Hadrons in a hot and dense medium: a holographic study

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What happens to hadrons?

supposed to melt in plasma
Spectral function is a key quantity to look at

Difficult task! QCD is non-perturbative!

Some techniques developed so far for spectral functions at finite temperature:

Heavy Quarkonium:
- Potential models
- EFT
- MEM on the lattice
- Sum Rules
  ....

Light mesons:
- Sum Rules
- (P)NJL
  ....

Also studies at finite density, see discussion in concluding remarks

In next slides, some examples and results (not exhaustive, only an idea)
Introduction

Lattice (1) MEM on lattice

\[ \gamma \]

\[ J/\psi \]

\[ \rho(\omega) \]

\[ T/T_c = 0.42 \quad T/T_c = 1.05 \quad T/T_c = 1.20 \quad T/T_c = 1.40 \]

\[ T = 0.78T_c \quad T = 1.38T_c \quad T = 1.62T_c \]

\[ T = 1.87T_c \quad T = 2.33T_c \]

\[ \omega \text{ [GeV]} \]

\[ \Delta \text{EM} \]

M. Asakawa, T. Hatsuda
PRL 92 012001 (2004)

JHEP 1111 (2011) 103
Introduction

EFT

\[ J/\psi \]

\[ -\frac{\rho}{M^2} \]

\[ \frac{-\rho}{m_Q^2} \]

\[ \frac{\Gamma(V)/\Gamma(0)}{\nu} \]

M. Laine
JHEP 0705, 028 (2007)

M.A. Escobedo, FG, M. Mannarelli, J. Soto
PRD 87, 114005 (2013)
Introduction

Lattice (2) Through quark-antiquark potential

Only real part of the potential

Including imaginary part

P. Petreczky, C. Miao, A. Mocsy
NPA 855, 125 (2011)
Introduction

QCD Sum Rules (1)

P. Gubler, K. Morita, M. Oka
PRL 107, 092003 (2011)
QCD Sum Rules (2)

\[ \rho \]

\begin{align*}
\text{T. Hatsuda, Y. Koike, S.H. Lee} \\
\text{NPB 394, 221 (1993)}
\end{align*}

\begin{align*}
\text{A. Ayala, C.A. Dominguez, M. Loewe, Y. Zhang} \\
\text{PRD 86, 114036 (2012)}
\end{align*}
Introduction

Hadrons in a hot and dense medium: a holographic study

PNJL ($\sigma$)

H. Hansen, W.M. Alberico, A. Beraudo, A. Molinari, M. Nardi, C. Ratti

PRD 75, 065004 (2007)
Introduction

Present talk: Soft wall

Scalar light mesons

\[ \omega^2 (\text{GeV}^2) \]

\[ n^2 (\text{GeV}^2) \]

\[ \Gamma (\text{GeV}) \]
Outline

- Introduction on AdS/CFT
- The AdS/QCD correspondence
- The soft-wall model
- The finite temperature case
- The finite density case
AdS/CFT correspondence

IIB string theory
in AdS$_5 \times S^5$

$\mathcal{N}=4$ SYM, SU(N)
in $M_4$

$g_s = g_{YM}^2$

$R^4 = 4\pi g_s N \alpha'^2$

Why interesting for us?

SUGRA limit

\[
\begin{cases}
    g_s \to 0 \\
    g_s N \to \infty
\end{cases}
\]

Large N + NP regime

\[
\begin{cases}
    N \to \infty \\
    \lambda = g_{YM} N \to \infty
\end{cases}
\]

J.M. Maldacena
AdS/CFT correspondence

How can the two theories be linked?

Holographic description + dictionary

AdS space has a boundary:
Minkowski space + point at infinity

Build gauge theory on
Minkowski space at boundary,
i.e. $z \to 0$

$$ds^2 = \frac{R^2}{z^2} (-dt^2 + d\vec{x}^2 + dz^2)$$
How can the two theories be linked?

1. **field** $\phi(x, z) \iff \text{operator} \ O(x)$
   - kind of field fixed by operator
   - mass is related to conformal dimension of operator
   - boundary value identified with source of operator

2. $Z_S[\phi_0(x)] = \left< e^{\int_{\partial AdS_{d+1}} \phi_0(x) O(x)} \right>_{\text{CFT}}$

   Example: 0-form $\iff$ scalar
   
   $m_5^2 R^2 = (\Delta - p)(\Delta + p - 4)$
   
   $\phi(x, z) = \int_{\partial AdS_{d+1}} d^d x' K(x - x', z) \phi_0(x')$
   
   $K(x - x', z) \xrightarrow{\partial AdS_{d+1}} z^\xi \delta^d(x - x')$

   Bulk-to-boundary propagator

   $Z_{\text{CFT}} = Z_S = e^{iS_{os}}$

   $\left. Z_{\text{CFT}} \right|_{\phi_0 = 0} \iff \frac{\delta^2 S_{os}}{\delta \phi_0(x_1) \delta \phi_0(x_1)} \bigg|_{\phi_0 = 0}$
**AdS/QCD correspondence**

**Apply to QCD!**

QCD is not:
1. supersymmetric
2. conformal (running coupling constant)

Introduce:
1. independent bosonic and fermionic states
2. mass scale

---

**Investigate the NP regime of QCD through a perturbative theory in a curved space with d>4**

**Two approaches:**

**top-down**
modify the supergravity theory such that it can describe QCD

Ex. glueballs

Ex. mesons

**bottom-up**
write a 5d model, inspired by supergravity, that can reproduce the properties of QCD

Ex. Lagrangian for $\chi_{SB}$:

$$\mathcal{L} = \sqrt{|g|} \text{Tr} \left[ DX^2 + m_5^2 X^2 + \frac{1}{4 g_5^2} F^2 \right]$$
Bottom-up: soft wall

To break conformal invariance, introduce a mass scale

\[
\text{Soft wall: insert } e^{c^2 z^2} \text{ in the action}
\]

\[
\text{Vector mesons: } m_n^2 = c^2 (4n+4)
\]

\[
\text{Scalar mesons: } m_n^2 = c^2 (4n+6)
\]

\[
\text{Scalar glueballs: } m_n^2 = c^2 (4n+8)
\]

\[
\text{Hybrid vector mesons: } m_n^2 = c^2 (4n+8)
\]

\[
m_\rho = 776 \text{ MeV}
\]

\[
c = 388 \text{ MeV}
\]

Regge trajectories:

Suggestion: J/\Psi in the same scheme, by fitting \( c_\psi = m_\psi / 2 = 1.55 \text{ GeV} \)

Our studies in the Soft-Wall model:

- Scalar glueballs
- Scalar mesons
- Finite temperature spectral functions for scalar glueballs and mesons
- Finite temperature free energy
- AVV vertex
- Chiral condensate at finite temperature and density
- Spectral functions at finite density
- Gluon condensate at finite temperature and density

M. Fujita, K. Fukushima, T. Misumi, M. Murata
PRD 80, 035001 (2009)
Soft wall, vector and scalar mesons

<table>
<thead>
<tr>
<th>$\mathcal{O}(x)$</th>
<th>$\phi(x, z)$</th>
<th>$p$</th>
<th>$\Delta$</th>
<th>$m_5^2 R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{q}_L \gamma^{\mu} T^a q_L$</td>
<td>$A^a_{L\mu}$</td>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>$\bar{q}_R \gamma^{\mu} T^a q_R$</td>
<td>$A^a_{R\mu}$</td>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>$\bar{q}^\alpha_R q^\beta_L$</td>
<td>$2 X^{\alpha\beta}/z$</td>
<td>0</td>
<td>3</td>
<td>-3</td>
</tr>
</tbody>
</table>

$$ds^2 = \frac{R^2}{z^2} (-dt^2 + d\vec{x}^2 + dz^2)$$

$$S = -\frac{1}{k} \int d^5 x \sqrt{|g|} e^{-\phi(z)} \text{Tr} \left\{ |D X|^2 + m_5^2 |X|^2 + \frac{1}{4 g_5^2} (F^2_L + F^2_R) \right\}$$

$$D_M X = \partial_M X + i [X, V_M] - i \{X, A_M\}$$

$$X(x, z) = (X_0(z) + S(x, z)) e^{2i \pi (x, z)}$$

$A = \frac{A_L - A_R}{2}$ (axial vector)

$V = \frac{A_L + A_R}{2}$ (vector)

Next slides: scalar mesons in vacuum and finite T

vector mesons at finite density
Scalar mesons

Spectrum:

\[
S_S = -\frac{1}{2k} \int d^5x \sqrt{|g|} \ e^{-\phi(z)} \left[ g^{MN} \partial_M S^A(x, z) \partial_N S^A(x, z) + m_s^2 S^A(x, z) S^A(x, z) \right]
\]

\[ \text{e.o.m.} \quad \partial_z \left( \frac{1}{z^3} e^{-\phi(z)} \partial_z \tilde{S} \right) + \frac{3}{z^5} e^{-\phi(z)} \tilde{S} - \frac{q^2}{z^3} e^{-\phi(z)} \tilde{S} = 0 \]

Bogoliubov transformation

\[ \tilde{S} = e^{c^2 z^2/2} z^{3/2} Y \]

\[-Y''(z) + V(z)Y(z) = -q^2 Y(z) \quad V(z) = \frac{3}{4z^2} + 2c^2 + c^4 z^2 \]

Eigenfunctions:

\[ \tilde{S}_n(c z) = \sqrt{\frac{2}{n + 1}} c^3 z^3 L_n^1(c^2 z^2) \]

\[ m_{S_n}^2 = c^2 (4n + 6) \quad \rightarrow \quad m_0 = 950 \text{ MeV} \quad \text{close to } f_0(980) \]
Scalar mesons

Two-point correlation function:

\[
\langle \mathcal{O}(x_1)\mathcal{O}(x_2) \rangle = \frac{1}{Z_{\text{CFT}}} \left( -i \frac{\delta}{\delta S_0(x_1)} \right) \left( -i \frac{\delta}{\delta S_0(x_2)} \right) Z_{\text{CFT}} \bigg|_{S_0=0}
\]

\[
\Pi_{\text{AdS}}^{AB}(q^2) = \delta^{AB} \frac{R^3}{\kappa z^3} e^{-\phi} S(q, z) \partial_z S(q, z) \bigg|_{z=0} = \tilde{S}(q, z) = S(q, z) \tilde{S}_0(q^2)
\]

\[
\Pi_{\text{AdS}}^{AB}(q^2) = \delta^{AB} \frac{R}{\kappa} \left[ \nu^2 + q^2 \log(c^2/\nu^2) + \frac{1}{2} q^2 \left( -1 + 4\gamma_E + 2\psi\left(\frac{q^2}{4c^2} + \frac{3}{2}\right) \right) + \right.
\]

\[
+ 2c^2 \left( -1 + 2\gamma_E + \psi\left(\frac{q^2}{4c^2} + \frac{3}{2}\right) + \log(c^2/\nu^2) \right) \left. \right]
\]

\[
F_n^2 = \frac{R}{\kappa} \text{Residue}[(2c^2 + q^2) \psi\left(\frac{q^2}{4c^2} + \frac{3}{2}\right)] = \frac{R}{\kappa} 16\, c^4 \,(n + 1)
\]
Scalar mesons

\[ \Pi^{AB}_{\text{AdS}}(q^2) \xrightarrow[q^2 \to \infty]{\delta^{AB}} \frac{R}{k} \left[ \frac{q^2}{\nu^2} \log \frac{q^2}{\nu^2} + q^2 \left( 2\gamma_E - \log 4 - \frac{1}{2} \right) + 2c^2 \left( \log \frac{q^2}{\nu^2} - \log 4 + 2\gamma_E + 1 \right) + \frac{2c^4}{3q^2} + \frac{4c^6}{3q^4} + \mathcal{O}(1/q^6) \right] \]

\[ \Pi^{AB}_{\text{QCD}}(q^2) \xrightarrow[q^2 \to \infty]{\delta^{AB}} \frac{3}{8\pi^2} \left( 1 + \frac{11\alpha_s}{3\pi} \right) q^2 \log \left( \frac{q^2}{\nu^2} \right) + \frac{3}{q^2} \langle m_q \bar{q}q \rangle + \frac{1}{8q^2} \frac{\alpha_s}{\pi} G^2 \]

\[ + \frac{m_q g_s}{2q^4} \langle (\bar{q} \gamma_{\mu\nu} \lambda^a q) G^a_{\mu\nu} \rangle + \frac{\pi \alpha_s}{q^4} \langle (\bar{q} \sigma_{\mu\nu} \lambda^a q)^2 \rangle + \frac{2\pi \alpha_s}{3q^4} \langle (\bar{q} \gamma_{\mu} \lambda^a q) \sum_{q=u,d} \bar{q} \gamma_{\mu} \lambda^a q \rangle + \mathcal{O}(1/q^6) \left[ \right] \]

\[ \frac{R}{k} = \frac{N_c}{16\pi^2} \]
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AdS space + black hole

$$ds^2 = \frac{R^2}{z^2} \left( f(z) \, d\tau^2 + d\vec{x}^2 + \frac{dz^2}{f(z)} \right)$$

$$f(z) = 1 - \frac{z^4}{z_h^4}$$

$$z_h = \frac{1}{\pi T}$$

Hawking temperature

**Application: spectral functions at finite temperature**

**Example: scalar mesons**

Same action, but different metric! Eom reads:

$$S''(q, z) - \frac{2c^2 z^2 f(z) + 3 + \frac{z^4}{z_h^4}}{zf(z)} S'(q, z) + \frac{3}{z^2 f(z)} S(q, z) + \left( \frac{q_0^2}{f(z)^2} - \frac{q^2}{f(z)} \right) S(q, z) = 0$$

rest frame
Scalar mesons at finite temperature

Low temperatures (high $\tilde{z}_h$)

- BH horizon far, does not affect eigenfunctions

Eigenfunctions can be found by solving a Schroedinger-type eq., as at $T=0$

$$Y(\omega^2, z) = \sqrt{f(z)} z^{-3/2} e^{-\omega^2 z^2/2} S(\omega^2, z)$$

- $u = z / \tilde{z}_h$
Scalar mesons at finite temperature

High temperatures → compute spectral functions
masses = position of peaks
finite width appears

Green's function:

$$\Pi(\omega^2) = \frac{R^3}{2k} \frac{f(u)}{u^3 z_h^4} e^{-\phi(u)} S(\omega^2, u) \partial_u S(\omega^2, u) \bigg|_{u=0}$$

BTBP is the solution of the e.o.m, with boundary conditions:

$$S(q, 0) = u$$

$$S(q, z) \xrightarrow[u \to 1]{} S_- + S_+$$

$$S_{\mp}(\omega^2, u) = (1 - u)^{\mp i \sqrt{\omega^2 z_h^2} / 4}$$

"falling-in" solution, to get the retarded Green's function

Spectral function:

$$\rho(\omega^2) = \Im \left( \Pi^R_G(\omega^2) \right)$$
Scalar mesons at finite temperature

Masses and widths from fit with modified Breit-Wigner function:

$$\rho(\omega^2) = \frac{am \Gamma \omega^b}{(\omega^2 - m^2)^2 + m^2 \Gamma^2}$$

from Sch.-type eq

![Graph showing mass-squared as a function of temperature](image1)

![Graph showing width as a function of temperature](image2)
Finite temperature

Glueballs in the AdS-BH model with $\bar{q} \neq 0$

T=30 MeV

peaks shift towards higher $q_0^2$ and become broader
Finite temperature and density

AdS + Charged Black Hole $\rightarrow$ AdS/RN metric

$$ds^2 = \frac{R^2}{z^2} \left( f(z) dt^2 - d\bar{x}^2 - \frac{dz^2}{f(z)} \right) \quad 0 < z < z_h$$

$$f(z) = 1 - \left( \frac{1}{z_h^4} + q^2 z_h^2 \right) z^4 + q^2 z^6 \quad f(z_h) = 0$$

$A_0(z) = \mu - \kappa q z^2$ dual to $q^+ q = \bar{q} \gamma^0 q$

Temperature and chemical potential are related to BH parameters by:

$$T = \frac{1}{4\pi} \left| \frac{df}{dz} \right|_{z=z_h} = \frac{1}{\pi z_h} \left( 1 - \frac{Q^2}{2} \right)$$

$$\mu = \kappa Q / z_h$$

$$Q = q z_h^3 \quad 0 \leq Q \leq \sqrt{2}$$

from $A_0(z_h) = 0$
Vector mesons in medium

Vector meson spectral functions

\[
S = - \frac{1}{2 k_V g_5^2} \int d^5 x \sqrt{g} e^{-\phi(z)} \operatorname{Tr} \left[ F^{MN}_V F_{VMN} \right]
\]

\[
F^{MN}_V = \partial^M V^N - \partial^N V^M
\]

\[
\partial_z \left( \frac{e^{-\phi(z)}}{z} f(z) \partial_z V_i(z, \omega^2) \right) + \frac{e^{-\phi(z)}}{z} \omega^2 V_i(z, \omega^2) = 0
\]

\[
V_i(z, \omega^2) = V(z, \omega^2) V^0_i(\omega^2)
\]

\[
V(0, \omega^2) = 1
\]

\[
V(z, \omega^2) \xrightarrow{z \to z_h} (1 - z/z_h) ^{-i \frac{\sqrt{\omega^2 z_h}}{2(2-Q^2)}} (1 + O(1 - z/z_h))
\]

\[
G_{ij}^R(p_0) = \frac{\delta^2 S}{\delta V_i^0(-\omega) \delta V_j^0(\omega)} = \delta_{ij} \frac{e^{-\phi(z)} f(z)}{g_5^2 k_V} V(z, \omega^2) \frac{\partial_z V(z, \omega^2)}{z} \bigg|_{z=0}
\]

\[
\rho(\omega^2) = \Re \left( \Pi_G^R(\omega^2) \right)
\]
Vector mesons in medium

- T=0.06
  - $\mu = 0.139$
  - $\mu = 0.225$
  - $\mu = 0.259$
  - $\mu = 0.455$

- $\mu = 0.139$
  - T=0.06
  - T=0.09
  - T=0.104
  - T=0.15

- $\rho \sim \omega^2$
- Peaks move towards lower masses
- Broadening of peaks
- Melting (earlier for excited states)
Vector mesons in medium

Masses and widths from fit with modified Breit-Wigner function:

\[
\rho(\omega^2) = \frac{am\Gamma\omega^b}{(\omega^2 - m^2)^2 + m^2\Gamma^2}
\]

Mass decrease effect:

\[
\frac{m|_{\mu=0} - m|_{\mu=\mu_c}}{m|_{\mu=0}} = 13\% \text{ at } T=0 \quad (25\% \text{ for squared mass})
\]

\[
18\% \text{ at } T=0.1c \quad (16\% \text{ for squared mass})
\]
Comparison with other models and experiments:

- **Width**
  Increases with temperature and chemical potential, in agreement with other models and experiments

- **Mass**
  - in SW decreases with temperature and chemical potential
  - Models suggest a mass increase or decrease
  - In experiments found a small decrease or no effect

In particular:

- Brown-Rho dropping: mass decrease related to chiral symmetry breaking parameters

- Hatsuda-Lee:
  \[ T = 0 : \quad m(\rho)/m(0) = 1 - \alpha \rho/\rho_0 \quad \alpha = 0.16 \pm 0.06 \]

Assuming this scaling we find \( \alpha = 0.012 \) \( (\mu_0 = 0.209c) \)
Concluding remarks

About hadrons' fate:

- Broadening of peaks in the spectral functions, as in models and experiments
- Downward mass shift at finite density, debated issue!
- Very small mass decrease at nuclear density, in agreement with experiments

About soft wall:

- Little analytical and numerical effort to compute spectral functions
- Reproduce key feature, in spite of the simplicity of the model
- Able to reach finite density
- Need to improve the model! i.e. fixing the mass scale

There are many indications that AdS/QCD can help in studying QCD, however the holographic description still needs some efforts