New developments in PHOKHARA MC generator: new tool to help to measure Lambda form factors at B-meson factories

H. CZYŻ, IF, UŚ, Katowice Frascati 2007

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Introduction and nucleon form factors

PHOKHARA 6.0 - $\Lambda\bar{\Lambda}$

Plans

From EVA to PHOKHARA and ...

EVA: $e^+e^- \to \pi^+\pi^-\gamma$

- tagged photon ($\theta_\gamma > \theta_{cut}$)
- ISR at LO + Structure Function
- FSR: point-like pions

[Binner et al.]

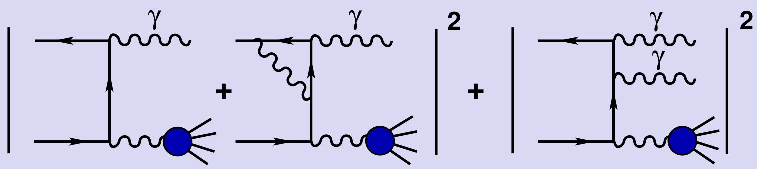
$e^+e^- \to 4\pi + \gamma$

- ISR at LO + Structure Function

[Czyż, Kühn,2000]

PHOKHARA 6.0: $\pi^+\pi^-$, $\mu^+\mu^-$, 4π, $\bar{N}N$, 3π, KK, $\Lambda(\to\cdots)\bar{\Lambda}(\to\cdots)$

- **ISR at NLO:** virtual corrections to one photon events and two photon emission at tree level

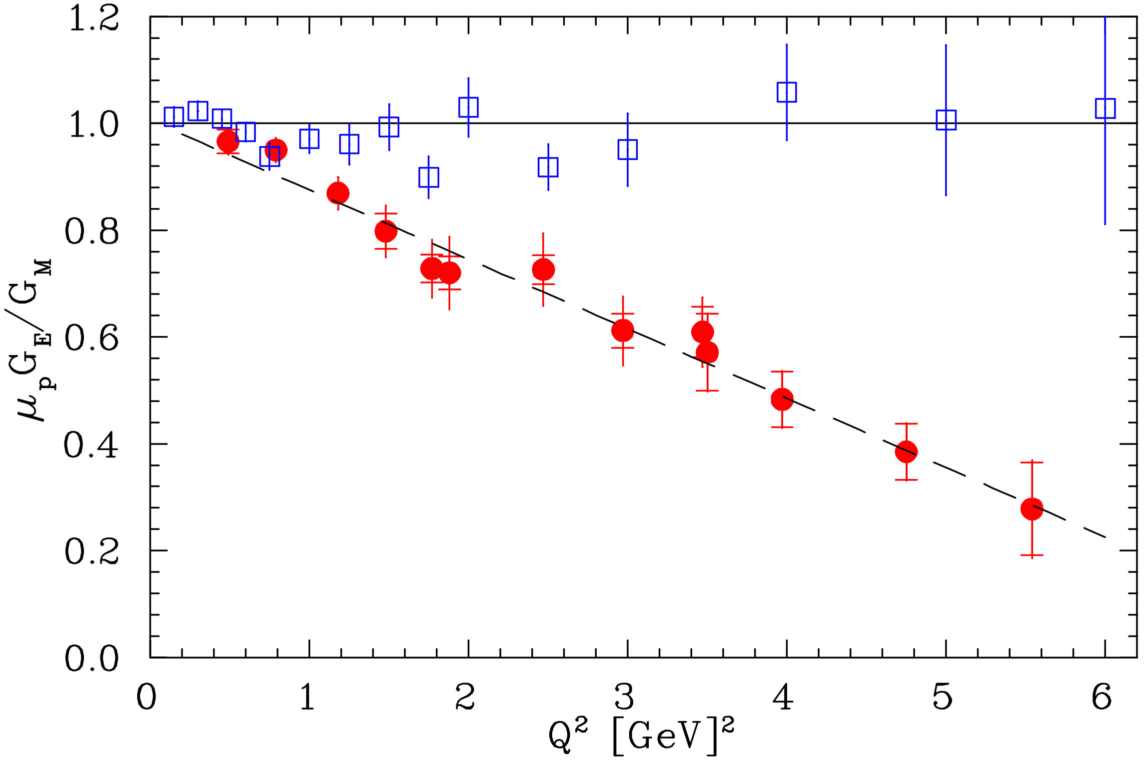


- FSR at NLO:$\pi^+\pi^-$, $\mu^+\mu^-$,K^+K^-
- tagged or untagged photons
- Modular structure

http://ific.uv.es/~rodrigo/phokhara/

FF separation at B-factories

H.C., J. H. Kühn, E. Nowak and G. Rodrigo, Eur.Phys.J.C35(2004)527



J. Arrington, Phys. Rev. **C 68** (2003) 034325

FF separation at B-factories

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Electromagnetic current describing production of baryon-antibaryon pair

$$J_\mu = -ie \cdot \bar{u}(q_2) \left(F_1^N(Q^2)\gamma_\mu - \frac{F_2^N(Q^2)}{4m_N} [\gamma_\mu, \not{Q}\,] \right) v(q_1) \ ,$$

$$G_M^N = F_1^N + F_2^N \ , \qquad G_E^N = F_1^N + \tau F_2^N \ ,$$

$$\tau = Q^2/4m_N^2, \, Q = q_1 + q_2$$

FF separation at B-factories

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AT LO ISR : $e^+ + e^- \to \bar{N} + N + \gamma.$

$$d\sigma = \frac{1}{2s} L_{\mu\nu} H^{\mu\nu} dLips(p_1 + p_2; q_1, q_2, k)$$

$$H_{\mu\nu} = 2|G_M^N|^2 (Q_\mu Q_\nu - g_{\mu\nu} Q^2) - \frac{8\tau}{\tau - 1}\left(|G_M^N|^2 - \frac{1}{\tau}|G_E^N|^2\right) q_\mu q_\nu$$

FF separation at B-factories

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$|G_E^N|$, $|G_M^N|$ can be extracted via angular distributions :

$$L_{\mu\nu}H^{\mu\nu} = \frac{(4\pi\alpha)^3}{Q^2}\Bigg\{\left(|G_M^N|^2 - \frac{1}{\tau}|G_E^N|^2\right) \times \frac{32s}{\beta_N^2(s-Q^2)}\left(\frac{1}{y_1}+\frac{1}{y_2}\right)\left(\frac{(p_1\cdot q)^2+(p_2\cdot q)^2}{s^2}\right) + 2\left(|G_M^N|^2 + \frac{1}{\tau}|G_E^N|^2\right)\left[\left(\frac{1}{y_1}+\frac{1}{y_2}\right)\frac{(s^2+Q^4)}{s(s-Q^2)} - 2\right]\Bigg\} ,$$

where $y_{1,2} = \frac{s-Q^2}{2s}(1 \mp \cos\theta_\gamma)$.

FF separation at B-factories

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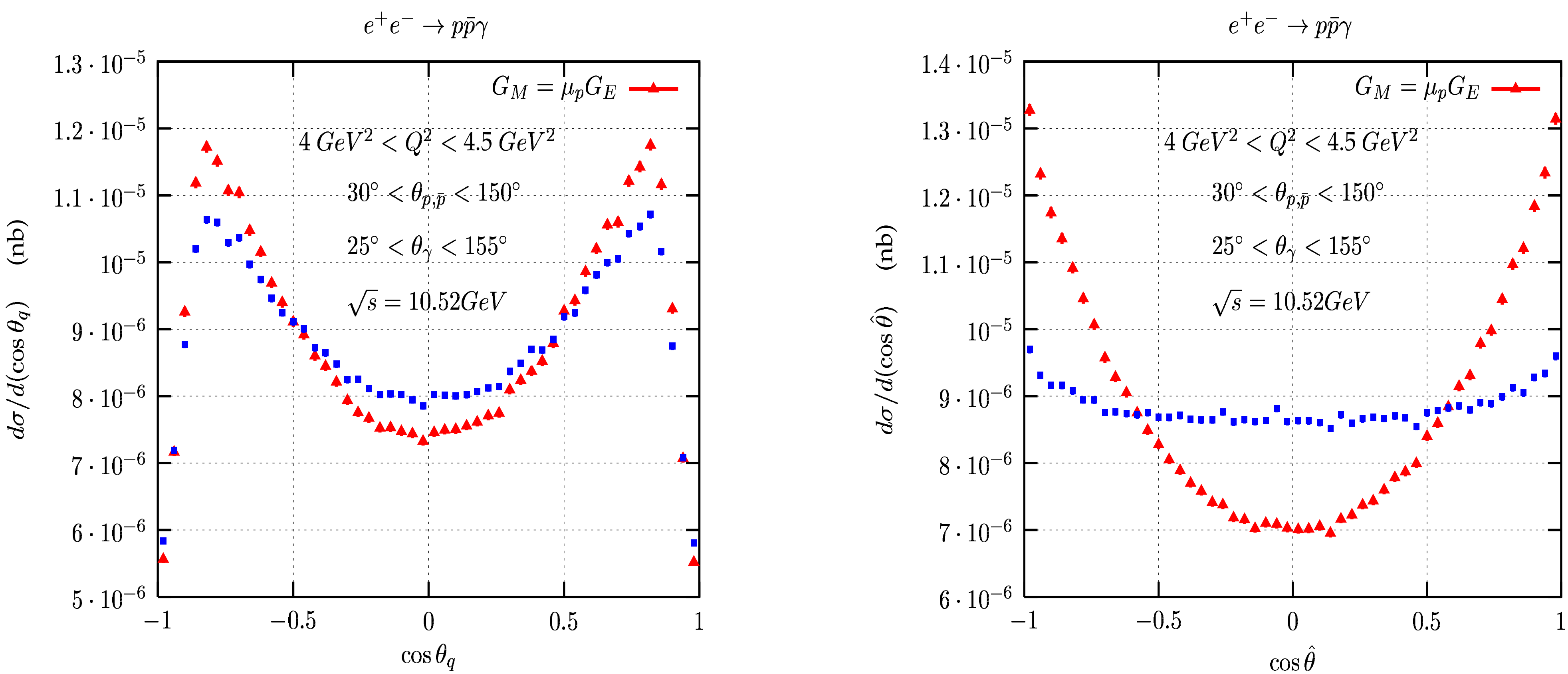
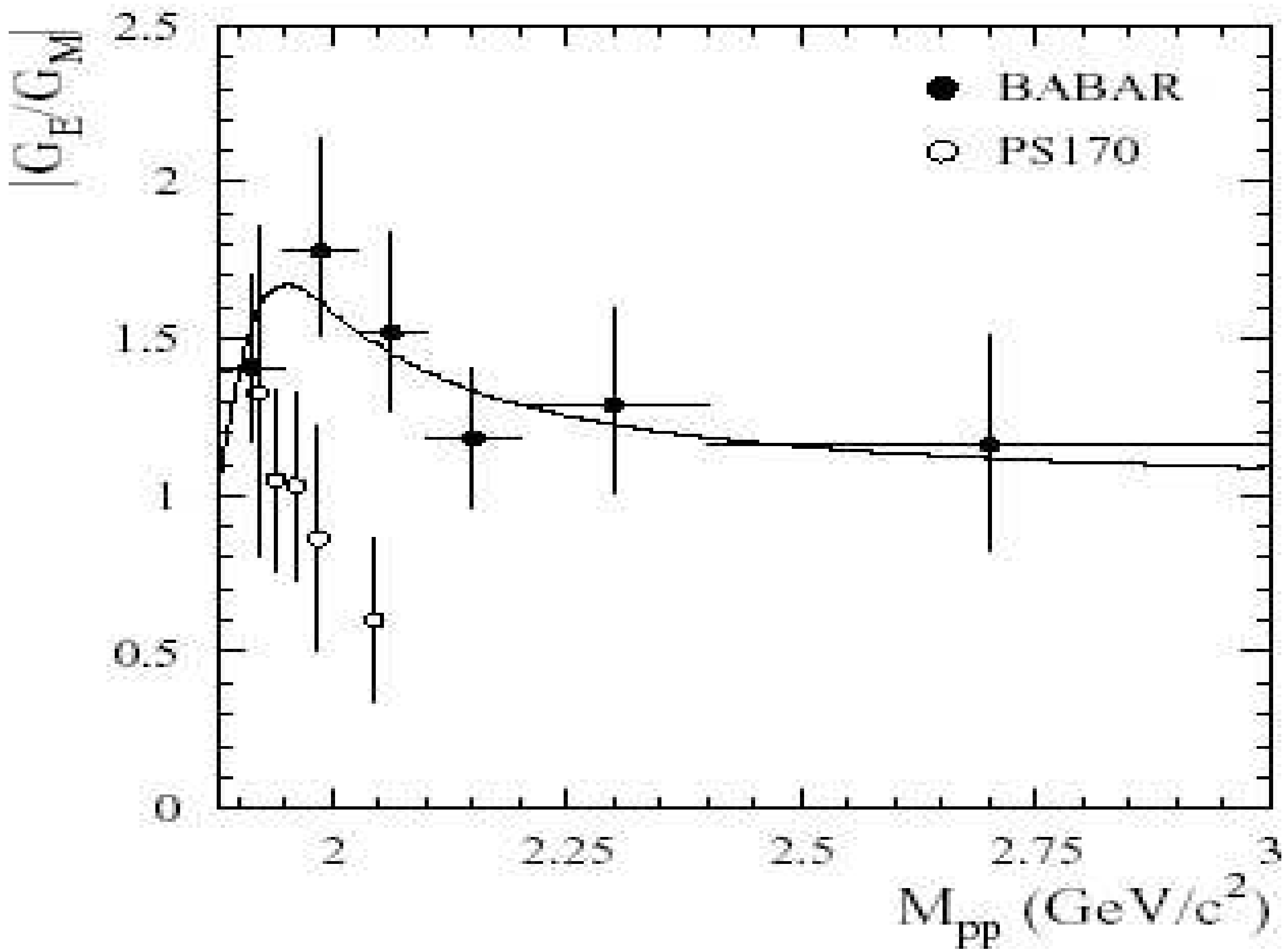


about 2000 events per 100 fb^{-1}

nucleon FF



BaBar: Phys.Rev.D73:012005,2006.

Λ formfactors

$$e^+e^- \rightarrow \Lambda(q_2, S_2)\bar{\Lambda}(q_1, S_1)$$

$$e^+e^- \rightarrow \Lambda(q_2, S_2)\bar{\Lambda}(q_1, S_1)\gamma_{ISR}$$

$$J_\mu = -ie \cdot \bar{u}(q_2, S_2)$$

$$\left(F_1^\Lambda(Q^2)\gamma_\mu - \frac{F_2^\Lambda(Q^2)}{4m_\Lambda}[\gamma_\mu, \not{Q}] \right) \; v(q_1, S_1)$$

The hadronic tensor

$$|M^{0,1}|^2 = L^{0,1}_{\mu\nu} \, H^{\mu\nu}$$

$$H^{\mu\nu} = J^{\mu\dagger} \, J^{\nu}$$

The hadronic tensor

$$\frac{1}{e^2}H^S_{\mu\nu} = \frac{|G_M|^2}{2}\Big(Q.S_2\ \{Q_\mu, S_{1\nu}\}_+ + Q.S_1\ \{Q_\mu, S_{2\nu}\}_+$$

$$-S_1.S_2\ Q_\mu\ Q_\nu - Q^2\ \{S_{1\mu}, S_{2\nu}\}_+\Big) + |G_M|^2(\frac{Q^2}{2}\ S_1.S_2$$

$$-Q.S_1\ Q.S_2)\ \ g_{\mu\nu} + \Big(\frac{|G_M - G_E|^2}{m_\Lambda^2(\tau-1)^2}\ Q.S_1\ Q.S_2\ + \frac{2\tau}{\tau-1}\Big(|G_M|^2$$

$$-\frac{1}{\tau}|G_E|^2\Big)S_1.S_2\Big)q_\mu\ q_\nu + \frac{\tau|G_M|^2 - Re(G_M G_E^*)}{\tau-1}(Q.S_2\ \{q_\mu, S_{1\nu}\}_+$$

$$-Q.S_1\ \{q_\mu, S_{2\nu}\}_+) + \frac{Im(G_M G_E^*)}{m_\Lambda(\tau-1)}\{\varepsilon_{\beta\gamma\delta\mu}\ Q^\beta q^\gamma\ (S_1^\delta + S_2^\delta), q_\nu\}_+$$

$$+\ \frac{1}{4}H^U_{\mu\nu}$$

The hadronic tensor

$$\frac{1}{e^2}H^A_{\mu\nu} = i\frac{Im(G_M G_E^*)}{\tau-1}(Q.S_2\{q_\mu,\ S_{1\nu}\}_- - Q.S_1\{q_\mu,\ S_{2\nu}\}_-)$$
$$- i\ \frac{Re(G_M G_E^*)}{m_\Lambda(\tau-1)}\ \{\varepsilon_{\beta\gamma\delta\mu}\ Q^\beta q^\gamma\ (S_1^\delta + S_2^\delta), q_\nu\}_-$$
$$+ i\ \frac{|G_M|^2}{2m_\Lambda(\tau-1)}\varepsilon_{\beta\gamma\mu\nu}\ Q^\beta q^\gamma\ (Q.S_2 - Q.S_1)$$

where $q = (q_2 - q_1)/2$, $\{a_\mu, b_\nu\}_\pm \equiv a_\mu b_\nu \pm b_\mu a_\nu$ and

$$\frac{1}{4}H^U_{\mu\nu} = 2|G_M|^2(Q_\mu Q_\nu - g_{\mu\nu}Q^2) - \frac{8\tau}{\tau-1}\left(|G_M|^2 - \frac{1}{\tau}|G_E|^2\right)q_\mu q_\nu$$

The polarized cross section

$$d\sigma(e^+e^- \to \bar{\Lambda}\Lambda) \;=\; \frac{1}{2s}\, L^0_{\mu\nu}\, H^{\mu\nu} d\Phi_2(p_1+p_2; q_1, q_2)$$

$$L^0_{\mu\nu}\, H^{\mu\nu} =$$

$$4\pi^2\alpha^2 \Bigg\{ |G_M|^2 \left(1+\cos^2\theta_{\bar{\Lambda}}\right) + \frac{1}{\tau}|G_E|^2 \sin^2\theta_{\bar{\Lambda}}$$

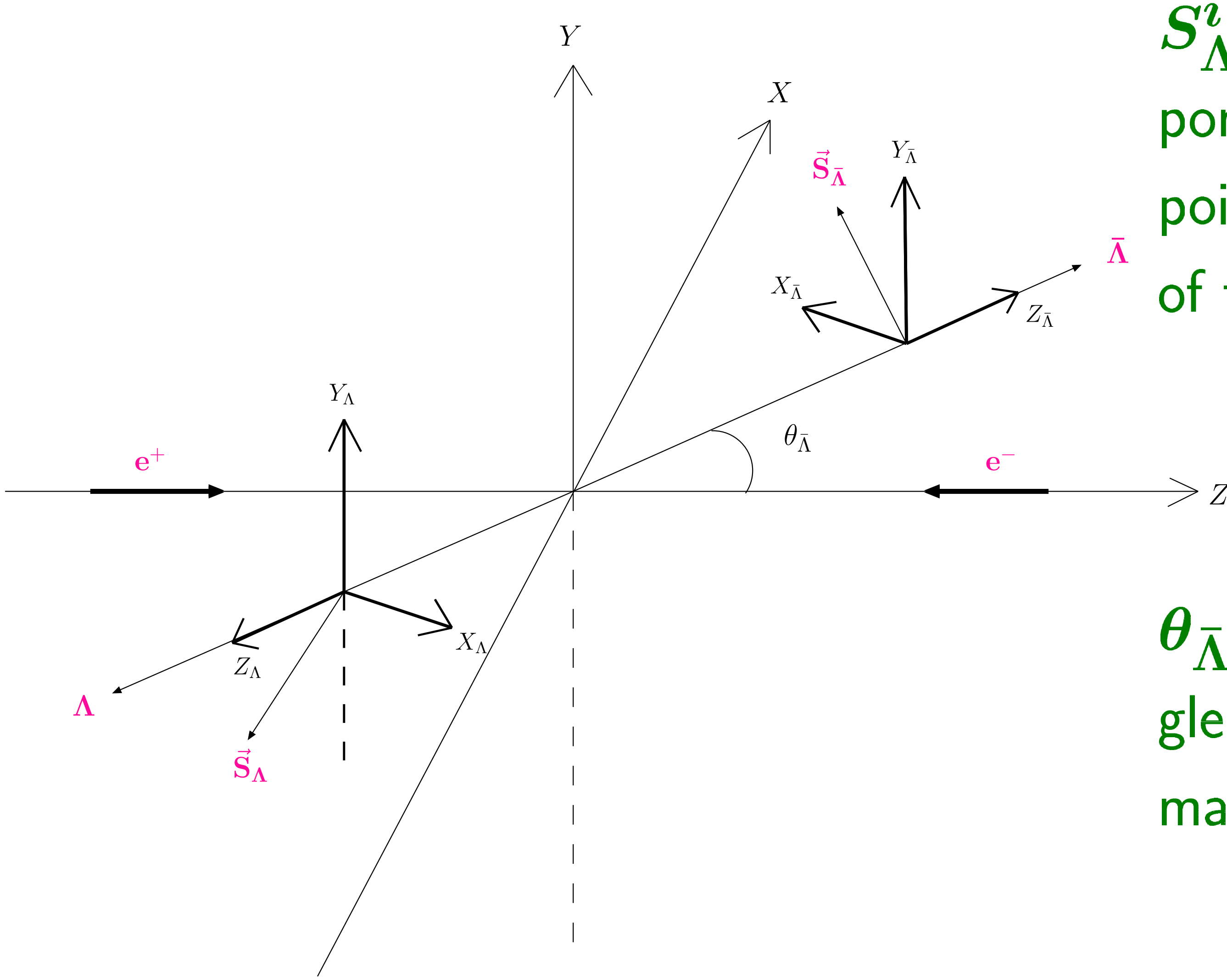
$$+ \;\; Im(G_M G_E^*)/\sqrt{\tau}\;\; \sin(2\theta_{\bar{\Lambda}}) \;\; \left(S^y_\Lambda + S^y_{\bar{\Lambda}}\right)$$

$$- \;\; Re(G_M G_E^*)/\sqrt{\tau}\;\; \sin(2\theta_{\bar{\Lambda}}) \left(S^z_\Lambda S^x_{\bar{\Lambda}} + S^z_{\bar{\Lambda}} S^x_\Lambda\right)$$

$$+ \left(\frac{1}{\tau}|G_E|^2 + |G_M|^2\right) \sin^2\theta_{\bar{\Lambda}} \;\; S^x_{\bar{\Lambda}} S^x_\Lambda$$

$$+ \left(\frac{1}{\tau}|G_E|^2 - |G_M|^2\right) \sin^2\theta_{\bar{\Lambda}} \;\; S^y_{\bar{\Lambda}} S^y_\Lambda$$

$$- \left(\frac{1}{\tau}|G_E|^2 \sin^2\theta_{\bar{\Lambda}} - |G_M|^2 \left(1+\cos^2\theta_{\bar{\Lambda}}\right)\right) \;\; S^z_{\bar{\Lambda}} S^z_\Lambda \Bigg\}$$



S^i_Λ and $S^i_{\bar{\Lambda}} \rightarrow i$th components of the unit vector pointing into the direction of the Λ or $\bar{\Lambda}$ spin

$\theta_{\bar{\Lambda}} \rightarrow$ the $\bar{\Lambda}$ polar angle in the e^+e^- center of mass frame

$$Im(G_M G_E^*)/\sqrt{\tau}\ \sin(2\theta_{\bar{\Lambda}})\ \left(S_\Lambda^y + S_{\bar{\Lambda}}^y\right)$$

and

$$Re(G_M G_E^*)/\sqrt{\tau}\ \sin(2\theta_{\bar{\Lambda}})\left(S_\Lambda^z S_{\bar{\Lambda}}^x + S_{\bar{\Lambda}}^z S_\Lambda^x\right)$$

$$G_M = |G_M|\ e^{i\phi_M}$$

$$G_E = |G_E|\ e^{i\phi_E}$$

$$Re(G_M G_E^*) = |G_M|\ |G_E| \cos(\phi_M - \phi_E)$$

$$Im(G_M G_E^*) = |G_M|\ |G_E| \sin(\phi_M - \phi_E)$$

$$\boxed{\phi_M - \phi_E = \Delta\phi}$$

- relative phase between electric and magnetic form factors

The subsequent two body decays of Λs

The measurement of the subsequent two body decays:

$$\Lambda \to \pi^- p$$

and

$$\bar{\Lambda} \to \pi^+ \bar{p}$$

allow for a spin analysis of the decaying Λs.

The decay distribution:

$$R_\Lambda = 1 - \alpha_\Lambda \, \bar{S}_\Lambda \cdot \bar{n}_{\pi^-}$$

The spin vector is replaced by:

$$\bar{S}_\Lambda \to -\alpha_\Lambda \bar{n}_{\pi^-} \quad \text{and} \quad \bar{S}_{\bar{\Lambda}} \to -\alpha_{\bar{\Lambda}} \bar{n}_{\pi^+}$$

$$e^+e^- \to \bar{\Lambda}(\to \pi^+\bar{p})\Lambda(\to \pi^- p)$$

using the narrow width approximation

$$d\sigma\left(e^+e^- \to \bar{\Lambda}(\to \pi^+\bar{p})\Lambda(\to \pi^- p)\right) =$$

$$d\sigma\left(e^+e^- \to \bar{\Lambda}\Lambda\right)\left(S_{\Lambda,\bar{\Lambda}} \to \mp\alpha_\Lambda n_{\pi^\mp}\right)$$

$$\times\ d\bar{\Phi}_2(q_1; p_{\pi^+}, p_{\bar{p}})d\bar{\Phi}_2(q_2; p_{\pi^-}, p_p)$$

$$\times\ \mathrm{Br}(\bar{\Lambda} \to \pi^+\bar{p})\mathrm{Br}(\Lambda \to \pi^- p)$$

$n_{\pi^+}(n_{\pi^-}) = (0, \bar{n}_{\pi^+})\ ((0, \bar{n}_{\pi^-}))$ in the $\bar{\Lambda}$ (Λ) rest frame

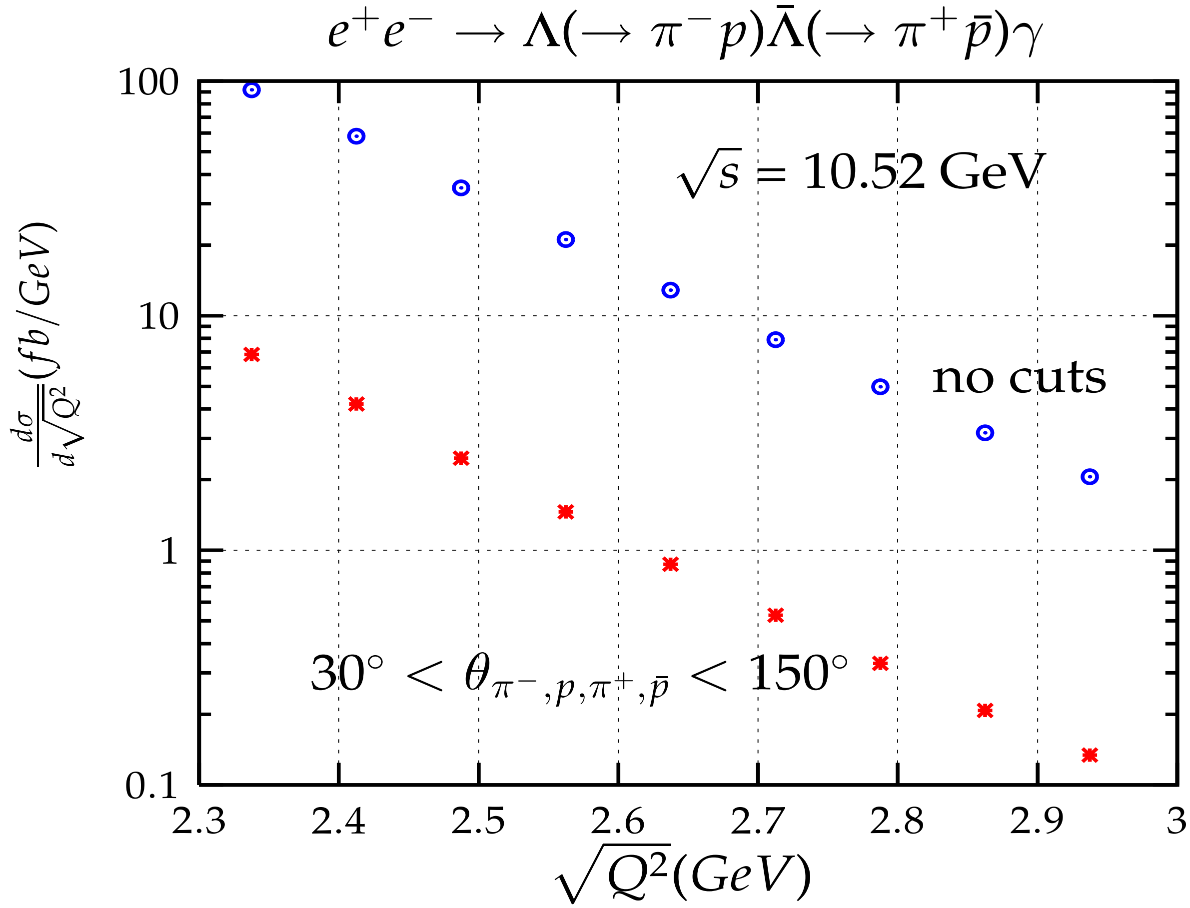
The cross section with ISR photon emision

$$
\begin{aligned}
L^{ij}H_{ij} \simeq \frac{(4\pi\alpha)^3}{4Q^2 y_1 y_2}\left(1+\cos^2\theta_\gamma\right) \Big\{ & |G_M|^2\left(1+\cos^2\theta_{\bar\Lambda}\right) \\
& + \frac{1}{\tau}|G_E|^2\sin^2\theta_{\bar\Lambda} - \alpha_\Lambda \frac{Im(G_M G_E^*)}{\sqrt{\tau}}\sin(2\theta_{\bar\Lambda})\left(n^y_{\pi^-} - n^y_{\pi^+}\right) \\
& + \alpha_\Lambda^2 \frac{Re(G_M G_E^*)}{\sqrt{\tau}}\sin(2\theta_{\bar\Lambda})\left(n^z_{\pi^-}n^x_{\pi^+} + n^z_{\pi^+}n^x_{\pi^-}\right) \\
& - \alpha_\Lambda^2\left(\frac{1}{\tau}|G_E|^2 + |G_M|^2\right)\sin^2\theta_{\bar\Lambda}\ \ n^x_{\pi^+}n^x_{\pi^-} \\
& - \alpha_\Lambda^2\left(\frac{1}{\tau}|G_E|^2 - |G_M|^2\right)\sin^2\theta_{\bar\Lambda}\ \ n^y_{\pi^+}n^y_{\pi^-} \\
& + \alpha_\Lambda^2\left(\frac{1}{\tau}|G_E|^2\sin^2\theta_{\bar\Lambda} - |G_M|^2\left(1+\cos^2\theta_{\bar\Lambda}\right)\right)\ \ n^z_{\pi^+}n^z_{\pi^-} \Big\}
\end{aligned}
$$

$\theta_{\bar\Lambda}$ - $\bar{Q}$ rest frame with the z-axis opposite to the photon direction

The cross section

FF from Körner et al. Phys. Rev. D 16 (1977) 2165



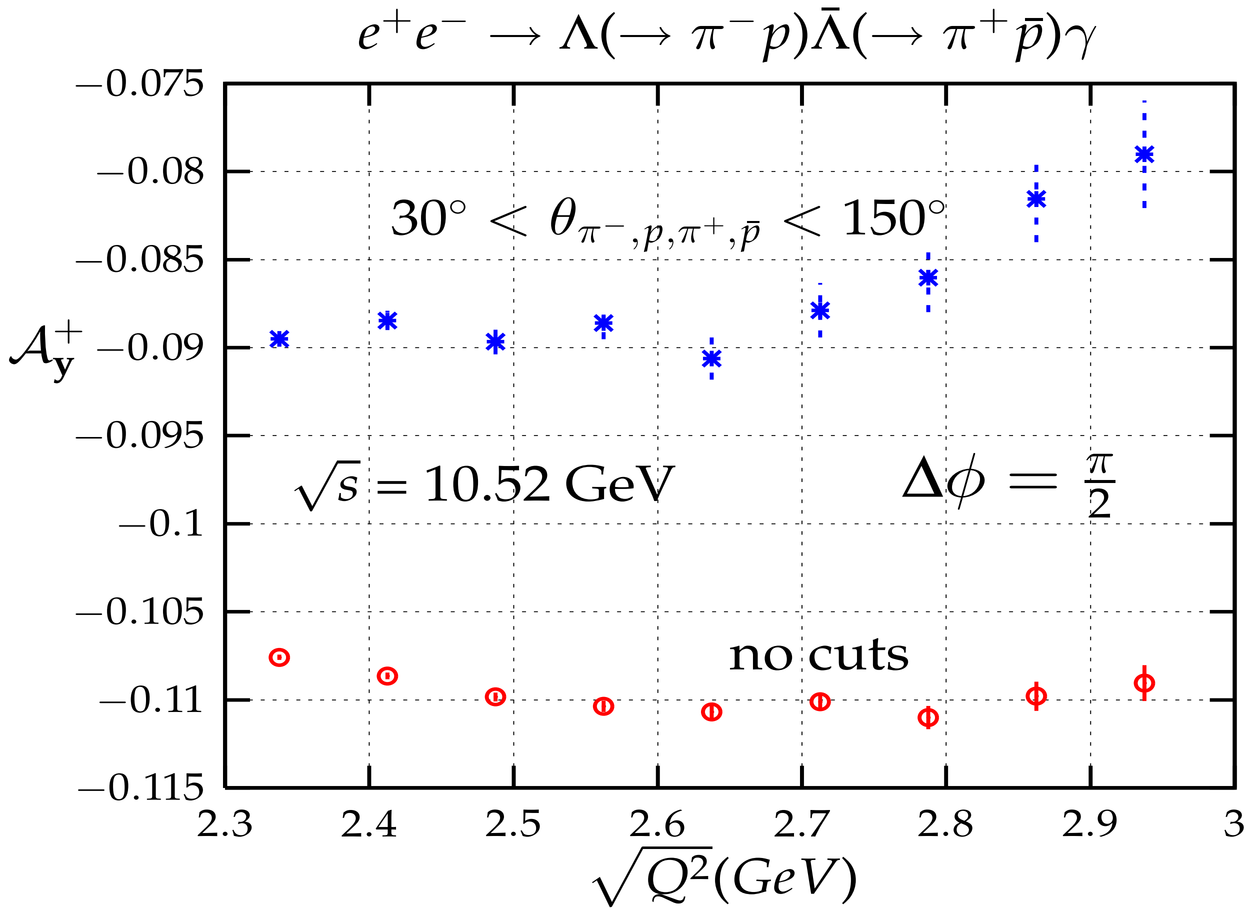
At $\boldsymbol{B}$-factories we expect about 130 events per 100 fb^{-1}.

Asymmetry

$$\mathcal{A}_y^{\pm} = \frac{d\sigma(a^{\pm}>0) - d\sigma(a^{\pm}<0)}{d\sigma(a^{\pm}>0) + d\sigma(a^{\pm}<0)}$$

$$a^{+(-)} = \sin(2\theta_{\bar{\Lambda}})\; n^y_{\pi^+(\pi^-)}$$

Asymmetry



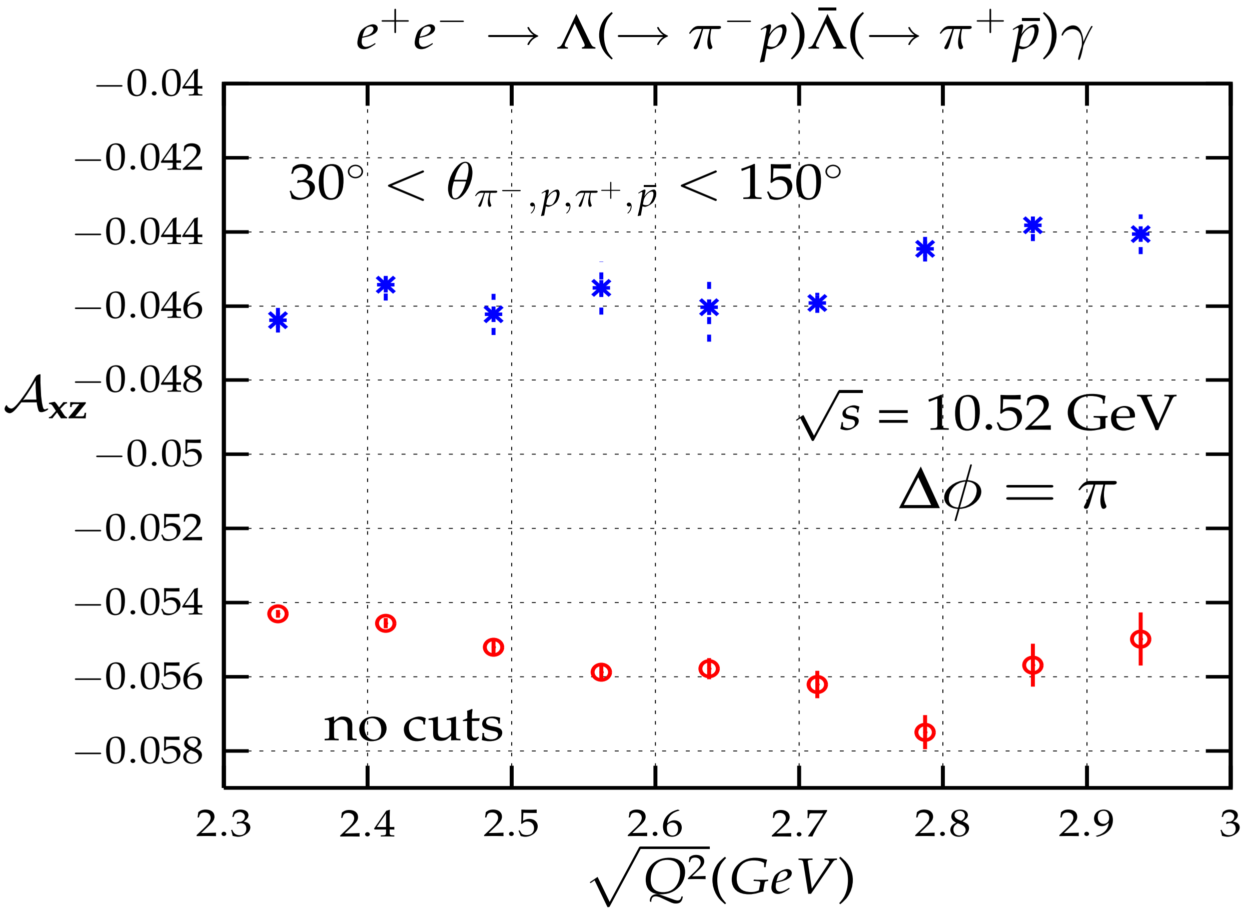
$e^+e^- \to \Lambda(\to \pi^- p)\bar{\Lambda}(\to \pi^+ \bar{p})\gamma$
$30^\circ < \theta_{\pi^-,p,\pi^+,\bar{p}} < 150^\circ$
$\sqrt{s} = 10.52$ GeV
$\Delta\phi = \frac{\pi}{2}$
no cuts
$\mathcal{A}_{\mathbf{y}}^+$
−0.075
−0.08
−0.085
−0.09
−0.095
−0.1
−0.105
−0.11
−0.115
2.3
2.4
2.5
2.6
2.7
2.8
2.9
3
$\sqrt{Q^2}(GeV)$

Spin correlations

$$\mathcal{A}_{xz} = \frac{d\sigma(\tilde{a}>0) - d\sigma(\tilde{a}<0)}{d\sigma(\tilde{a}>0) + d\sigma(\tilde{a}<0)}$$

$$\tilde{a} = \sin(2\theta_{\bar{\Lambda}}) \times \left(n^z_{\pi^-} n^x_{\pi^+} + n^z_{\pi^+} n^x_{\pi^-} \right)$$

Spin correlations



PHOKHARA: near future developments

- 4π revisited

- $J/\psi, \psi(2S)$ with the radiative return

Conclusions

- The baryon form factors can be measured using the radiative return method

- Only investments in theoretical and experimental analysis necessary