

# Top quark pair production cross section in $e^+e^-$ and hadronic collisions

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LC13: Exploring QCD from the infrared regime to ... LHC and Linear Colliders  
Trento, Italy, Sep 16-20, 2013

## Outline

- Top production at the LHC – NNLL resummed

MB, P. Falgari, S. Klein, C. Schwinn, 1109.1536 [hep-ph];

MB, P. Falgari, S. Klein, J. Piclum, C. Schwinn, M. Ubiali, F. Yan, 1206.2454 [hep-ph] (TOPIXS program)

- Top production near threshold in  $e^+e^-$  collisions

MB, Kiyo, Schuller, hep-ph/0501289; MB, Kiyo, 0801.3464 [hep-ph];

MB, Kiyo, Schuller, in preparation.

- Effective field theory of unstable particle production and non-resonant effects

MB, Jantzen, Ruiz-Femenia 1004.2188 [hep-ph]; Jantzen, Ruiz-Femenia 1307.4337 [hep-ph]



- ... could be like any other quark, just happens to be a bit more massive.
    - SM
    - simple, perhaps unexciting
- ⇒ Precision studies of top quark properties (mass, width) through inclusive production, FB and charge asymmetries, invariant mass,  $p_T$  (momentum) spectra etc.

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- or to a strongly coupled sector (partial compositeness)
- have new flavour-changing interactions
- determine the fate of the Universe

Search for anomalous couplings, anomalies in production and decay.

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This is not a review ... focus on recent progress in inclusive production.

## Hadronic production (LHC)

- Benchmark for QCD calculations of hadronic processes. Now the most precisely predicted purely hadronic cross section.
- Theoretically clean but not very precise mass determination.
- Template for searches for unknown particles with missing transverse momentum.
- Constraint on gluon distribution in the proton in a momentum fraction region relevant to Higgs production.

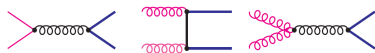
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$e^+e^-$

- Very precise and clean mass measurement and width measurement at threshold
- Couplings above threshold

# I. Hadronic production



dominant at Tevatron

dominant at LHC

NLO correction is large (50% at LHC)

[NLO QCD: Nason, Dawson, Ellis (1988), Beenakker et al. (1989)]

Data requires precision beyond NLO

	NLO	CDF	D0
Tevatron	$6.68^{+0.36+0.23}_{-0.75-0.22}$	$7.50^{+0.48}_{-0.48}$	$7.56^{+0.63}_{-0.56}$
	NLO	ATLAS	CMS
LHC (7 TeV)	$158.1^{+19.5+6.2}_{-21.2-6.8}$	$177 \pm 3^{+8}_{-7} \pm 7$	$166 \pm 2 \pm 11 \pm 8$
LHC (8 TeV)	$226.2^{+27.8+9.8}_{-29.7-9.1}$	$241 \pm 2 \pm 31 \pm 9$	$227 \pm 3 \pm 11 \pm 10$

$\sigma_{t\bar{t}}$  (in pb). NLO:  $m_t = 173.3$  GeV. MSTW2008NLO, PDF+ $\alpha_s$  error at 68% CL; independent  $\mu_r$  and  $\mu_f$  variations.



$$\sigma_{\text{had}}(\sqrt{s}, m_t) = \sum_{i,j=q,\bar{q},g} \int dx_i dx_j f_i(x_i) f_j(x_j) \hat{\sigma}_{ij}(\sqrt{\hat{s}}, m_t) \quad \hat{s} = x_i x_j s$$
$$\beta = \sqrt{1 - 4m_t^2/\hat{s}}$$

# Theoretical progress beyond NLO

$$\sigma_{\text{had}}(\sqrt{s}, m_t) = \sum_{i,j=q,\bar{q},g} \int dx_i dx_j f_i(x_i) f_j(x_j) \hat{\sigma}_{ij}(\sqrt{\hat{s}}, m_t) \quad \hat{s} = x_i x_j s$$
$$\beta = \sqrt{1 - 4m_t^2/\hat{s}}$$

**NLO** [NLO QCD: Nason, Dawson, Ellis, 1988, Beenakker et al., 1989]

**NLL resummed** [Catani et al.; Berger, Contopanagos; Kidonakis, Smith, Vogt, 1996; Bonciani et al., 1998]



**NNLO<sub>approx</sub>** [MB, Czakon, Falgari, Mitov, Schwinn, 2009; MB, Falgari, Klein, Schwinn, 2010; Aliev et al., 2010; from 1PI/1PM distributions: Ahrens et al., 2010/11; Kidonakis, 2010]



**NNLL resummed** [MB, Falgari, Klein, Schwinn, 2011 (soft and Coulomb); Cacciari, Czakon, Mangano, Mitov, Nason, 2011 (soft) (**Top++**); MB, Falgari, Klein, Piclum, Schwinn, Ubiali, Yan, 2012 (**TOPIXS**); from 1PI/1PM distributions: Ahrens et al., 2010/11]



**NNLO** [Bärnreuther, Czakon, Mitov, 2012 ( $q\bar{q}$ ); Czakon, Fiedler, Mitov, 2013 ( $gg$ )]

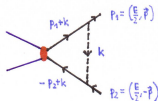


**NNLO + NNLL** [2013 v2 versions of (**Top++**) and (**TOPIXS**)]

# Resummation: breakdown of perturbation theory at small $\beta$

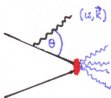
Fixed-order PT not applicable for threshold production (non-relativistic)

- Coulomb force



$$A \sim A_0 \times \frac{g^2}{\beta}$$

- Inhibited (soft) radiation

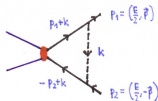


$$A \sim A_0 \times g^2 \ln^2 \beta$$

# Resummation: breakdown of perturbation theory at small $\beta$

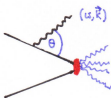
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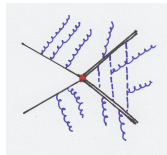


$$A \sim A_0 \times g^2 \ln^2 \beta$$

Perturbation theory breaks down due to the **emergence of small scales**  $M\beta, M\beta^2 \ll M, \sqrt{s}$ .  
Sum the series of enhanced quantum fluctuations to all orders:

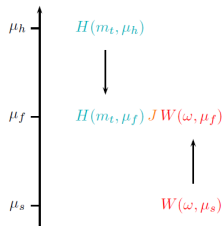
$$\begin{aligned} \sigma &= \sigma_0 \left[ 1 + g^2 \left\{ \frac{1}{\beta}, \ln^{2,1} \beta \right\} + g^4 \left\{ \frac{1}{\beta^2}, \frac{\ln^{2,1} \beta}{\beta}, \ln^{4,3,2,1} \beta \right\} + \dots \right] \\ &= \sigma_0 \sum_{k=0} \left( \frac{\alpha_s}{\beta} \right)^k \exp \left[ \underbrace{\ln \beta g_0(\alpha_s \ln \beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln \beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln \beta)}_{\text{(NNLL)}} + \dots \right] \\ &\quad \times \left\{ 1 \text{ (LL,NLL)}; \alpha_s, \beta \text{ (NNLL)}; \alpha_s^2, \alpha_s \beta, \beta^2 \text{ (NNNLL)}; \dots \right\} \end{aligned}$$

$$\hat{\sigma}(\beta, \mu) = \sum_i H_i(M, \mu) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(E - \frac{\omega}{2}) W_i^{R_\alpha}(\omega, \mu)$$



- Factorization and resummation with non-relativistic and soft-collinear EFT.
- Summation of  $\ln \beta$  terms by SCET RGE equation in  $x$ -space, not moment space. [Becher, Neubert (2006); Becher, Neubert, Xu (2007)]
- Match to fixed-order result to describe the region  $\beta \sim 1$

$$\hat{\sigma}_{\text{matched}}^{\text{NNLL}} = [\hat{\sigma}^{\text{NNLL}} - \hat{\sigma}_{\text{exp. to NNLO}}^{\text{NNLL}}] + \hat{\sigma}^{\text{NNLO}}$$

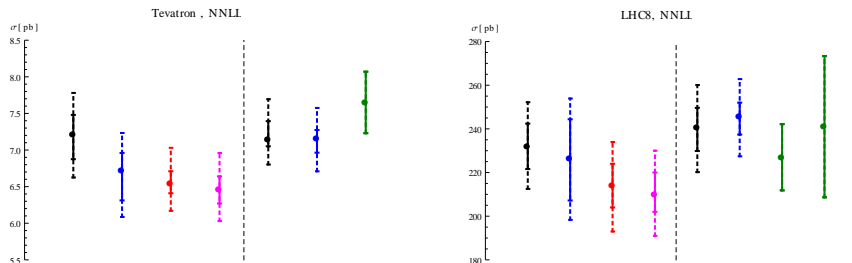


$\sigma_{t\bar{t}}$ [pb]	Tevatron	LHC ( $\sqrt{s} = 7$ TeV)	LHC ( $\sqrt{s} = 8$ TeV)
NLO	$6.68^{+0.36+0.23}_{-0.75-0.22}$	$158.1^{+19.5+6.8}_{-21.2-6.2}$	$226.2^{+27.8+9.2}_{-29.7-8.3}$
NNLO <sub>app</sub>	$7.06^{+0.26+0.29}_{-0.34-0.24}$	$161.1^{+12.3+7.3}_{-11.9-6.7}$	$230.0^{+16.7+9.7}_{-15.7-9.0}$
NNLO	$7.01^{+0.27+0.29}_{-0.37-0.24}$	$167.1^{+6.7+7.7}_{-10.7-7.1}$	$239.1^{+9.2+10.4}_{-14.8-9.6}$
<b>NNLL</b>	<b><math>7.15^{+0.24+0.30}_{-0.10-0.25}</math></b>	<b><math>168.5^{+6.3+7.7}_{-7.5-7.2}</math></b>	<b><math>241.0^{+8.7+10.5}_{-11.1-9.7}</math></b>

$m_t = 173.3$  GeV,  $\alpha_s(M_Z) = 0.1171$ , (N)NLO MSTW08 PDFs, first error (theoretical uncertainty), second PDF+ $\alpha_s$  at 68% CL.  
 Theoretical error: independent soft/hard/Coulomb scale variations, resummation ambiguities.

- **Tevatron ( $q\bar{q}$ )**  
 significant resummation/NNLO effect [+8%],  
 reduction of theoretical uncertainty [8%  $\rightarrow$  3%] (excluding PDF +  $\alpha_s$  error),  
 threshold approximation worked well.
- **LHC ( $gg$ )**  
 small resummation [+1%]/significant NNLO effect [+4%],  
 significant reduction of theoretical uncertainty [13%  $\rightarrow$  4.5%],  
 threshold expansion did not work that well.

# Comparison of NNLL results



## left of dashed: NNLO<sub>app</sub> + NNLL

MB, Falgari, Klein, Schwinn 2011; TOPIXS 1.0 ( $m_t = 173.3$  GeV)

Czakov, Mangano, Mitov, Nason 2011 ( $m_t = 173.3$  GeV)

Ahrens, Ferroglia, Neubert, Pecjak, Yang 2011 (1PI,  $m_t = 173.1$  GeV)

Ahrens, Ferroglia, Neubert, Pecjak, Yang 2011 (PIM,  $m_t = 173.1$  GeV)

## right of dashed: NNLO + NNLL

Piclum; 2013 update of MB, Falgari, Klein, Piclum, Schwinn, Ubiali, Yan 2012; TOPIXS 2.0 ( $m_t = 173.3$  GeV)

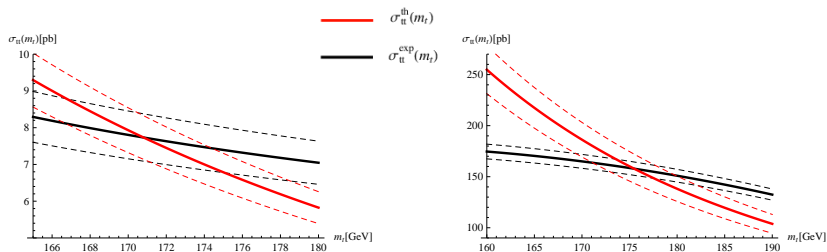
Czakov, Fidler, Mitov, 2013  $\tau_{\text{top}++2.0}$  ( $m_t = 173.3$  GeV)

Experiments: CDF/D0 2012 combined; CMS/ATLAS-CONF-2012-149

**Error bars:** solid – theory, dashed – PDF ( $+\alpha_s$ ) added

**PDF set:** MSTW 2008 NNLO

Experimental cross section depends on assumed  $m_t$ .



Maximize joint likelihood  $f(m_t) = \int d\sigma f_{\text{th}}(\sigma|m_t) \cdot f_{\text{exp}}(\sigma|m_t)$ .

$$m_t = (171.5^{+5.4}_{-6.2}) \text{ GeV} \quad (\text{Tevatron data})$$

$$m_t = (176.1^{+5.3}_{-4.7}) \text{ GeV} \quad (\text{CMS data [1208.2671]})$$

Consistent with top decay  
mass reconstruction!

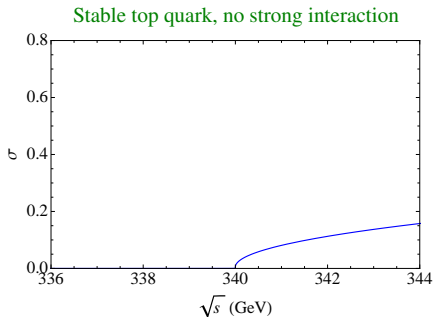
$$(m_t = 173.2 \pm 0.8 \text{ GeV})$$

See also Langenfeld et al. (2009), D0, CDF, ...

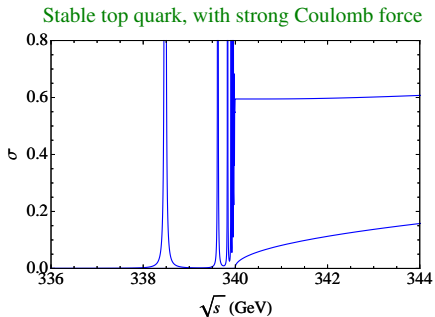


## II. Threshold production in $e^+e^-$

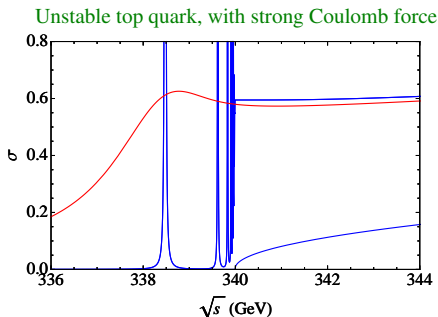
Ultra-precise mass measurement  
Unique QCD dynamics



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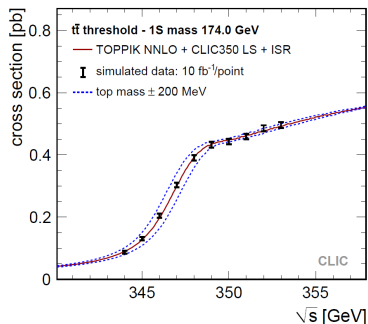
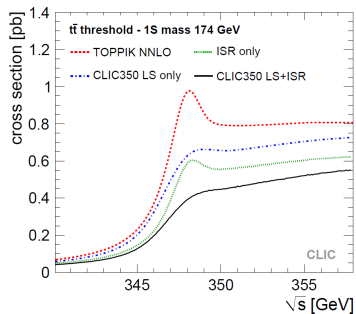


Ultra-precise mass measurement  
Unique QCD dynamics



Smallest structure in particle physics known to exist ( $10^{-17}$  m).  
Direct “spectroscopic” width measurement.

Additional smearing of the resonance due to beam luminosity spectrum (collider-specific) and ISR

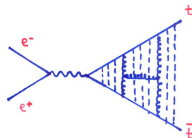


Most recent study for ILC/CLIC [Seidel, Simon, Tesai, Poss, 2013] assumes 10 fb<sup>-1</sup> at 10 points.

$$[\delta m_t]_{\text{thr}} = 27 \text{ MeV} \quad [\text{simultaneous fit of } \alpha_s]$$

# Key items for theoretical precision

- **All-order summation** since  $\alpha_s/v$  can be large.  
Well-defined framework (EFT) to combine relativistic and boundstate effects.
- **Mass definition:** abandon the pole mass!  
Potential-subtracted mass [MB, 1998]



$$m_{\text{PS}}(\mu_f) \equiv m_{\text{pole}} + \frac{1}{2} \int_{|\vec{q}| < \mu_f} \frac{d^3 \vec{q}}{(2\pi)^3} \tilde{V}_{\text{Coulomb}}(\vec{q})$$

Cancellation of large perturbative contributions from the IR.

- **High orders:**  
NNLO calculations find large uncertainty [up  $\pm 25\%$ ] in the cross section in the resonance peak region (MB, Signer, Smrinov; Hoang, Teubner; Melnikov, Yelkovsky; Yakovlev; Nagano et al.; Penin, Pivovarov, 1998/99).  
Maybe less after  $\log(v)$  resummation [ $\pm 3\%$ ] (Hoang et al., 2002; Hoang, Stahlhofen, 2011)

**Basic techniques worked out end of 1990s but the next order is hard!**

In the absence of electroweak corrections

$$(q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T(j_\mu(x) j_\nu(0)) | 0 \rangle, \quad j^\mu(x) = [\bar{t} \gamma^\mu t](x)$$

$$R \equiv \frac{\sigma_{\bar{t}X}}{\sigma_0} = 12\pi e_t^2 \text{Im} \Pi(s)$$

Relevant scales:  $m_t \approx 175 \text{ GeV}$  (**hard**),  $m_t \alpha_s \approx 30 \text{ GeV}$  (**soft, potential**) and the **ultrasoft scale (us)**  $m_t \alpha_s^2 \approx 2 \text{ GeV}$ .

$$\begin{array}{ccc} \mathcal{L}_{\text{QCD}} [Q(h, s, p), g(h, s, p, us)] & & \mu > m_t \\ \downarrow & & \\ \mathcal{L}_{\text{PNRQCD}} [Q(p), g(us)] & & \mu < m_t v \end{array}$$

The ultrasoft scale appears explicitly only at NNNLO. A complete calculation of the (non-logarithmic) NNNLO correction is therefore needed.

Matching of currents and interactions (potentials and ultrasoft)

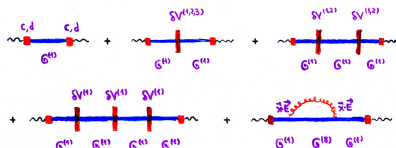
$$j^i = c_v \psi^\dagger \sigma^i \chi + \frac{d_v}{6m_t^2} \psi^\dagger \sigma^i \mathbf{D}^2 \chi + \dots$$

$$\mathcal{L}_{\text{QCD}} \rightarrow \mathcal{L}_{\text{PNRQCD}}$$

$$\begin{aligned} \mathcal{L}_{\text{PNRQCD}} = & \psi^\dagger \left( iD_0 + i\frac{\Gamma_t}{2} + \frac{\partial^2}{2m} + \frac{\partial^4}{8m^3} \right) \psi + \chi^\dagger \left( iD_0 - i\frac{\Gamma_t}{2} - \frac{\partial^2}{2m} - \frac{\partial^4}{8m^3} \right) \chi \\ & + \int d^{d-1} \mathbf{r} \left[ \psi^\dagger \psi \right] (x + \mathbf{r}) \left( -\frac{\alpha_s C_F}{r} + \delta V(r, \partial) \right) \left[ \chi^\dagger \chi \right] (x) \\ & - g_s \psi^\dagger(x) \mathbf{x} \mathbf{E}(t, \mathbf{0}) \psi(x) - g_s \chi^\dagger(x) \mathbf{x} \mathbf{E}(t, \mathbf{0}) \chi(x) \end{aligned}$$

- Three-loop matching required for the hard vertex  $c_v$  [Marquard, Piclum, Seidel, Steinhauser, unpublished] and the Coulomb potential [Anzai, Kiyo, Sumino, 2009; Smirnov, Smirnov, Steinhauser, 2009].





where

$$G_c^{(1,8)}(r, r', E) = \frac{my}{2\pi} e^{-y(r+r')} \sum_{l=0}^{\infty} (2l+1)(2yr)^l (2yr')^l P_l \left( \frac{r \cdot r'}{rr'} \right) \sum_{s=0}^{\infty} \frac{s! L_s^{(2l+1)} (2yr) L_s^{(2l+1)} (2yr')}{(s+2l+1)!(s+l+1-\lambda)}$$

$$y = \sqrt{-m(E+i\epsilon)}, \lambda = \frac{m\alpha_s}{2y} \times \{C_F \text{ (singlet); } C_F - C_A/2 \text{ (octet)}\}$$

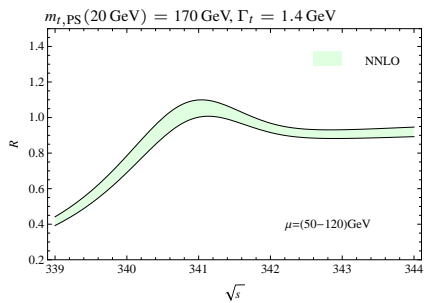
The ultrasoft contribution is ( $D = d - 1$ )

$$\delta G_{\text{us}} = (-i)(ig_s)^2 C_F \int \frac{d^d k}{(2\pi)^d} \frac{-i}{k^2} \left( \frac{k^i k^j}{k_0^2} - \delta^{ij} \right) \int \prod_{n=1}^6 \frac{d^D p_n}{(2\pi)^D} i\tilde{G}^{(1)}(p_1, p_2; E) i\tilde{G}^{(8)}(p_3, p_4; E + k^0) i\tilde{G}^{(1)}(p_5, p_6; E)$$

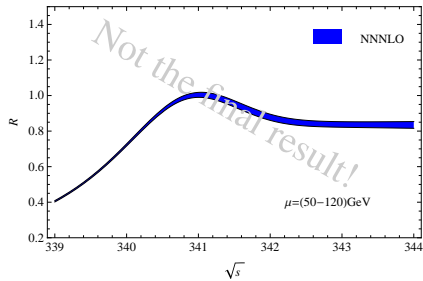
$$\times i \left[ \frac{2p_3^i}{m_t} (2\pi)^D \delta^{(D)}(p_3 - p_2) + (ig_s)^2 \frac{C_A}{2} \frac{2(p_2 - p_3)^i}{(p_2 - p_3)^4} \right] i \left[ -\frac{2p_4^j}{m_t} (2\pi)^D \delta^{(D)}(p_4 - p_5) + (ig_s)^2 \frac{C_A}{2} \frac{2(p_4 - p_5)^j}{(p_4 - p_5)^4} \right]$$

Difficulty is the extraction of the divergence in dim. reg.

# Cross section near threshold, from 2nd to 3rd order



[MB, Signer, Smirnov, 1999]



[MB, Kiyo, Schuller, in progress]

### III. Non-resonant effects

# Finite-width divergences and “electroweak effects”

The QCD-only result usually discussed is far from reality

- Finite-width divergences (overall divergence, already at NNLO):

$$[\delta G(E)]_{\text{overall}} \propto \frac{\alpha_s}{\epsilon} \cdot E$$

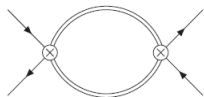


Since  $E = \sqrt{s} - 2m_t + i\Gamma$ , the divergence survives in the imaginary part:

$$\text{Im} [\delta G(E)]_{\text{overall}} \propto m_t \times \frac{\alpha_s \alpha_{ew}}{\epsilon}$$

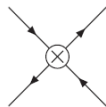
- **Electroweak effect. Must consider  $e^+ e^- \rightarrow W^+ W^- b \bar{b}$ . Significant “non-resonant” background from off-shell top decay (unless tight invariant mass cuts are applied), not described by NRQCD. Starts at NLO [MB, Jantzen, Ruiz-Femenia, 2010; Penin, Piclum, 2011]**
- **QED effects [Pineda, Signer, 2006; MB, Jantzen, Ruiz-Femenia, 2010], electroweak absorptive parts [Hoang, Reisser, 2004], initial state radiation (formalism in [MB, Falgari, Schwinn, Signer, Zanderighi]).**

Unstable particle EFT provides a systematic expansion of the amplitude in powers of  $\Gamma/m$ .



### Resonant contributions

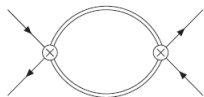
Production of an on-shell, non-relativistic  $t\bar{t}$  pair and subsequent decay  $t \rightarrow W^+ b$ .



### Non-resonant contributions

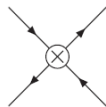
Off-shell lines. Full theory diagrams expanded around  $s = 4m_t^2$ .

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Production of an on-shell, non-relativistic  $t\bar{t}$  pair and subsequent decay  $t \rightarrow W^+ b$ .



### Non-resonant contributions

Off-shell lines. Full theory diagrams expanded around  $s = 4m_t^2$ .

$$i\mathcal{A} = \sum_{k,l} C_p^{(k)} C_p^{(l)} \int d^4x \langle e^- e^+ | T[i\mathcal{O}_p^{(k)\dagger}(0) i\mathcal{O}_p^{(l)}(x)] | e^- e^+ \rangle + \sum_k C_{4e}^{(k)} \langle e^- e^+ | i\mathcal{O}_{4e}^{(k)}(0) | e^- e^+ \rangle$$

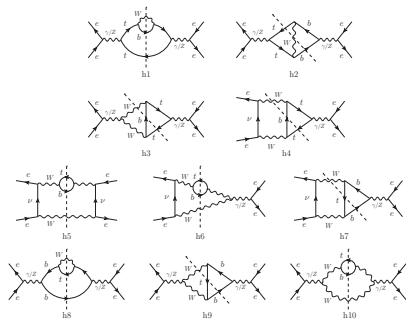
$$\mathcal{O}_p^{(v)} = \bar{e}_{c_2} \gamma_i e_{c_1} \psi_t^\dagger \sigma^i \chi_t$$

$$\mathcal{O}_p^{(a)} = \bar{e}_{c_2} \gamma_i \gamma_5 e_{c_1} \psi_t^\dagger \sigma^i \chi_t$$

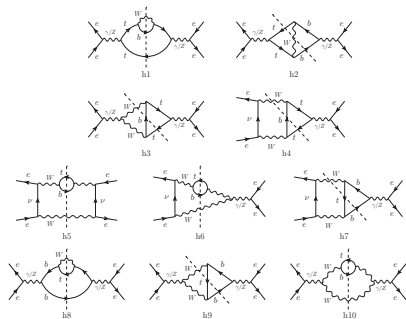
$$\mathcal{O}_{4e}^{(k)} = \bar{e}_{c_1} \Gamma_1 e_{c_2} \bar{e}_{c_2} \Gamma_2 e_{c_1},$$

$$\sigma_{\text{non-res}} = \frac{1}{s} \sum_k \text{Im} [C_{4e}^{(k)}] \langle e^- e^+ | i\mathcal{O}_{4e}^{(k)}(0) | e^- e^+ \rangle$$

# Non-resonant corrections at NLO

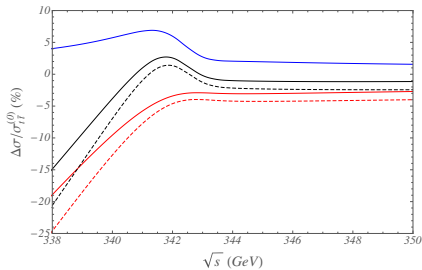


Equivalent to the dimensionally regulated  $e^+e^- \rightarrow bW^+ \bar{t}$  process with  $\Gamma_t = 0$ , expanded in the hard region around  $s = 4m_t^2$ .



Equivalent to the dimensionally regulated  $e^+e^- \rightarrow bW^+\bar{\tau}$  process with  $\Gamma_\tau = 0$ , expanded in the hard region around  $s = 4m_t^2$ .

[MB, Jantzen, Ruiz-Femenia, 2010]



QED and non-resonant corrections relative to the  $t\bar{t}$  LO cross section in percent:  $\sigma_{\text{QED}}^{(1)} / \sigma_{t\bar{t}}^{(0)}$  (upper solid blue),  $\sigma_{\text{non-res}}^{(1)} / \sigma_{t\bar{t}}^{(0)}$  for the total cross section (lower solid red) and  $\Delta M_t = 15$  GeV (lower dashed red). The relative size of the sum of the QED and non-resonant corrections is represented by the middle (black) lines, for  $\Delta M_{t,\text{max}}$  (solid) and  $\Delta M_t = 15$  GeV (dashed)

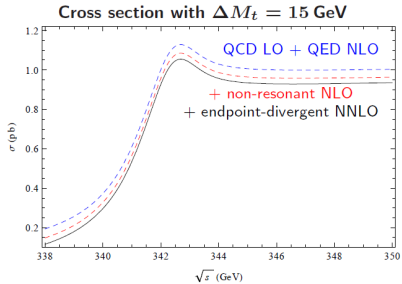
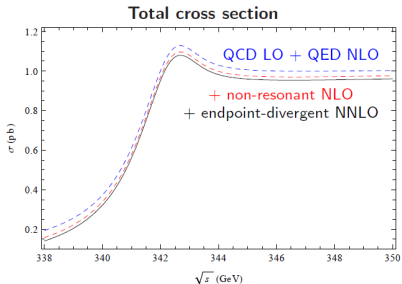
Large correction below threshold.

Invariant mass cuts can be applied within the EFT framework.



$$e^+e^- \rightarrow W^+W^-b\bar{b} \text{ near } s = 4m_t^2$$

NLO + NNLO singular terms [Jantzen, Ruiz-Femenia, 2013]



“Realistic” cross section calculations almost completed.  
ISR corrections should be applied in the theoretical prediction, not the experimental simulation.

## I Hadronic $t\bar{t}$ cross section now computed at NNLO+NNLL (soft + Coulomb)

Theoretical uncertainty: 3% (Tevatron), 4.5% (LHC)

PDF +  $\alpha_s$  uncertainty (68% CL): 5%

Best predicted purely hadronic cross section. PDF error dominates. Solid constraint on “light” new physics.

Well-defined determination of  $m_t$  with a present error of (4 – 5) GeV, consistent with the top mass reconstruction from decay products.

Outlook: differential NNLO; NNNLL resummation.

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Outlook: differential NNLO; NNNLL resummation.

## II $e^+e^- \rightarrow t\bar{t}X$ cross section near threshold now computed at NNNLO.

Sizeable 3rd order corrections and reduction of theoretical uncertainty.

Ultra-precise mass measurement.

Realistic predictions for  $e^+e^- \rightarrow W^+W^-b\bar{b}$  near top-pair threshold available with sufficient accuracy, including cuts.

Outlook: NNNLO code; complete NNLO non-resonant.