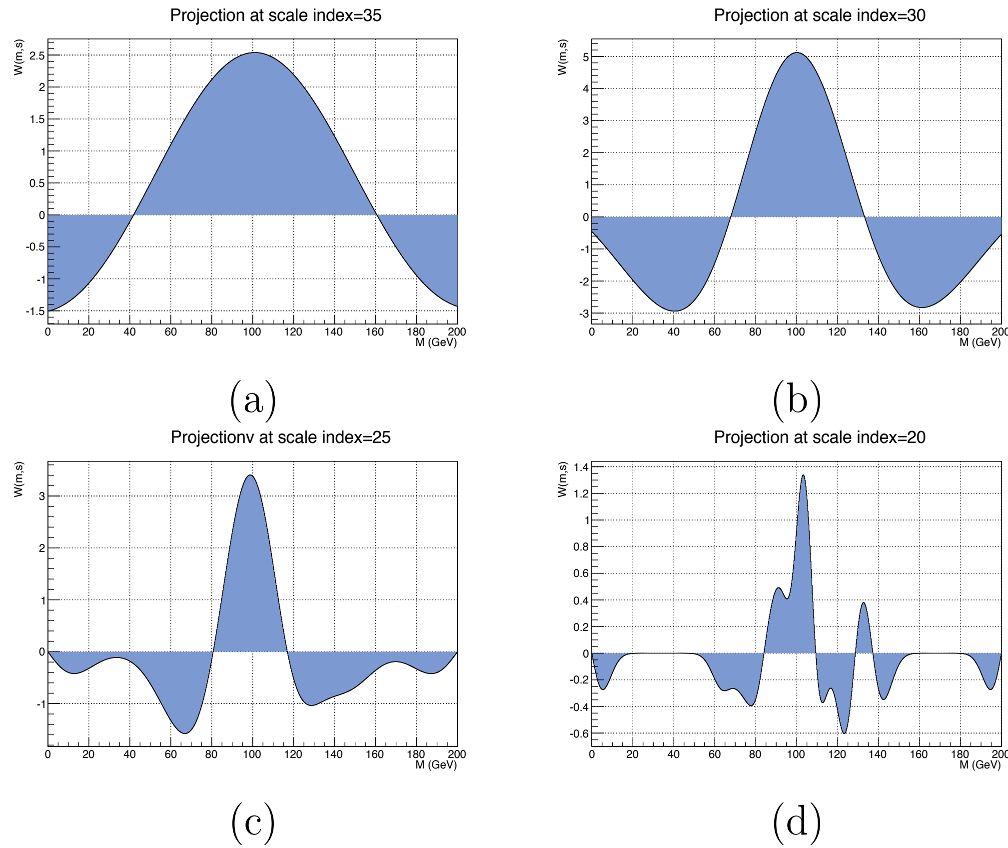
Fit with a gaussian signal superimposed to a constant term.

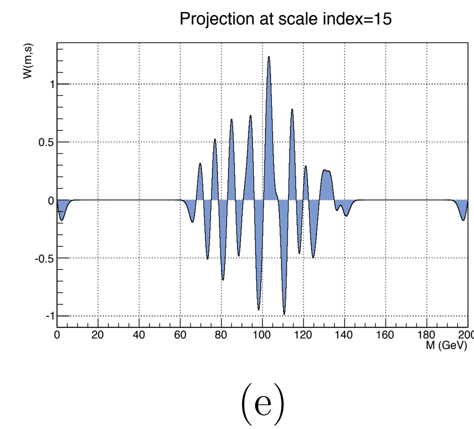
Wavelet transform.

WAVELET TRANSFORM OF A GAUSSIAN SIGNAL: PROJECTIONS AT FIXED SCALE.

- At larger scale *W(m,s)* has a DoG-like shape, with mean corresponding to the signal mean.

- at low scale the DoG shape is lost and W presents various narrower peaks, corresponding to statistical fluctuations of groups of bins.





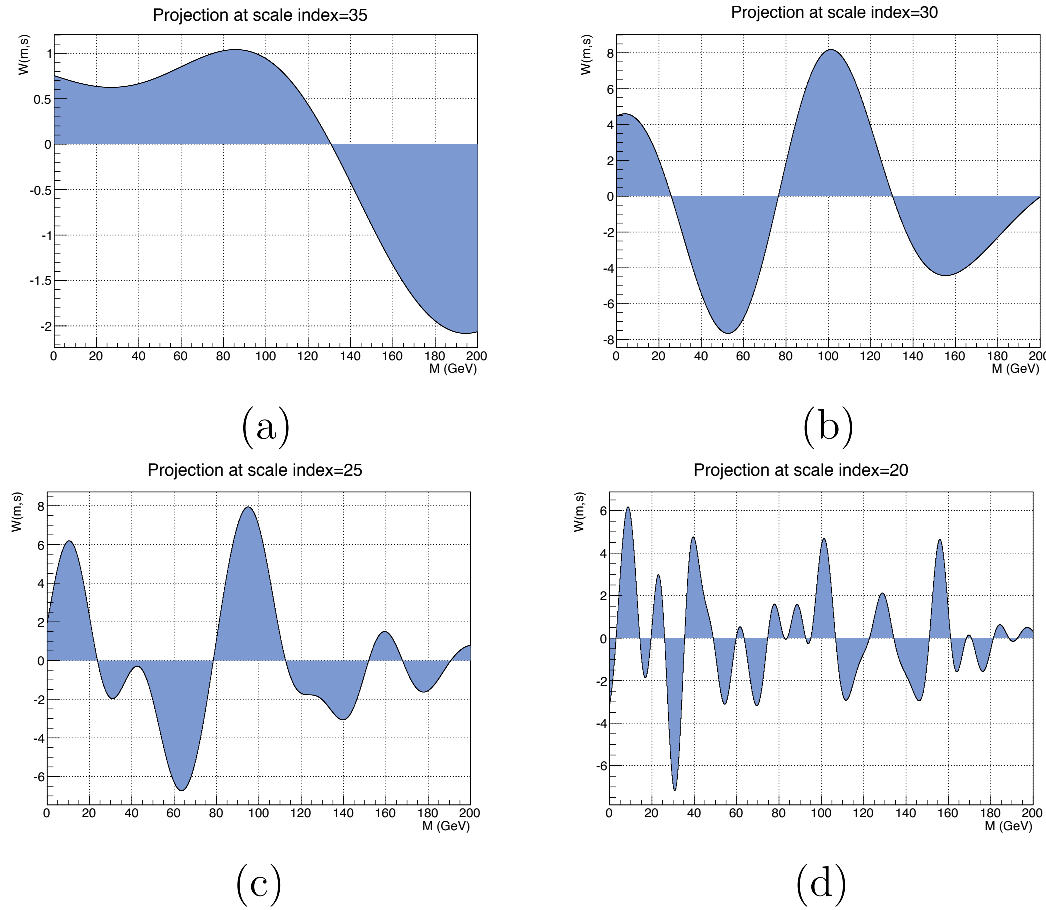
Wavelet transform of a gaussian signal (100 events, mean=100 GeV, standard deviation = 15 GeV): projection at fixed scale. (a): scale index=35. (b): scale index=30. (c): scale index=25. (d): scale index=20. (e): scale index=15.

BACKGROUND EFFECTS, UNIFORM BACKGROUND

- A flat background is the condition in which wavelet analysis has been applied in most of literature. *W(m,s)* is computed considering variations with respect to arithmetic mean of the data.

- Wavelet transform of a gaussian signal over a uniform background at fixed scale (from the example of slide 11-16).
  - At scale index j_s = 30, the wavelet transform has a DoG shape in the region of the signal.
  - At higher scale, *W(m,s) is* hardly sensitive to the signal.
  - At lower scale it is dominated by statistical fluctuation: only the scale region $j_s \geq 25$ is used for the analysis.

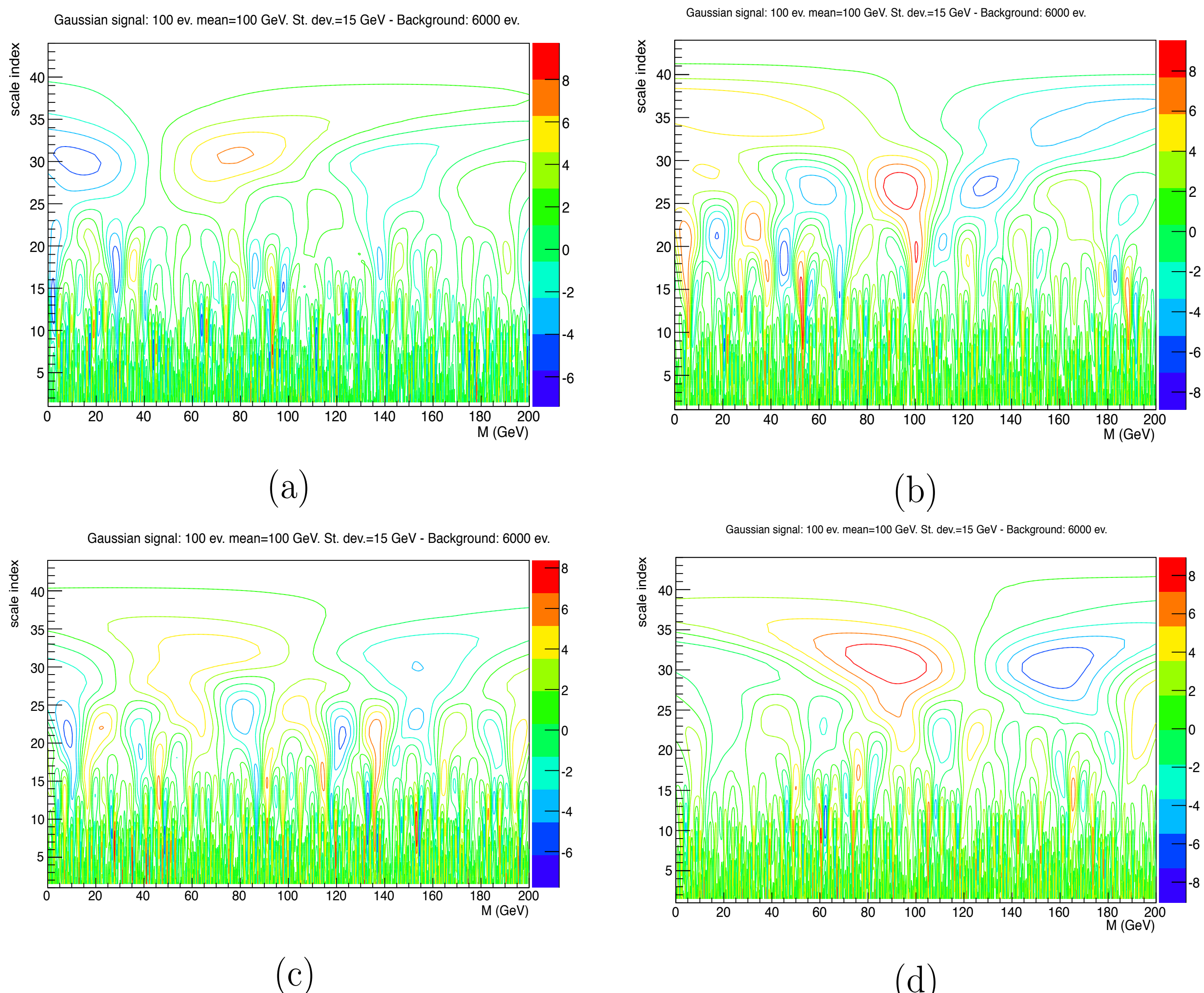
- The signal is not always detected as clearly as in this example.
  - The wavelet transform peak can be moved in mass and scale, change in shape or eventually not be detected at all.



Wavelet transform of a gaussian signal (100 events, mean=100 GeV, standard deviation = 15 GeV) over a flat background (6000 events): projection at fixed scale. (a): scale index=35. (b): scale index=30. (c): scale index=25. (d): scale index=20.

Wavelet transform of a gaussian signal (100 events, mean=100 GeV, standard deviation=15 GeV) over a flat background (6000 events):

EXAMPLES OF HOW THE PEAK CAN VARY.



THE *CONTOUR ALGORITHM*

- To develop a quantitative analysis, the efficiency of wavelet analysis and the dependence of wavelet transform on signal intensity should be evaluated.
- To do this we must define an appropriate algorithm to find a peak in the *W(m,s)* plot.
  - The contour algorithm is the basic strategy for the search of a signal in a *W(m,s)* plot.
  - It starts from the contour level representation of the wavelet transform.

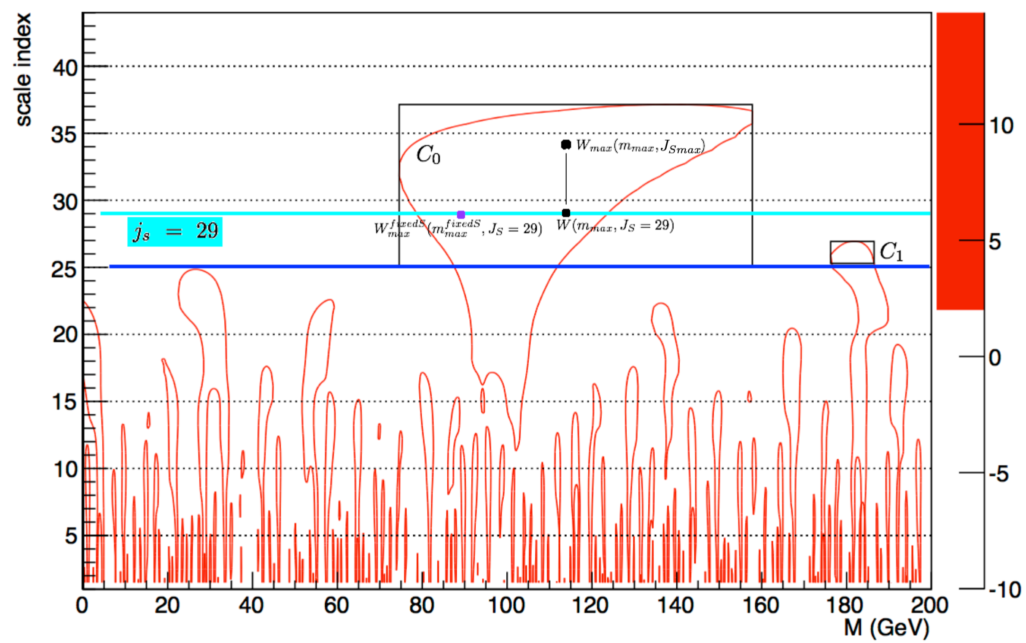
1. Fixed a single contour level W_0, the algorithm searches for contours at W_0.

2. Loop on the contours: given a contour, check if at least a part of it is contained in the scale region $j_s \geq 25$. If not, the contour is discarded.

3. The maximum value of W is searched. The search is limited to the region of $m \times j_s$ plane which is both inside the contour and contained in the scale region $j_s \geq 25$.

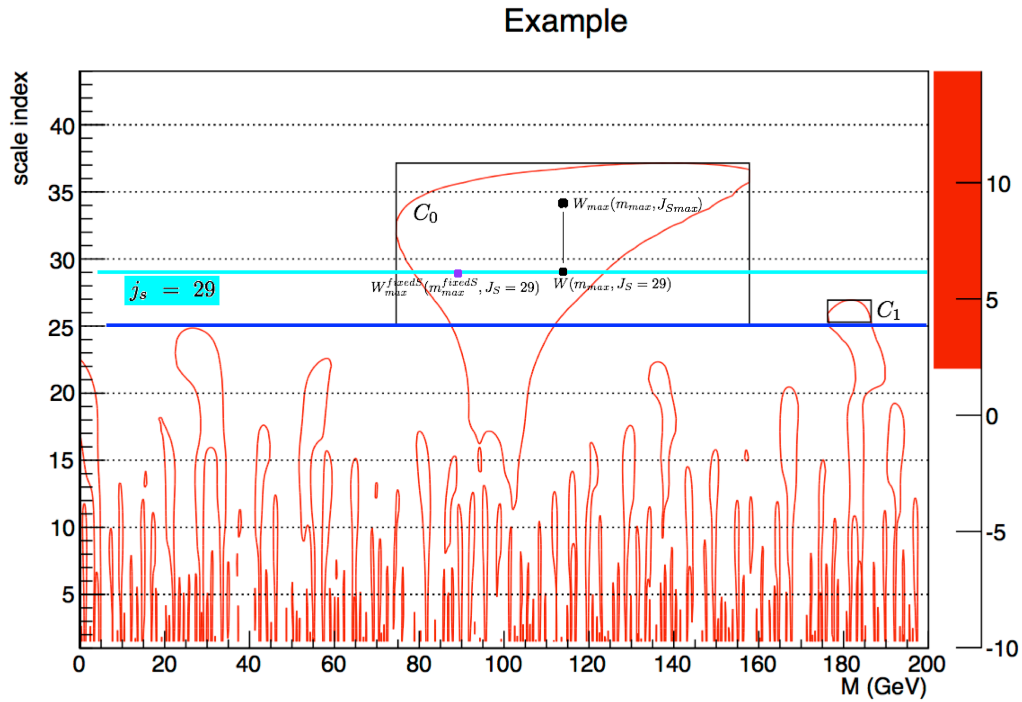
4. Assume the maximum W_{max} has been found in a certain point (m_{max}, Js_{max}): s_{max} is used to define the acceptance region for the calculation of efficiency, if
   the signal have been found and the loop is interrupted.

Example



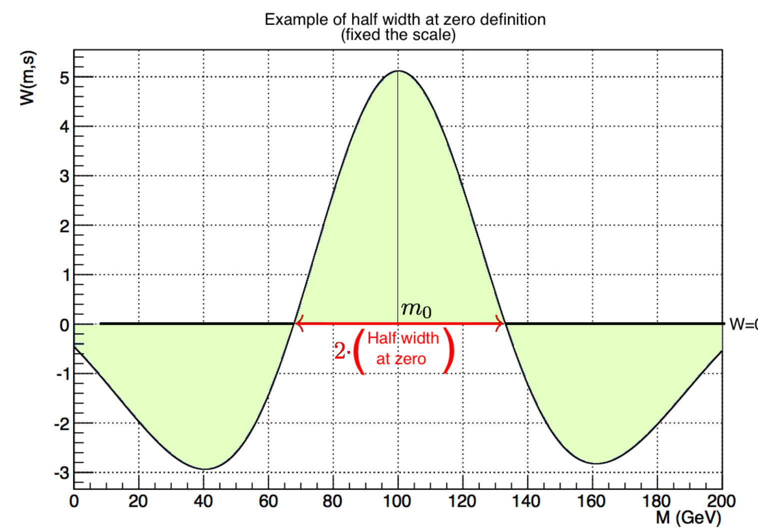
Definition of variables used to evaluate $W(m,s)$ dependence on signal parameters.

- Variable scale: the *contour algorithm* finds the maximum W_{max} over a certain contour, at (m_{max},s_{max}).
  - The half width at zero (HW_{max}) is found taking the $W(m,s)$ projection at fixed $s = s_{max}$: the first two zeros at $m>m_{max}$ and $m<m_{max}$ are found, HW_{max} is the half difference between them.
- Fixed scale.



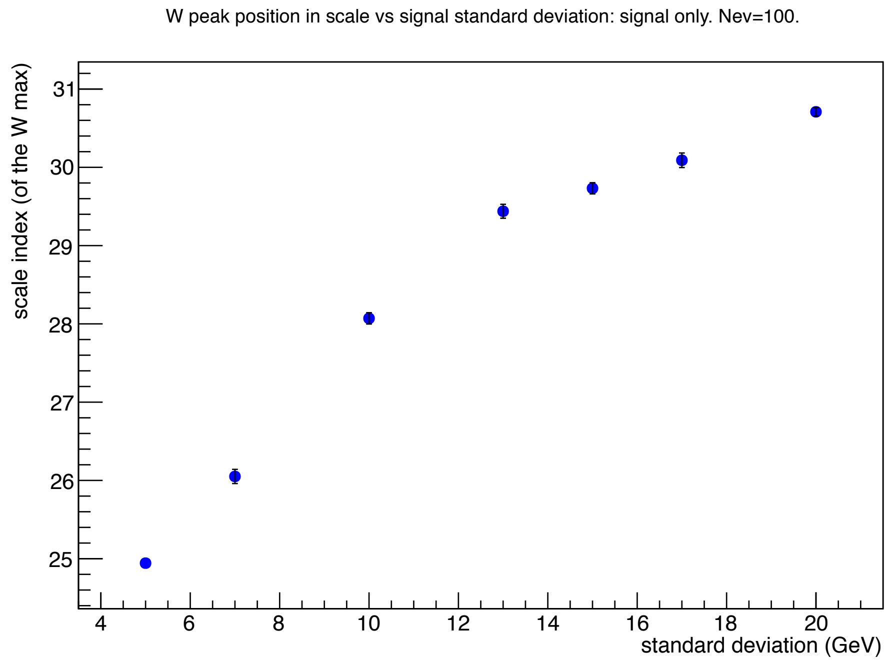
1. Consider $W_{max}(m_{max},s_{max})$: fixed a scale s_0, the variable used to evaluate N_{ev} is $W(m_{max},s_0)$. The corresponding half width HW_{s0} is found as before, taking the projection at $s = s_0$.

2. An alternative variable is found searching for the maximum of $W(m,s)$ inside the contour at the fixed scale s_0. If the found maximum is: $W_m^{fixedS}(m_{fixedS},s_0)$ the half width (HW_m^{fixedS}) is found considering the projection at $s = s_0$ and referring to m_{fixedS} instead of m_{max} .



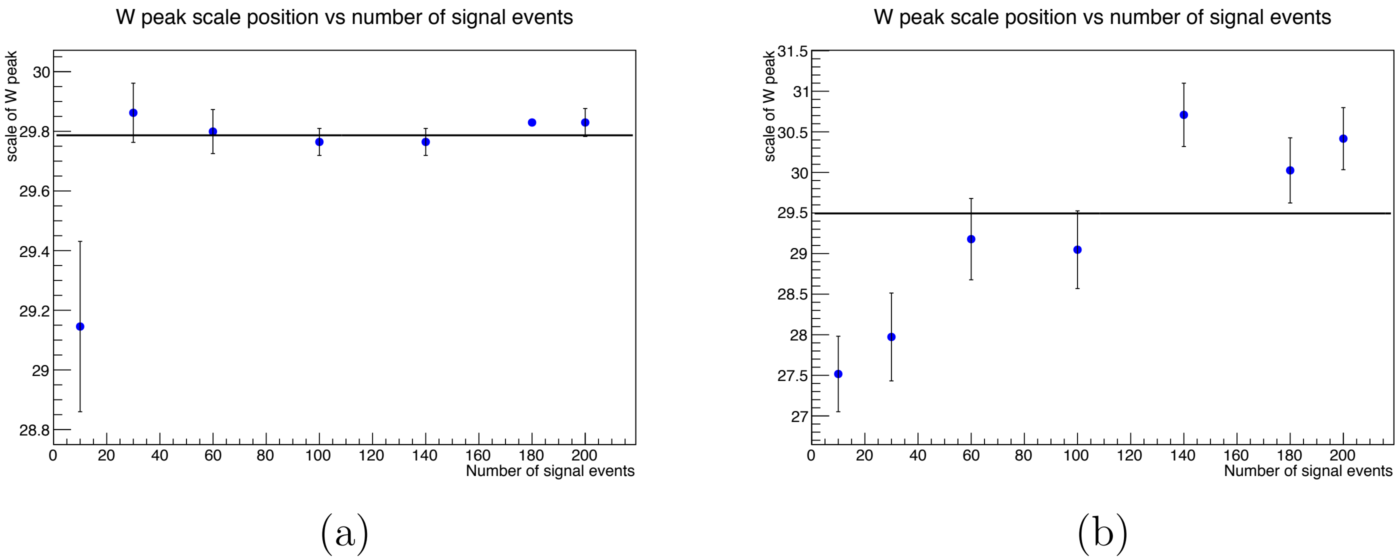
- We use $j_{s0}=29$

Signal only: scale index of wavelet transform maxima as a function of the signal standard deviation



Scale position of the wavelet transform peak for a signal of 100 events, mean $\mu = 100\ GeV$ *and varying it's standard deviation. No background is present.*

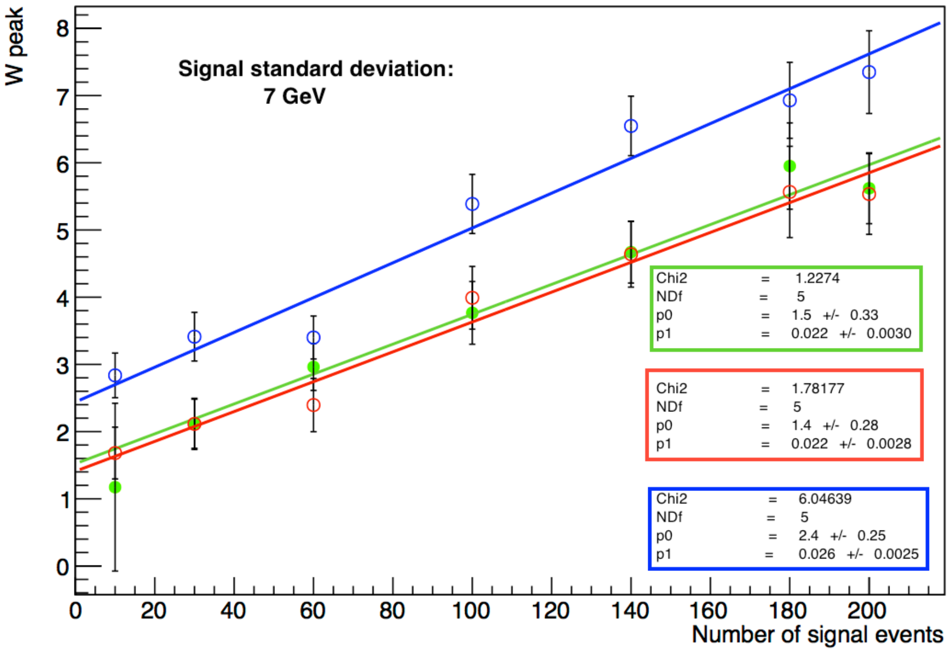
Flat background: scale index of wavelet transform maxima as a function of the number of signal events

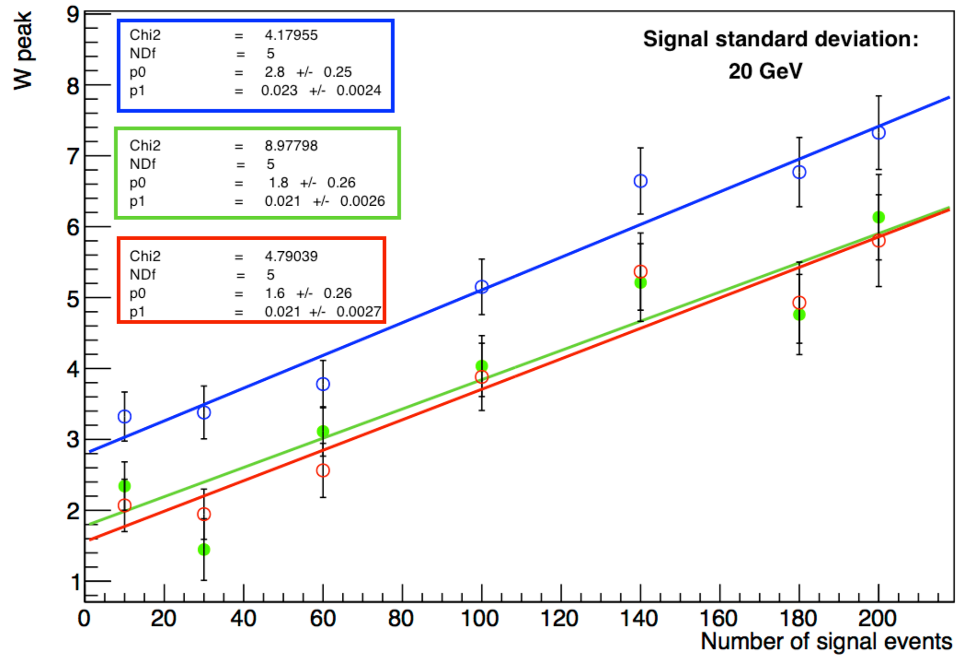


Scale index of wavelet transform maxima for varying number of signal events. (a): no background is added. (b): 6000 events of uniform background are added. To show the mean value over the whole intensity range, the two plots have been fitted with a constant, the result is shown by the black line.

W_{max}: blue. $W(m_{max}, s_0)$: green. W_m^{fixedS}: red

The linear dependence has been checked using signals of σ=7 GeV and σ=20 GeV: the linearity is conserved and the slope has only a slight variation. This calibration will be considered independent of signal standard deviation.

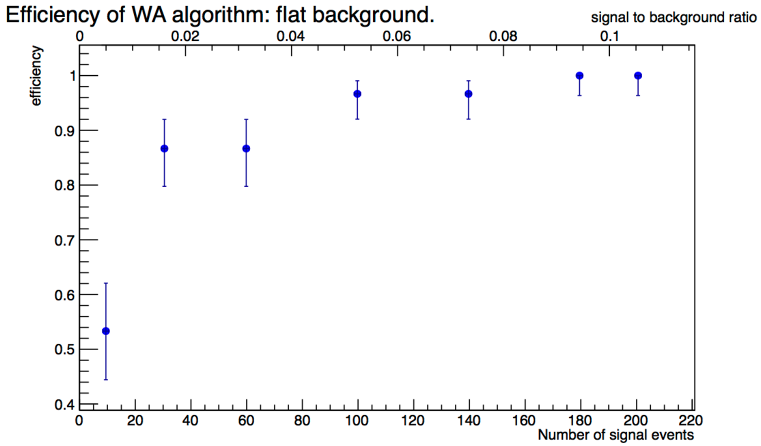




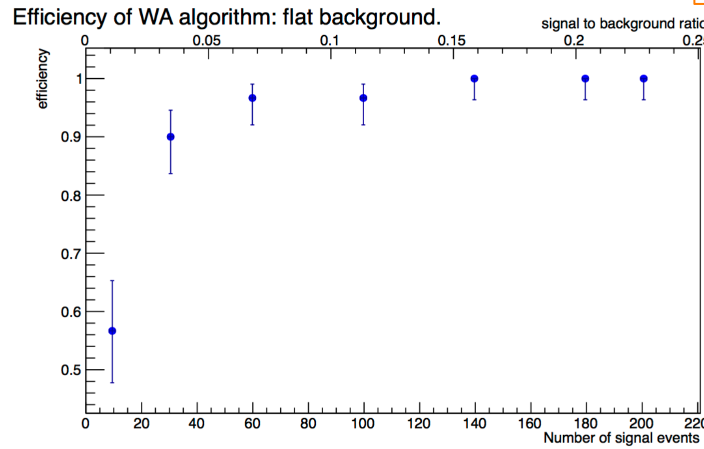
$W(N_{ev})$: slope

	no background $\sigma = 15$ GeV	flat background $\sigma = 15$ GeV	flat background $\sigma = 7$ GeV	flat background $\sigma = 20$ GeV
W_{max}	0.0495 ± 0.00027	0.028 ± 0.0024	0.026 ± 0.0025	0.023 ± 0.0024
W_{max}^{fixedS}	0.0505 ± 0.0002	0.028 ± 0.0029	0.022 ± 0.0028	0.021 ± 0.0027
$W(m_{max}, s_0)$	0.0505 ± 0.0002	0.029 ± 0.0030	0.022 ± 0.0030	0.021 ± 0.0027

EFFICIENCY:
FLAT BACKGROUND

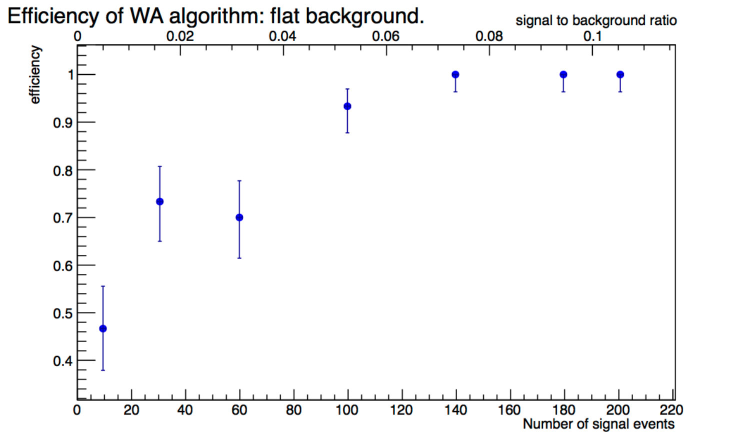


(a)

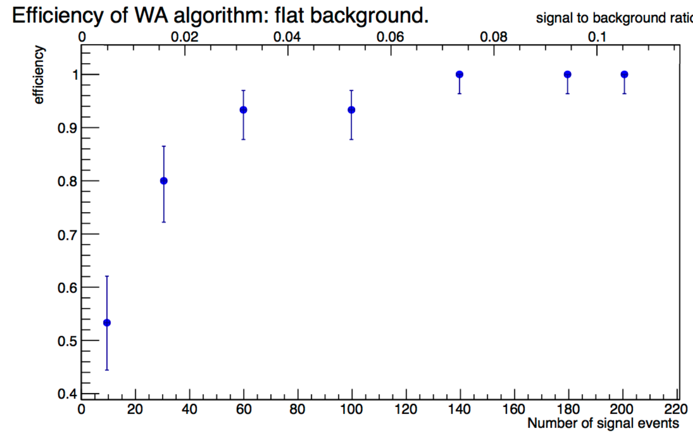


(b)

Flat background: 6000 events.
Signal: 100 events.

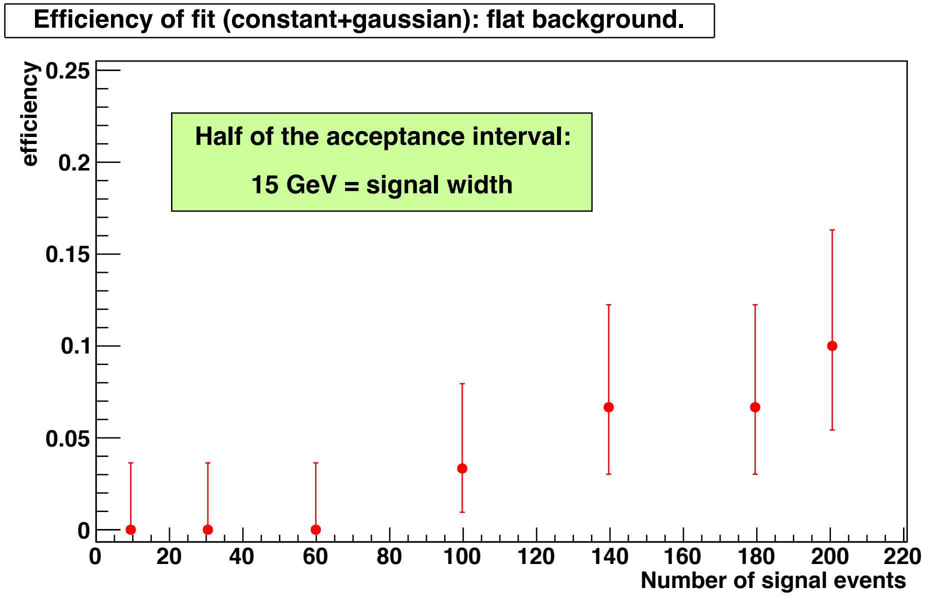


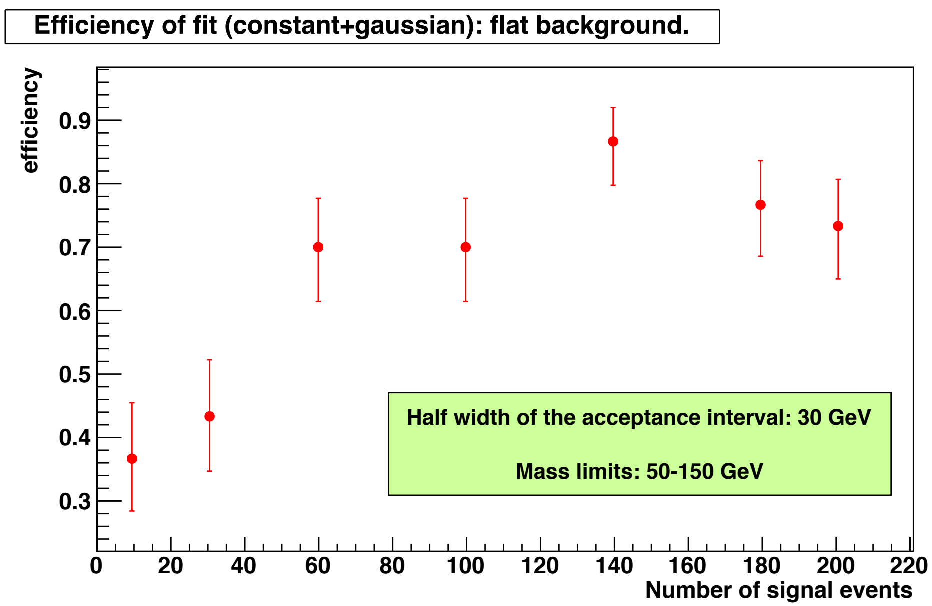
(c)



(d)

The efficiency is calculated for different signal mean (μ) and standard deviation (σ). (a): $\mu = 100$ GeV $\sigma = 15$ GeV. (b): $\mu = 100$ GeV $\sigma = 7$ GeV. (c): $\mu = 40$ GeV $\sigma = 15$ GeV. (d): $\mu = 160$ GeV $\sigma = 15$ GeV.





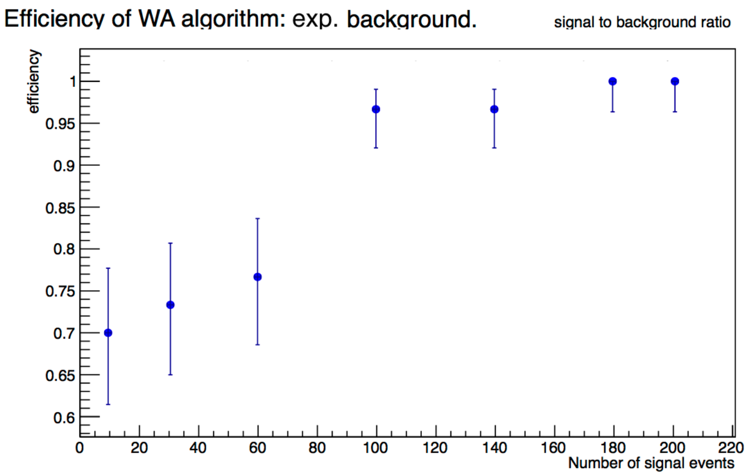
Efficiency in identifying a gaussian signal over a flat background, by fitting the data with a gaussian function superimposed to a constant term.

Flat background: 6000 events.
Signal: 100 events, μ=100 GeV, σ=15 GeV.

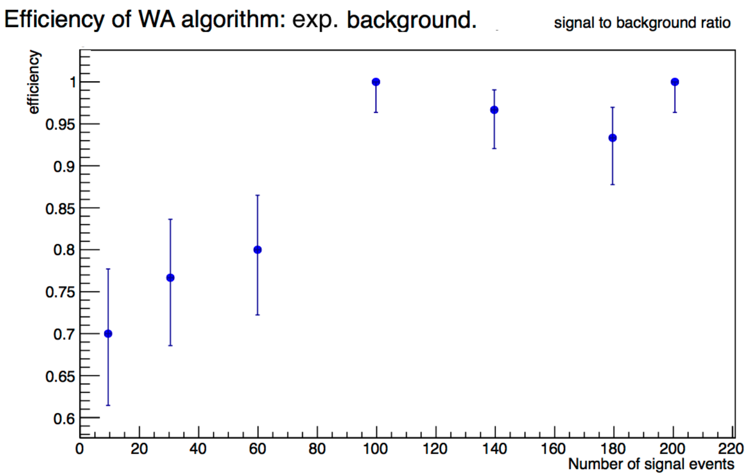
The signal width have been fixed to 15 GeV in the fit.

EFFICIENCY: EXPONENTIAL BACKGROUND

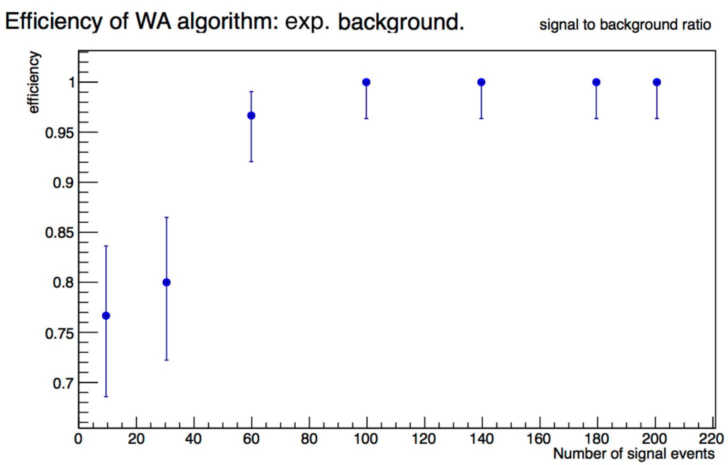
Exponential background: 6000 events.
Signal: 100 events.



(a)

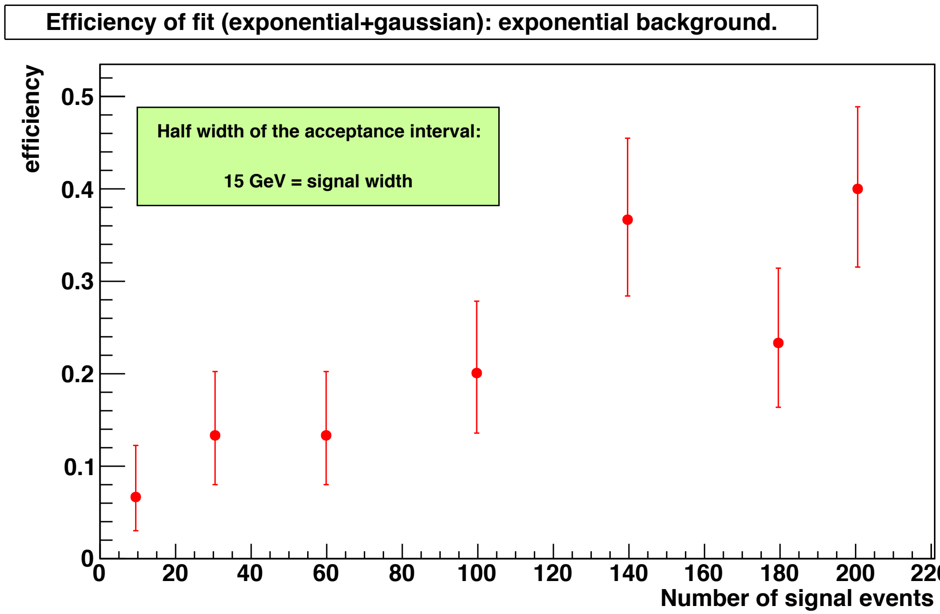


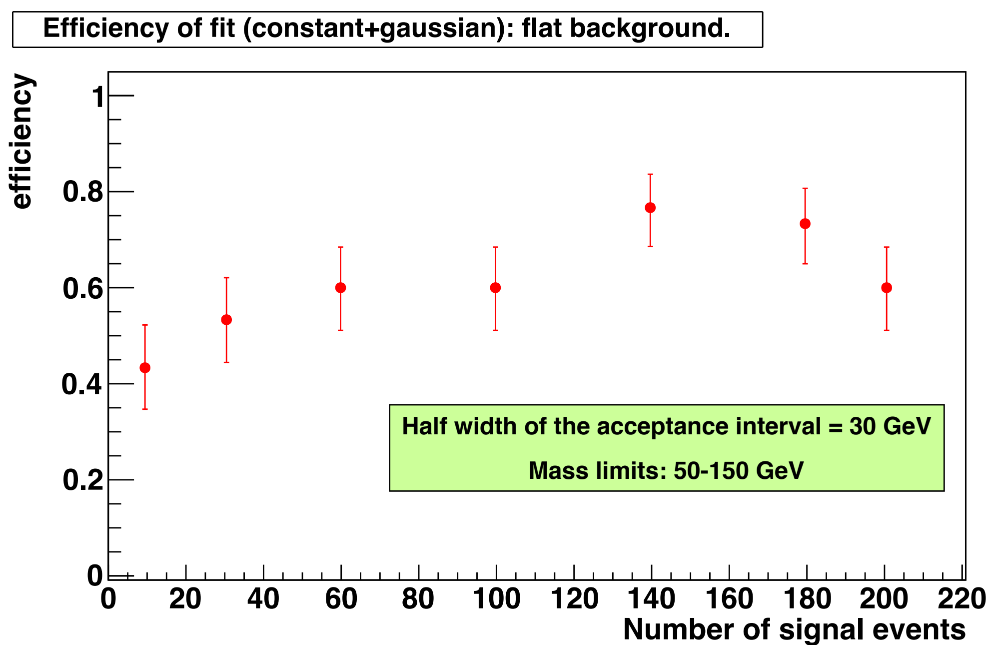
(b)



(c)

The efficiency is calculated for different signal mean (μ). *(a):* $\mu = 100$ *GeV (b):* $\mu = 40$ *GeV. (c):* $\mu = 160$ *GeV. The standard deviation is* $\sigma = 15$ *GeV.*



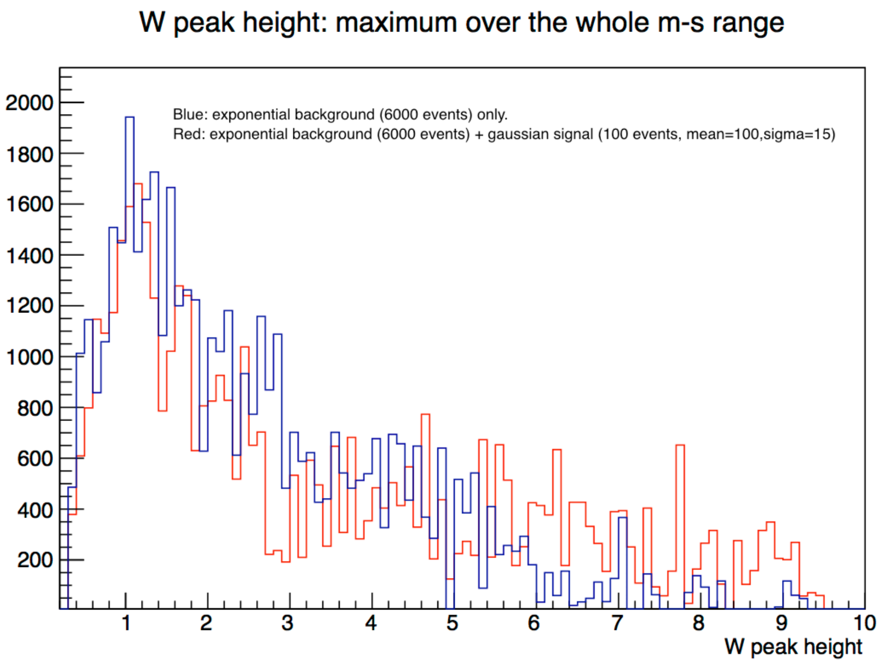


Efficiency in identifying a gaussian signal over an exponential background, by fitting the data with a gaussian function superimposed to a decreasing exponential.

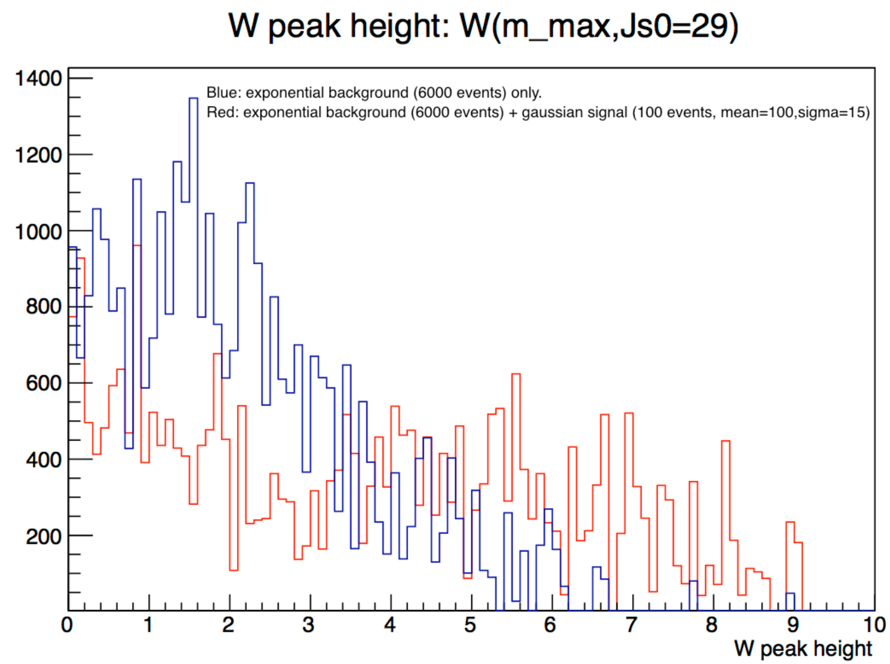
Exponential background: 6000 events.
Signal: 100 events, μ=100 GeV, σ=15 GeV.

The signal width have been fixed to 15 GeV in the fit.

Wavelet transform maximum in the case of a background only sample (blue) and background plus signal sample (red). Plot (a) shows W_{max}, plot (b) shows $W(m_{max}, s_0)$. Exponential background: 6000 events. Gaussian signal: 100 events, mean $\mu = 100$ GeV, standard deviation $\sigma = 15$ GeV.



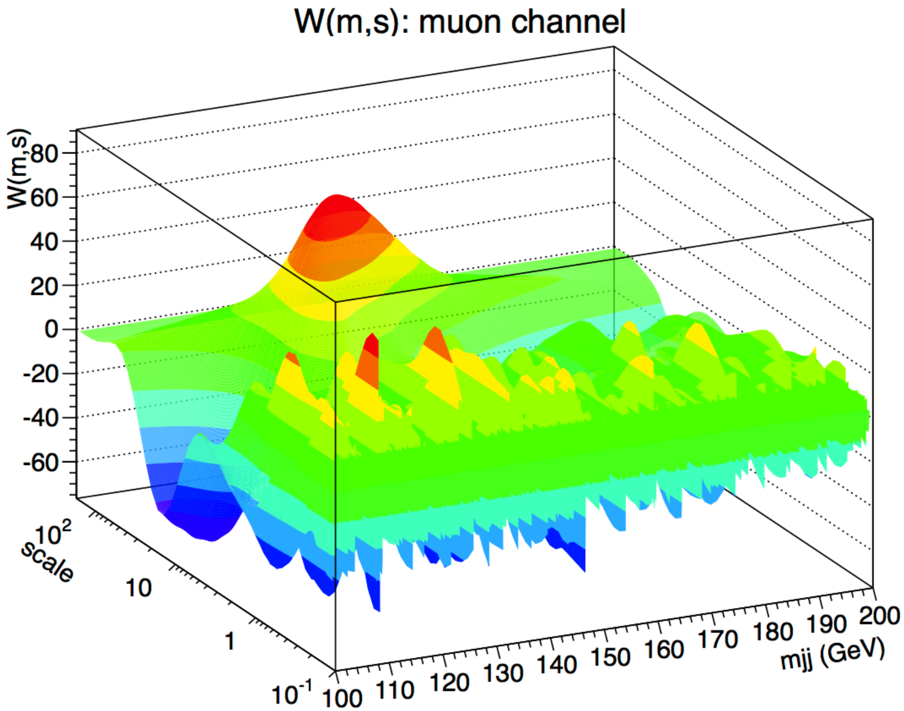
(a)

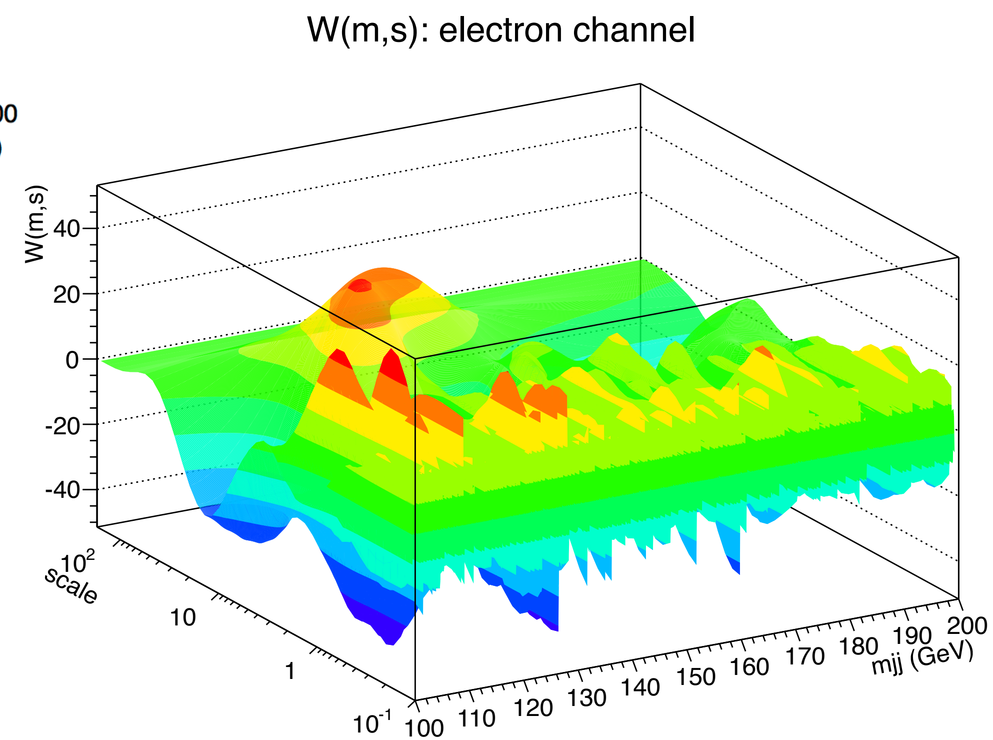


(b)

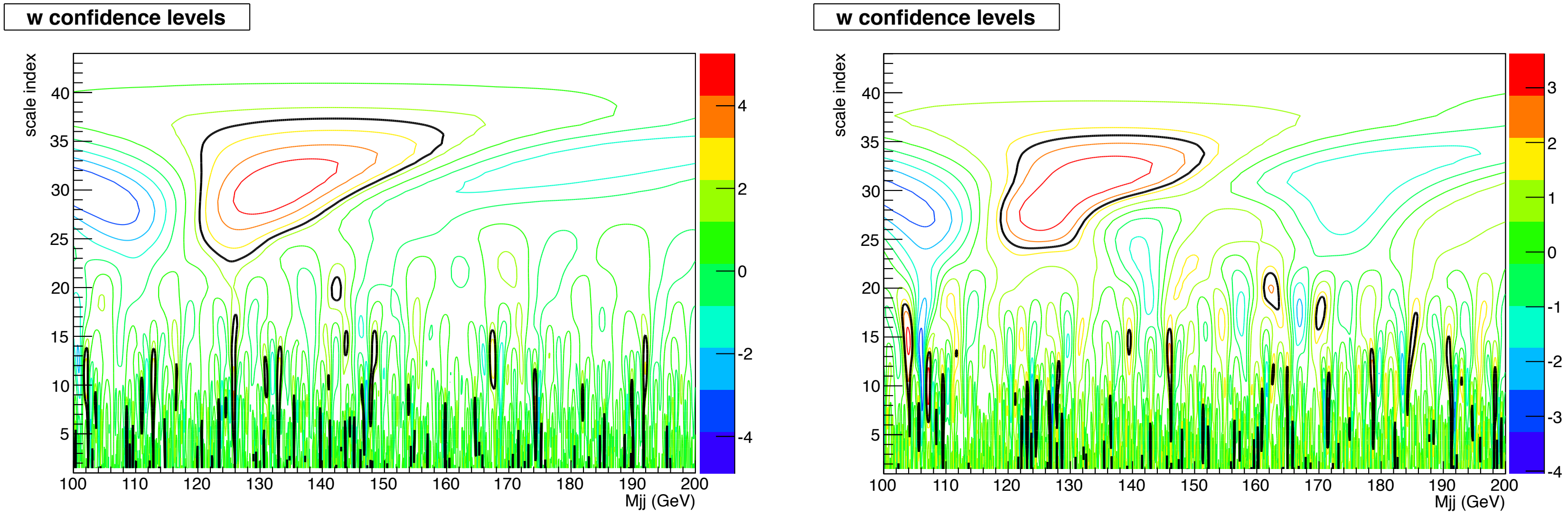
Background only: blue.
Background plus signal: red.

RESULTS IN [100,200] GeV MASS REGION: TRIDIMENSIONAL VIEW





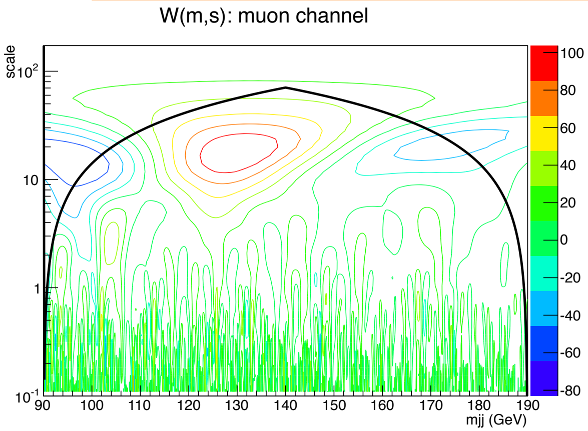
RESULTS IN [100,200] GeV MASS REGION: CONFIDENCE LEVEL, BI-DIMENSIONAL PLOT



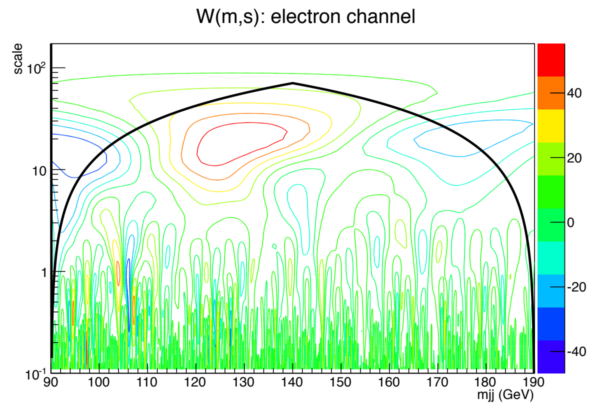
(a) (b)

Wavelet transform of the jet-jet invariant mass spectrum for different mass ranges divided by its standard deviation ($W(m,s)/\sigma_{m,s}$) as a function of mass and scale. The wavelet transform has been computed after background subtraction. (a): Muon channel. (b): Electron channel. The 95% confidence level is indicated by a black contour.

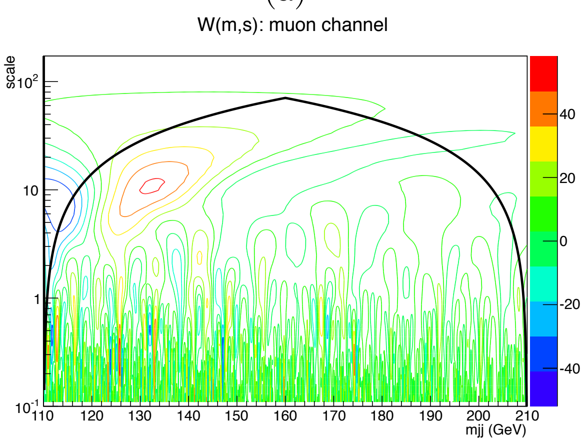
RESULTS IN [100,200] GeV MASS REGION: MOVED MASS INTERVAL



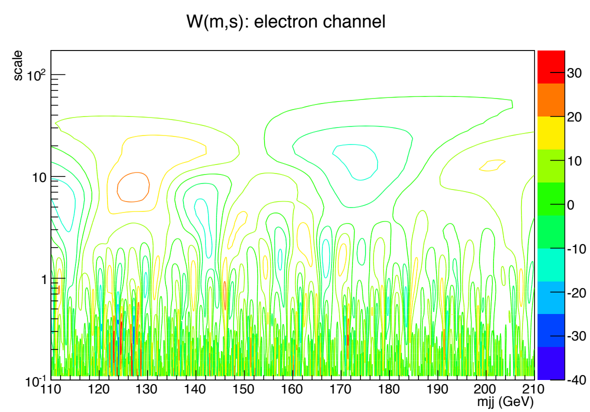
(a)



(b)



(c)



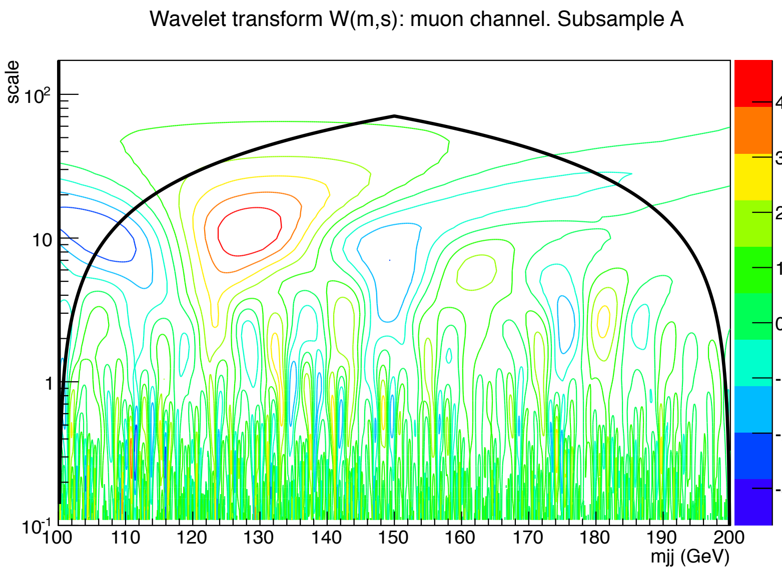
(d)

Wavelet transform ($W(m,s)$) of the jet-jet invariant mass spectrum for different mass ranges. The wavelet transform has been computed after background subtraction, it is represented as a function of mass and scale. (a): Muon channel, mass range $m_{jj} \in [90,190]$ GeV. (b): Electron channel, mass range $m_{jj} \in [90,190]$ GeV. (c): Muon channel, mass range $m_{jj} \in [110,210]$ GeV. (b): Electron channel, mass range $m_{jj} \in [110,210]$ GeV.

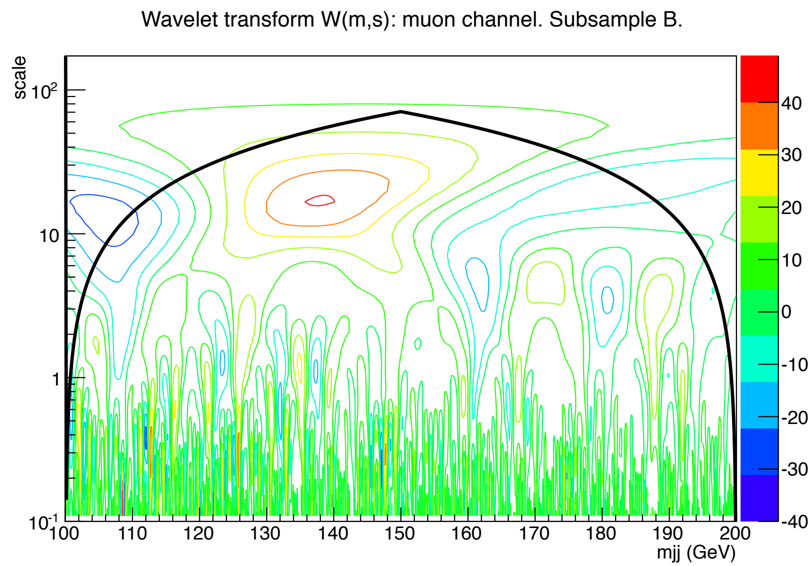
- At [110,210] GeV the wavelet transform peak is affected by the edge effects.
- At [90,190] GeV the fitted mass region is closer to the background peak: slight effects on the fit quality are possible.

RESULTS IN [100,200] GeV MASS REGION: SUBSAMPLES OF MUON CHANNEL

Subsample A

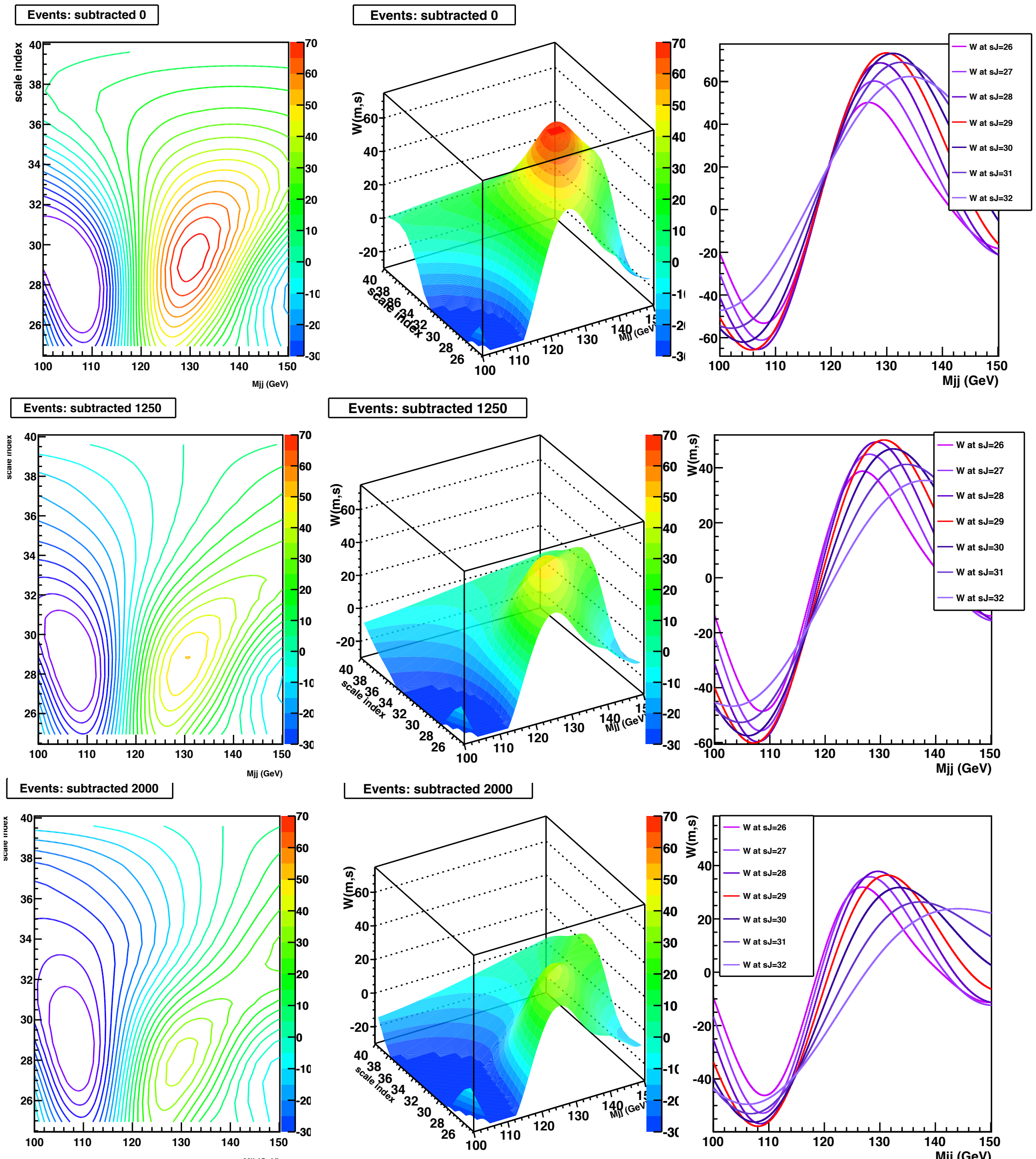


Subsample B



Wavelet transform ($W(m,s)$) of the jet-jet invariant mass spectrum for different mass ranges: muon channel. The wavelet transform has been computed after background subtraction, it is represented as a function of mass and scale. The analysis has been repeated independently using the subsamples A and B, each containing half of the original muon channel sample.

SIGNAL SUBTRACTION: MUON CHANNEL

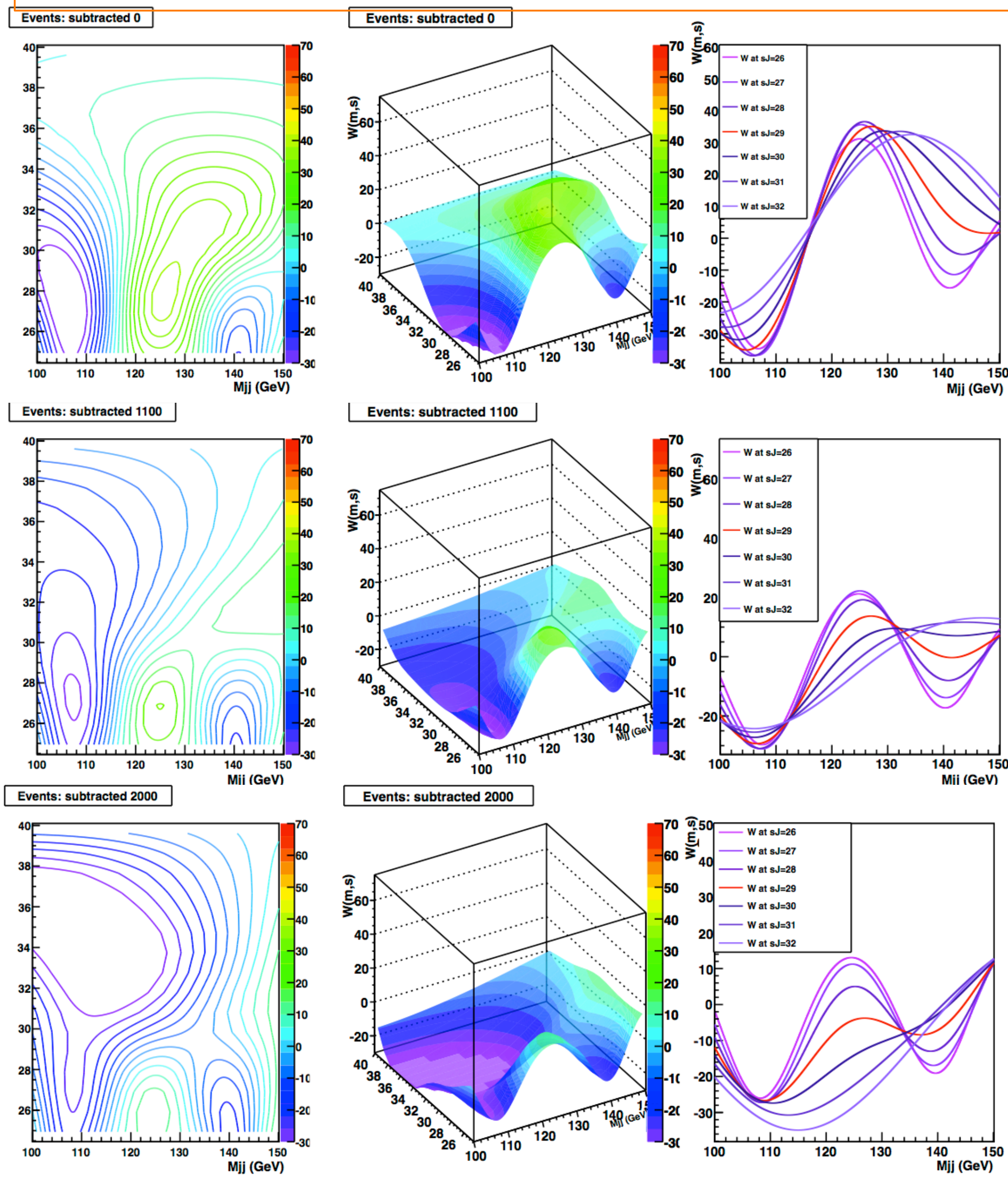


No signal subtracted

1250 events subtracted

2000 events subtracted

SIGNAL SUBTRACTION: ELECTRON CHANNEL

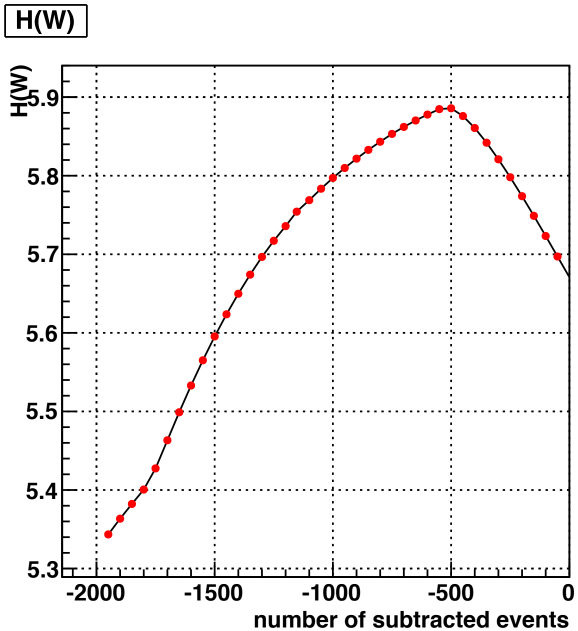


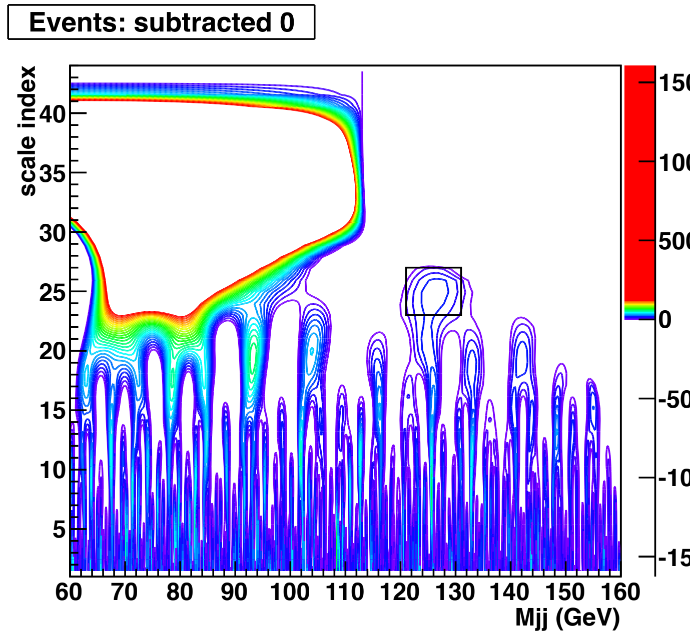
No signal subtracted

1100 events subtracted

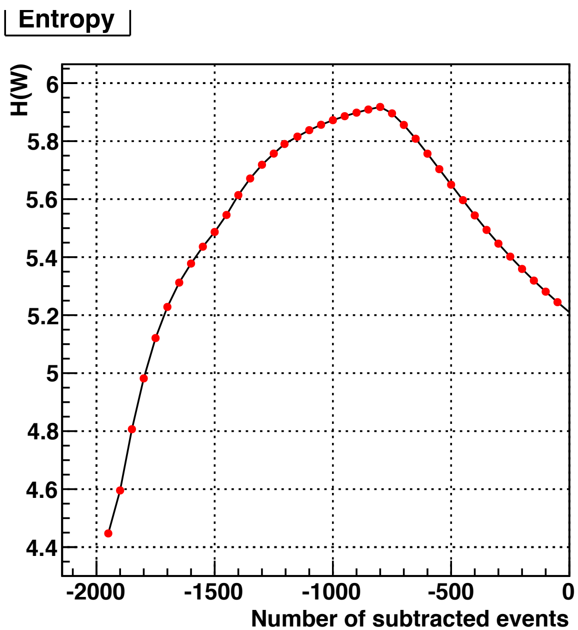
2000 events subtracted

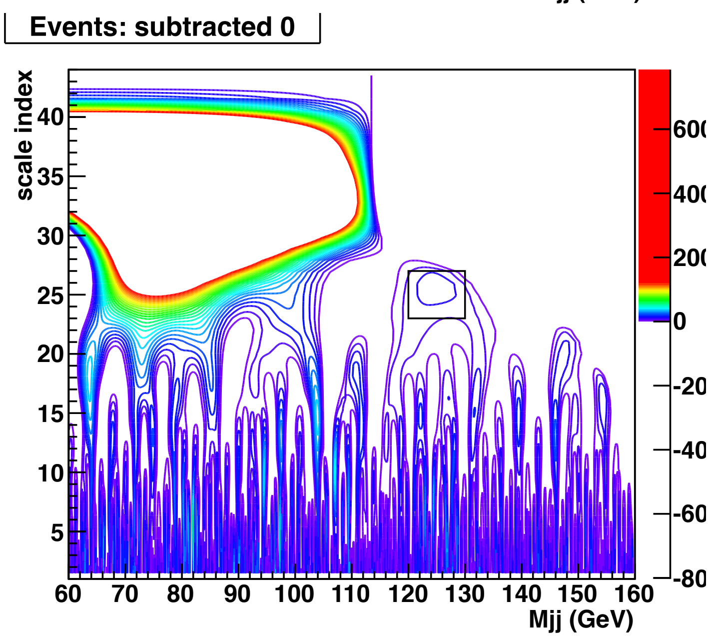
SIGNAL SUBTRACTION WITHOUT BACKGROUND SUBTRACTION





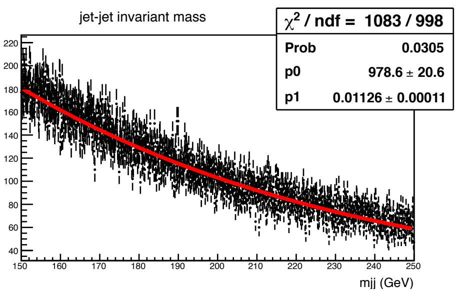
Muon channel

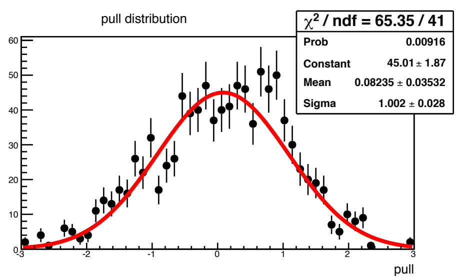


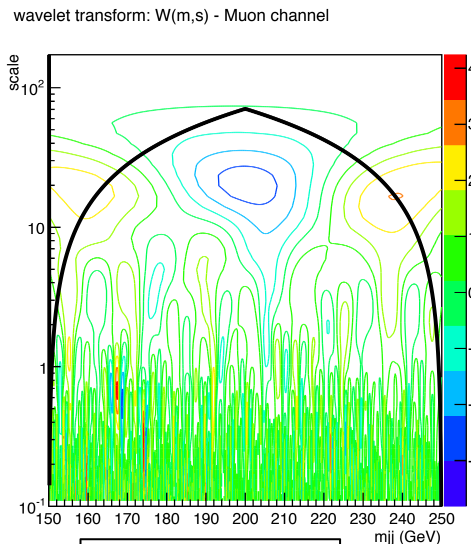


Electron channel

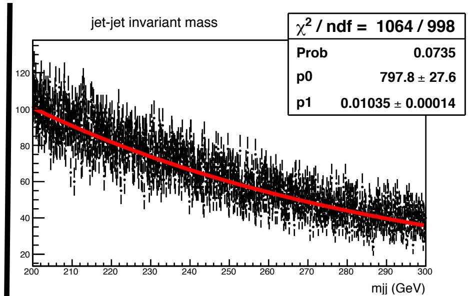
SEARCH OF NEW PHYSICS VIA THE WAVELET ANALYSIS: MUON CHANNEL 1

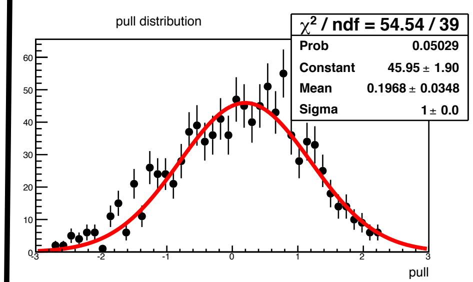


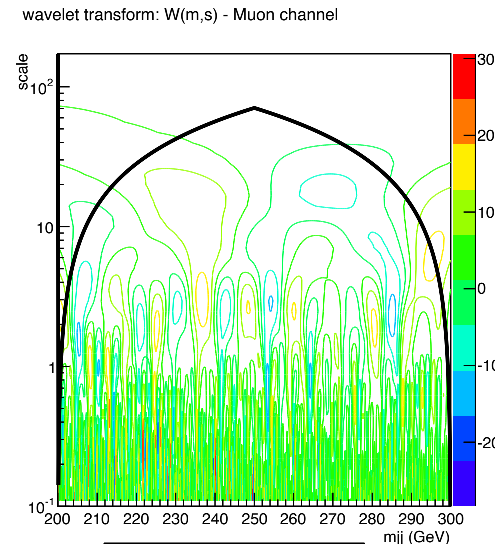




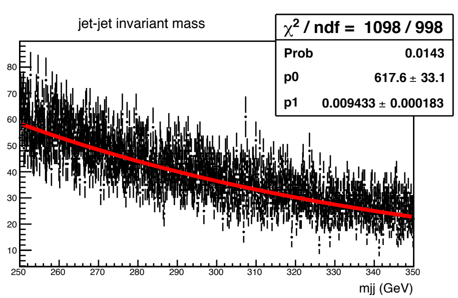
[150,250] GeV

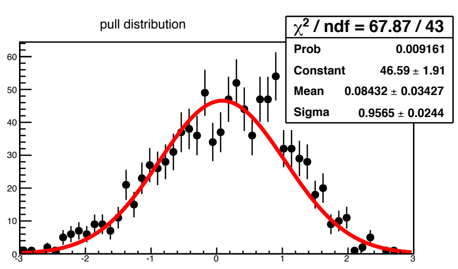


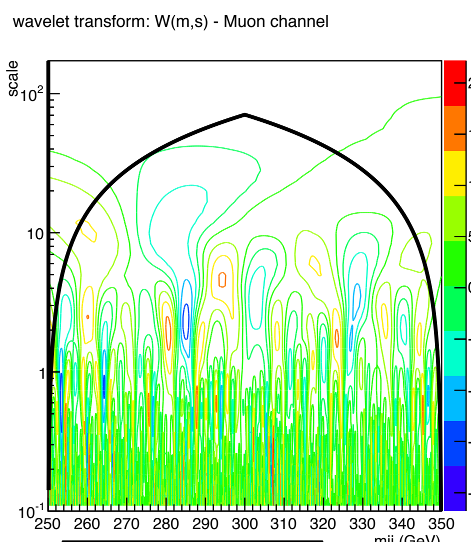




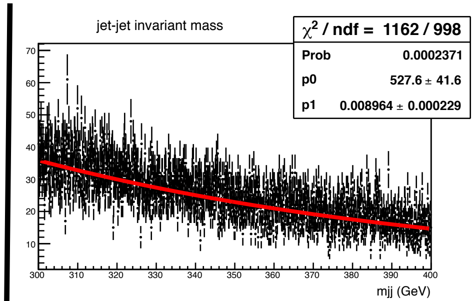
[200,300] GeV

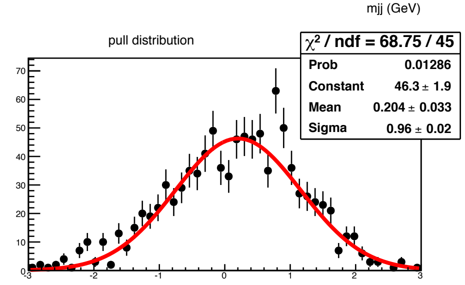


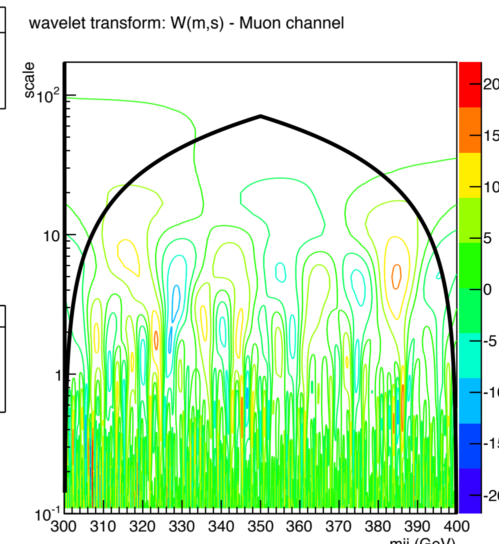




[250,350] GeV

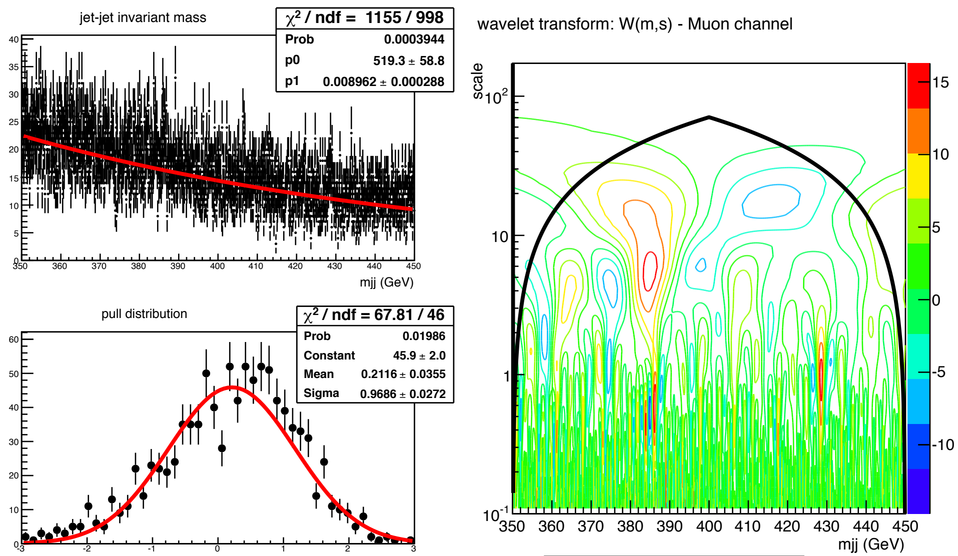




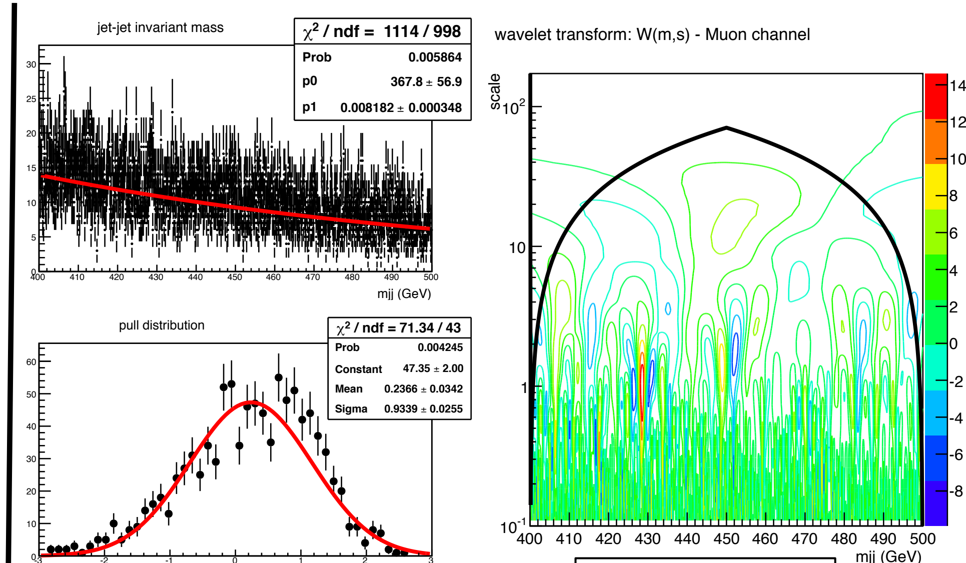


[300,400] GeV

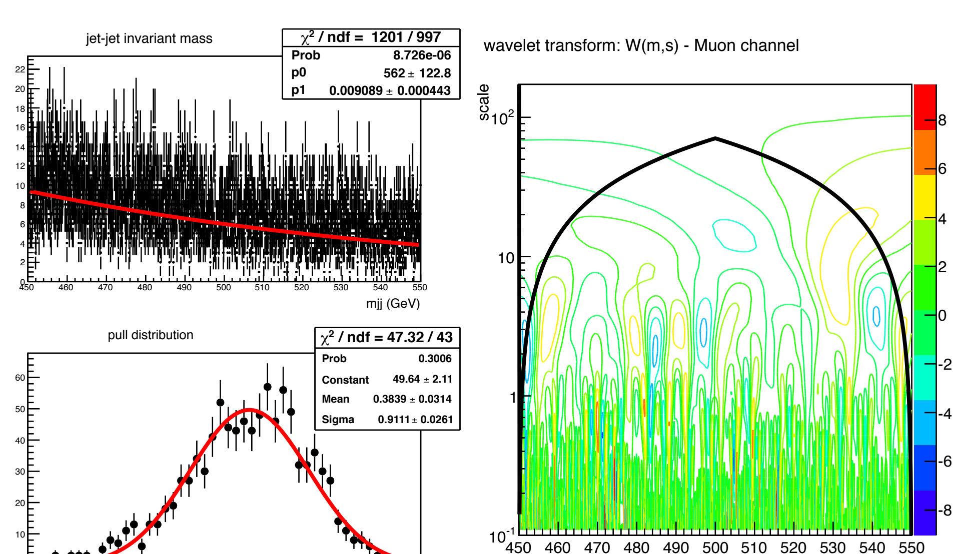
SEARCH OF NEW PHYSICS VIA THE WAVELET ANALYSIS: MUON CHANNEL 2



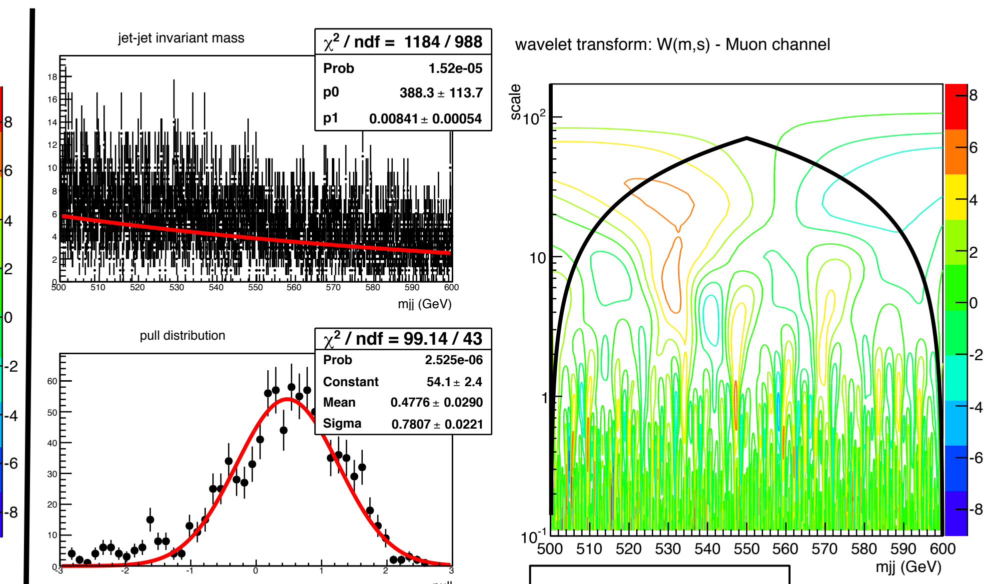
[350,450] GeV



[400,500] GeV

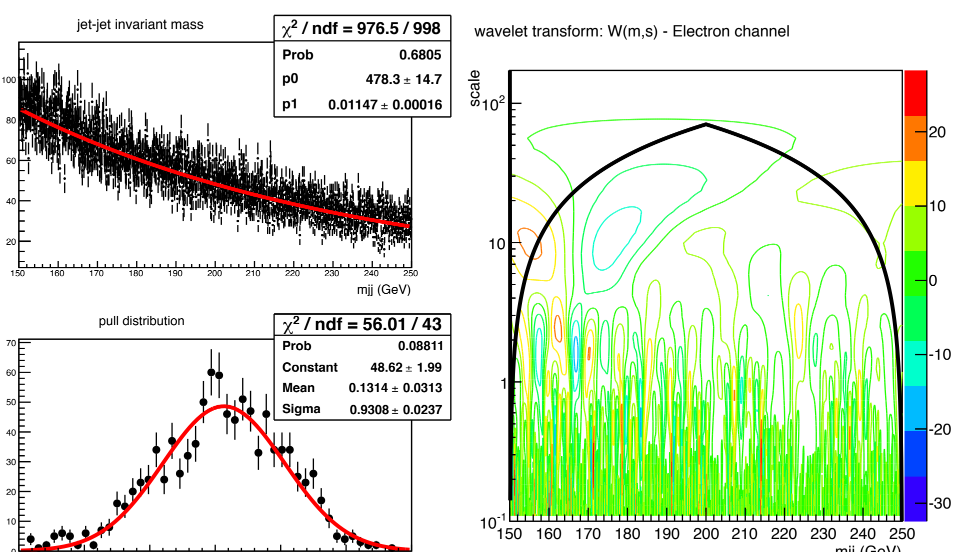


[450,550] GeV

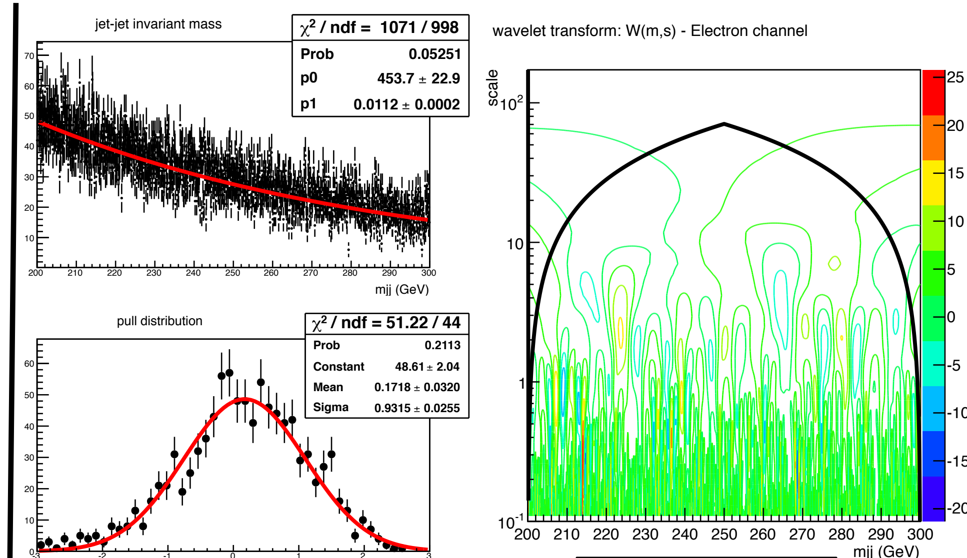


[500,600] GeV

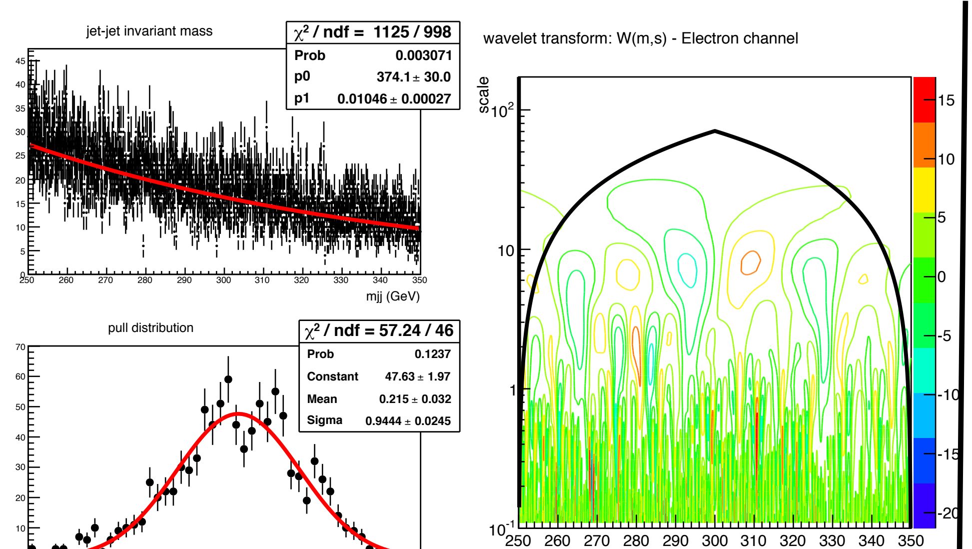
SEARCH OF NEW PHYSICS VIA THE WAVELET ANALYSIS: ELECTRON CHANNEL 1



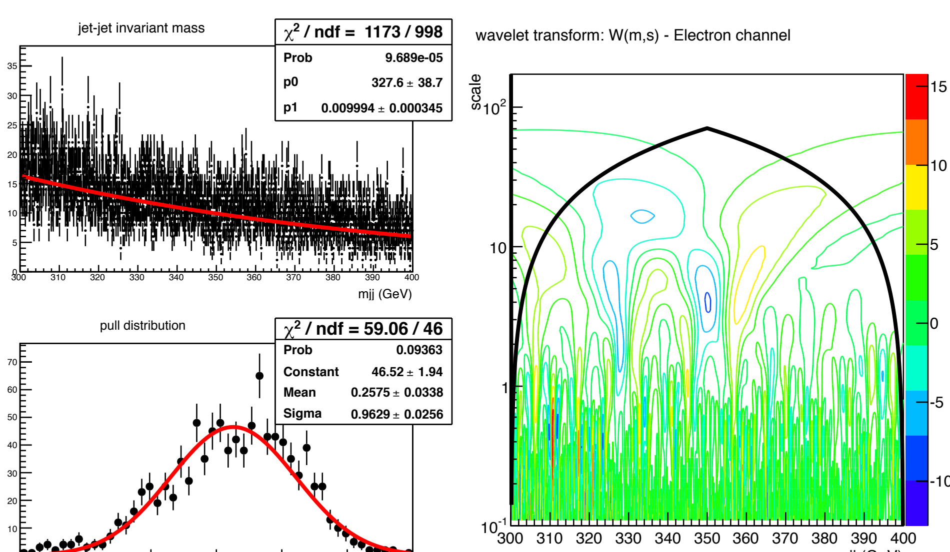
[150,250] GeV



[200,300] GeV

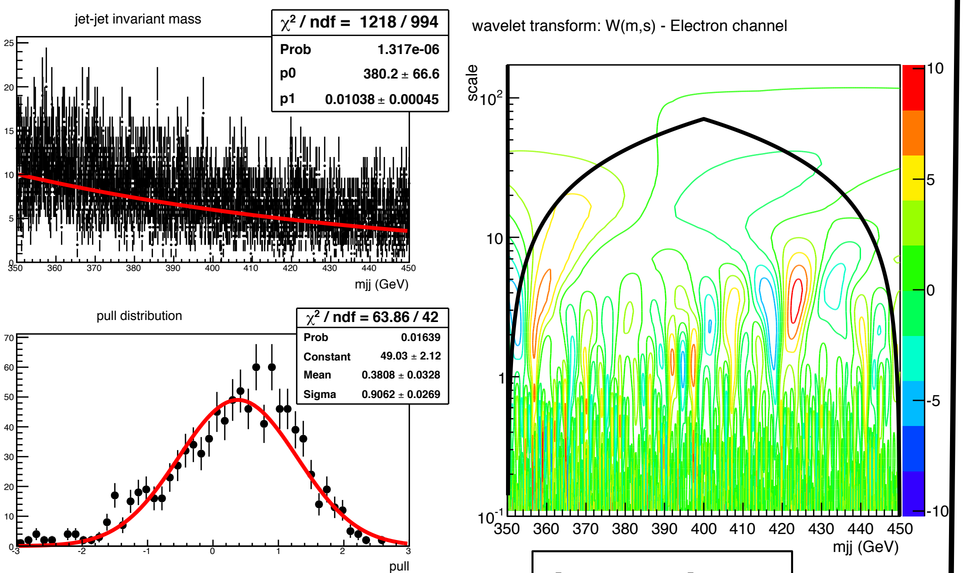


[250,350] GeV

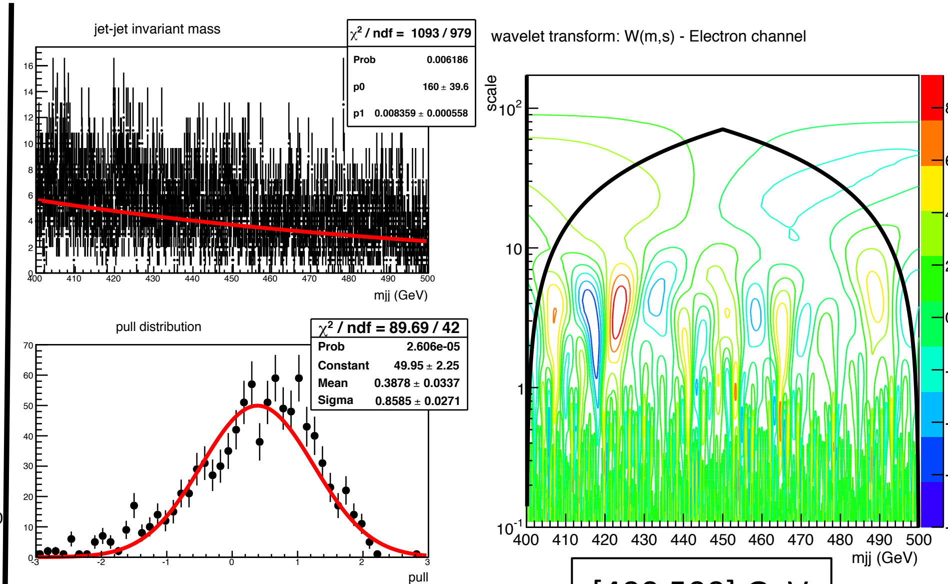


[300,400] GeV

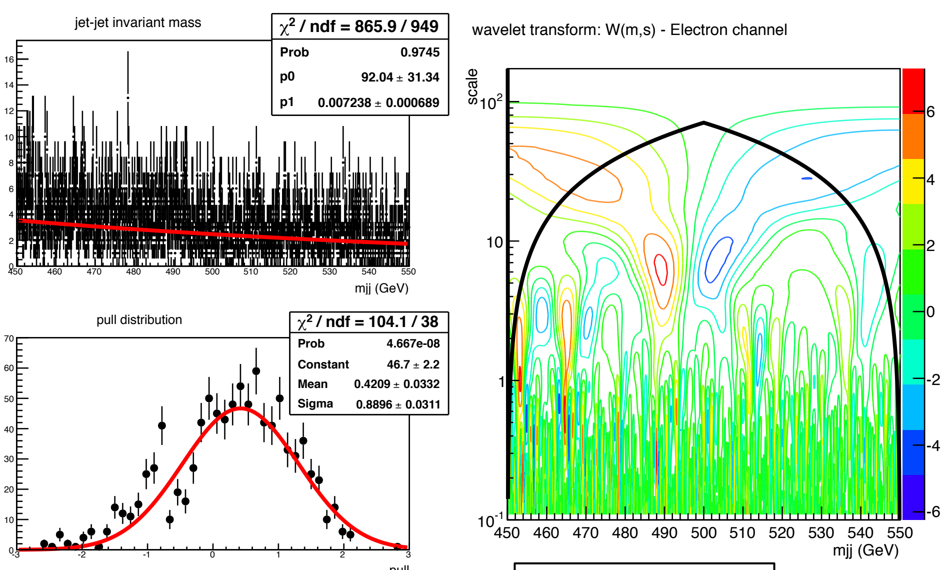
SEARCH OF NEW PHYSICS VIA THE WAVELET ANALYSIS: ELECTRON CHANNEL 2



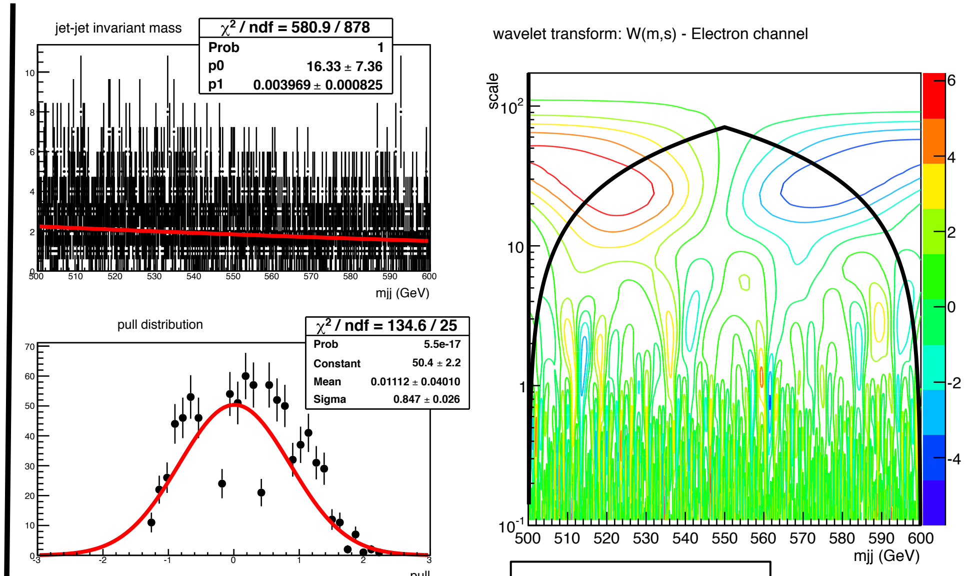
[350,450] GeV



[400,500] GeV



[450,550] GeV



[500,600] GeV