The role of the top quark in the stability of the SM Higgs potential

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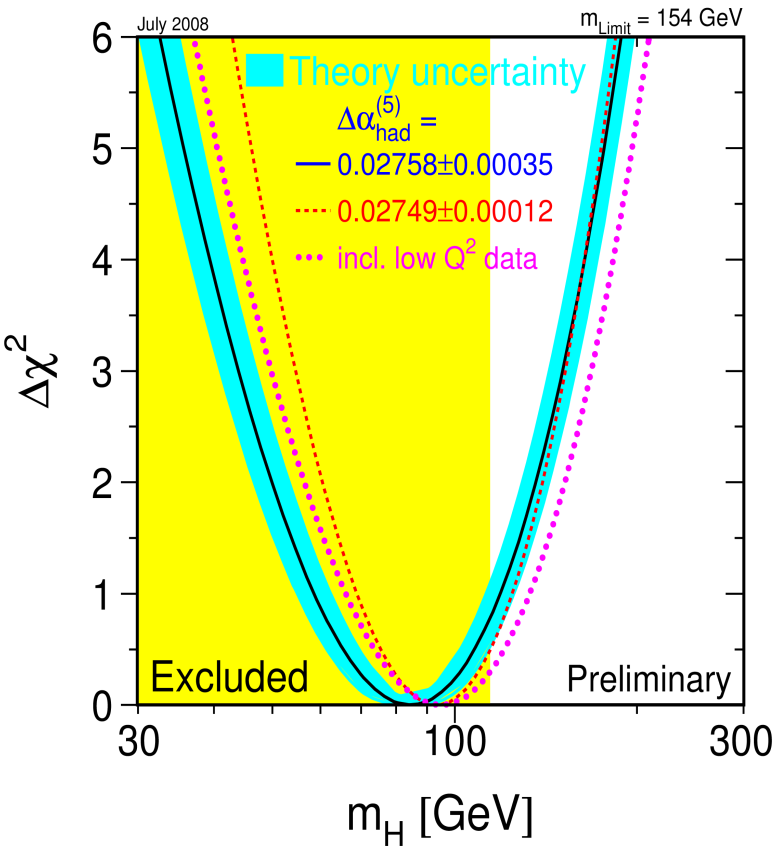
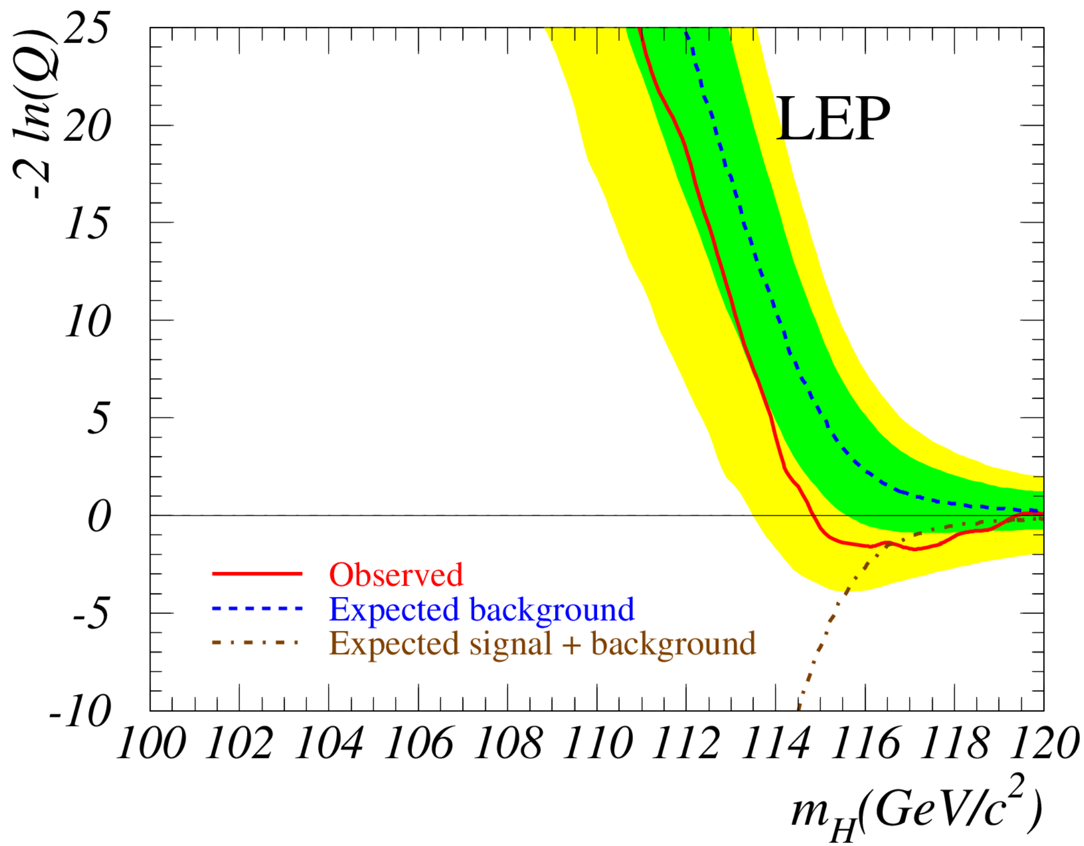
Outline

- Past and present informations on the Higgs boson
- Implications of $M_h \sim 125$ GeV in the analysis of the vacuum stability: the role of the top
- Conclusions

The past: LEP



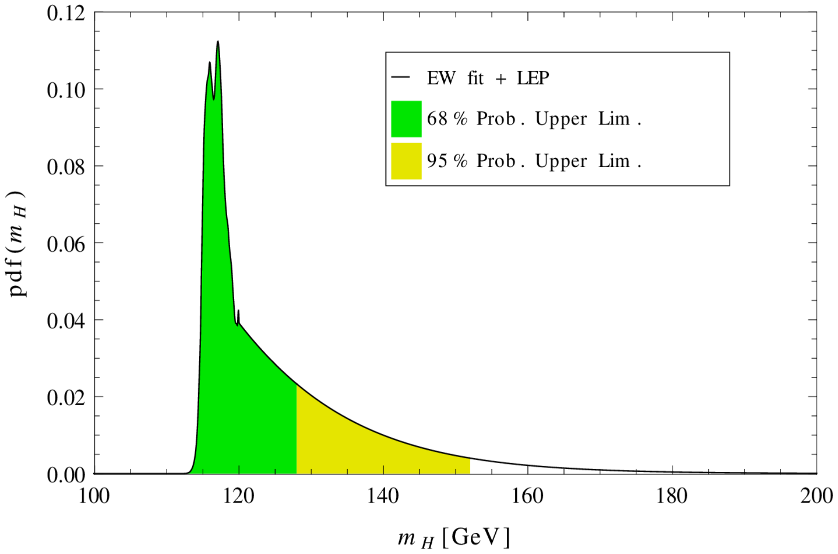


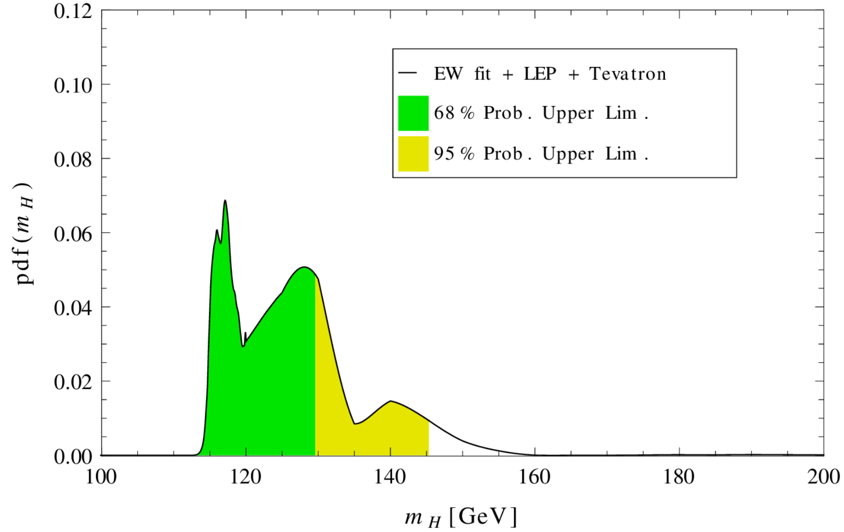
$$Q = \frac{\mathcal{L}(s+b)}{\mathcal{L}(b)}$$

The past: LEP+ Tevatron

Combining direct and indirect information:
D'Agostini, G.D.1999

$$\mathrm{pdf}(M_h) \propto \frac{Q(M_h)e^{-(\chi^2/2)}}{M_h}$$

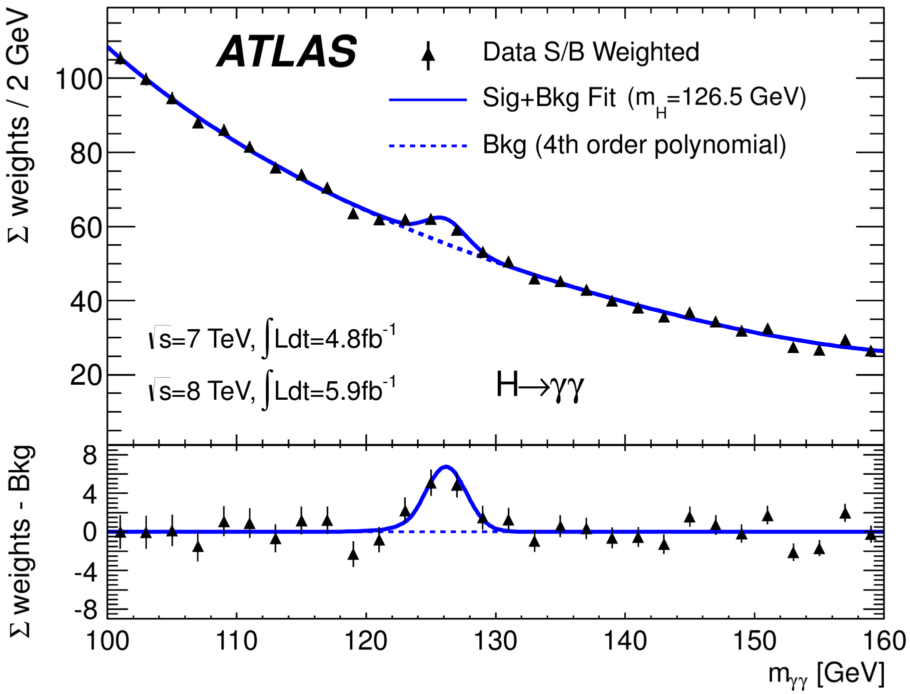


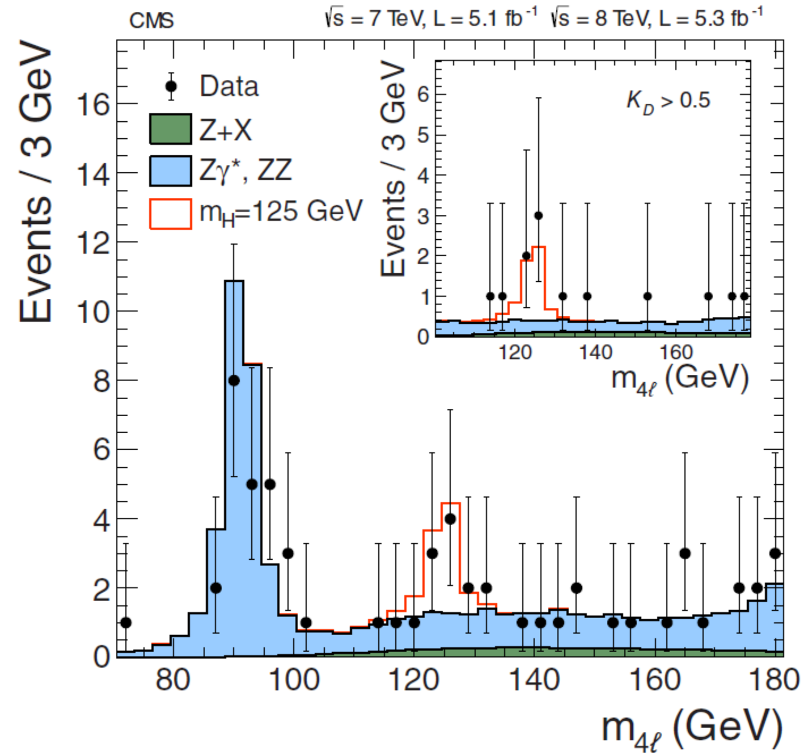


courtesy of S. Di Vita

The consistency of the (minimal) SM at the quantum level predicts a Higgs boss with mass between 110 and 160 GeV

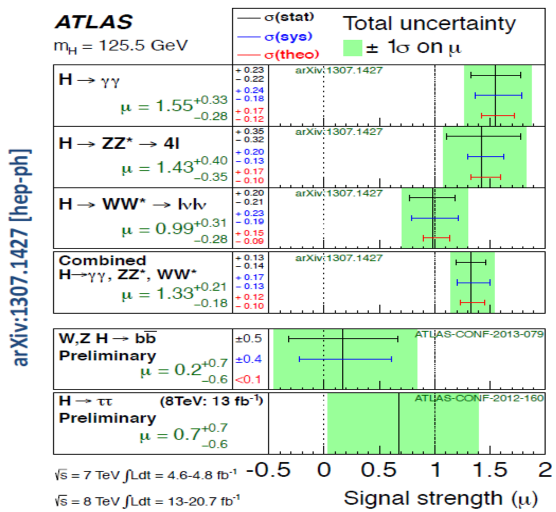
The present: LHC 4th of July 2012 news

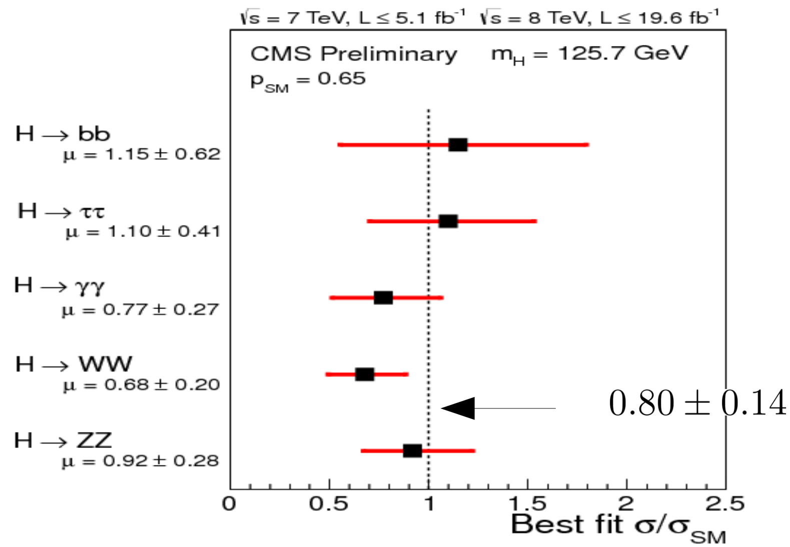


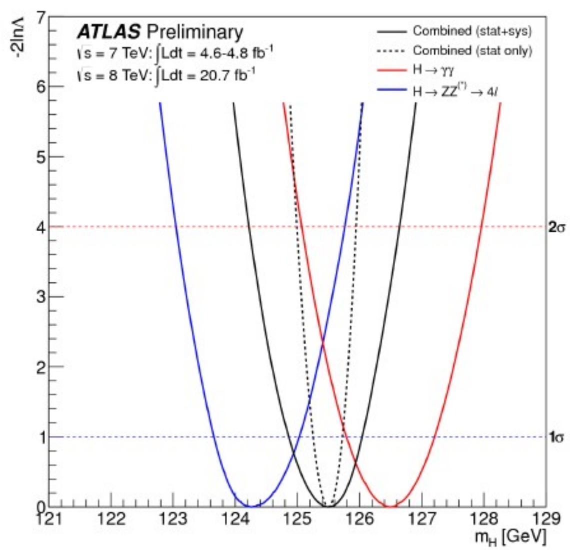


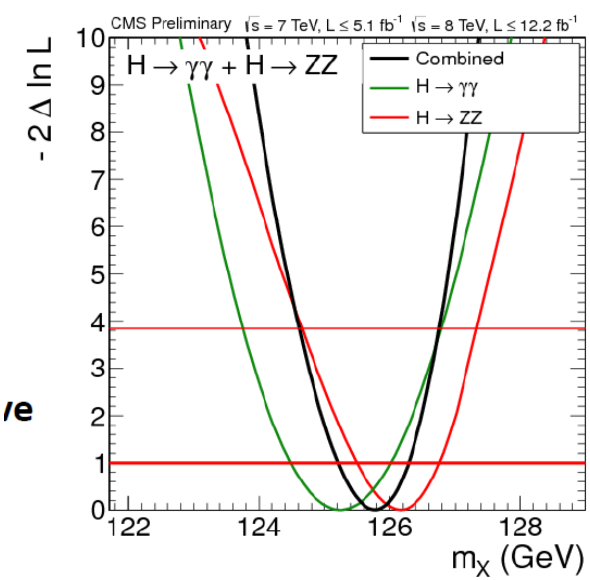
Clear evidence of a new particle
with properties compatible with those of the SM Higgs boson

The present: LHC
Studying the properties of the new particle





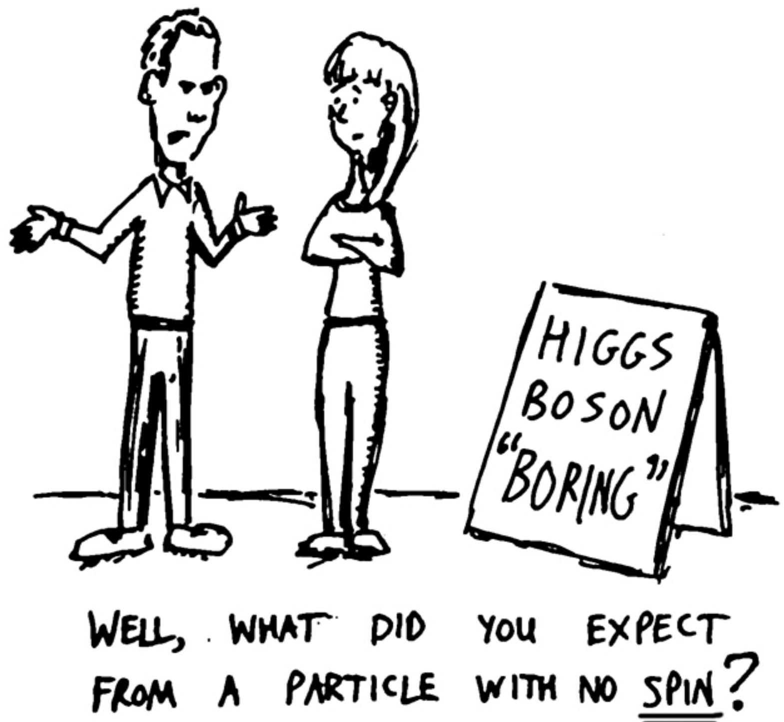




$M_h = 125.5 \pm 0.2(stat)\ ^{+0.5}_{-0.6}(syst)$ GeV

$M_h = 125.7 \pm 0.3(stat) \pm 0.3(syst)$ GeV

Implications of M_h ~ 125 GeV



Reversing the heavy Higgs argument

Specific type of NP could allow a heavy Higgs in the EW fit ("conspiracy"). Take

$$\hat\rho = \rho_0 + \delta\rho \; (\rho_0^{\rm SM} = 1, \delta\rho \leftrightarrow (\epsilon_1, T))$$
$$\Delta\hat r_W \leftrightarrow (\epsilon_3, S)$$

$$\sin^2\theta_{\rm eff}^{\rm lep} \sim \frac{1}{2}\left\{1 - \left[1 - \frac{4A^2}{M_Z^2\,\hat\rho\,(1-\Delta\hat r_W)}\right]^{1/2}\right\}$$
$$\sim (\sin^2\theta_{\rm eff}^{\rm lep})^\circ + c_1 \ln\left(\frac{M_H}{M_H^\circ}\right) + c_2\left[\frac{(\Delta\alpha)_h}{(\Delta\alpha)_h^\circ} - 1\right] - c_3\left[\left(\frac{M_t}{M_t^\circ}\right)^2 - 1\right] + \ldots$$

$c_i > 0$

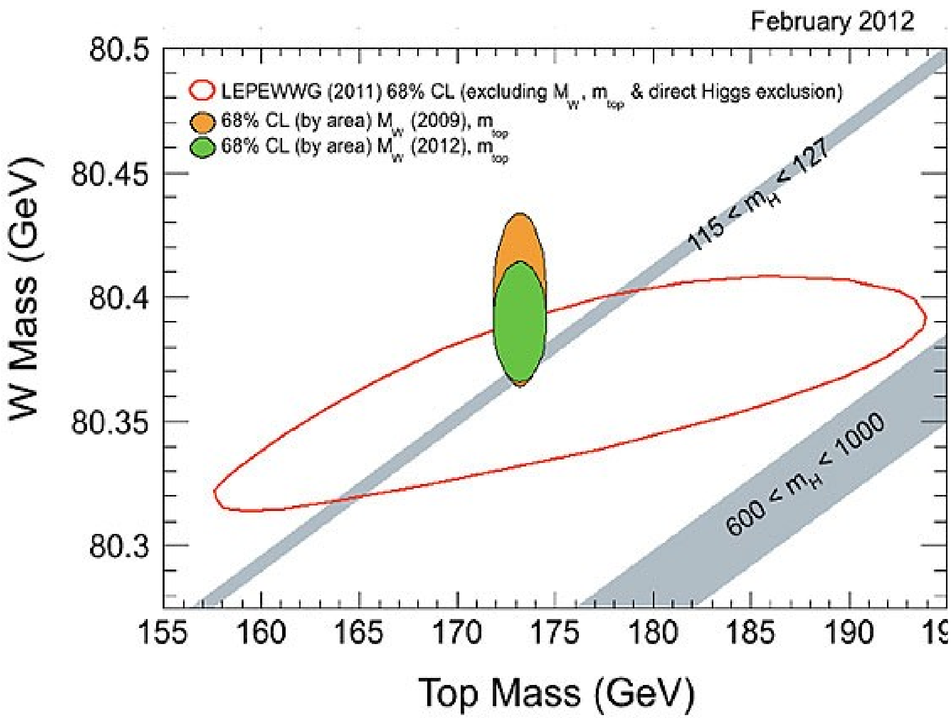
To increase the fitted M_H : $\left\{ \begin{array}{l} \hat\rho > 1 \to \left\{ \begin{array}{l} \rho_0 > 1 \\ \delta\rho > 0 \end{array} \right. \\ \Delta\hat r_W < 0 \end{array} \right.$

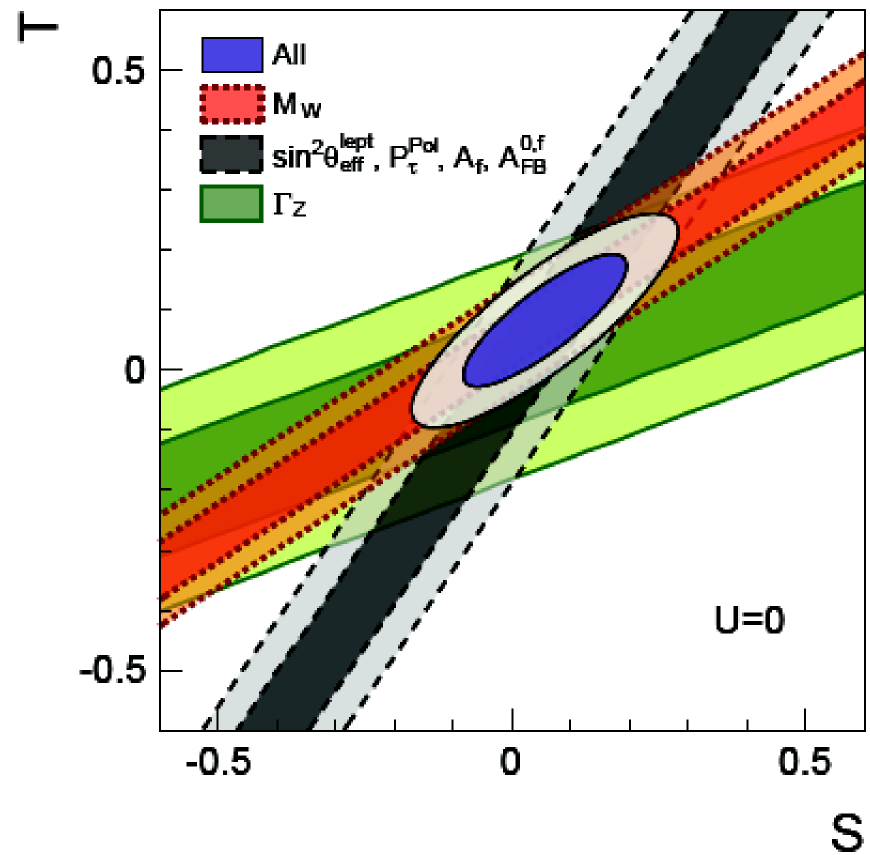
$\rho_0 > 1$ ← Extra Z

$\delta\rho > 0$ ← Isosplitt (s)fermions, Multi Higgs models,

$\Delta\hat r_W < 0$ ← Light sleptons

NP (if there) seems to be of the decoupling type





Ciuchini, Franco, Mishima, Silvestrini (13)

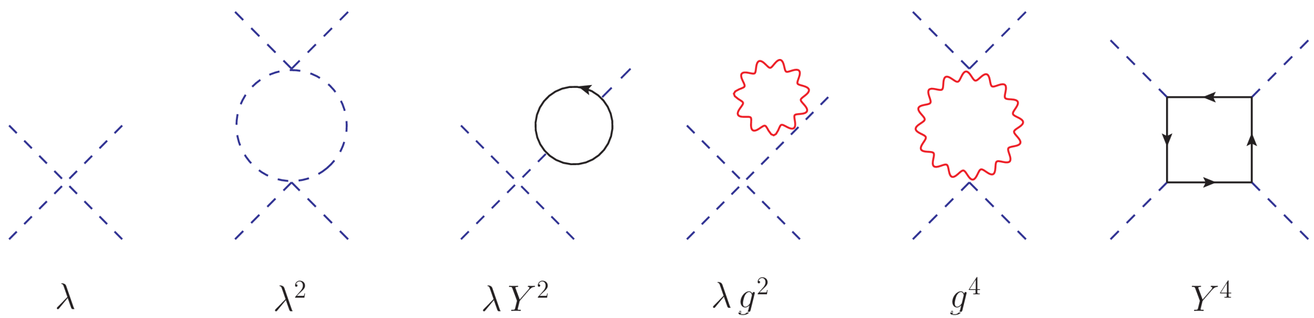
(Meta)Stability bound

Quantum corrections to the classical Higgs potential can modify its shape

$$V^{class}(\phi) = -\frac{1}{2}m^2\phi^2 + \lambda\phi^4 \longrightarrow V^{\text{eff}} \approx -\frac{1}{2}m^2(\mu)\phi^2(\mu) + \lambda(\mu)\phi^4(\mu) \sim \lambda(\mu)\phi^4(\mu)$$

$$\phi \sim \mu \gg v$$

λ runs



$$\frac{d\lambda}{d\ln\mu} = \frac{1}{16\pi^2}\left[+24\lambda^2 + \lambda\left(4N_cY_t - 9g^2 - 3g'^2\right) - 2N_cY_t^4 + \frac{9}{8}g^4 + \frac{3}{8}g'^4 + \frac{3}{4}g^2g'^2 + \ldots\right]$$

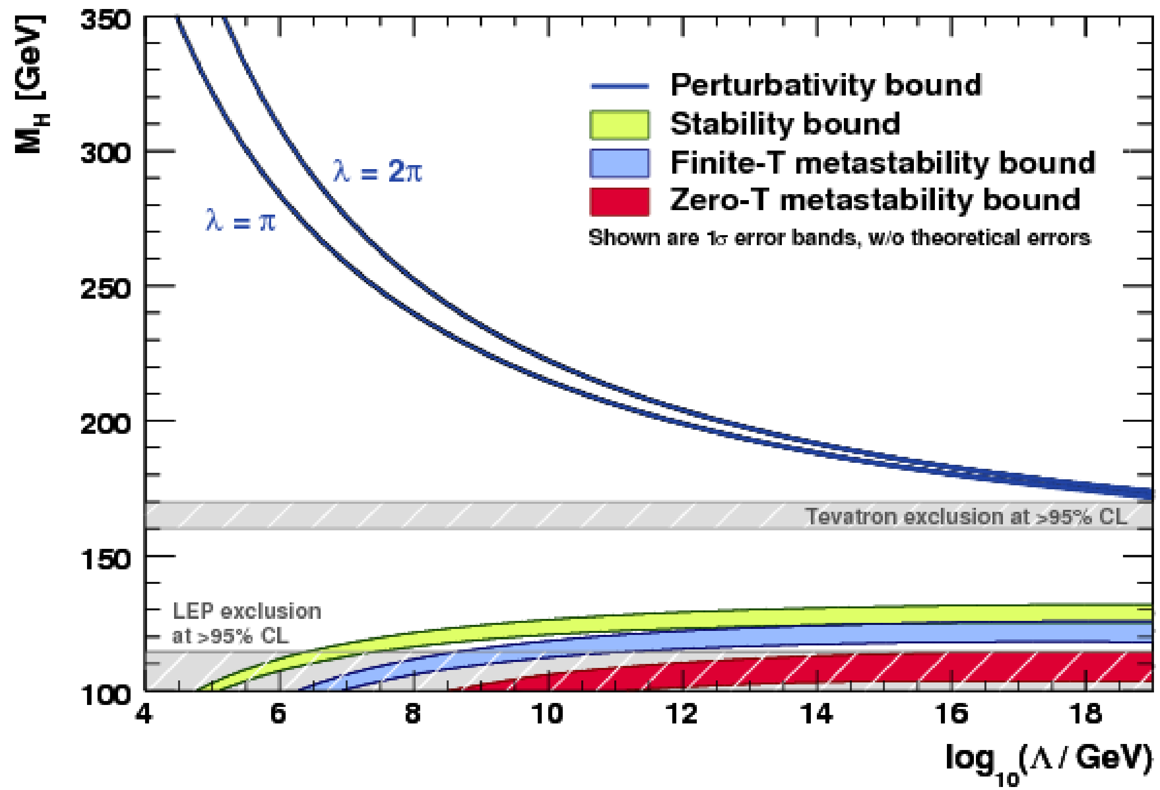
M_H large: λ^2 wins $\quad \lambda(M_t) \to \lambda(\mu) \gg 1 \quad$ non-perturbative regime, Landau pole

M_H small: $-Y_t^4$ wins $\quad \lambda(M_t) \to \lambda(\mu) \ll 1$

Ellis et al. 09

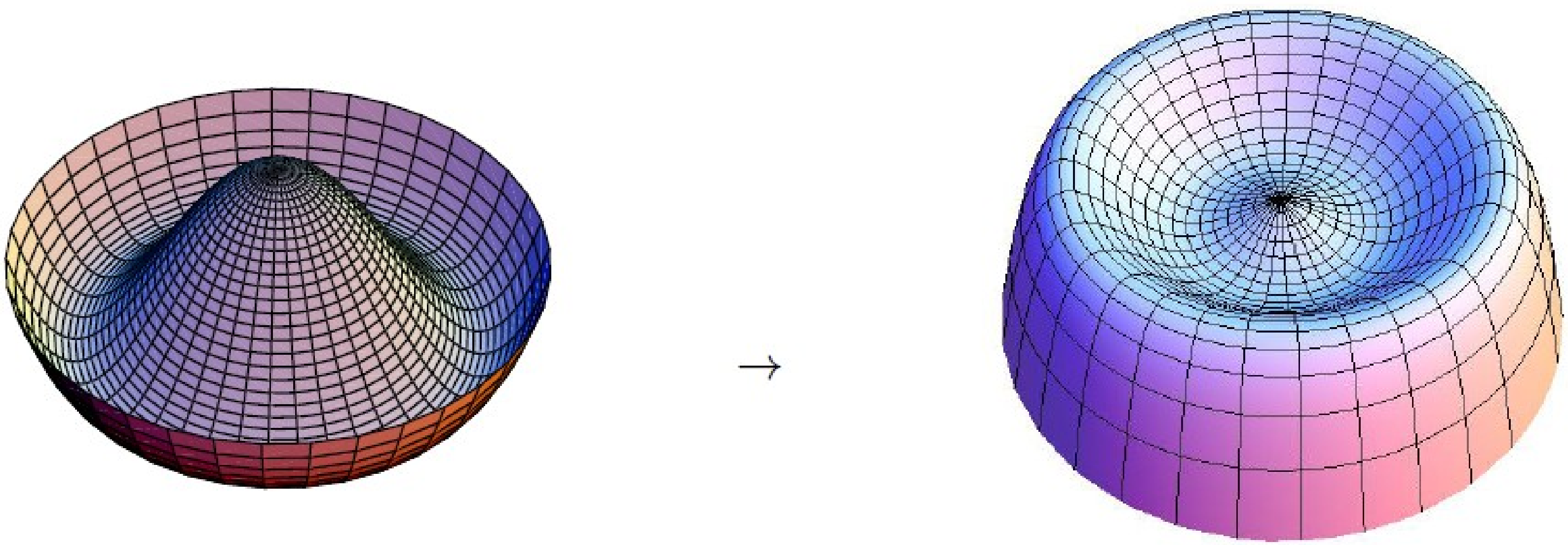
M_H ~ 125-126 GeV: $-Y_t^4$ wins
no problem with the Landau pole

Running depends on M_t, α_s ….



M_H ~ 125-126 GeV: $-Y_t^4$ wins: $\lambda(M_t)$ ~ 0.14 runs towards smaller values and can eventually become negative. If so the potential is either unbounded from below or can develop a second (deeper) minimun at large field values

Illustrative

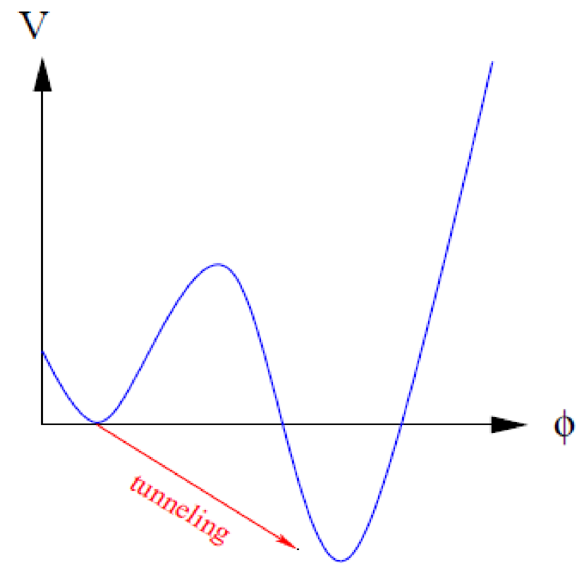


If your mexican hat turns out to be a dog bowl you have a problem...

from A. Strumia

The problem

There is a transition probability between the false and true vacua



It is really a problem ?

It is a problem that must be cured via the appearance of New Physics at a scale below that where the potential become unstable ONLY if the transition probability is smaller than the life of the universe.

Metastability condition: if λ becomes negative provided it remains small in absolute magnitude the SM vacuum is unstable but sufficiently long-lived compared to the age of the Universe

Transition probability: $p \sim e^{-\frac{8\pi^2}{3|\lambda|}}$

Vacuum stability analyses

Long history, back to the middle seventies

Linde (76); Weinberg (76); Cabibbo, Maiani, Parisi, Petronzio (79); Hung (79); Lindner (86); Sher(89) ....

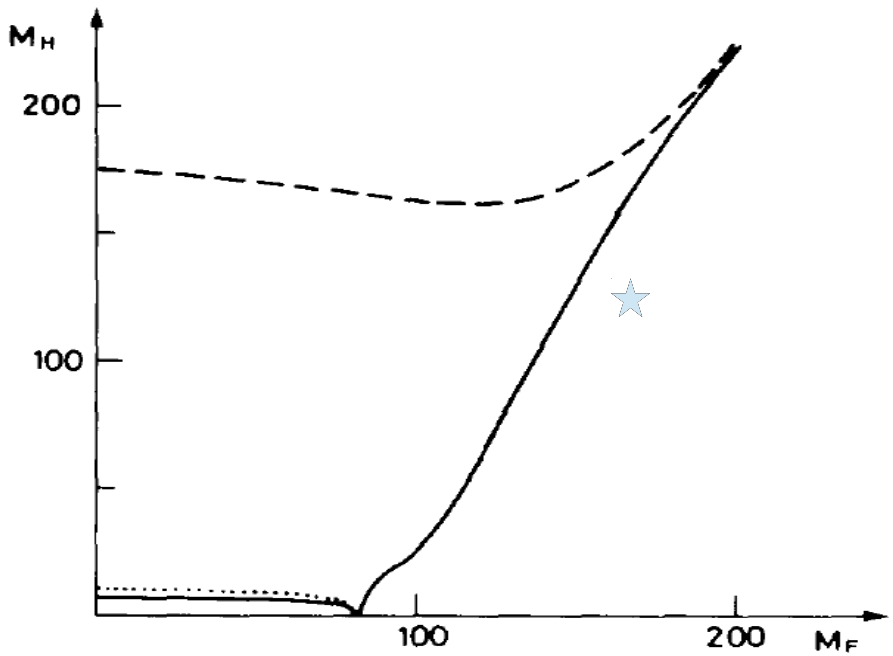


Fig. 1. Bounds on the mass of the Higgs boson (m_H) as a function of the top quark mass (M_t) in the case of three generations. We have taken $\sin^2\theta_W = 0.2$. The dashed line and the full line represent the upper and the lower bound, respectively. The dotted line is the prediction of the massless theory. The curves end in correspondence to the upper bound on M_t, eq. (4.2).

Cabibbo, Maiani, Parisi, Petronzio (79);

Vacuum stability analyses

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NNLO

- Two-loop effective potential
  (complete) Ford, Jack, Jones 92,97; Martin (02)

- Three-loop beta functions
  - gauge — Mihaila, Salomon, Steinhauser (12)
  - Yukawa, Higgs — Chetyrkin, Zoller (12, 13,); Bednyakov et al. (13)

- Two-loop threshold corrections at the weak scale
  - y_t: gauge x QCD — Bezrukov, Kalmykov, Kniehl, Shaposhnikov (12)
  - λ: Yuk x QCD, — Bezrukov et al. (12), Di Vita et al. (12)
  - SM gaugeless — Di Vita, Elias-Miro', Espinosa, Giudice, Isidori, Strumia, G.D. (12)

Dominant theory uncertainty on the Higgs mass value that ensures vacuum stability comes from the residual missing two-loop threshold corrections for λ at the weak scale

Full SM two-loop threshold corrections to λ, y_t and m

Buttazzo,Giardino, Giudice, Sala, Salvio, Strumia, G.D. (13)

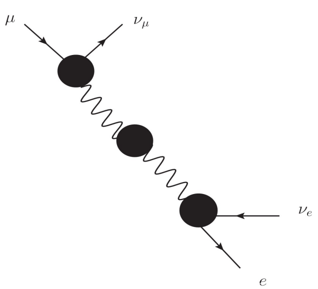
$\lambda(\mu)$ in terms of G_μ, $\alpha(M_Z)$, M_h, M_t, M_z, M_w (pole masses)

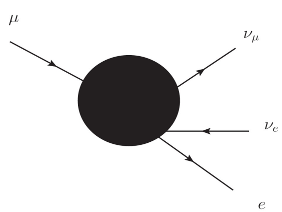
$$\lambda(\mu) = \frac{G_\mu}{\sqrt{2}} M_h^2 - \delta\lambda^{(1)}(\mu) - \delta\lambda^{(2)}(\mu)$$

Sirlin, Zucchini (86)

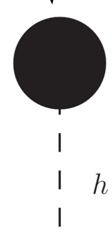
$$\frac{G_\mu}{\sqrt{2}} = \frac{1}{2v_0^2}(1 + \Delta r_0)$$

$$\delta\lambda^{(2)}(\mu) = \frac{G_\mu}{\sqrt{2}} M_h^2 \left\{ -\frac{\Delta r_0^{(1)}}{M_h^2} \left[M_h^2 \Delta r_0^{(1)} + \frac{3\,T^{(1)}}{2\,v_{\rm ren}} + {\rm Re}\,\Pi_{hh}^{(1)}(M_h^2) \right] + \Delta r_0^{(2)} + \frac{1}{M_h^2} \left[\frac{T^{(2)}}{v_{\rm ren}} + {\rm Re}\,\Pi_{hh}^{(2)}(M_h^2) \right] \right\}_{\rm fin} + \Delta_\lambda$$

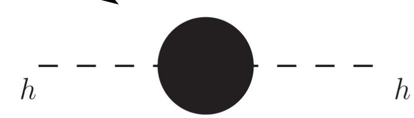




analytical



analytical

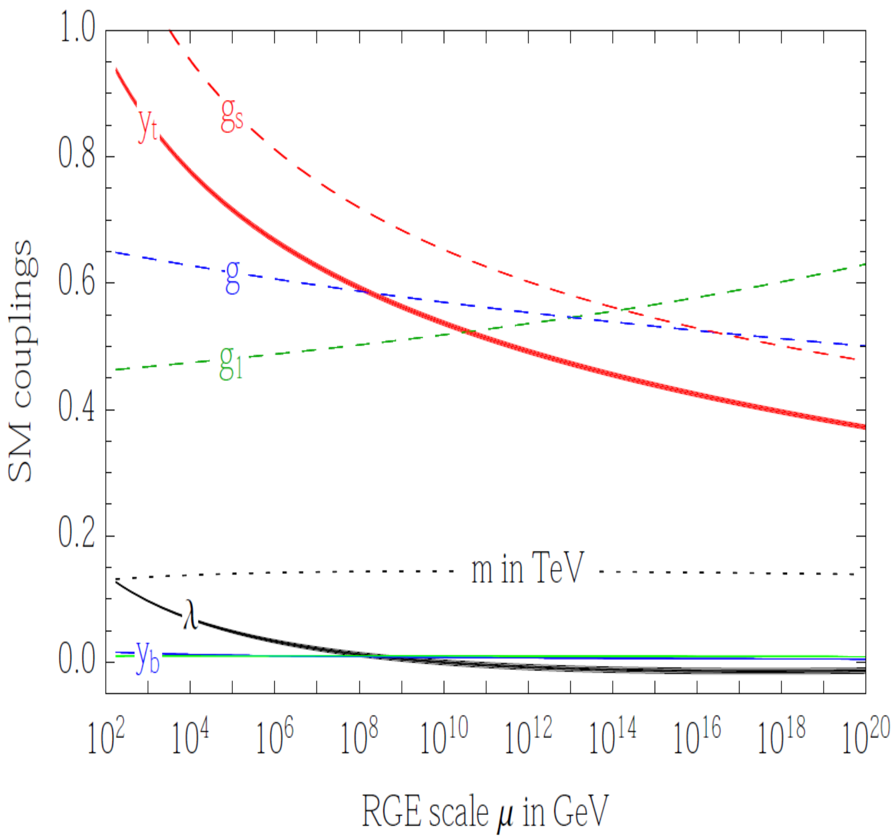


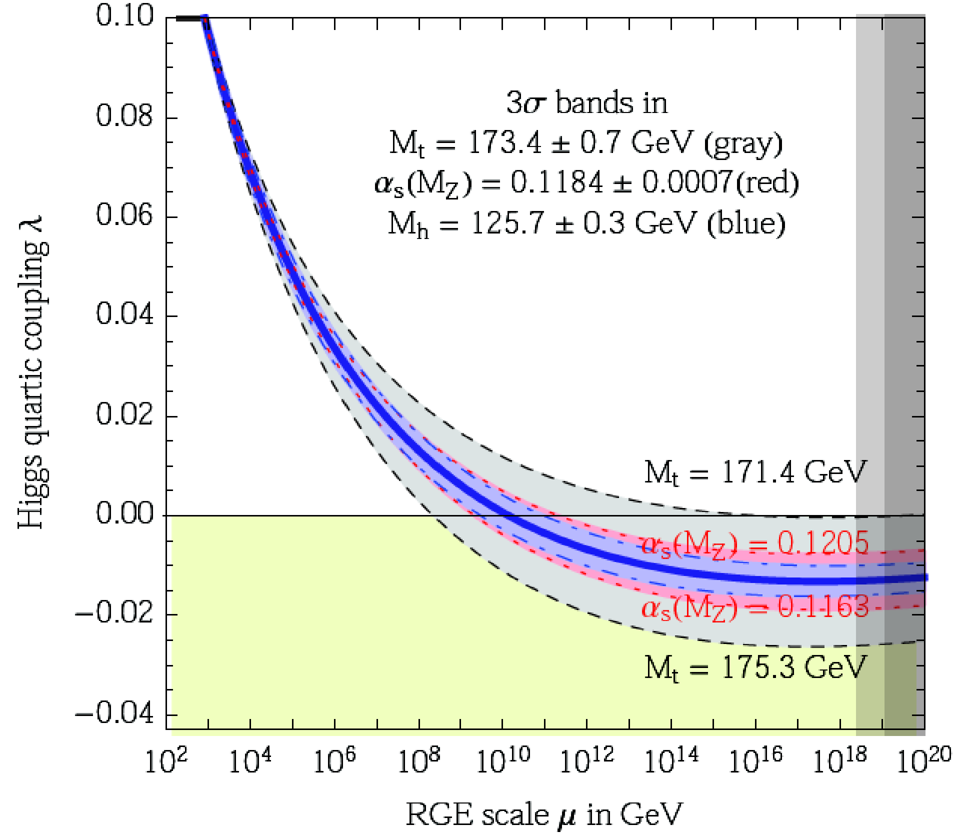
numerical,
Martin's loop functions

Martin (02,03)

$$\delta\lambda^{(2)}_{SM}(\mu = M_t) = -\frac{9.545}{(4\pi)^4}, \qquad \delta\lambda^{(2)}_{G.L.}(\mu = M_t) = -\frac{9.605}{(4\pi)^4}$$

$$\lambda(\mu = M_t) = 0.12709 + 0.00206\left(\frac{M_h}{\rm GeV} - 125.66\right) - 0.00004\left(\frac{M_t}{\rm GeV} - 173.15\right) \pm 0.00035_{\rm th}$$



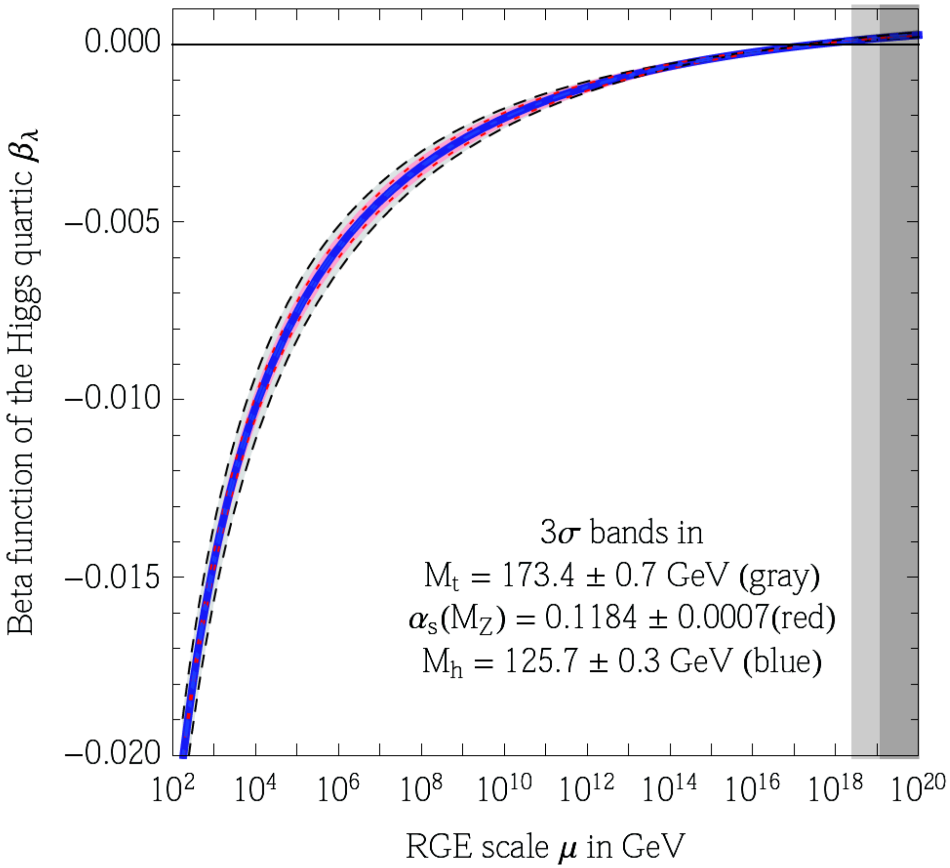


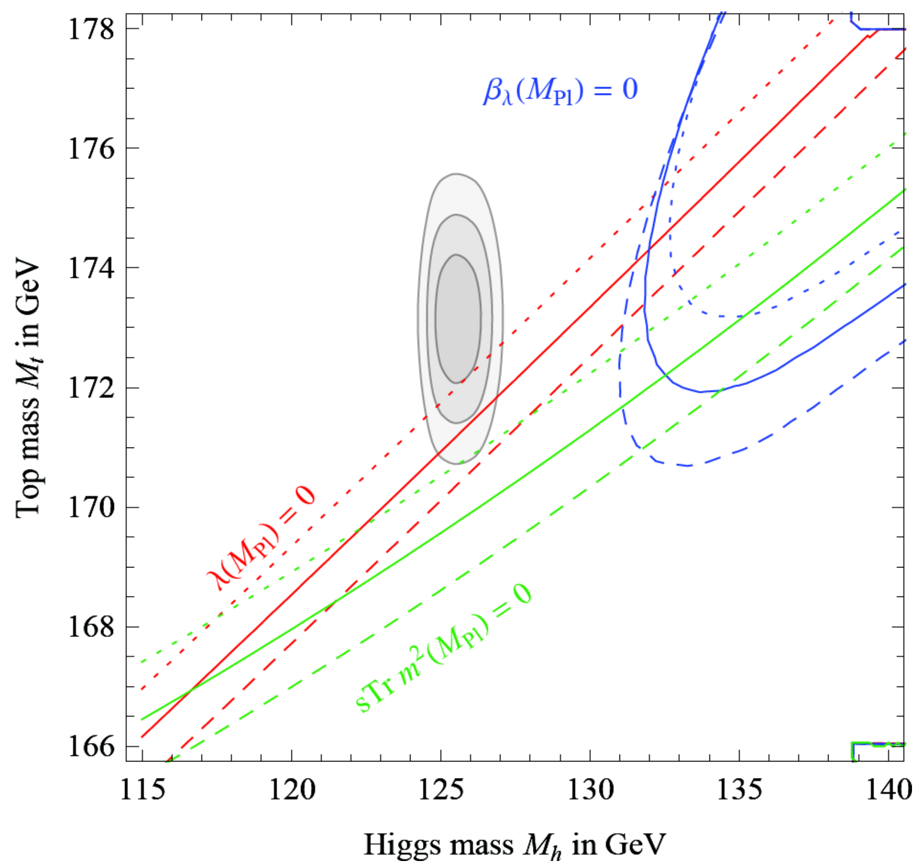
Full stability is lost at $\Lambda \sim 10^{10}$-10^{11} GeV but λ never becomes too negative

$$\lambda(M_{Pl.}) = -0.0128 + 0.0010\left(\frac{M_h - 125.66\,\text{GeV}}{0.34\,\text{GeV}}\right) - 0.0043\left(\frac{M_t - 173.35\,\text{GeV}}{0.65\,\text{GeV}}\right) + 0.0018\left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007}\right)$$

Both λ and β_λ are very close to zero around the Planck mass

Are they vanishing there?





$\lambda(M_{Pl})=0 \rightarrow M_t \sim 171$ GeV

Veltman's condition $\rightarrow M_t \sim 170$ GeV

$M_t = 173.4 \pm 0.7$ GeV

Pole mass

Top pole vs. $\overline{\text{MS}}$ mass

Is the Tevatron number really the "pole" (what is?) mass?
Monte Carlo are used to reconstruct the top pole mass form its decays products that contain jets, missing energy and initial state radiation.

$$M_t = M_t^{MC} + \Delta, \qquad \delta M_t^{MC} = \pm 0.7\ \text{GeV}, \qquad \Delta = ?$$

$M_t^{\overline{MS}}$ can be extracted from total production cross section

$$M_t^{\overline{MS}}(M_t) = 163.3 \pm 2.7\,\text{GeV} \rightarrow M_t = 173.3 \pm 2.8\,\text{GeV}$$

Alekhin, Djouadi, Moch, 12

Consistent with the standard value albeit with a larger error. $(\Delta = 0\ ?)$

N.B.
Fermion masses are parameters of the QCD Lagrangian, not of the EW one, Yukawas are.
$\overline{\text{MS}}$ masses are gauge invariant objects in QCD, not in EW, Yukawas are.
The vacuum is not a parameter of the EW Lagrangian. Its definition is not unique:

- Minimum of the tree-level potential
  $\rightarrow$ $M_t^{\overline{MS}}$ g.i. but large EW corrections in the relation pole-$\overline{\text{MS}}$ mass (~ M_t^4)

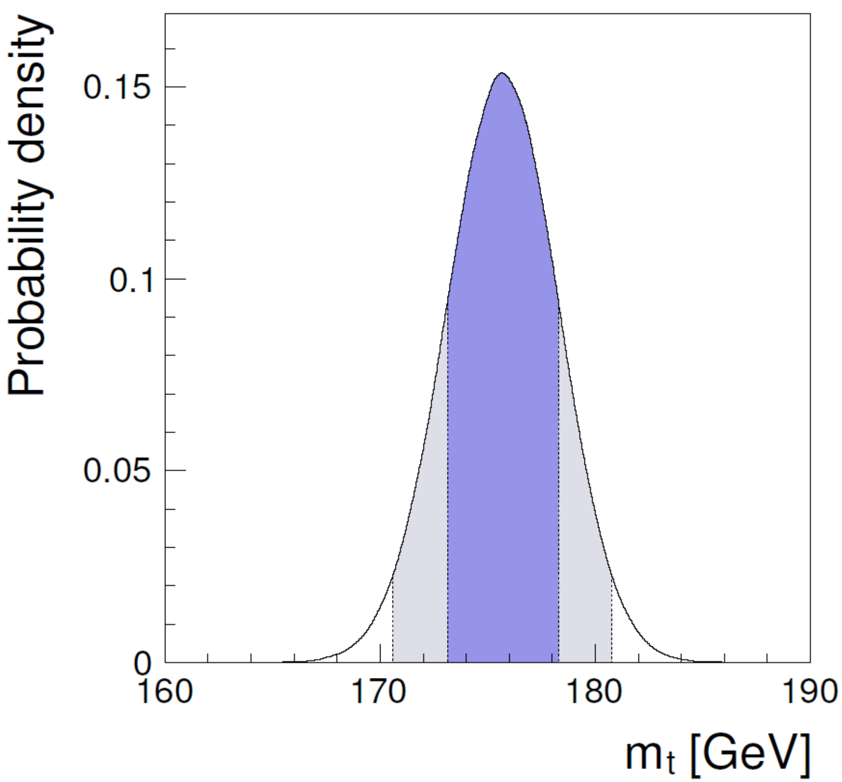
  Jegerlehner, Kalmykov, Kniehl, 12

  But direct extraction of $M_t^{\overline{MS}}$ requires EW correction

- Minimum of the radiatively corrected potential
  $\rightarrow$ $M_t^{\overline{MS}}$ not g.i. (problem? $\overline{\text{MS}}$ mass is not a physical quantity)
  no large EW corrections in the relation pole-$\overline{\text{MS}}$ mass

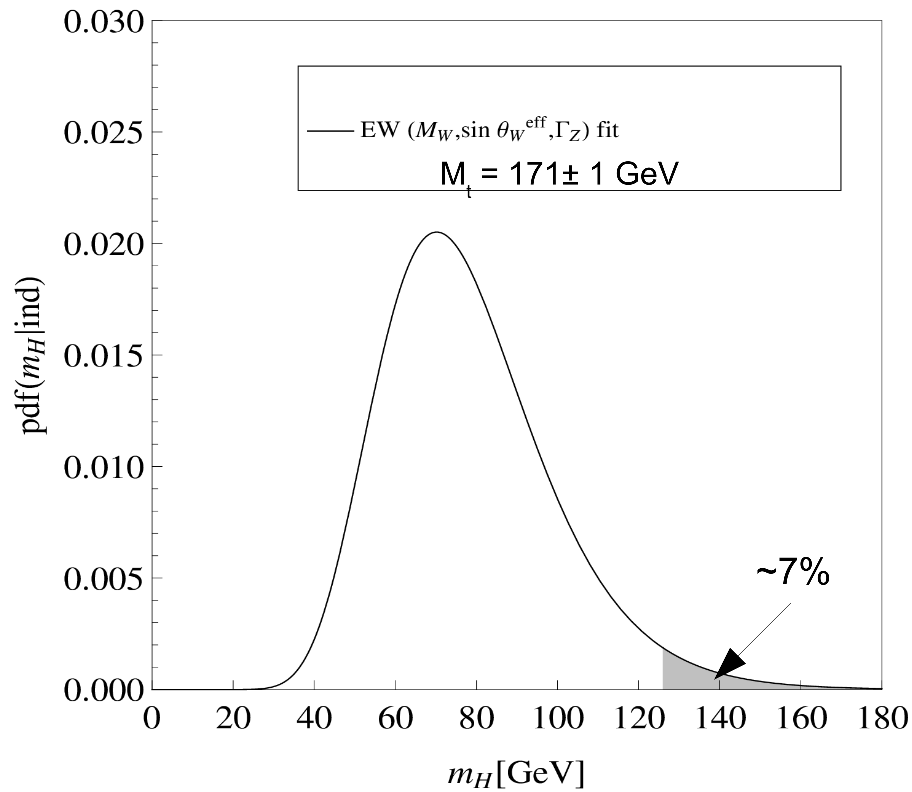
Is M_t ~ 171 GeV compatibile?

Indirect determination of M_t



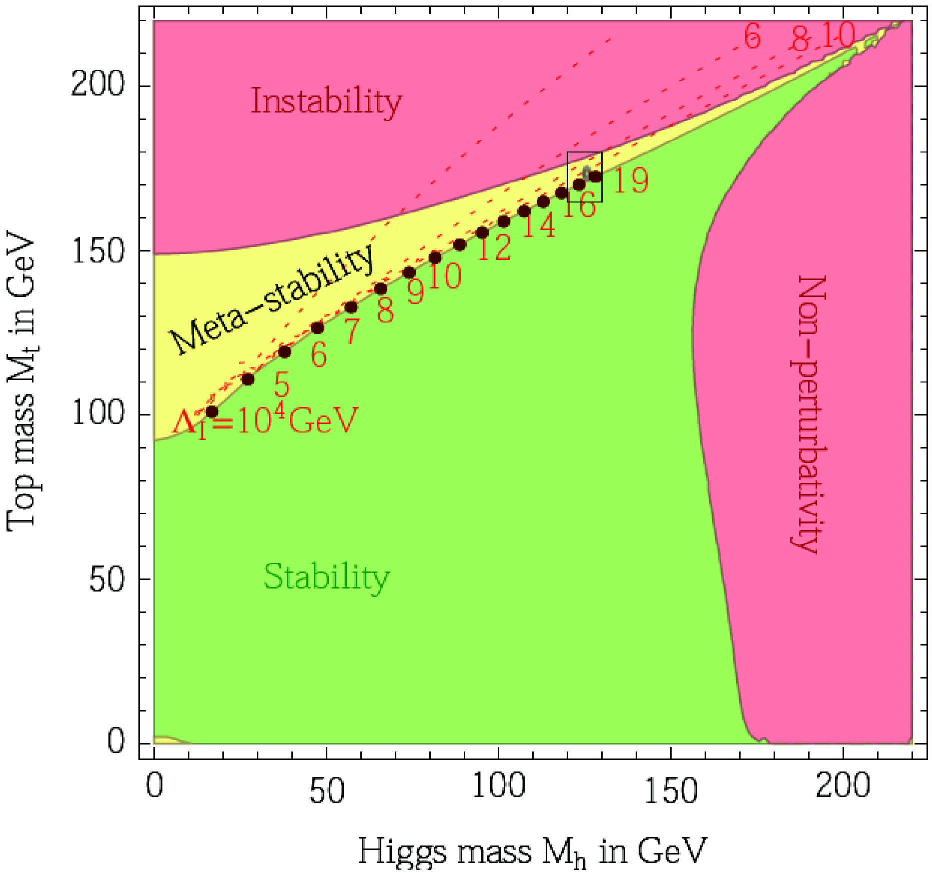
Ciuchini, Franco, Mishima, Silvestrini (13)

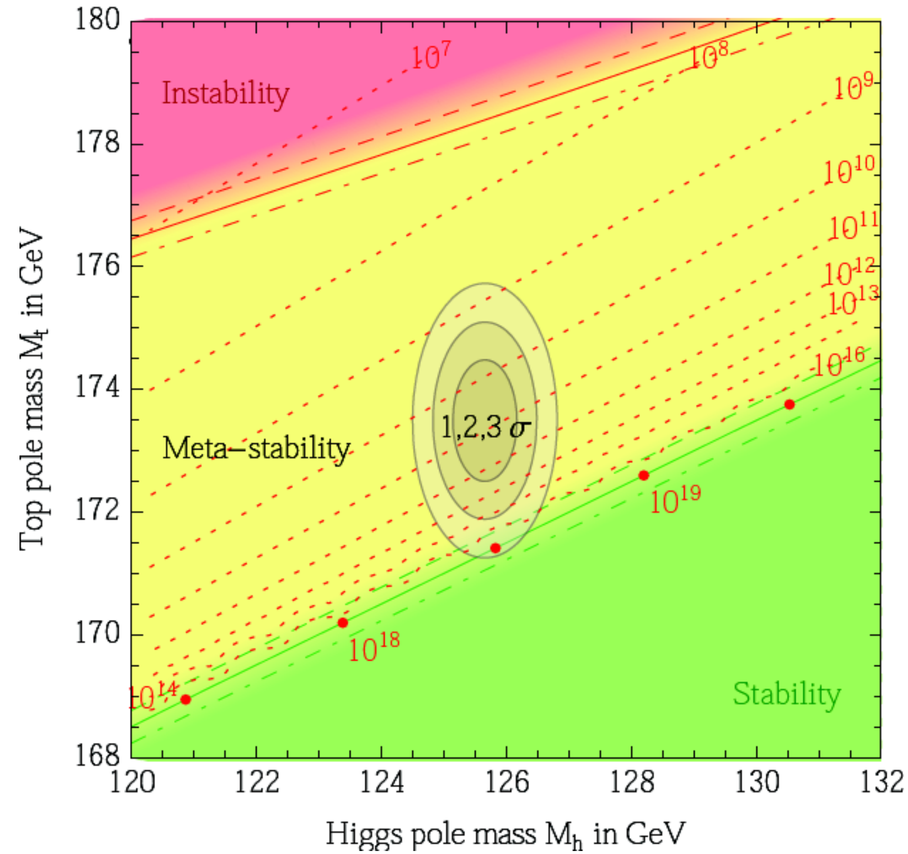
Indirect determination of M_h



courtesy of S. Di Vita

SM phase diagram





We live in a metastable universe close to the border with the stability region.

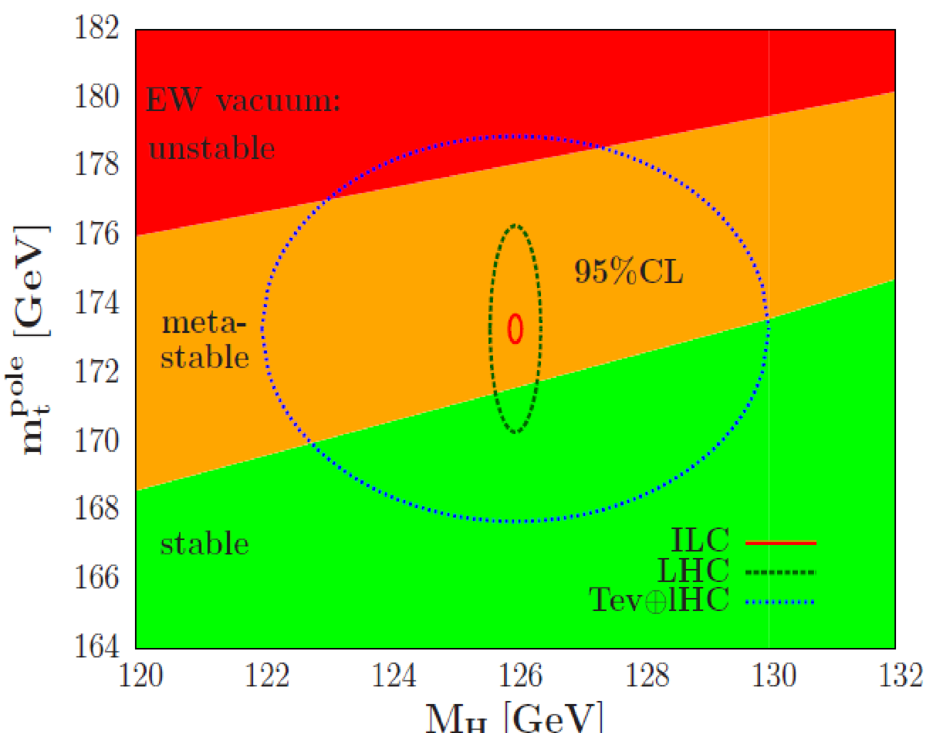
Stability condition:

$$\frac{M_h}{\text{GeV}} > 129.6 + 1.3\left(\frac{M_t - 173.35\,\text{GeV}}{0.65\,\text{GeV}}\right) - 0.5\left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007}\right) \pm 0.3_{\text{pert.}} \pm 0.6_{\text{non-pert.}}$$

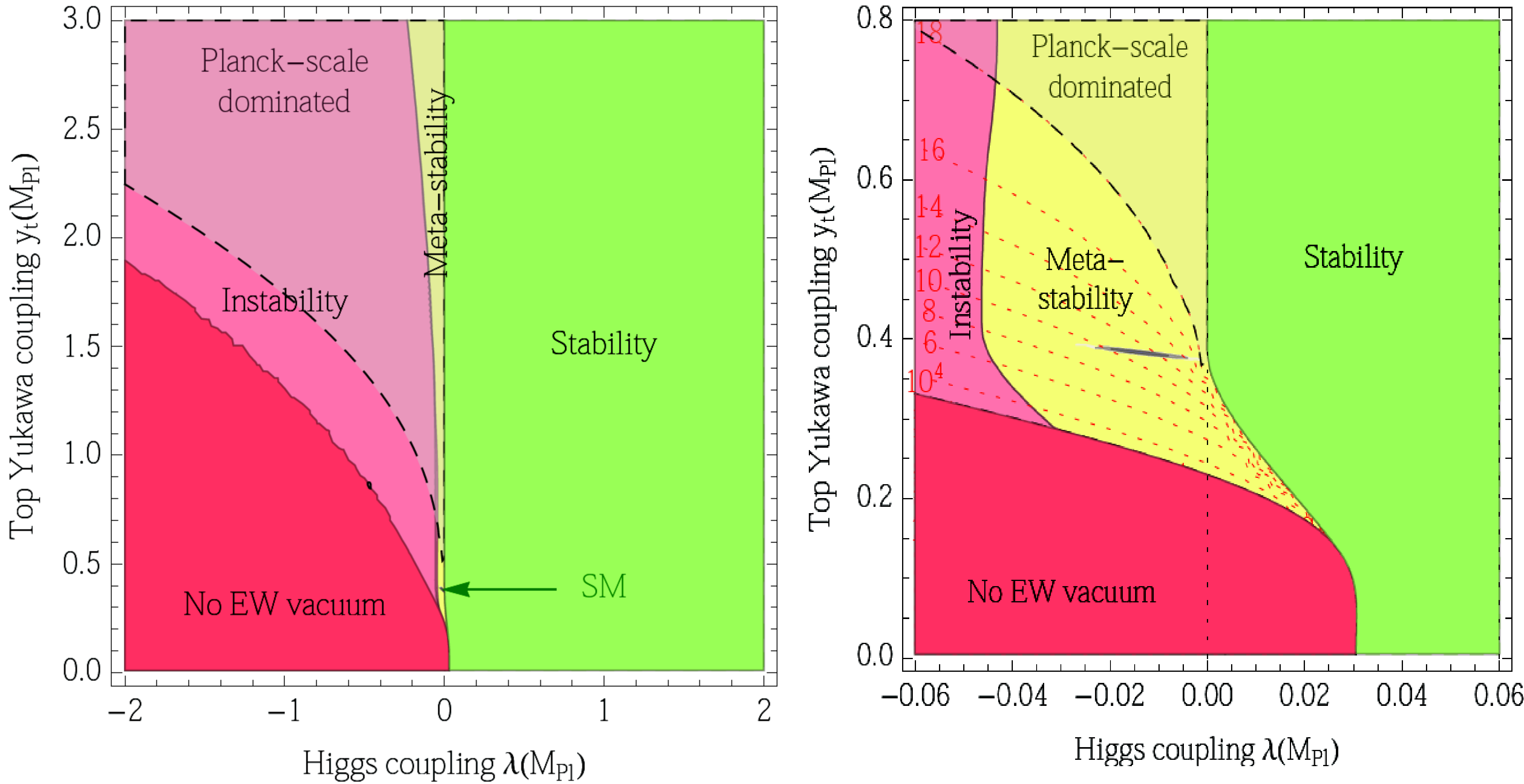
reduced

$$M_t < (171.36 \pm 0.15_{\text{pert.}} \pm 0.30_{\text{non-pert.}} \pm 0.25_{\alpha_s} \pm 0.17_{M_h})\,\text{GeV}$$

Type of error	Estimate of the error	Impact on M_h
M_t	experimental uncertainty in M_t	± 1.4 GeV
α_s	experimental uncertainty in α_s	± 0.5 GeV
Experiment	**Total combined in quadrature**	± 1.5 GeV
λ	scale variation in λ	~~± 0.7 GeV~~
h_t	$\mathcal{O}(\Lambda_{QCD})$ correction to M_t	± 0.6 GeV
h_t	QCD threshold at 4 loops	± 0.3 GeV
RGE	EW at 3 loops + QCD at 4 loops	~~± 0.2 GeV~~
Theory	**Total combined in quadrature**	~~± 1.0 GeV~~ ±0.7 GeV



Alekhin, Djouadi, Moch, 12



$\lambda(M_{Pl})$ and $y_t(M_{Pl})$ almost at the minimum of the funnel

An accident or deep meaning?

λ(μ) as a result of a matching with a high-scale theory

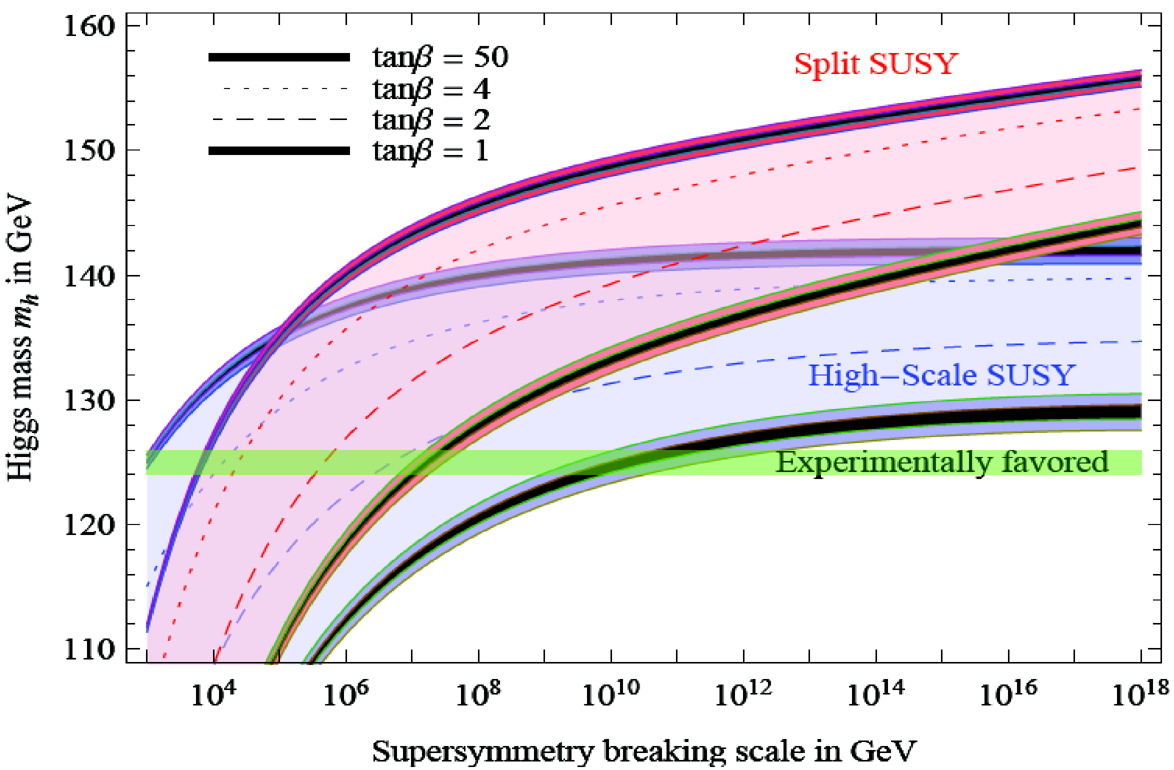
MSSM variant: High-Scale Supersymmetry: All SUSY particle with mass $\tilde{m}$

Split SUSY: Susy fermions at the weak scale; Susy scalars with mass $\tilde{m}$

($\tilde{m}$: Supersymmetry breaking scale)

$$\lambda(\tilde{m}) = \frac{1}{8}\left[g^2(\tilde{m}) + g'^2(\tilde{m})\right]\cos^2 2\beta$$

Predicted range for the Higgs mass



Supersymmetry broken at very large scale is disfavored

Conclusions

SM is quite OK

M_h−125/6   GeV is a very intriguing value.

The SM potential is at the “border” of the stability region.
The exact value of the top mass plays the central role between the full stability or metastability (preferred) options.

Model-independent conclusion about the scale of NP cannot be derived.
λ is small at high energy: NP (if exists) should have a *weakly interacting* Higgs particle
λ and β_λ are very close to zero around the Planck mass:
deep meaning or coincidence?

If Susy is realized in its minimal version the scale of its breaking cannot be too high