Status of hadronic light-by-light scattering in the muon $g-2$

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Outline

- Muon $g-2$: current status
- Hadronic light-by-light scattering in the muon $g-2$
- Status 2010
- Recent developments: Quark-loop, pion-loop
- Status 2013: how to proceed now ?
- Conclusions

Muon $g-2$: current status

- Experimental value (world average dominated by BNL experiment '06; shifted $+9.2 \times 10^{-11}$ due to new $\lambda = \mu_\mu/\mu_p$ from CODATA '08):

  $a_\mu^{\text{exp}} = (116\ 592\ 089 \pm 63) \times 10^{-11}$

- Theory: total SM contribution (based on various recent papers):

  $$a_\mu^{\text{SM}} = (116\ 591\ 795 \pm \underbrace{47}_{\text{VP}} \pm \underbrace{40}_{\text{LbyL}} \pm \underbrace{1.8}_{\text{QED + EW}} \quad [\pm 62]) \times 10^{-11}$$

  Hadronic contributions are largest source of error: vacuum polarization (VP) and light-by-light (LbyL) scattering.

  $a_\mu^{\text{had. LbyL}} = (116 \pm 40) \times 10^{-11}$ (Nyffeler '09; Jegerlehner, Nyffeler '09)
  Sometimes used: $a_\mu^{\text{had. LbyL}} = (105 \pm 26) \times 10^{-11}$ (Prades, de Rafael, Vainshtein '09)

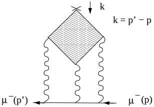
- $\Rightarrow a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (294 \pm 88) \times 10^{-11}$ $\qquad [3.3\ \sigma]$

- Other evaluations: $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} \sim (250 - 400) \times 10^{-11}$ $\quad [2.9 - 4.9\ \sigma]$
  (Jegerlehner, Nyffeler '09; Davier et al. '10; Jegerlehner, Szafron '11; Hagiwara et al. '11; Aoyama et al. '12; Benayoun et al. '13)

- Discrepancy a sign of New Physics ?

- Note: Hadronic contributions need to be better controlled in order to fully profit from future muon $g-2$ experiments at Fermilab or JPARC with $\delta a_\mu = 16 \times 10^{-11}$

Hadronic light-by-light scattering in the muon $g-2$



$\mathcal{O}(\alpha^3)$ hadronic contribution to muon $g-2$: four-point function $\langle VVVV \rangle$ projected onto a_μ (soft external photon $k \to 0$).

Had. LbyL: not directly related to experimental data, in contrast to had. VP which can be obtained from $\sigma(e^+e^- \to \text{hadrons}) \Rightarrow$ need hadronic model (or lattice QCD)

Current approach: use some hadronic model at low energies with exchanges and loops of resonances and some form of (dressed) "quark-loop" at high energies.
Problem: $\langle VVVV \rangle$ depends on several invariant momenta $\Rightarrow$ distinction between low and high energies is not as easy as for two-point function $\langle VV \rangle$ (had. VP).

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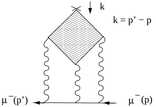

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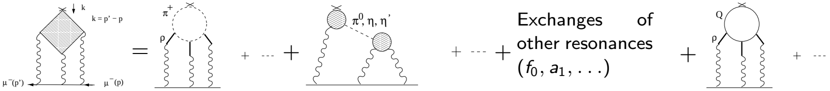
Classification of de Rafael '94: Chiral counting p^2 (ChPT) and large-N_C counting as guideline (all higher orders in p^2 and N_C contribute):



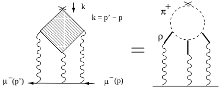
Chiral counting:	p^4	p^6	p^8	p^8
N_C-counting:	1	N_C	N_C	N_C
	pion-loop (dressed)	pseudoscalar exchanges		quark-loop (dressed)

Constrain models using experimental data (form factors of hadrons with photons) and theory (ChPT at low energies; short-distance constraints from pQCD / OPE at high momenta).

Relevant scales in had. LbyL ($\langle VVVV \rangle$ with off-shell photons): $0-2$ GeV, i.e. larger than m_μ !

Issue: on-shell versus off-shell form factors.

Had. LbyL scattering: anno 2010



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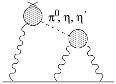
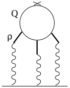


+ ··· + Exchanges of other resonances ($f_0, a_1, \ldots$) +



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Contribution to $a_\mu \times 10^{11}$:

BPP:	+83 (32)	-19 (13)	+85 (13)	-4 (3) [f_0, a_1]	+21 (3)
HKS:	+90 (15)	-5 (8)	+83 (6)	+1.7 (1.7) [a_1]	+10 (11)
KN:	+80 (40)		+83 (12)		
MV:	+136 (25)	0 (10)	+114 (10)	+22 (5) [a_1]	0
2007:	+110 (40)				
PdRV:	+105 (26)	-19 (19)	+114 (13)	+8 (12) [f_0, a_1]	+2.3 [c-quark]
N,JN:	+116 (40)	-19 (13)	+99 (16)	+15 (7) [f_0, a_1]	+21 (3)
		ud.: -45	ud.: $+\infty$		ud.: +60

ud. = undressed, i.e. point vertices without form factors

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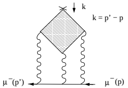

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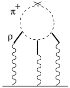

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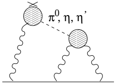

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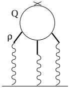

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BPP = Bijnens, Pallante, Prades '96, '02: Extended Nambu-Jona-Lasinio (ENJL) model; but for some contributions also other models used (in particular for pseudoscalars, pion-loop)

HKS = Hayakawa, Kinoshita, Sanda '96, '98, '02: Hidden Local Symmetry (HLS) model (often = VMD)

KN = Knecht, Nyffeler '02: large-N_C QCD for pion-pole (lowest meson dominance LMD, LMD+V)

MV = Melnikov, Vainshtein '04: large-N_C QCD, short-distance constraint from $\langle VVVV \rangle$ on pion-pole and axial-vector contribution, mixing of two axial-vector nonets

2007 = Bijnens, Prades; Miller, de Rafael, Roberts; PdRV = Prades, de Rafael, Vainshtein '09 (compilation)

N = Nyffeler '09: large-N_C for pion-exchange with off-shell LMD+V form factor, new short-distance constraint at external vertex; JN = Jegerlehner, Nyffeler '09 (compilation)

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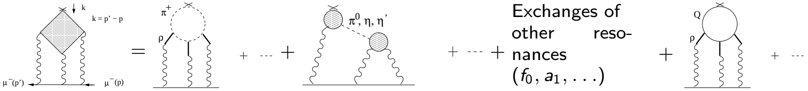

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- 2001: sign change in dominant pseudoscalar contribution: $a_\mu^{\text{had. LbyL}} \sim 85 \times 10^{-11}$ with discussion about estimate of error (adding errors of individual contributions linearly or in quadrature).
- 2004: MV $\Rightarrow$ enhanced pion-pole and axial-vector contributions. Estimate shifted upwards.
- 2010: (almost) consensus reached on central value $a_\mu^{\text{had. LbyL}} \sim 110 \times 10^{-11}$, still discussion about error estimate. Conservative in N, JN: $\pm 40 \times 10^{-11}$, more progressive in PdRV: $\pm 26 \times 10^{-11}$.

Other recent partial evaluations (mostly pseudoscalars)

- Nonlocal chiral quark model (off-shell) [Dorokhov et al.]

  2008: $a_\mu^{\mathrm{LbyL};\pi^0} = 65(2) \times 10^{-11}$

  2011: $a_\mu^{\mathrm{LbyL};\pi^0} = 50.1(3.7) \times 10^{-11}, \quad a_\mu^{\mathrm{LbyL;PS}} = 58.5(8.7) \times 10^{-11}$

  2012: $a_\mu^{\mathrm{LbyL};\pi^0+\sigma} = 54.0(3.3) \times 10^{-11}, \quad a_\mu^{\mathrm{LbyL};a_0+f_0} \sim 0.1 \times 10^{-11}$

  $a_\mu^{\mathrm{LbyL;PS+S}} = 62.5(8.3) \times 10^{-11}$

  Strong damping for off-shell form factors. Positive and small contribution from scalar $\sigma(600)$, differs from other estimates (BPP '96, '02; Blokland, Czarnecki, Melnikov '02).

- Holographic (AdS/QCD) model 1 (off-shell ?) [Hong, Kim '09]

  $a_\mu^{\mathrm{LbyL};\pi^0} = 69 \times 10^{-11}, \quad a_\mu^{\mathrm{LbyL;PS}} = 107 \times 10^{-11}$

- Holographic (AdS/QCD) model 2 (off-shell) [Cappiello, Cata, D'Ambrosio '10]

  $a_\mu^{\mathrm{LbyL};\pi^0} = 65.4(2.5) \times 10^{-11}$

  Used AdS/QCD to fix parameters in ansatz by D'Ambrosio et al. '98.

- Resonance saturation in odd-intrinsic parity sector (off-shell) [Kampf, Novotny '11]

  $a_\mu^{\mathrm{LbyL};\pi^0} = 65.8(1.2) \times 10^{-11}$

- Padé approximants (on-shell, but not constant FF at external vertex)

  $a_\mu^{\mathrm{LbyL};\pi^0} = 54(5) \times 10^{-11}$ [Masjuan '12 (using on-shell LMD+V FF)]

  $a_\mu^{\mathrm{LbyL};\pi^0} = 64.9(5.6) \times 10^{-11}, \quad a_\mu^{\mathrm{LbyL;PS}} = 89(7) \times 10^{-11}$

  [Escribano, Masjuan, Sanchez-Puertas '13]
  Fix parameters in Padé approximants from data on transition form factors.

Relevant momentum regions in $a_\mu^{\mathrm{LbyL};\pi^0}$

- In Knecht, Nyffeler '02, a 2-dimensional integral representation was derived for a certain class (VMD-like) of form factors (schematically):

$$a_\mu^{\mathrm{LbyL};\pi^0} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \sum_i w_i(Q_1, Q_2)\, f_i(Q_1, Q_2)$$

  with universal weight functions w_i. Dependence on form factors resides in the f_i.
- Expressions with on-shell form factors are in general not valid as they stand. One needs to set form factor at external vertex to a constant to obtain pion-pole contribution (Melnikov, Vainshtein '04). Expressions valid for WZW and off-shell VMD form factors.

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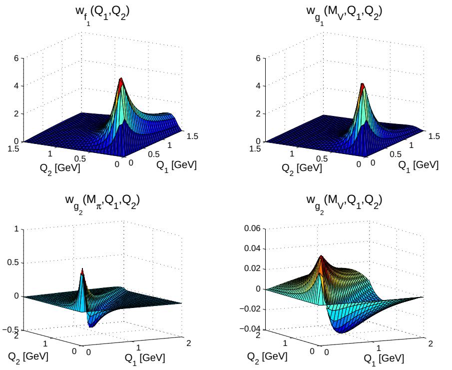
- Plot of weight functions w_i from Knecht, Nyffeler '02:

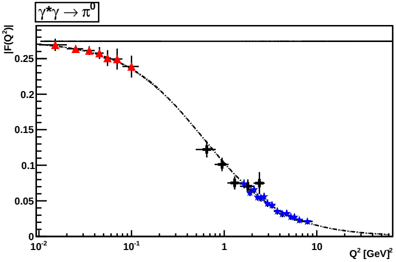


- Relevant momentum regions around $0.25 - 1.25$ GeV. As long as form factors in different models lead to damping, expect comparable results for $a_\mu^{\mathrm{LbyL};\pi^0}$, at level of 20%.
- Jegerlehner, Nyffeler '09 derived 3-dimensional integral representation for general (off-shell) form factors (hyperspherical approach). Integration over $Q_1^2, Q_2^2, \cos\theta$, where $Q_1 \cdot Q_2 = |Q_1||Q_2|\cos\theta$.
- Idea recently taken up by Dorokhov et al. '12 (for scalars) and Bijnens, Zahiri Abyaneh '12, '13 (for all contributions, work in progress).

Impact of form factor measurements: example KLOE-2

On the possibility to measure the $\pi^0 \to \gamma\gamma$ decay width and the $\gamma^*\gamma \to \pi^0$ transition form factor with the KLOE-2 experiment

Babusci et al. '12



Simulation of KLOE-2 measurement of $F(Q^2)$ (red triangles). MC program EKHARA 2.0 (Czyż, Ivashyn '11) and detailed detector simulation.
Solid line: $F(0)$ given by chiral anomaly (WZW).
Dashed line: form factor according to on-shell LMD+V model (Knecht, Nyffeler '01).
CELLO (black crosses) and CLEO (blue stars) data at higher Q^2.

Within 1 year of data taking, collecting 5 fb^{-1}, KLOE-2 will be able to measure:

- $\Gamma_{\pi^0 \to \gamma\gamma}$ to 1% statistical precision.
- $\gamma^*\gamma \to \pi^0$ transition form factor $F(Q^2)$ in the region of very low, space-like momenta $0.01 \text{ GeV}^2 \leq Q^2 \leq 0.1 \text{ GeV}^2$ with a statistical precision of less than 6% in each bin.
  KLOE-2 can (almost) directly measure slope of form factor at origin (note: logarithmic scale in Q^2 in plot !).

Impact of form factor measurements: example KLOE-2 (continued)

- Error in $a_\mu^{\mathrm{LbyL};\pi^0}$ related to the model parameters determined by $\Gamma_{\pi^0\to\gamma\gamma}$ (normalization of form factor; not taken into account in most papers) and $F(Q^2)$ will be reduced as follows:

  - $\delta a_\mu^{\mathrm{LbyL};\pi^0} \approx 4 \times 10^{-11}$ (with current data for $F(Q^2) + \Gamma_{\pi^0\to\gamma\gamma}^{\mathrm{PDG}}$)
  - $\delta a_\mu^{\mathrm{LbyL};\pi^0} \approx 2 \times 10^{-11}$ ($+\ \Gamma_{\pi^0\to\gamma\gamma}^{\mathrm{PrimEx}}$)
  - $\delta a_\mu^{\mathrm{LbyL};\pi^0} \approx (0.7 - 1.1) \times 10^{-11}$ (+ KLOE-2 data)

- Note that this error does not account for other potential uncertainties in $a_\mu^{\mathrm{LbyL};\pi^0}$, e.g. related to the off-shellness of the pion or the choice of model.

- Simple models with few parameters, like VMD (two parameters: F_π, M_V), which are completely determined by the data on $\Gamma_{\pi^0\to\gamma\gamma}$ and $F(Q^2)$, can lead to very small errors in $a_\mu^{\mathrm{LbyL};\pi^0}$. For illustration:

  $a_{\mu;\mathrm{VMD}}^{\mathrm{LbyL};\pi^0} = (57.3 \pm 1.1) \times 10^{-11}$

  $a_{\mu;\mathrm{LMD+V}}^{\mathrm{LbyL};\pi^0} = (72 \pm 12) \times 10^{-11}$ (off-shell LMD+V form factor, including all errors)

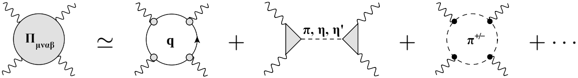
  But this might be misleading ! Results differ by about 20% ! VMD form factor has wrong high-energy behavior $\Rightarrow$ too strong damping.

Recent development: Dressed quark-loop

Dyson-Schwinger equation (DSE) approach [Fischer, Goecke, Williams '11, '13]

Claim: no double-counting between quark-loop and pseudoscalar exchanges (or exchanges of other resonances)

Had. LbyL in Effective Field Theory (hadronic) picture:



Quarks here may have different interpretation than below !

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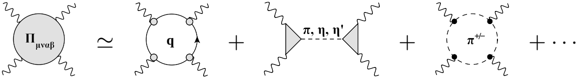

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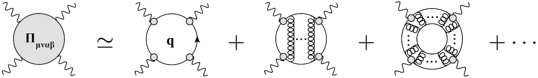
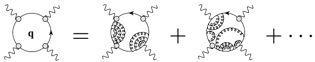
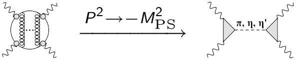
Had. LbyL using functional methods (all propagators and vertices fully dressed):



Expansion of quark-loop in terms of planar diagrams (rainbow-ladder approx.):



Pole representation of ladder-exchange contribution:



Truncate DSE using well tested model for dressed quark-gluon vertex (Maris, Tandy '99).

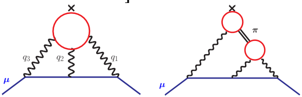
Large contribution from quark-loop (even after recent correction), in contrast to all other approaches, where coupling of (constituent) quarks to photons is dressed by form factors ($\rho - \gamma$-mixing, VMD).

Recent development: Dressed quark-loop (continued)

- Dyson-Schwinger equation approach [Fischer, Goecke, Williams '11, '13]
  $a_\mu^{\mathrm{LbyL};\pi^0} = 57.5(6.9) \times 10^{-11}$ (off-shell), $a_\mu^{\mathrm{LbyL;PS}} = 81(2) \times 10^{-11}$
  $a_\mu^{\mathrm{LbyL;quark-loop}} = 107(2) \times 10^{-11}$, $a_\mu^{\mathrm{had.\ LbyL}} = 188(4) \times 10^{-11}$

  Error for PS, quark-loop and total only from numerics. Quark-loop: still some parts are missing. Systematic error ? Not yet all contributions calculated.
  Note: numerical error in quark-loop in earlier paper (GFW PRD83 '11):
  $a_\mu^{\mathrm{LbyL;quark-loop}} = 136(59) \times 10^{-11}$, $a_\mu^{\mathrm{had.\ LbyL}} = 217(91) \times 10^{-11}$
- Constituent quark loop [Boughezal, Melnikov '11]
  $a_\mu^{\mathrm{had.\ LbyL}} = (118 - 148) \times 10^{-11}$

  Consider ratio of had. VP and had. LbyL with pQCD corrections. Paper was reaction to earlier results using DSE yielding large values for the quark-loop and the total.
- Constituent Chiral Quark Model [Greynat, de Rafael '12]
  $a_\mu^{\mathrm{LbyL;CQloop}} = 82(6) \times 10^{-11}$
  $a_\mu^{\mathrm{LbyL};\pi^0} = 68(3) \times 10^{-11}$ (off-shell)
  $a_\mu^{\mathrm{had.\ LbyL}} = 150(3) \times 10^{-11}$



  Error only reflects variation of constituent quark mass $M_Q = 240 \pm 10$ MeV, fixed to reproduce had. VP in $g-2$. Determinations from other quantities give larger value for $M_Q \sim 300$ MeV and thus smaller value for quark-loop. 20%-30% systematic error estimated. Not yet all contributions calculated.
- Padé approximants [Masjuan, Vanderhaeghen '12]
  $a_\mu^{\mathrm{had.\ LbyL}} = (76(4) - 125(7)) \times 10^{-11}$
  Quark-loop with running mass $M(Q) \sim (180 - 220)$ MeV, where the average momentum $\langle Q \rangle \sim (300 - 400)$ MeV is fixed from relevant momenta in 2-dim. integral representation for pion-pole in Knecht, Nyffeler '02.

Recent development: Dressed pion-loop

1. ENJL/VMD versus HLS

Model	$a_\mu^{\pi-\text{loop}} \times 10^{11}$
scalar QED (no FF)	-45
HLS	-4.5
ENJL	-19
full VMD	-15

Strong damping if form factors are introduced, very model dependent: compare ENJL (BPP '96) versus HLS (HKS '96). See also discussion in Melnikov, Vainshtein '04.

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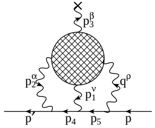
Origin: different behavior of integrands in contribution to $g-2$ (Zahiri Abyaneh '12; Bijnens, Zahiri Abyaneh '12; Talk by Bijnens at MesonNet 2013, Prague)

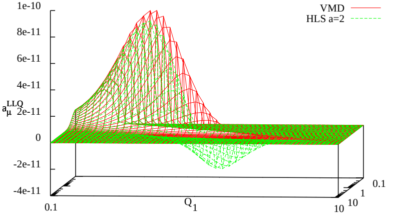


π loop

One can do 5 of the 8 integrations in the 2-loop integral for $g-2$ analytically, using the hyperspherical approach / Gegenbauer polynomials (Jegerlehner, Nyffeler '09; taken up in Bijnens, Zahiri Abyaneh '12):

$$a_\mu^{\mathrm{X}} = \int dl_{P_1} dl_{P_2}\, a_\mu^{\mathrm{XLL}} = \int dl_{P_1} dl_{P_2} dl_Q\, a_\mu^{\mathrm{XLLQ}}, \quad \text{with} \quad l_P = \ln(P/\mathrm{GeV})$$

Contribution of type X at given scale P_1, P_2, Q is directly proportional to volume under surface when a_μ^{XLL} and a_μ^{XLLQ} are plotted versus the energies on a logarithmic scale.



Momentum distribution of the full VMD and HLS pion-loop contribution for $P_1 = P_2$.

HLS: Integrand changes from positive to negative at high momenta. Leads to cancellation and therefore smaller absolute value. Usual HLS model ($a=2$) known to not fullfill certain QCD short-distance constraints.

Recent development: Dressed pion-loop (continued)

2. Role of pion polarizability and a_1 resonance

- Engel, Patel, Ramsey-Musolf '12: ChPT analysis of LbyL up to order p^6 in limit $p_1, p_2, q \ll m_\pi$. Identified potentially large contributions from pion polarizability ($L_9 + L_{10}$ in ChPT) which are not fully reproduced in ENJL / HLS models used by BPP '96 and HKS '96. Pure ChPT approach is not predictive. Loops not finite, would need new a_μ counterterm (Knecht et al. '02).

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- Engel, Ph.D. Thesis '13; Engel, Ramsey-Musolf '13: tried to include a_1 resonance explicitly in EFT. Problem: contribution to $g-2$ in general not finite (loops with resonances) $\Rightarrow$ Form factor approach with a_1 that reproduces pion polarizability at low energies, has correct QCD scaling at high energies and generates a finite result in a_μ:

$$\mathcal{L}_I = -\frac{e^2}{4} F_{\mu\nu}\pi^+ \left(\frac{1}{D^2 + M_A^2}\right) F^{\mu\nu}\pi^- + \mathrm{h.c.} + \cdots$$

$$\mathcal{L}_{II} = -\frac{e^2}{2M_A^2}\pi^+\pi^- \left[\left(\frac{M_V^2}{\partial^2 + M_V^2}\right) F^{\mu\nu}\right]^2 + \cdots$$

$a_\mu^{\pi-\mathrm{loop}} \times 10^{11}$:

Model	(a)	(b)
I	-11	-34
II	-40	-71

Second and third columns in Table correspond to different values for the polarizability LECs, $(\alpha_9^r + \alpha_{10}^r)$: (a) $(1.32 \pm 1.4) \times 10^{-3}$ (from radiative pion decay $\pi^+ \to e^+\nu_e\gamma$) and (b) $(3.1 \pm 0.9) \times 10^{-3}$ (from radiative pion photoproduction $\gamma p \to \gamma' \pi^+ n$).

Potentially large results (absolute value): $a_\mu^{\pi-\mathrm{loop}} \sim -(11 - 71) \times 10^{-11}$. Variation of 60×10^{-11} ! Uncertainty underestimated in earlier calculations ?

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- Issue taken up in Zahiri Abyaneh '12; Bijnens, Zahiri Abyaneh '12; Bijnens, Relefors (to be published); Talk by Bijnens at MesonNet 2013, Prague. Tried various ways to include a_1, but again no finite result for $g-2$ achieved. With a cutoff of 1 GeV:

$$a_\mu^{\pi-\text{loop}} = (-20 \pm 5) \times 10^{-11} \qquad \text{(preliminary)}$$

Summary of recent developments

- Recent partial evaluations (mostly pseudoscalars):

$$a_\mu^{\mathrm{LbyL};\pi^0} \sim (50 - 69) \times 10^{-11}$$
$$a_\mu^{\mathrm{LbyL;PS}} \sim (59 - 107) \times 10^{-11}$$

Most evaluations agree at level of 15%, but some estimates are quite low or high.

- Open problem: Dressed quark-loop
Dyson-Schwinger equation (DSE) approach (Fischer, Goecke, Williams '11, '13):

$$a_\mu^{\mathrm{LbyL;quark-loop}} = 107 \times 10^{-11} \qquad \text{(still incomplete)}$$

Large contribution, no damping seen, in contrast to BPP '96, HKS '96.

- Open problem: Dressed pion-loop
Potentially important effect from pion polarizability and a_1 resonance (Engel, Patel, Ramsey-Musolf '12; Engel '13; Engel, Ramsey-Musolf '13):

$$a_\mu^{\mathrm{LbyL};\pi-\mathrm{loop}} = -(11 - 71) \times 10^{-11}$$

Large negative contribution, no damping seen, in contrast to BPP '96, HKS '96.

Had. LbyL scattering: anno 2013

- If we take those newer estimates of the quark-loop and the pion-loop seriously and combine the extreme estimates:

$$a_\mu^{\mathrm{had.\ LbyL}} = (64 - 202) \times 10^{-11}$$

or: $$a_\mu^{\mathrm{had.\ LbyL}} = (133 \pm 69) \times 10^{-11}$$

$\Rightarrow$ We do not understand had. LbyL scattering at all !?

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- Option 1: Wait for final result from Lattice QCD ...

  One idea: put QCD + QED on the lattice !
  Blum et al. '05, '08, '09; Chowdhury '09; Blum, Hayakawa, Izubuchi '12 + poster at Lattice 2013 (private communication):

$$F_2(0.18\ \text{GeV}^2) = (127 \pm 29) \times 10^{-11} \qquad \text{(result } 4.4\sigma \text{ from zero)}$$

$$F_2(0.11\ \text{GeV}^2) = (-15 \pm 39) \times 10^{-11} \qquad \text{(result consistent with zero)}$$

$$a_\mu^{\text{had. LbyL;models}} = F_2(0) = (116 \pm 40) \times 10^{-11} \qquad \text{(Jegerlehner, Nyffeler '09)}$$

  For $m_\mu = 190$ MeV, $m_\pi = 329$ MeV. Still large statistical errors, systematic errors not yet under control, still quenched QED, potentially large "disconnected" contributions missing !

- Option 2: Maybe non-Lattice theorists and experimentalists can still do some work in the coming years, as far as had. LbyL scattering in muon $g-2$ is concerned !

Outlook

- Need more information from experiment for various form factors of photons with hadrons at small and intermediate momenta $|Q| \leq 2$ GeV, decays like $\pi^0 \to \gamma\gamma$ to fix normalization of form factors and from cross-section measurements like $\gamma\gamma \to \pi\pi$ to gain information on the relevant $\gamma\pi\pi$ and $\gamma\gamma\pi\pi$ form factors (with off-shell pions !). Also needed as input for dispersion relations. In this way one can hopefully test the models.

- Need more theoretical constraints on form factors and $\langle VVVV \rangle$ at low energies from ChPT and short-distance constraints from OPE and pQCD.

  Also useful to constrain models: sum rules for the (on-shell) hadronic light-by-light scattering (Pascalutsa, Pauk, Vanderhaeghen '12)

- Pseudoscalars: under control at level of 15%. Issue: off-shell form factors (pion-exchange) versus on-shell form factors (pion-pole; Melnikov, Vainshtein '04).

- Quark-loop: more work needed. Problem for theory only !
  First let Fischer et al. complete DSE calculation of quark-loop and the rest of the contributions !?

- Pion-loop: more work needed. Theory and experiment have to work together.

  Need more information on pion-polarizability, e.g. from radiative pion decay $\pi^+ \to e^+\nu_e\gamma$, radiative pion photoproduction $\gamma p \to \gamma'\pi^+ n$, the hadronic Primakoff process $\pi A \to \pi'\gamma A$ (with some heavy nucleus A) or $\gamma A \to \pi^+\pi^- A$. Conflicting values from previous experiments, some new measurements are ongoing or planned.
  Also the properties of the a_1 resonance should be better determined, e.g. its decay modes $a_1 \to \rho\pi$ and $a_1 \to \pi\gamma$. Also important for axial-vector exchange contribution !

Conclusions

- Hadronic light-by-light scattering in muon $g-2$: not directly related to data $\Rightarrow$ need hadronic model (or lattice QCD).
  Goal: to match precision of new muon $g-2$ experiments $\delta a_\mu = 16 \times 10^{-11}$.
- Note: only Bijnens, Pallante, Prades '96, '02 and Hayakawa, Kinoshita, Sanda '96, '98, '02 are "full" calculations so far ! But the models used have their deficiencies.
  Need one consistent (as much as possible) hadronic model !
- Error estimates for individual contributions: a small error does not necessarily imply that the estimate is "better", maybe the model used is too simple !

  Overall uncertainty: combine errors from different contributions, where different models are used, linearly or in quadrature ?
  Small error of $\pm 26 \times 10^{-11}$ in Prades, de Rafael, Vainshtein '09 from adding errors in quadrature might again be misleading.
- Recent developments for the quark-loop and the pion-loop: those authors raised important questions about the validity of the models used so far, but more work is needed to confirm those numbers.
- We think that the estimate

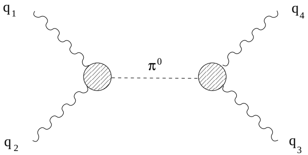
$$a_\mu^{\text{had. LbyL}} = (116 \pm 40) \times 10^{-11} \qquad \text{(Nyffeler '09; Jegerlehner, Nyffeler '09)}$$

  still gives a fair description of the current situation.

Backup

Pion-pole in $\langle VVVV \rangle$ versus pion-exchange in $a_\mu^{\mathrm{LbyL};\pi^0}$

- To uniquely identify contribution of exchanged neutral pion π^0 in Green's function $\langle VVVV \rangle$, we need to pick out pion-pole:

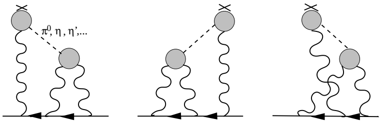


$+$ crossed diagrams

$$\lim_{(q_1+q_2)^2 \to m_\pi^2} ((q_1+q_2)^2 - m_\pi^2)\langle VVVV \rangle$$

Residue of pole: on-shell vertex function $\langle 0|VV|\pi\rangle \to$ on-shell form factor $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$

- But in contribution to muon $g-2$, we evaluate Feynman diagrams, integrating over photon momenta with exchanged off-shell pions.
  For all the pseudoscalars:



Shaded blobs represent off-shell form factor $\mathcal{F}_{\mathrm{PS}^*\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2)$ where $\mathrm{PS} = \pi^0, \eta, \eta', \pi^{0\prime}, \ldots$

Off-shell form factors are either inserted "by hand" starting from constant, pointlike Wess-Zumino-Witten (WZW) form factor or using e.g. some resonance Lagrangian.

- Similar statements apply for exchanges (or loops) of other resonances.

Off-shell pion form factor from $\langle VVP \rangle$

- Following Bijnens, Pallante, Prades '96; Hayakawa, Kinoshita, Sanda '96, '98, we can define off-shell form factor for π^0:

$$\int d^4x\, d^4y\, e^{i(q_1\cdot x + q_2\cdot y)} \langle 0|T\{j_\mu(x) j_\nu(y) P^3(0)\}|0\rangle$$

$$= \quad \varepsilon_{\mu\nu\alpha\beta}\, q_1^\alpha q_2^\beta \frac{i\langle\overline{\psi}\psi\rangle}{F_\pi} \frac{i}{(q_1+q_2)^2 - m_\pi^2} \mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2) + \ldots$$

Up to small mixing effects of P^3 with η and η' and neglecting exchanges of heavier states like $\pi^{0\prime}, \pi^{0\prime\prime}, \ldots$

$$j_\mu(x) = (\overline{\psi}\hat{Q}\gamma_\mu\psi)(x), \quad \psi \equiv \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \quad \hat{Q} = \mathrm{diag}(2,-1,-1)/3$$

(light quark part of electromagnetic current)

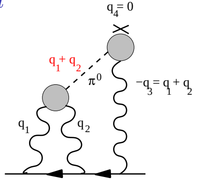
$$P^3 = \overline{\psi} i\gamma_5 \frac{\lambda^3}{2}\psi = \left(\overline{u} i\gamma_5 u - \overline{d} i\gamma_5 d\right)/2, \quad \langle\overline{\psi}\psi\rangle = \text{single flavor quark condensate}$$

Bose symmetry: $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2) = \mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_2^2, q_1^2)$

- Note: for off-shell pions, instead of $P^3(x)$, we could use any other suitable interpolating field, like $(\partial^\mu A_\mu^3)(x)$ or even an elementary pion field $\pi^3(x)$! Off-shell form factor is therefore model dependent and not a physical quantity !

Pion-exchange versus pion-pole contribution to $a_\mu^{\mathrm{LbyL};\pi^0}$

- Off-shell form factors have been used to evaluate the pion-exchange contribution in Bijnens, Pallante, Prades '96 and Hayakawa, Kinoshita, Sanda '96, '98. "Rediscovered" by Jegerlehner in '07, '08. Consider diagram:



$$\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2) \;\times\; \mathcal{F}_{\pi^{0*}\gamma^*\gamma}((q_1+q_2)^2, (q_1+q_2)^2, 0)$$

- On the other hand, Knecht, Nyffeler '02 used on-shell form factors:

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2, q_1^2, q_2^2) \;\times\; \mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, (q_1+q_2)^2, 0)$$

- But form factor at external vertex $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, (q_1+q_2)^2, 0)$ for $(q_1+q_2)^2 \neq m_\pi^2$ violates momentum conservation, since momentum of external soft photon vanishes !
  Often the following misleading notation was used:
  $\mathcal{F}_{\pi^0\gamma^*\gamma^*}((q_1+q_2)^2, 0) \equiv \mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2, (q_1+q_2)^2, 0)$
  At external vertex identification with transition form factor was made (wrongly !).
- Melnikov, Vainshtein '04 had observed this inconsistency and proposed to use

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2, q_1^2, q_2^2) \;\times\; \mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, m_\pi^2, 0)$$

  i.e. a constant form factor at the external vertex given by the WZW term.
- However, this prescription will only yield the so-called pion-pole contribution and not the full pion-exchange contribution !
- The pion-exchange contribution with off-shell pions is model dependent. Only the sum of all contributions in a given model is relevant.

The VMD form factor

Vector Meson Dominance:

$$\mathcal{F}^{\mathrm{VMD}}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2) = \frac{N_C}{12\pi^2 F_\pi} \frac{M_V^2}{q_1^2 - M_V^2} \frac{M_V^2}{q_2^2 - M_V^2}$$

on-shell = off-shell form factor !

Only two model parameters even for off-shell form factor: F_π and M_V

Transition form factor:

$$F^{\mathrm{VMD}}(Q^2) = \frac{N_C}{12\pi^2 F_\pi} \frac{M_V^2}{Q^2 + M_V^2}$$

The LMD+V form factor (off-shell)

Knecht, Nyffeler, EPJC '01; Nyffeler '09

- Ansatz for $\langle VVP \rangle$ and thus $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}$ in large-N_C QCD in chiral limit with 1 multiplet of lightest pseudoscalars (Goldstone bosons) and 2 multiplets of vector resonances, ρ, ρ' (lowest meson dominance (LMD) + V)
- $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}$ fulfills all leading (and some subleading) QCD short-distance constraint from Operator Product Expansion (OPE)
- Reproduces Brodsky-Lepage (BL): $\lim_{Q^2\to\infty} \mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(m_\pi^2, -Q^2, 0) \sim 1/Q^2$

  (OPE and BL cannot be fulfilled simultaneously with only one vector resonance)
- Normalized to decay width $\Gamma_{\pi^0\to\gamma\gamma}$

Off-shell LMD+V form factor:

$$\mathcal{F}^{\mathrm{LMD+V}}_{\pi^{0*}\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2) = -\frac{F_\pi}{3}\,\frac{q_1^2\, q_2^2\,(q_1^2+q_2^2+q_3^2) + P_H^V(q_1^2, q_2^2, q_3^2)}{(q_1^2 - M_{V_1}^2)\,(q_1^2 - M_{V_2}^2)\,(q_2^2 - M_{V_1}^2)\,(q_2^2 - M_{V_2}^2)}$$

$$P_H^V(q_1^2, q_2^2, q_3^2) = h_1\,(q_1^2+q_2^2)^2 + h_2\, q_1^2\, q_2^2 + h_3\,(q_1^2+q_2^2)\, q_3^2 + h_4\, q_3^4 + h_5\,(q_1^2+q_2^2) + h_6\, q_3^2 + h_7$$

$$q_3^2 = (q_1+q_2)^2$$

$F_\pi = 92.4$ MeV, $M_{V_1} = M_\rho = 775.49$ MeV, $M_{V_2} = M_{\rho'} = 1.465$ GeV

Free parameters: h_i

Relevant momentum regions in $a_\mu^{\mathrm{LbyL;PS}}$

Result for pseudoscalar exchange contribution $a_\mu^{\mathrm{LbyL;PS}} \times 10^{11}$ for off-shell LMD+V and VMD form factors obtained with momentum cutoff Λ in 3-dimensional integral representation of Jegerlehner, Nyffeler '09 (integration over square). In brackets, relative contribution of the total obtained with $\Lambda = 20$ GeV.

Λ [GeV]	π^0 LMD+V ($h_3 = 0$)	π^0 LMD+V ($h_4 = 0$)	π^0 VMD	η VMD	η' VMD
0.25	14.8 (20.6%)	14.8 (20.3%)	14.4 (25.2%)	1.76 (12.1%)	0.99 (7.9%)
0.5	38.6 (53.8%)	38.8 (53.2%)	36.6 (64.2%)	6.90 (47.5%)	4.52 (36.1%)
0.75	51.9 (72.2%)	52.2 (71.7%)	47.7 (83.8%)	10.7 (73.4%)	7.83 (62.5%)
1.0	58.7 (81.7%)	59.2 (81.4%)	52.6 (92.3%)	12.6 (86.6%)	9.90 (79.1%)
1.5	64.9 (90.2%)	65.6 (90.1%)	55.8 (97.8%)	14.0 (96.1%)	11.7 (93.2%)
2.0	67.5 (93.9%)	68.3 (93.8%)	56.5 (99.2%)	14.3 (98.6%)	12.2 (97.4%)
5.0	71.0 (98.8%)	71.9 (98.8%)	56.9 (99.9%)	14.5 (99.9%)	12.5 (99.9%)
20.0	71.9 (100%)	72.8 (100%)	57.0 (100%)	14.5 (100%)	12.5 (100%)

π^0:

- Although weight functions plotted earlier are not applicable to off-shell LMD+V form factor, region below $\Lambda = 1$ GeV gives the bulk of the result: 82% for LMD+V, 92% for VMD.
- No damping from off-shell LMD+V form factor at external vertex since $\chi \neq 0$ (new short-distance constraint). Note: VMD falls off too fast, compared to OPE.

η, η':

- Mass of intermediate pseudoscalar is higher than pion mass $\rightarrow$ expect a stronger suppression from propagator.
- Peak of relevant weight functions shifted to higher values of Q_i. For η', vector meson mass is also higher $M_V = 859$ MeV. Saturation effect and the suppression from the VMD form factor only fully set in around $\Lambda = 1.5$ GeV: 96% of total for η, 93% for η'.