Vacuum polarization calculation in VEPP-2M energy range

G.V.Fedotovich

Budker Institute of Nuclear Physics

Novosibirsk

Outline

I. Why the accuracy better than 0.05% is required to calculate vacuum polarization effects in photon propagator at low energies.

II. How today the leptons and hadrons vacuum polarization contributions to virtual photon propagator are calculated.

III. Main factors affecting on the final accuracy. What should be done to improve the calculation accuracy to the level 0.05%.

IV. Cross sections of the processes $e^+e^- \to e^+e^- + \gamma$, $e^+e^- \to \mu^+\mu^- + \gamma$, $e^+e^- \to \tau^+\tau^- + \gamma$, $e^+e^- \to \pi^+\pi^- + \gamma$, $e^+e^- \to K^+K^- + \gamma$.

V. Vacuum polarization calculation with "dress" and "bare" cross sections.

VI. Summary

Why is the accuracy of "bare" hadronic cross sections better 0.2 – 0.3% required?

$R(s) = \sigma_{bare}(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)$,
where $\sigma(e^+e^- \to \mu^+\mu^-) = (4\pi/3)(\alpha^2/s)$

$$\sigma_{bare}(hadr) = \sigma_{dress}(hadr)[1-P(s)]^2$$

To keep "bare" cross section accuracy at the same level accuracy
$P(s) < 0.05\%$

$$a_\mu^{had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{R(s)K(s)}{s^2} ds \sim 60 \text{ ppm}$$

- 60 ppm • 0.3 % ≈ 0.2 ppm (BNL – 0.5 ppm)
- Experimental accuracy for muon anomalous 0.1÷0.2 ppm is expected in the nearest future.
- The accuracy better 0.3 % for hadronic cross sections will be

What does P(s) consist of ?

$$P(s) = P_e(s) + P_\mu(s) + P_\tau(s) + P_h(s)$$

Expression for electron $P_l(s)$ is calculated in α power series up to fourth order.

$$P_l(s) = \left(\frac{\alpha}{\pi}\right) P_1(s) + \left(\frac{\alpha}{\pi}\right)^2 P_2(s) + \left(\frac{\alpha}{\pi}\right)^3 P_3(s) + .$$

$P_2(s) = (L - i\pi)/4 + \zeta(3) - 5/24 \approx 10$, $(\alpha/\pi)^2 P_2(s) \sim 0.01\%$

Hadronic contribution can be evaluated through integration only

$$P_h(s) = \frac{s}{4\pi^2\alpha}\left[\int_{4m^2_\pi}^{\infty} \frac{\sigma^h{}_{bare}(s')ds'}{s - s'} - i\pi\sigma^h{}_{bare}(s)\right]$$

Lepton contribution first order in α

$$P_1(s) = L/3 - 5/9 + f(x_\mu) + f(x_\tau) -$$
$$i\pi[1/3 + \phi(x_\mu)\Theta(1-x_\mu) + \phi(x_\tau)\Theta(1-x_\tau)]$$

Where L = ln(s/m_e^2) is a large logarithm (~15), x = $4m^2/s$

Functions f(x) и Φ(x) are calculated in many papers:

for x < 1 $$f(x) = -5/9 - x/3 + \frac{2+x}{6}\sqrt{1-x}\,Ln\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}}$$

for x > 1 $$f(x) = -5/9 - x/3 + \frac{2+x}{3}\sqrt{x-1}\arctan\left(\frac{1}{\sqrt{x-1}}\right)$$

$$\Phi(x) = \frac{2+x}{6}\sqrt{1-x}$$

How has P(s) been calculated in past ?

- Well known analytical expressions for lepton Born cross sections were used. Today it is not enough to get P(s) with 0.05% accuracy.
- Experimental values for hadronic cross sections were used to evaluate hadronic contribution to P(s). It was done early a little not correctly!

How (on our opinion) lepton and hadron parts of P(s) should be calculated to improve it's accuracy?

- It is necessary to take into account all first order in α QED corrections. Two effects – namely: FSR and Coulomb interaction in final state should be incorporated into Born cross sections (it was done early for electrons only).
- For hadronic cross sections it is enough to do for $\pi^+\pi^-$ and K^+K^- channels only. They gives dominant contribution at low energies.
- To obtain hadronic Born cross section VP effects must be removed from

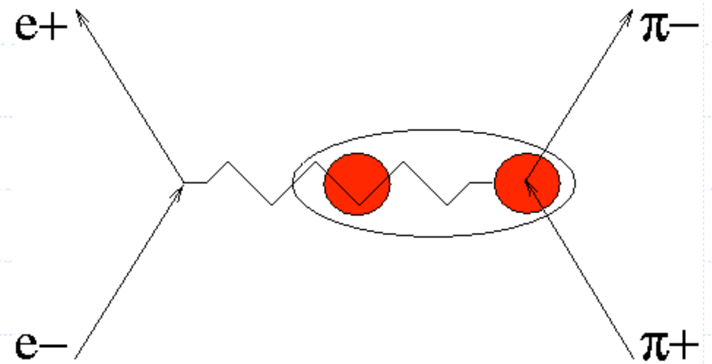
Definition of $\sigma(e^+e^- \to hadrons)$ depends on the application

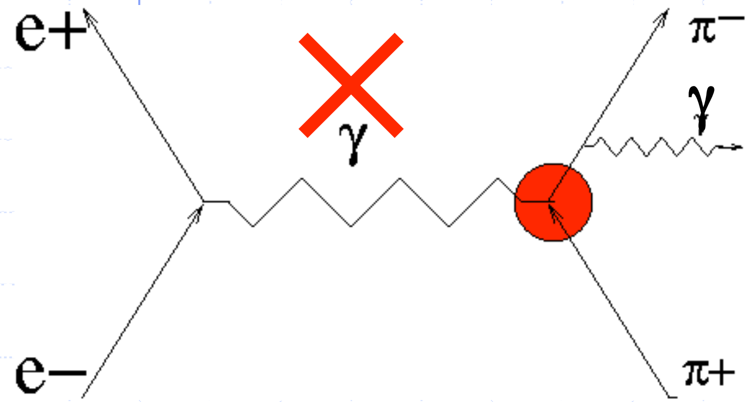
- Hadron spectroscopy (used to get rho-meson mass, wdth, …): the vacuum polarization (VP) is the part of the "dress" cross-section and is left untouched. Final state radiation (FSR) and Coulomb interaction (CI) are not and should be removed (they are in RC).

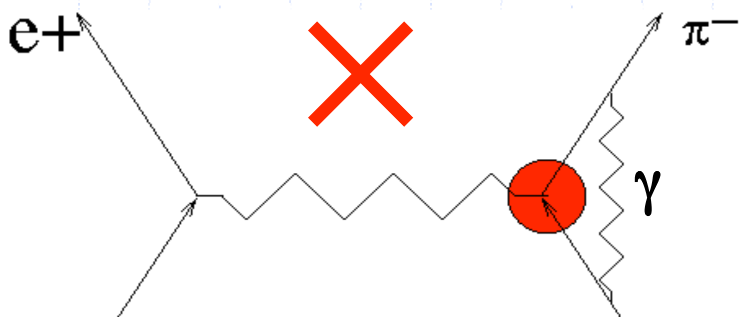
- "Bare" cross-section used in R: vice versa – FSR and CI are the part of the cross-section, VP is not.

- VP calculation: for this aim to "dress" cross section FSR and CI must be added. Iteration approach is needed.

Feynman graphs for process $e^+e^- \rightarrow \pi^+\pi^-$ visually demonstrate what are talking about







- Dynamics of hadron production (form-factor properties, mass, width, peak cr. sect.) must define by strong interactions ⇒virtual VP effects in photon propagator must glue to the vertex production of two pions. It is so called "dress" cross section correctly describe energy dependence behavior.

- "Dress" cross section does not contain FSR by pions. Their contribution to cross section is taken into account in RC.

- Coulomb interaction of pions in final state is extracted from "dress" cross section, because this effect is in RC.
- "Dress" cross section does not contain any QED effects.

"Bare" cross sections for processes $e^+e^- \to e^+e^- + \gamma,\ \mu^+\mu^- + \gamma,\ \tau^+\tau^- + \gamma,\ \pi^+\pi^- + \gamma$ Used for R calculations

$$\sigma^{bare}_{ee\to\mu\mu+\gamma} = \sigma^{Born}_{ee\to\mu\mu}\left(1+\frac{2\alpha}{\pi}\delta^{FSR}\right)f_c(z)$$

$$\sigma^{Born}_{ee\to\mu\mu} = \frac{4\pi}{3}\frac{\alpha^2}{s}\beta(3-\beta^2)/2$$

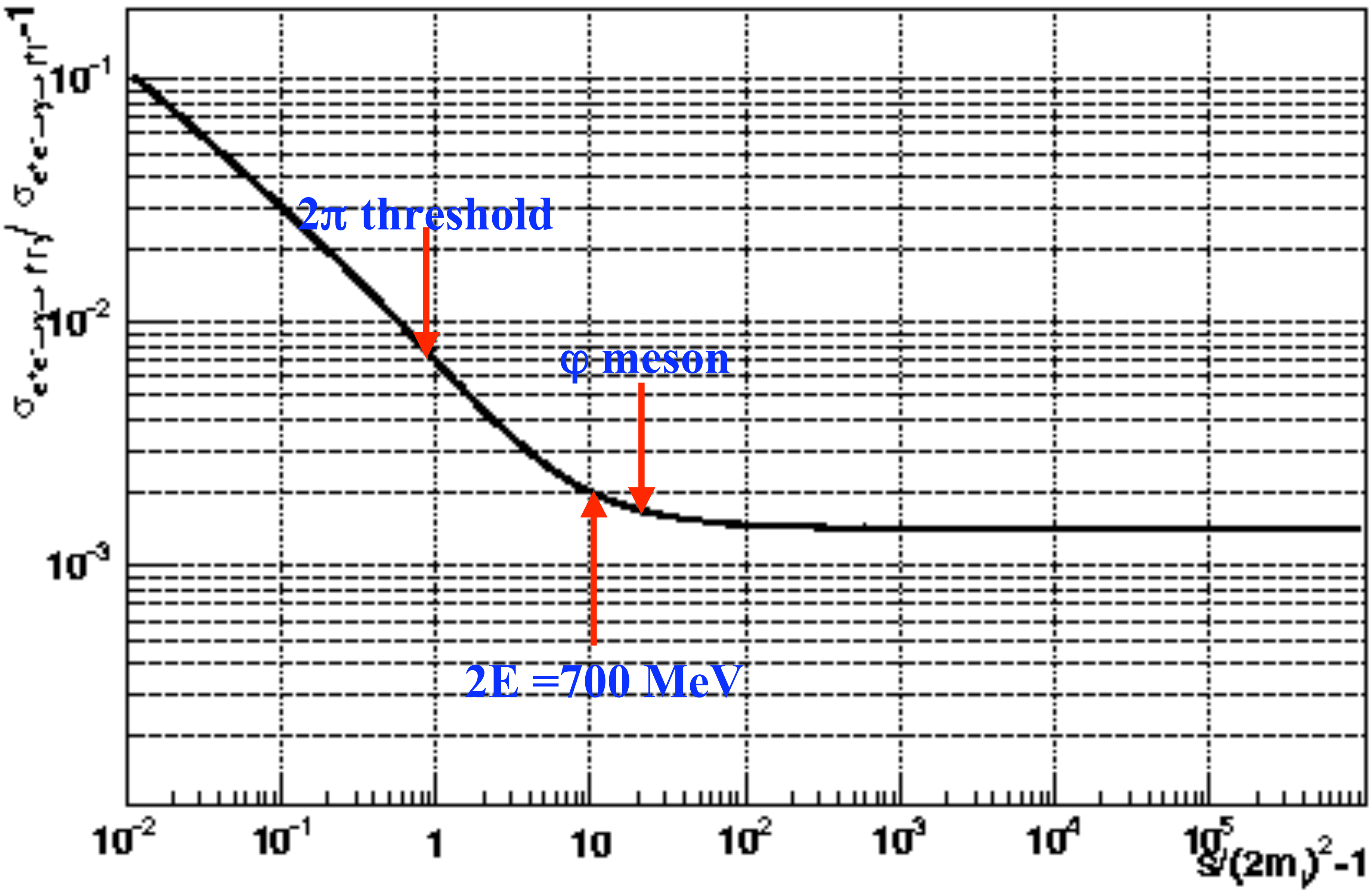
$$\delta^{FSR}_{\mu}|_{u.l.} \to \frac{3}{8} \qquad \delta^{FSR}_{\pi}|_{u.l.} \to \frac{3}{2}$$

Coulomb interaction factor

$$f_c(z) = \frac{z}{1-\exp(-z)}, \quad \text{where } z = \frac{2\pi\alpha}{v_{rel}}, \quad v_{rel} = \frac{2\beta}{1+\beta^2}$$

$$f_c(z)| \to 1 + z/2 \approx 1.023 \qquad f_c(z)| \to \pi\alpha/2\beta$$

Relative corrections to process $e^+e^- \to \mu^+\mu$



“Dress” lepton cross sections

$$\sigma^{dress}_{ee\to\mu\mu}(s)=\frac{\sigma^{bare}_{ee\to\mu\mu+\gamma}(s)}{|1-P(s)|^2}$$

At narrow resonances hadronic contribution to P(s) becomes huge, whereas it must be much less 1!!! In this case expression for P(s) (according to Landau, Lifshits) should be modified a little.

$$\Pi^{dress}(s)=\frac{P(s)}{1-P(s)}=\Pi_e(s)+\Pi_\mu(s)+\Pi_\tau(s)+\Pi_h(s)$$

$$\sigma^{dress}_{ee\to\mu\mu}(s)=\sigma^{bare}_{ee\to\mu\mu+\gamma}(s)|1+\Pi(s)|^2$$

So kind redefinition should be done for all processes of electron-positron annihilation

VP evaluation using "dress" cross sections

$$\Pi_e(s)=\frac{s}{4\pi^2\alpha}\left[P\int_{4m_e^2}^{\infty}\frac{\sigma_{ee\to ee}^{dress}(s')ds'}{s-s'}-i\pi\sigma_{ee\to ee}^{dress}(s)ds\right]$$

$$\Pi_\mu(s)=\frac{s}{4\pi^2\alpha}\left[P\int_{4m_\mu^2}^{\infty}\frac{\sigma_{ee\to \mu\mu}^{dress}(s')ds'}{s-s'}-i\pi\sigma_{ee\to \mu\mu}^{dress}(s)ds\right]$$

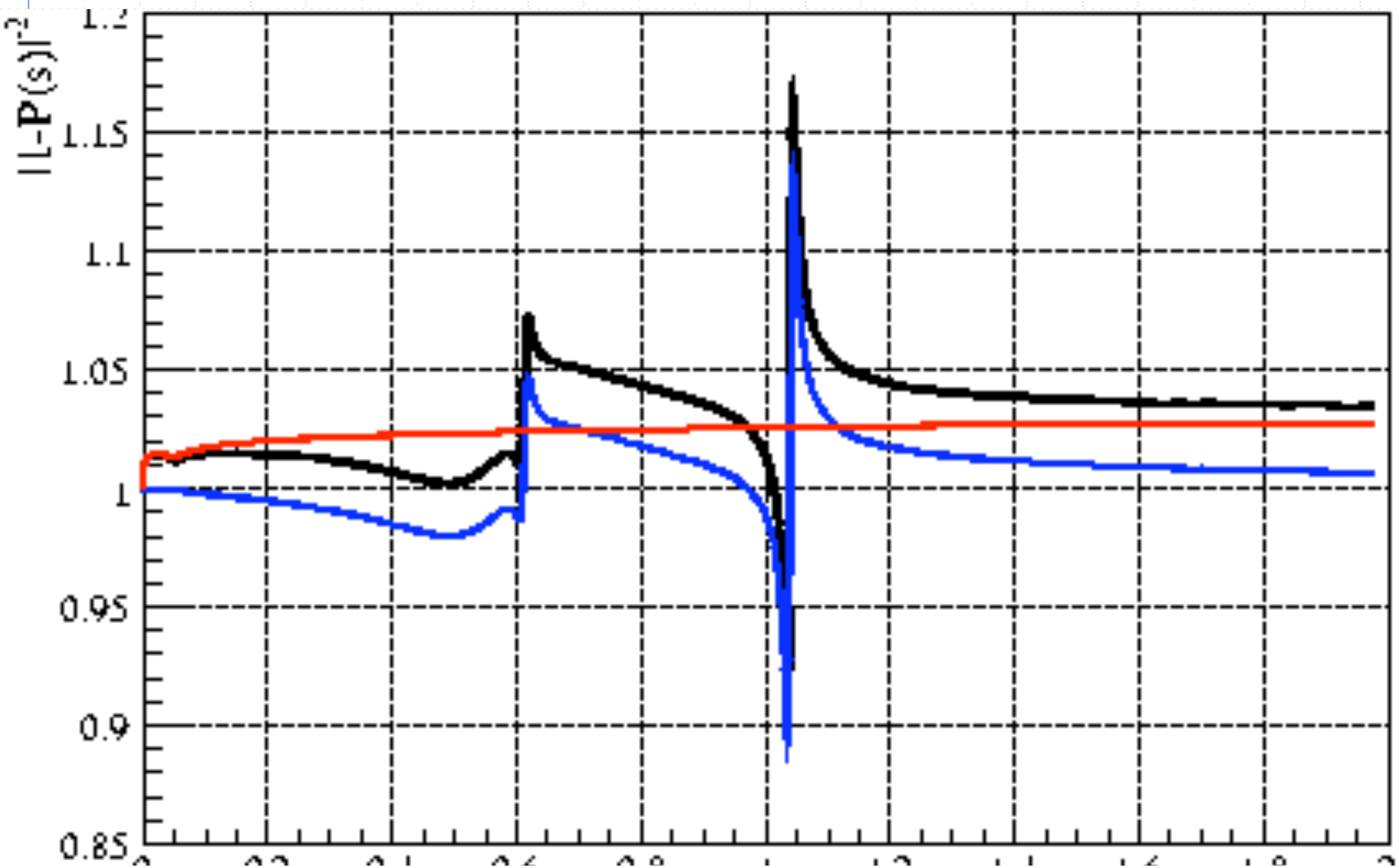
$$\Pi_\tau(s)=\frac{s}{4\pi^2\alpha}\left[P\int_{4m_\tau^2}^{\infty}\frac{\sigma_{ee\to \tau\tau}^{dress}(s')ds'}{s-s'}-i\pi\sigma_{ee\to \tau\tau}^{dress}(s)ds\right]$$

$$\Pi_h(s)=\frac{s}{4\pi^2\alpha}\left[P\int_{4m_\pi^2}^{\infty}\frac{\sigma_{ee\to had}^{dress}(s')ds'}{s-s'}-i\pi\sigma_{ee\to had}^{dress}(s)ds\right]$$

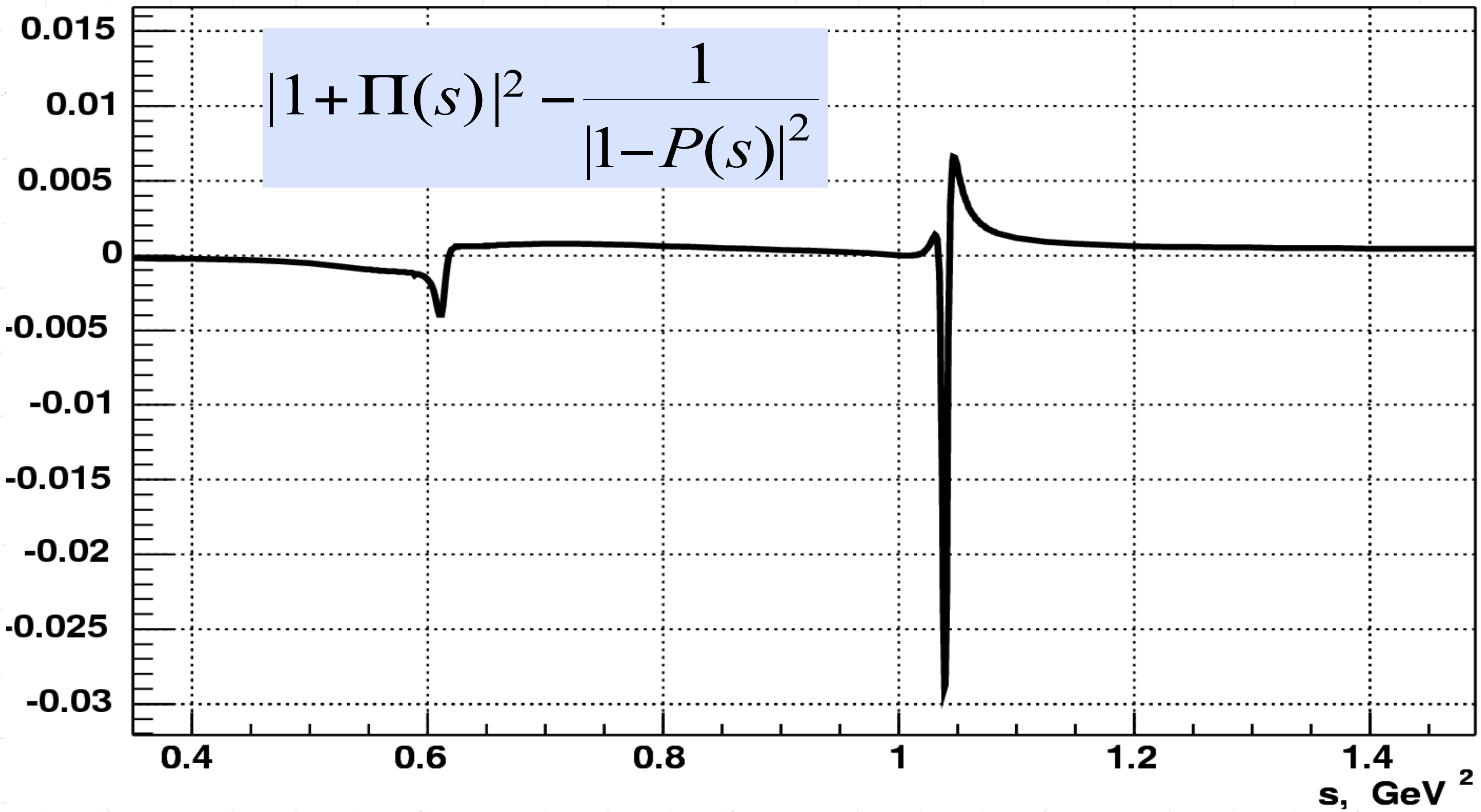
Iteration technics is necessary in this approach.

Value of the quantity $|1-P(s)|^{-2}$ in VEPP-2M energy range

Born cross sections were exploited – traditional approach



Comparison of "dress + FSR + CI" muon cross sections in relative units (divided on Born)



- The same is valid for any "bare" hadronic cross sections too.
- Near ω and ϕ resonanses deviations reach ~0.5% and ~2.5% respectively.
- Out of resonance the difference are inside corridor with width ±0.1% as expected – main result!!!
- Central value displacement of a (had) for redefine VP calculation (and as

Analytical expression for narrow resonance contribution to VP operator P(s)

$$\sigma_X^{res}(s) = \frac{12\pi B_X^{ee}}{M_X^2} \frac{M_X^2 \Gamma_X^2}{|s - M_X^2 - iM_X \Gamma_X|^2}$$

$$\Pi_X^{res}(s) = \frac{3B_X^{ee}}{\alpha} \frac{M_X \Gamma_X}{s - M_X^2 + iM_X \Gamma_X}$$

$$\delta m_X^{shift} = \frac{3\Gamma_X^{ee}}{2\alpha}$$

Simple analytical expression for mass shift. It can be used for quick bench evaluation.

Using PDG values for Γ^{ee} the difference between "dress" and "bare" masses for ϕ and J/ψ mesons mass gives ~262 (253) keV and ~1080 (1040) keV. The difference between estimation and

Factors determining accuracy evaluation of VP operator P(s)

Unaccounted corrections to cross sections more high order:

- Weak interactions contribute to cross sections 0.1% for energies

2E < 3 GeV and we can omitted in our approach.

2. Second order and higher RC (NLO), which that proportional to $(\alpha/\pi)^2\ln(s/m^2) \sim 10^{-4}$, fortunately small with respect to 0.1% level.

3. The uncertainty of about 0.1% is related to experimental systematic error. For example, 1% error in hadronic cross sections change "bare" cross sections at scale 0.03%.

4. Fourth source uncertainty due to theoretical models which describe cross sections energy dependence.

5. In paper Smith and Voloshin was concluded that the combine effect of all parametrically enhanced $O(\alpha^2)$ corrections limited by $2 \cdot 10^{-4}$ and it is beyond intended accuracy.

Considering the uncertainty sours as independent $\Rightarrow$ total systematic error

Summary

- **For processes $e^+e^- \to e^+e^- + \gamma$, $e^+e^- \to \mu^+\mu^- + \gamma$, $e^+e^- \to \tau^+\tau^- + \gamma$, $e^+e^- \to \pi^+\pi^- + \gamma$, $K^+K^- + \gamma$ total cross section are calculated with first order α corrections.**

- **Coulomb interaction in FS taken into account correctly without expansion in series α/v (integration from threshold).**

- **Hadron contribution to operator P(s) is calculated using "dress + FSR +CI" cross sections with first order α corrections.**

- **"Bare" cross sections with first order α corrections are required for different dispersion evaluations (VP is removed).**

- **Analytical expression for narrow resonance contribution to operator P(s) is derived in compact form.**

- **VP effects in propagator of virtual photon have been evaluated with accuracy better 0.05% in VEPP-2M energy**

А как можно продвинуться дальше и проверить точность данного подхода?

Измерять сечение $e^+e^- \rightarrow \mu^+\mu^-$

$$\frac{\sigma_{\exp}^{\mu\mu}}{\sigma_{Born}^{\mu\mu}} = |1 + \Pi(s)|^2$$

Точность 0.1%, число событий 10^6. На VEPP-2M было набрано ~ 10^6 мюонов.

При светимости ~ 10^{32} за тот же период может быть набрано ~ 10^8 событий

Сканирование по энергии ~ 100 точек (избыток).

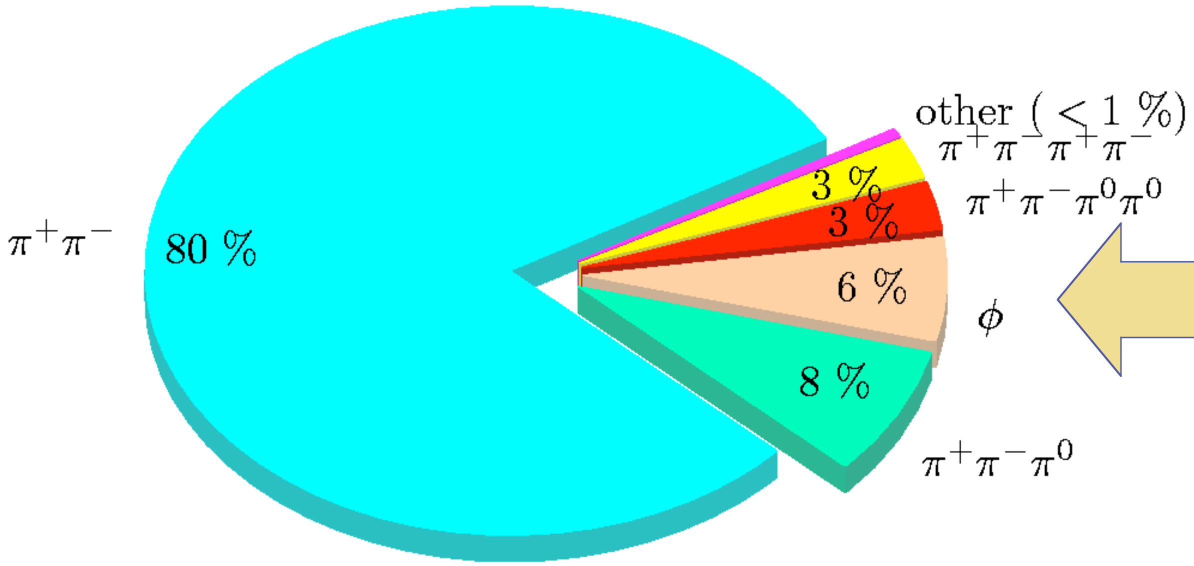
Специализированный детектор для измерения процесса $e^+e^- \rightarrow \mu^+\mu^-$.

Процесс двухквантовой аннигиляции для измерения светимости: нет FSR, нет интерфер., нет вклада от поляризации вакуума, 9/4 от сечения Bhabha.

Радиационные поправки для сечений с ISR вычислены с точностью лучше 0.1%.

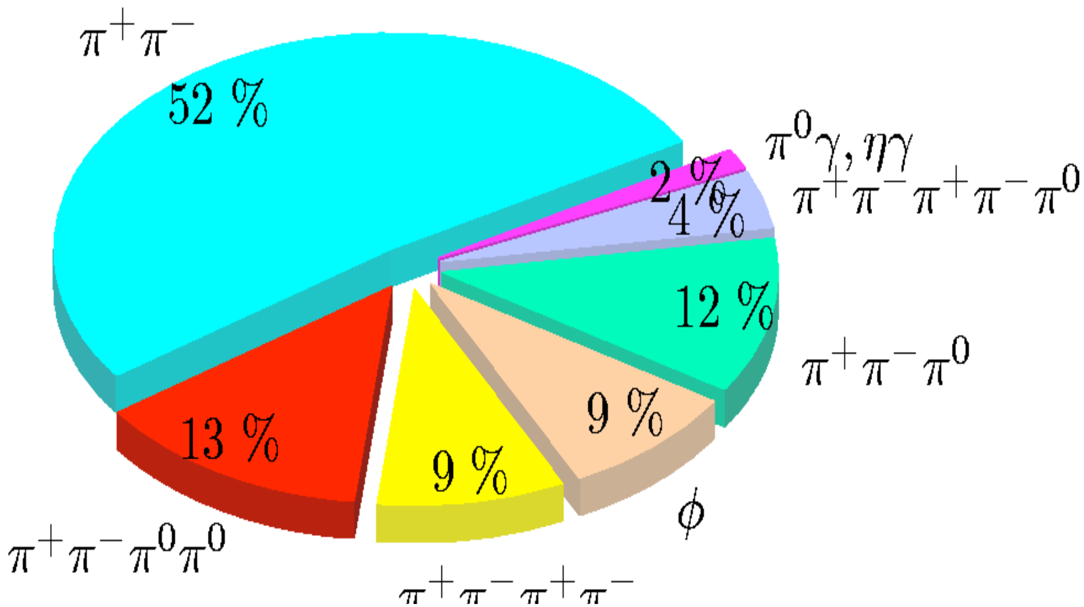
Нужен детектор для измерения координат конверсии фотона в калориметре (LXe, LKr). Координатная точность ~ 2 мм на базе ~ 0.5 м эквивалентна параметрам ZK.

Èç ÷åãî ïîñò∂îåíî a_μ è åå òî÷íîñòü?



Îòíîñèòåëüíûå âêëàäû â a_μ

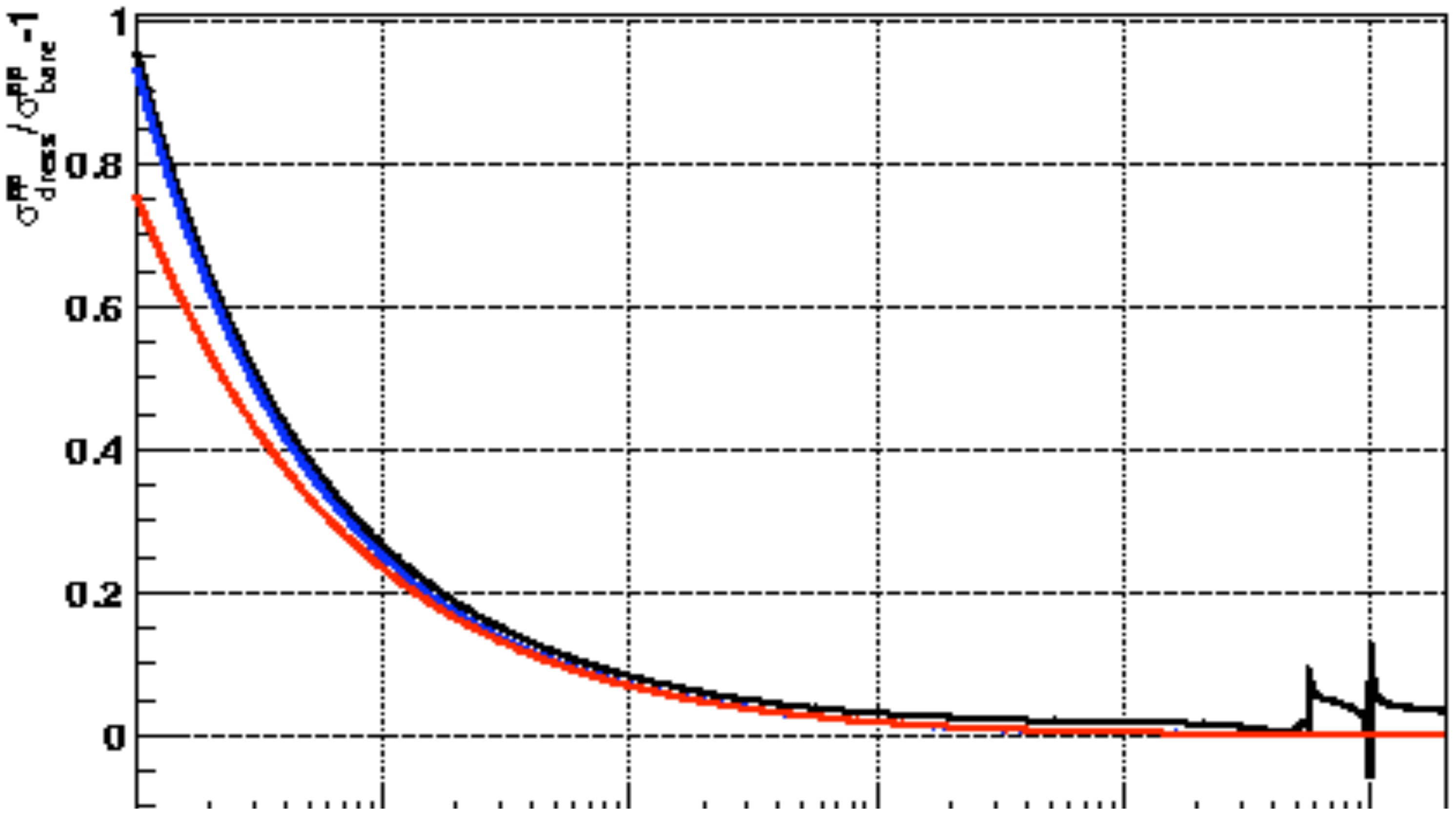
Îòíîñèòåëüíûå âêëàäû â îøèáêó a_μ

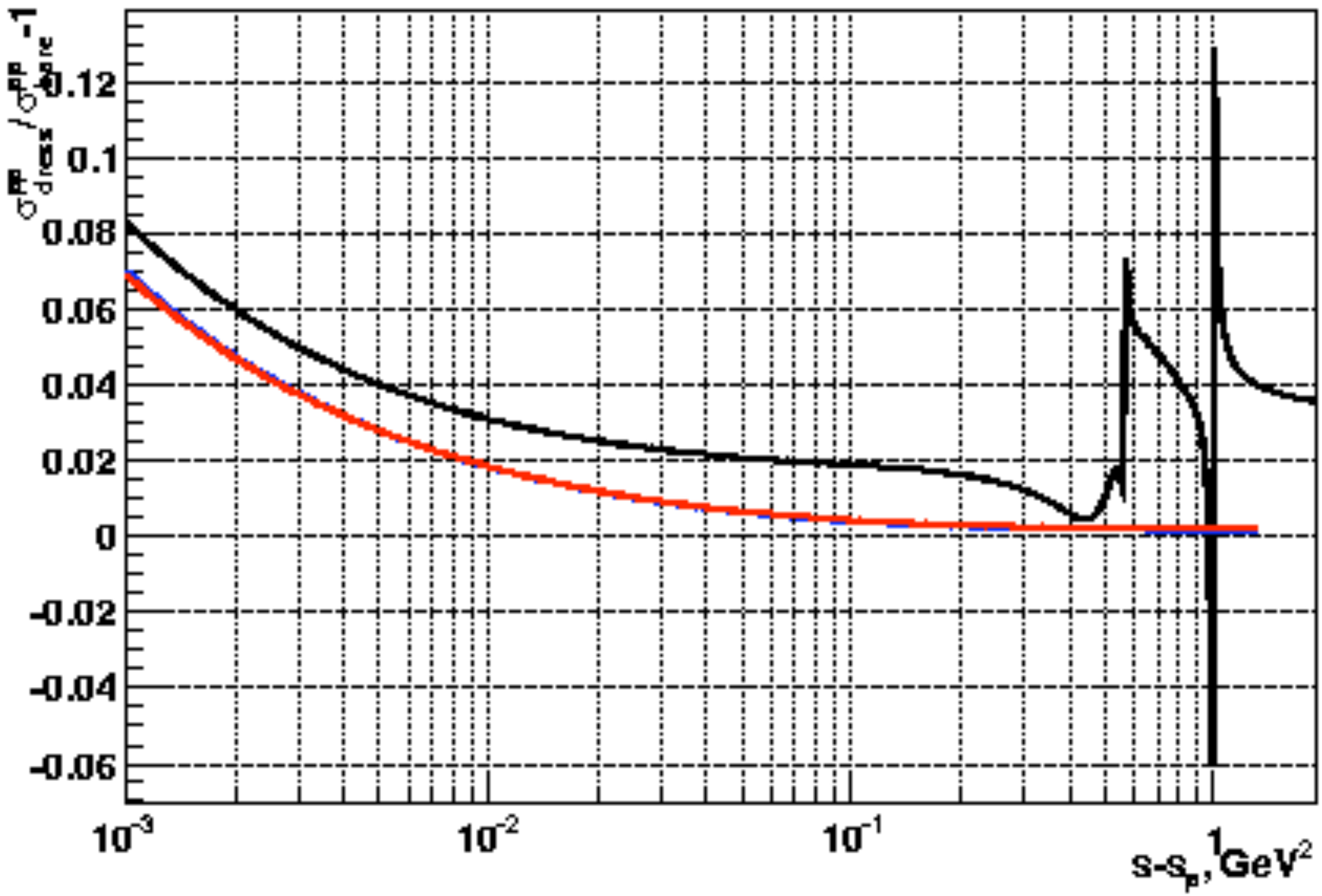


--- излучение фотона в конечном состоянии

--- точный учет Кулоновского взаимодействия

--- с учетом всех факторов ("одетые" сечения)





σ^pp_dress / σ^pp_bare -1
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
10^-3
10^-2
10^-1
1
s-s_p, GeV^2

$$|1+\Pi(s)|^2 - \frac{1}{|1-P(s)|^2}$$