



ECT\*



## Total, elastic and inelastic cross-sections at LHC



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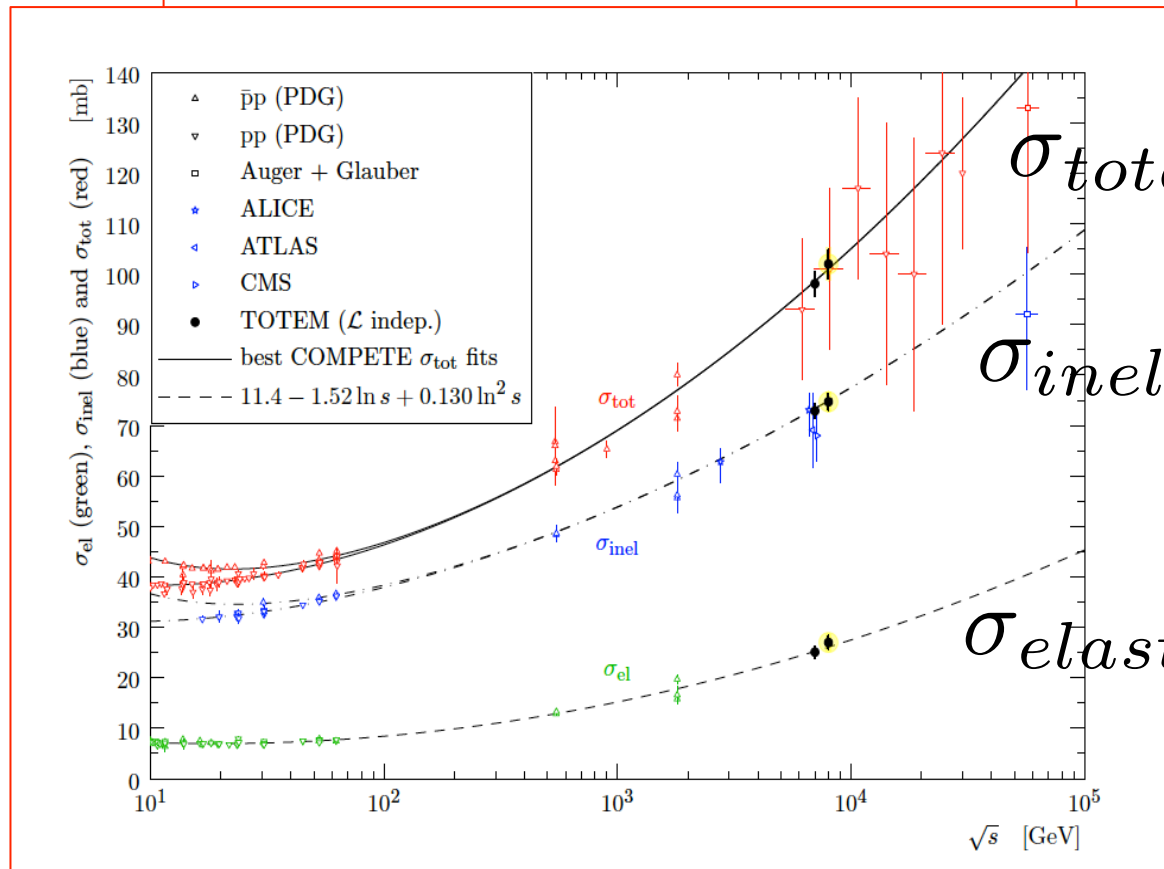
Trento, September 16-20, 2013

# Outline

- Overview of data at 7 and 8 TeV (TOTEM)
- The total cross-section
  - eikonalized minijets with IR gluon resummation model
- The inelastic cross-section
- The elastic differential cross-section
  - empirical model a' la Barger&Phillips+ proton FF
- The black disk limit with empirical model

# Total cross-sections: do we understand them?

**TOTEM** plot : total, elastic, inelastic

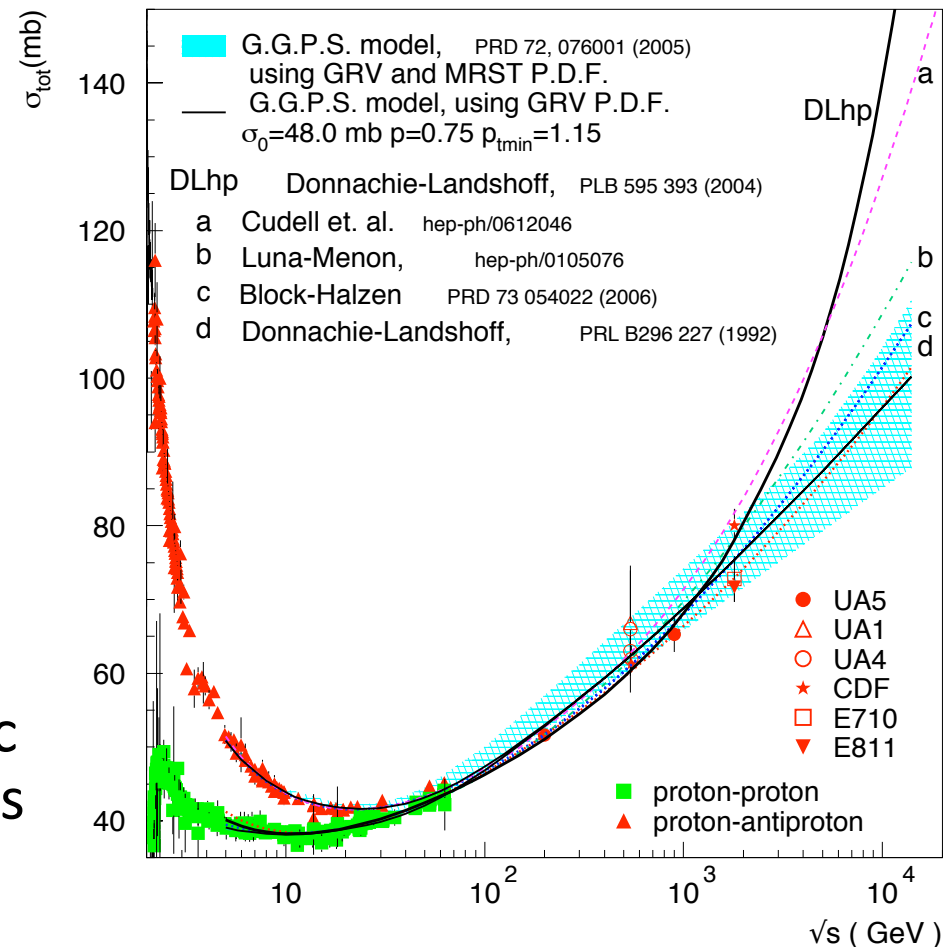


# The total pp cross-section structure

- Not much structure
- Very slow initial descent  $\sim$  constant
- A moderate rise
- General theorems help to model it, i.e. Froissart bound, optical theorem

# A 2008 compilation PLB2008

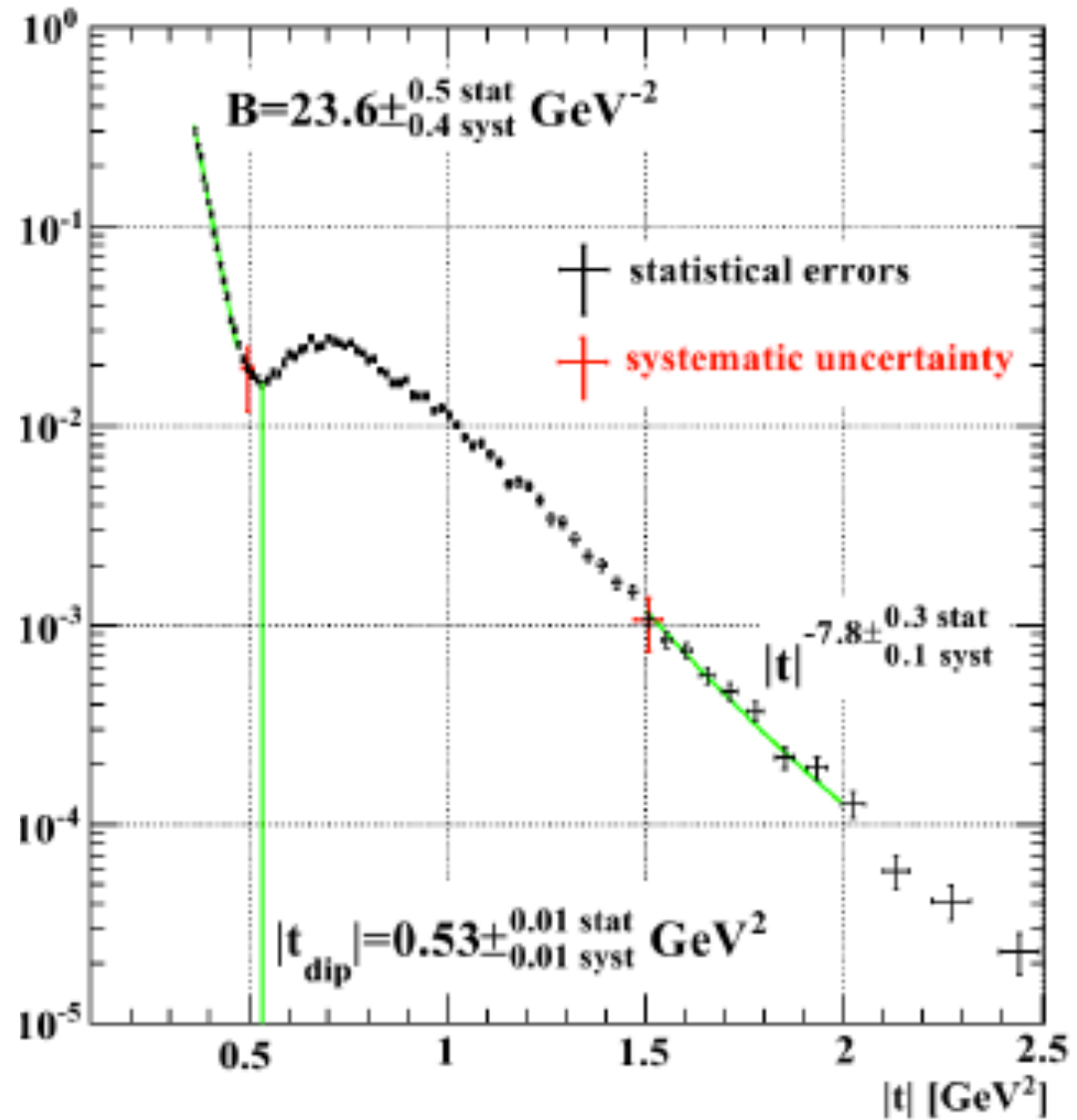
- Many models combine Regge-Pomeron behaviour and eikonal representation
- it is relatively easy to describe the total cross-section with few parameters, but a microscopic QCD description is lacking
- Not so easy for the inelastic or the elastic differential (as Laszlo Jenkovszki showed)



# TOTEM data 2011: the elastic differential cross-section

The return of the  
**dip!** It had not  
been seen since ISR

[because it not really present in pbarp]



# The 4 components of the elastic scattering amplitude :

- The **optical** point

$$\frac{d\sigma}{dt} \Big|_{t=0} \propto \sigma_{tot}^2$$

- The **forward**  
precipitous descent

$$\frac{d\sigma}{dt} \Big|_{t \sim 0} \propto e^{-Bt}$$

- The **dip** in pp (and not in pbarp) *a phase ?*

- The **tail**

$$\frac{d\sigma}{dt} \sim t^{-(7 \div 8)}$$

The total pp cross-section:

$$\sqrt{s} \sim (0.002 - 57) \text{ TeV}$$

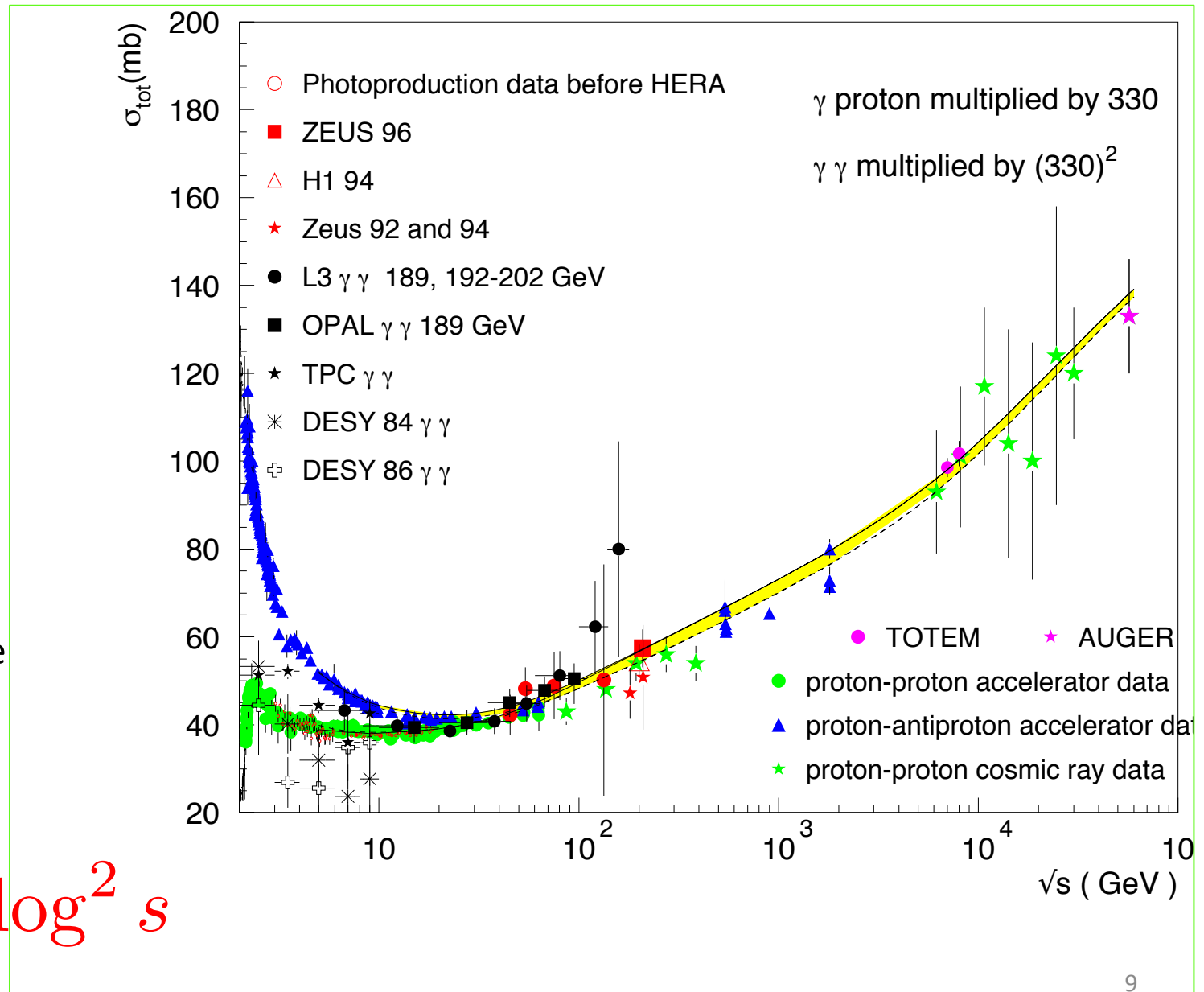
5 decades in energy!

~ 50 years of measurements



# The total cross-section

- Past ISR energies all total cross-sections rise
- The rise is fast at the beginning
- General theorems restrict the rate at which the cross-section rises



# Models since ~1950

- Heisenberg 1952 *constant*  $\simeq \sigma_{total} \lesssim \log^2 s$
- Froissart limit *cut – off in b – space*
- Regge theory + optical theorem
- Eikonal models a' la Glauber
- Regge+eikonal
- Pomeron 1+2+3 ...
- QCD Minijets

# Basic tension between Regge vs. eikonal: total x-section

- Regge + optical theorem:

t-space

$$\mathcal{A}(s, t) \simeq i\beta(t) s^{\alpha(t)-1}$$

$$\sigma_{tot} = 4\pi \Im m \mathcal{A}(s, t = 0)$$

$$\simeq s^{\alpha(0)-1}$$

- Rise  $\sim \alpha(0) = 1 + \epsilon > 1$

Donnachie Landshoff

$$\sigma_{total} = X s^{-\eta} + Y s^{\epsilon}$$

- NO Froissart bound

- Eikonal models: b-space

$$\mathcal{A}(s, t) = \int d^2\mathbf{b} e^{i\mathbf{q}\cdot\mathbf{b}} [1 - e^{i\chi(b, s)}]$$

Simplest : Black Disk Limit

$$i\chi(b, s) = -\theta(R(s) - b)$$

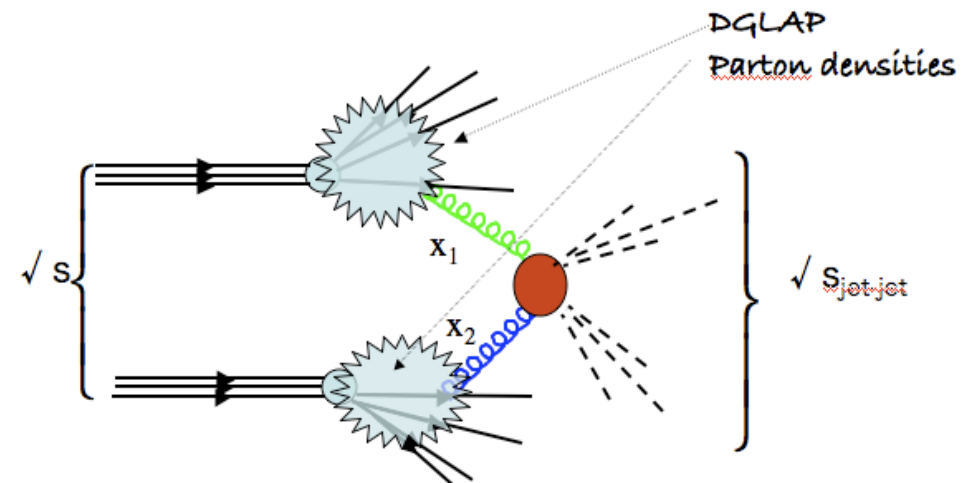
$$\sigma_{total} = \pi R^2(s)$$

Expanding radius  $\sim \log s$

Froissart bound OK because  
of cut-off in b-space

New entry when rise was first seen:  
**partons** (Halzen 1973) , later aka mini-jet models

- **Rise** is obtained from QCD minijets because of  $1/x$  **gluon** distributions
- Low energy behaviour is parametrized
- To satisfy **unitarity**, minijets are embedded into an **eikonal** and then **impact** parameter distributions are needed as extra input



Our QCD model for the total cross-section  
R. Godbole, A. Grau, GP, YN Srivastava

$$\sigma_{total} \simeq 2 \int d^2\vec{b} [1 - e^{-\chi_I(b,s)}]$$
$$2\chi_I(b,s) = \sigma_{soft} + A(b,s)\sigma_{jet}$$

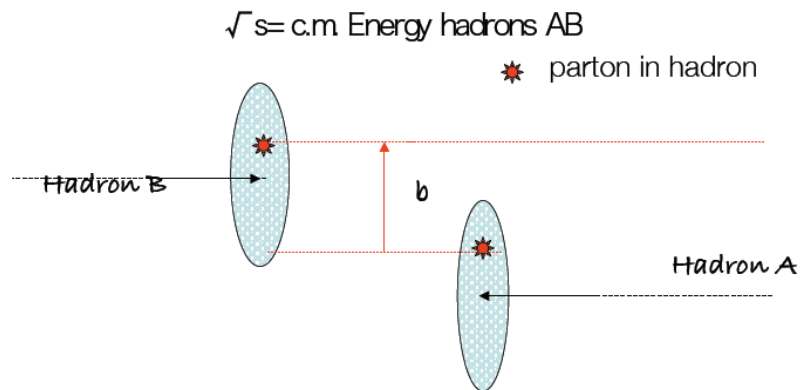
- **Minijets** to drive the rise
- Soft kt-**resummation** to tame the rise and introduce the cut-off needed to satisfy the Froissart bound
- Phenomenological singular but integrable soft gluon coupling to relate confinement with the rise
- Interpolation between soft and asymptotic freedom region

We model the impact parameter distribution as the Fourier-transform of ISR soft  $k_t$  distribution and thus obtain a cut-off at large distances : Froissart bound?

$$A_{BN}(b, s) = N \int d^2\mathbf{K}_\perp e^{-i\mathbf{K}_\perp \cdot \mathbf{b}} \frac{d^2 P(\mathbf{K}_\perp)}{d^2\mathbf{K}_\perp} = \frac{e^{-h(b, q_{max})}}{\int d^2\mathbf{b} e^{-h(b, q_{max})}}$$

$$h(b, E) = \frac{16}{3\pi} \int_0^{q_{max}} \frac{dk_t}{k_t} \alpha_{eff}(k_t) \ln\left(\frac{2q_{max}}{k_t}\right) [1 - J_0(bk_t)]$$

$$\alpha_{eff}(k_t \rightarrow 0) \sim k_t^{-2p}$$



$$A_{BN}(b, s) \sim e^{-(b\bar{\Lambda})^{2p}}$$

$q_{tmax}$

?

Fixed by single gluon emission kinematics

In our model, the emission of singular infrared gluons tames low-x gluon-gluon scattering (mini-jets) and restores the Froissart bound

$$\sigma_{tot}(s) \approx 2\pi \int_0^\infty db^2 [1 - e^{-C(s)e^{-\epsilon} e^{-(b\bar{\Lambda})^{2p}}}]$$

$$\sigma_{tot}(s) \rightarrow [\epsilon \ln(s)]^{(1/p)} \quad \frac{1}{2} < p < 1$$

# The inelastic total cross-section



# The inelastic total cross-section in eikonal models

$$\sigma_{total} = \sigma_{elastic} + \sigma_{inelastic}$$

$$t = 0$$

$$-t \geq 0$$

$$\mathcal{A}(s, t)$$

$$\sigma_{inelastic} = \int d^2\mathbf{b} [1 - e^{-2\Im m\chi(b,s)}]$$

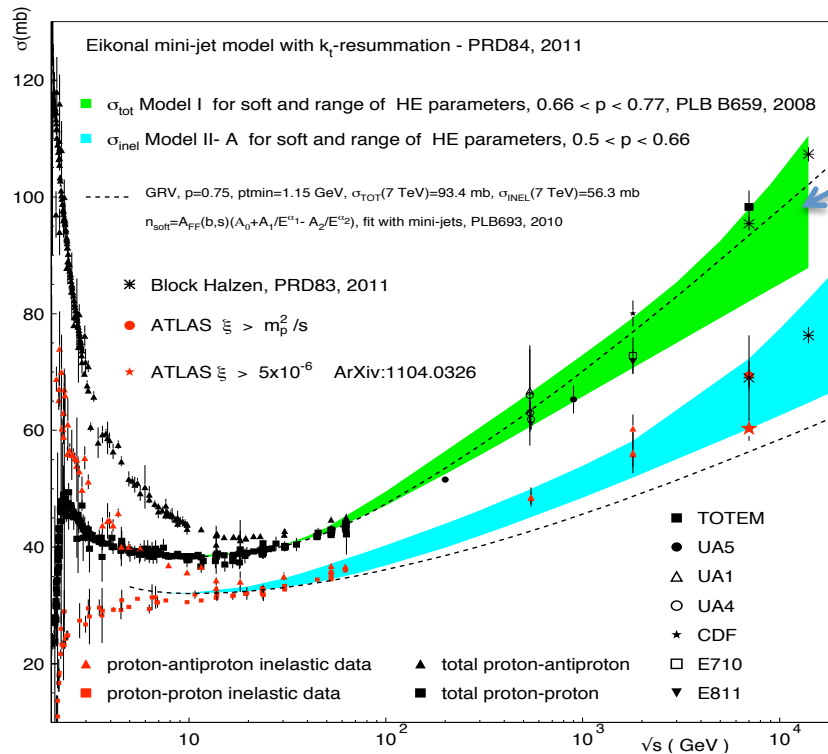
Only Poisson – independent-collisions-nodiffraction

# Inelastic and elastic in eikonal models

- One channel: diffraction, i.e. correlated events, is added into the expression for elastic (PRD2012)
- Two channels, Khoze et al. +
- Three channels...Gostman et al. +
- Continuous distributions, Lipari and Lusignoli

# The inelastic cross-section

PLB 2008  
Band  
for the total

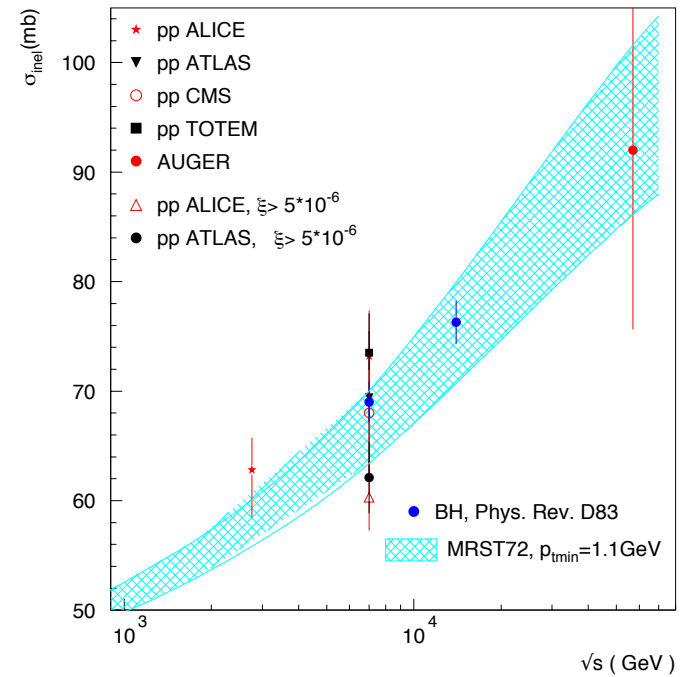
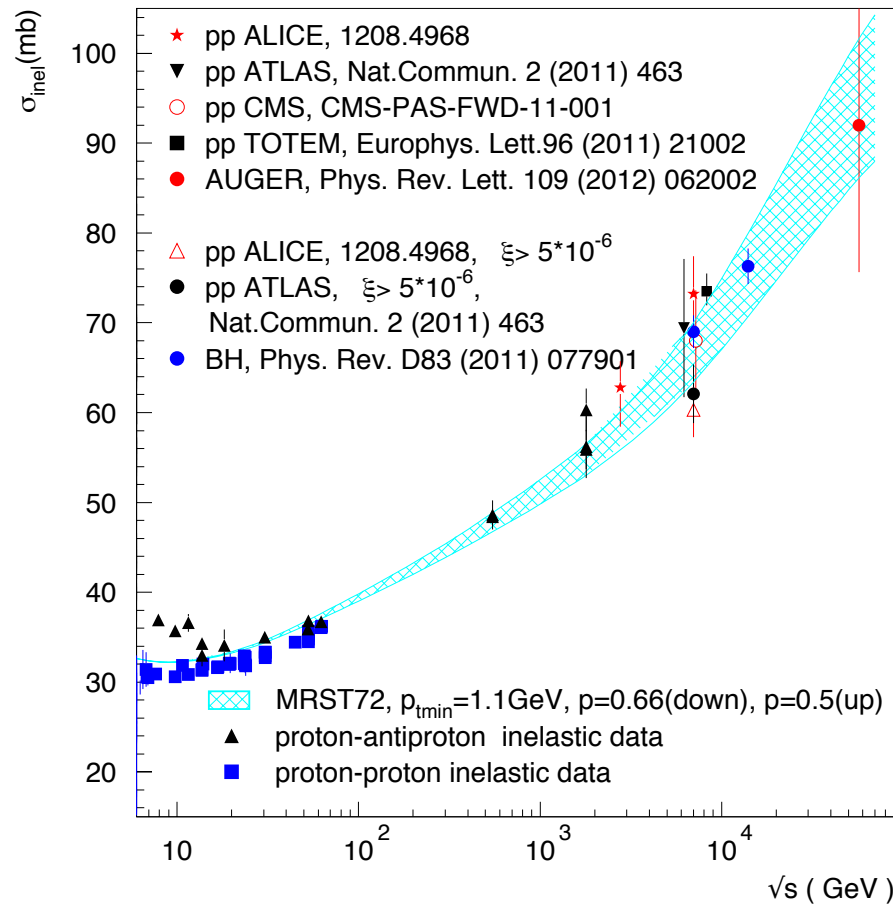


Inelastic cross-section PRD2012

It is not so clear experimentally nor theoretically: need cuts and models or parametrizations for diffraction

# Update of PRD2012 analysis

With Olga Shekhovtsova



Why the uncertainty in the inelastic? Models for diffraction 20

# The elastic differential cross-section

# Basic tension between Regge vs. eikonal: differential elastic

- Regge and Pomeron exchange:

t-space

$$\mathcal{A}(s, t) \simeq i\beta(t) s^{\alpha(t)-1}$$

- Donnachie and Landshoff parametrization

$$\sigma_{total} = X s^{-\eta} + Y s^{\epsilon}$$

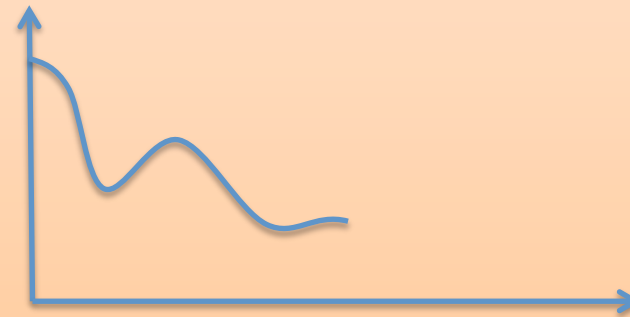
- Diffraction cone behaviour

$$d\sigma/dt \sim e^{(\alpha' \log s)t}$$

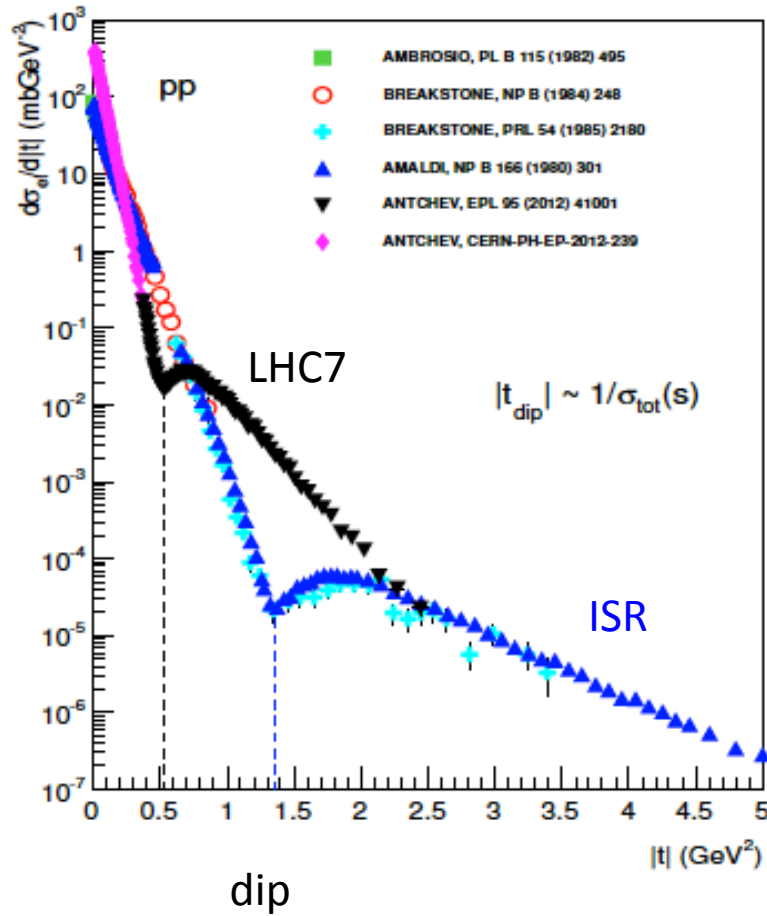
- Eikonal models: b-space

$$\mathcal{A}(s, t) = \int d^2\mathbf{b} e^{i\mathbf{q}\cdot\mathbf{b}} [1 - e^{i\chi(b,s)}]$$

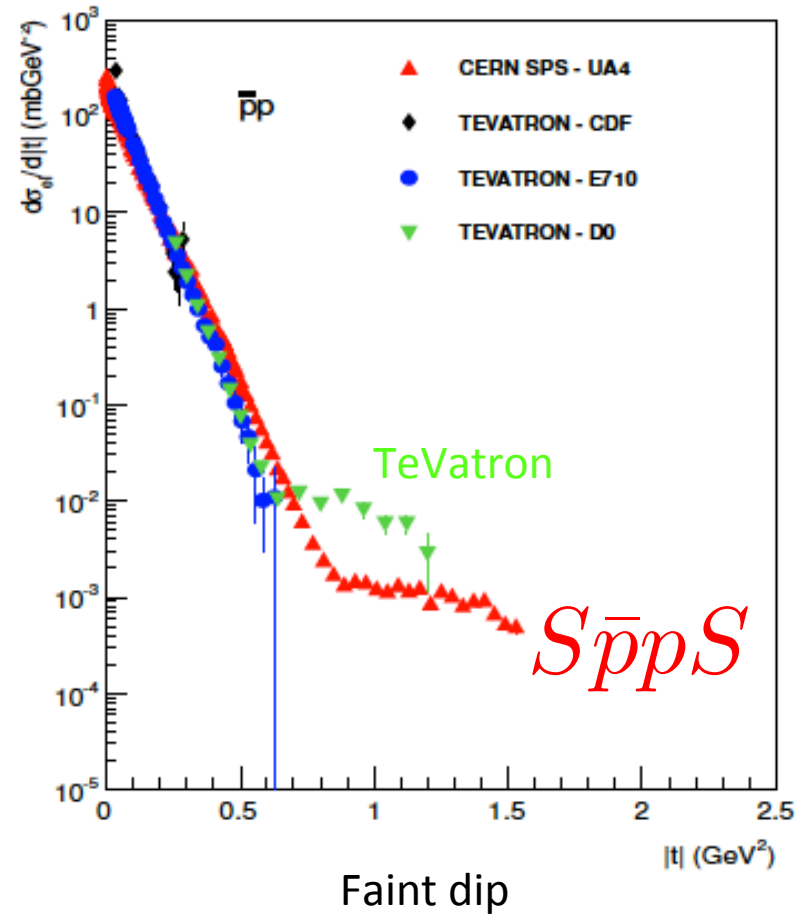
Predict a diffraction pattern in t-space



# Elastic ISR, LHC $pp$

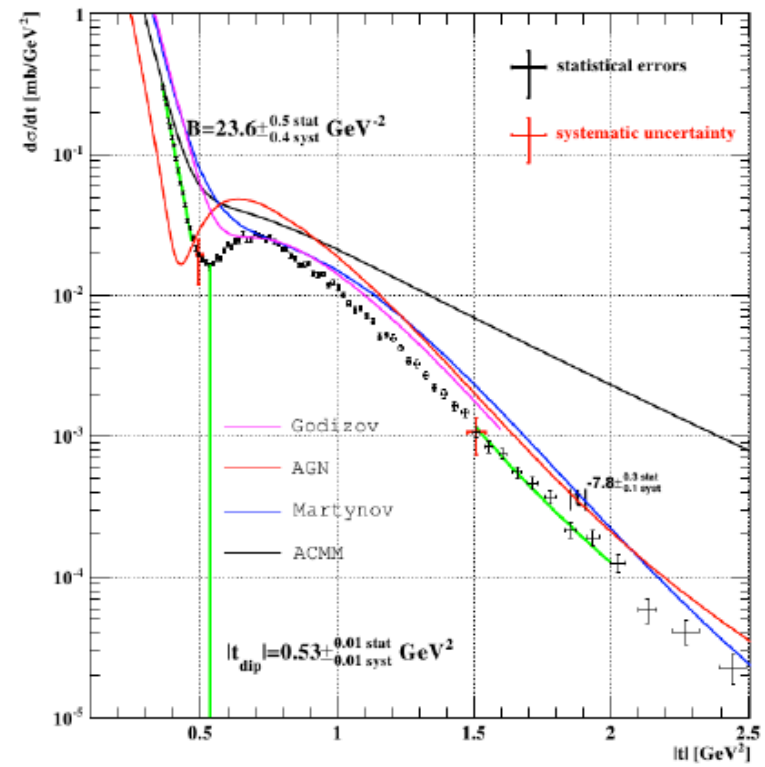
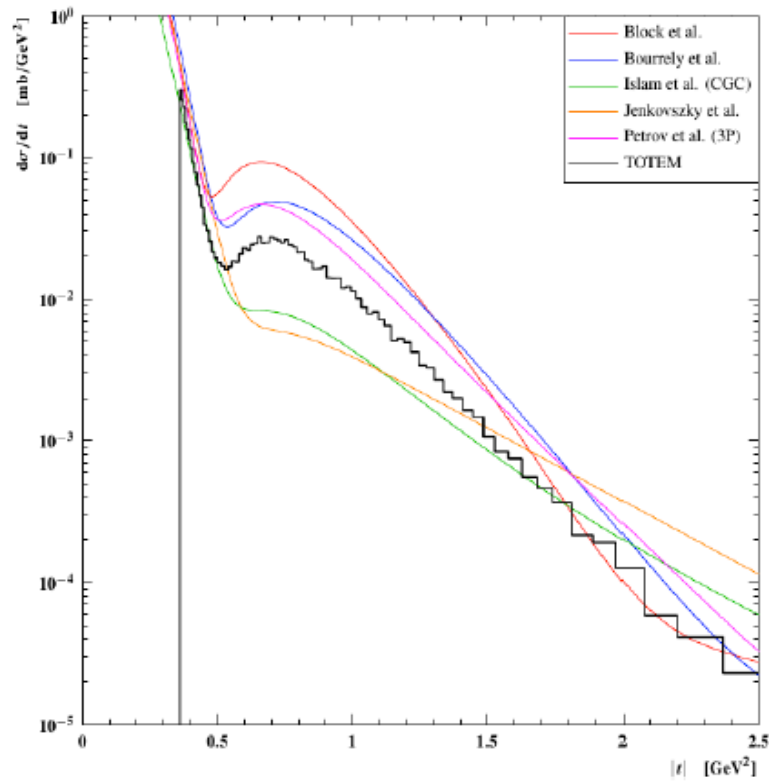


# $S p\bar{p} S$ , Tevatron $p\bar{p}$



From D. Fagundes, DIS 2013 Marseille

# LHC run at $\sqrt{s} = 7$ TeV from TOTEM<sup>2</sup>

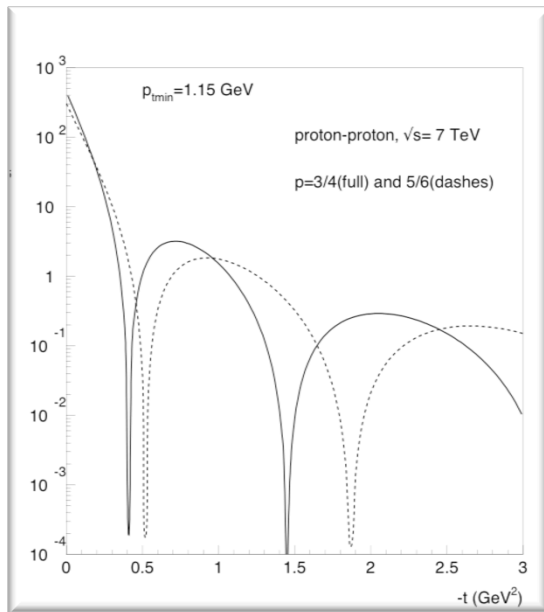


none of the representative models reproduce the data

<sup>2</sup>Left: G. Antchev *et al.*, *Europhys.Lett.* 95 (2011) 41001. Right: A.A. Godizov, *PoS (IHEP-LHC-2011) 005*.

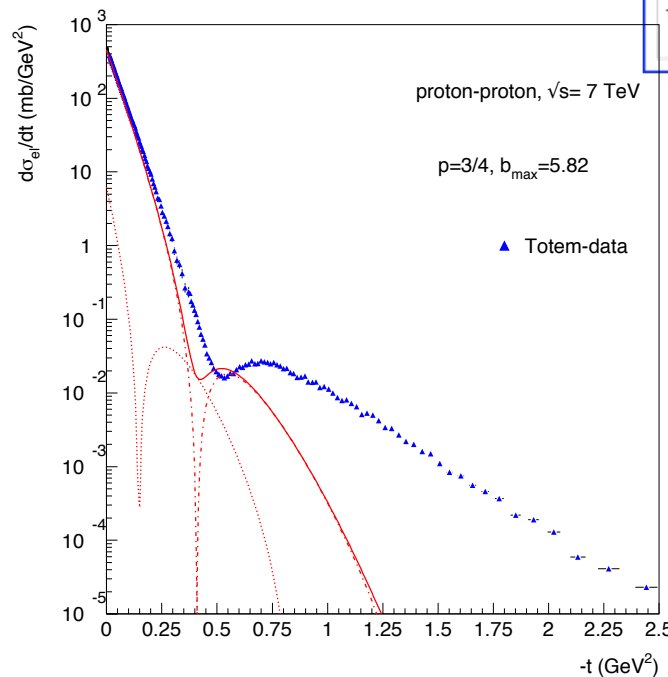
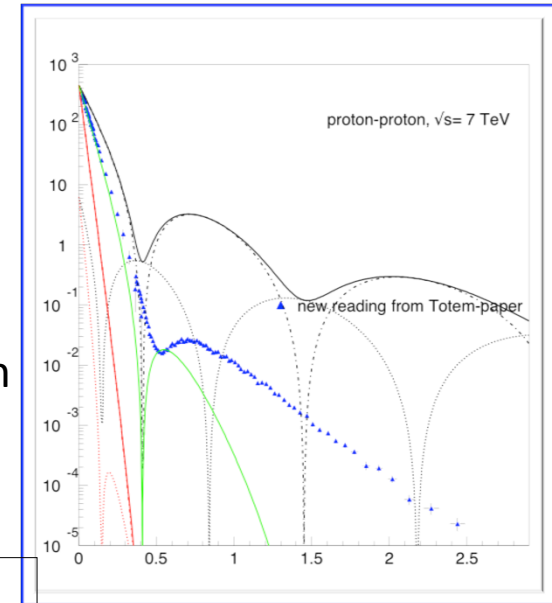


# Our QCD one-channel eikonal model with mini-jets and resummation



Purely imaginary eikonal

With real part a' la Martin



Complex eikonal  
 With a gaussian cutoff  
 In b-space: but the dip and tail are still wrong  
 Parameters could be added  
 But choose to change strategy

# Back to the 4 components of the elastic scattering amplitude : an empirical analysis

- The optical point

$$\frac{d\sigma}{dt} \Big|_{t=0} \propto \sigma_{tot}^2$$

- The forward precipitous descent

$$\frac{d\sigma}{dt} \Big|_{t \sim 0} \propto e^{-Bt}$$

- The dip in pp (and not in pbarp) *a phase ?*

- The tail

$$\frac{d\sigma}{dt} \sim t^{-(7 \div 8)}$$

# Empirical model for pp scattering from ISR to LHC, from the optical point to past the dip

$$\mathcal{A}(s, t) = i[G(s, t) \sqrt{A(s)} e^{B(s)t/2} + e^{i\phi(s)} \sqrt{C(s)} e^{D(s)t/2}].$$

$$G(s, t) \equiv 1$$

Barger-Phillis 1973  
ISR data

Grau, GP, Pacetti, Srivastava 2012  
ISR & LHC7

This work, 2013, with D. Fagundes

$$G(s, 0) = 1$$

Pion-loop singularity  
Anselm&gribov, KMR,  
Jenkowszki

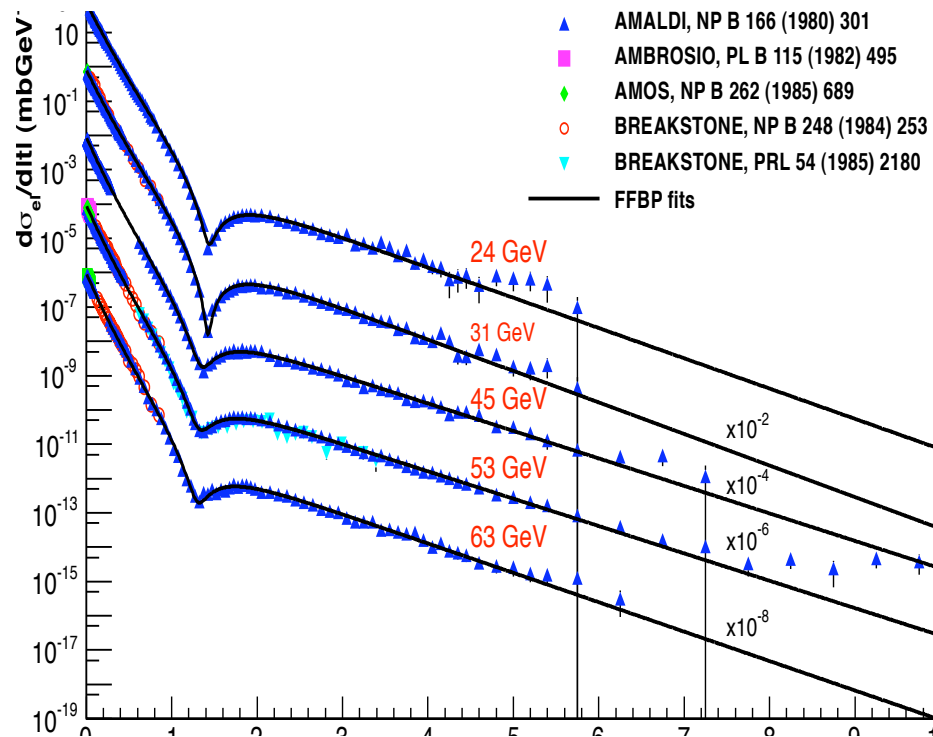


$$G(s, t) = \left[ \frac{1}{(1 - t/t_0)^2} \right]^2$$

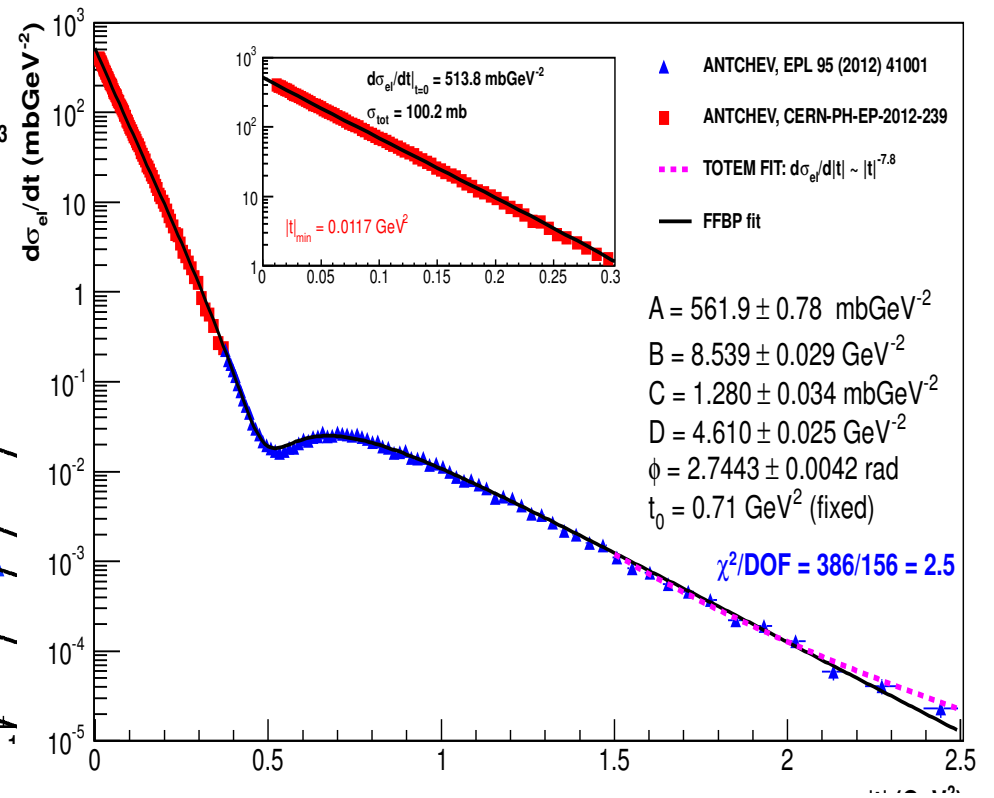
Proton form factor

# BP model with Proton Form Factor

## ISR for pp



## TOTEM LHC7 for pp



# How about physical meaning and predictions for higher energies?

$$\mathcal{A}(s, t) = i \left[ \underbrace{G(s, t) \sqrt{A(s)} e^{B(s)t/2}}_{\text{Leading term}} + \underbrace{e^{i\phi(s)} \sqrt{C(s)} e^{D(s)t/2}}_{\text{Non leading at } t=0} \right].$$

Form factor

Total x-section

Forward slope

Mixture of C=+1 and C=-1

# Can one make predictions?

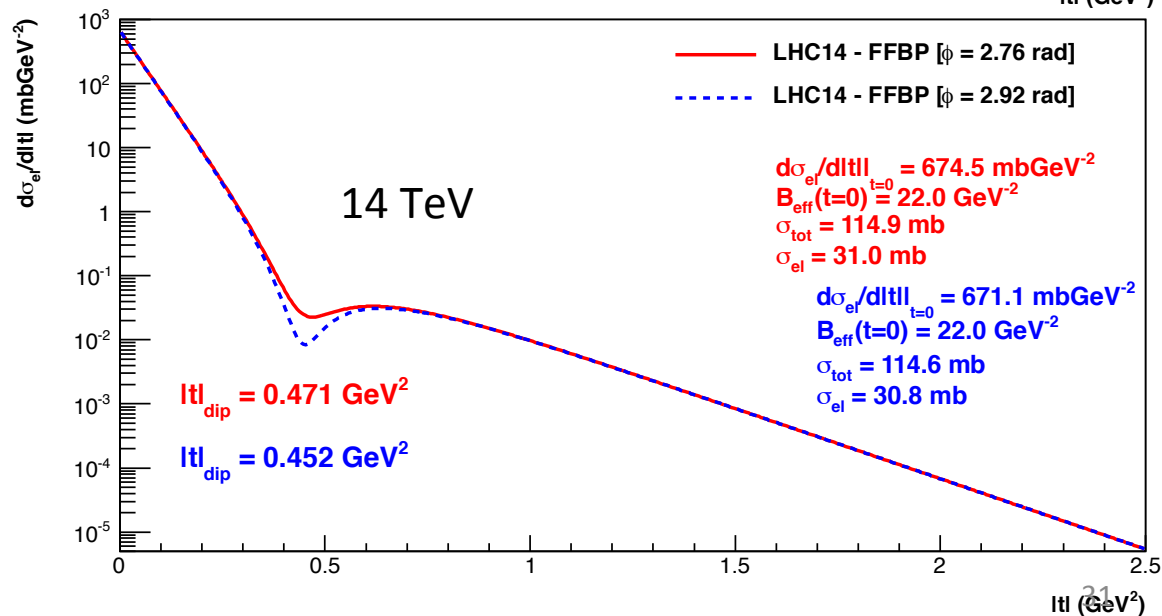
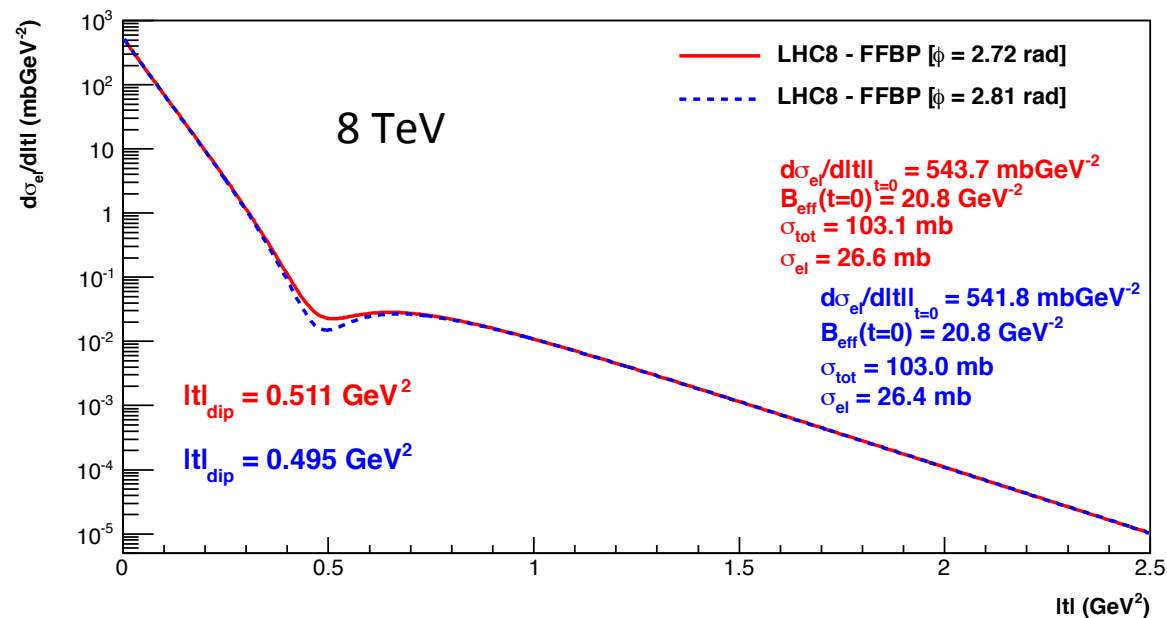
## An asymptotic model of maximal saturation

Fagundes, Grau, Pacetti, GP, Srivastava [arXiv:1306.0452](https://arxiv.org/abs/1306.0452) (to be published)

- Froissart-Martin bound
- Khuri-Kinoshita
- Total absorption at  $b=0$

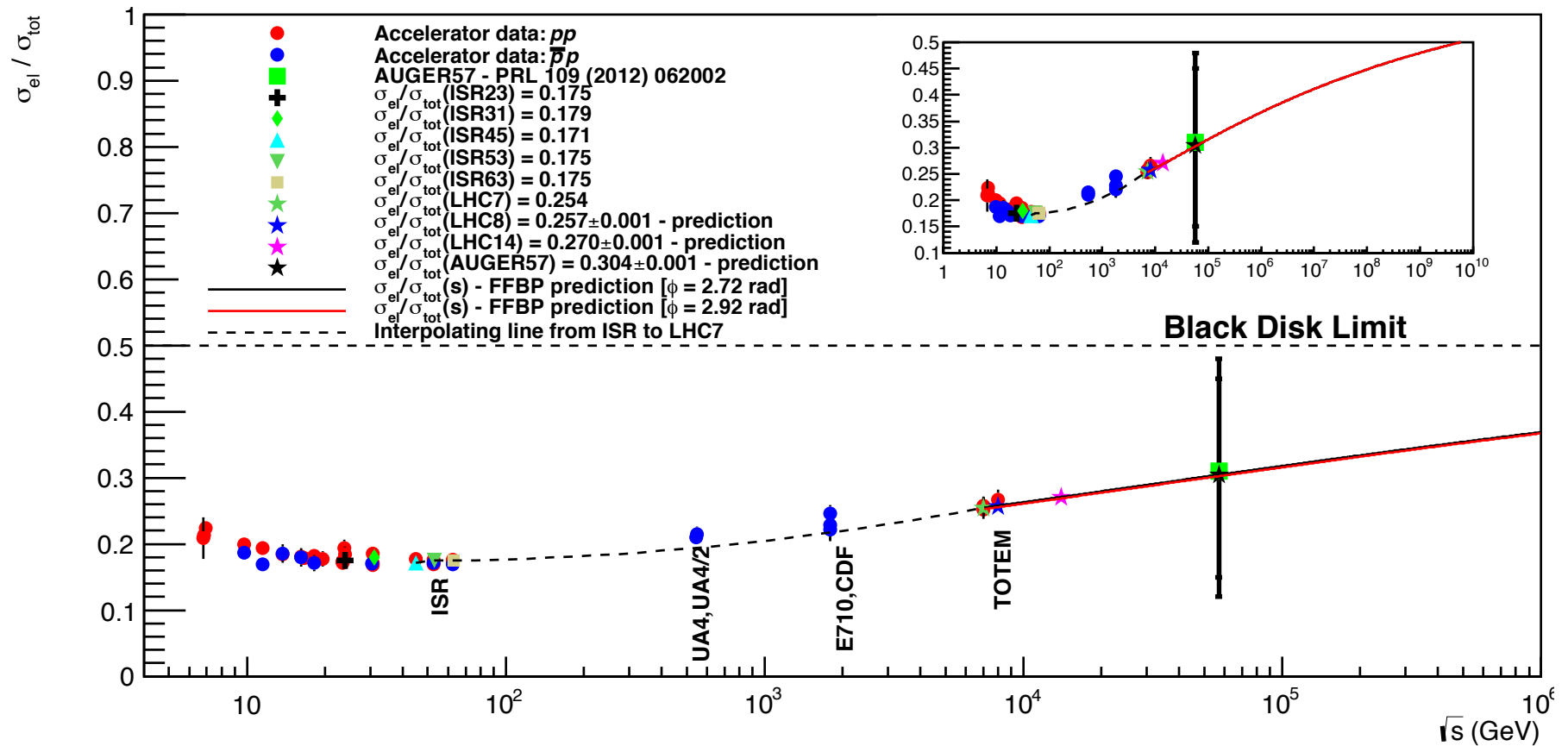
# Predictions from asymptotic model

At any given energy the difference is in the phase, which is so far unconstrained



The black disk limit in the asymptotic model

$$R_{el} = \frac{\sigma_{elastic}}{\sigma_{total}}$$



The black disk limit in this asymptotic extrapolation is not reached until

$$\sqrt{s} \sim 10^5 \text{ TeV}_{32}$$



# Outlook and conclusions

- W. Heisenberg, Z. Phys.133 (1952) 65

$$\sigma_{tot} \approx \frac{\pi}{m_{\pi}^2} \ln^2 \frac{\sqrt{s}}{\langle E_0 \rangle} \quad !!!$$

- Include Diffraction in our QCD model
- Compare with empirical BP model to understand role of non-leading term
- Wait for LHC8 and LHC14 (mostly) new data

**SPARES**

# Our proposal for running $\alpha_s(k_t)$ in the infrared region

*One gluon exchange*  $\sim r^{2p-1}$

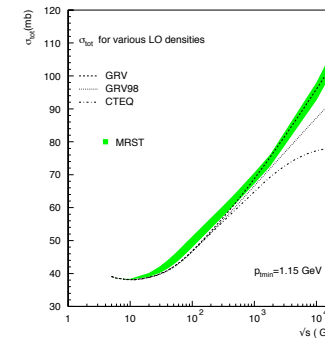
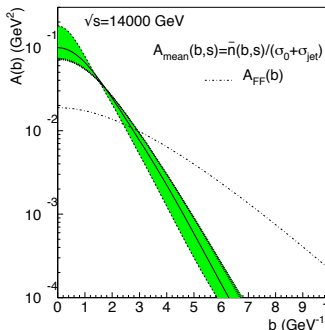
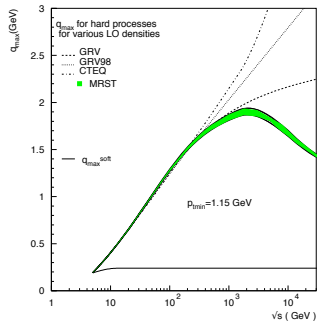
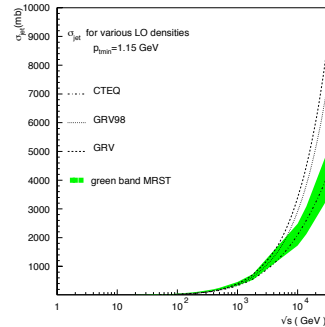
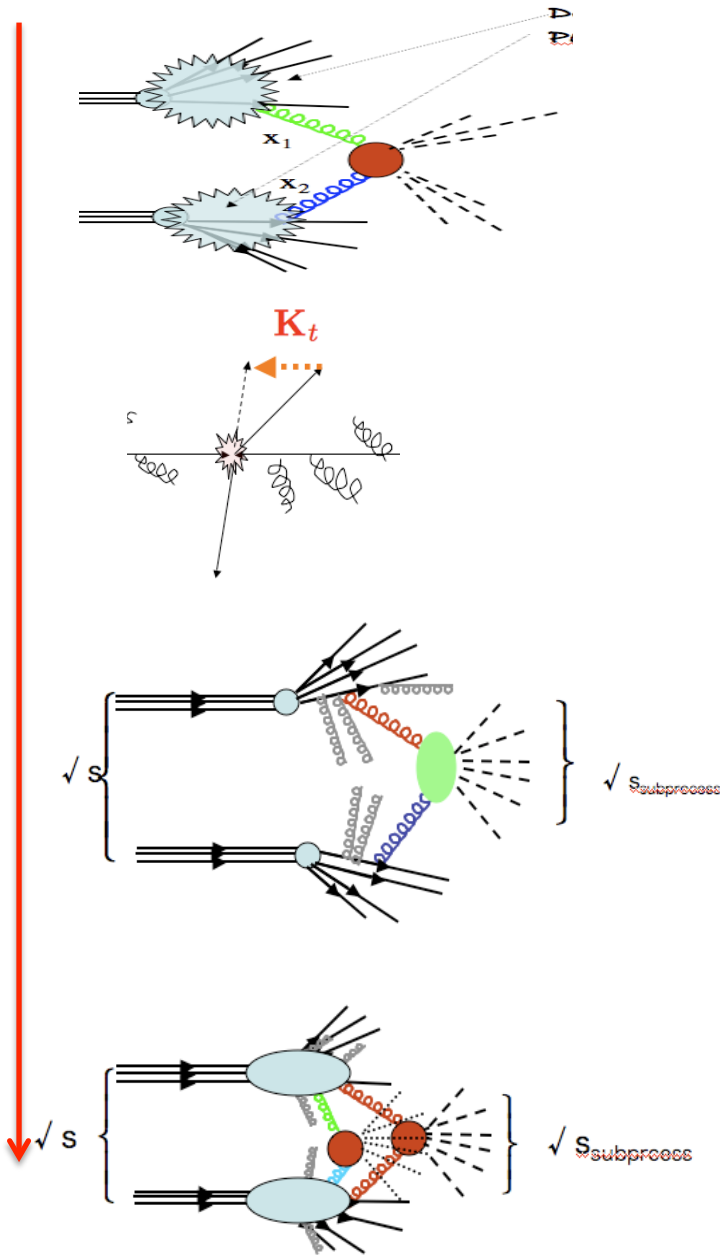
$$\propto k_t^{-2p} \quad k_t \ll \Lambda$$

To reconcile with asymptotic  
Freedom

$$\propto \frac{1}{\log k_t^2 / \Lambda^2} \quad k_t \gg \Lambda$$

A phenomenological  
interpolation

$$\alpha_{eff}(k_t) = \frac{12\pi}{11N_c - 2N_f} \frac{p}{\log[1 + p(k_t/\Lambda_{QCD})^{2p}]}$$



1. Calculate mini-jet cross-section  
Choosing densities and ptmin

$$\sigma_{mini-jet} \simeq s^\epsilon$$

$$\epsilon \simeq 0.3 - 0.4$$

2. Calculate qmax: single soft gluon  
upper scale, for given PDF, ptmin

$$q_{max} \simeq p_{tmin}$$

$$\lesssim 2 - 3 \text{ GeV}$$

3. Calculate impact parameter  
distribution for given qmax and  
given infrared parameter p

$$\chi(b, s) = \chi_{low \text{ energy}} +$$

$$+ A(b, q_{max}) \sigma_{jet}$$

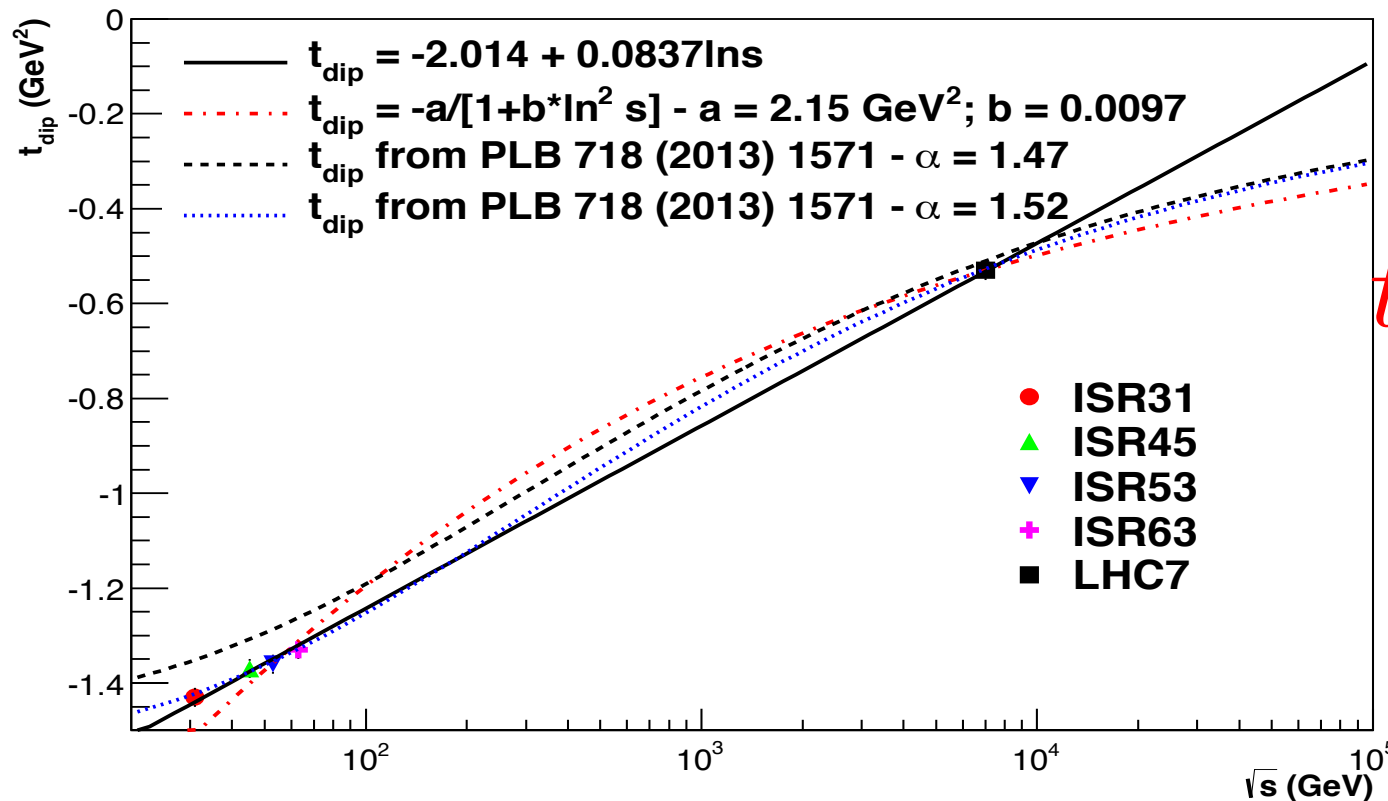
4. Eikonalize

$$\sigma_{total} = 2 \int d^2\mathbf{b} [1 - e^{-\chi(b,s)}]$$

How about  $\phi$  ?

$\phi$

- $t$ -independent in the model – probably average over  $t$
- approximately constant from ISR to LHC7
- determines the dip position (together with the other parameters)



$t_{dip} \text{ vs. } \sqrt{s}$

Fix  $\phi$  for higher energies

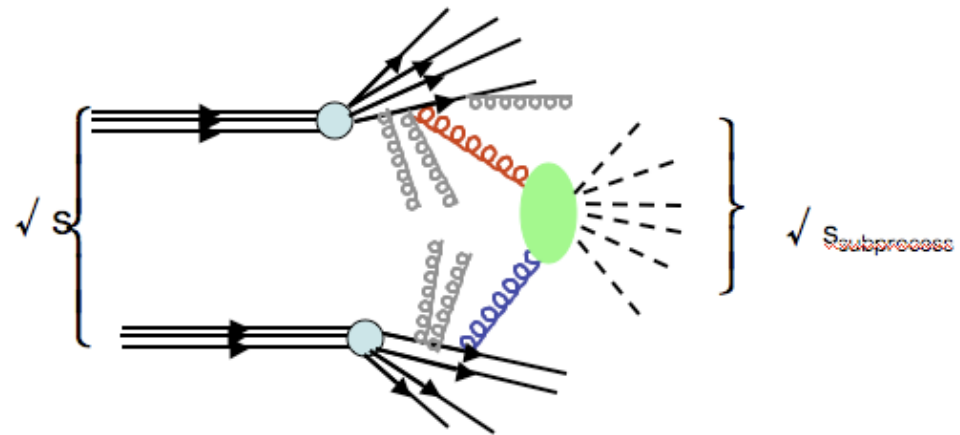
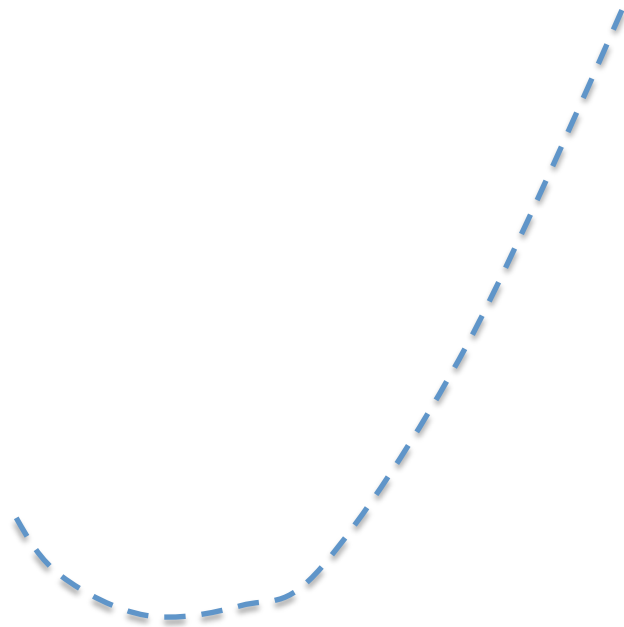
New data from LHC7 and LHC8 for  $pp$   
total, inelastic and elastic cross-  
sections

$pp$  : not seen since ISR, 40 years ago  
the cleanest way to study the proton

Given  $S_{pp\bar{p}S}$  and TeVatron for  $p\bar{p}$ ,  $pp$  at  
these energies was not so important for the  
total (Pomeranchuk theorem), but very  
important for the differential elastic x-section

# Beyond 5-10 GeV cm, all total cross-sections show same behaviour

- The RISE
  - Initial fast rise first observed at ISR
  - Slowing down of the rise confirmed at



# Outline

LHC7 and LHC8: new data for elastic and total pp scattering

Ultimate chance to study large and small distances QCD

Total cross-section: confinement dominates

Still far from understanding

A soft  $k_t$ -resummation model for total cross-section (BN model)

Application to elastic differential cross-section and difficulties

Change of perspective: find a good parametrization to analyze data

The Barger and Phillips model

Fits and facts

Asymptotic predictions

The dip

The black Disk limit

Outlook



The eikonal mini-jet model with infrared soft gluon resummation links confinement to the total cross-section

1. One channel eikonal format (to be improved next) with real profile function

$$\sigma_{total} = 2 \int d^2\mathbf{b} [1 - e^{-\chi_I(b,s)}]$$

1. Profile  $\chi_I(b,s)$  function built with

- QCD Minijets to get the rise: use actual PDF (LO)

$$\sigma_{mini-jet} \sim s^{0.3-0.4}$$

- b-distribution from soft gluon emission in parton-parton scattering leading to **saturation**

$$A(b,s) = \mathcal{F}[soft\ gluons]$$

The problem with the inelastic: the **extrapolation** to **diffractive** region where particles are correlated

$$F(s, t) = i \int d^2\mathbf{b} e^{i\mathbf{q}\cdot\mathbf{b}} [1 - e^{i\chi(b, s)}]$$

$$\sigma_{total} = 2 \int d^2\mathbf{b} [1 - \cos \Re\chi(b, s) e^{-\Im m\chi(b, s)}]$$

$$\sigma_{elastic} = \int d^2\mathbf{b} |[1 - e^{i\chi(b, s)}]|^2$$

$$\sigma_{inel} = \sigma_{total} - \sigma_{elastic} = \int d^2\mathbf{b} [1 - e^{-2\Im m\chi(b, s)}]$$

but

$$P(\{n, \bar{n}(b, s)\}) = \frac{e^{-\bar{n}(b, s)}}{n!} \bar{n}(b, s)^n$$

$$\sigma_{independent\ collisions} = \int d^2\mathbf{b} [1 - e^{-\bar{n}(b, s)}]$$

# Zero Degrees: elastic scattering, total inelastic, total cross-section

- What do we have from a theoretical point of view? A large variety of theorems based on analyticity, crossing, and unitarity, basically
- For **TOTAL CROSS-SECTION**  
**Optical theorem**, only assumption is **unitarity**,  
**Froissart bound** with assumptions

- For **ELASTIC** amplitude  $A(s, t)$   
 –  $t=0$  ok  
 Asymptotic theorems with assumptions : such as Froissart bound for **Imaginary part at  $t=0$** , Kinoshita-Khuri for  $\rho(s, t = 0)$

- Martin suggestion for

$$\Re F_+(s, t) \simeq \rho(s) \frac{d}{dt} [t \Im F_+(s, t)]$$

- for the inelastic?

$$\sigma_{inelastic} \equiv \sigma_{total} - \sigma_{elastic}$$

In Eikonal models

$$\sigma_{inel} = \sigma_{total} - \sigma_{elastic} = \int d^2\mathbf{b} [1 - e^{-2\Im\chi(b,s)}]$$

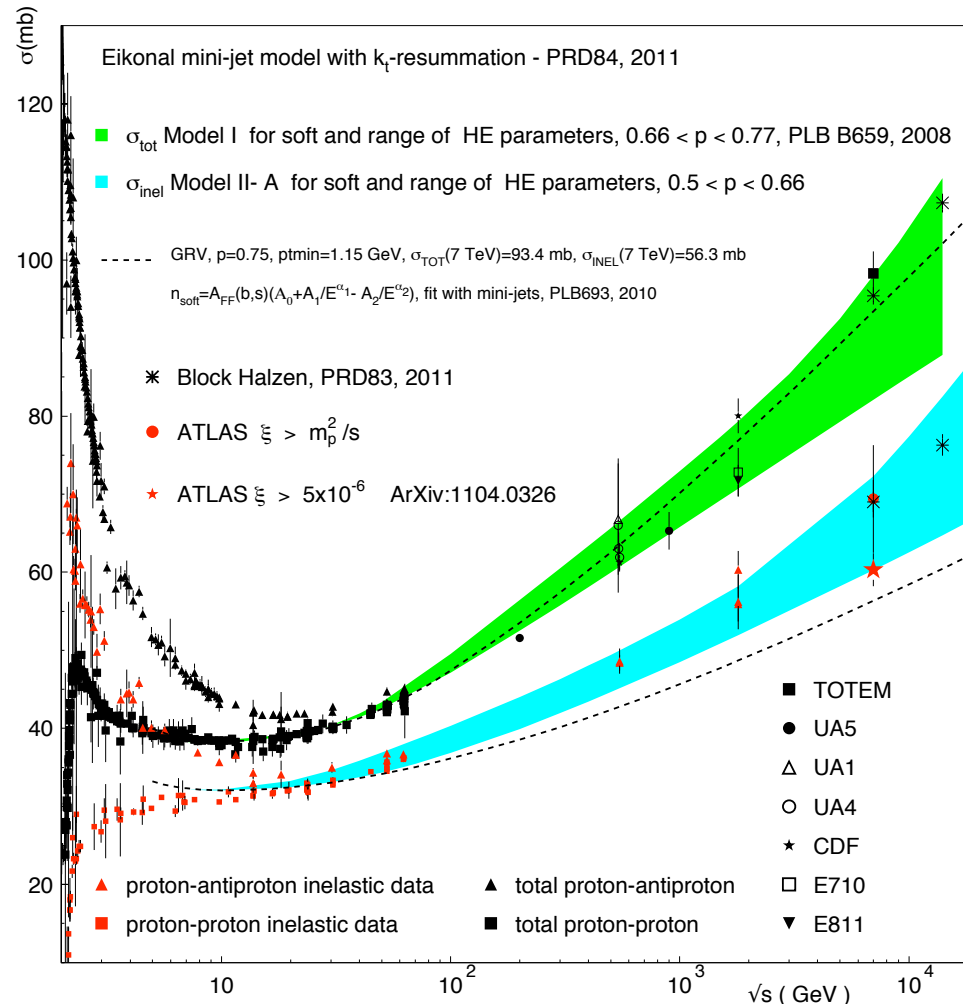
one channel eikonal approach: formula interpretation

advantage: once you have the imaginary part, you do not need further modeling, but you miss the two channel eikonal: needs further modeling

Our approach for the time being: the singularity parameter of our QCD model can span the region and then use  $\sigma_{total} = \sigma_{inel} + \sigma_{elastic}$  work is in progress, FIGURE

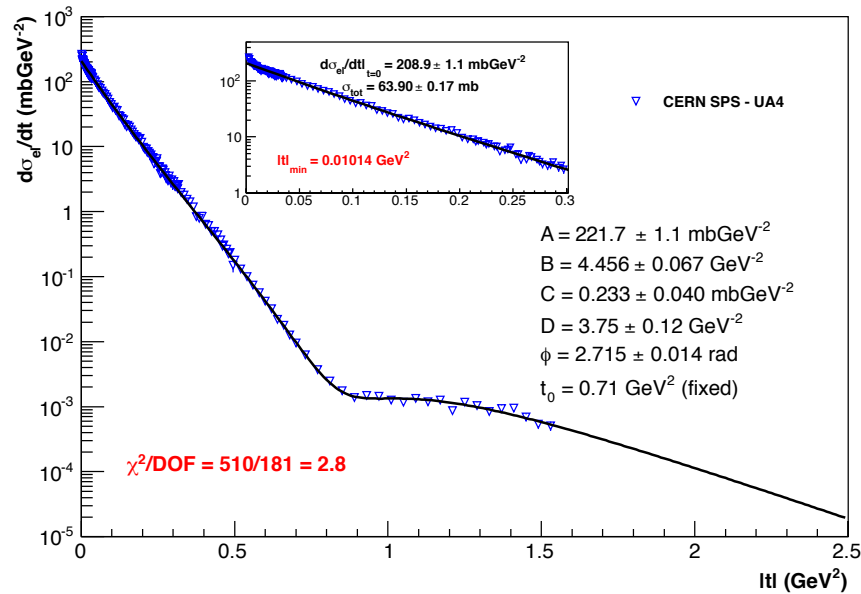
let me describe our model, whose aim is to give a QCD partonic interpretation to all the components of forward scattering, elastic, total and inelastic non-diffractive. At present clear ideas about total, some ideas about inelastic, lots of work in progress for the elastic.

# The inelastic cross-section

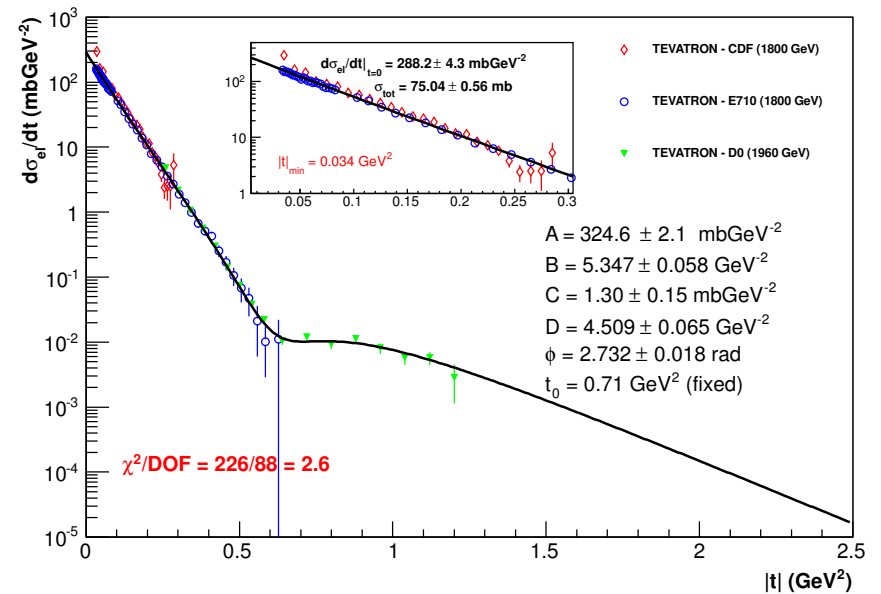


# Empirical model applied to pbarp

## UA4 data



## CD-E710-D0 data



# Asymptotic model

From asymptotic theorems

$$4\sqrt{\pi A(s)}(mb) = 47.8 - 3.8 \log s + 0.398(\log s)^2$$

$$B(s)(GeV^{-2}) = 11.04 + 0.028(\log s)^2 - \frac{8}{0.71} = -0.23 + 0.028(\log s)^2$$

$$D(s)(GeV^{-2}) = -0.41 + 0.29 \log s$$

Empirical

$$4\sqrt{\pi C(s)}(mb) = \frac{9.6 - 1.8 \log s + 0.01(\log s)^3}{1.2 + 0.001(\log s)^3}$$