Radiative corrections to the Higgs couplings in the triplet model

Mariko Kikuchi (Univ. of TOYAMA)

M. Aoki, S. Kanemura, M. K., K. Yagyu,
Phys.Lett. B714 (2012) 279-285
Phys.Rev. D87 (2013) 015012

LC13 @ Trento , Sept. 16th -20th 2013

Contents

I. Introduction

Higgs coupling measurement is a probe of new physics

Future precision measurement ⇔ Radiative Corrections

II. Higgs triplet model

Motivation, Particle entries, Some characteristics of the model

Renormalization and loop calculations for Higgs couplings

III. Results

Deviation from SM in $h\gamma\gamma$, hWW, hZZ, hhh

IV. Summary

Higgs sector

m_h=126 GeV

Higgs boson was discovered last year !

Data indicate that it is a SM-like Higgs boson (h).

What is the shape of the Higgs sector?

No principle for minimal Higgs with one doublet

All extended Higgs sectors can predict the SM-like Higgs boson

Many new physics models predict specific extended Higgs sectors

Hierarchy, Neutrino Masses, Dark Matter,
Baryon Asymmetry, …

Higgs sector is a window of new physics !

Extended Higgs models

What kind of extended Higgs sectors are possible?

Simplest models are

1. Higgs Singlet Model $\Phi + S$ (B-L Higgs, ...)
2. 2Higgs Doublet Model $\Phi + \Phi$ (SUSY, EW Baryogenesis, …)
3. Higgs Triplet Model $\Phi + \Delta$ (Type II seesaw, ….)

…

All these extended Higgs models can contain the 126 GeV SM-like Higgs boson *h*, and satisfy current LEP/LHC data.

Therefore, we may be able to test these models by measuring the deviation in *h*-coupling constants from the SM predictions

hγγ, hWW, hZZ, hττ, hbb, htt, hhh, ….

Future precision measurements of h couplings at LHC, HL-LHC, ILC, ……

Test of Extended Higgs

Direct search

Extended Higgs sectors can be tested by direct searches of extra Higgs bosons

H, A, H^{+}, H^{++}, ….

Indirect search

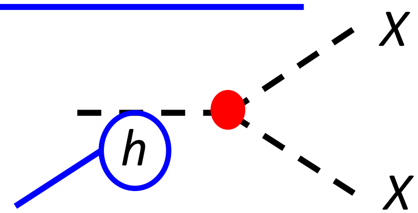
Physics of *h*

Deviation in coupling constants of the *h* due to heavy Higgs bosons (or NP particles)

There are **two possibilities** to change couplings of *h*

Mixing among scalar fields

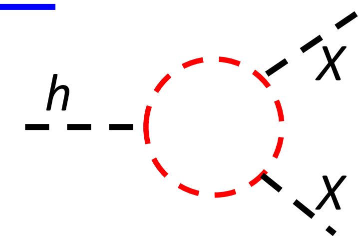
EX) 2HDM



$h_1 \cos\alpha - h_2 \sin\alpha$

Radiative corrections

Deviation due to the loop contributions of additional new particles.



We can discriminate an extended Higgs model by precisely measuring the pattern of deviations in the *h boson* couplings.

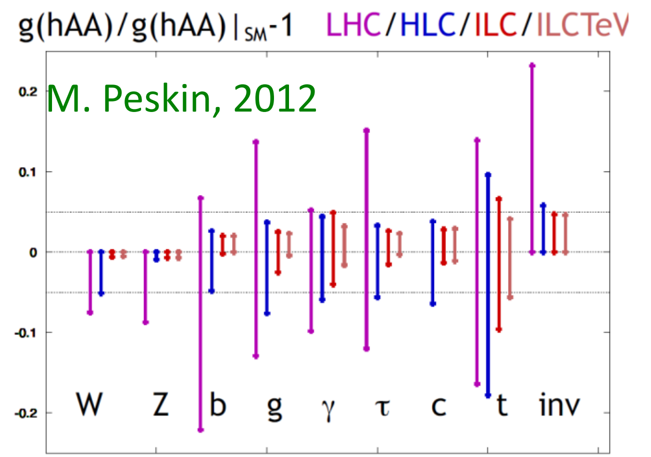
Determination of *h* couplings

◆ LHC data for signal strength slightly deviates from SM predictions.

EX) $h \to \gamma\gamma$ 0.8(±0.3) (CMS) 1.6 (±0.3) (ATLAS)

$h \to WW^*$, 0.75(±0.2) (CMS) 0.95(±0.2) (ATLAS)

◆ Clear deviations may appear at future measurements at LHC, and especially at ILC.



ILC 250-1000GeV

hZZ, *hWW* : O(1)% or better
hhh : O(10)%

Technical design report of ILC (2013)

HL-LHC & ILC

$h\gamma\gamma$: about 5%(HL-LHC&LC500)

Markus, Remi, Tilman, Michael,and Dirk Zerwas(2013)

◆ In order to compare to such precision measurements, we must evaluate the theory predictions at the one-loop level. This is essentially important!

In this talk

- We here consider the Higgs triplet model (Φ+Δ), and study radiative corrections to the h-couplings in this model.

- Why the Higgs triplet model?
  - Neutrino mass can be explained
  - Theoretical interest for 1-loop calculation of such an exotic model with $\rho \neq 1$.

Higgs triplet model (HTM)

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} + \Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & \frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$$

$$\phi^0 = \frac{1}{\sqrt{2}}(\phi + v_\phi + i\,\chi)$$

$$\Delta^0 = \frac{1}{\sqrt{2}}(\delta + v_\Delta + i\,\eta)$$

	$SU(2)_L$	$U(1)_Y$
Φ	2	1/2
Δ	3	1

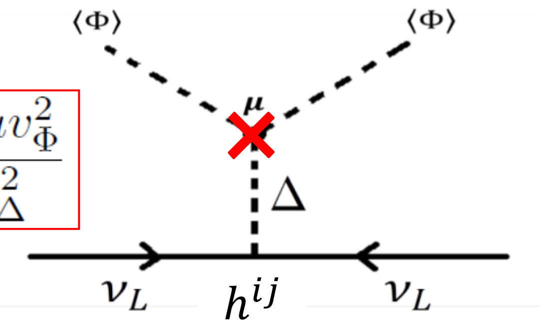
◆ Neutrino mass Type-II seesaw

Cheng, Li (1980), Mohapatra, Senjanovic(1981)

$$\mathcal{L}_\nu = h_{ij}\overline{L_L^i}^c i\tau_2 \Delta L_L^j + hc$$

Majorana neutrino masses are generated via the LNV parameter μ .

$$M_\nu^{ij} = \frac{h^{ij}\mu v_\Phi^2}{M_\Delta^2}$$



v_ϕ : doublet VEV
v_Δ : triplet VEV

$v^2 = v_\varphi^2 + v_\Delta^2 \simeq 246$GeV

◆ The rho parameter $\rho_{exp} \simeq 1.0008^{+0.0017}_{-0.0007}$

ρ is not equal to unity at tree.

We need to set $v_\Delta / v_\phi \ll 1$

$$\rho \equiv \frac{m_W{}^2}{\cos^2\theta_W m_Z{}^2} = \frac{v_\phi{}^2 + 2v_\Delta{}^2}{v_\phi{}^2 + 4v_\Delta{}^2} \neq 1$$

Higgs potential

$$V(\Phi,\Delta) = m^2\Phi^\dagger\Phi + M^2\mathrm{Tr}(\Delta^\dagger\Delta) + \left[\mu\Phi^T i\tau_2\Delta^\dagger\Phi + \mathrm{h.c.}\right]$$
$$+ \lambda_1(\Phi^\dagger\Phi)^2 + \lambda_2\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2 + \lambda_3\mathrm{Tr}[(\Delta^\dagger\Delta)^2] + \lambda_4(\Phi^\dagger\Phi)\mathrm{Tr}(\Delta^\dagger\Delta) + \lambda_5\Phi^\dagger\Delta\Delta^\dagger\Phi.$$

◆ Mass eigenstates

$$\begin{pmatrix}\phi\\ \delta\end{pmatrix} = \begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}h\\ H\end{pmatrix}, \quad \begin{pmatrix}\phi^\pm\\ \Delta^\pm\end{pmatrix} = \begin{pmatrix}\cos\beta & -\sin\beta\\ \sin\beta & \cos\beta\end{pmatrix}\begin{pmatrix}G^\pm\\ H^\pm\end{pmatrix}, \quad \begin{pmatrix}\chi\\ \eta\end{pmatrix} = \begin{pmatrix}\cos\beta' & -\sin\beta'\\ \sin\beta' & \cos\beta'\end{pmatrix}\begin{pmatrix}G^0\\ A\end{pmatrix},$$

Mass eigenstates: h, $H^{\pm\pm}, H^\pm, A, H$

SM-like Higgs boson — triplet-like Higgs bosons

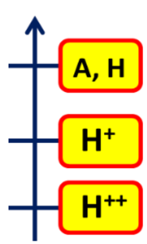
◆ Mass hierarchy

$v_\Delta{}^2 \ll v_\phi{}^2$ ⇒ Relation among masses

$$m_{H^{++}}{}^2 - m_{H^+}{}^2 \simeq m_{H^+}{}^2 - m_A{}^2 \simeq -\frac{\lambda_5}{4}v_\Phi{}^2$$

Three patterns for mass spectrum for the triplet-like Higgs bosons.

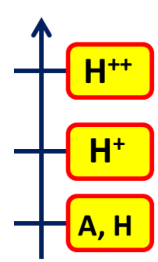
$\Delta m = m_{H^\pm} - m_{\text{lightest}}$



CaseI

$\lambda_5 > 0$

$\Delta m = m_{H^\pm} - m_{H^{\pm\pm}}$

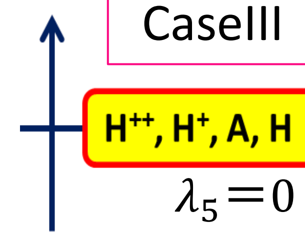


CaseII

$\lambda_5 < 0$

$\Delta m = m_{H^\pm} - m_A$

CaseIII



$\lambda_5 = 0$

$\Delta m = 0$

Current Bound from LHC

Decay of H++ depends on v_Δ

- $v_\Delta < 1$ MeV

  Mainly decay into dilepton $H^{++} \to l^+ l^+$

  LHC excluded:

  $m_{H^{++}} < 400$ GeV

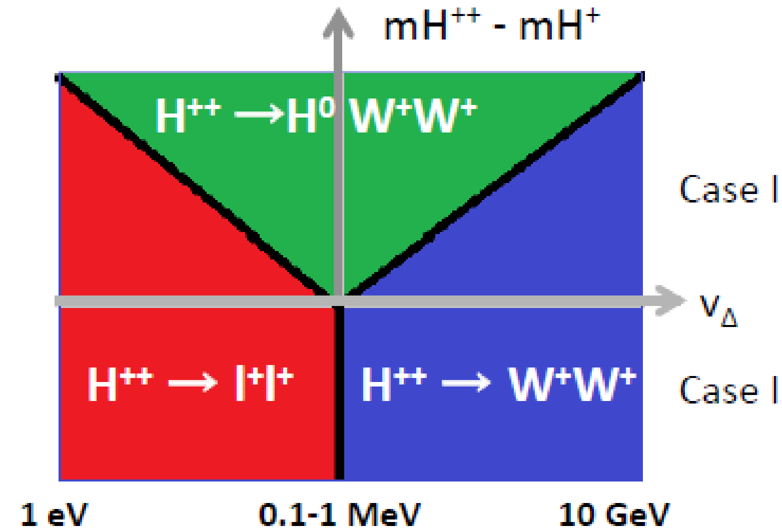
- $v_\Delta > 1$ MeV

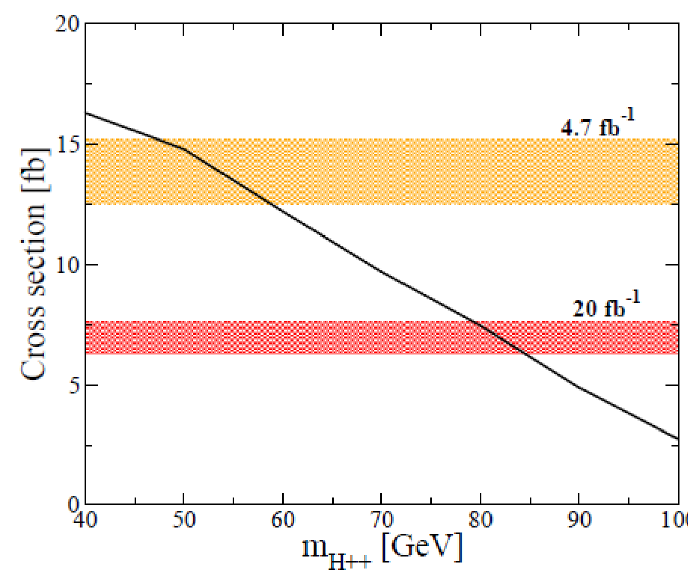
  Mainly decay into diboson $H^{++} \to W^+ W^+$

  LHC excluded:

  $m_{H^{++}} < 80$ GeV

In this case, we can consider the light H^{++}, which can be tested at ILC etc.



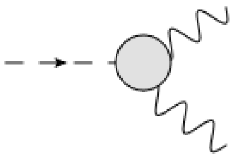


Kanemura, Yagyu, Yokoya (2013)

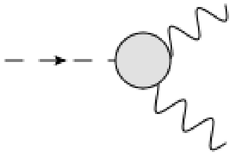
What do we calculate?

h-couplings

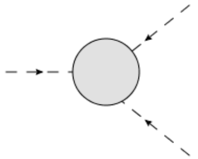
- ***hγγ*** *(1-loop induced, new loop contributions)*
  10% at LHC 5% LHC3000+ILC500



- ***hWW, hZZ*** *(Nature of Higgs mechanism)*
  10% at LHC 1% or better at ILC500



- ***hhh*** *(Structure of the Higgs potential)*
  rather difficult at LHC O(10)% at ILC1000



➢ How do we calculate these coupling constants in the HTM?

- The renormalization in HTM is different from the one in the SM because the relation $\rho=1$ does not hold in HTM.
- So we must construct a new renormalization scheme in the HTM.
- Then we calculate these coupling constants in the HTM

Renormalization

T. Blank, W. Hollik (1998), S. Kanemura, K. Yagyu (2012), P. H. Chankowski, S. Pokorski, J. Wagner, (2007); M. -C. Chen, S. Dawson, C. B. Jackson (2008).

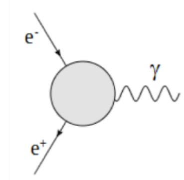
➢ The model with SM $\rho_{tree}=1$

- Lagrangian parameters • • • g, g', v
- Physical parameters • • • $m_W, m_Z, \sin\theta_W, G_F, \alpha_{em}$.
- Renormalization conditions

On-shell condition

$$Re\Pi_{ZZ}(p^2)|_{p^2=m_Z^2}=0, \quad \delta m_Z^2 = Re\Pi_{ZZ}^{1PI}(m_Z^2),$$

$$Re\Pi_{WW}(p^2)|_{p^2=m_W^2}=0, \quad \delta m_W^2 = Re\Pi_{WW}^{1PI}(m_W^2),$$



$$\Big|_{p^2=0} = -ie\gamma^\mu$$

$$\frac{\delta\alpha_{em}}{\alpha_{em}} = \frac{d}{dp^2}\Pi_{\gamma\gamma}^{1PI}(p^2)|_{p^2=0} - \frac{2s_W}{c_W}\frac{\Pi_{\gamma Z}^{1PI}(0)}{m_Z^2}$$

➢ The model with HTM $\rho_{tree}\neq 1$

$$\rho = \frac{m_W{}^2}{m_Z{}^2\cos^2\theta_W} \neq 1$$

- Lagrangian parameters • • • g, g', v, v_Δ
- Physical parameters • • • $m_W, m_Z, \sin\theta_W, G_F, \alpha_{em}$.
- Renormalization conditions

$$\sin^2\theta_W \neq 1 - \frac{m_W{}^2}{m_Z{}^2} \Rightarrow \frac{\delta s_W^2}{s_W^2} = \frac{c_W^2}{s_W^2}\left(\frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2}\right)$$

$$\cos\theta_W{}^2 = \frac{m_W{}^2}{m_Z{}^2}\frac{2}{(1+\cos\beta'^2)} \Rightarrow \begin{aligned}\delta\bar{s}_W^2 &= -\delta\bar{c}_W^2 \\ &= \frac{2m_W^2}{m_Z^2(1+c_{\beta'}^2)}\left(\frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2} - \frac{2s_{\beta'}c_{\beta'}}{(1+c_{\beta'}^2)}\delta\beta'\right)\end{aligned}$$

β’ is a mixing angle among CP-odd scalar field.

δβ’ is determined in the next slide

Renormalization in Higgs potential

(α: Mixing angle among CP- even Higgs bosons,
β: Mixing angle among charged Higgs bosons)

➢ Higgs potential

Parameters ・・・ $v,\ \alpha,\ \beta,\ \beta',\ m_{H\pm\pm},\ m_{H\pm},\ m_A,\ m_H,\ m_h$

➢ Counter-terms

$\delta v,\ \delta\alpha,\ \delta\beta,\ \delta\beta',\ \delta m_{H++}^2,\ \delta m_{H+}^2,\ \delta m_A^2,\ \delta m_h^2,\ \delta m_H^2$

Tadpole: $\delta T_{\varphi},\ \delta T_{\Delta},$

Wave function renormalization : $\delta Z_h,\ \delta Z_H,\ \delta Z_A,\ \delta Z_{G0},\ \delta Z_{H+},\ \delta Z_{G+},\ \delta Z_{H++},\ \delta C_{hH},\ \delta C_{AG0},\ \delta C_{H+G+}$

➢ Renormalization conditions

δT_φ ・・・ $\delta T_\varphi[m_\varphi{}^2] = T_\varphi{}^{tree} + T_\varphi{}^{1PI}[m_\varphi{}^2] + \delta T_\varphi = 0$

$\delta m_\varphi{}^2$ ・・・ $\Pi_{\varphi\varphi}[m_\varphi{}^2] = 0,$

$\Pi_{\varphi\varphi}[p^2] =$ 

δv ・・・ Renormalizations in the gauge sector, $v^2 = \frac{m_W{}^2 \sin^2\theta_W}{\pi^2 \alpha_{em}}$, ➡ $\frac{\delta v}{v} = \frac{1}{2}\left(\frac{\delta m_W^2}{m_W^2} - \frac{\delta\alpha_{\rm em}}{\alpha_{\rm em}} + \frac{\delta \bar{s}_W^2}{\bar{s}_W^2}\right)$

$\delta\alpha$ ・・・ $\Pi_{Hh}[m_h{}^2] = 0, \Pi_{Hh}[m_H{}^2] = 0,$ No mixing on-shell

$\delta\beta'$ ・・・ $\Pi_{AG}[m_A{}^2] = 0, \Pi_{AG}[m_G{}^2] = 0,$ No mixing on-shell

$\delta\beta$ ・・・ $\delta\beta = \frac{1+s_\beta^2}{\sqrt{2}}\delta\beta'$, ⬅ $\tan\beta = \frac{\sqrt{2}v_\Delta}{v_\phi}$, $\tan\beta' = \frac{2v_\Delta}{v_\phi}$,

Results of one-loop calculations

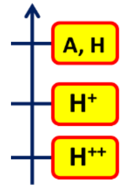
$h\gamma\gamma$, hWW, hZZ, hhh

$h\gamma\gamma$

- Ratio of the event rate for $h\to\gamma\gamma$

A. Arhrib, R. Benbrik, M. Chabab, G. Moultaka (2012);
A. G. Akeroyd, S. Moretti (2012);

$$R_{\gamma\gamma} \equiv \frac{\sigma(gg\to h)_{\rm HTM}\times {\rm BR}(h\to\gamma\gamma)_{\rm HTM}}{\sigma(gg\to h)_{\rm SM}\times {\rm BR}(h\to\gamma\gamma)_{\rm SM}}$$

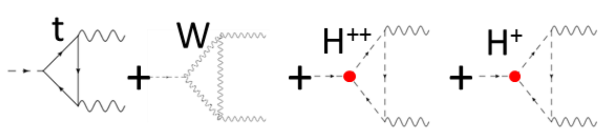


Results at LHC

$R_{\gamma\gamma}^{\rm exp}$ = 0.5 – 1.1 (CMS)

$R_{\gamma\gamma}^{\rm exp}$ = 1.3 – 1.9 (ATLAS)

$\Gamma(h\to\gamma\gamma)_{HTM} = \Big|$



$\Big|^2$

$R_{\gamma\gamma}$ depends on λ_4.

$\lambda_{hH^{++}H^{--}} \approx -\lambda_4 v$

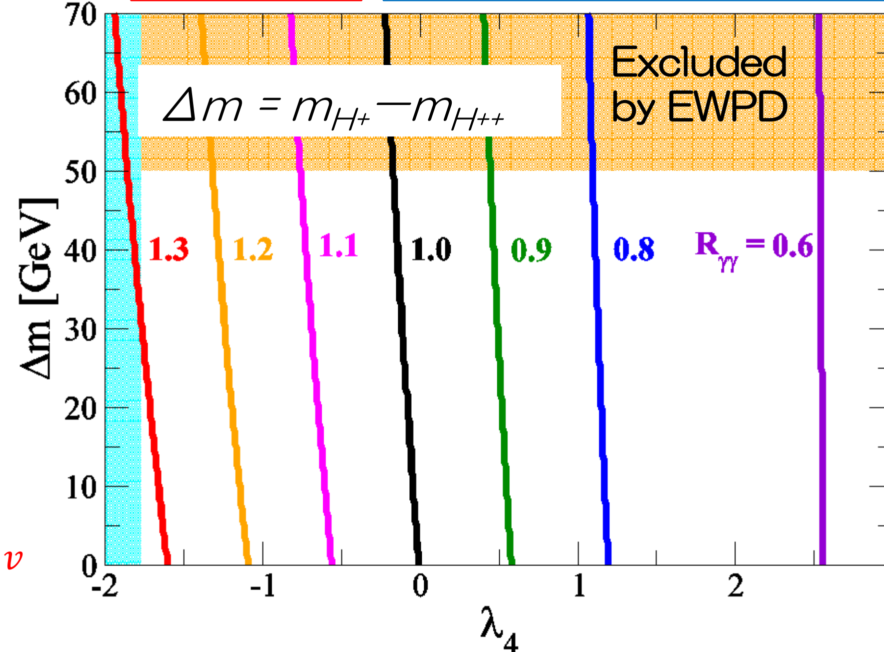
$\lambda_{hH^{+}H^{-}} \approx -(\lambda_4 + \frac{\lambda_5}{2})v$

Not excluded

$\lambda_5 \geq 0$

CaseI

$m_{H^{++}}$=300GeV, v_Δ=1MeV



M. Aoki, S. Kanemura, M. Kikuchi, K. Yagyu (2013)

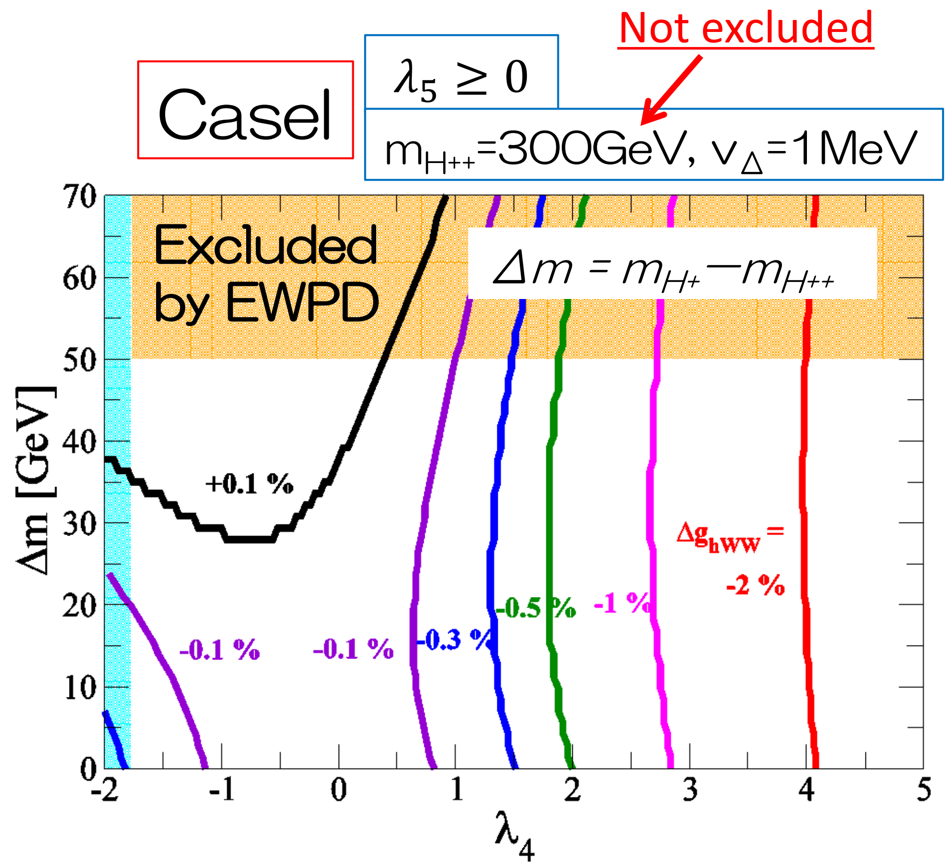
$R_{\gamma\gamma}$ can be enhanced or reduced depending on the sign of $\lambda_{hH^{++}H^{--}}$.

hWW

- Deviations for *hWW* from the SM predictions

$$\Delta g_{hVV} \equiv \frac{\mathrm{Re}M_1^{hVV} - \mathrm{Re}M_1^{hVV}(\mathrm{SM})}{\mathrm{Re}M_1^{hVV}(\mathrm{SM})}$$

$$\Delta g_{hWW} \simeq -\frac{v^2}{32\pi^2}\left[\frac{\lambda_4{}^2}{6m_{H++}{}^2} + \frac{\left(\lambda_4+\frac{\lambda_5}{2}\right)^2}{6m_{H+}{}^2} + \frac{(\lambda_4+\lambda_5)^2}{6m_A{}^2} + \frac{(\lambda_4+\lambda_5)^2}{6m_H{}^2}\right] + \frac{1}{4\pi^2}\frac{2(c_W{}^2-s_W{}^2)}{3s_W{}^2}\left[\frac{(m_{H++}-m_{H+})^2}{v^2} + \frac{(m_{H+}-m_A)^2}{v^2}\right]$$



M. Aoki, S. Kanemura, M. Kikuchi, K. Yagyu (2013)

Deviations for *hWW* from the SM predictions can be several %.

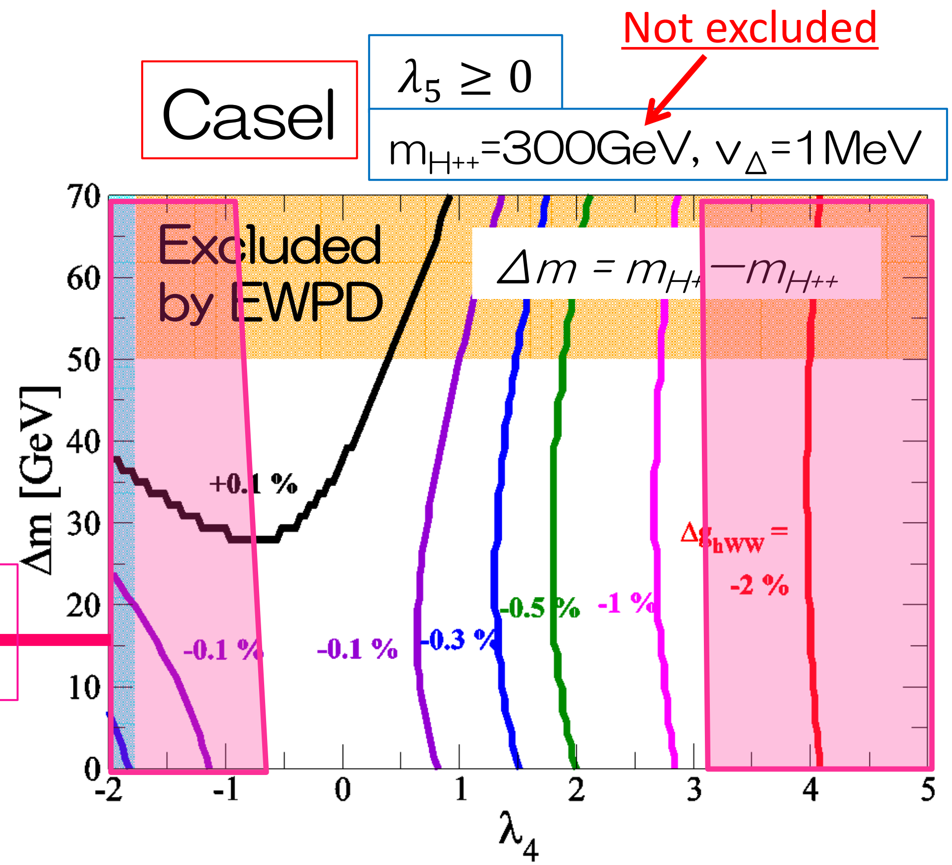
hWW

- Deviations for *hWW* from the SM predictions

$$\Delta g_{hVV} \equiv \frac{\mathrm{Re}M_1^{hVV} - \mathrm{Re}M_1^{hVV}(\mathrm{SM})}{\mathrm{Re}M_1^{hVV}(\mathrm{SM})}$$

$$\Delta g_{hWW} \simeq -\frac{v^2}{32\pi^2}\left[\frac{{\lambda_4}^2}{6{m_{H++}}^2} + \frac{\left(\lambda_4+\frac{\lambda_5}{2}\right)^2}{6{m_{H+}}^2} + \frac{(\lambda_4+\lambda_5)^2}{6{m_A}^2} + \frac{(\lambda_4+\lambda_5)^2}{6{m_H}^2}\right] + \frac{1}{4\pi^2}\frac{2({c_W}^2-{s_W}^2)}{3{s_W}^2}\left[\frac{(m_{H++}-m_{H+})^2}{v^2} + \frac{(m_{H+}-m_A)^2}{v^2}\right]$$

If we take into account the CMS data for $R_{\gamma\gamma}$, pink regions are excluded.



M. Aoki, S. Kanemura, M. Kikuchi, K. Yagyu (2013)

Δg_{hVV} can be 1%.

Deviation is detectable at ILC !

hZZ

- Deviations for *hZZ* from the SM predictions

$$\Delta g_{hVV} \equiv \frac{\mathrm{Re}M_1^{hVV} - \mathrm{Re}M_1^{hVV}(\mathrm{SM})}{\mathrm{Re}M_1^{hVV}(\mathrm{SM})}$$

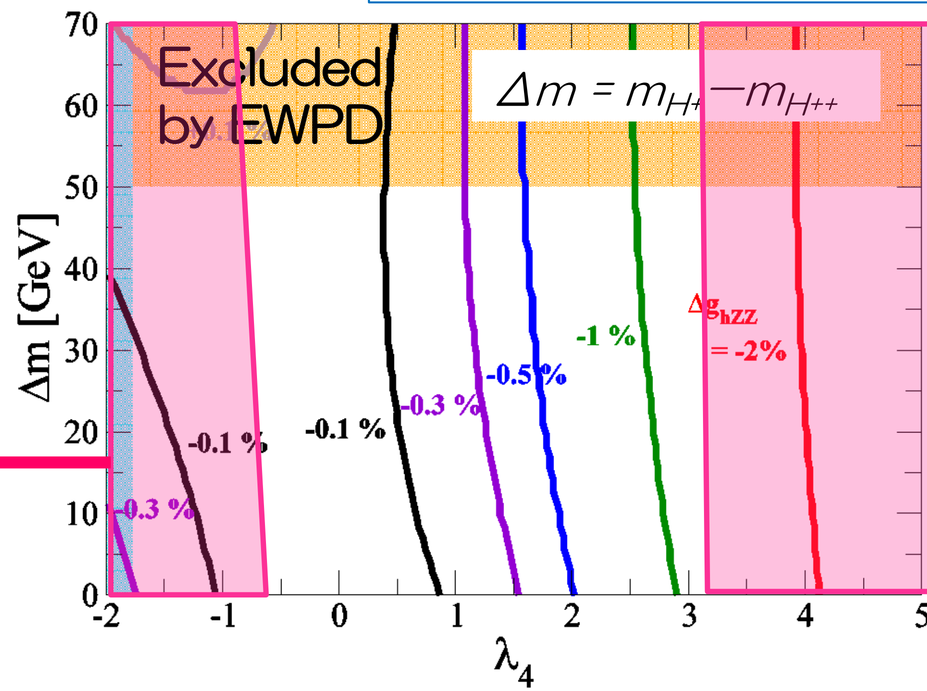
$$\Delta g_{hWW} \simeq -\frac{v^2}{32\pi^2}\left[\frac{\lambda_4{}^2}{6m_{H++}{}^2} + \frac{\left(\lambda_4+\frac{\lambda_5}{2}\right)^2}{6m_{H+}{}^2} + \frac{(\lambda_4+\lambda_5)^2}{6m_A{}^2} + \frac{(\lambda_4+\lambda_5)^2}{6m_H{}^2}\right] + \frac{1}{4\pi^2}\frac{2(c_W{}^2-s_W{}^2)}{3s_W{}^2}\left[\frac{(m_{H++}-m_{H+})^2}{v^2} + \frac{(m_{H+}-m_A)^2}{v^2}\right]$$

CaseI

$\lambda_5 \geq 0$

m_{H++}=300GeV, v_Δ=1MeV

Not excluded



If we take into account results of $R_{\gamma\gamma}$ a, pink region is excluded.

Δg_{hVV} can be 1%.

M. Aoki, S. Kanemura, M. Kikuchi, K. Yagyu (2013)

Deviations are detectable at ILC !

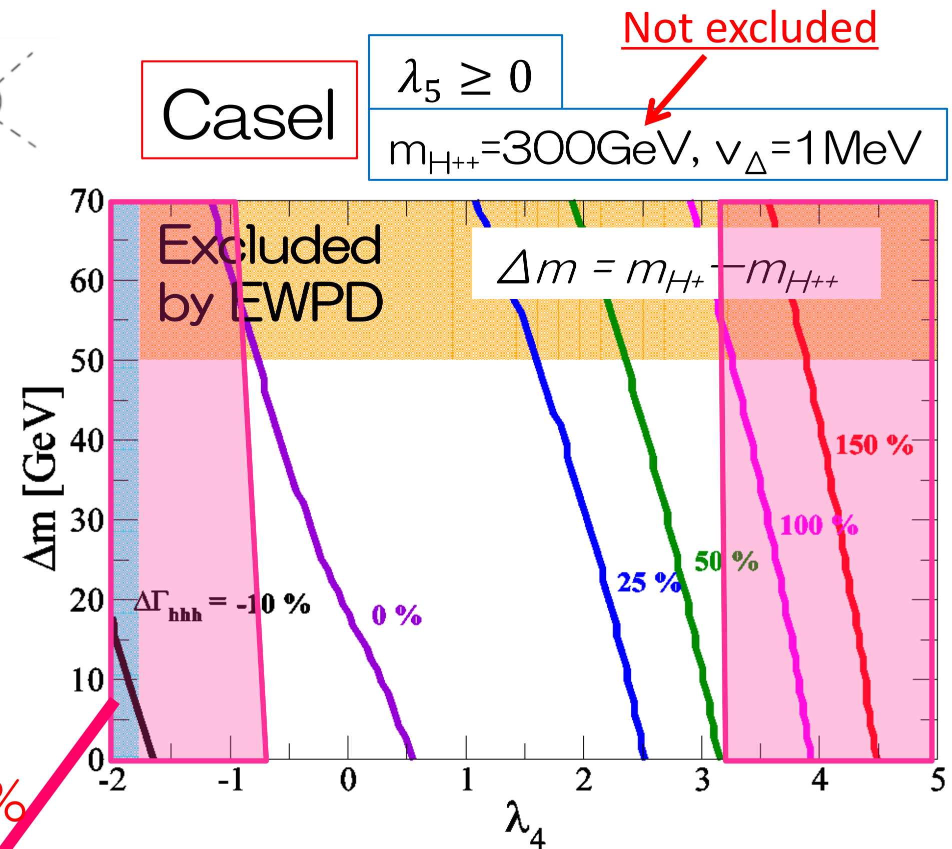
hhh

➢ Deviations for *hhh* from the SM predictions

$$\Delta\Gamma_{hhh} \equiv \frac{\mathrm{Re}\Gamma_{hhh} - \mathrm{Re}\Gamma_{hhh}^{\mathrm{SM}}}{\mathrm{Re}\Gamma_{hhh}^{\mathrm{SM}}}$$

$$\Delta\Gamma_{hhh} \simeq \frac{-v}{48\pi^2 m_h^2}\left(\frac{\lambda_{H^{++}H^{--}h}^{3}}{m_{H^{++}}^{2}} + \frac{\lambda_{H^+H^-h}^{3}}{m_{H^+}^{2}} + \frac{4\lambda_{hAA}^{3}}{m_A^{2}} + \frac{4\lambda_{hHH}^{3}}{m_H^{2}}\right)$$

$$\simeq \frac{v^4}{48 m_h^2 \pi^2}\left[\frac{\lambda_4^{3}}{m_{H^{++}}^{2}} + \frac{\left(\lambda_4+\frac{\lambda_5}{2}\right)^{3}}{m_{H^+}^{2}} + \frac{(\lambda_4+\lambda_5)^{3}}{2m_A^{2}} + \frac{(\lambda_4+\lambda_5)^{3}}{2m_H^{2}}\right]$$



Deviations in hhh coupling from the SM prediction can be −10 %~ +150%

M. Aoki, S. Kanemura, M. Kikuchi, K. Yagyu (2013)

If we take into account results of $R_{\gamma\gamma}$, pink region is excluded.

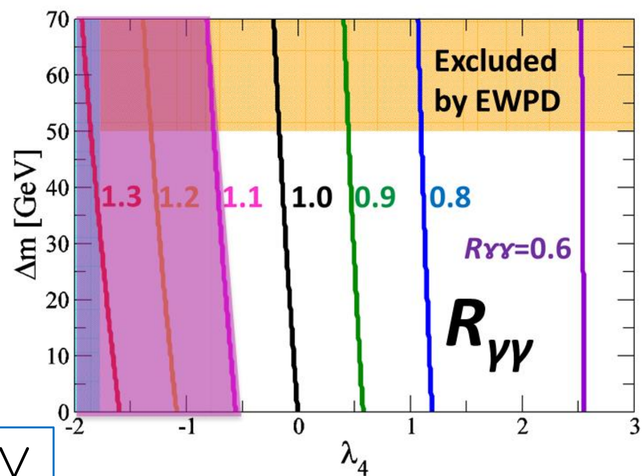
$\Delta\Gamma_{hhh}$ can be 50%. ➡ Deviations are detectable at ILC !

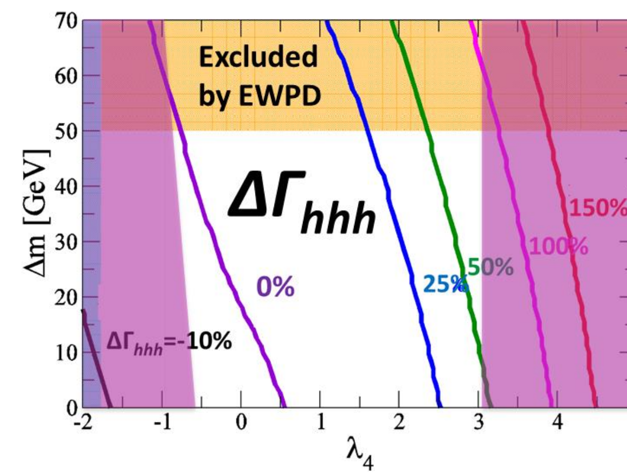
Correlations of the deviation

Contributions to $R_{\gamma\gamma}$ is opposite to one of $\Delta\Gamma_{hhh}$.

Case-I

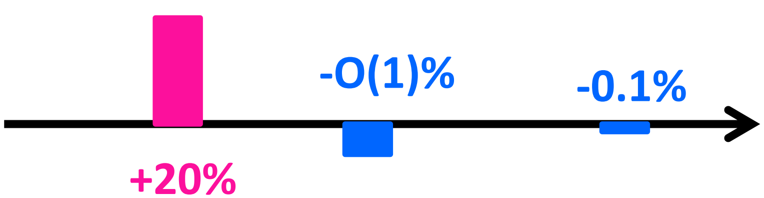
$\Delta m = m_{H+} - m_{H++}$



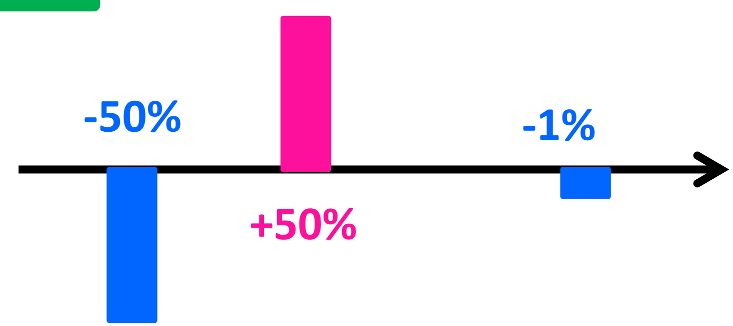


$m_{lightest}$=300GeV, v_Δ=1MeV

$\lambda_4 = -0.5$ $R_{\gamma\gamma}$ $\Delta\Gamma_{hhh}$ $\Delta g_{hWW}, \Delta g_{hZZ}$



$\lambda_4 = 3$ $R_{\gamma\gamma}$ $\Delta\Gamma_{hhh}$ $\Delta g_{hWW}, \Delta g_{hZZ}$



There is a correlation in deviations in $h\gamma\gamma$ and hhh.
By detecting this, we can discriminate the model from the others.

Summary

- We calculate radiative corrections to the *h*-coupling constants in the Higgs triplet model to compare future precision measurements.

- Results :

In the region where LHC (CMS) data($\Gamma(h \to \gamma\gamma)$) allows,

  - deviations for *hWW (hZZ)* can be about -0.1% to -1%.
  - deviations for *hhh* can be about -5% to +50 %.

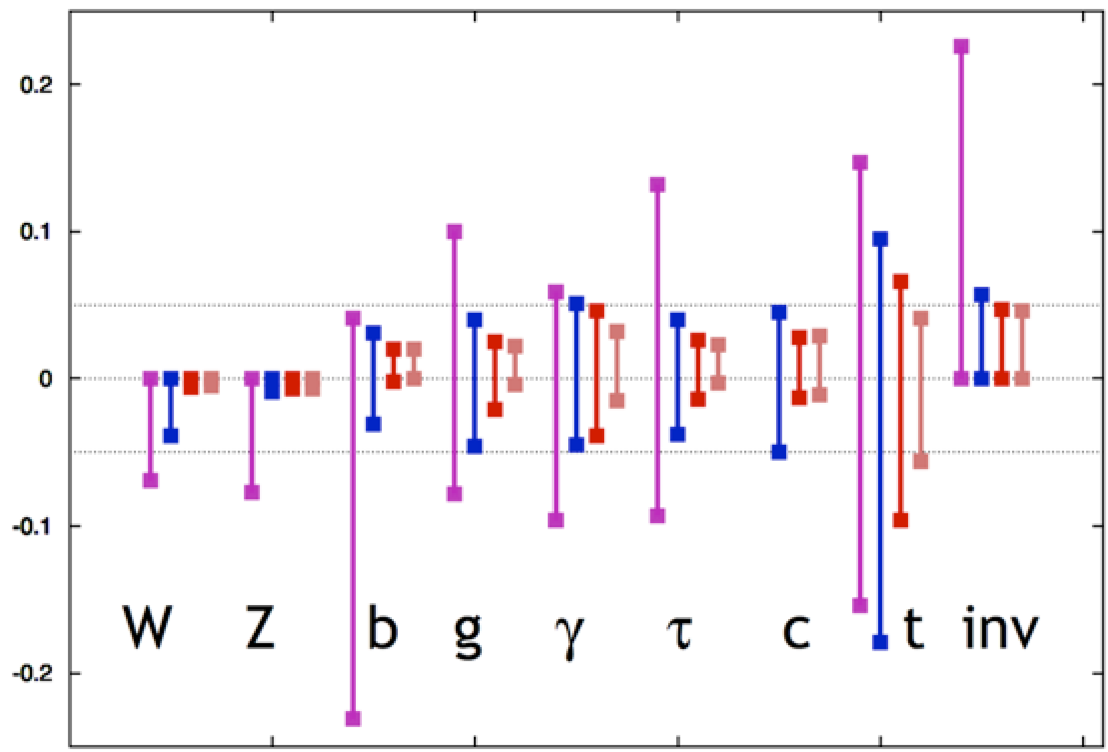
- Conclusion:

Deviations for these coupling constants can be large enough to be detected at ILC.

By detecting the pattern of deviations in coupling constants, we can discriminate the HTM from the other models.

Thank you for your attention !

g(hAA)/g(hAA)|$_{SM}$-1 LHC/HLC/ILC/ILCTeV



LHC : $\sqrt{s}$=14 TeV, L=300fb^{-1} in LHC

HLC : $\sqrt{s}$=250 GeV, L=250fb^{-1} in ILC

ILC : $\sqrt{s}$=500 GeV, L=500fb^{-1} in ILC

ILCTeV : $\sqrt{s}$=1000 GeV, L=1000fb^{-1} in ILC

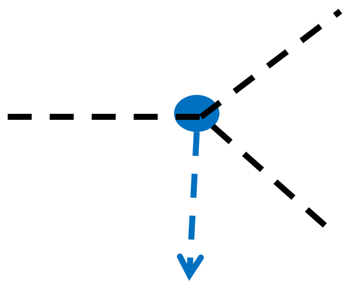
hhh

$$v_\Delta{}^2 \ll v_\phi{}^2$$
$$v^2 = v_\phi{}^2 + 2v_\Delta{}^2$$

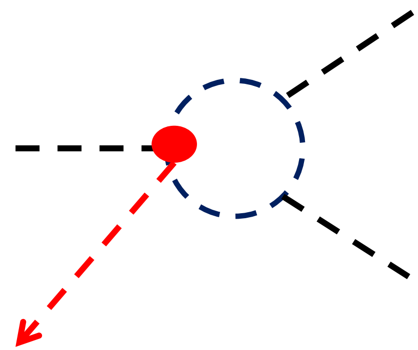
$$m_{H++}{}^2 \simeq M^2 + \frac{1}{2}\lambda_4 v^2$$
$$m_{H+}{}^2 \simeq M^2 + (\tfrac{1}{2}\lambda_4 + \tfrac{1}{4}\lambda_5)v^2$$
$$m_A{}^2 \simeq m_H{}^2 \simeq M^2 + \frac{1}{2}(\lambda_4 + \lambda_5)v^2$$

$$\Delta\Gamma_{hhh} \simeq \frac{v}{48\pi^2 m_h{}^2}\left(\frac{\lambda_{H++H--h}{}^3}{m_{H++}{}^2} + \frac{\lambda_{H+H-h}{}^3}{m_{H+}{}^2} + \frac{4\lambda_{hAA}{}^3}{m_A{}^2} + \frac{4\lambda_{hHH}{}^3}{m_H{}^2}\right)$$
$$\simeq \frac{v^4}{48 m_h{}^2\pi^2}\left[\frac{\lambda_4{}^3}{m_{H++}{}^2} + \frac{\left(\lambda_4 + \frac{\lambda_5}{2}\right)^3}{m_{H+}{}^2} + \frac{(\lambda_4+\lambda_5)^3}{2m_A{}^2} + \frac{(\lambda_4+\lambda_5)^3}{2m_H{}^2}\right]$$

Coupling parameters of the loop diagrams are different from the one of the tree diagram.
So, deviations by loop correction can be large.



$$\lambda_{hhh} \simeq -\lambda_1 v$$



$$\lambda_{hH++H--} \simeq -\lambda_4 v$$
$$\lambda_{hH+H-} \simeq -\left(\lambda_4 + \frac{1}{2}\lambda_5\right)v$$
$$\lambda_{hAA} \simeq \lambda_{hH+H-} \simeq -\frac{1}{2}(\lambda_4 + \lambda_5)v$$

Global symmetries

This potential respects additional global symmetries in some limits.

$$V_{Higgs} = m^2\Phi^\dagger\Phi + M^2 Tr(\Delta^\dagger\Delta) + [\mu\Phi^T i\tau_2\Delta^\dagger\Phi + h.c.]$$
$$+ \lambda_1(\Phi^\dagger\Phi)^2 + \lambda_2[Tr(\Delta^\dagger\Delta)]^2 + \lambda_3 Tr(\Delta^\dagger\Delta)^2$$
$$+ \lambda_4(\Phi^\dagger\Phi)Tr(\Delta^\dagger\Delta) + \lambda_5\Phi^\dagger\Delta\Delta^\dagger\Phi$$

- When the μ term is absent, there is the global U(1) symmetry in the potential.
  This symmetry conserves the lepton number.

Mass formula appear in this limit.

- When both the μ term and the λ5 term are zero, an additional global SU(2) symmetry appears.
  This is the symmetry which rotate the doublet field and the triplet field with different angle.

In this case, all triplet-like Higgs bosons are degenerate in mass.

$$m_{H++}{}^2 = m_{H+}{}^2 = m_A{}^2 = m_H{}^2$$

Renormalization in EW

- Parameters ; $m_W, m_Z, sin\theta_W, G_F, \alpha_{em}$

- Relation ; $G_F = \frac{\pi\alpha_{em}}{\sqrt{2}m_W{}^2 sin\theta_W 2}$, $cos^2\theta_W = \frac{2m_W{}^2}{m_Z{}^2(1+cos^2\beta\prime)}$

- Input parameters ; $m_W, m_Z, \alpha_{em}, \underline{sin\theta_W}$

  $\Rightarrow$ β' (mixing angle among the CP-odd scalar field)

- Parameter shift ;

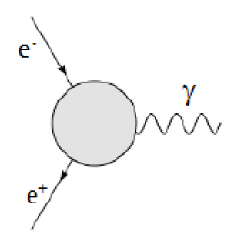
$$m_W{}^2 \to m_W{}^2 + \delta m_W{}^2,$$
$$m_Z{}^2 \to m_Z{}^2 + \delta m_Z{}^2,$$
$$\alpha_{em} \to \alpha_{em} + \delta\alpha_{em},$$
$$sin^2\theta_W \to sin^2\theta_W + \delta sin^2\theta_W$$

- Renormalization conditions ;

$\mathrm{Re}\hat{\Pi}_{WW}[m_W^2] = 0,$ $\to \delta m_W{}^2$ $\mathrm{Re}\hat{\Pi}_{ZZ}[m_Z^2] = 0,$ $\to \delta m_Z{}^2,$

$$\hat{\Gamma}_\mu^{\gamma ee}[q^2 = 0, \not{p}_1 = \not{p}_2 = m_e] = ie\gamma_\mu,$$

$\to \delta\alpha_{em},$



Renormalization in Higgs potential

➢ Higgs potential

(α: Mixing angle among CP- even Higgs bosons,
β: Mixing angle among charged Higgs bosons)

- Parameters ･･･ $v, \alpha, \beta, \beta', m_{H\pm\pm}, m_{H\pm}, m_A, m_H, m$

➢ Counter-terms

$\delta v, \delta\alpha, \delta\beta, \delta\beta', \delta m_{H++}^2, \delta m_{H+}^2, \delta m_A^2, \delta m_h^2, \delta m_H^2$

Tadpole : $\delta T_\varphi, \delta T_\Delta,$

Wave function renormalization : $\delta Z_h, \delta Z_H, \delta Z_A, \delta Z_{G0}, \delta Z_{H+}, \delta Z_{G+}, \delta Z_{H++}, \delta C_{hH}, \delta C_{AG0}, \delta C_{H+G+}$

➢ Renormalization conditions

$\Pi_{\varphi\varphi}[p^2]$ =

$\delta m_\varphi{}^2$ ･･･ $\Pi_{\varphi\varphi}[m_\varphi{}^2] = 0,$

$v^2 = \frac{m_W{}^2 \sin^2\theta_W}{\pi^2 \alpha_{em}},$

δv ･･･ Renormalizations in the gauge sector, ➡ $\frac{\delta v}{v} = \frac{1}{2}\left(\frac{\delta m_W^2}{m_W^2} - \frac{\delta\alpha_{\rm em}}{\alpha_{\rm em}} + \frac{\delta \bar{s}_W^2}{\bar{s}_W^2}\right)$

$\delta\alpha$ ･･･ $\Pi_{Hh}[m_h{}^2] = 0, \Pi_{Hh}[m_H{}^2] = 0,$ No mixing on-shell

$\delta\beta'$ ･･･ $\Pi_{AG}[m_A{}^2] = 0, \Pi_{AG}[m_G{}^2] = 0,$ No mixing on-shell

$\delta\beta$ ･･･ $\delta\beta = \frac{1 + s_\beta^2}{\sqrt{2}}\delta\beta',$ ⬅ $\tan\beta = \frac{\sqrt{2}v_\Delta}{v_\phi}, \quad \tan\beta' = \frac{2v_\Delta}{v_\phi},$

$h\gamma\gamma$

A. Arhrib, R. Benbrik, M. Chabab, G. Moultaka (2012);
A. G. Akeroyd, S. Moretti (2012);

- Ratio of the event rate for $h \to \gamma\gamma$

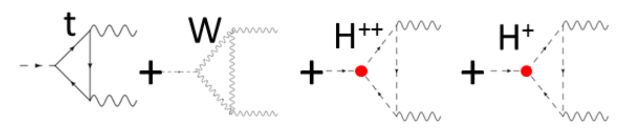
$$R_{\gamma\gamma} \equiv \frac{\sigma(gg \to h)_{\rm HTM} \times {\rm BR}(h \to \gamma\gamma)_{\rm HTM}}{\sigma(gg \to h)_{\rm SM} \times {\rm BR}(h \to \gamma\gamma)_{\rm SM}}$$

R_{gg} depend on λ_4.

$$\lambda_{hH^{++}H^{--}} \approx -\lambda_4 v$$

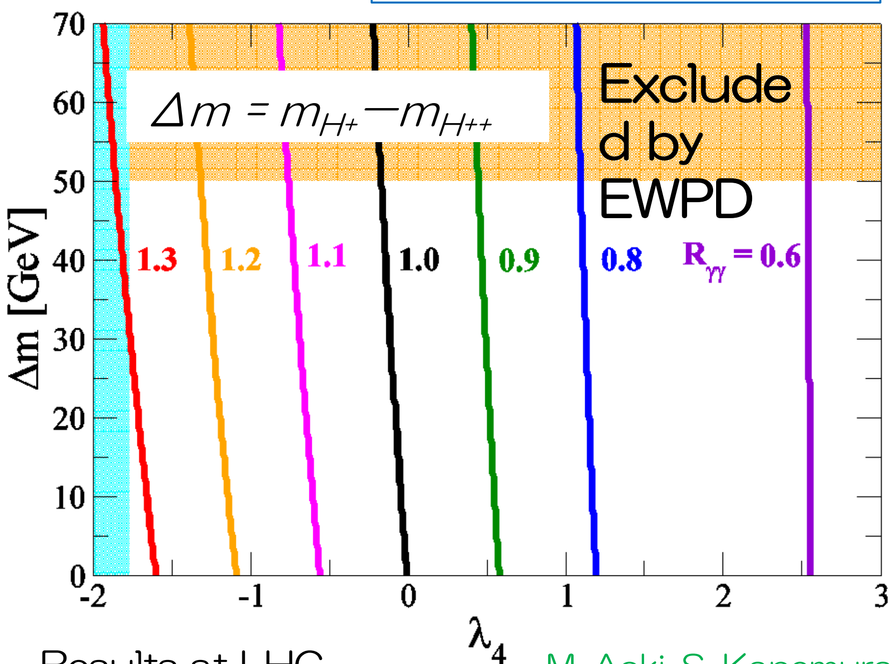
$$\lambda_{hH^{+}H^{-}} \approx -(\lambda_4 + \frac{\lambda_5}{2})v$$

$\Gamma(h \to \gamma\gamma)_{HTM} = \left| \quad \right|^2$



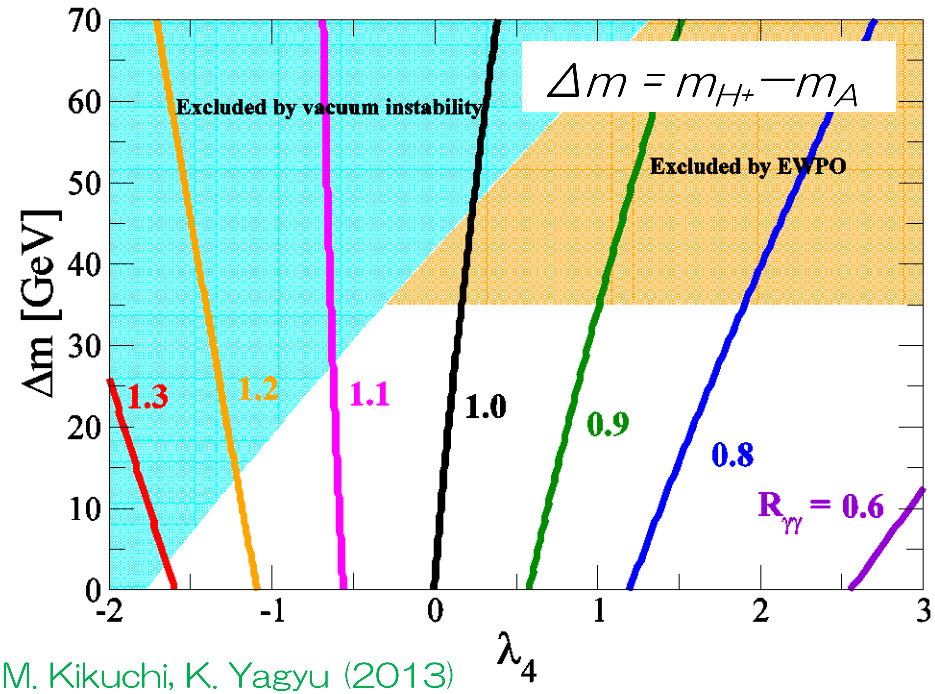
CaseI $\lambda_5 \geq 0$

$m_{H^{++}}$=300GeV, v_Δ=1MeV



CaseII $\lambda_5 \leq 0$

m_A=300GeV, v_Δ=1MeV



Results at LHC

M. Aoki, S. Kanemura, M. Kikuchi, K. Yagyu (2013)

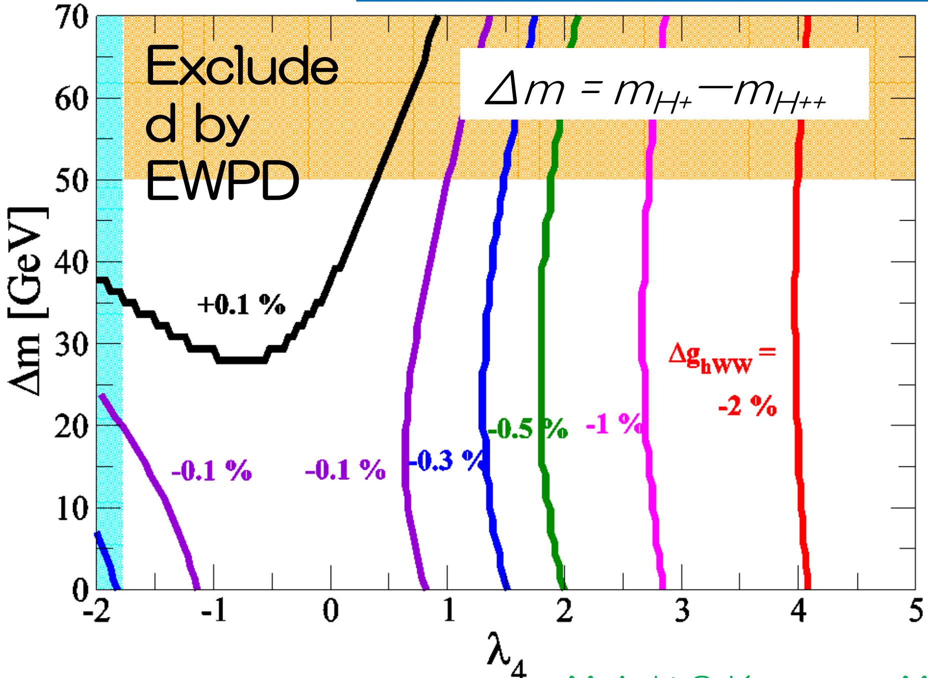
hWW

- Deviations for hWW from the SM predictions

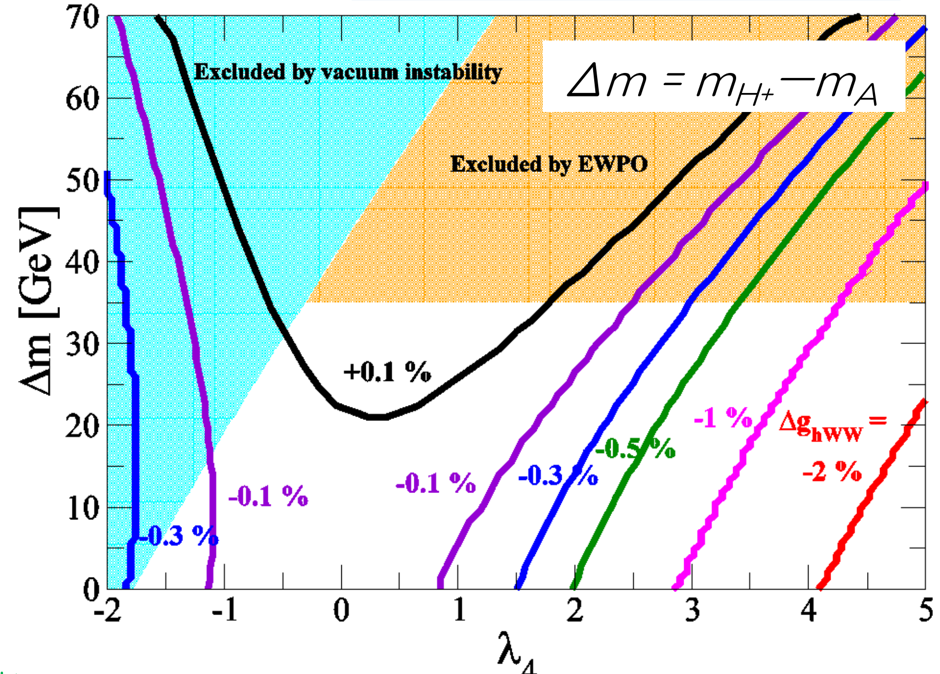
$$\Delta g_{hVV} \equiv \frac{\mathrm{Re}M_1^{hVV} - \mathrm{Re}M_1^{hVV}(\mathrm{SM})}{\mathrm{Re}M_1^{hVV}(\mathrm{SM})}$$

$$\Delta g_{hWW} \simeq -\frac{v^2}{32\pi^2}\left[\frac{{\lambda_4}^2}{6{m_{H^{++}}}^2} + \frac{\left(\lambda_4+\frac{\lambda_5}{2}\right)^2}{6{m_{H^+}}^2} + \frac{(\lambda_4+\lambda_5)^2}{6{m_A}^2} + \frac{(\lambda_4+\lambda_5)^2}{6{m_H}^2}\right] + \frac{1}{4\pi^2}\frac{2({c_W}^2-{s_W}^2)}{3{s_W}^2}\left[\frac{(m_{H^{++}}-m_{H^+})^2}{v^2} + \frac{(m_{H^+}-m_A)^2}{v^2}\right]$$

CaseI $\lambda_5 \geq 0$ $m_{H^{++}}$=300GeV, v_Δ=1MeV



CaseII $\lambda_5 \leq 0$ m_A=300GeV, v_Δ=1MeV



M. Aoki, S. Kanemura, M. Kikuchi, K. Yagyu (2013)

Deviations for hWW from the SM predictions can be several %.

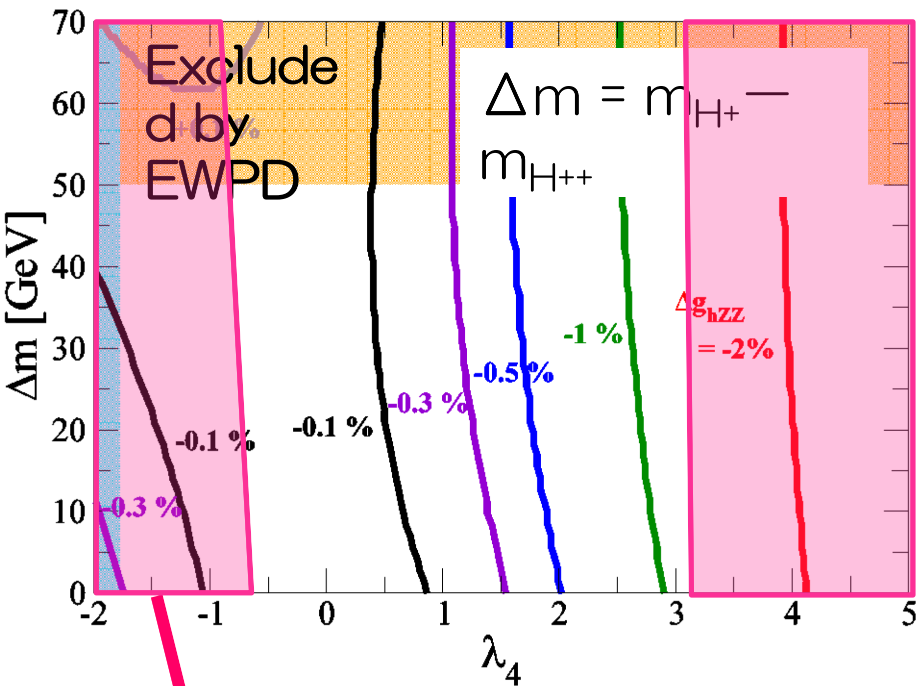
hZZ

- Deviations for hWW from the SM predictions

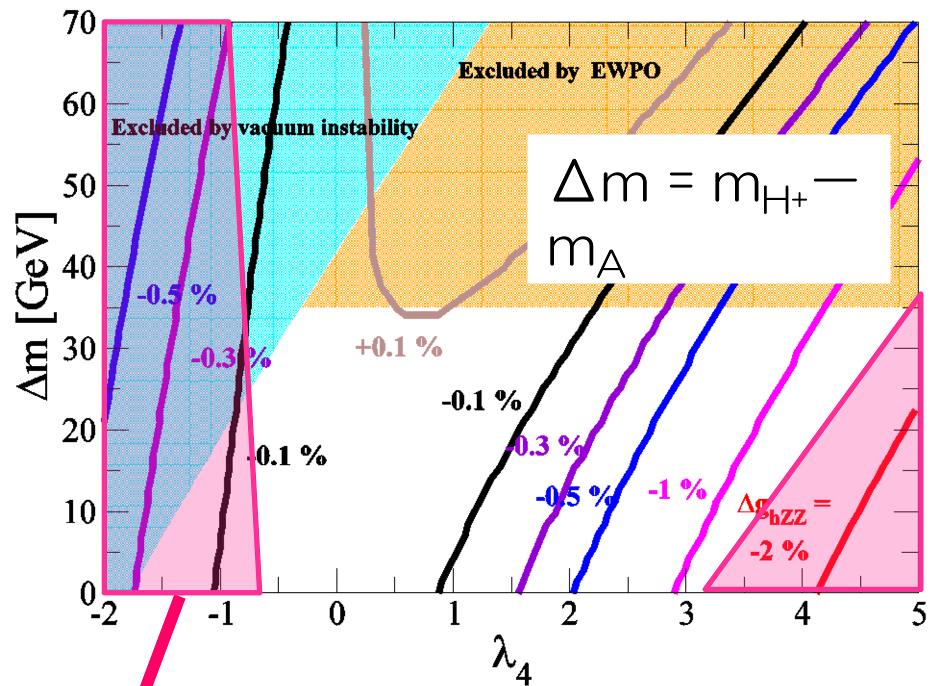
$$\Delta g_{hVV} \equiv \frac{\mathrm{Re}M_1^{hVV} - \mathrm{Re}M_1^{hVV}(\mathrm{SM})}{\mathrm{Re}M_1^{hVV}(\mathrm{SM})}$$

$$\Delta g_{hWW} \simeq -\frac{v^2}{32\pi^2}\left[\frac{{\lambda_4}^2}{6{m_{H++}}^2} + \frac{\left(\lambda_4+\frac{\lambda_5}{2}\right)^2}{6{m_{H+}}^2} + \frac{(\lambda_4+\lambda_5)^2}{6{m_A}^2} + \frac{(\lambda_4+\lambda_5)^2}{6{m_H}^2}\right] + \frac{1}{4\pi^2}\frac{2({c_W}^2-{s_W}^2)}{3{s_W}^2}\left[\frac{(m_{H++}-m_{H+})^2}{v^2} + \frac{(m_{H+}-m_A)^2}{v^2}\right]$$

CaseI m_{H++}=300GeV, v_Δ=1MeV



CaseII m_A=300GeV, v_Δ=1MeV



Excluded region from the LHC (CMS) data ($h \to \gamma\gamma$)

Δg_{hVV} can be 1%. ➡ Deviations are detectable at ILC !

hhh

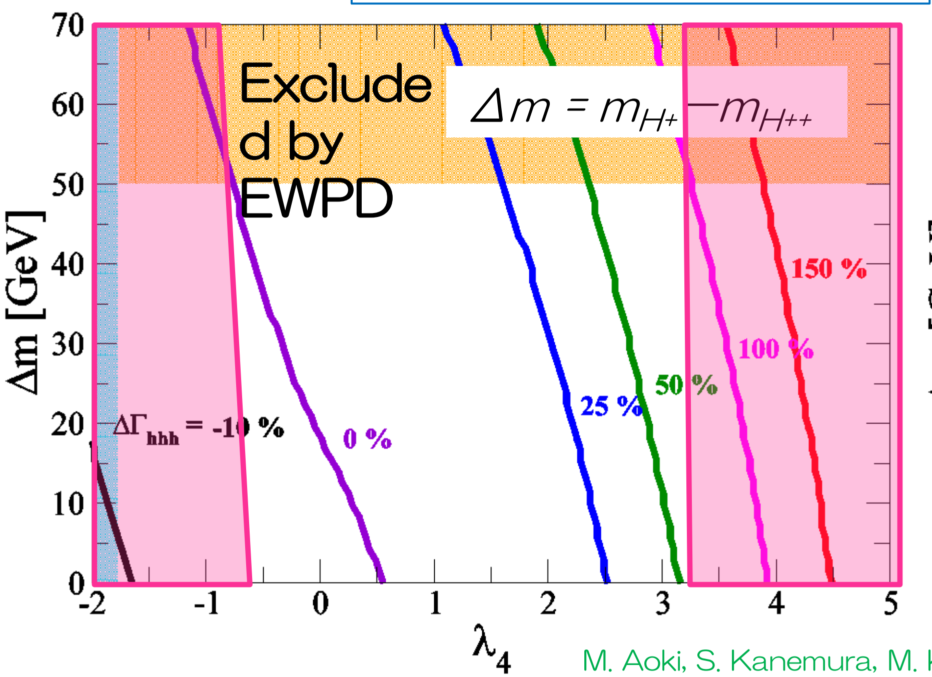
$$\Delta\Gamma_{hhh} \equiv \frac{\mathrm{Re}\Gamma_{hhh} - \mathrm{Re}\Gamma_{hhh}^{\mathrm{SM}}}{\mathrm{Re}\Gamma_{hhh}^{\mathrm{SM}}}$$

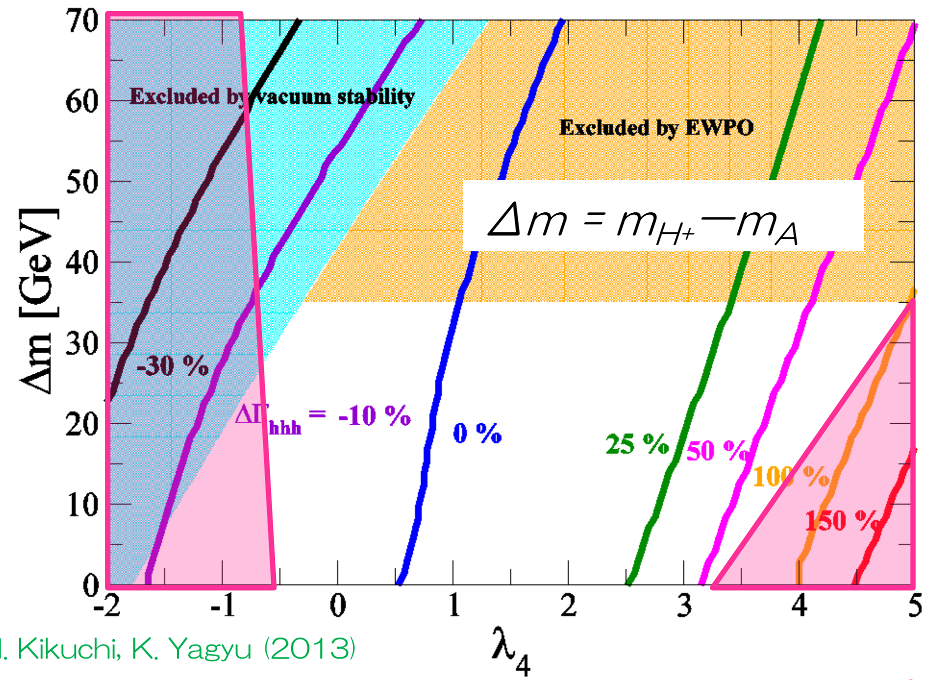
$$\Delta\Gamma_{hhh} \simeq \frac{v}{48\pi^2 m_h^2}\left(\frac{\lambda_{H^{++}H^{--}h}^3}{m_{H^{++}}^2}+\frac{\lambda_{H^+H^-h}^3}{m_{H^+}^2}+\frac{4\lambda_{hAA}^3}{m_A^2}+\frac{4\lambda_{hHH}^3}{m_H^2}\right)$$

$$\simeq \frac{v^4}{48 m_h^2\pi^2}\left[\frac{\lambda_4^3}{m_{H^{++}}^2}+\frac{\left(\lambda_4+\frac{\lambda_5}{2}\right)^3}{m_{H^+}^2}+\frac{(\lambda_4+\lambda_5)^3}{2m_A^2}+\frac{(\lambda_4+\lambda_5)^3}{2m_H^2}\right]$$

CaseI — $\lambda_5 \geq 0$ — $m_{H^{++}}$=300GeV, v_Δ=1MeV

CaseII — $\lambda_5 \leq 0$ — m_A=300GeV, v_Δ=1MeV





M. Aoki, S. Kanemura, M. Kikuchi, K. Yagyu (2013)

$\Delta\Gamma_{hhh}$ can be 50%. → Deviations are detectable at ILC !