QCD effects at small transverse momentum

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M. García Echevarría, A. Idilbi, **(EIS)** A. Schaefer, arXive:1208.1281

EIS:Our final definition of the TMD is given in arXive:1211.1947

Initial definition, calculation and properties of TMDs in JHEP 07(2012)002
Work in progress with U. D`Alesio, S. Melis,..

Transverse Momentum Dependent … Physics

- Qt dependent Higgs and vector boson production

(with/without an associated jet)

- Semi Inclusive DIS: TMD PDF/FF (Fragmentation Functions)
- 3-D picture of the nucleon and spin origin
- TMD fragmentation in lepton-lepton colliders
- For all these we need to formulate a consistent definition of matrix elements called TMD within factorization theorems

TMDPDFs at Leading Twist

[Mulders, Tangerman'96]
[Boer, Mulders '98]

Momentum

Helicity

Transversity

Boer-Mulders

Quark Polarization

Nucleon Polarization

	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

[Higher twist structures in Mulders, Buffing, Mukherjee.'12]

Worm-Gear

Sivers

Worm-Gear

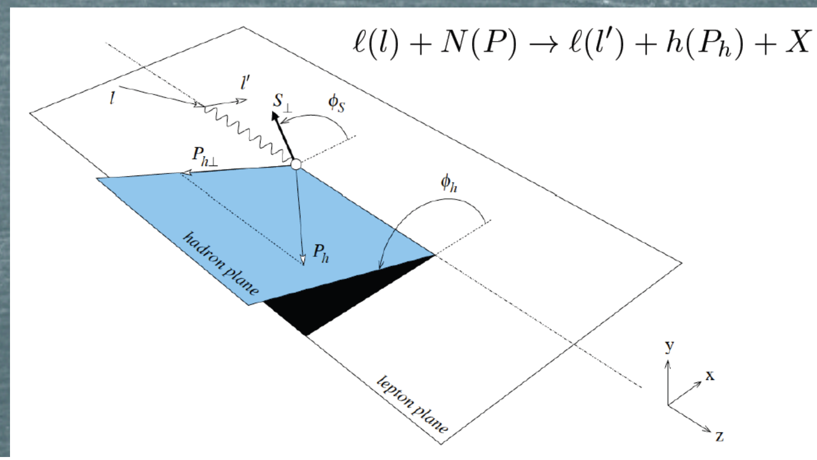
Pretzelosity

- The only ones that survive in the collinear limit (when we integrate over qT)
  - They are T-odd $f_{1T}^{\perp,DIS} = -f_{1T}^{\perp,DY}$
- There are similar families for gluon-TMDPDFs and quark/gluon-TMDFFs
- They give us information about the inner structure of the nucleons

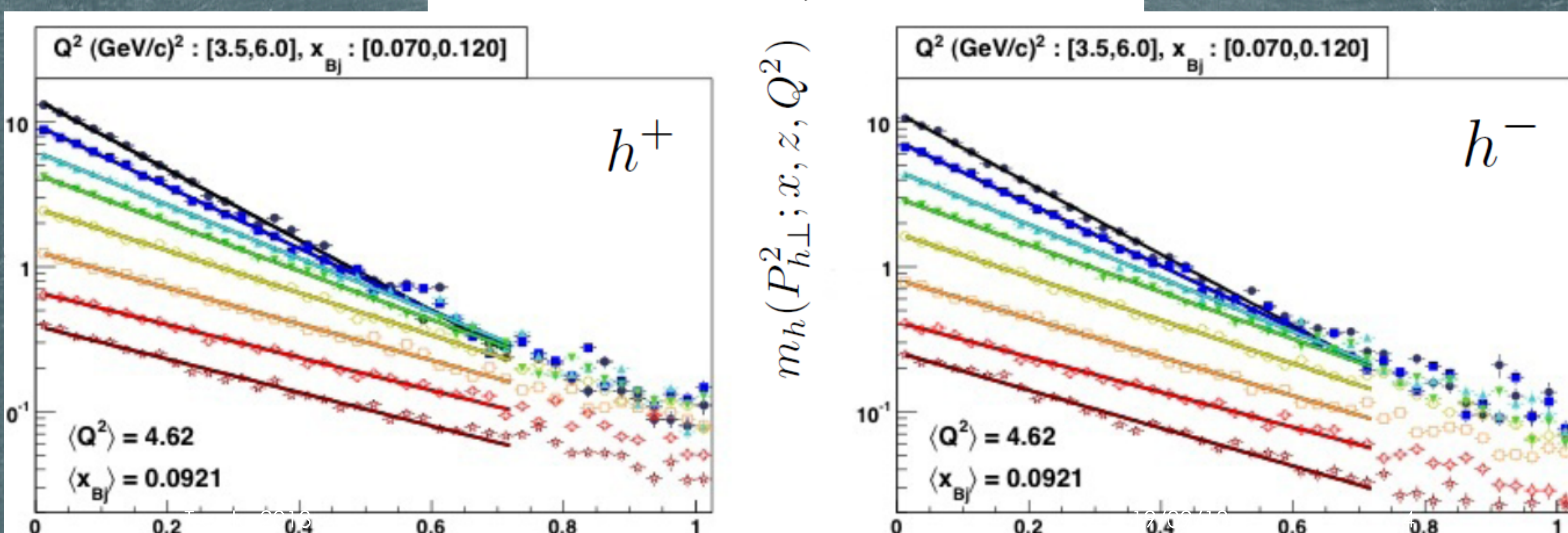
Experimental interests…

Current and past Experiments: Hermes, Belle, LHC, Compass

New experiments: Jlab, LHeC, EIC



Compass Coll. Cern [arXive:1305.7317]

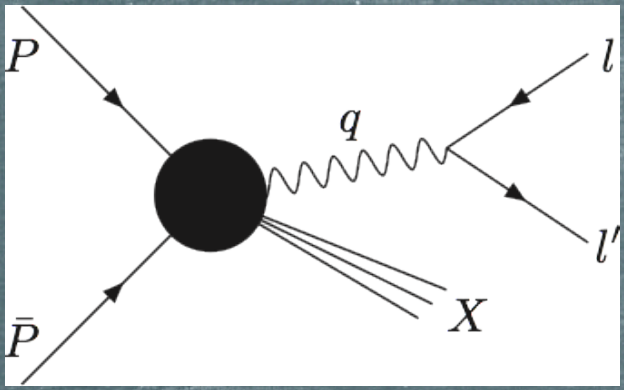


Some…topics

- Transverse Momentum distributions are fundamental in the ***factorization*** of DY at small qT and SIDIS and e+e- to 2j
- We formulate the TMD definition independently of the IR/collinear regulators that we use
- ***Universality*** of TMDs (the same in DY and DIS)
- ***Evolution of TMDs up to NNLL..***
- ***The spin independence of the evolution of all quark TMDs***
- ***Model independent evolution of the TMDs.***
- ***Extraction of TMDs***

Factorization in QCD

- Let's consider the inclusive Drell-Yan process:



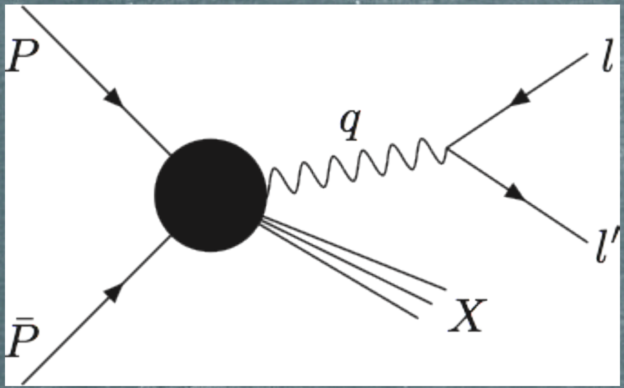
$$\boxed{q^2 = Q^2}$$

$$q_\perp = q_\perp^\ell + q_\perp^{\ell'}$$

- **Goal**: explore the internal structure of initial and/or final states (beyond PDF/FF)
- Example: how is the nucleon spin originated by partons?

Factorization in QCD

- Let's consider the inclusive Drell-Yan process:



$$\boxed{q^2 = Q^2}$$

Collins-Soper-Sterman '85, '88

$$\frac{d\sigma}{dQ^2} = \sum_{i,j=q,\bar{q},g} \int_0^1 dx_1\, dx_2\, \mathcal{H}_{ij}(x_1, x_2, Q^2, \mu^2)\, f_{i/P}(x_1, \mu^2)\, f_{j/\bar{P}}(x_2, \mu^2)$$

Short-distance physics.
Perturbative coefficient

Long-distance physics.
Non-perturbative PDFs

Naive TMDPDF...

- One could naively think of defining the TMDPDF by extending the PDF (Collins, Soper):

$$F_n^{naive}(0^+, y^-, \vec{y}_\perp) = \frac{1}{2}\sum_\sigma \langle P,\sigma| \, [\bar{\xi}_n W_n]\,(0^+, y^-, \vec{y}_\perp)\,\frac{\bar{\not n}}{2}\,[W_n^\dagger \xi_n]\,(0)\,|P,\sigma\rangle$$

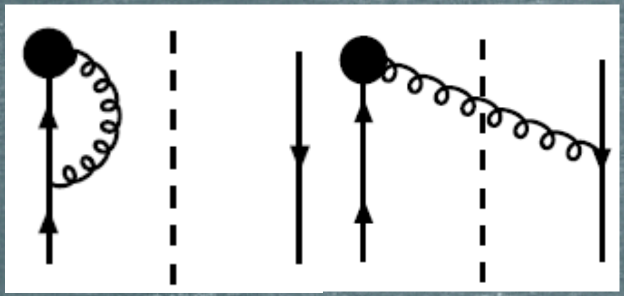
*We also need **transverse** gauge links to maintain gauge invariance EIS'11*

- If we calculate this matrix element we get:

$$\tilde{F}_n^{naive} = \delta(1-x) + \frac{\alpha_s C_F}{2\pi}\left\{\delta(1-x)\left[\frac{2}{\varepsilon_{\mathrm{UV}}}\ln\frac{\Delta^+}{Q^2} + \frac{3}{2\varepsilon_{\mathrm{UV}}} - \frac{1}{4} + \frac{3}{2}L_T + 2L_T\ln\frac{\Delta^+}{Q^2}\right] - (1-x)\ln(1-x) - \mathcal{P}_{q/q}\ln\frac{\Delta^-}{\mu^2} - L_T\mathcal{P}_{q/q}\right\}$$

- **It is ill-defined!! We cannot renormalize this quantity...**

Naive TMDPDF…



The problem resides in the light-cone divergences

Collinear part:

$$\mu^{2\epsilon}\int \frac{d^dk}{(2\pi)^d}\frac{p^+ + k^+}{[k^+ - i\varepsilon][(p+k)^2 + i\varepsilon][k^2 + i0]} \Longrightarrow \frac{1}{\varepsilon_{UV}} \times \int_0^1 \frac{1}{t}$$

In the PDF these divergences are canceled between real and virtual emissions, but this does not happen now

The rapidity divergence of the soft part is the double of the collinear part
IN QCD THE HADRONIC TENSOR (M) HAS NO RAPIDITY DIVERGENCES!!

The Definition

One can find many definitions of TMDPDF "in the market":

- *Collins, Soper '82: just collinear (off-the-LC)*
- *Ji, Ma, Yuan '05: collinear with subtraction of complete soft function (off-the-LC)*
- *Cherednikov, Stefanis '08: collinear with subtraction of complete soft function (LC gauge)*
- *Mantry, Petriello '10: fully unintegrated collinear matrix element*
- *Collins '11: collinear with subtraction of square root of 3 soft functions (off-the-LC)*
- *M.G. Echevarría. A. Idilbi, I. Scimemi ´11-´12: collinear with subtraction of square root of soft function (on-the-LC)*
- *Chiu, Jain, Neill, Rothstein '12: collinear matrix element (rapidity renormalization group)*

The criteria of the proper definition of the TMDPDF:

- **A well-defined TMDPDF should:**

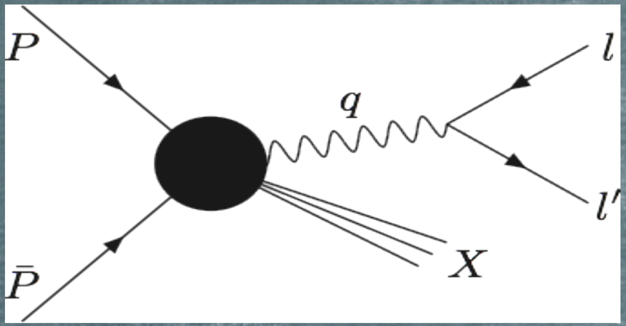
1. Be compatible with a factorization theorem.
2. Have no mixed UV/nUV divergencies, i.e., be renormalizable
3. Have a matching coefficient onto PDFs independent of nUV regulators.

* By "nUV" I mean non-ultraviolet, i.e., infrared (IR) and rapidity.

Our definition fulfills all of them

DY Factorization at Small q_T:

General Overview



$$q^2 = Q^2 \gg q_T^2$$

Problem with different scales... Perfect for Effective Field Theories approach!

QCD → SCET_{q_T} → $\mathrm{SCET}_{\Lambda_{\mathrm{QCD}}}$

$$\tilde{M} = H(Q^2/\mu^2)\, \tilde{F}_n(x_n, b; Q^2, \mu^2)\, \tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q^2, \mu^2)$$

$$q_T^2 \sim \Lambda_{QCD}^2$$

$$\tilde{M} = H(Q^2/\mu^2)\, \tilde{C}_n(b^2\mu^2, Q^2/\mu^2)\, \tilde{C}_{\bar{n}}(b^2\mu^2, Q^2/\mu^2)\, f_n(x_n; \mu^2)\, f_{\bar{n}}(x_{\bar{n}}; \mu^2)$$

$$q_T^2 \gg \Lambda_{QCD}^2$$

The IR has to be regulated consistently in the theories above and below every matching scale in order to properly extract the matching (Wilson) coefficients.

Effective field theory of QCD:SCET

- We can distinguish two energy scaling regimes.

$$Q \gg q_T \sim \Lambda_{QCD} \qquad Q \gg q_T \gg \Lambda_{QCD}$$

- One defines a power counting of operators in terms of

$$\lambda_1 \sim \frac{q_T}{Q} \qquad \lambda_2 \sim \frac{\Lambda_{QCD}}{q_T}$$

- We need an effective field theory for each energy scale

Factorization of Modes (1/2)

The factorization of the relevant modes is ... [Manohar-Stewart '06]

<u>**Each mode has its own Lagrangian**</u> (if one cannot write an on-shell interaction mixing the modes)

$$\lambda \sim \frac{q_T}{Q}$$

$$\begin{aligned} k_n &\sim Q(1, \lambda^2, \lambda) && \to && y \gg 0 \\ k_{\bar{n}} &\sim Q(\lambda^2, 1, \lambda) && \to && y \ll 0 \\ k_s &\sim Q(\lambda, \lambda, \lambda) && \to && y \approx 0 \end{aligned}$$

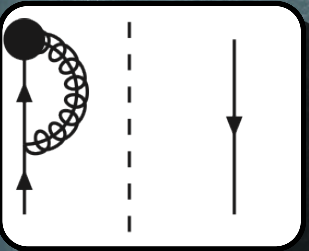
$$k_n^2 \sim k_{\bar{n}}^2 \sim k_s^2 \sim q_T^2$$

$$y = \frac{1}{2} \ln \left| \frac{k^+}{k^-} \right|$$

Soft and Collinear Modes can be mixed under boosts, (they have the same invariant mass.)

We need rapidity cuts

(modes can be distinguished only by their relative rapidities):



- Rapidity divergence when k^+ goes to 0
- We need rapidity cuts

$$\mu^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} \frac{p^+ + k^+}{[k^+ - i\varepsilon][(p+k)^2 + i\varepsilon][k^2 + i\varepsilon]}$$

Drell-Yan at small qT

$$d\Sigma = \frac{4\pi\alpha}{3q^2 s}\frac{d^4q}{(2\pi)^4}\boxed{\frac{1}{4}\sum_{\sigma_1\sigma_2}\int d^4y e^{-iqy}(-g_{\mu\nu})\langle N_1(P,\sigma_1)N_2(\bar{P},\sigma_2)|J^{\mu\dagger}(y)J^{\nu}(0)|N_1(P,\sigma_1)N_2(\bar{P},\sigma_2)\rangle}$$

M : Hadronic Tensor

$$J^{\mu} = \sum_q e_q \bar{\psi}\gamma^{\mu}\psi \xrightarrow[SCETqT]{QCD} J^{\mu}_{SCET} = C(Q^2,\mu)\sum_q e_q \bar{\chi}_{\bar{n}} S_{\bar{n}}^{T\dagger}\gamma^{\mu} S_n^T \chi_n$$

$$\chi_n = W^{T\dagger}\xi_n$$

However this is not the end of the story…

Collinear, anti-collinear and soft act on different Hilbert spaces!! (SCET)

Drell-Yan at small qT

$$M = |C(Q^2,\mu)|^2 \int d^4 y e^{-iq\cdot y} \sum_q e_q^2 J_n^{(0)}(0^+, y^-, y_\perp) J_{\bar{n}}^{(0)}(y^+, 0^-, y_\perp) S(0^+, 0^-, y_\perp)$$

The collinear currents are PURE!! (zero bin subtracted)

$$J_n^{(0)}(0^+, y^-, y_\perp) = \frac{1}{2} \sum_{\sigma_1} \langle N_1(P,\sigma_1) | \bar{\chi}_n(0^+, y^-, y_\perp) \frac{\bar{\not{n}}}{2} \chi_n(0) | N_1(P,\sigma_1) \rangle$$

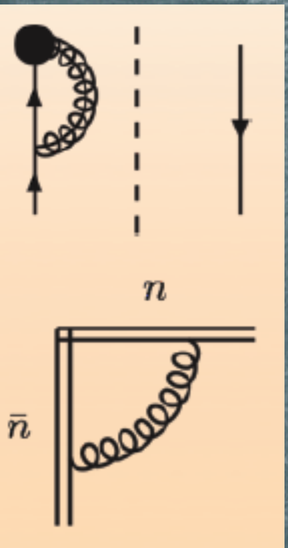
$$J^{(0)}{}_{\bar{n}}(y^+, 0^-, y_\perp) = \frac{1}{2} \sum_{\sigma_1} \langle N_2(\bar{P},\sigma_2) | \bar{\chi}_{\bar{n}}(0) \frac{\not{n}}{2} \chi_{\bar{n}}(y^+, 0^-, y_\perp) | N_2(\bar{P},\sigma_2) \rangle$$

$$S(0^+, 0^-, y_\perp) = \langle 0 | \mathrm{Tr}\, \bar{\mathbf{T}}\left[S_n^{T\dagger} S_{\bar{n}}^T \right](0^+, 0^-, y_\perp) \mathbf{T}\left[S_{\bar{n}}^{T\dagger} S_n^T \right](0) | 0 \rangle, \qquad \chi = W^{T\dagger} \xi$$

Double Counting (definition of pure collinear currents)

Taking the soft limit of collinear graphs one can get the soft contribution

Manohar, Stewart 2006



$$= -2ig^2 C_F \delta(1-x)\delta^{(2)}(\vec{k}_{n\perp})\mu^{2\varepsilon}\int \frac{d^d k}{(2\pi)^d}\frac{p^+ + k^+}{[k^+ - i0^+][(p+k)^2 + i0^-][k^2 + i0]}$$

$$= -2ig^2 C_F \delta^{(2)}(\vec{k}_{n\perp})\mu^{2\varepsilon}\int \frac{d^d k}{(2\pi)^d}\frac{1}{[k^+ - i0^+][k^- + i0^-][k^2 + i0]}$$

FOR PURE COLLINEAR f's WE HAVE TO SUBTRACT THE OVERLAPPING WITH THE SOFT PART : THIS CAN ALWAYS BE DONE BUT THE ANSWER IS REGULATOR DEPENDENT

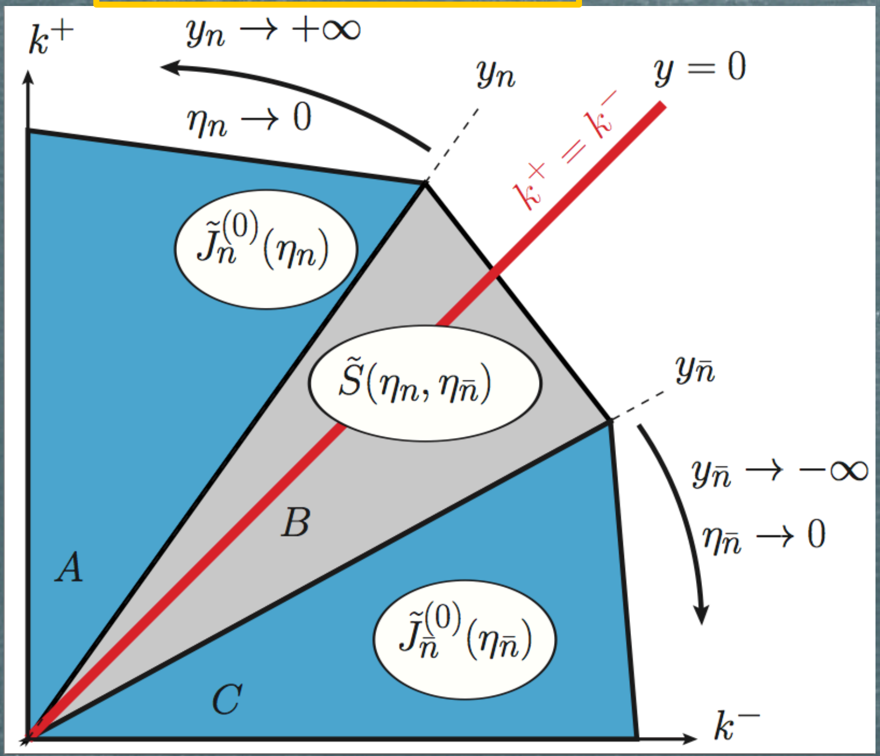
The factorization theorem must be written in terms of PURE COLLINEAR objects

Factorization of Modes (2/2)

We need to impose rapidity cutoffs to separate the modes:

$$H(Q^2)\,\tilde{J}_n^{(0)}(\eta_n)\,\tilde{S}(\eta_n,\eta_{\bar{n}})\,\tilde{J}_{\bar{n}}^{(0)}(\eta_{\bar{n}})$$

Pure collinear!



$$y = \frac{1}{2}\ln\left|\frac{k^+}{k^-}\right|$$

A. A is collinear
B. B is soft
C. C is anti-collinear
D. Soft function is NOT symmetric w.r.t. the "separating line" k+=k-

- We proved that the soft function can be split in two "hemispheres"
- We identify positive & negative rapidity quanta with each TMDPDF!!

Rapidity Divergences

- We need a rapidity regulator.
- All properties of TMDPDF are regulator independent

(Only the zero bin subtraction is regulator dependent).

- We did our calculations staying **on-the-light-cone** and using the Δ-regulator (Chiu, Fuhrer, Hoang, Manohar,'09), and we have checked that our results are consistent with all other regulators.

$$\frac{i(\not{p}+\not{k})}{(p+k)^2+i\Delta^-} \longrightarrow \frac{1}{k^-+i\delta^-}, \; \delta^- = \frac{\Delta^-}{p^+}$$

$$\frac{i(\not{\bar{p}}-\not{k})}{(\bar{p}-k)^2+i\Delta^+} \longrightarrow \frac{1}{-k^++i\delta^+}, \; \delta^+ = \frac{\Delta^+}{\bar{p}^-}$$

$$\delta^\pm \to 0$$

THE TMD MUST BE FREE FROM RAPIDITY DIVERGENCES

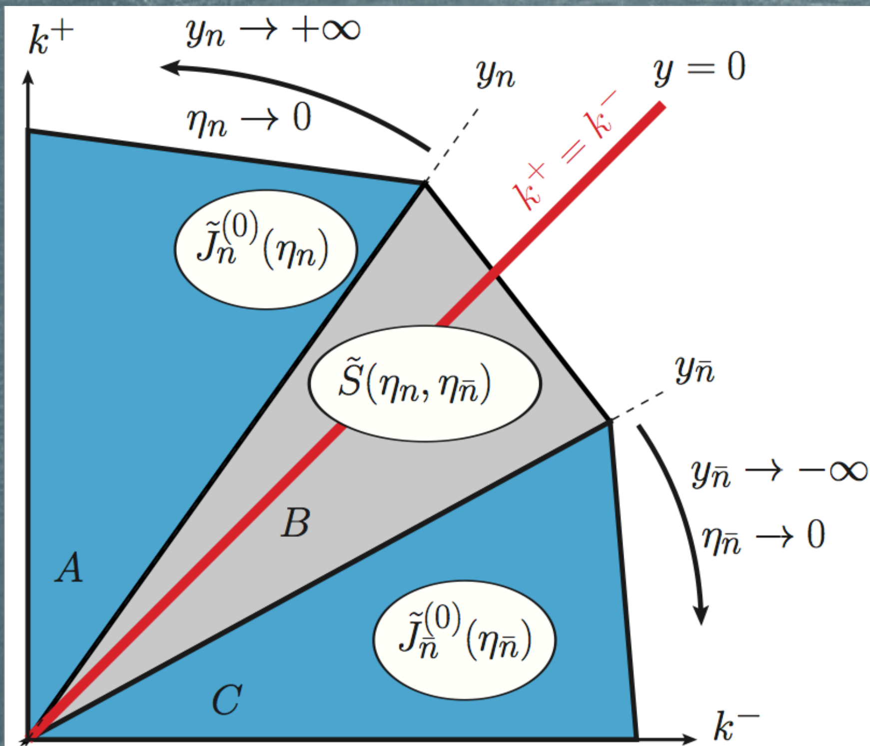
A subtle problem related to this cancellation.

In JHEP 1207 (2012) 002 we used the anstatz $\delta^+ = \delta^- = \delta$

In arXiv:1211.1947 we have removed the ansatz!!! Subtle property of Soft funct.

Definition of TMDPDF

Positive and negative rapidity quanta can be collected into two different TMDs because of ***<u>the splitting of the soft function</u>***



$$\tilde{S}(\Delta^+, \Delta^-) = \sqrt{\tilde{S}(\Delta^-, \Delta^-)\, \tilde{S}(\Delta^+, \Delta^+)}$$

$$\tilde{F}_n(x_n, b; Q, \mu) = \tilde{J}_n^{(0)}(\Delta^-) \sqrt{\tilde{S}\left(\frac{\Delta^-}{p^+}, \frac{\Delta^-}{\bar{p}^-}\right)}$$

$$\tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q, \mu) = \tilde{J}_n^{(0)}(\Delta^+) \sqrt{\tilde{S}\left(\frac{\Delta^+}{p^+}, \frac{\Delta^+}{\bar{p}^-}\right)}$$

$$\tilde{M} = H(Q^2/\mu^2)\, \tilde{F}_n(x_n, b; Q^2, \mu^2)\, \tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q^2, \mu^2)$$

No soft function in the factorization theorem!!

TMDPDF: One loop results

$$\tilde{M} = H(Q^2/\mu^2)\tilde{F}_n(x_n, b^2; Q^2, \mu^2)\tilde{F}_{\bar{n}}(x_{\bar{n}}, b^2; Q^2, \mu^2)$$

$$\boxed{\begin{aligned}\tilde{F}_n &= \tilde{J}_n^{(0)}(\Delta^-)\sqrt{\tilde{S}(\Delta^-, \Delta^-)}\\ \tilde{F}_{\bar{n}} &= \tilde{J}_{\bar{n}}^{(0)}(\Delta^+)\sqrt{\tilde{S}(\Delta^+, \Delta^+)}\end{aligned}}$$

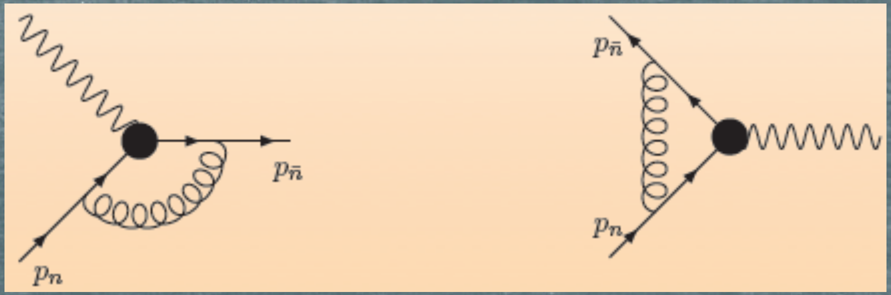
$$\tilde{F}_{n1} = \frac{\alpha_s C_F}{2\pi}\Big\{ \boxed{\delta(1-x)[\frac{1}{\varepsilon_{UV}^2} - \frac{1}{\varepsilon_{UV}}\ln\frac{Q^2}{\mu^2} + \frac{3}{2\varepsilon_{UV}}} \longrightarrow \text{NO MIXED DIVERGENCES}$$

$$\boxed{-\frac{1}{2}L_T^2 + \frac{3}{2}L_T - L_T\ln\frac{Q^2}{\mu^2} - \frac{\pi^2}{12} + (1-x) - L_T P_{q/q}]} \longrightarrow \text{MATCHING COEFF. TO PDF}$$

$$\boxed{-P_{q/q}\ln\frac{\Delta^-}{\mu^2} - \frac{1}{4}\delta(1-x) - (1-x)[1+\ln(1-x)] \Big\}} \longrightarrow \text{PDF}$$

Universality of unpolarized TMDPDF

- The collinear and soft matrix element are the same in DY and SIDIS



- The definition of Wilson lines in DY and SIDIS is different

DY

$$W_n(x) = \bar{P}\exp\left[ig\int_{-\infty}^{0} ds\bar{n}\cdot A_n(x+s\bar{n})\right]$$

$$S_n(x) = P\exp\left[ig\int_{-\infty}^{0} ds\bar{n}\cdot A_s(x+s\bar{n})\right]$$

SIDIS

$$\widetilde{W}_n(x) = \bar{P}\exp\left[-ig\int_{-\infty}^{0} ds\bar{n}\cdot A_n(x+s\bar{n})\right]$$

$$\widetilde{S}_n(x) = P\exp\left[-ig\int_{-\infty}^{0} ds\bar{n}\cdot A_s(x+s\bar{n})\right]$$

Universality of unpolarized TMDPDF

✓ Universality of the Soft Function

$$S_1^{r,DIS} = S_1^{r,DY} + \frac{\alpha_s C_F}{2\pi}\delta^{(2)}(\vec{k}_T)\pi^2; \qquad S_1^{vDIS} = S_1^{v,DY} - \frac{\alpha_s C_F}{2\pi}\delta^{(2)}(\vec{k}_T)\pi^2;$$

✓ Universality of the Collinear Function

$$\hat{j}_{n1}^{r,DIS} = \hat{j}_{n1}^{r,DY} - \frac{\alpha_s C_F}{2\pi}\delta^{(2)}(\vec{k}_T)\pi^2; \qquad \hat{j}_{n1}^{vDIS} = \hat{j}_{n1}^{v,DY} + \frac{\alpha_s C_F}{2\pi}\delta^{(2)}(\vec{k}_T)\pi^2;$$

Both Naive Collinear And Soft ME Are Universal!

The TMDPDF is Universal

Evolution of the TMDPDF

- The hadronic tensor is RG scale independent

$$\tilde{M} = H(Q^2/\mu^2)\widetilde{F}_n(x;\vec{b}_\perp,Q,\mu)\widetilde{F}_{\bar{n}}(z;\vec{b}_\perp,Q,\mu)$$

$$\boxed{\frac{d\ln\tilde{M}}{d\ln\mu} = 0 = \gamma_H + \gamma_n + \gamma_{\bar{n}} = \gamma_H + 2\gamma_{\bar{n}} = \gamma_H + 2\gamma_n}$$

$$\gamma_H = A(\alpha_s)\ln\frac{Q^2}{\mu^2} + B(\alpha_s); \quad \widetilde{F}_n(x;\vec{b}_\perp,Q,\mu) = \exp\left[\int_{\mu_I}^{\mu}\frac{d\mu'}{\mu'}\gamma_n\right]\widetilde{F}_n(x;\vec{b}_\perp,Q,\mu_I)$$

$$\boxed{H(Q^2/\mu^2) = |C(Q^2/\mu^2)|^2}$$

Comes from the matching of currents: It is spin independent

The hard coefficient is the same as for inclusive DY!

WE KNOW THE AD of the 8 TMDPDF up to 3-LOOPS

OPE of the TMDPDF onto the PDF

When qT is in the perturbative region the TMDPDF can be factorized in a Wilson coefficient and a PDF like in OPE

$$\widetilde{F}_f(x;\vec{b}_\perp,Q,\mu)=\sum_{j=q,g}\int_x^1\frac{dx'}{x'}\tilde{C}_{f/j}\left(\frac{x}{x'};b,Q,\mu\right)f_{j/P}(x';\mu)$$

The coefficient C works as any other Wilson coefficient
IT IS INDEPENDENT OF IR-SCALES

BUT THERE IS STILL A Q^2 DEPENDENCE

$$\tilde{C}_n(x;b,Q,\mu)=\delta(1-x)+\frac{\alpha_s C_F}{2\pi}\left[-P_{q/q}L_T+(1-x)-\delta(1-x)\left(\frac{1}{2}L_T^2-\frac{3}{2}L_T+\ln\frac{Q^2}{\mu^2}L_T+\frac{\pi^2}{12}\right)\right]$$

THESE TERMS HAVE TO BE RESUMMED!!

$$L_T=\ln\frac{\mu^2 b^2}{4e^{-2\gamma_E}}$$

Q^2-Resummation

Using Lorentz invariance and dimensional analysis

$$\ln \widetilde{F}_n = \ln \widetilde{\widetilde{j}}_n - \frac{1}{2}\ln \widetilde{S}$$

$$\ln \widetilde{\widetilde{j}}_n = R_n\left(x;\alpha_s, L_T, \ln\frac{\Delta}{Q^2}\right), \qquad \ln \widetilde{S} = R_\phi\left(\alpha_s, L_T, \ln\frac{\Delta^2}{Q^2\mu^2}\right)$$

Since the TMDPDF (Wilson coefficients and PDFs) is free from rapidity divergences to all orders in perturbation theory:

$$\frac{d}{d\ln\Delta}\ln\tilde{F}_n = -\mathcal{P}_{q/q}$$

Q^2-Resummation

- From the fact that the TMDPDF is free from rapidity divergencies we can extract and exponentiate the Q^2-dependence.

- But we can also extract it just applying the RGE to the hadronic tensor:

$$\frac{d\ln\tilde{F}_n}{d\ln\mu} = -\frac{1}{2}\gamma_H = -\frac{1}{2}A(\alpha_s)\ln\frac{Q^2}{\mu^2} - \frac{1}{2}B(\alpha_s)$$

$$\ln\tilde{F}_n = \ln\tilde{F}_n^{\not{Q}} - D(\alpha_s, L_T)\ln\frac{Q^2}{\mu^2}$$

Independent of Q^2!!

$$\tilde{F}_n(x,b;Q^2,\mu) = \left(\frac{Q^2}{\mu^2}\right)^{-D(b;\mu)} \tilde{F}_n^{\not{Q}}(x,b;\mu)$$

$$\frac{dD(b;\mu)}{d\ln\mu} = \Gamma_{cusp}(\alpha_s)$$

$$A(\alpha_s) = 2\Gamma_{cusp}$$

- The Q2-factor is extracted for each TMDPDF individually.
- We do not need Collins-Soper evolution equation to resum the logs of Q2.
- We know cusp AD at 3-loops, so we know D at order α^2!!

Q^2-Resummation

The final form of the TMD in IPS is

$$\ln \widetilde{F}_n = \ln \widetilde{F}_n^{sub} - D(\alpha_s, L_T)\left(\ln \frac{Q^2}{\mu^2} + L_T\right)$$

$$\widetilde{F}_n(x; \vec{b}_\perp, Q, \mu) = \left(\frac{Q^2 b^2 e^{2\gamma_E}}{4}\right)^{-D(\alpha_s, L_T)} \widetilde{C}_n(x; \vec{b}_\perp, \mu) \otimes f_n(x; \mu)$$

$$\boxed{\frac{dD(\alpha_s, L_T)}{d \ln \mu} = \Gamma_{\text{cusp}}(\alpha_s)}$$

$$D(\alpha_s, L_T) = \sum_{n=1}^{\infty} d_n(L_T)\left(\frac{\alpha_s}{4\pi}\right)^n$$

$$d_n'(L_\perp) = \frac{1}{2}\Gamma_{n-1} + \sum_{m=1}^{n-1} m\beta_{n-1-m} d_m(L_\perp)$$

The cusp AD is known at 3-loops!!

→ The function D is known up to order α^2

Evolution Kernel

- If we want to connect two TMDPDFs at two different scales:

$$\tilde{F}_n(x,b;Q_f^2) = \tilde{F}_n(x,b;Q_i^2)\,\tilde{R}(b;Q_i,Q_f)$$

$$\tilde{R}(b;Q_i,Q_f) = \left(\frac{Q_f^2}{Q_i^2}\right)^{-D(\alpha_s(Q_i),L_T(Q_i))} \exp\left[\int_{Q_i}^{Q_f}\frac{d\mu'}{\mu'}\gamma_F\left(\alpha_s(\mu'),\ln\frac{Q_f^2}{\mu'^2}\right)\right]$$

- The evolution is given in terms of the function D and the AD

- When we Fourier transform back, we **need** to resum large logs in the D...

$$L_T = \ln\frac{Q^2 b^2}{4e^{-2\gamma_E}}$$

- We propose A NEW METHOD TO RESUM THESE LOGS

D-Resummation

- We are going to write D as a <u>series</u> and resum it directly:

$$\frac{dD(b;\mu)}{d\ln\mu} = \Gamma_{cusp}(\alpha_s)$$

$$D(b;\mu) = \sum_{n=1}^{\infty} d_n(L_\perp)\left(\frac{\alpha_s}{4\pi}\right)^n$$

$$\Gamma_{\rm cusp}(\alpha_s) = \sum_{n=1}^{\infty} \Gamma_{n-1}\left(\frac{\alpha_s}{4\pi}\right)^n$$

$$\beta(\alpha_s) = -2\alpha_s \sum_{n=1}^{\infty} \beta_{n-1}\left(\frac{\alpha_s}{4\pi}\right)^n$$

$$\frac{d}{dL_\perp} d_n(L_\perp) = \frac{1}{2}\Gamma_{n-1} + \sum_{m=1}^{n-1} m\beta_{n-1-m} d_m(L_\perp)$$

Recurrence relation

D-Resummation

$$X = a\beta_0 L_\perp$$

$$X = 1 \rightarrow b_X = \frac{2e^{-\gamma_E}}{Q_i}\exp\frac{2\pi}{\beta_0\alpha_s(Q_i)}$$

In the IR region X~1

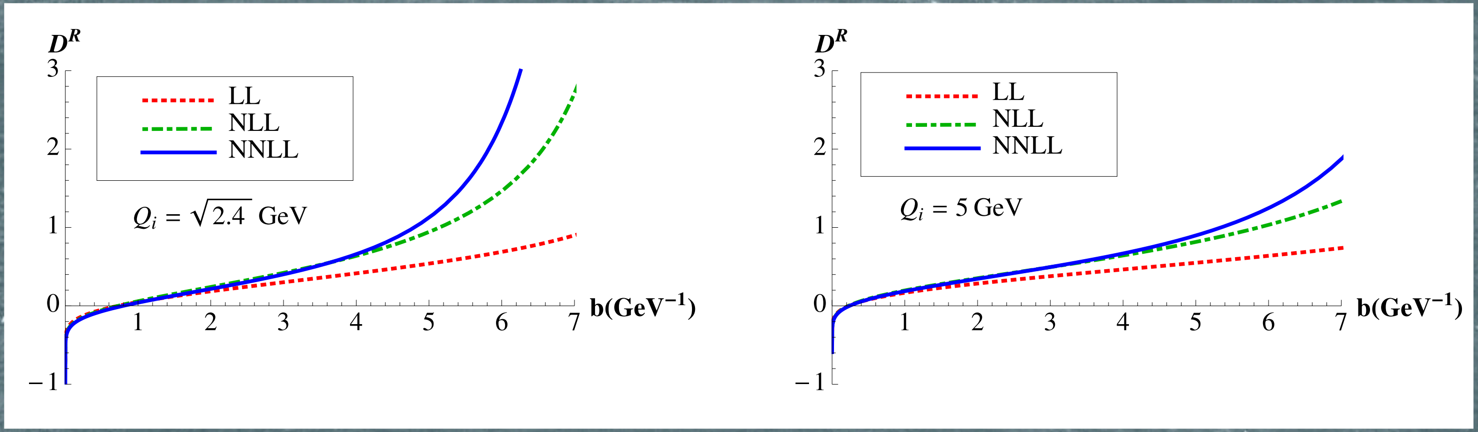
Properties of DR:

- The resummation works for X<1

New expansion!

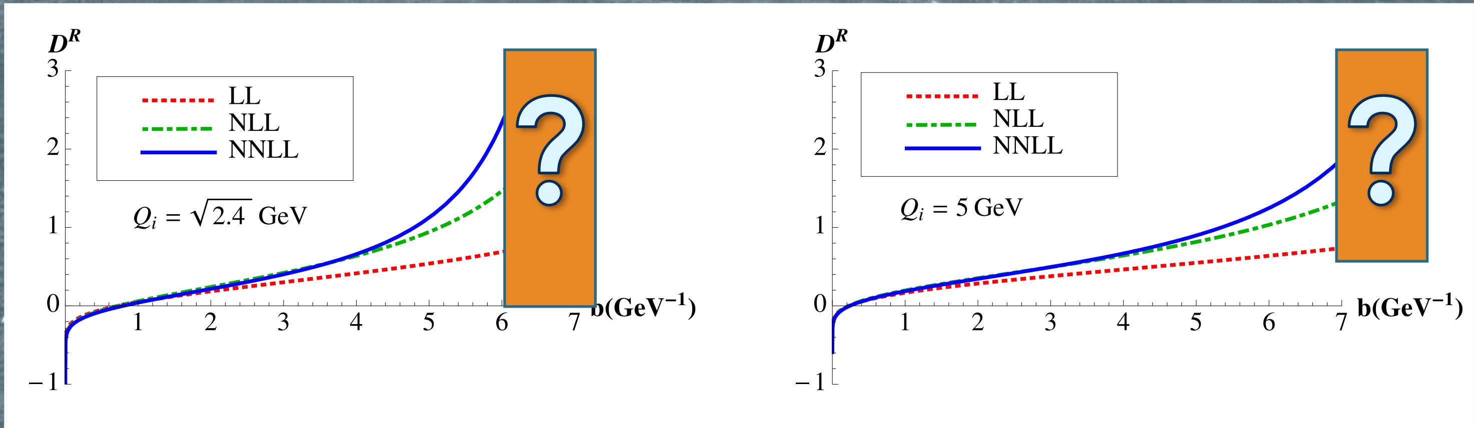
$$D^R(b;\mu_i) = -\frac{\Gamma_0}{2\beta_0}\ln(1-X) + \boxed{\frac{1}{2}\left(\frac{a}{1-X}\right)}\left[-\frac{\beta_1\Gamma_0}{\beta_0^2}(X+\ln(1-X)) + \frac{\Gamma_1}{\beta_0}X\right]$$
$$+\boxed{\frac{1}{2}\left(\frac{a}{1-X}\right)^2}\left[2d_2(0) + \frac{\Gamma_2}{2\beta_0}(X(2-X)) + \frac{\beta_1\Gamma_1}{2\beta_0^2}(X(X-2)-2\ln(1-X)) + \frac{\beta_2\Gamma_0}{2\beta_0^2}X^2\right.$$
$$\left.+\frac{\beta_1^2\Gamma_0}{2\beta_0^3}(\ln^2(1-X)-X^2)\right] + \dots,$$

D-Resummation



The resummed D is convergent almost up to the Landau pole (bL~7).

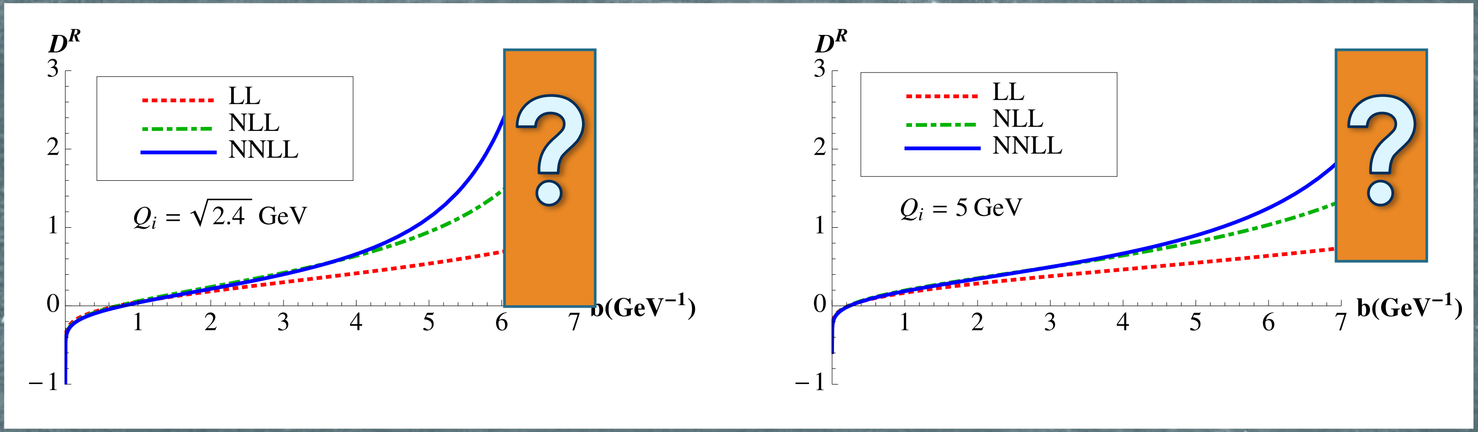
D-Resummation



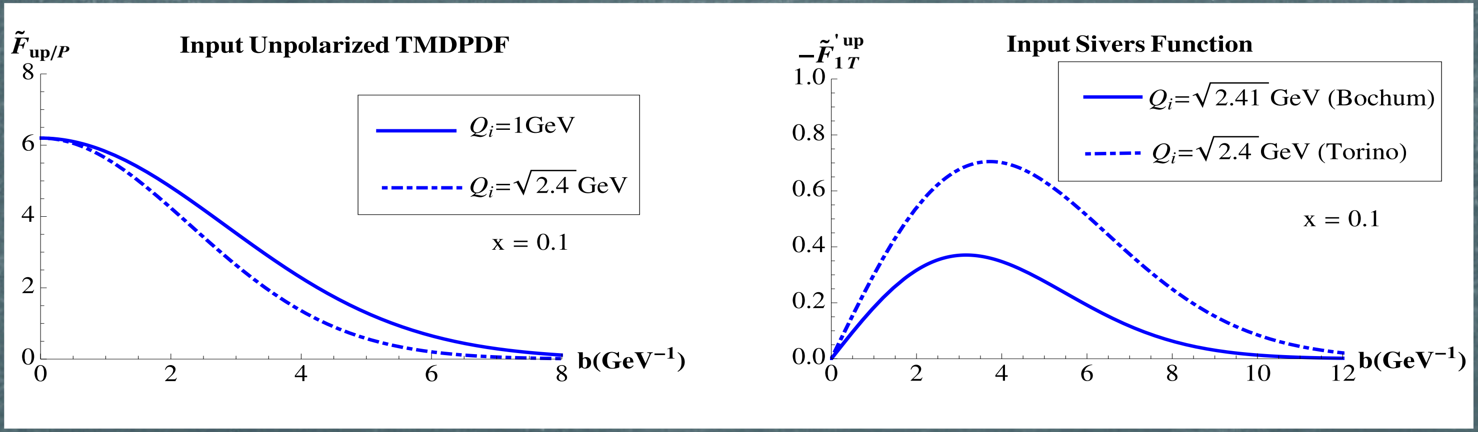
We cannot define the evolution beyond the range of convergence of the resummed series, nor beyond the Landau pole.

As a result we cut the evolution kernel where the resummation fails.

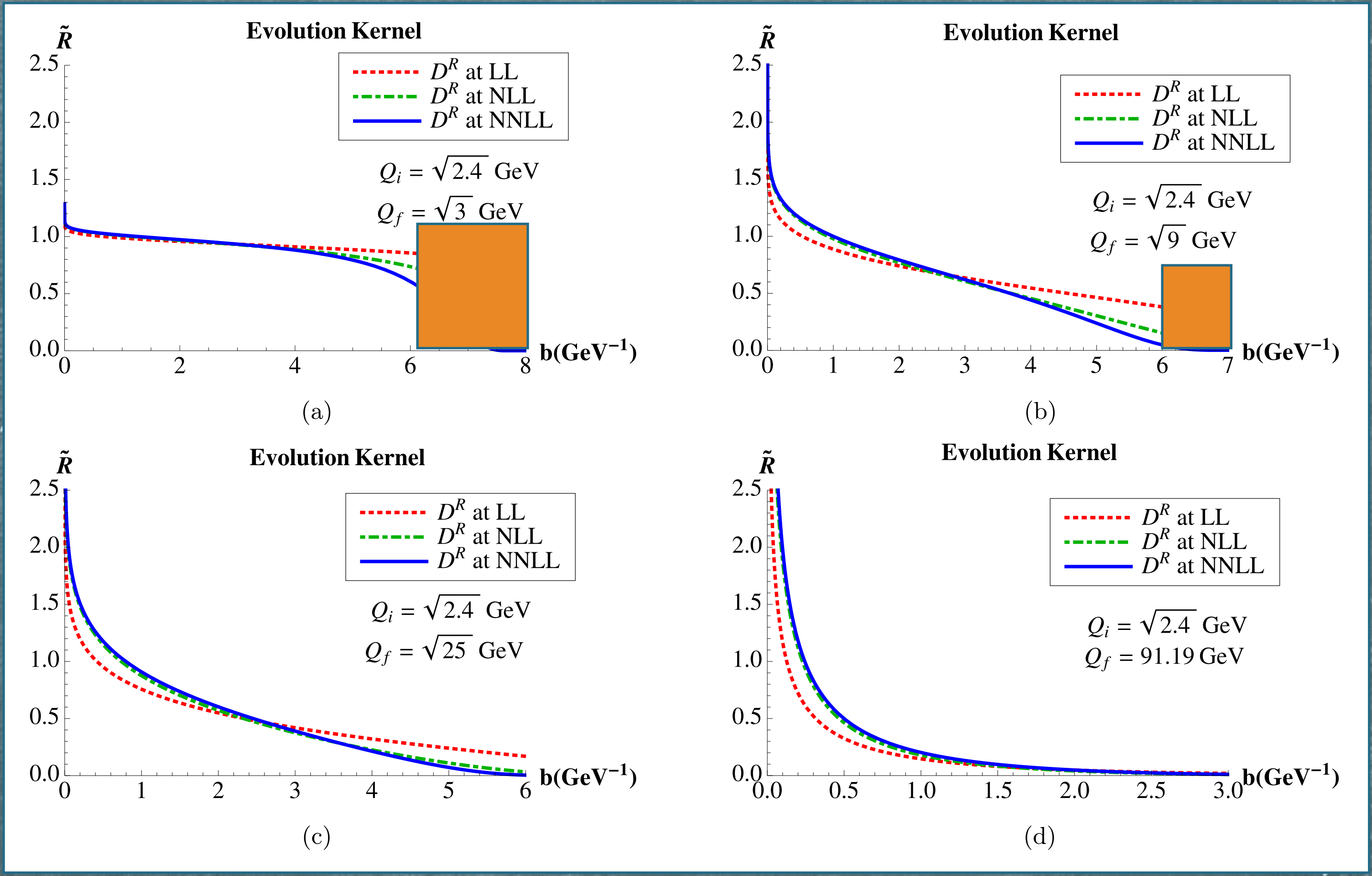
D-Resummation



The input TMDs turn out to be small in the tail, so no effect from the cut.
The highers is Qi the narrower is the TMD in IPS



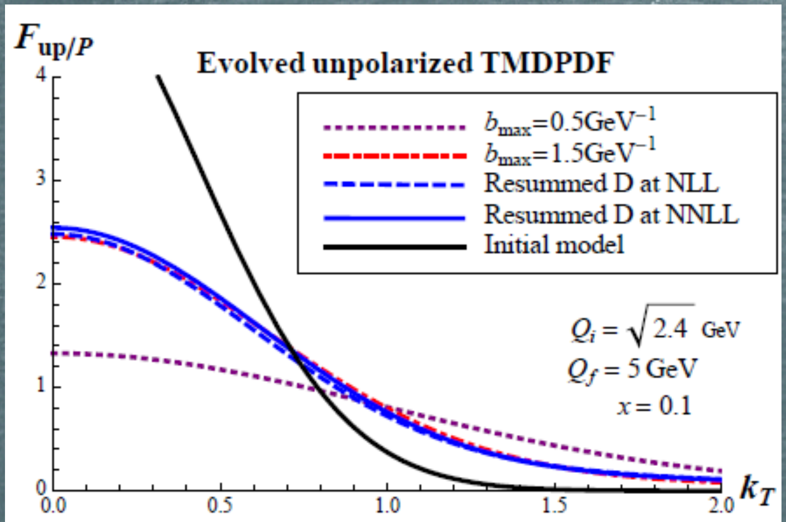
Results: Evol. Kernel at LL, NLL, NNLL

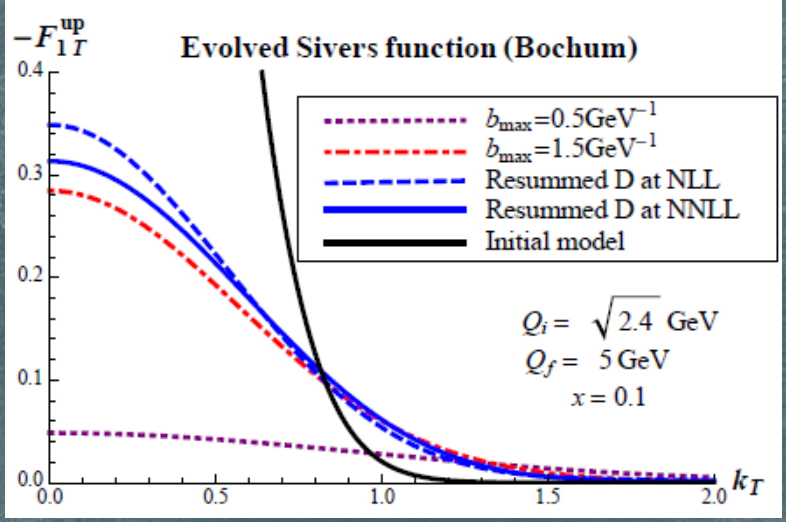


The evolution kernel so defined is completely parameter free and model independent

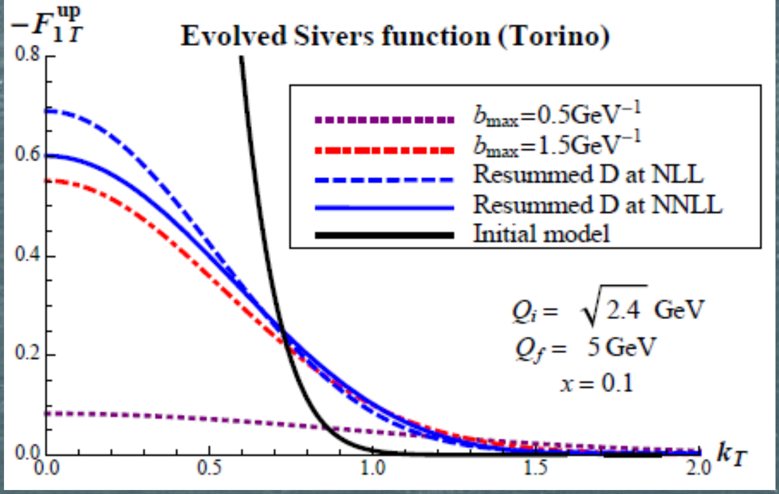
Results

We compare with CSS and
bmax=0.5, Collins ideal
bmax=1.5, fitted from
Phenomenology
(Konychev, Nadolsky'06)





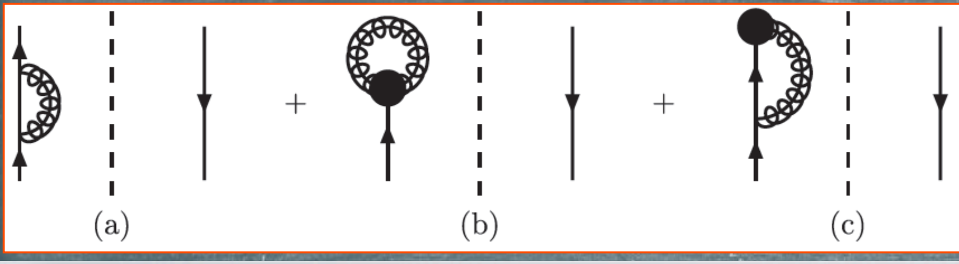
The value of bmax~1.5 is a by-product in our resummation

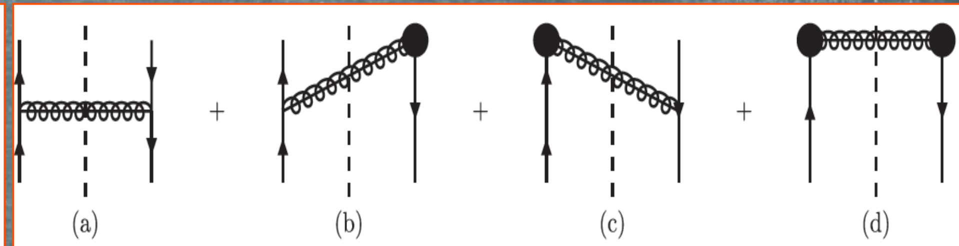


CONCLUSIONS

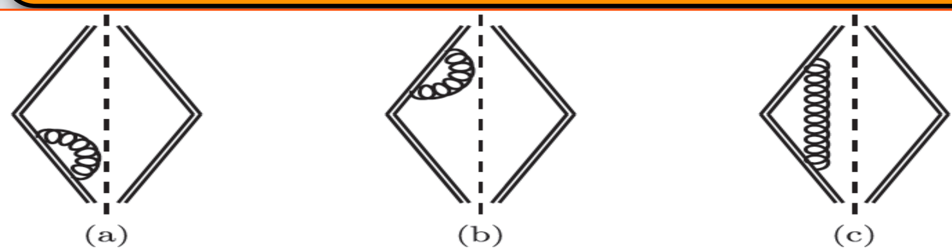
- ✓ We have a formulation of factorization on-the-light-cone (no parameters on any matching coefficient!)
- ✓ We can relate the AD of the hard matching coefficient to the AD of the TMDPD's ➡ WE KNOW THE EVOLUTION OF ALL TMDPDF UP TO NNLL
- ✓ We can build an evolutor for TMDPDF treating consistently the problem of the Landau pole in a model independent way (we predict bmax=1.5 in the usual CSS evolution)
- ✓ Unpolarized TMDs and Sivers under study now
- ✓ Effects of TMDs on vector boson production and Higgs production are under investigation (including ϕ^* distribution at LHC)

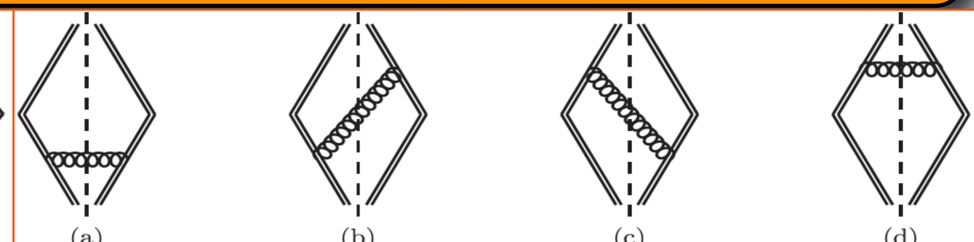
BACKUP SLIDES





$$\tilde{J}_{n1}^{(0)}(\Delta^-) = \frac{\alpha_s C_F}{2\pi}\left\{\delta(1-x)\left[\frac{2}{\varepsilon_{\mathrm{UV}}^2} - \frac{2}{\varepsilon_{\mathrm{UV}}}\ln\frac{\Delta^-}{\mu^2} + \frac{3}{2\varepsilon_{\mathrm{UV}}} - \frac{1}{4} - \frac{2\pi^2}{12} - L_T^2 + \frac{3}{2}L_T - 2L_T\ln\frac{\Delta^-}{\mu^2}\right] - (1-x)\ln(1-x) - \mathcal{P}_{q/q}\ln\frac{\Delta^-}{\mu^2} - L_T\mathcal{P}_{q/q}\right\}$$





$$\tilde{S}_1(\Delta^+,\Delta^-) = \frac{\alpha_s C_F}{2\pi}\left[-\frac{2}{\varepsilon_{\mathrm{UV}}^2} + \frac{2}{\varepsilon_{\mathrm{UV}}}\ln\frac{\Delta^+\Delta^-}{\mu^2 Q^2} + L_T^2 + 2L_T\ln\frac{\Delta^+\Delta^-}{\mu^2 Q^2} + \frac{\pi^2}{6}\right]$$
$$= \frac{1}{2}\left[\tilde{S}_1(\Delta^+,\Delta^+) + \tilde{S}_1(\Delta^-,\Delta^-)\right]$$

Resumming! (Unpolarized case)

$$\tilde{F}_{f/P}(x;\vec{b}_\perp,Q^2,\mu=Q)=\sum_{j=q,g}\exp\left[\int_{\mu_I}^{\mu}\frac{d\mu'}{\mu'}\gamma_n\right]\left(\frac{Q^2}{\mu^2}\right)^{-D(b,\mu_I)}\tilde{C}_{f/j}\left(x;\vec{b}_\perp,\mu_I\right)\otimes f_{j/P}(x;\mu_I)$$

Order	γ	Γcusp	C	D
LL	-	α	tree	-
NLL	α	α^2	tree	α
NNLL	α^2	α^3	α	α^2
NNNLL	α^3	α^4	α^2	α^3

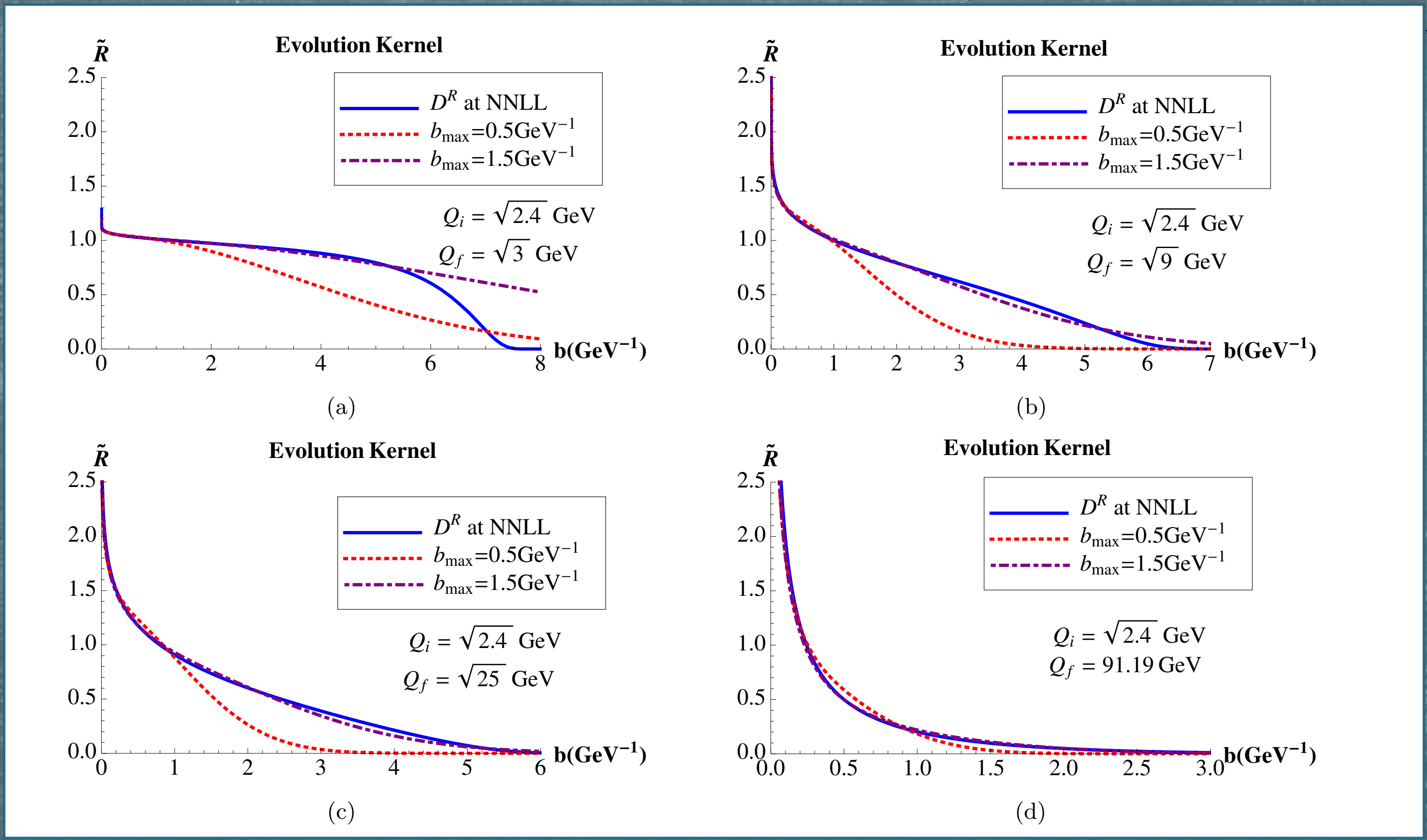
Aybat, Collins , Qiu, Rogers; Aybat, Rogers; Anselmino, Boglione,Melis

<u>EIS</u>

Known pieces: C for unpolarized TMDs from Catani et al. ' 12 And Gehrmann et al. '12

Polarized case: Some new results from Bacchetta, Prokudin '12,
and work in progress by EIS

Results: EISS vs CSS



CSS: The evolution is modeled with a bmax and a gaussian. In this way it is defined also BEYOND the Landau pole