Reaching NNLOPS accuracy with POWHEG and MiNLO

Emanuele Re*
Rudolf Peierls Centre for Theoretical Physics, University of Oxford

LC13 Workshop

ECT*, Trento, 19 September 2013

*based mainly on [1206.3572, 1212.4504, 1309.0017]

Outline: “explain the acronyms in the title”

- POWHEG in a nutshell
- MiNLO: Multiscale Improved NLO

  [1206.3572]
- NLOPS merging of X @ NLO and $X + 1j$ @ NLO

  [1212.4504]
- NNLOPS simulation of Higgs production

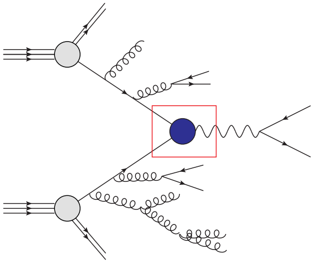
  [1309.0017]

NLOPS: intro

- NLO is important: first order where rates are reliable, and possible to attach sensible theoretical uncertainties
- In some phase-space regions, NLO accuracy is not enough:
  $\hookrightarrow$ need to resum to all orders the dominant terms
- A parton shower is an algorithm to resum (some classes of) collinear/soft logs in a "fully-exclusive" way

- NLO is important: first order where rates are reliable, and possible to attach sensible theoretical uncertainties
- In some phase-space regions, NLO accuracy is not enough:
  $\hookrightarrow$ need to resum to all orders the dominant terms
- A parton shower is an algorithm to resum (some classes of) collinear/soft logs in a "fully-exclusive" way

$$d\sigma_{\rm SMC} = B(\Phi_n)\, d\Phi_n \left[\Delta(t_{\max}, t_0) + \Delta(t_{\max}, t) \underbrace{\frac{\alpha_s}{2\pi}\frac{1}{t} P(z)\, d\Phi_r}_{\rm coll.approximation}\right]$$



$$\Delta(t_{\max}, t) = \exp\left\{-\int_t^{t_{\max}} d\Phi'_r\, \frac{\alpha_s}{2\pi}\frac{1}{t'} P(z')\right\}$$

This is LOPS

Idea: *Modify $d\sigma_{\rm SMC}$ in such a way that*, expanding in $\alpha_{\rm S}$, *one recovers the NLO cross section*.

$$B(\Phi_n) \quad \Rightarrow \quad \bar{B}(\Phi_n) = B(\Phi_n) + \frac{\alpha_s}{2\pi}\left[V(\Phi_n) + \int R(\Phi_{n+1})\, d\Phi_r\right]$$

$$\Delta(t_{\rm m}, t) \quad \Rightarrow \quad \Delta(\Phi_n; k_{\rm T}) = \exp\left\{-\frac{\alpha_s}{2\pi}\int \frac{R(\Phi_n, \Phi_r')}{B(\Phi_n)}\theta(k_{\rm T}' - k_{\rm T})\, d\Phi_r'\right\}$$

POWHEG "master formula" for the hardest emission:

$$d\sigma_{\rm POW} = d\Phi_n\ \bar{B}(\Phi_n)\left\{\Delta(\Phi_n; k_{\rm T}^{\rm min}) + \Delta(\Phi_n; k_{\rm T})\frac{\alpha_s}{2\pi}\frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)}\, d\Phi_r\right\}$$

+ $p_{\rm T}$*-vetoing subsequent emissions*, to avoid double-counting.

Idea: *Modify $d\sigma_{\rm SMC}$ in such a way that*, expanding in $\alpha_{\rm S}$, *one recovers the NLO cross section*.

$$B(\Phi_n) \quad \Rightarrow \quad \bar{B}(\Phi_n) = B(\Phi_n) + \frac{\alpha_s}{2\pi}\left[V(\Phi_n) + \int R(\Phi_{n+1})\, d\Phi_r\right]$$

$$\Delta(t_{\rm m}, t) \quad \Rightarrow \quad \Delta(\Phi_n; k_{\rm T}) = \exp\left\{-\frac{\alpha_s}{2\pi}\int \frac{R(\Phi_n, \Phi'_r)}{B(\Phi_n)}\theta(k'_{\rm T} - k_{\rm T})\, d\Phi'_r\right\}$$

POWHEG "master formula" for the hardest emission:

$$d\sigma_{\rm POW} = d\Phi_n\ \bar{B}(\Phi_n)\left\{\Delta(\Phi_n; k_{\rm T}^{\rm min}) + \Delta(\Phi_n; k_{\rm T})\frac{\alpha_s}{2\pi}\frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)}\ d\Phi_r\right\}$$

+ $p_{\rm T}$*-vetoing subsequent emissions*, to avoid double-counting.

- inclusive observables: @NLO
- first hard emission: full tree level ME
- (N)LL resummation of collinear/soft logs

This is NLOPS

Idea: *Modify $d\sigma_{\rm SMC}$ in such a way that*, expanding in $\alpha_{\rm S}$, *one recovers the NLO cross section*.

$$B(\Phi_n) \quad \Rightarrow \quad \bar{B}(\Phi_n) = B(\Phi_n) + \frac{\alpha_s}{2\pi}\left[V(\Phi_n) + \int R(\Phi_{n+1})\, d\Phi_r\right]$$

$$\Delta(t_{\rm m}, t) \quad \Rightarrow \quad \Delta(\Phi_n; k_{\rm T}) = \exp\left\{-\frac{\alpha_s}{2\pi}\int \frac{R(\Phi_n, \Phi'_r)}{B(\Phi_n)}\theta(k'_{\rm T} - k_{\rm T})\, d\Phi'_r\right\}$$

POWHEG "master formula" for the hardest emission:

$$d\sigma_{\rm POW} = d\Phi_n\ \bar{B}(\Phi_n)\left\{\Delta(\Phi_n; k_{\rm T}^{\rm min}) + \Delta(\Phi_n; k_{\rm T})\frac{\alpha_s}{2\pi}\frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)}\, d\Phi_r\right\}$$

+ $p_{\rm T}$*-vetoing subsequent emissions*, to avoid double-counting.

- inclusive observables: @NLO
- first hard emission: full tree level ME
- (N)LL resummation of collinear/soft logs

This is NLOPS

☞ Notice: when doing $X+$ jet(s) @ NLO, $\bar{B}(\Phi_n)$ is not finite !
↪ need of a generation cut on Φ_n (or variants thereof)
↪ POWHEG for $H+1$ jet cannot be used for inclusive Higgs production

MiNLO: intro

MiNLO: Multiscale Improved NLO

- goal: method to a-priori choose scales in NLO computation
- relevant for processes with widely different scales (*e.g.* $X+$ jets close to Sudakov regions)

How?

- At LO, the CKKW procedure allows to take these effects into account: modify the LO weight $B(\mathbf{\Phi}_n)$ in order to include (N)LL effects.

$\Rightarrow$ "Use CKKW" on top of NLO computation that potentially involves many scales

---

Next-to-Leading Order accuracy needs to be preserved

MiNLO: Multiscale Improved NLO

- goal: method to a-priori choose scales in NLO computation
- relevant for processes with widely different scales (*e.g.* $X+$ jets close to Sudakov regions)

How?

- At LO, the CKKW procedure allows to take these effects into account: modify the LO weight $B(\mathbf{\Phi}_n)$ in order to include (N)LL effects.

> $\Rightarrow$ "Use CKKW" on top of NLO computation that potentially involves many scales

---

Next-to-Leading Order accuracy needs to be preserved

1. Scale dependence shows up at NNLO ["scale compensation"]:

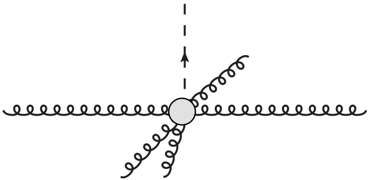
$$O(\mu') - O(\mu) = \mathcal{O}(\alpha_S^{n+2}) \quad \text{if} \quad O \sim \alpha_S^n \quad \text{at LO}$$

2. Away from soft-collinear regions, exact NLO recovered:

$$O_{\mathrm{MiNLO}} = O_{\mathrm{NLO}} + \mathcal{O}(\alpha_S^{n+2}) \qquad [\, i.e.\ \alpha_S^n \ \& \ \alpha_S^{n+1} \text{ as in NLO} \,]$$

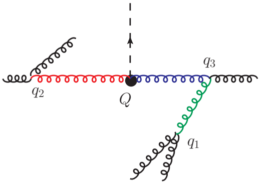
From CKKW to MiNLO

- Find “most-likely” shower history (via k_T-algo): $Q > q_3 > q_2 > q_1 \equiv Q_0$



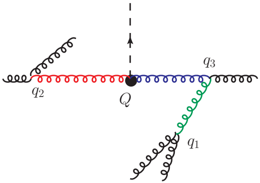
From CKKW to MiNLO

- Find “most-likely” shower history (via k_T-algo): $Q > q_3 > q_2 > q_1 \equiv Q_0$



From CKKW to MiNLO

- Find "most-likely" shower history (via k_T-algo): $Q > q_3 > q_2 > q_1 \equiv Q_0$



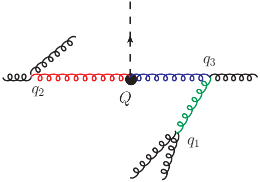
- Evaluate α_S at nodal scales

$$\alpha_S^n(\mu_R)B(\mathbf{\Phi}_n) \Rightarrow \alpha_S(q_1)\alpha_S(q_2)...\alpha_S(q_n)B(\mathbf{\Phi}_n)$$

* scale compensation requires $\bar{\mu}_R^2 = (q_1 q_2 ... q_n)^{2/n}$ in V

From CKKW to MiNLO

- Find "most-likely" shower history (via k_T-algo): $Q > q_3 > q_2 > q_1 \equiv Q_0$



- Evaluate α_S at nodal scales

$$\alpha_S^n(\mu_R)B(\mathbf{\Phi}_n) \Rightarrow \alpha_S(q_1)\alpha_S(q_2)...\alpha_S(q_n)B(\mathbf{\Phi}_n)$$

* scale compensation requires $\bar{\mu}_R^2 = (q_1 q_2 ... q_n)^{2/n}$ in V

- Sudakov FFs in internal and external lines of Born "skeleton"

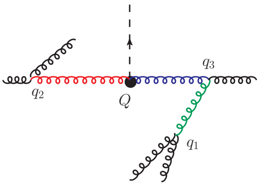
$$B(\mathbf{\Phi}_n) \Rightarrow B(\mathbf{\Phi}_n) \times \{\Delta(Q_0, Q)\Delta(Q_0, q_i)...\}$$

* Upon expansion, $\mathcal{O}(\alpha_S^{n+1})$ (log) terms are introduced, and need to be removed

$$B(\mathbf{\Phi}_n) \Rightarrow B(\mathbf{\Phi}_n)\Big(1 - \Delta^{(1)}(Q_0, Q) - \Delta^{(1)}(Q_0, q_i) + ...\Big)$$

From CKKW to MiNLO

- Find "most-likely" shower history (via k_T-algo): $Q > q_3 > q_2 > q_1 \equiv Q_0$



- Evaluate α_S at nodal scales

$$\alpha_S^n(\mu_R)B(\mathbf{\Phi}_n) \Rightarrow \alpha_S(q_1)\alpha_S(q_2)...\alpha_S(q_n)B(\mathbf{\Phi}_n)$$

* scale compensation requires $\bar{\mu}_R^2 = (q_1 q_2 ... q_n)^{2/n}$ in V

- Sudakov FFs in internal and external lines of Born "skeleton"

$$B(\mathbf{\Phi}_n) \Rightarrow B(\mathbf{\Phi}_n) \times \{\Delta(Q_0, Q)\Delta(Q_0, q_i)...\}$$

* Upon expansion, $\mathcal{O}(\alpha_S^{n+1})$ (log) terms are introduced, and need to be removed

$$B(\mathbf{\Phi}_n) \Rightarrow B(\mathbf{\Phi}_n)\left(1 - \Delta^{(1)}(Q_0, Q) - \Delta^{(1)}(Q_0, q_i) + ...\right)$$

☞ $X+$ jets cross-section finite without generation cuts
$\hookrightarrow$ $\bar{B}$ with MiNLO prescription: ideal starting point for NLOPS (POWHEG) for $X+$ jets

MiNLO: example

Example, in 1 line: $H+1$ jet

- Pure NLO:

$$d\sigma = \bar{B}\, d\mathbf{\Phi}_n = \alpha_{\mathrm{S}}^3(\mu_R)\Big[B + \alpha_{\mathrm{S}} V(\mu_R) + \alpha_{\mathrm{S}} \int d\Phi_{\mathrm{rad}} R\Big]\, d\mathbf{\Phi}_n$$

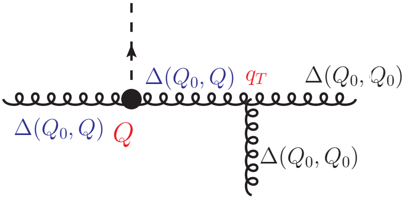
Example, in 1 line: $H+1$ jet

- Pure NLO:

$$d\sigma = \bar{B}\, d\boldsymbol{\Phi}_n = \alpha_S^3(\mu_R)\Big[B + \alpha_S V(\mu_R) + \alpha_S \int d\Phi_{\rm rad} R\Big]\, d\boldsymbol{\Phi}_n$$

- MiNLO:

$$\bar{B} = \alpha_S^2(M_H)\alpha_S(q_T)\Delta_g^2(q_T, M_H)\Big[B\Big(1 - 2\Delta_g^{(1)}(q_T, M_H)\Big) + \alpha_S V(\bar{\mu}_R) + \alpha_S \int d\Phi_{\rm rad} R\Big]$$



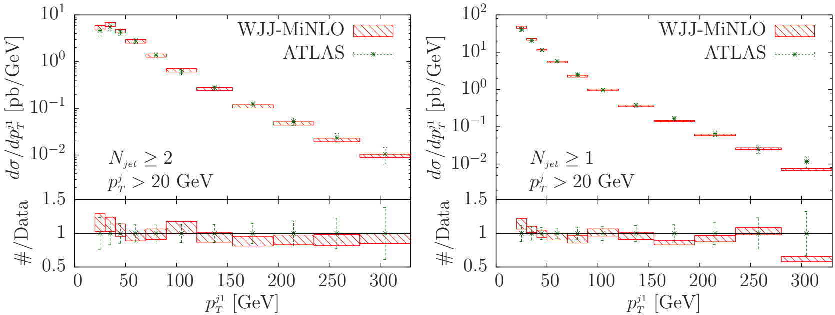
* $\bar{\mu}_R = (M_H^2 q_T)^{1/3}$

* $\log \Delta_{\rm f}(q_T, Q) = -\int_{q_T^2}^{Q^2} \frac{dq^2}{q^2}\frac{\alpha_S(q^2)}{2\pi}\Big[A_f \log\frac{Q^2}{q^2} + B_f\Big]$

* $\Delta_{\rm f}^{(1)}(q_T, Q) = -\frac{\alpha_S}{2\pi}\Big[\frac{1}{2}A_{1,\rm f}\log^2\frac{Q^2}{q_T^2} + B_{1,\rm f}\log\frac{Q^2}{q_T^2}\Big]$

* $\mu_F = Q_0 (= q_T)$

VJJ−MiNLO [Campbell,Ellis,Nason,Zanderighi, 1303.5447]



☞ Start from $W + 2$ jets @ NLO, good agreement with data also when requiring $N_{\mathrm{jet}} \geq 1$!

Improved MiNLO (NLOPS merging)

- Accuracy of BJ+MiNLO for inclusive observables carefully investigated
- BJ+MiNLO describes inclusive boson observables at relative order α_S wrt $B + 0j$ at LO
- However, to reach genuine NLO, higher terms must be order α_S^2, *i.e.*

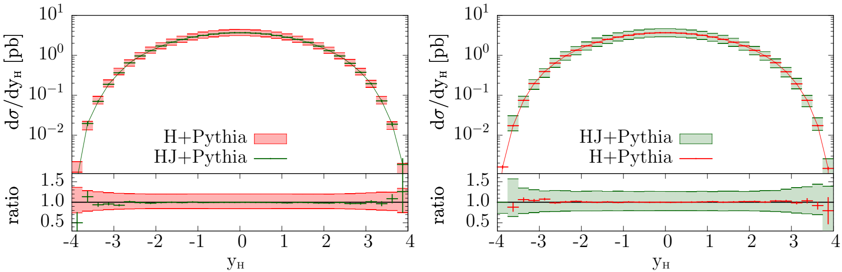
$$O_{\mathrm{VJ+MiNLO}} = O_{\mathrm{V@NLO}} + \mathcal{O}(\alpha_S^2)$$

  if O is inclusive. "Original MiNLO" contains ambiguous $\mathcal{O}(\alpha_S^{3/2})$ terms
- Possible to improve BJ+MiNLO such that NLO $B + 0j$ is recovered, without spoiling NLO accuracy for $B + 1j$.
  - proof based on careful comparisons of general resummation formula with MiNLO ingredients
  - need to include B_2 in Sudakovs
  - need to evaluate $\alpha_S{}^{(\mathrm{NLO})}$ in BJ+MiNLO at scale q_T, and $\mu_F = q_T$

Effectively it is like if we merged NLO$^{(0)}$ and NLO$^{(1)}$ samples, without merging different samples (no merging scale used).

Other NLOPS-merging approaches: [Hoeche,Krauss, et al.,1207.5030] [Frederix,Frixione,1209.6215] [Lonnblad,Prestel,1211.7278 - Platzer,1211.5467] [Alioli,Bauer, et al.,1211.7049]

MiNLO merging: results 1

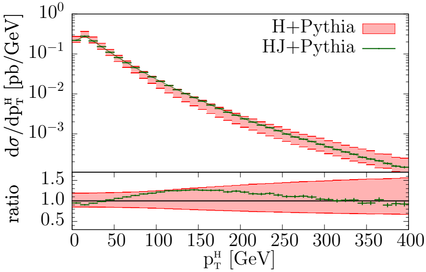


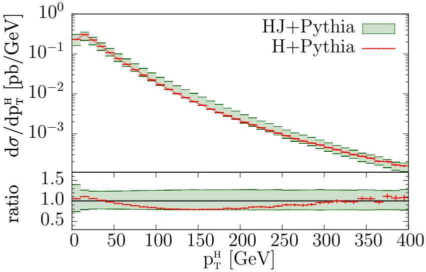
- “H+Pythia”: standalone POWHEG ($gg \to H$) + PYTHIA (PS level) [7pts band, $\mu = m_H$]
- “HJ+Pythia”: HJ−MiNLO* + PYTHIA (PS level) [7pts band, μ from MiNLO]

✓ very good agreement (both value and band)

☞ Notice: band is $\sim 20 - 30\%$

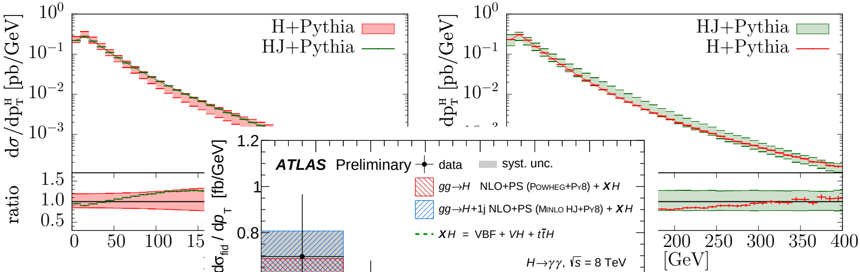
MiNLO merging: results 2

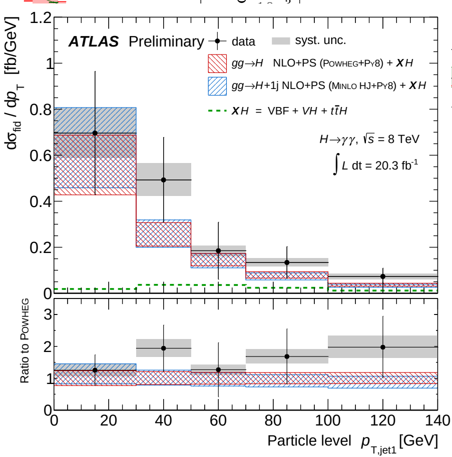




- Good agreement
- At high p_T, bands as expected (LO vs NLO)
  (POWHEG $(gg \to H)$ with $\texttt{hfact} = m_H/1.2$, YR2)

MiNLO merging: results 2





- Good agreement
- At high p_T, bands (POWHEG ($gg \to I$

- HJ−MiNLO* differential cross section $(d\sigma/dy)_{\mathrm{HJ-MiNLO}}$ is NLO accurate

$$W(y) = \frac{\left(\frac{d\sigma}{dy}\right)_{\mathrm{NNLO}}}{\left(\frac{d\sigma}{dy}\right)_{\mathrm{HJ-MiNLO}}} = \frac{c_2\alpha_S^2 + c_3\alpha_S^3 + c_4\alpha_S^4}{c_2\alpha_S^2 + c_3\alpha_S^3 + d_4\alpha_S^4} \simeq 1 + \frac{c_4 - d_4}{c_2}\alpha_S^2 + \mathcal{O}(\alpha_S^3)$$

- thus, reweighting each event with this factor, we get NNLO+PS
  * obvious for y_H, by construction
  * α_S^4 accuracy of HJ−MiNLO* in 1-jet region not spoiled, because $W(y) = 1 + \mathcal{O}(\alpha_S^2)$
  * if we had NLO$^{(0)}$ $+ \alpha_S^{3/2}$, 1-jet region spoiled because

$$[\text{NLO}^{(1)}]_{\text{NNLOPS}} = \text{NLO}^{(1)} + \mathcal{O}(\alpha_S^{4.5})$$

NNLO+PS

- HJ−MiNLO* differential cross section $(d\sigma/dy)_{\mathrm{HJ-MiNLO}}$ is NLO accurate

$$W(y) = \frac{\left(\frac{d\sigma}{dy}\right)_{\mathrm{NNLO}}}{\left(\frac{d\sigma}{dy}\right)_{\mathrm{HJ-MiNLO}}} = \frac{c_2\alpha_S^2 + c_3\alpha_S^3 + c_4\alpha_S^4}{c_2\alpha_S^2 + c_3\alpha_S^3 + d_4\alpha_S^4} \simeq 1 + \frac{c_4 - d_4}{c_2}\alpha_S^2 + \mathcal{O}(\alpha_S^3)$$

- thus, reweighting each event with this factor, we get NNLO+PS
  - * obvious for y_H, by construction
  - * α_S^4 accuracy of HJ−MiNLO* in 1-jet region not spoiled, because $W(y) = 1 + \mathcal{O}(\alpha_S^2)$
  - * if we had NLO$^{(0)}$ $+ \alpha_S^{3/2}$, 1-jet region spoiled because

$$[\mathrm{NLO}^{(1)}]_{\mathrm{NNLOPS}} = \mathrm{NLO}^{(1)} + \mathcal{O}(\alpha_S^{4.5})$$

---

* Variants for W are possible: with

$$W(y, p_T) = h(p_T)\frac{\int d\sigma^{\mathrm{NNLO}}\delta(y - y(\mathbf{\Phi})) - \int d\sigma_B^{\mathrm{MiNLO}}\delta(y - y(\mathbf{\Phi}))}{\int d\sigma_A^{\mathrm{MiNLO}}\delta(y - y(\mathbf{\Phi}))} + (1 - h(p_T))$$

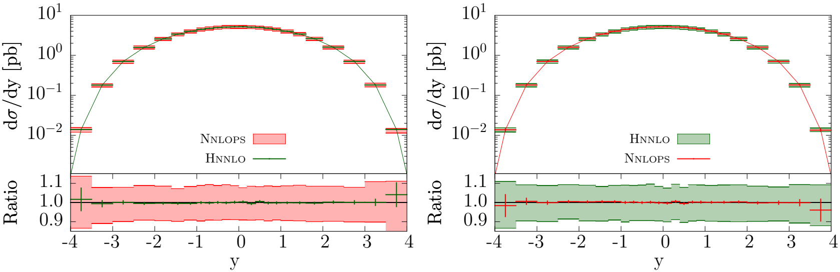
$$d\sigma_A = d\sigma\; h(p_T), \qquad d\sigma_B = d\sigma\;(1 - h(p_T)), \qquad h = \frac{(\beta m_H)^2}{(\beta m_H)^2 + p_T^2}$$

we get exactly $(d\sigma/dy)_{\mathrm{NNLOPS}} = (d\sigma/dy)_{\mathrm{NNLO}}$ (no α_S^5 terms)

* h essentially controls where the NNLO/NLO K-factor is spread.

NNLO+PS (fully incl.)

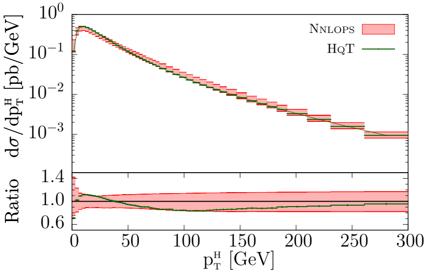
- NNLO with $\mu = m_H/2$, HJ-MiNLO "core scale" m_H [NNLO from HNNLO, Catani,Grazzini]
- events reweighted at the LH level, then showered with PYTHIA (PS level)
- (7×3) pts scale var. in NNLOPS, 7pts in NNLO



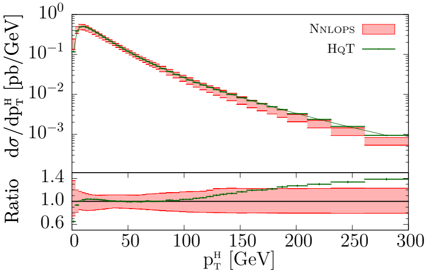
☞ Notice: band is 10%

[Until and including $\mathcal{O}(\alpha_S^4)$, PS effects don't affect y_H (first 2 emissions controlled properly at $\mathcal{O}(\alpha_S^4)$ by MiNLO+POWHEG)]

$\beta = \infty$ (W indep. of p_T)

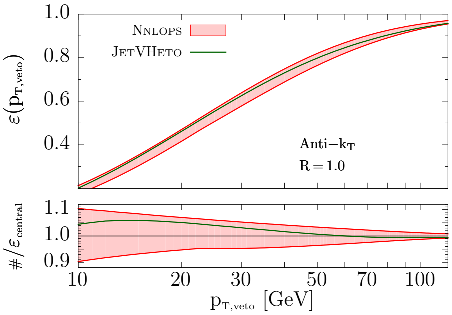


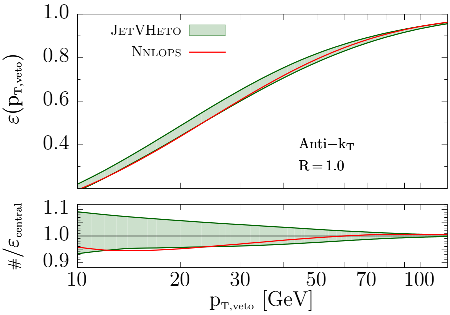
$\beta = 1/2$



- HqT: NNLL+NNLO, $\mu_R = \mu_F = m_H/2$ [7pts], $Q_{\rm res} \equiv m_H/2$
- $\beta = 1/2$ & ∞: uncertainty bands of HqT (not shown) contain NNLOPS at low-/moderate p_T
- $\beta = 1/2$: NNLOPS tail $\rightarrow$ NLOPS tail [$W(y, p_T \gg m_H) \rightarrow 1$]
  larger band (affected just marginally by NNLO, so it's $\sim$ genuine NLO band)
- $\beta = 1/2$: HqT tail harder than NNLOPS tail ($\mu_{\rm HqT} <$ "$\mu_{\rm MiNLO}$")
- $\beta = 1/2$: very good agreement with HqT resummation

NNLO+PS ($p_T^{j_1}$)





$$\varepsilon\left(p_{\mathrm{T,veto}}\right) = \frac{\Sigma(p_{\mathrm{T,veto}})}{\sigma^{\mathrm{tot}}} = \frac{1}{\sigma^{\mathrm{tot}}} \int d\sigma\, \theta\left(p_{\mathrm{T,veto}} - p_{\mathrm{T}}^{\mathrm{j_1}}\right)$$

- `JetVHeto`: NNLL resum, $\mu_R = \mu_F = m_H/2$ [7pts], $Q_{\mathrm{res}} \equiv m_H/2$, (a)-scheme only
- nice agreement, differences never more than 5-6 %

☞ Separation of $H \to WW$ from $t\bar{t}$ bkg: x-sec binned in N_{jet}
0-jet bin ⇔ jet-veto accurate predictions needed !

Conclusions

1. MiNLO:
   - assign scales and Sudakov FF in $B + n$ jets NLO computations
   - well-behaved in Sudakov regions
   - NLO away from Sudakov regions
   - ideal as starting point for POWHEG
2. Improved MiNLO:
   - $B + 1$ jet improved MiNLO allows to merge NLO$^{(0)}$ and NLO$^{(1)}$ samples, without merging (no merging scale used)
   - merging for higher multiplicity requires further study, it'll take some time
3. NNLOPS:
   - MiNLO allows to define a procedure to reach NNLOPS
   - Shown results for Higgs production

Conclusions

1. MiNLO:
   - assign scales and Sudakov FF in $B + n$ jets NLO computations
   - well-behaved in Sudakov regions
   - NLO away from Sudakov regions
   - ideal as starting point for `POWHEG`
2. Improved MiNLO:
   - $B + 1$ jet improved MiNLO allows to merge NLO$^{(0)}$ and NLO$^{(1)}$ samples, without merging (no merging scale used)
   - merging for higher multiplicity requires further study, it'll take some time
3. NNLOPS:
   - MiNLO allows to define a procedure to reach NNLOPS
   - Shown results for Higgs production

Thanks for your attention!

POWHEG BOX: news and technical improvements

Number of processes still increasing (~ 30)

- Drell-Yan with QED/EW and QCD effects available

Automation:

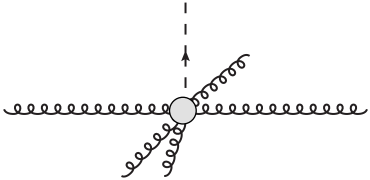
- Interface to `MadGraph 4` [Frederix]: automatically builds subprocesses list, B, B_{ij}, $B^{\mu\nu}$, R and large-N Born color structures.
  - Used to build the code for Hj and Hjj [Campbell, Ellis, Frederix, Nason, Oleari], with virtuals from MCFM.
- interface to `GoSam` [Luisoni, Nason, Oleari, Tramontano]: automatically write the code for 1-loop amplitudes, and interface it via BLHA - Used to study VH and VHj

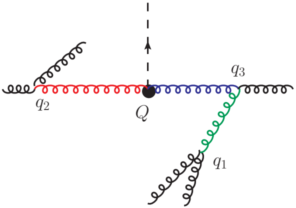
PDF and scale uncertainties:

- Generate MC samples for different scale choices, and, even more, for different PDFs, is very time consuming
- Primitive reweighting facility now superseded by new mechanism [Hamilton,Nason,ER]

V2 ready and under testing; MiNLO will also be the default for $X+$jets processes.

- Start from ME weight: $B(\mathbf{\Phi}_n)$
- Find "most-likely" shower history (via k_T-algo): $Q > q_3 > q_2 > q_1 \equiv Q_0$





- New weight:

$$\alpha_S^5(Q)B(\mathbf{\Phi}_3) \quad \rightarrow \quad \alpha_S^2(Q)B(\mathbf{\Phi}_3)\frac{\Delta_g(Q_0,Q)}{\Delta_g(Q_0,q_2)}\frac{\Delta_g(Q_0,Q)}{\Delta_g(Q_0,q_3)}\frac{\Delta_g(Q_0,q_3)}{\Delta_g(Q_0,q_1)}$$
$$\Delta_g(Q_0,q_2)\Delta_g(Q_0,q_2)\Delta_g(Q_0,q_3)\Delta_g(Q_0,q_1)\Delta_g(Q_0,q_1)$$
$$\alpha_S(q_1)\alpha_S(q_2)\alpha_S(q_3)$$

where typically

$$\log \Delta_f(q_T,Q) = -\int_{q_T^2}^{Q^2}\frac{dq^2}{q^2}\frac{\alpha_S(q^2)}{2\pi}\left[A_{1,f}\log\frac{Q^2}{q^2}+B_{1,f}\right]$$

- Fill phase space below Q_0 with vetoed shower

Backup (2)

MiNLO: All α_S in Born term are chosen with CKKW (local) scales $q_1, ..., q_n$

$$\alpha_S^n(\mu_R)B \Rightarrow \alpha_S(q_1)\alpha_S(q_2)...\alpha_S(q_n)B$$

- Normal NLO structure ($\mu = \mu_R$):

$$\sigma(\mu) = \underbrace{\alpha_S^n(\mu)B}_{\text{Born}} + \underbrace{\alpha_S^{n+1}(\mu)\Big(C + nb_0\log(\mu^2/Q^2)B\Big)}_{\text{Virtual}} + \underbrace{\alpha_S^{n+1}(\mu)R}_{\text{Real}}$$

- Explicit μ dependence of virtual term as required by RG invariance:

$$\alpha_S^n(\mu')B = \Big[\alpha_S(\mu) - nb_0\alpha_S^{n+1}(\mu)\log(\mu'^2/\mu^2)\Big]B + \mathcal{O}(\alpha_S^{n+2})$$

$$\text{Virtual}(\mu') = \text{Virtual}(\mu) + \alpha_S^{n+1}(\mu)nb_0\log(\mu'^2/\mu^2)B + \mathcal{O}(\alpha_S^{n+2})$$

$\Rightarrow \sigma(\mu') - \sigma(\mu) = \mathcal{O}(\alpha_S^{n+2})$

- In MiNLO "scale compensation" kept if

$$\Big(C + nb_0\log(\mu_R^2/Q^2)B\Big) \Rightarrow \Big(C + nb_0\log(\bar{\mu}_R^2/Q^2)B\Big)$$

with $\bar{\mu}_R^2 = (q_1 q_2 ... q_n)^{2/n}$

Backup (3)

Few technicalities for original MiNLO:

- $\mu_F = Q_0$ (as in CKKW)
- Cluster with CKKW also V and R kinematics
  - Actual implementation uses FKS mapping for first cluster of $\mathbf{\Phi}_{n+1}$
  - Ignore CKKW Sudakov for 1^{st} clustering of $\mathbf{\Phi}_{n+1}$ (inclusive on extra radiation)
- Some freedom in choice of ${\alpha_{\mathrm{S}}}^{(n+1)}$ (entering V, R and $\Delta^{(1)}$) (not free for MiNLO merging)
- Used full NLL-improved Sudakovs (A_1, B_1, A_2)

Backup (4): Improved MiNLO & merging

- Resummation formula

$$\frac{d\sigma}{dq_T^2 dy} = \sigma_0 \frac{d}{dq_T^2}\Big\{[C_{ga}\otimes f_a](x_A, q_T) \times [C_{gb}\otimes f_b](x_B, q_T) \times \exp S(q_T, Q)\Big\} + R_f$$

- NLO$^{(0)}$ if $C_{ij}^{(1)}$ included and R_f is LO$^{(1)}$
- Take derivative, then compare with MiNLO:

$$\sim \sigma_0 \frac{1}{q_T^2}[\alpha_S, \alpha_S^2, \alpha_S^3, \alpha_S^4, \alpha_S L, \alpha_S^2 L, \alpha_S^3 L, \alpha_S^4 L] \exp S(q_T, Q) \qquad L = \log(Q^2/q_T^2)$$

- can be shown that

$$\int^{Q^2} \frac{dq_T^2}{q_T^2} L^m \alpha_S{}^n(q_T) \exp S \sim \big(\alpha_S(Q^2)\big)^{n-(m+1)/2}$$

- if I drop B_2 in MiNLO Δ_g, I miss a term $(1/q_T^2)\alpha_S^2 B_2 \exp S$
- upon integration, violate NLO$^{(0)}$ by a term $\mathcal{O}(\alpha_S^{3/2})$
- “wrong” scale in $\alpha_S^{(\mathrm{NLO})}$ in MiNLO produces again same error

Alternative proof also available in the paper.

p_T^H spectrum:

- "$\mu_{\mathrm{HJ-MiNLO}} = m_H, m_H, p_T$"
- At high p_T, $\mu_{\mathrm{HJ-MiNLO}} = p_T$
- If $\beta = 1/2$, `NNLOPS` $\rightarrow$ `HJ-MiNLO` at high p_{T}
- NNLO/NLO ~ 1.5, because HNNLO with $\mu = m_H/2$, $\mu_{\mathrm{HJ-MiNLO,core}} = m_H$