BCS: 50 Years

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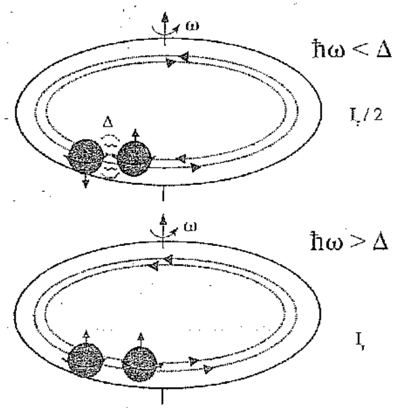
Fifty Years of Nuclear BCS

Pairing in Finite Systems



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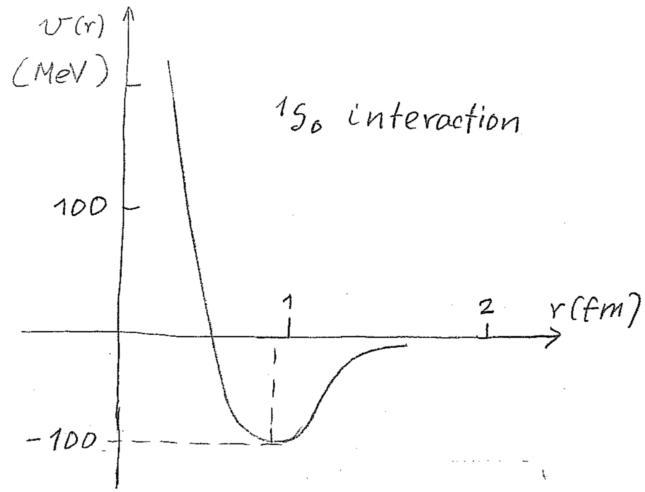


quantality parameter

$$q = \left(\frac{\hbar^2}{M a^2}\right) \frac{1}{|v_0|}$$

$q \ll 1$ crystalline structure ($T=0$)

$q \approx 1$ quantum fluid ($T=0$)

$q \approx 0.4$ nuclei

Interplay potential energy and
quantal (ZPM) fluctuations (single-particle)

$$H = T + v = \underbrace{T + U + V_p}_{\text{mean field}} + (v - U - V_p)$$

diagon. Kramers deg. $\nu\bar{\nu}$

$$\alpha^{+}_{\nu} = U_\nu a^{+}_{\nu} - V_\nu a_{\bar{\nu}} \;;$$

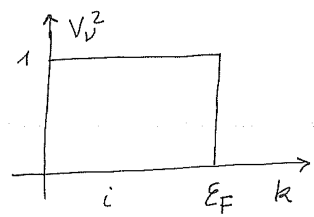
ground state $\alpha_\nu |\tilde{0}\rangle = 0$

$$|\tilde{0}\rangle = \prod_{\nu>0} \alpha_\nu \alpha_{\bar{\nu}} |0\rangle \sim \prod_{\nu>0} (U_\nu + V_\nu a^{+}_{\nu} a^{+}_{\bar{\nu}}) |0\rangle$$

$a_\nu |0\rangle = 0$

<u>Ansatz 1</u> : $|\tilde{0}\rangle$ sharp step-funct. occ.

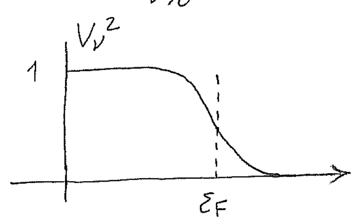
$$|HF\rangle = \prod_{i>0} a^{+}_{i} a^{+}_{\tilde{i}} |0\rangle = \prod_{i} a^{+}_{i} |0\rangle$$



independent particle motion (fermions)

<u>Ansatz 2</u> : $|\tilde{0}\rangle$ sigmoidal distr. occ.

$$|BCS\rangle = \prod_{\nu>0} (U_\nu + V_\nu a^{+}_{\nu} a^{+}_{\bar{\nu}}) |0\rangle$$



independent pair motion (bosons)

$(X>1)$

$$\alpha_0 = \langle P^+ \rangle = \frac{\Delta}{G} \approx 7 \qquad P^+ = \sum_{\nu>0} a^+_\nu a^+_{\bar\nu}$$

$$\left(X = \frac{G 2\Omega}{D} = GN(0)\right)$$

$$\alpha_{dyn} = \frac{1}{G} \; \frac{\langle PP^+ \rangle^{1/2} + \langle P^+P \rangle^{1/2}}{2}$$

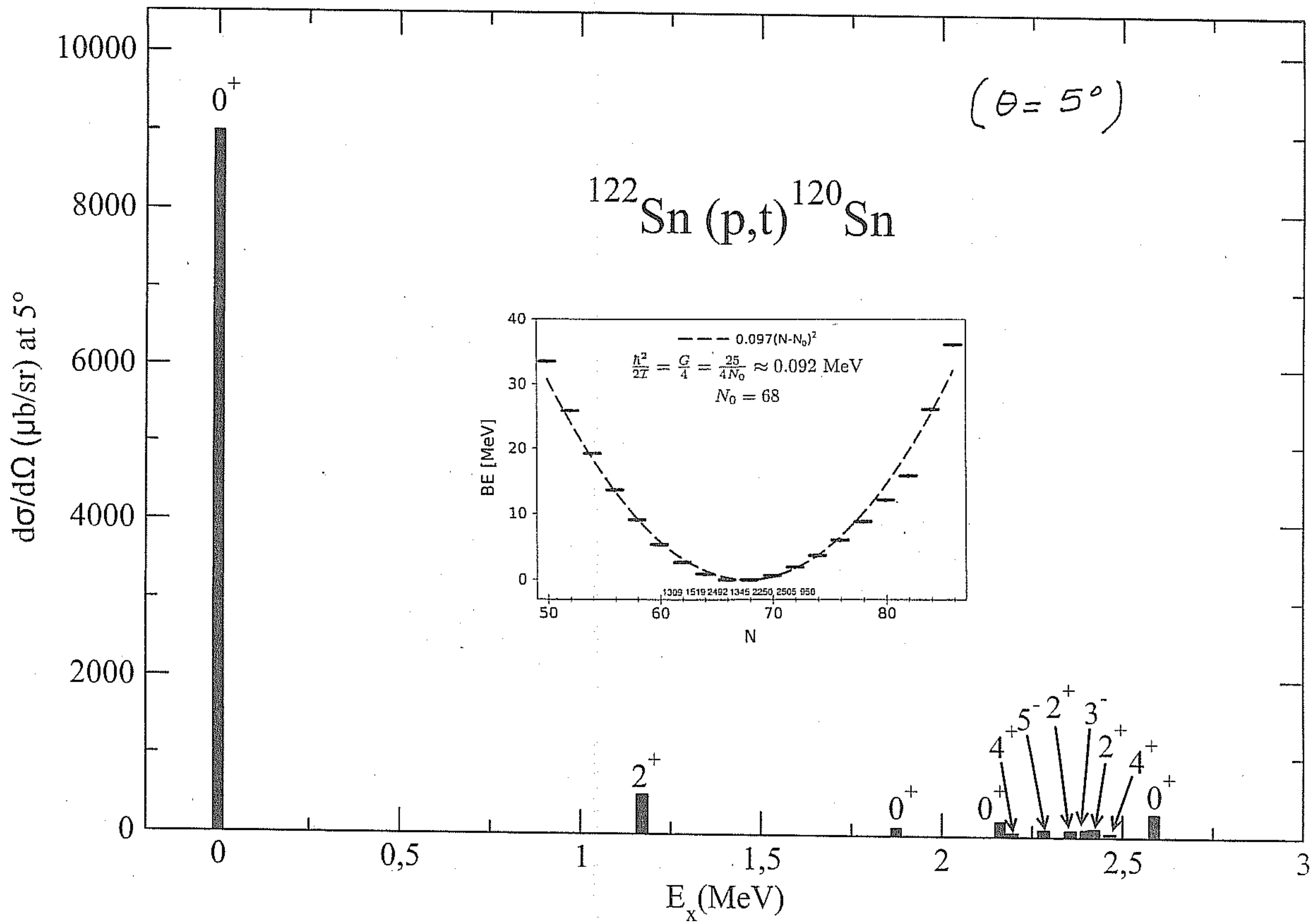
$(X<1)$

$$\approx \frac{1}{2}\left(\frac{E_{corr}(A+2)}{G} + \frac{E_{corr}(A-2)}{G}\right)$$

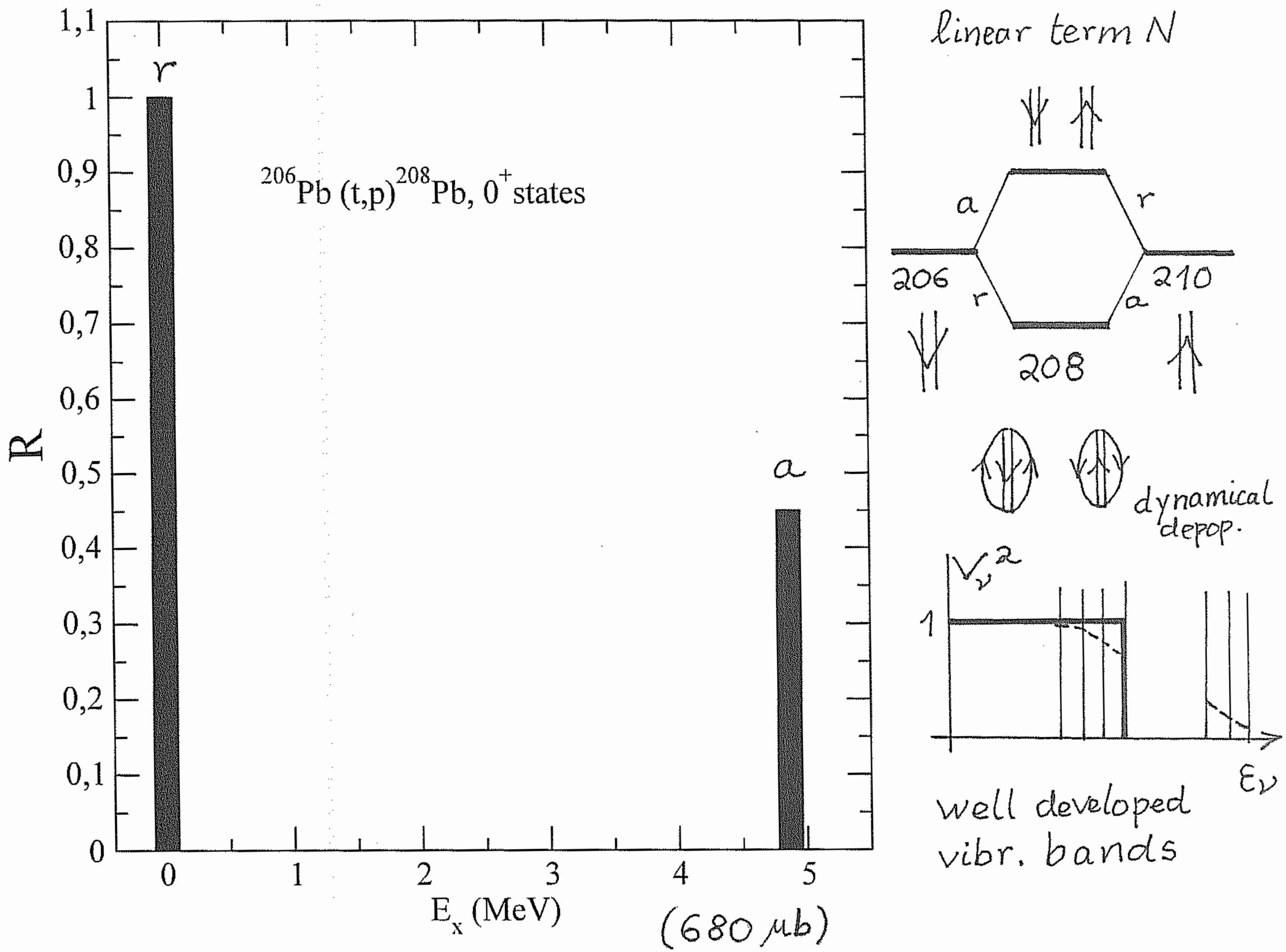
$$\approx 10$$

$$\frac{\alpha_0}{\alpha_{dyn}} \approx 0.7$$

$$\frac{\beta_2}{(\beta_2)_{dyn}} \approx 3-6$$



Potel et al., Rep. Prog. Phys. **76** (2013) 106301



^{206}Pb (t,p)^{208}Pb, 0$^+$ states
R
E_x (MeV)
r
a
(680 μb)
linear term N
a
r
206
210
r
a
208
dynamical depop.
V_ν^2
1
ε_ν
well developed vibr. bands

Spectroscopic amplitudes

pairing rotations $({}^{A+2}Sn(p,t)\,{}^{A}Sn(gs))$

$U_\nu V_\nu$ (BCS)

pairing vibrations (e.g. ${}^{206}Pb(t,p)\,{}^{208}Pb(gs)$)

$X_r(i)$, $Y_r(k)$ (RPA)

coherent states: essentially exact

systematic (silent revolution)

$$^{A+2}Sn(p,t)\,^{A}Sn$$

$(A+2) = 112, 114, 116, 118, 120,$
$122, 124$

Guazzoni et al PRC
1999 (122), 2004 (116), 2006 (112),
2008 (120), 2011 (118, 124),
2012 (114)

Major breakthrough

$$^{1}H(^{11}Li, ^{9}Li)^{3}H$$

Tanihata et al
PRL 2008

$$\xi = \frac{\hbar v_F}{E_{corr}} \approx 30\text{-}36\ fm$$

$$r_{np} \approx 0.4\ fm$$

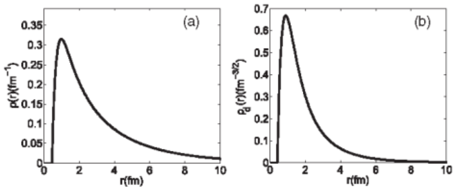


FIG. 6. (a) Radial function $\rho(r)$ (hard core 0.45 fm) entering the triton wave function. (b) Radial function $\rho_d(r)$ entering the deuteron wave function.

different channels ($a + A \to f + F \to b + B$). Within this context, n and n' are possible not only owing to the fact that partners of a Cooper pair feel different mean fields ($\phi^f_{n'ljm}, \phi^F_{nljm}$), but also because a general nuclear structure treatment of pairing will include Cooper-like correlations associated with multipole pairing (see, e.g., Ref. [10], Sec. 5.3, and references therein), correlations which, in the present case, have not a dynamical origin (one works with $H_p = -GP^\dagger P$), but only a trivial kinematical one.

III. REACTION MECHANISM

In what follows we present the elements which enter the calculation of the absolute two-particle transfer differential cross section in terms of the reaction

$$A + t \to B(\equiv A + 2) + p, \tag{32}$$

in which $A + 2$ and A denote the mass number of even nuclei in their ground state. In other words, one concentrates on $L = 0$ transfer. The wave function of nucleus $A + 2$ is written as

$$\Psi_{A+2}(\xi_A, \mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2) = \psi_A(\xi_A) \sum_{l_i, j_i} \left[\phi^{A+2}_{l_i, j_i}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)\right]^0_0, \tag{33}$$

the product of the wave function describing the ground state of the nucleus A, the corresponding relative (intrinsic) $3A - 3$ radial coordinates being denoted ξ_A, and of the wave function of two-correlated nucleons,

$$\left[\phi^{A+2}_{l_i, j_i}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)\right]^0_0 = \sum_{nm} a_{nm} \left[\varphi^{A+2}_{n, l_i, j_i}(\mathbf{r}_{A1}, \sigma_1) \varphi^{A+2}_{m, l_i, j_i}(\mathbf{r}_{A2}, \sigma_2)\right]^0_0, \tag{34}$$

the wave functions $\varphi^{A+2}_{n, l_i, j_i}(\mathbf{r}, \sigma)$ describing the single-particle motion of a nucleon in a mean-field potential, e.g., a Saxon-Woods potential. The two-neutron wave function in the triton can be written as $\phi_t(r_{p1}, \sigma_1, r_{p2}, \sigma_2) = \rho(r_{p1})\rho(r_{p2})\rho(r_{12})[\chi(\sigma_1)\chi(\sigma_2)]^0_0$, r_{p1} and r_{p2} denoting the modulus of the relative coordinate of each of the two neutrons involved in the transfer process, measured with respect to the proton, while r_{12} denotes the modulus of the relative coordinate of the two neutrons in the triton. The deuteron wave function is written as $\phi_d(r_{p1}, \sigma_1) = \rho_d(r_{p1})\chi(\sigma_1)$. The functions $\rho(r)$ and $\rho_d(r)$, as depicted in Fig. 6, are generated with the p-n Tang-Herndon interaction [37],

$$v(r) = -v_0 \exp(-k(r - r_c)), \quad r > r_c, \tag{35}$$

$$v(r) = \infty \quad r < r_c, \tag{36}$$

where $k = 2.5\ \mathrm{fm}^{-1}$ and $r_c = 0.45$ fm denotes the radius of the hard core. The depth v_0 is adjusted so as to reproduce the binding energy of the triton and of the deuteron, respectively. This hard-core potential is also used in the above expressions as the n-p interaction potential responsible for neutron transfer.

The two-particle transfer differential cross section is written as

$$\frac{d\sigma}{d\Omega} = \frac{\mu_i \mu_f}{(4\pi\hbar^2)^2} \frac{k_f}{k_i} \left|T^{(1)} + T^{(2)}_{\mathrm{succ}} - T^{(2)}_{\mathrm{NO}}\right|^2. \tag{37}$$

The amplitudes appearing in it describe the simultaneous,

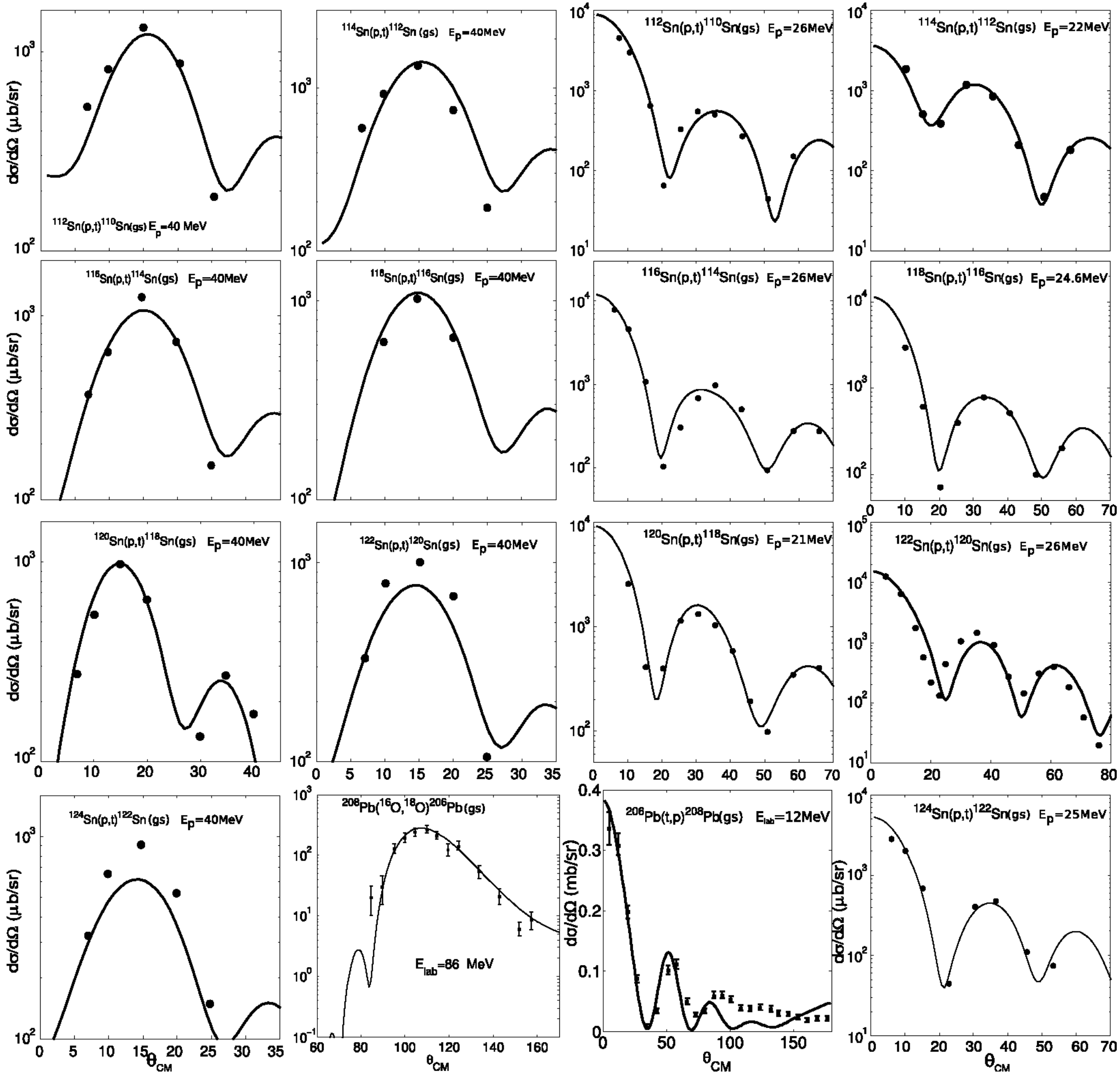
$$T^{(1)} = 2 \sum_{l_i, j_i} \sum_{\sigma_1 \sigma_2} \int d\mathbf{r}_{tA} d\mathbf{r}_{p1} d\mathbf{r}_{A2} \left[\phi^{A+2}_{l_i, j_i}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)\right]^{0*}_0 \chi^{(-)*}_{pB}(\mathbf{r}_{pB}) v(r_{p1}) \phi_t(r_{p1}, \sigma_1, r_{p2}, \sigma_2) \chi^{(+)}_{tA}(\mathbf{r}_{tA}), \tag{38a}$$

successive,

$$\begin{aligned} T^{(2)}_{\mathrm{succ}} = {} & 2 \sum_{l_i, j_i} \sum_{l_f, j_f, m_f} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma'_1 \sigma'_2}} \int d\mathbf{r}_{dF} d\mathbf{r}_{p1} d\mathbf{r}_{A2} \left[\phi^{A+2}_{l_i, j_i}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)\right]^{0*}_0 \chi^{(-)*}_{pB}(\mathbf{r}_{pB}) v(r_{p1}) \phi_d(r_{p1}, \sigma_1) \varphi^{A+1}_{l_f, j_f, m_f}(\mathbf{r}_{\mathbf{A2}}, \sigma_2) \\ & \times \int d\mathbf{r}'_{dF} d\mathbf{r}'_{p1} d\mathbf{r}'_{A2} G(\mathbf{r}_{dF}, \mathbf{r}'_{dF}) \phi_d(r'_{p1}, \sigma'_1)^* \varphi^{A+1*}_{l_f, j_f, m_f}(\mathbf{r}'_{\mathbf{A2}}, \sigma'_2) \frac{2\mu_{dF}}{\hbar^2} v(r'_{p2}) \phi_d(r'_{p1}, \sigma'_1) \phi_d(r'_{p2}, \sigma'_2) \chi^{(+)}_{tA}(\mathbf{r}'_{tA}), \end{aligned} \tag{38b}$$

and nonorthogonal,

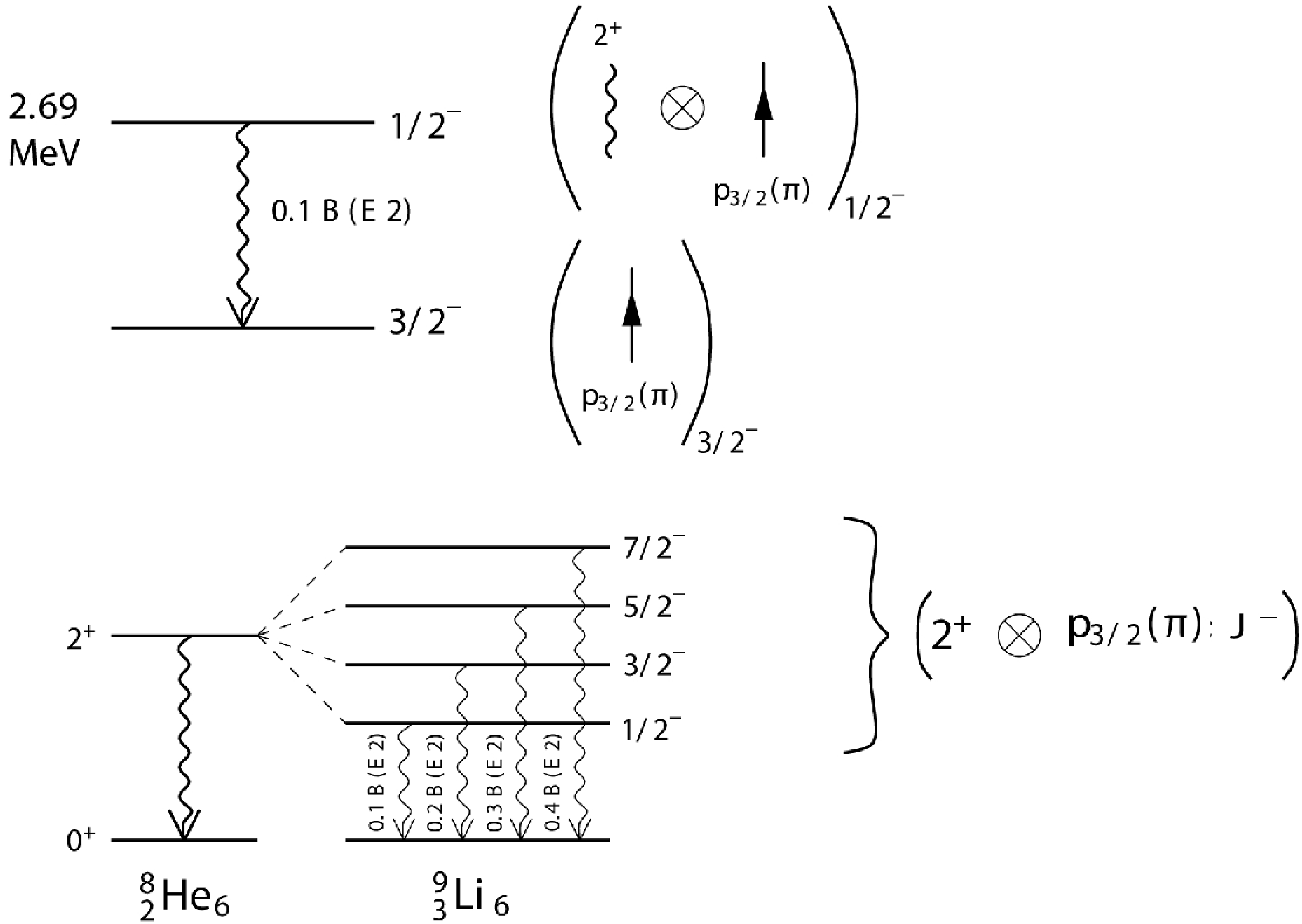
$$\begin{aligned} T^{(2)}_{\mathrm{NO}} = {} & 2 \sum_{l_i, j_i} \sum_{l_f, j_f, m_f} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma'_1 \sigma'_2}} \int d\mathbf{r}_{dF} d\mathbf{r}_{p1} d\mathbf{r}_{A2} \left[\phi^{A+2}_{l_i, j_i}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)\right]^{0*}_0 \chi^{(-)*}_{pB}(\mathbf{r}_{pB}) v(r_{p1}) \phi_d(r_{p1}, \sigma_1) \varphi^{A+1}_{l_f, j_f, m_f}(r_{A2}, \sigma_2) \\ & \times \int d\mathbf{r}'_{p1} d\mathbf{r}'_{A2} d\mathbf{r}'_{dF} \phi_d(r'_{p1}, \sigma'_1)^* \varphi^{A+1*}_{l_f, j_f, m_f}(\mathbf{r}'_{\mathbf{A2}}, \sigma'_2) \phi_d(r'_{p1}, \sigma'_1) \phi_d(r'_{p2}, \sigma'_2) \chi^{(+)}_{tA}(\mathbf{r}'_{tA}), \end{aligned} \tag{38c}$$

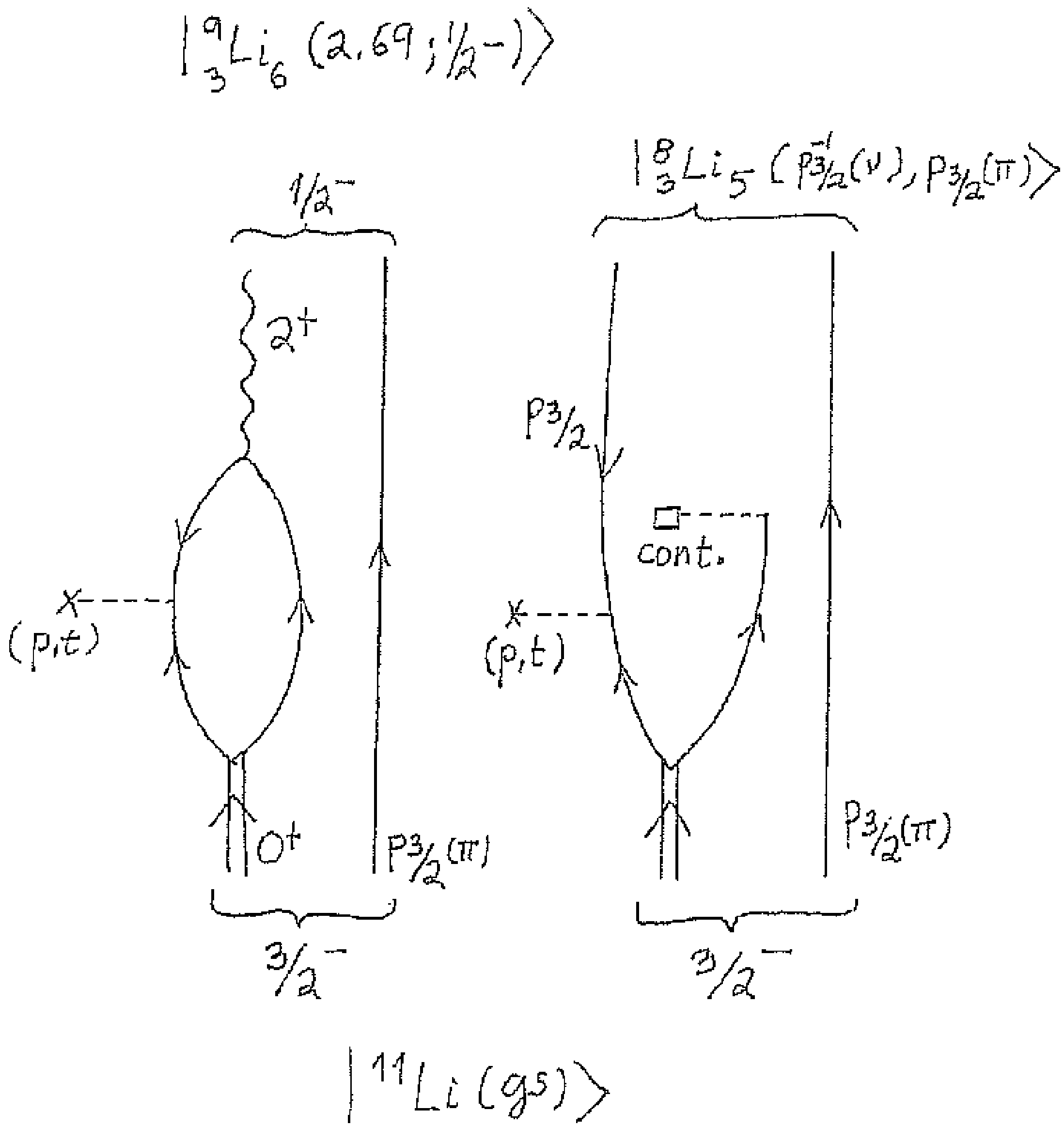

^{112}Sn(p,t)^{110}Sn(gs) E$_p$=40 MeV
^{114}Sn(p,t)^{112}Sn (gs) E$_p$=40MeV
^{112}Sn(p,t)^{110}Sn(gs) E$_p$=26MeV
^{114}Sn(p,t)^{112}Sn(gs) E$_p$=22MeV
^{116}Sn(p,t)^{114}Sn(gs) E$_p$=40MeV
^{118}Sn(p,t)^{116}Sn(gs) E$_p$=40MeV
^{116}Sn(p,t)^{114}Sn(gs) E$_p$=26MeV
^{118}Sn(p,t)^{116}Sn(gs) E$_p$=24.6MeV
^{120}Sn(p,t)^{118}Sn(gs) E$_p$=40MeV
^{122}Sn(p,t)^{120}Sn(gs) E$_p$=40MeV
^{120}Sn(p,t)^{118}Sn(gs) E$_p$=21MeV
^{122}Sn(p,t)^{120}Sn(gs) E$_p$=26MeV
^{124}Sn(p,t)^{122}Sn (gs) E$_p$=40MeV
^{208}Pb(^{16}O,^{18}O)^{206}Pb(gs)
E$_{lab}$=86 MeV
^{206}Pb(t,p)^{208}Pb(gs) E$_{lab}$=12MeV
^{124}Sn(p,t)^{122}Sn(gs) E$_p$=25MeV
dσ/dΩ (μb/sr)
dσ/dΩ (mb/sr)
θ_{CM}

New Technical Achievement

	σ(gs→f)		
	f	Theory $^{a)\ b)\ f)}$	Experiment$^{f-m)}$
^{7}Li$(t,p)^9$Li	gs	14.3 $^{c)}$	14.7 ± 4.4 $^{c,i)}$ $[9.4° < \theta < 108.7°]$
^{1}H(^{11}Li,^{9}Li)^{3}H	gs	6.1 $^{c)}$	5.7 ± 0.9 $^{c,b)}$ $[20° < \theta < 154.5°]$
	$1/2^-$	0.7 $^{c)}$	1.0 ± 0.36 $^{c,b)}$ $[30° < \theta < 100°]$
^{10}Be$(t,p)^{12}$Be	gs	2.3 $^{c)}$	1.9 ± 0.57 $^{c,j)}$ $[4.4° < \theta < 57.4°]$
^{48}Ca$(t,p)^{50}$Ca	gs	0.55 $^{c)}$	0.56 ± 0.17 $^{c,m)}$ $[4.5° < \theta < 174°]$
^{112}Sn$(p,t)^{110}$Sn, $E_{CM} = 26$ MeV	gs	1301 $^{d)}$	$1309 \pm 200(\pm 14)$ $^{d,g)}$ $[6° < \theta < 62.2°]$
^{114}Sn$(p,t)^{112}$Sn, $E_{CM} = 22$ MeV	gs	1508 $^{d)}$	$1519 \pm 456(\pm 16.2)$ $^{d,g)}$ $[7.64° < \theta < 62.24°]$
^{116}Sn$(p,t)^{114}$Sn, $E_{CM} = 26$ MeV	gs	2078 $^{d)}$	$2492 \pm 374(\pm 32)$ $^{d,g)}$ $[4° < \theta < 70°]$
^{118}Sn$(p,t)^{116}$Sn, $E_{CM} = 24.4$ MeV	gs	1304 $^{d)}$	$1345 \pm 202(\pm 24)$ $^{d,g)}$ $[7.63° < \theta < 59.6°]$
^{120}Sn$(p,t)^{118}$Sn, $E_{CM} = 21$ MeV	gs	2190 $^{d)}$	$2250 \pm 338(\pm 14)$ $^{d,g)}$ $[7.6° < \theta < 69.7°]$
^{122}Sn$(p,t)^{120}$Sn, $E_{CM} = 26$ MeV	gs	2466 $^{d)}$	$2505 \pm 376(\pm 18)$ $^{d,g)}$ $[6° < \theta < 62.2°]$
^{124}Sn$(p,t)^{122}$Sn, $E_{CM} = 25$ MeV	gs	838 $^{d)}$	$958 \pm 144(\pm 15)$ $^{d,g)}$ $[4° < \theta < 57°]$
^{112}Sn$(p,t)^{110}$Sn, $E_p = 40$ MeV	gs	3349 $^{e)}$	3715 ± 1114 $^{e,h)}$
^{114}Sn$(p,t)^{112}$Sn, $E_p = 40$ MeV	gs	3790 $^{e)}$	3776 ± 1132 $^{e,h)}$
^{116}Sn$(p,t)^{114}$Sn, $E_p = 40$ MeV	gs	3085 $^{e)}$	3135 ± 940 $^{e,h)}$
^{118}Sn$(p,t)^{116}$Sn, $E_p = 40$ MeV	gs	2563 $^{e)}$	2294 ± 668 $^{e,h)}$
^{120}Sn$(p,t)^{118}$Sn, $E_p = 40$ MeV	gs	3224 $^{e)}$	3024 ± 907 $^{e,h)}$
^{122}Sn$(p,t)^{120}$Sn, $E_p = 40$ MeV	gs	2339 $^{e)}$	2907 ± 872 $^{e,h)}$
^{124}Sn$(p,t)^{122}$Sn, $E_p = 40$ MeV	gs	1954 $^{e)}$	2558 ± 767 $^{e,h)}$
^{206}Pb$(t,p)^{208}$Pb	gs	0.52 $^{c)}$	0.68 ± 0.21 $^{c,k)}$ $[4.5° < \theta < 176.5°]$
^{208}Pb(^{16}O,^{18}O)^{206}Pb	gs	0.80 $^{c)}$	0.76 ± 0.18 $^{c,f)}$ $[84.6° < \theta < 157.3°]$

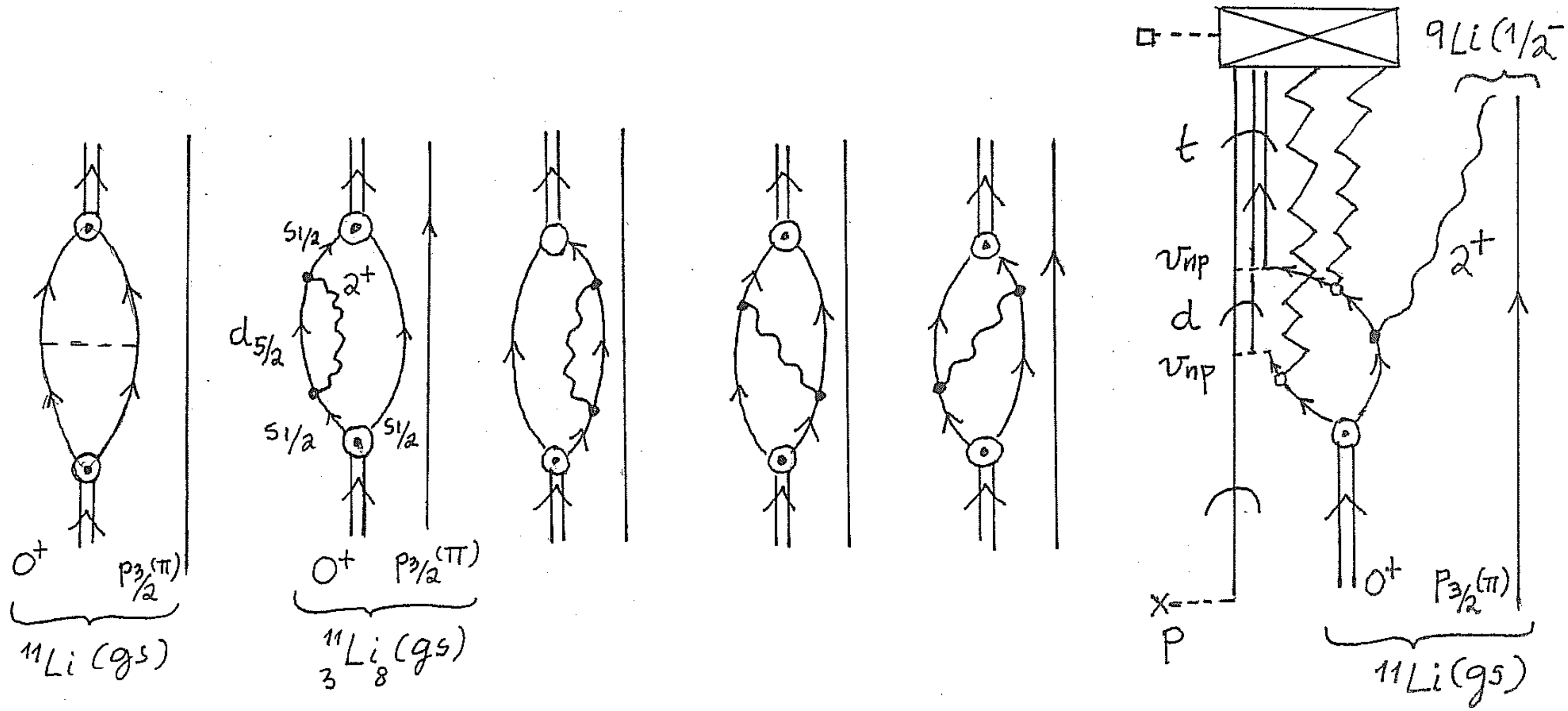
Table 4:
It is of notice that the number in parenthesis (last column) corresponds to the statistical errors.
$^{a)}$ G. Potel et al., Phys. Rev. Lett. **107**, (2011) 092501.
$^{b)}$ G. Potel et al., Phys. Rev. Lett. **105**, (2010) 172502.
$^{c)}$ mb
$^{d)}$ μb
$^{e)}$ μb/sr ($\sum_{i=1}^{N}(d\sigma/d\Omega)$; differential cross section summed over the few, $N = 3-7$ experimental points).
$^{f)}$ B. Bayman and J. Chen, Phys. Rev. **C 26** (1982) 1509 and refs. therein.
$^{g)}$ P. Guazzoni, L. Zetta, et al., Phys. Rev. **C 60**, 054603 (1999).
P. Guazzoni, L. Zetta, et al., Phys. Rev. **C 69**, 024619 (2004).
P. Guazzoni, L. Zetta, et al., Phys. Rev. **C 74**, 054605 (2006).
P. Guazzoni, L. Zetta, et al., Phys. Rev. **C 83**, 044614 (2011).
P. Guazzoni, L. Zetta, et al., Phys. Rev. **C 78**, 064608 (2008).
P. Guazzoni, L. Zetta, et al., Phys. Rev. **C 85**, 054609 (2012).
$^{h)}$ G. Bassani et al. Phys. Rev. **139**, (1965)B830.
$^{i)}$ P.G. Young and R.H. Stokes, Phys. Rev. **C 4**, (1971) 1597.
$^{j)}$ H.T. Fortune, G.B. Liu and D.E. Alburger, Phys. Rev. **C 50**, (1994) 1355.
$^{k)}$ J.H. Bjerregaard et al., Nucl. Phys. **89**, (1966) 337.
$^{l)}$ J.H. Bjerregaard et al., Nucl. Phys. **A 113**, (1968) 484.
$^{m)}$ J. H. Bjerregaard et al., Nucl. Phys. **A 103**, (1967) 33.


2.69
MeV
1/ 2⁻
0.1 B (E 2)
3/ 2⁻
2+
p3/2(π)
1/2⁻
p3/2(π)
3/2⁻
7/ 2⁻
5/ 2⁻
3/ 2⁻
1/ 2⁻
2+
0+
0.1 B (E 2)
0.2 B (E 2)
0.3 B (E 2)
0.4 B (E 2)
(2+ ⊗ p3/2(π): J⁻)
8 2 He6
9 3 Li 6



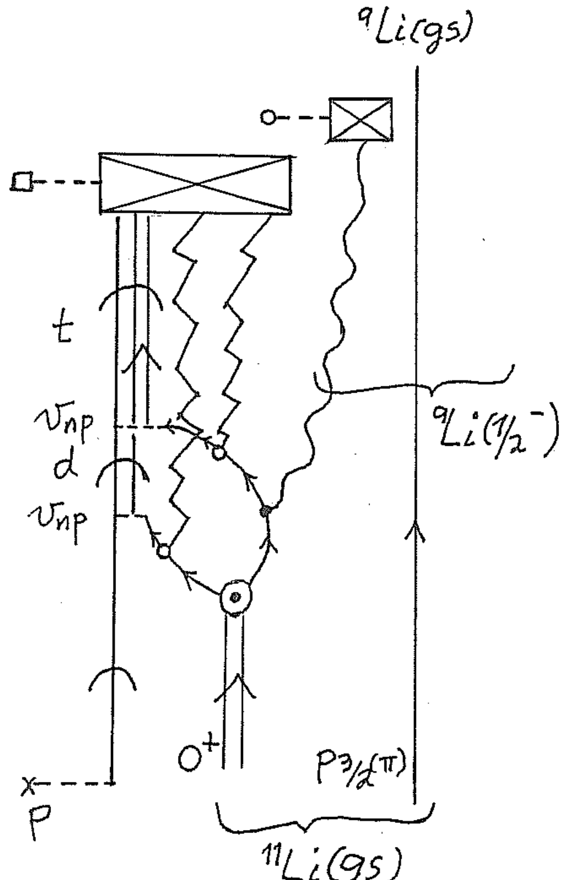
$|{}^{9}_{3}Li_{6}(2.69; 1/2^{-})\rangle$
$|{}^{8}_{3}Li_{5}(p^{-1}_{3/2}(\nu), p_{3/2}(\pi))\rangle$
$1/2^{-}$
2^{+}
$p_{3/2}$
cont.
(p,t)
(p,t)
0^{+}
$p_{3/2}(\pi)$
$p_{3/2}(\pi)$
$3/2^{-}$
$3/2^{-}$
$|{}^{11}Li(gs)\rangle$

$^{1}H(^{11}Li, ^{9}Li(1/2^-))^{3}H$



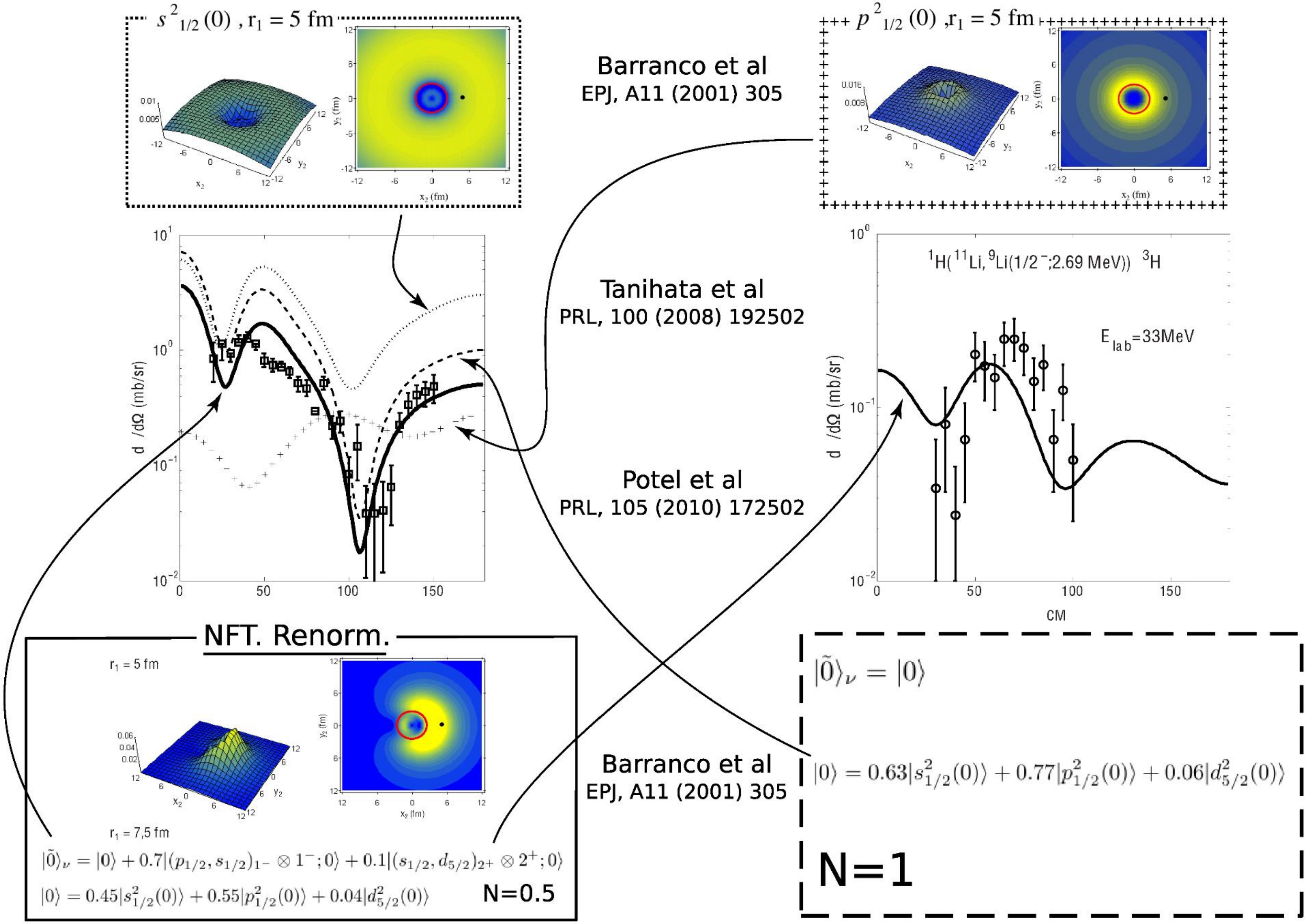
Variety of pv-coupling vertices (NFT)

⊙ pair
• surface
○ recoil



variety of pv-coupling vertices (NFT)

⊙ pair
• surface
∘ recoil



$s^2_{1/2}(0)$, $r_1 = 5$ fm
$p^2_{1/2}(0)$, $r_1 = 5$ fm
Barranco et al
EPJ, A11 (2001) 305
Tanihata et al
PRL, 100 (2008) 192502
Potel et al
PRL, 105 (2010) 172502
Barranco et al
EPJ, A11 (2001) 305
^{1}H(^{11}Li, ^{9}Li(1/2$^-$;2.69 MeV)) ^{3}H
E_{lab}=33MeV
d /dΩ (mb/sr)
CM
NFT. Renorm.
$r_1 = 5$ fm
$r_1 = 7{,}5$ fm
$|\tilde{0}\rangle_\nu = |0\rangle + 0.7|(p_{1/2}, s_{1/2})_{1^-} \otimes 1^-; 0\rangle + 0.1|(s_{1/2}, d_{5/2})_{2^+} \otimes 2^+; 0\rangle$
$|0\rangle = 0.45|s^2_{1/2}(0)\rangle + 0.55|p^2_{1/2}(0)\rangle + 0.04|d^2_{5/2}(0)\rangle$
N=0.5
$|\tilde{0}\rangle_\nu = |0\rangle$
$|0\rangle = 0.63|s^2_{1/2}(0)\rangle + 0.77|p^2_{1/2}(0)\rangle + 0.06|d^2_{5/2}(0)\rangle$
N=1

$$\mathcal{R} = \sqrt{\frac{3}{R_0^3}}\, \Theta(r - R_0)\,; \quad R_0 = 1{,}2\, A^{1/3}\, \mathrm{fm} \quad \text{(systematics)}$$

$$\int_0^{R_0} r^2 dr\, \mathcal{R}^2 = \frac{3}{R_0^3} \int_0^{R_0} \frac{dr^3}{3} = 1$$

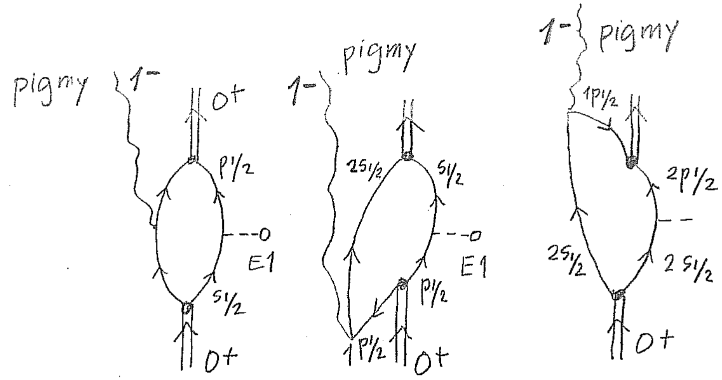
$$\mathcal{R}_{halo} = \sqrt{\frac{3}{R^3}}\, \Theta(r - R)$$

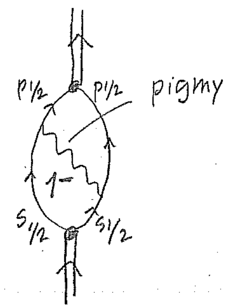
Two-nucleon overlap $(\mathcal{O} = |\langle \varphi_{halo} | \varphi_{syst} \rangle|^2)$

$$\mathcal{O} = \left(\int_0^{R_0} r^2 dr\, \mathcal{R}_{halo}\, \mathcal{R}_{syst} \right)^2$$

$$= \left(\sqrt{\frac{3}{R^3}} \sqrt{\frac{3}{R_0^3}} \int_0^{R_0} \frac{dr^3}{3} \right)^2 = \left(\frac{R_0}{R} \right)^3$$

R: halo radius





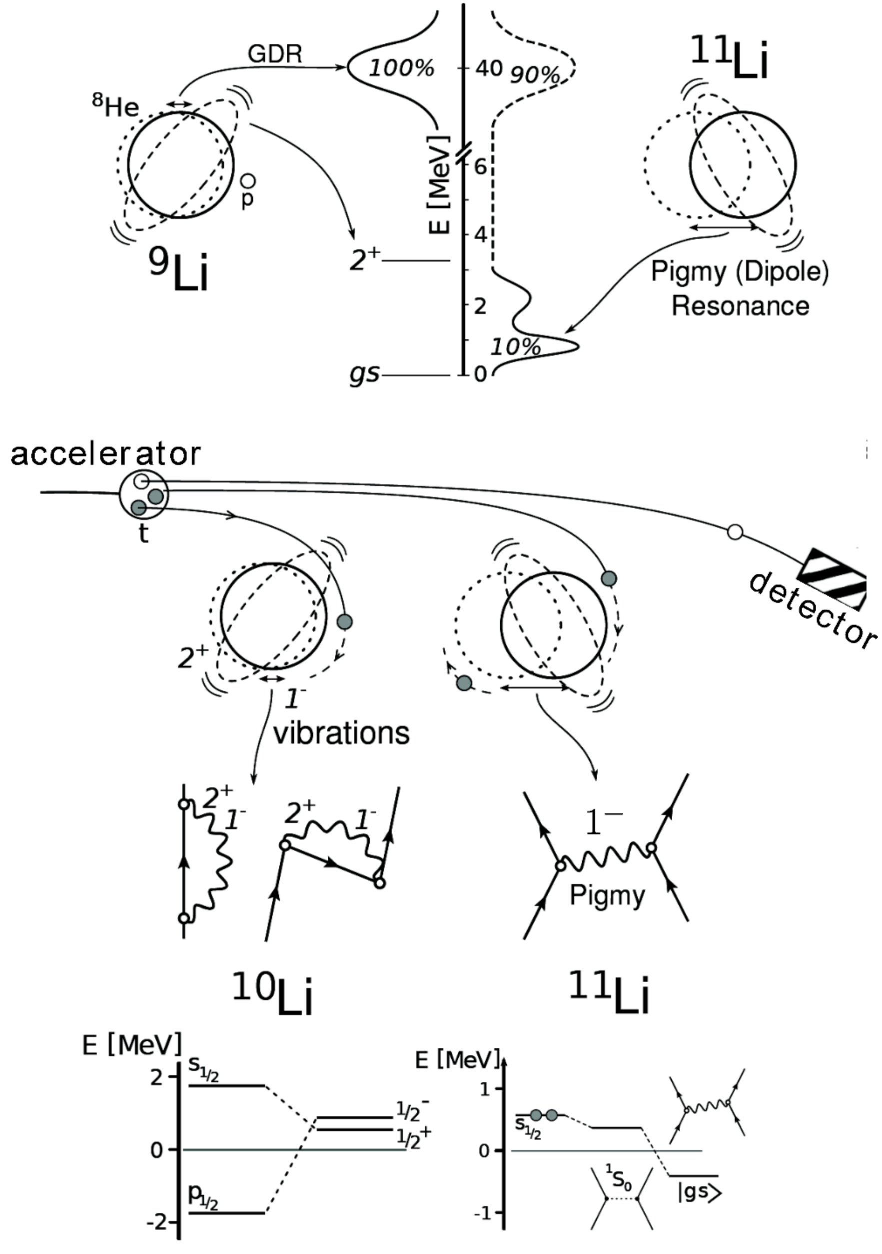
$$\theta \approx \left(\frac{R_0}{R}\right)^3 \approx 0.16$$

(screening of symm. term)

The pigmy resonance can hardly be viewed but in symbiosis with the ^9Li pair addition mode (virtual)

The pigmy resonance is built on a ground state with little overlap with the gs on which the GDR is built. Thus, it is a new mode of excitation

extreme inhomogeneous damping (radial degree of freedom instead of quadrupole def.)



Bootstrap pairing correlations

New physics

$r \ll 1$

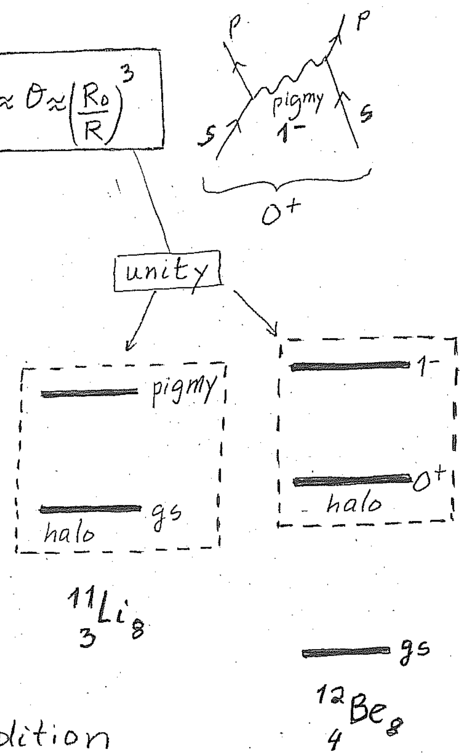
bare NN-pairing
screened
subcritical ($v_{NN} < G_c$)

$r \approx \theta \approx \left(\frac{R_0}{R}\right)^3$

$\theta \ll 1$

strongly screened
isospin interaction

sof E1-mode
(pigmy 1-)



pair addition
halo mode

$$V(r_{12}) = -4\pi V_0\, \delta(|\vec{r}_1 - \vec{r}_2|)$$

$$M_j = \langle (j)^2_0 | V | (j)^2_0 \rangle = -\frac{2j+1}{2} V_0\, \mathbb{I}(j)$$

$$\mathbb{I}(j) = \int_0^{R_0} \mathcal{R}_j^4\, r^2 dr \qquad \left(R_0 = 1{,}2 A^{1/3}\,\mathrm{fm}\ (\text{systematics}) \right)$$

$$\mathcal{R}_j \approx \sqrt{\frac{3}{R_0^3}}\, \Theta(r - R_0); \; \mathbb{I} \approx \frac{3}{R_0^3}$$

Ratio

$$r = \frac{(M_j)_{halo}}{(M_j)_{syst}} = \left(\frac{R_0}{R} \right)^3$$

R : halo radius

(halo anti-pairing effect)

dual origin of pairing in nuclei

v_{bare}(3N-corrs.) + v_{ind}

^{11}Li v_{ind} very important

direct and circumstantial evidence

nuclei along stability valley

50% 50%

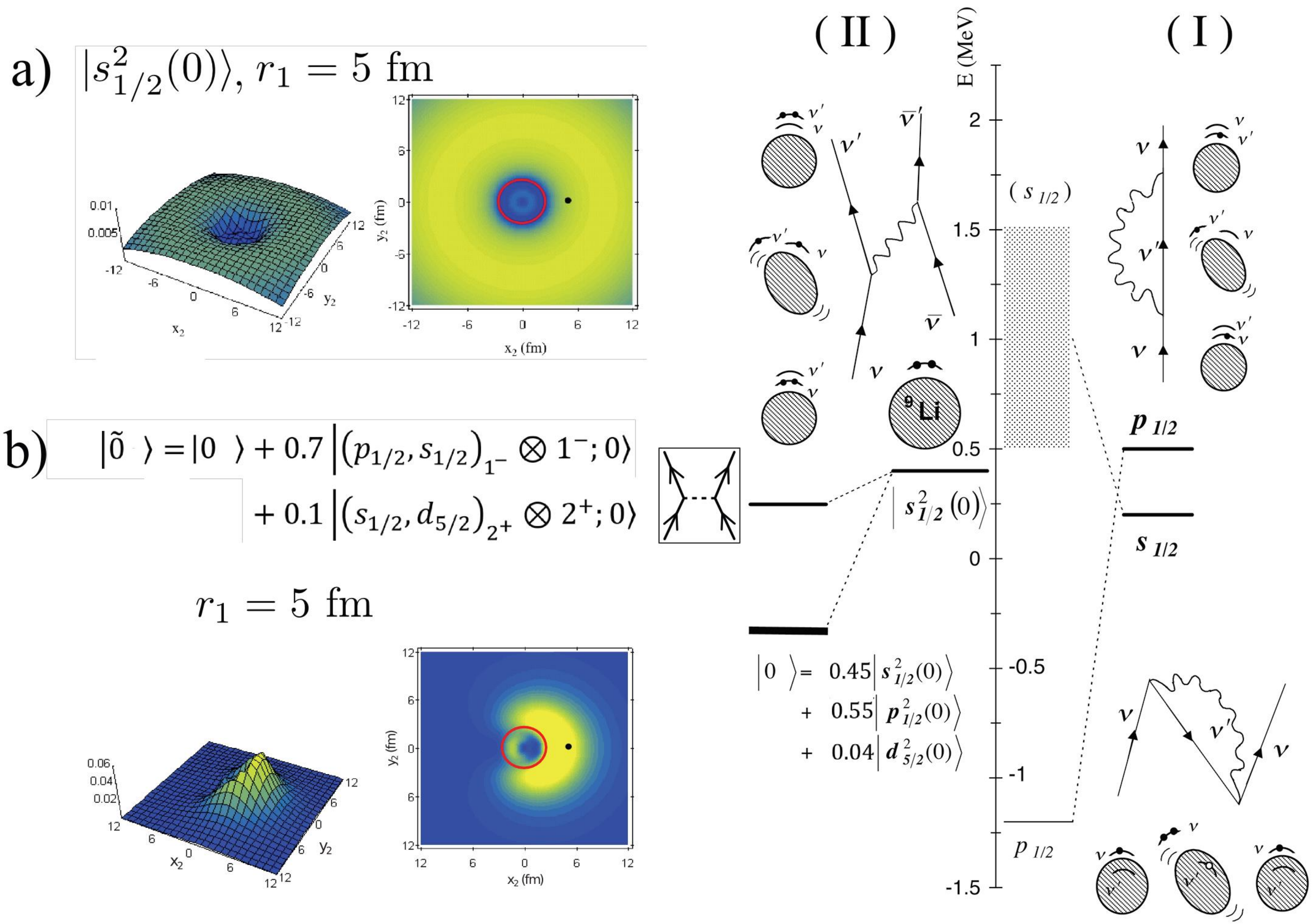
open problem

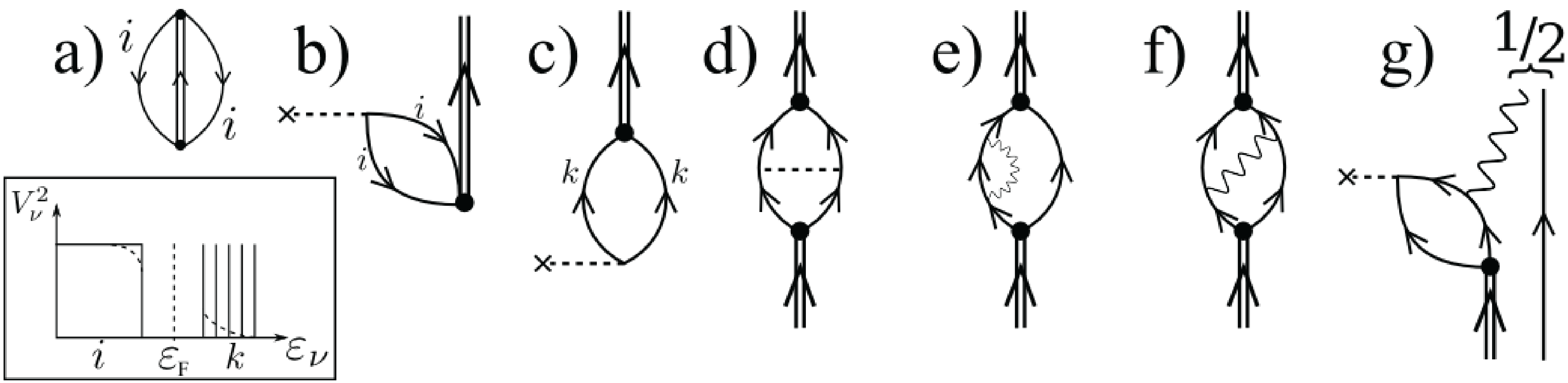
G. Potel (Livermore)

A. Idini (Darmstadt)

F. Barranco (Sevilla)

E. Vigezzi (Milan)


a) $|s^2_{1/2}(0)\rangle$, $r_1 = 5$ fm
b) $|\tilde{0}\rangle = |0\rangle + 0.7\left|(p_{1/2}, s_{1/2})_{1^-} \otimes 1^-; 0\right\rangle + 0.1\left|(s_{1/2}, d_{5/2})_{2^+} \otimes 2^+; 0\right\rangle$
$r_1 = 5$ fm
x2 (fm)
y2 (fm)
(II)
(I)
E (MeV)
$(s_{1/2})$
^{9}Li
$p_{1/2}$
$s_{1/2}$
$p_{1/2}$
$|s^2_{1/2}(0)\rangle$
$|0\rangle = 0.45|s^2_{1/2}(0)\rangle + 0.55|p^2_{1/2}(0)\rangle + 0.04|d^2_{5/2}(0)\rangle$



a)
b)
c)
d)
e)
f)
g)
i
i
k
k
1/2⁻
V_ν^2
ε_{F}
ε_ν