Towards holography for asymptotically flat spacetimes

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Holography

- Any gravitational theory is expected to be holographic, *i.e.* it should have a description in terms of a non-gravitational theory in one dimension less.
- The current formulation of holographic dualities depends sensitively on the detailed structure of asymptotics: the case we understand best is that with AdS asymptotics.
- Here we will present a map between AdS and Ricci-flat solutions that may allow us to develop holography for Ricci-flat spacetimes.

References

- Based on work with
  Marco Caldarelli, Joan Camps and Blaise Goutéraux:
  1211.2815, 1311.xxxx

Outline

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AdS holography

- The best understood holographic dualities are those obtained in string theory via decoupling limits of branes.
- The prototype example is the near-horizon limit of D3 brane, which yields the duality between string theory on $AdS_5 \times S^5$ and $N = 4$ SYM at the conformal boundary of AdS_5.
- CFT's contain a universal sector describing the correlators of the energy-momentum tensor $T_{\mu\nu}$.
- This universal sector is described in AdS/CFT correspondence by *AdS gravity*,

$$S = \int d^{d+1}x\sqrt{G}(R - 2\Lambda)$$

AdS asymptotics and holography

- The AdS solution,

$$ds^2 = \frac{1}{r^2}(dr^2 + \eta_{ij} dx^i dx^j)$$

represents the vacuum of the CFT.

- The spacetime has a conformal boundary at $r = 0$.
- We need to impose boundary conditions there. The Dirichlet problem in AdS is to fix a conformal class:

$$g_{(0)ij}(x) \sim e^{2\sigma(x)} g_{(0)ij}(x)$$

- The corresponding bulk metric has the following form [Fefferman-Graham (1985)]

$$ds^2 = \frac{1}{r^2}\left(dr^2 + (g_{(0)ij} + r^2 g_{(2)ij} + \cdots + r^d g_{(d)ij} + \cdots)dx^i dx^j\right)$$

Correlation functions

In gauge/gravity duality:

- $g_{(0)ij}$ is identified with the source for T_{ij}.
- The expectation value of the T_{ij} in the presence of sources is [de Haro, Solodukhin, KS (2000)]

$$\langle T_{ij} \rangle \sim g_{(d)ij}$$

- To compute correlation functions we consider

$$g_{(0)ij} = \eta_{ij} + h_{ij}$$

  and compute exact bulk solutions perturbatively in h_{ij}.
- Regularity in the interior results in

$$g_{(d)ij} = g^{B}_{(d)ij} + \mathcal{T}_{ijkl} h^{kl} + \frac{1}{2} \mathcal{T}_{ijklmn} h^{kl} h^{mn} + \cdots$$

- $\mathcal{T}_{ijkl} \sim \langle T_{ij}(k) T_{kl}(-k) \rangle$,
  $\mathcal{T}_{ijklmn} \sim \langle T_{ij}(k_1) T_{kl}(k_2) T_{mn}(-k_1 - k_2) \rangle$, etc.

2-point function

- To compute 2-point function one needs a solution of the linearized equation.
- Regular linear perturbations around AdS are given by

$$h_{ij}(k) = h_{(0)ij}(k)\frac{1}{2^{d/2-1}\Gamma(d/2)}(kr)^{d/2}K_{d/2}(kr)$$

- ... and using the asymptotic expansion

$$\frac{1}{2^{d/2-1}\Gamma(d/2)}(kr)^{d/2}K_{d/2}(kr) = 1 + \cdots + r^d k^d + \cdots$$

- we find

$$\langle T_{ij}(k)T_{mn}(-k)\rangle = \Pi_{ijmn}k^d$$

  where Π_{ijmn} is a projection to transverse traceless tensors.
- This is precisely the correct 2-point function for the stress energy tensor of a d-dimensional CFT (when d even $k^d \to k^d \log k$)..

Outline

Non-conformal branes

- A decoupling limit for non-conformal branes (D0, D1, D2, D4), similar to the one leading to the AdS/CFT conjecture was presented in [Itzhaki et al (1998)].
- For these branes the geometry is *conformal* to $AdS_{(p+2)} \times S^{8-p}$ and after reducing over the sphere one obtains an asymptotically power-law geometry with a running dilaton [Boonstra, KS, Townsend (1998)].
- A precise holographic framework for these cases was only recently established [Kanitscheider, KS, Taylor (2008)] [Wiseman, Withers (2008)].

Holographic dictionary: non-conformal branes

- For the non-conformal branes, the starting point is the Lagrangian obtained by consistent reduction over the transverse sphere S^{8-p}:

$$S = \int d^{p+2}x \sqrt{-g} e^{\phi} \left[R - \frac{2\sigma - p - 2}{2\sigma - p - 1} (\partial\phi)^2 - 2\sigma(2\sigma - 1) \right]$$

The parameter σ takes the values:

$$\{D0, D1, D2, D4\} = \{7/5, 3/2, 5/3, 3\}$$

- The dual theory is not conformal: it has a dimensionful coupling constant and ϕ encodes this coupling.
- This action captures holographically correlators of $T_{\mu\nu}, \mathcal{O}$. This is the universal sector of the corresponding QFTs.

Holography and asymptotics

- Since the solutions are not asymptotically AdS one needs to develop the dictionary from scratch.
- Extending the results of Fefferman-Graham to this system one finds similar asymptotic expansions but with unusual powers,

$$ds^2 = \frac{1}{r^2}\left(dr^2 + (g_{(0)ij} + r^2 g_{(2)ij} + \cdots + r^{\sigma} g_{(\sigma)ij} + \cdots)dx^i dx^j\right)$$

  and similar for the scalar field.
- Holographic renormalization leads to holographic formulae:

$$\langle T_{ij}\rangle \sim g_{(\sigma)ij}$$

  and similar for $\langle \mathcal{O}\rangle$.
- One can also work out linear fluctuations

$$h_{ij}(k) = h_{(0)ij}(k)\frac{1}{2^{\sigma/2-1}\Gamma(\sigma/2)}(kr)^{\sigma/2}K_{\sigma/2}(kr)$$

  and from here the corresponding 2-point functions ....
  [Kanitscheider, KS, Taylor (2008)]

Generalized dimensional reduction [Kanitscheider, KS (2009)]

The starting action for the non-conformal branes can be obtained from $AdS_{2\sigma+1}$ gravity by

1. reducing over $T^{2\sigma-p-1}$ torus, keeping only the overall size of the torus,
2. analytically continuing on σ

$$S = \int d^{2\sigma+1}x\,\sqrt{-g}\,[R - 2\Lambda] \quad \text{with} \quad ds^2_{\Lambda} = ds^2_{p+2}(r,x) + e^{\frac{2\phi(r,x)}{2\sigma-p-1}} d\vec{y}^2$$

$$\rightarrow \qquad S = \int d^{p+2}x\,\sqrt{-\hat{g}}e^{\phi}\left[\hat{R} + \frac{2\sigma-p-2}{2\sigma-p-1}(\partial\phi)^2 - 2\sigma(2\sigma-1)\right].$$

- ➢ The reduced action and equations of motion depend smoothly on σ, provided $2\sigma - p - 1 > 0$.
- ➢ This is a consistent reduction: all solutions of the reduced theory originate from solutions of AdS gravity.

Holography

- All results needed for establishing a holographic dictionary are inherited from the corresponding AdS results via the generalized dimensional reduction:
  - Asymptotic solutions
  - Counterterms
  - Renormalized 1-point functions in the presence of sources.
- This links also black hole solutions and the nearby hydrodynamic regime and explains their conserved charges, thermodynamics and the values of the transport coefficients:
  - Equation of state: $P = \epsilon/(2\sigma - 1)$
  - Transport coefficients: $\eta/s = 1/4\pi$,
    $\zeta/\eta = 2(1/(d-1) - c_s^2)$

Holography for asymptotically flat spacetime?

In the two classes of holographic spacetimes we have just reviewed:

- The fields that parametrize the boundary conditions at infinity act as QFT sources.
- The gravitational on-shell action is the generating functional of QFT correlation functions.

This requires that

- The fields that parametrize the boundary conditions are unconstrained.
- The infinities in the on-shell action are local on these fields.

➠ Both of these conditions fail for asymptotically flat spacetimes [de Haro, Solodukhin, KS (2001)]

➠ A straightforward extension of the holographic methodology to AF spacetimes does not work.

Outline

AdS/Ricci-flat correspondence

This correspondence relates a class of Ricci-flat spacetimes with a class of Asymptotically AdS solutions.

- Asymptotically locally AdS in $(d+1)$ dimensions, $R_{\mu\nu} = -\frac{d}{\ell} g_{\mu\nu}$,

$$ds^2_\Lambda = ds^2_{p+2}(r,x) + e^{\frac{2\phi(r,x)}{d-p-1}} d\vec{y}^2,$$

  where $\vec{y}$ are coordinates on a $(d-p-1)$ diagonal torus.

- Ricci-flat in $D = n+p+3$ dimensions, $R_{AB} = 0$,

$$ds^2_0 = e^{\frac{2\phi(r,x)}{n+p+1}} \left(ds^2_{p+2}(r,x) + \ell^2 d\Omega^2_{n+1} \right),$$

  where $d\Omega^2_{n+1}$ is the metric of the unit round $(n+1)$-sphere.

The two solutions are mapped to each other under

$$d \leftrightarrow -n$$

Generalized dimensional reduction

In both cases there is consistent truncation to the the $(p+2)$ dimensional metric $\hat{g}$ and the scalar field ϕ. Reducing yields:

$$S = \int d^{d+1}x\,\sqrt{-g}\,[R - 2\Lambda]$$

$$\rightarrow \qquad S = \int d^{p+2}x\,\sqrt{-\hat{g}}e^{\phi}\left[\hat{R} + \frac{d-p-2}{d-p-1}\,(\partial\phi)^2 - 2\Lambda\right].$$

and

$$S = \int d^{n+p+3}x\,\sqrt{-g}R$$

$$\rightarrow \qquad S = \int d^{p+2}x\,\sqrt{-\hat{g}}e^{\phi}\left[\hat{R} + \frac{n+p+2}{n+p+1}\,(\partial\phi)^2 - 2\Lambda\right].$$

In the reduced action d and n appear as parameters.

Example: AdS on torus

- AdS spacetime on a torus

$$ds^2_\Lambda = \frac{\ell^2}{r^2}\left(dr^2 + \eta_{ab}dx^a dx^b + d\vec{y}^2\right)$$

- Comparing with

$$ds^2_\Lambda = ds^2_{p+2}(r,x) + e^{\frac{2\phi(r,x)}{d-p-1}} d\vec{y}^2,$$

we find

$$ds^2_{p+2}(r,x) = \frac{\ell^2}{r^2}(dr^2 + \eta_{ab}dx^a dx^b), \qquad \exp(2\phi/(d-p-1)) = \ell^2/r^2$$

- Apply the map: $d \to -n$

$$\exp(-2\phi/(n+p+1)) = \ell^2/r^2$$

AdS on a torus

- Recall that on the Ricci-flat side

$$ds_0^2 = e^{\frac{2\phi(r,x)}{n+p+1}}\left(ds^2_{p+2}(r,x) + \ell^2 d\Omega^2_{n+1}\right),$$

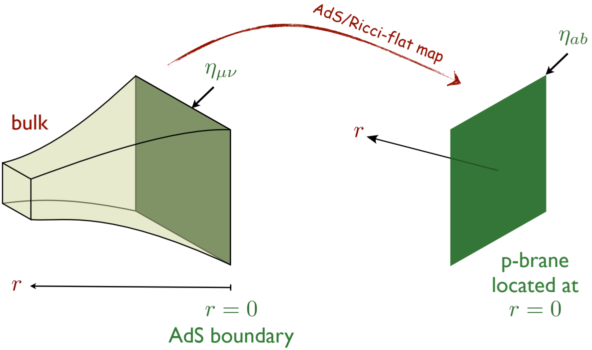
and we now have

$$ds^2_{p+2}(r,x) = \frac{\ell^2}{r^2}(dr^2 + \eta_{ab}dx^a dx^b), \qquad \exp(2\phi/(n+p+1)) = r^2/\ell^2$$

- leading to the Ricci-flat solution:

$$\begin{aligned} ds_0^2 &= e^{2\phi/(n+p+1)}\left(\frac{\ell^2}{r^2}\left(dr^2 + \eta_{ab}dx^a dx^b\right) + \ell^2 d\Omega^2_{n+1}\right) \\ &= \left(dr^2 + r^2 d\Omega^2_{n+1}\right) + \eta_{ab}dx^a dx^b \end{aligned}$$

AdS on a torus $\leftrightarrow$ Minkowski



AdS spacetime with $(d-p-1)$ of the boundary directions compactified on a torus is mapped to Minkowski spacetime

$$ds^2_\Lambda = \frac{\ell^2}{r^2}\left(dr^2 + \eta_{ab}dx^a dx^b + d\vec{y}^2\right) \leftrightarrow ds^2_0 = (dr^2 + r^2 d\Omega^2_{n+1}) + \eta_{ab}dx^a dx^b,$$

Holographic dictionary 1

- On AdS, the boundary condition was to choose a metric at $r = 0$. In this example we chose $g_{(0)ab} = \eta_{ab}$
- This translates on the Ricci-flat side into a choice of a metric at the location of a p-brane.

Non-trivial states

- Let us now add an small excitation on AdS. The metric near the boundary looks like:

$$ds^2 = \frac{1}{r^2}\left(dr^2 + (\eta_{\mu\nu} + r^d g_{(d)\mu\nu} + \cdots)dx^\mu dx^\nu\right)$$

- Compactifying on a $(d-p-1)$ torus:

$$\begin{aligned} ds^2_{p+2} &= \frac{dr^2}{r^2} + \frac{1}{r^2}\left(\eta_{ab} + r^d g_{(d)ab} + \ldots)\right) dx^a dx^b, \\ \phi &= (p+1-d)\ln r + r^d \phi_{(d)} + \ldots, \end{aligned}$$

- In the dual theory we have:

$$T_{ab} = \frac{d}{16\pi G_{p+2}} g_{(d)ab}\,, \quad \mathcal{O}_\phi = -\frac{d(d-p-1)}{32\pi G_{p+2}}\phi_{(d)}$$

which satisfy the expected Ward identities,

$$\partial^a T_{ab} = 0\,, \qquad T_a{}^a = (d-p-1)\mathcal{O}_\phi\,.$$

Non-trivial states: Ricci-flat

- Applying the AdS/Ricci-flat correspondence one now gets:

$$ds_0^2 = (\eta_{AB} + h_{AB} + \dots)dx^A dx^B =$$
$$\left(1 - \frac{16\pi G_{p+2}}{n\, r^n}\mathcal{O}_\phi(x)\right)\left(dr^2 + \eta_{ab}dx^a dx^b + r^2 d\Omega_{n+1}^2\right)$$
$$-\frac{16\pi G_{p+2}}{n\, r^n} T_{ab}(x) dx^a dx^b + \dots$$

- The linearized perturbation $\bar{h}_{AB} = h_{AB} - \frac{h^C{}_C}{2}\eta_{AB}$ satisfies,

$$\Box \bar{h}_{AB} = 16\pi G_{p+2}\Omega_{n+1}\delta_A{}^a \delta_B{}^b T_{ab}(x)\delta^{n+2}(r)$$

through second order terms in (boundary) derivatives.

Correlation functions

- As discussed earlier, to compute correlation functions we set $g_{(0)\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$.
- We have seen that the linear transverse-traceless fluctuations are given by

$$h^{\Lambda}_{ij}(k) = h_{(0)ij}(k) \frac{1}{2^{d/2-1}\Gamma(d/2)} (kr)^{d/2} K_{d/2}(kr)$$

- Applying to AdS/Ricci-flat flat correspondence, $d \to -n$, leads to

$$h^{0}_{ij}(k) = h_{(0)ij}(k) \frac{2^{n/2+1}}{\Gamma(-n/2)} \frac{K_{n/2}(kr)}{(kr)^{n/2}}$$

- This is indeed the linearized gravitational field produced by a p-brane with worldvolume metric $\eta_{\mu\nu} + h_{\mu\nu}$.
- This perturbation fall-off at infinity, so the metric is asymptotically flat (transverse to the p-brane).

Holographic dictionary 2

- At linear order, the holographic stress energy tensor becomes the stress energy tensor due to a p-brane located at $r = 0$ that sources the linearized gravitational field h_{AB}.

Symmetries

- The AdS metric

$$ds_\Lambda^2 = \frac{\ell^2}{r^2}\left(dr^2 + \eta_{\mu\nu} dz^\mu dz^\nu\right)$$

is invariant under isometries, which in particular geometrize dilatations and special conformal transformations:

$$\delta_\lambda x^M = \lambda x^M, \qquad \delta_b z^\mu = b^\mu z^2 - 2 z^\mu (z \cdot b) + r^2 b^\mu, \quad \delta_b r = -2(z \cdot b) r$$

- This is reflected on the fact that the holographic stress energy tensor is conserved and traceless,

$$\partial^\mu T_{\mu\nu} = 0\,, \qquad T_\mu{}^\mu = 0\,,$$

Broken symmetries

- Compactification over a torus breaks dilatations and special conformal transforms. The corresponding transformations in the reduced theory are:

$$\delta_\lambda x^a = \lambda x^a,\ \delta_\lambda r = \lambda r; \quad \delta_b x^a = b^a x^2 - 2x^a (x\cdot b) + r^2 b^a,\ \delta_b r = -2(x\cdot b) r$$

- These act as isometries in the metric but transform the scalar field:

$$\delta_\lambda \phi = (p+1-d)\lambda; \qquad \delta_b \phi = -2(p+1-d)(x\cdot b)$$

- This is reflected on the fact that the holographic stress energy tensor is conserved but tracefull,

$$\partial^a T_{ab} = 0\,, \qquad T_a{}^a = (d-p-1)\mathcal{O}_\phi\,.$$

- The dual QFT has a generalized conformal structure [Kanitscheider, Taylor, KS (2008)].
- On the gravitational side, these transformations act as solution generating transformations.

“Hidden” symmetries

- On Minkowski side, these transformations act as conformal transformations:

  $$\delta g_{0AB} = 2\sigma(x) g_{0AB}$$

  where $\sigma = \lambda$ for dilatations and $\sigma = -2(x \cdot b)$ for special conformal transformation.
- Although these are not isometries of Minkowski, they still act as solution generating transformations: δg_{0AB} is still Ricci-flat.

Outline

Holography and long wavelength behavior

- An important case is that of a thermal state on the boundary QFT. This corresponds to a black hole in AdS.
- A generic feature of QFTs is the existence of a *hydrodynamic description* capturing the long-wavelength behavior near to thermal equilibrium.
- Gauge/gravity duality then implies that there should exist a bulk solution corresponding to the thermal state, and nearby solutions corresponding to the hydrodynamic regime.
- Global solutions corresponding to non-equilibrium configurations should be well-approximated by the solutions describing the hydrodynamic regime at sufficiently long distances and late times.

Hydrodynamics and AdS/CFT

This picture is indeed beautifully realized in AdS/CFT:

Thermal state	$\Leftrightarrow$	AdS black hole
Relativistic hydrodynamics of a conformal fluid	$\Leftrightarrow$	Bulk solution in a relativistic gradient expansion

- Solutions describing non-equilibrium configurations are well approximated by hydrodynamics at late times.

[Witten (1998)] ... [Policastro, Son, Starinets (2001)] ... [Janik, Peschanski (2005)] ... [Bhattacharyya, Hubeny, Minwalla, Rangamani (2007)] ... [Chestler, Yaffe (2010)] ...

Hydrodynamics and vacuum Einstein gravity

A similar picture holds for vacuum Einstein gravity, $R_{ij} = 0$:

Thermal state	$\Leftrightarrow$	Rindler space
Relativistic hydrodynamics of a "peculiar" fluid	$\Leftrightarrow$	Bulk solution in a relativistic gradient expansion

[I. Bredberg, C. Keeler, V. Lysov, A. Strominger (2011)] [G.Compère, P. McFadden, KS, M. Taylor (2011) (2012)][C. Eling, A. Meyer, Y. Oz (2012)]

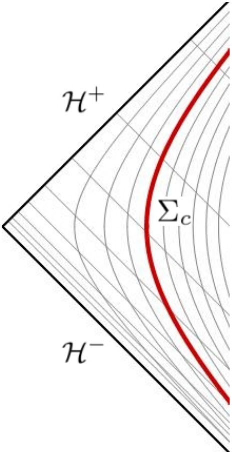
Rindler spacetime

- Flat spacetime in ingoing Rindler coordinates is given by:

$$ds^2 = -r d\tau^2 + 2 d\tau dr + dx_i dx^i$$

i.e. Minkowski space parametrised by timelike hyperbolae $X^2 - T^2 = 4r$ and ingoing null geodesics $X + T = e^{\tau/2}$.

- We consider the portion of spacetime between $r = r_c$ and the future horizon, $\mathcal{H}^+$, the null hypersurface $X = T$.



Black branes and the AdS/Ricci-flat correspondence

- The Planar AdS black brane

$$ds_\Lambda^2 = \frac{1}{r^2}(-f(r)d\tau^2 + d\vec{x}^2 + d\vec{y}^2) + \frac{dr^2}{r^2 f(r)}, \qquad f = 1 - (r/b)^d$$

- ... is mapped to the Schwarzschild black p-brane

$$ds_0^2 = -f(r)d\tau^2 + \frac{dr^2}{r^2 f(r)} + r^2 d\Omega_{n+1}^2 + d\vec{x}^2, \qquad f = 1 - (b/r)^n$$

- ... and the special case $n = -1$ (no sphere) is just Rindler space

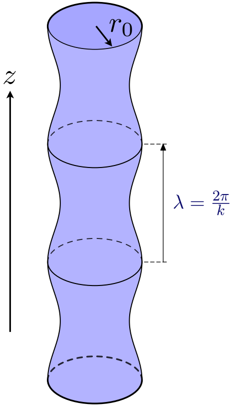
$$ds_0^2 = -r d\tau^2 + 2d\tau dr + d\vec{x}^2$$

$\left(\tau \to \tau/b + \log f(r),\ r \to b^2 f(r)\right)$

Implications

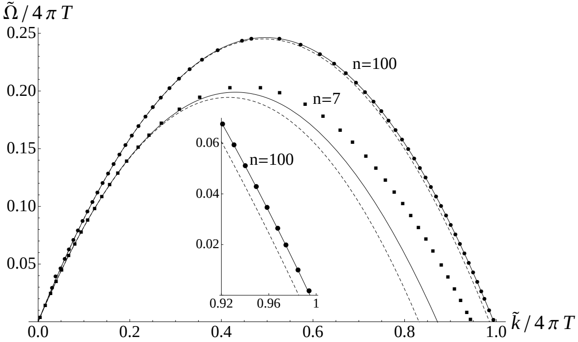
- On the AdS side conformal invariance dictates the equation of state:
$$T_i^i = 0 \qquad \Rightarrow \qquad \varepsilon = (d-1)P$$
- Under the map this becomes:
$$\varepsilon = -(n+1)P \quad \Rightarrow \quad c_s^2 = \frac{\partial P}{\partial \varepsilon} = -\frac{1}{n+1},$$
There is an instability for the sound modes. This is the Gregory-Laflamme instability. [Emparan, Harmark, Niarchos, Obers (2009)]
- When $n = -1$:
$$\varepsilon = 0, \qquad P > 0.$$
This is the Rindler fluid. [G.Compère, P. McFadden, KS, M. Taylor (2011)]

Gregory-Laflamme instability [Gregory, Laflamme (1994)]



- Linealized perturbation: $\delta r_0 \sim e^{\Omega t + ikz}$
- Instability for $\lambda \gtrsim r_0$
- Under the map the evolution of the instability for small Ω and k is mapped to the AdS hydrodynamic regime.
- This leads to a metric constructed to second order in derivatives describing the non-linear evolution of the GL instability.
- This metric to first order was studied earlier in [Camps,Emparan,Haddad (2010)].

Dispersion relation



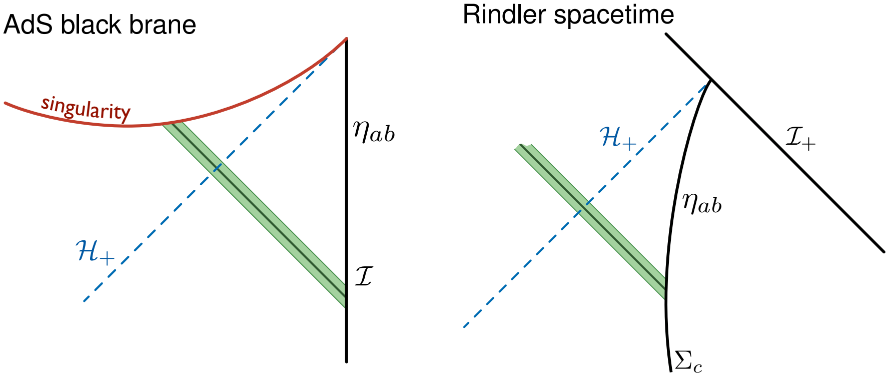
- The dots/squares represent numerical data for $n = 100/n = 7$ (courtesy: P. Figueras)
- The solid line the cubic approximation and the dashed line the quadratic approximation.
- In the insert we zoom into the region close to the threshold mode ($n = 100$)

The Rindler/fluid correspondence

Back to the $n = -1$ case, one can check that the correspondence maps:

- the AdS boundary to the position of the accelerating observer.
- second order hydrodynamics correction are mapped to each other.
- the transport coefficients are the same.
- the position of the horizons (which are corrected by higher derivative terms) and mapped to each other.

The Rindler/fluid correspondence



The conformal boundary of AdS is mapped to the surface Σ_c.

Outline

Conclusions/Outlook

- We used the AdS/Ricci-flat correspondence to start developing holography for asymptotically flat spacetimes.
- Holographic data are encoded in an unexpected way:
  - The source for dual operators are located at a position of a p-brane.
  - The stress energy tensor due to this p-brane is holographic.
- So far we only turned on sources infinitesimally. A full-fledged holography requires turning on finite sources.
- Ricci-flat spacetimes inherit a generalized conformal structure from AdS. It would interesting to extract the implications of this *hidden conformal invariance*.