# The Bessel-Weighting Strategy

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2nd Workshop on

"Probing Strangeness in Hard Processes"

Frascati, November 12, 2013

KVI

university of groningen

# Outline

**I.** Introduction to azimuthal asymmetries and TMD factorization, Sivers effect example

**II.** Weighted asymmetries, Sivers effect example

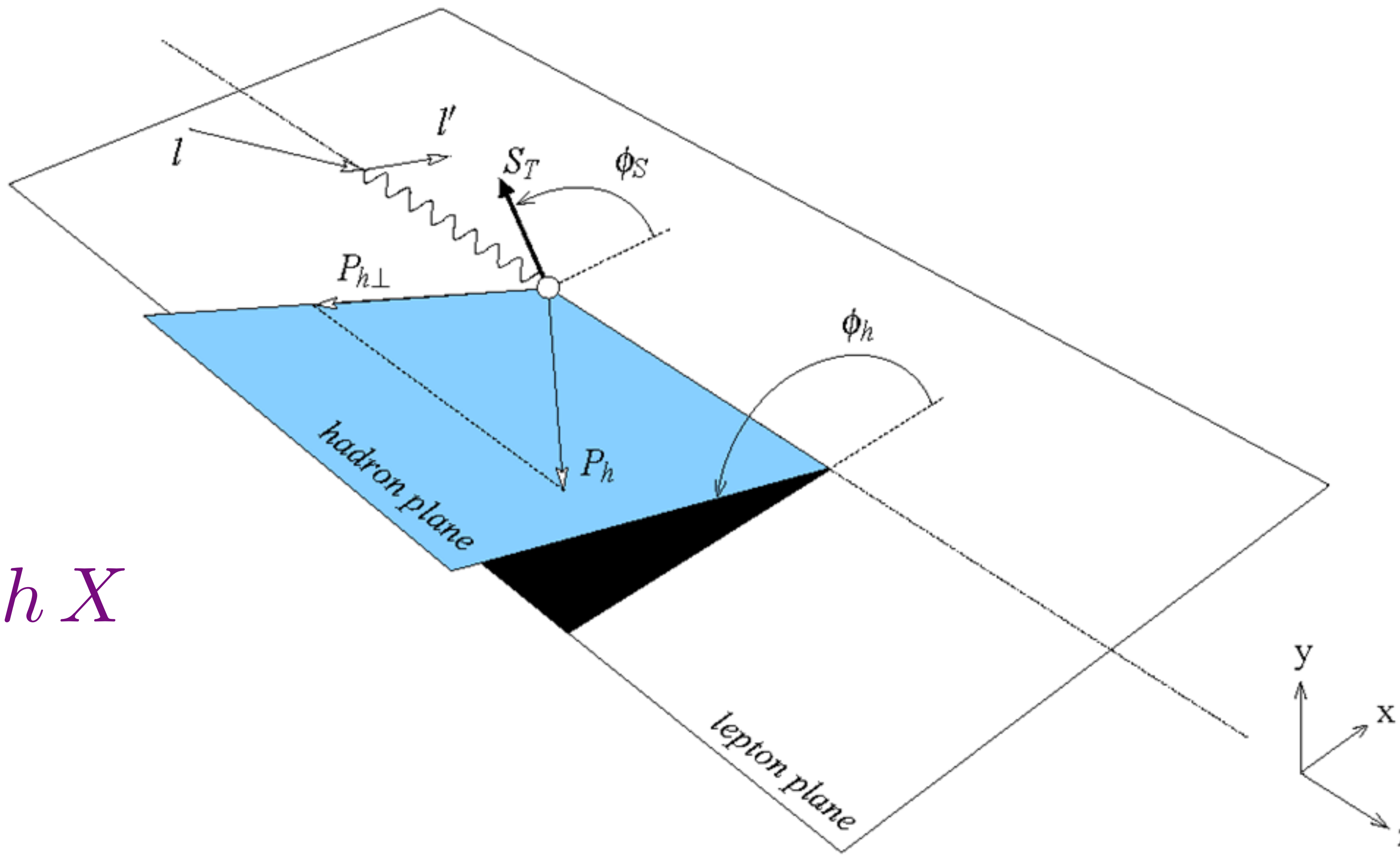
**III.** Very different example: $\cos(2\phi)$ asymmetry

**IV.** Average transverse momentum and transverse momentum broadening

# Introduction

# Azimuthal asymmetries

$$e\,p \to e'\,h\,X$$




Semi-inclusive DIS is a multi-scale process: M (hadronic), Q (large), and $P_{h\perp}$ (or $Q_T$)
$|P_{h\perp}|$ could be anywhere from small to large

Scattering does not happen in one plane generally → out-of-plane angles

Many azimuthal asymmetries in semi-inclusive DIS have been observed by HERMES, COMPASS, and JLab experiments

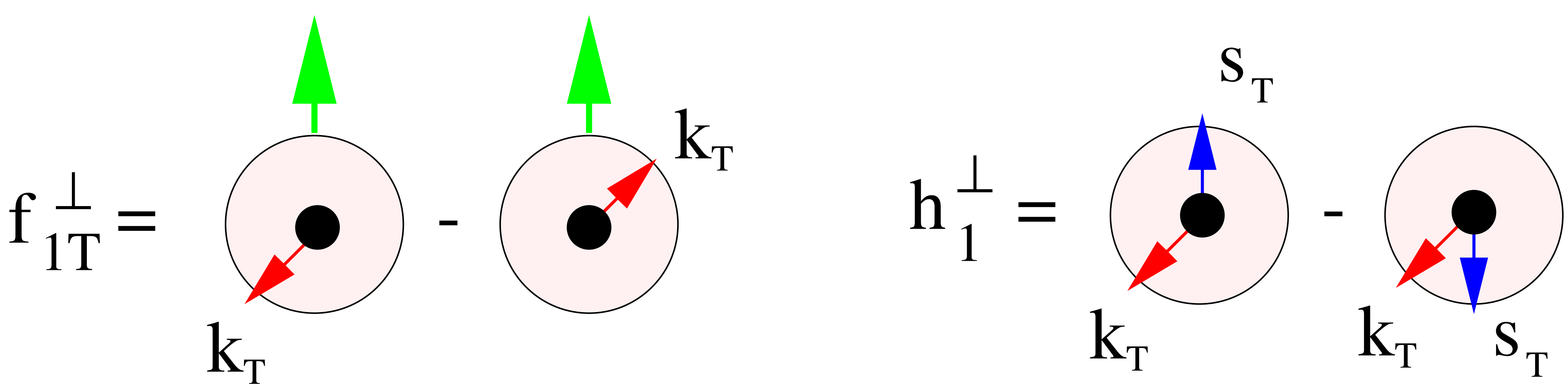
# Transverse Momentum of Quarks

Azimuthal asymmetries are most naturally described in terms of
transverse momentum distributions (TMDs)

[Ralston, Soper '79; Sivers '90; Collins '93; Kotzinian '95; Mulders, Tangerman '95; DB, Mulders '98]

Transverse momentum dependence can be correlated with spin dependence:

spin-orbit correlations




Azimuthal asymmetries can test the TMD framework, based on TMD factorization

[Collins & Soper '81; Ji, Ma & Yuan '04 & '05; Collins '11]

# TMD factorization

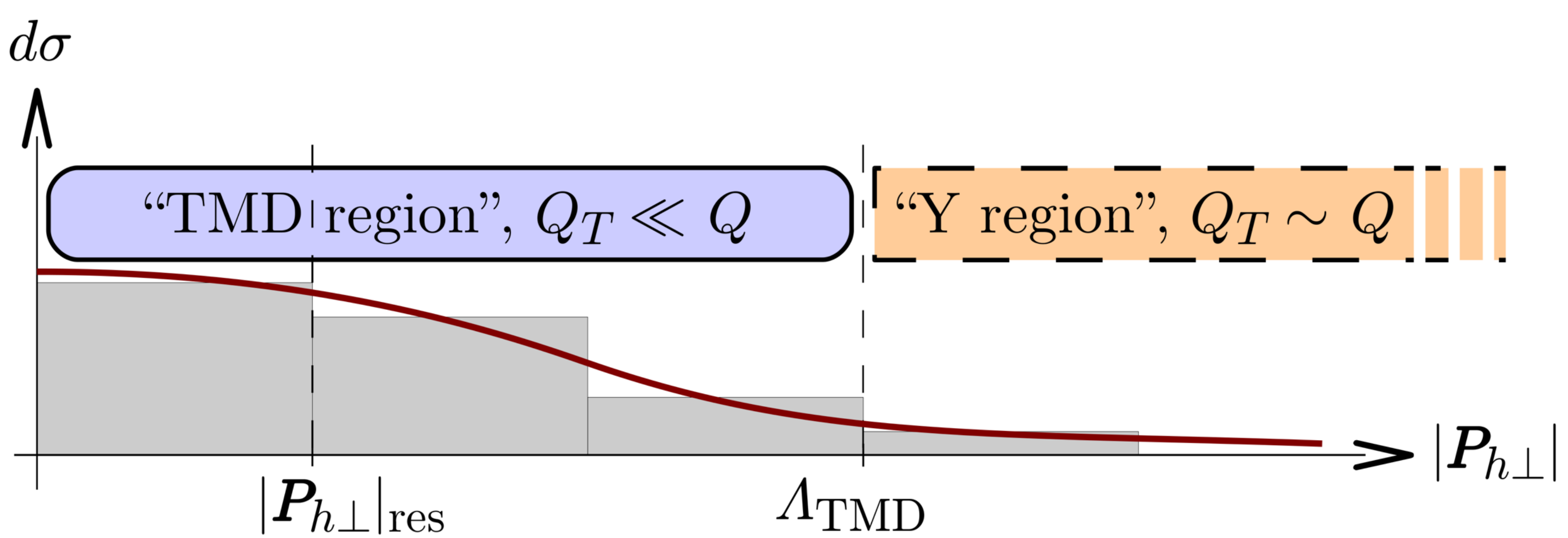
TMD factorization applies to SIDIS, but also $e^+e^- \to h_1\, h_2\, X$ and Drell-Yan (DY)

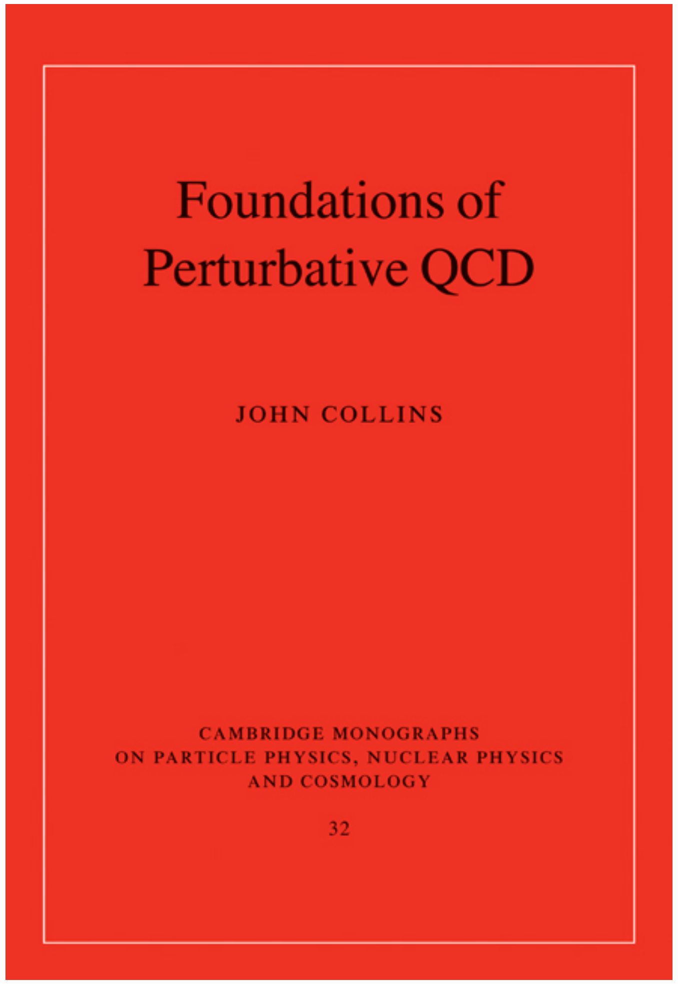
Schematic form of (new) TMD factorization [Collins 2011]:

$$d\sigma = H \times \text{convolution of } A\, B + \text{high-}q_T \text{ correction } (Y) + \text{power-suppressed}$$

A & B are TMD pdfs or FFs

Details in book by J.C. Collins
Summarized in arXiv:1107.4123




Convolution in terms of A and B best deconvoluted by Fourier transform

# TMD factorization expressions

Fourier transforms of A and B are functions of the momentum fraction *x*, the transverse coordinate $b_T$, a rapidity ζ , and the renormalization scale μ

$$\frac{d\sigma}{d\Omega d^4q} = \int d^2b\, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T}\tilde{W}(\boldsymbol{b}, Q; x, y, z) + \mathcal{O}\left(Q_T^2/Q^2\right)$$

$$\tilde{W}(\boldsymbol{b}, Q; x, y, z) = \sum_a \tilde{f}_1^a(x, \boldsymbol{b}^2; \zeta_F, \mu)\tilde{D}_1^a(z, \boldsymbol{b}^2; \zeta_D, \mu)H\left(y, Q; \mu\right)$$

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Y term

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# TMD factorization expressions

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Including partonic transverse momentum is more than just $f_1(x) \rightarrow f_1(x, k_T^2)$

$k_T$-odd functions may arise, that vanish upon integration over all $k_T$

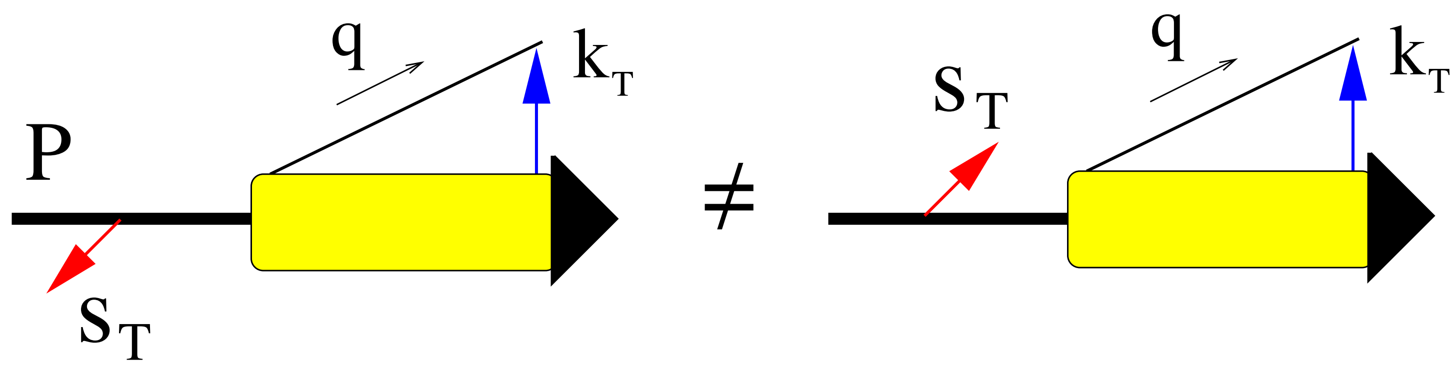
For unpolarized hadrons with momentum P and partons with $k \approx xP + k_T$:

$$\Phi(x, \boldsymbol{k}_T) = \frac{M}{2}\left\{ f_1(x, \boldsymbol{k}_T^2)\, \frac{\not{P}}{M} + h_1^{\perp}(x, \boldsymbol{k}_T^2)\, \frac{i\, \not{k}_T\, \not{P}}{M^2} \right\}$$

# Sivers effect

Also new hadron spin dependent terms arise, such as the Sivers function

D. Sivers ('89/'90):




$k_T \times S_T$

The leading twist TMDs:

$$\Phi(x, \boldsymbol{k}_T) = \frac{M}{2} \left\{ f_1(x, \boldsymbol{k}_T^2) \frac{\not{P}}{M} + f_{1T}^{\perp}(x, \boldsymbol{k}_T^2) \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k_T^\rho S_T^\sigma}{M^2} + g_{1s}(x, \boldsymbol{k}_T^2) \frac{\gamma_5 \not{P}}{M} \right.$$

$$\left. + h_{1T}(x, \boldsymbol{k}_T^2) \frac{\gamma_5 \not{S}_T \not{P}}{M} + h_{1s}^{\perp}(x, \boldsymbol{k}_T^2) \frac{\gamma_5 \not{k}_T \not{P}}{M^2} + h_1^{\perp}(x, \boldsymbol{k}_T^2) \frac{i \not{k}_T \not{P}}{M^2} \right\}$$

There are lots of TMDs (including TMD fragmentation functions),
but in this talk only 2 will be discussed: the leading twist T-odd ones

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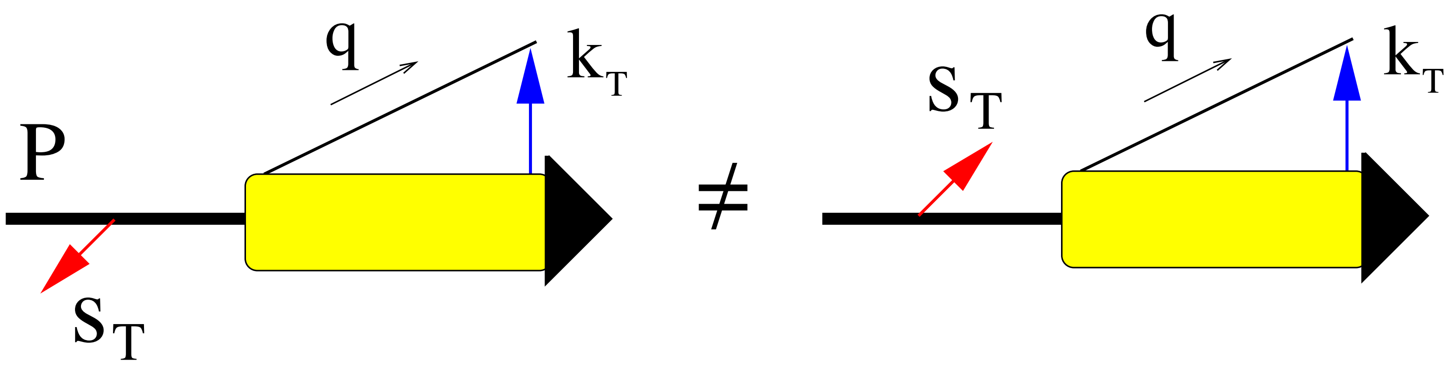


$k_T \times S_T$

The leading twist TMDs:

Sivers function

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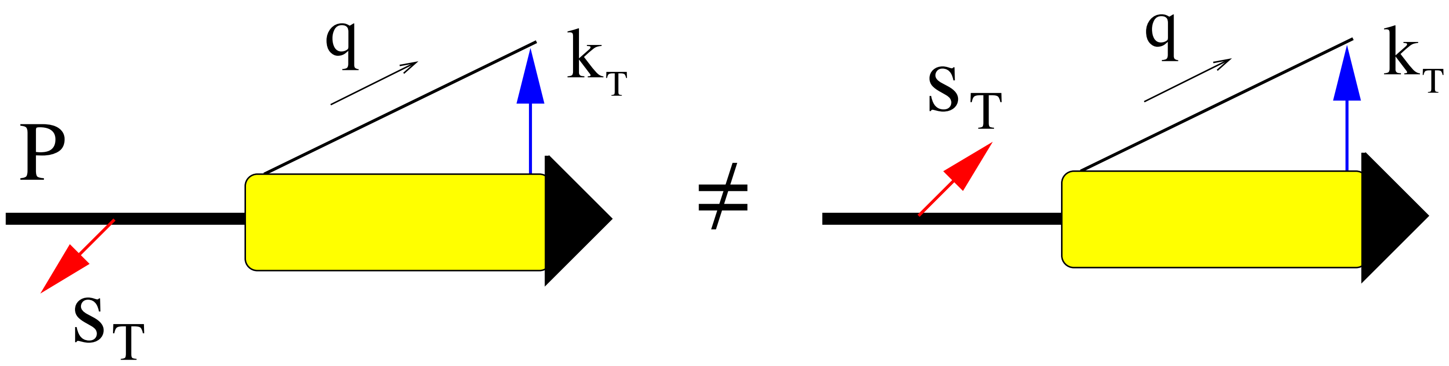


$k_T \times S_T$

The leading twist TMDs:

Sivers function

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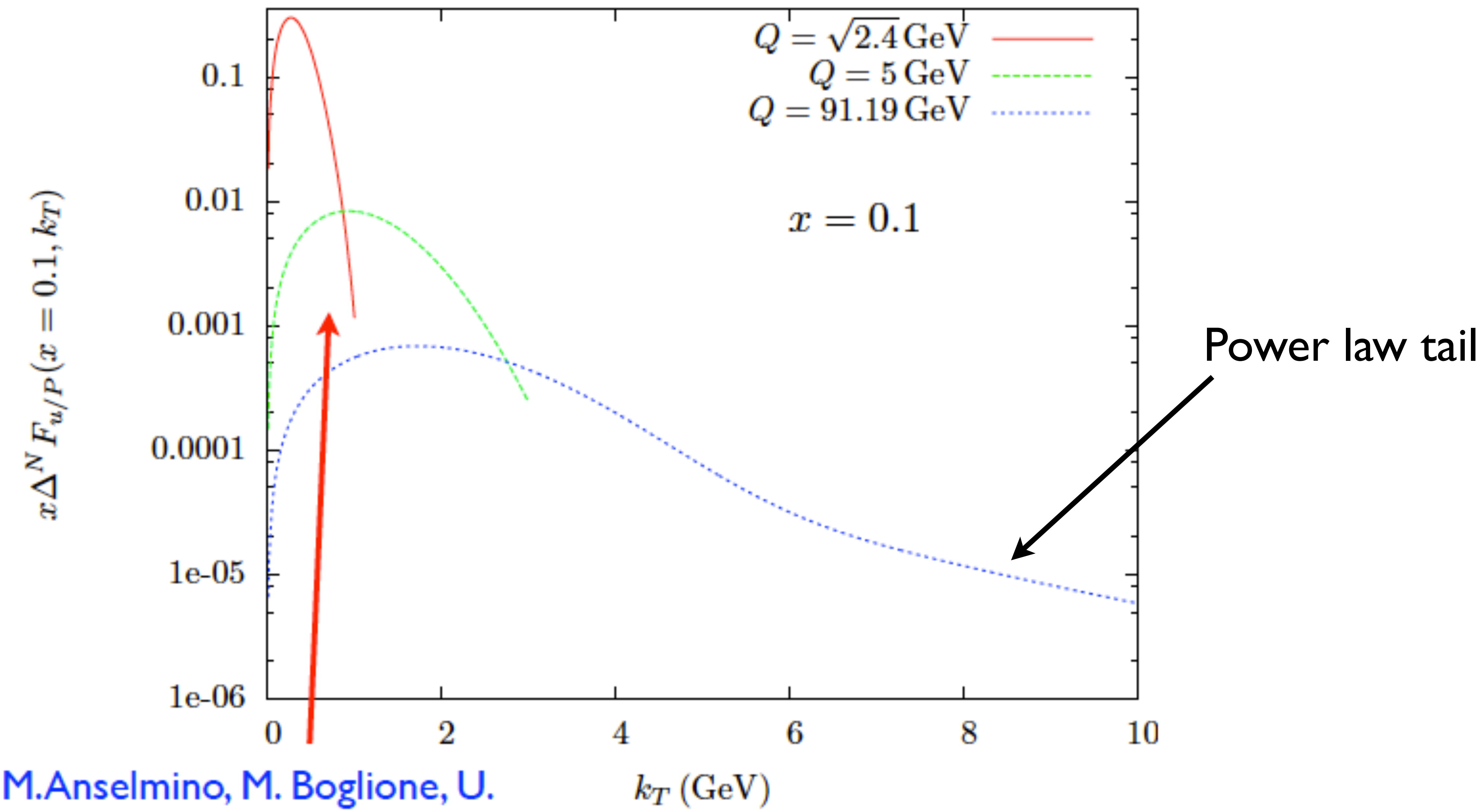
Boer-Mulders function

There are lots of TMDs (including TMD fragmentation functions),
but in this talk only 2 will be discussed: the leading twist T-odd ones

# Evolution of Sivers function

Factorization dictates the evolution:
TMDs and their asymmetries become broader and smaller with increasing energy




M.Anselmino, M. Boglione, U. D'Alesio, A.Kotzinian, S.Melis, F. Murgia, A. Prokudin, C.Turk; 2009

Aybat & Rogers, PRD 83 (2011) 114042
Aybat, Collins, Qiu, Rogers, PRD 85 (2012) 034043

# TMDs and collinear pdfs

Large transverse momentum (perturbative) tail of TMD determined by collinear pdf

$$f_1(x,\boldsymbol{p}_T^2) \overset{\boldsymbol{p}_T^2 \gg M^2}{\sim} \alpha_s \frac{1}{\boldsymbol{p}_T^2} \left(K \otimes f_1\right)(x)$$

Tail of Sivers function determined by the collinear twist-3 Qiu-Sterman function

$$f_{1T}^{\perp}(x,\boldsymbol{p}_T^2) \overset{\boldsymbol{p}_T^2 \gg M^2}{\sim} \alpha_s \frac{M^2}{\boldsymbol{p}_T^4} \left(K' \otimes T_F\right)(x)$$

[Ji, Qiu, Vogelsang, Yuan, PRL 97 (2006) 082002; PLB 638 (2006) 178;
Koike, Vogelsang, Yuan, PLB 659 (2008) 878]

$$T_F(x,x) \overset{A^+=0}{\propto} \mathrm{F.T.} \langle P|\, \overline{\psi}(0) \int d\eta^- \; F^{+\alpha}(\eta^-)\, \gamma^+ \psi(\xi^-)\, |P\rangle$$

[Qiu & Sterman, PRL 67 (1991) 2264]

One has to be careful when considering integrals over all transverse momenta
Convergence issue and does not automatically yield collinear pdfs

# Weighted asymmetries

# Actual extraction of TMDs

A problem common to all TMD asymmetries: TMDs appear in convolution integrals
For example, the expression for the Sivers asymmetry in SIDIS:

$$\frac{d\sigma(e\,p^\uparrow \to e'\,\pi\,X)}{d^2\boldsymbol{q}_T} \propto \frac{|\boldsymbol{S}_T|}{Q_T} \sin(\phi_\pi^e - \phi_S^e)\,\mathcal{F}\left[\frac{\boldsymbol{q}_T\cdot\boldsymbol{p}_T}{M}\,f_{1T}^\perp D_1\right]$$

$$\mathcal{F}\left[w\,f\,D\right] \equiv \int d^2\boldsymbol{p}_T\,d^2\boldsymbol{k}_T\,\delta^2(\boldsymbol{p}_T+\boldsymbol{q}_T-\boldsymbol{k}_T)\,w(\boldsymbol{p}_T,\boldsymbol{q}_T,\boldsymbol{k}_T)\,f(x,\boldsymbol{p}_T^2)D(z,z^2\boldsymbol{k}_T^2)$$

One solution (in this particular case) would be to measure ``jet SIDIS":

$$\frac{d\sigma(e\,p^\uparrow \to e'\,\mathrm{jet}\,X)}{d^2\boldsymbol{q}_T} \propto |\boldsymbol{S}_T|\,\sin(\phi_{\mathrm{jet}}^e - \phi_S^e)\,\frac{Q_T}{M} f_{1T}^\perp(x,Q_T^2), \qquad Q_T^2 = |\boldsymbol{P}_\perp^{\mathrm{jet}}|^2$$

One can probe the $k_T$-dependence of the Sivers function directly in this way

One can probe the $k_T$-broadening of the Sivers asymmetry with increasing $Q^2$

A more general solution is to consider weighted asymmetries

# Weighted asymmetries

Cross sections integrated & weighted with a power of observed transverse momentum

$$\langle W \rangle \equiv \int dz\, d^2\boldsymbol{P}_{h\perp}\, W\, \frac{d\sigma^{[e\,p \to e'\,h\,X]}}{dx\,dy\,dz\,d\phi_h^e d|\boldsymbol{P}_{h\perp}|^2}$$

$$A_{UT}^{\frac{|P_{h\perp}|}{zM}\sin(\phi_h - \phi_S)}(x,z,y)$$

$$= 2\,\frac{\int d|P_{h\perp}|\,|P_{h\perp}|\,d\phi_h\,d\phi_S\,\frac{|P_{h\perp}|}{zM}\sin(\phi_h-\phi_S)\left(d\sigma^{\uparrow}-d\sigma^{\downarrow}\right)}{\int d|P_{h\perp}|\,|P_{h\perp}|\,d\phi_h\,d\phi_S\,\left(d\sigma^{\uparrow}+d\sigma^{\downarrow}\right)}$$

$$= -2\,\frac{\sum_a e_a^2\,H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\mu^2)\,f_{1T}^{\perp(1)a}(x;\mu^2,\zeta)\,D_1^{(0)a}(z;\mu^2,\hat{\zeta})}{\sum_a e_a^2\,H_{UU,T}(Q^2,\mu^2)\,f_1^{(0)a}(x;\mu^2,\zeta)\,D_1^{(0)a}(z;\mu^2,\hat{\zeta})}$$

Contains a weighted Sivers function:

$$f_{1T}^{\perp(1)}(x) \equiv \int d^2k_T\,\frac{k_T^2}{2M^2}\,f_{1T}^{\perp}(x,k_T^2)$$

Such transverse moments appear in different asymmetries in exactly the same form

# Why (Bessel) weight?

Convolution expressions of TMDs that appear in different processes in different ways
Weighting projects out "portable" functions

[Kotzinian, Mulders, PLB 406 (1997) 373; DB, Mulders, PRD 57 (1998) 5780]

Conventional weighting with powers of transverse momentum assumes that:
- integral converges
- integral over TMD expression (without large $Q_T$ "Y term") is fine

To by-pass these tricky issues, both due to the perturbative tail of the asymmetries, one can consider a modified weighting: Bessel weighting

[DB, Gamberg, Musch, Prokudin, JHEP 10 (2011) 021]

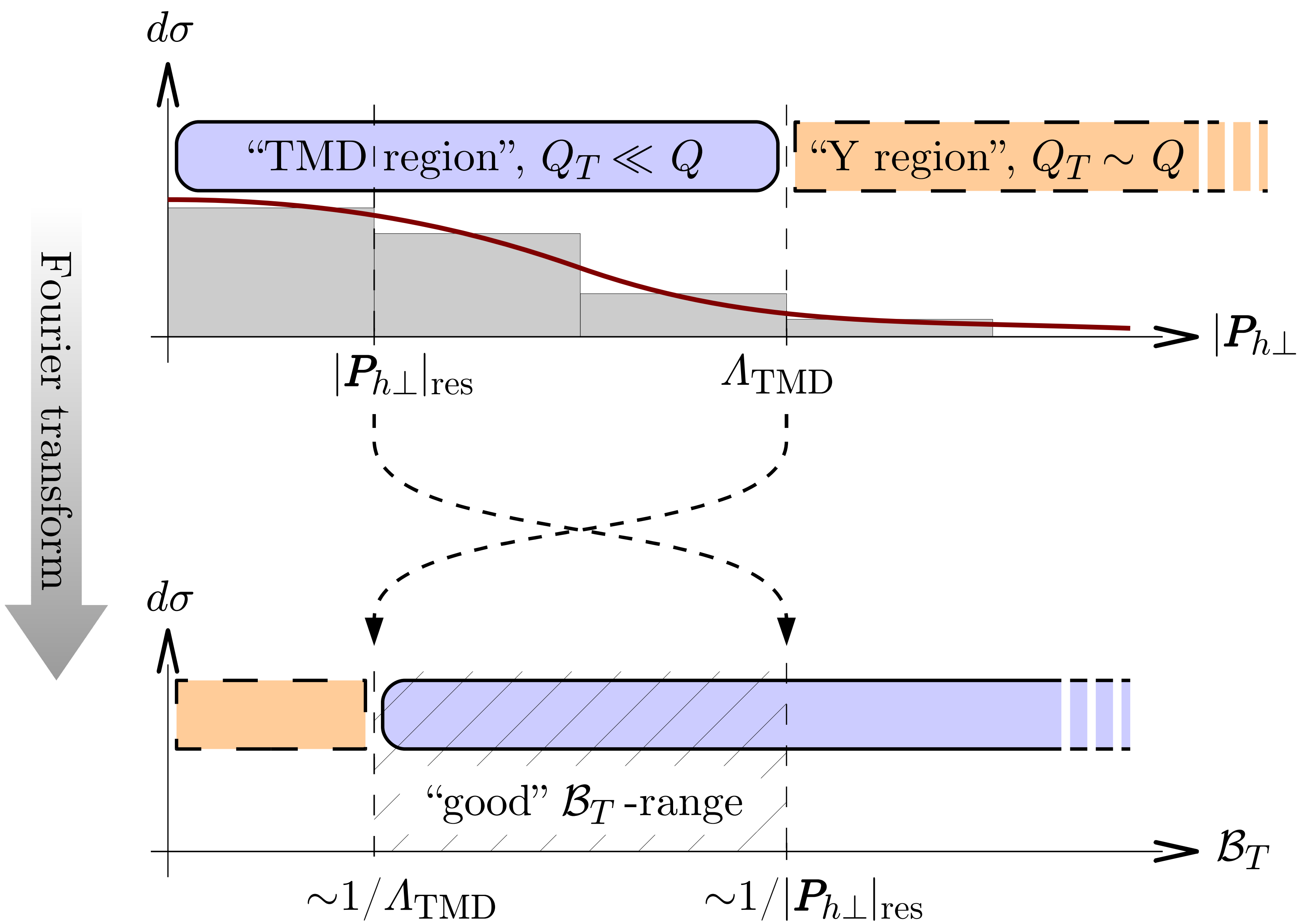
Conventional weight for Sivers asymmetry: $\mathcal{W} \equiv |P_{h\perp}|/zM \sin(\phi_h - \phi_S)$

Bessel weighting: $|P_{h\perp}|^n \to J_n(|P_{h\perp}|\mathcal{B}_T)\, n! \left(\frac{2}{\mathcal{B}_T}\right)^n$

In the limit $\mathcal{B}_T \to 0$ conventional weights are retrieved

First studies for Bessel-weighted $A_{LL}$ from CLAS (arXiv:1307.3500→Mher Aghasyan's talk)

# Why Bessel weight?




If $\mathcal{B}_T$ is not too small, the TMD region should dominate

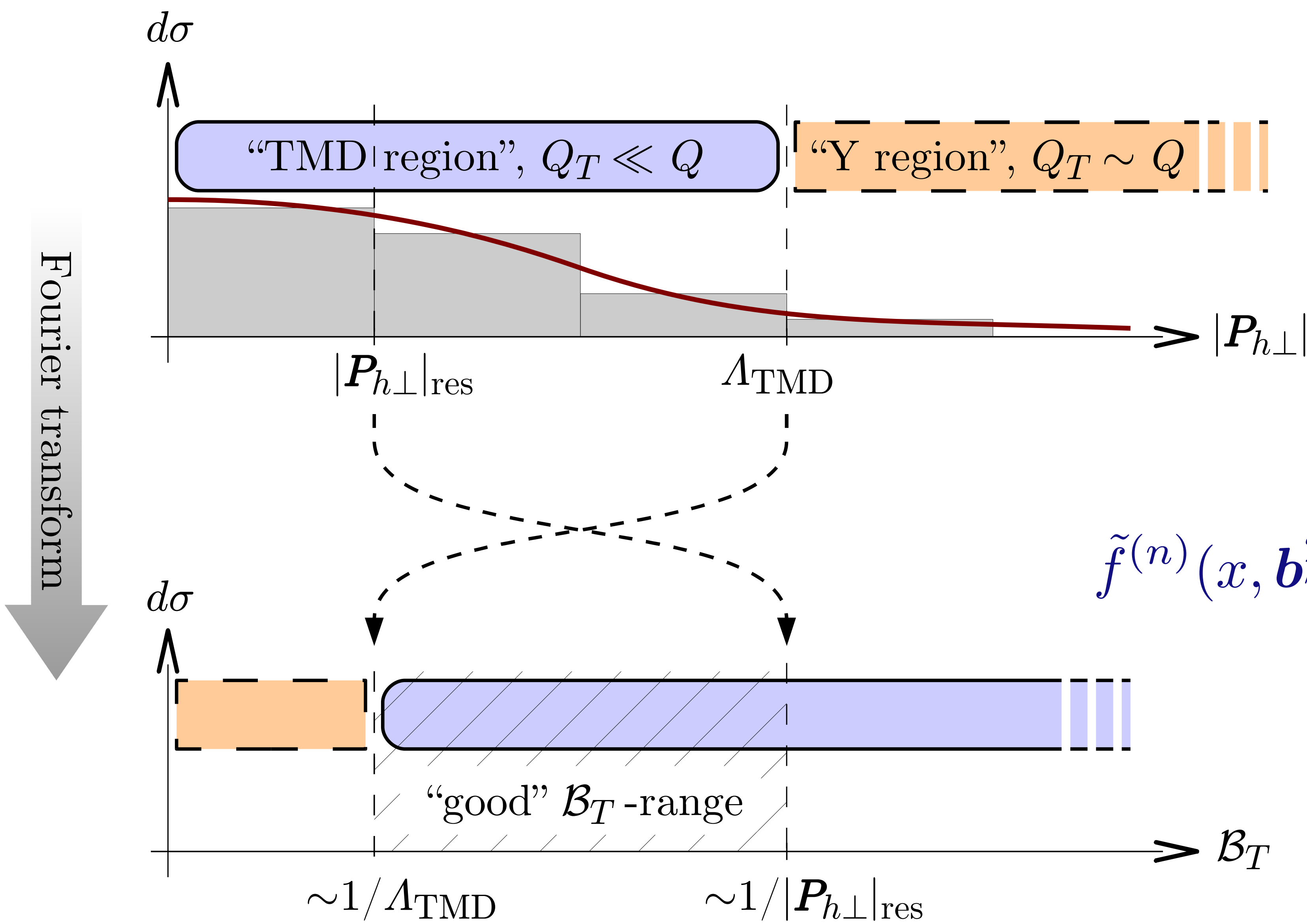
Allows to suppress Y term contribution & allows calculation of TMDs on the lattice!

(→Michael Engelhardt’s talk)

Weighted asymmetries for $\mathcal{B}_T$ in the TMD region have reduced scale dependence

(→ Leonard Gamberg’s talk)

# Why Bessel weight?




$$\tilde{f}^{(n)}(x, \boldsymbol{b}_T^2) = n! \left( -\frac{2}{M^2} \partial_{\boldsymbol{b}_T^2} \right)^n \tilde{f}(x, \boldsymbol{b}_T^2)$$

If $\mathcal{B}_T$ is not too small, the TMD region should dominate

Allows to suppress Y term contribution & allows calculation of TMDs on the lattice!

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Weighted asymmetries for $\mathcal{B}_T$ in the TMD region have reduced scale dependence

(→ Leonard Gamberg's talk)

In the limit $\mathcal{B}_T \to 0$ of conventional weights, Y term becomes very important and divergences may arise

# Bessel-weighted Sivers asymmetry

$$A_{UT}^{\frac{2\,J_1(|P_{h\perp}|\mathcal{B}_T)}{z_h M \mathcal{B}_T}\sin(\phi_h-\phi_S)}(\mathcal{B}_T)$$

$$= 2\frac{\int d|P_{h\perp}|\,|P_{h\perp}|\,d\phi_h\,d\phi_S\,\frac{2\,J_1(|P_{h\perp}|\mathcal{B}_T)}{z_h M \mathcal{B}_T}\sin(\phi_h-\phi_S)\left(d\sigma^{\uparrow}-d\sigma^{\downarrow}\right)}{\int d|P_{h\perp}|\,|P_{h\perp}|\,d\phi_h\,d\phi_S\;J_0(|P_{h\perp}|\,\mathcal{B}_T)\;\left(d\sigma^{\uparrow}+d\sigma^{\downarrow}\right)}$$

$$= -2\frac{\sum_a e_a^2\,H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\mu^2)\,\tilde{f}_{1T}^{\perp(1)a}(x,z^2\mathcal{B}_T^2;\mu^2,\zeta)\,\tilde{D}_1^{(0)a}(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta})}{\sum_a e_a^2\,H_{UU,T}(Q^2,\mu^2)\,\tilde{f}_1^{(0)a}(x,z^2\mathcal{B}_T^2;\mu^2,\zeta)\,\tilde{D}_1^{(0)a}(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta})}$$

Bessel-weighted asymmetries involves generalized transverse moments:

$$\tilde{f}^{(n)}(x,\boldsymbol{b}_T^2) = \frac{2\pi\;n!}{(M^2)^n}\int d|\boldsymbol{p}_T||\boldsymbol{p}_T|\left(\frac{|\boldsymbol{p}_T|}{|\boldsymbol{b}_T|}\right)^n J_n(|\boldsymbol{b}_T||\boldsymbol{p}_T|)\;f(x,\boldsymbol{p}_T^2)$$

$$\tilde{D}^{(n)}(z,\boldsymbol{b}_T^2) = \frac{2\pi\;n!}{(z^2M_h^2)^n}\int d|\boldsymbol{K}_T||\boldsymbol{K}_T|\left(\frac{|\boldsymbol{K}_T|}{|\boldsymbol{b}_T|}\right)^n J_n(|\boldsymbol{b}_T||\boldsymbol{K}_T|)\;D(z,\boldsymbol{K}_T^2)$$

In the limit $\mathcal{B}_T \rightarrow 0$ conventional weighted expression and moments are retrieved

# Bessel-weighted Sivers asymmetry

$$A_{UT}^{\frac{2\,J_1(|P_{h\perp}|\mathcal{B}_T)}{z_h M \mathcal{B}_T}\sin(\phi_h-\phi_S)}(\mathcal{B}_T)$$

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$$= -2\,\frac{\sum_a e_a^2\,H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\mu^2)\,\tilde{f}_{1T}^{\perp(1)a}(x,z^2\mathcal{B}_T^2;\mu^2,\zeta)\,\tilde{D}_1^{(0)a}(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta})}{\sum_a e_a^2\,H_{UU,T}(Q^2,\mu^2)\,\tilde{f}_1^{(0)a}(x,z^2\mathcal{B}_T^2;\mu^2,\zeta)\,\tilde{D}_1^{(0)a}(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta})}$$

This is the result of Bessel-weighting the TMD expression ($\mathcal{B}_T$ not small)

For small $\mathcal{B}_T$ the Y terms need to be considered, but give same type contribution

They should not be added though (would be double counting)

Bacchetta, DB, Diehl, Mulders, JHEP 0808 (2008) 023

Low-$Q_T$ Sivers asymmetry expression matches onto high-$Q_T$ Qiu-Sterman asymmetry!

Ji, Qiu, Vogelsang, Yuan, PRL 97 (2006) 082002; PLB 638 (2006) 178;

Koike, Vogelsang, Yuan, PLB 659 (2008) 878

In this case the Y term in the numerator falls off faster than in the denominator

Their ratio is twist-3, when weighted their contribution is not suppressed

# Sivers shift

The average transverse momentum shift orthogonal to a given transverse polarization:

$$\langle p_y(x)\rangle_{TU} = \left.\frac{\int d^2p_T\, p_y\ \Phi^{[\gamma^+]}(x,p_T,P,S,\mu^2,\zeta)}{\int d^2p_T\, \Phi^{[\gamma^+]}(x,p_T,P,S,\mu^2,\zeta)}\right|_{S^\pm=0,\, S_T=(1,0)}$$

$$= M\,\frac{f_{1T}^{\perp(1)}(x;\mu^2,\zeta)}{f_1^{(0)}(x;\mu^2,\zeta)}$$

And its Bessel-weighted analogue:

$$\langle p_y(x)\rangle_{TU}^{\mathcal{B}_T} = \left.\frac{\int d|p_T|\,|p_T|\int d\phi_p \frac{2J_1(|p_T|\mathcal{B}_T)}{\mathcal{B}_T}\ \sin(\phi_p-\phi_S)\,\Phi^{[\gamma^+]}(x,p_T,P,S,\mu^2,\zeta)}{\int d|p_T|\,|p_T|\int d\phi_p J_0(|p_T|\mathcal{B}_T))\,\Phi^{[\gamma^+]}(x,p_T,P,S,\mu^2,\zeta)}\right|_{|S_T|=1}$$

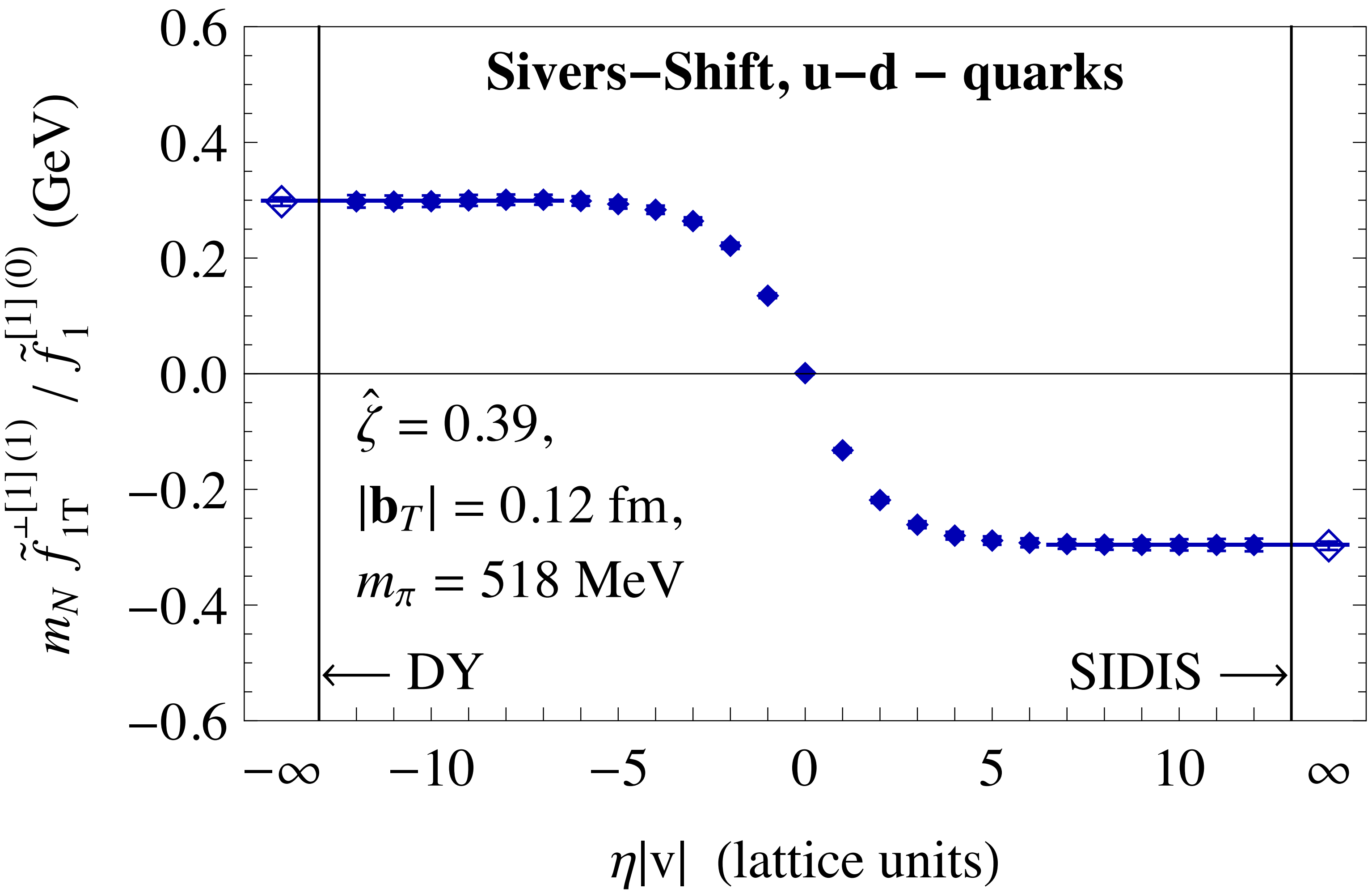
$$= M\,\frac{\tilde{f}_{1T}^{\perp(1)}(x,\mathcal{B}_T;\mu^2,\zeta)}{\tilde{f}_1^{(0)}(x,\mathcal{B}_T;\mu^2,\zeta)}$$

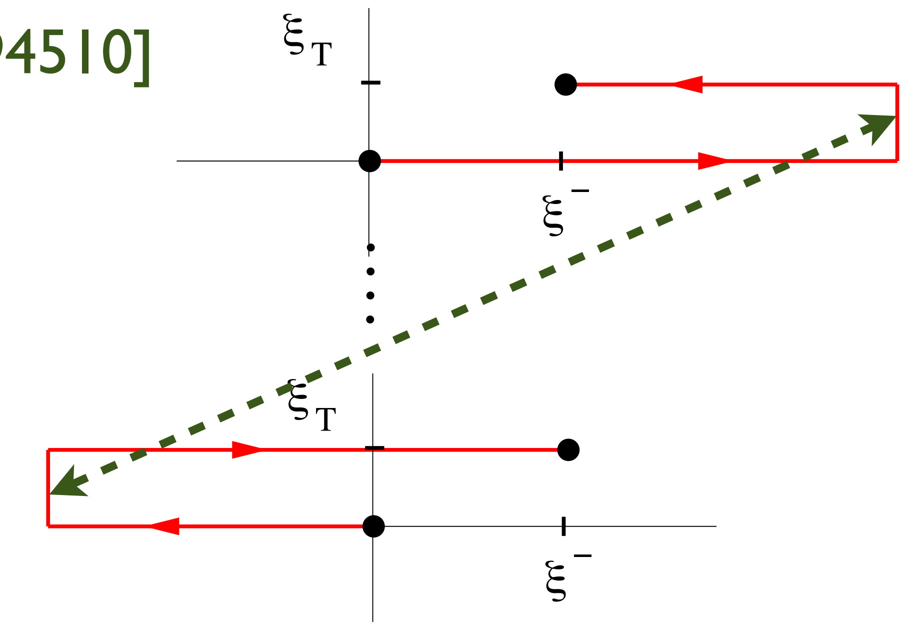
For nonzero $\mathcal{B}_T$ this involves well-defined (finite) quantities, with Wilson lines that are off the lightcone (spacelike)

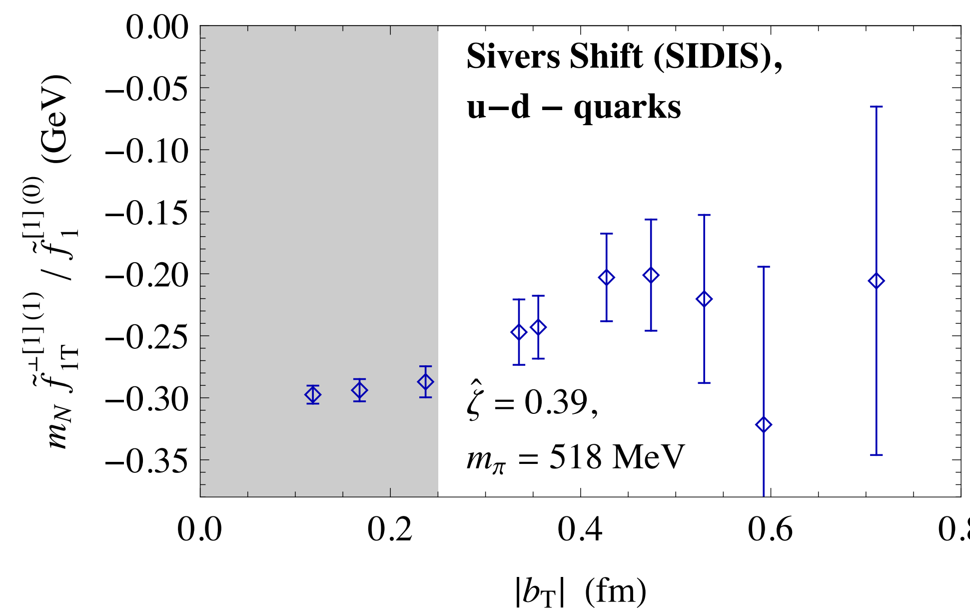
After taking Mellin moments *and* Bessel transverse moments of the Sivers function, one has a well-defined quantity $<k_T \times S_T>(n,\mathcal{B}_T)$, that can be evaluated on the lattice

# Sivers function on the lattice

[Musch, Hägler, Engelhardt, Negele & Schäfer, PRD 85 (2012) 094510]




The first `first-principle' demonstration in QCD that the Sivers function is nonzero
It clearly corroborates the sign change relation!

$$f_{1T}^{\perp[\mathrm{SIDIS}]} = -f_{1T}^{\perp[\mathrm{DY}]}$$

compatible with fits and models:
up Sivers ($f_{1T}^{\perp}$) of SIDIS < 0 and down Sivers of SIDIS > 0 and smaller

# Qiu-Sterman function

The limit $\mathcal{B}_T \to 0$ tells us something about the Qiu-Sterman function

$$f_{1T}^{\perp(1)}(x) \equiv \int d^2k_T \frac{k_T^2}{2M^2} f_{1T}^{\perp}(x, k_T^2) \propto T_F(x,x)$$

[DB, Mulders & Pijlman, NPB 667 (2003) 201]

$$\lim_{b_T \to 0} \tilde{f}_{1T}^{(1)[+]}(x, b_T^2; \mu, \zeta) \stackrel{?}{=} \frac{T_F(x,x;\mu)}{2M}$$

'?' because of rapidity dependence of r.h.s., identification meaningful when viewed as part of the full cross section expression, just like for:

$$\lim_{b_T \to 0} \tilde{f}_1^{(0)}(x, b_T^2; \mu, \zeta) \stackrel{?}{=} f_1(x;\mu)$$

Nevertheless, a very interesting limit to consider, since Qiu-Sterman function itself is intrinsically non-local along the lightcone and cannot be evaluated on the lattice

$$T_F(x,x) \stackrel{A^+=0}{\propto} \text{F.T.} \langle P | \overline{\psi}(0) \int d\eta^- \; F^{+\alpha}(\eta^-) \, \gamma^+ \, \psi(\xi^-) \, |P\rangle$$

But first Bessel-moment of Sivers function *can* be evaluated (for given rapidity)

# Qiu-Sterman function

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$$f_{1T}^{\perp(1)}(x) \equiv \int d^2k_T \frac{k_T^2}{2M^2} f_{1T}^{\perp}(x, k_T^2) \propto T_F(x,x)$$

[DB, Mulders & Pijlman, NPB 667 (2003) 201]

$$\lim_{b_T \to 0} \tilde{f}_{1T}^{(1)[+]}(x, b_T^2; \mu, \zeta) \stackrel{?}{=} \frac{T_F(x,x;\mu)}{2M}$$

'?' because of rapidity dependence of r.h.s., identification meaningful when viewed as part of the full cross section expression, just like for:

$$\lim_{b_T \to 0} \tilde{f}_1^{(0)}(x, b_T^2; \mu, \zeta) \stackrel{?}{=} f_1(x;\mu)$$

Nevertheless, a very interesting limit to consider, since Qiu-Sterman function itself is intrinsically non-local along the lightcone and cannot be evaluated on the lattice

$$T_F(x,x) \overset{A^+=0}{\propto} \text{F.T.} \langle P|\, \overline{\psi}(0) \int d\eta^-\ F^{+\alpha}(\eta^-) \gamma^+ \psi(\xi^-)\, |P\rangle$$

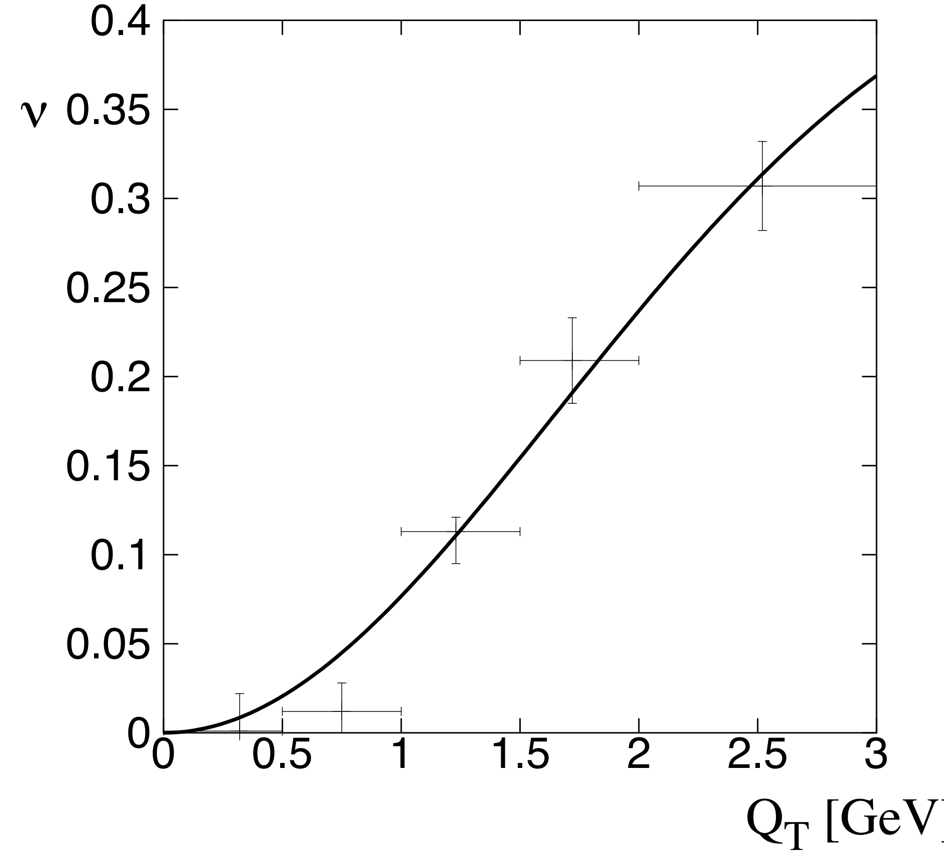
But first Bessel-moment of Sivers function *can* be evaluated (for given rapidity)

# $\cos(2\phi)$ asymmetry

# Quark polarization inside unpolarized hadrons

cos 2φ asymmetry in unpolarized DY ($\pi\, D(W) \to \mu^+\mu^- X$) is *incompatible* with NNLO collinear pQCD

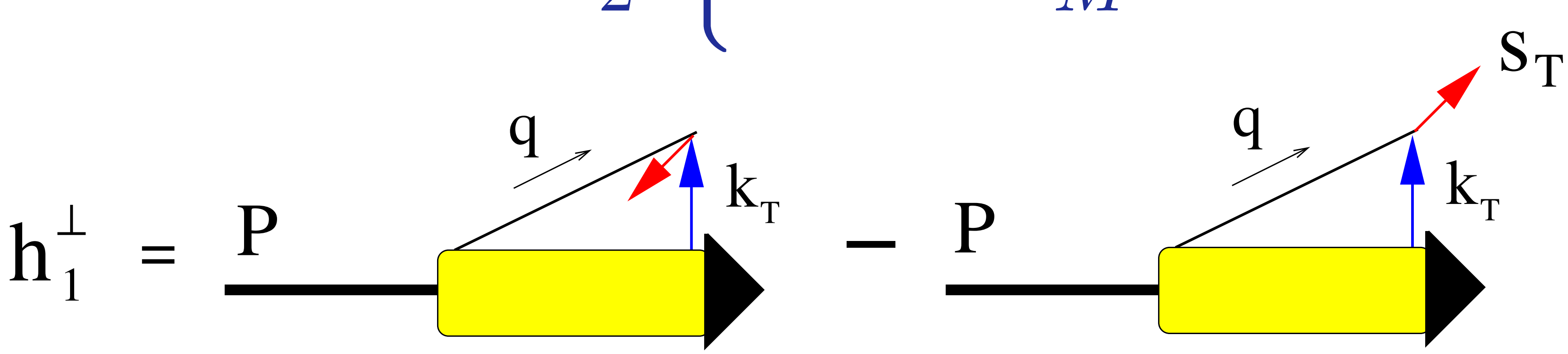
[Collins '79; Brandenburg, Nachtmann & Mirkes '93; Mirkes & Ohnemus '95]




Naturally explained within TMD framework
[DB '99]

It allows for transversely polarized quarks inside an unpolarized hadron:

$$\Phi(x, \boldsymbol{k}_T) = \frac{M}{2}\left\{ f_1(x, \boldsymbol{k}_T^2)\, \frac{\not{P}}{M} + h_1^\perp(x, \boldsymbol{k}_T^2)\, \frac{i\, \not{k}_T\, \not{P}}{M^2} \right\}$$



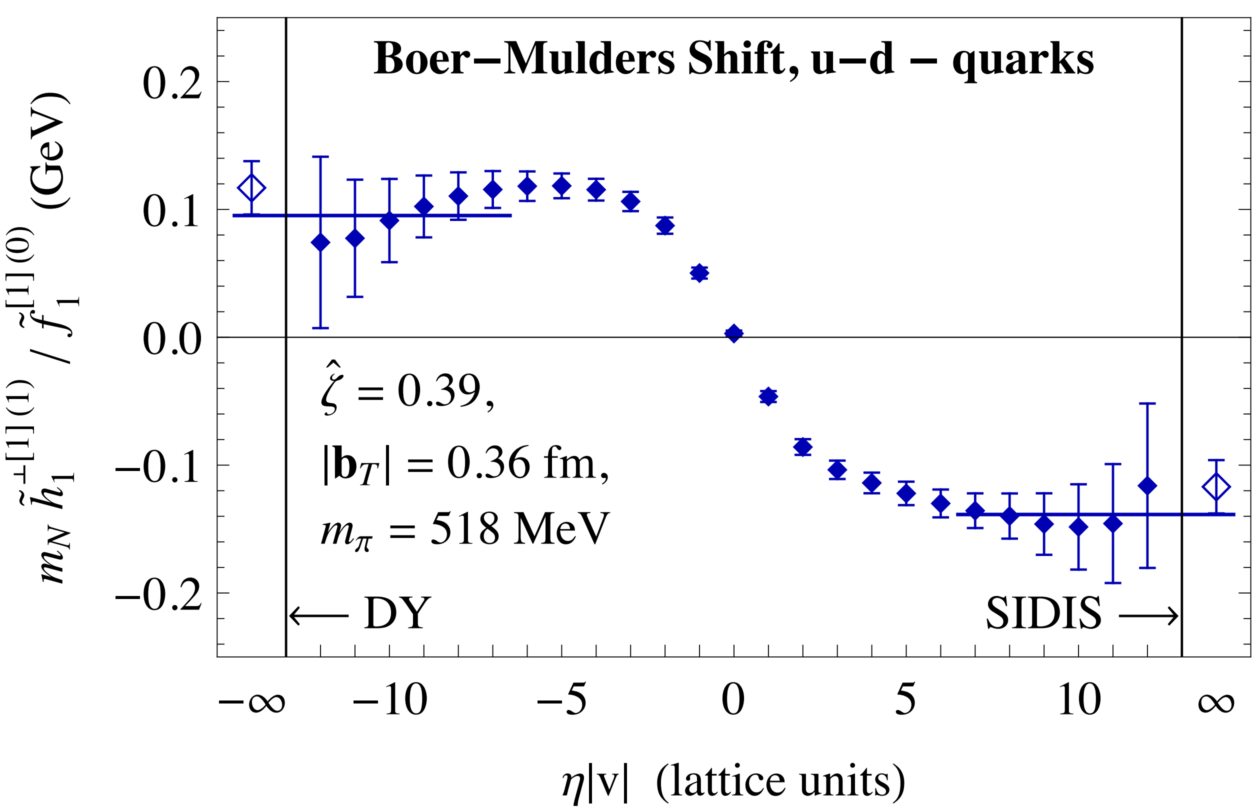

DB & Mulders ('98)

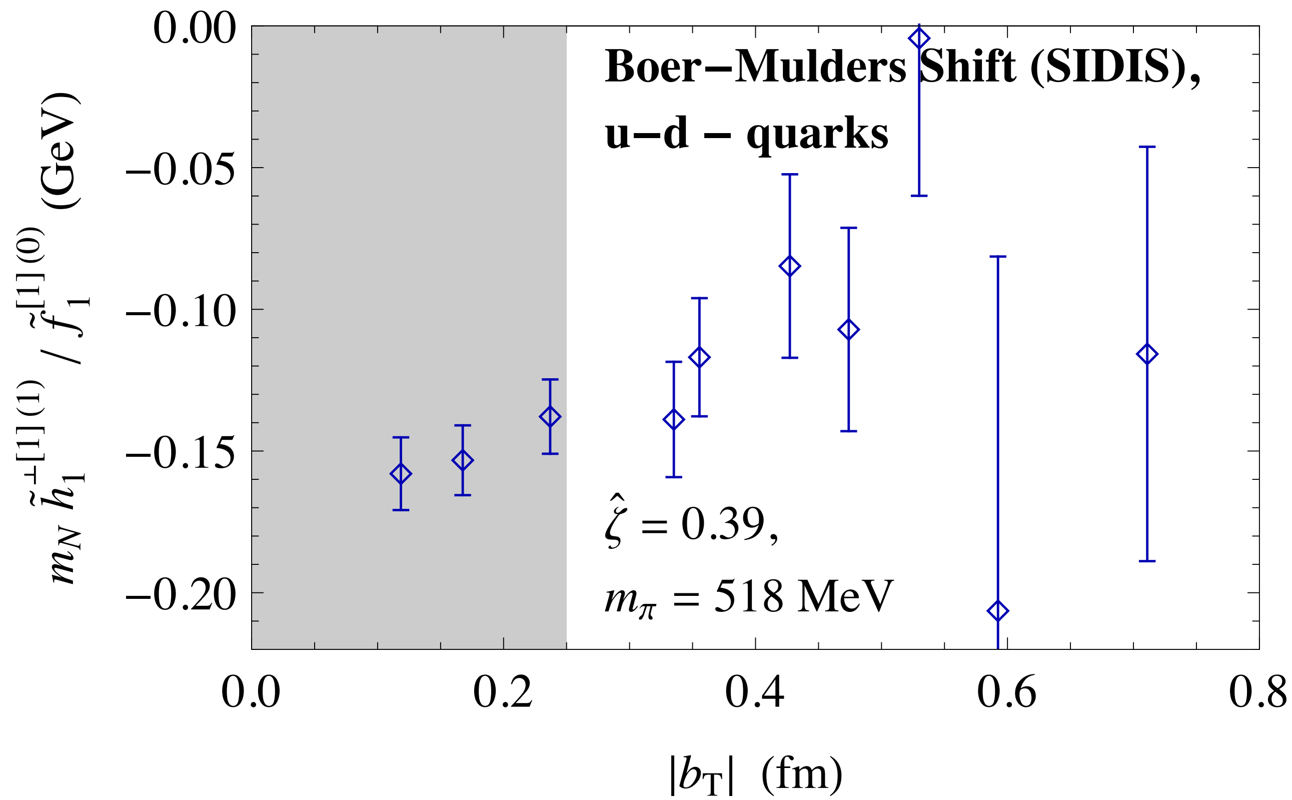
It generates azimuthal asymmetries in unpolarized collisions, like cos 2φ in DY

# Lattice calculation

Taking Mellin moments *and* Bessel transverse moments of the BM function, yields a well-defined quantity $\langle k_T \times s_T \rangle(n, \mathcal{B}_T)$, that can be evaluated on the lattice

[Musch, Hägler, Engelhardt, Negele & Schäfer, PRD 85 (2012) 094510]




Compatible with SIDIS $h_1^{\perp u,d}$ both negative and $|\, h_1^{\perp u}\,|$ quite a bit larger than $|\, h_1^{\perp d}\,|$ (comes possibly in addition to u-quark dominance due to the electric charge)

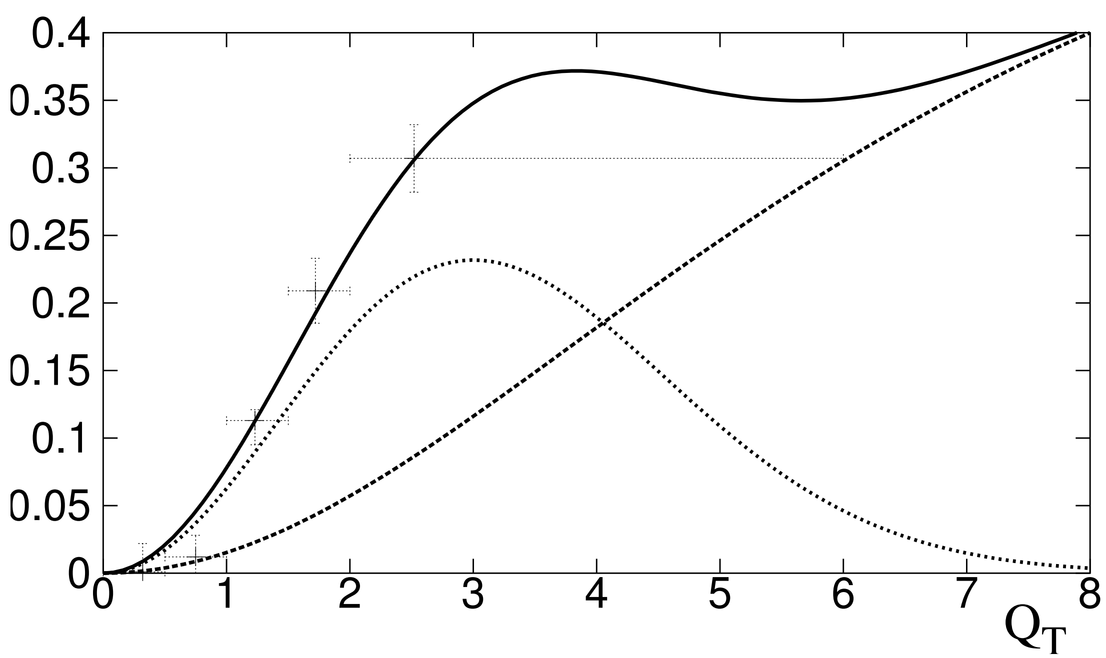
up and down same sign: Pobylitsa, hep-ph/0301236
both negative in SIDIS: Burkardt & Hannafious, PLB 658 (2008) 130

# cos 2ϕ in SIDIS

The cos 2φ asymmetry has different high and low $Q_T$ contributions

At low $Q_T$: ~ $h_1^\perp H_1^\perp$, with $M^2/Q_T^2$ suppressed high-$Q_T$ tail
At high $Q_T$: ~ $f_1 D_1$, which is $Q_T^2/Q^2$ suppressed at low $Q_T$

The two contributions both need to be included, which is not double counting




$$\nu = \nu_{h_1^\perp} + \nu_{\rm pert} + \mathcal{O}(\frac{Q_T^2}{Q^2} \text{ or } \frac{M^2}{Q_T^2})$$

Nontrivial since a ratio of sums becomes approximately a sum of ratios

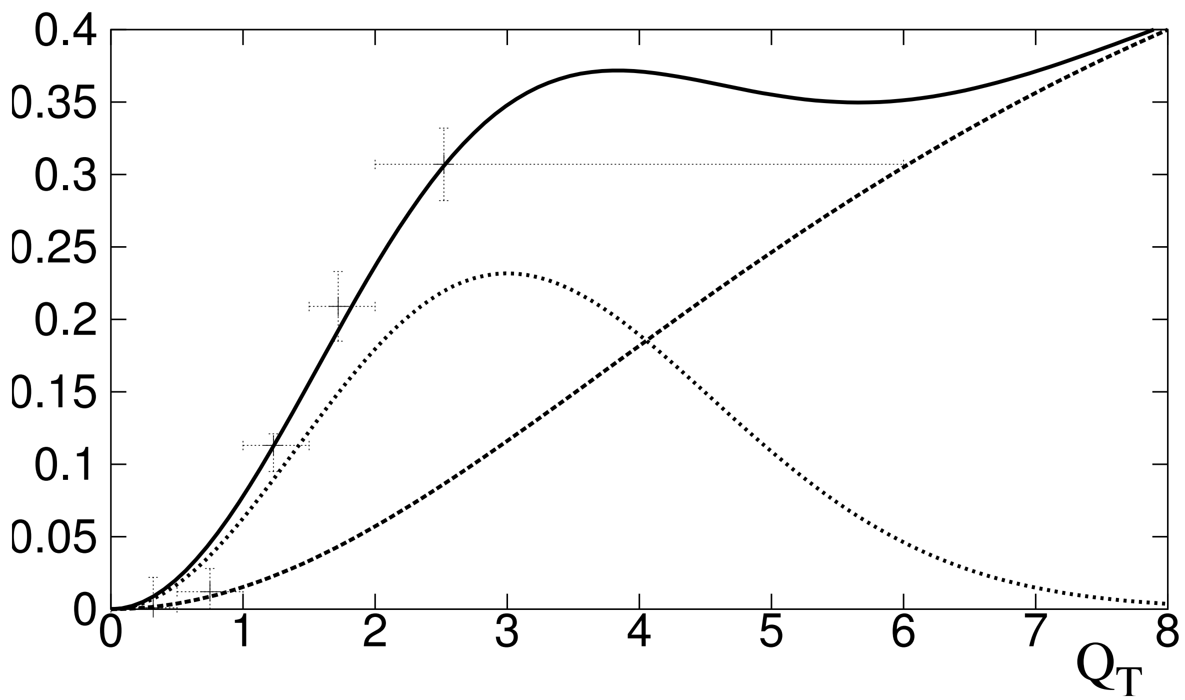
Bacchetta, DB, Diehl, Mulders, JHEP 0808 (2008) 023

At low $Q^2$ the twist-4 Cahn effect (~$M^2/Q^2$) also enters

# Weighted cos 2ϕ asymmetry

The cos 2φ asymmetry as function of $Q_T$ has different high and low $Q_T$ contributions

At low $Q_T$: ~ $h_1^\perp H_1^\perp$
At high $Q_T$: ~ $f_1 D_1$, i.e. dominated by the perturbative contribution




For TMD part of the cos 2φ asymmetry the appropriate weighting would be with $Q_T^2$:

$$\int d^2\boldsymbol{q}_T\, \boldsymbol{q}_T^2\, \frac{d\sigma}{d^2\boldsymbol{q}_T} \to h_1^{\perp(1)} H_1^{\perp(1)}$$

Unfortunately sensitive mainly to the high $Q_T$ part of the asymmetry (Y-terms)
Teaches us little about TMD part

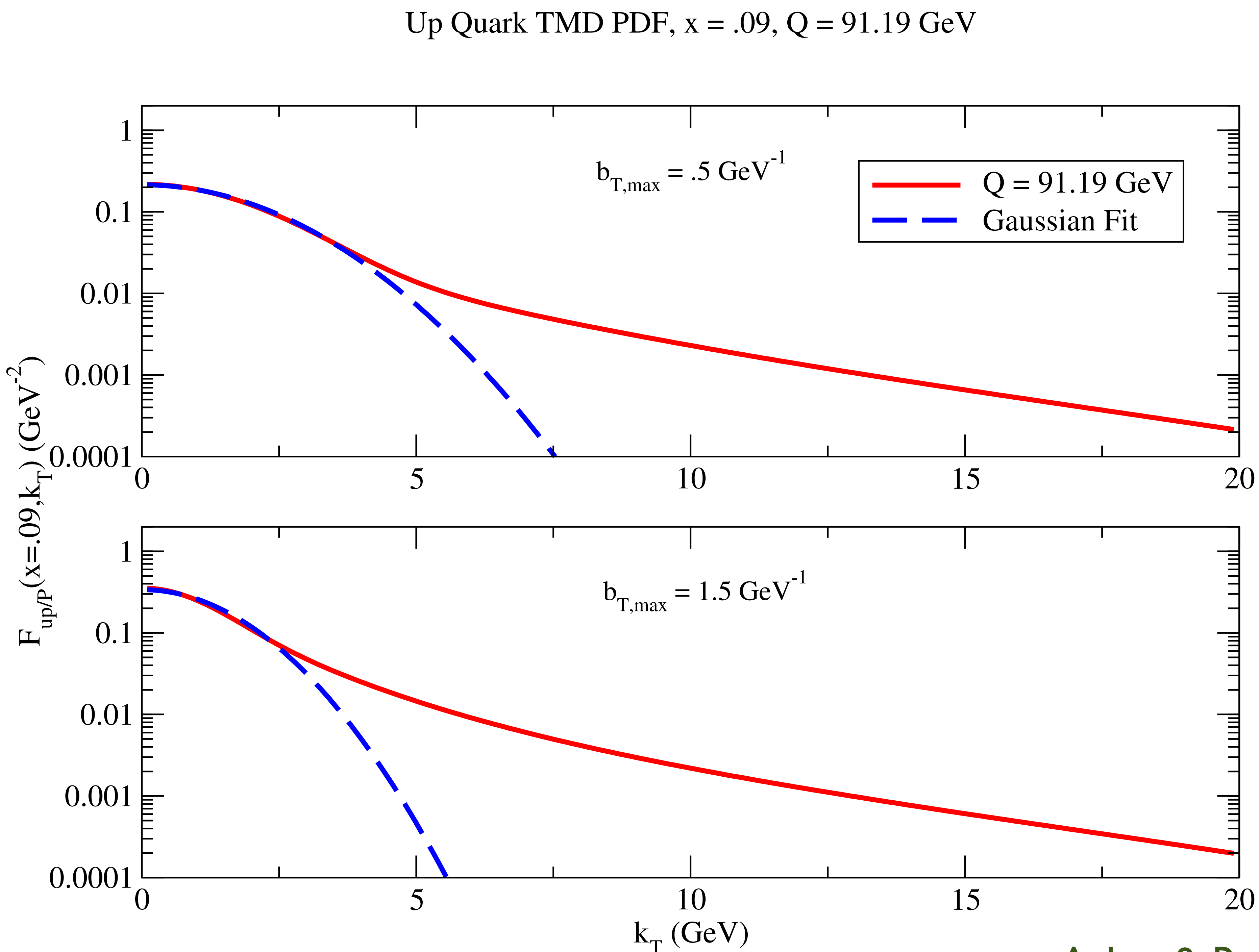
Bacchetta, DB, Diehl, Mulders, JHEP 0808 (2008) 023

Solutions: use Lam-Tung relation to largely cancel Y or calculate and subtract Y or do Bessel weighting with sufficiently large $\mathcal{B}_T$ in order to suppress Y

# Average transverse momentum

# Defining the average $p_T$

Higher transverse moments in general diverge due to power law tail




$$\frac{\langle p_T^2 \rangle}{2M^2} = f_1^{(1)}(x)$$

Aybat & Rogers, PRD 83 (2011) 114042

Average $p_T$ can be defined by Gaussian fit

# Defining the average $p_T$

$$\frac{\langle p_T^2 \rangle}{2M^2} = f_1^{(1)}(x)$$

$$\rightarrow \tilde{f}_1^{(1)}(x, \boldsymbol{b}_T^2) = \frac{2\pi}{M^2} \int d|\boldsymbol{p}_T| \frac{|\boldsymbol{p}_T|^2}{|\boldsymbol{b}_T|} J_1(|\boldsymbol{b}_T||\boldsymbol{p}_T|)\ f_1(x, \boldsymbol{p}_T^2)$$

With respect to cutting off the perturbative tail any regularization will do, but Bessel weighting is natural from the perspective of deconvoluting and:

$$\tilde{f}^{(n)}(x, \boldsymbol{b}_T^2) = n! \left( -\frac{2}{M^2} \partial_{\boldsymbol{b}_T^2} \right)^n \tilde{f}(x, \boldsymbol{b}_T^2)$$

It suggests a lattice study of the gauge link dependence of $\tilde{f}_1^{(1)[\mathcal{U}]}(x, \boldsymbol{b}_T^2)$

It can be shown that for *U*=+ (SIDIS) and *U*=− (DY) the answer is the same, but not for TMD-factorizing processes with more complicated links (e p→e' jet jet X?)

In all cases there will be contributions from double gluonic pole matrix elements

Buffing, Mukherjee, Mulders, PRD 83 (2011) 114042

# $p_T$-broadening

$p_T$ broadening involves a $p_T^2$ weighting, which theoretically yields divergent quantities, hence usually it is defined as a (finite) difference: $\Delta p_T^2 \equiv \langle p_T^2 \rangle_A - \langle p_T^2 \rangle_p$

An alternative is to consider Bessel weighting:

$$\tilde{f}_1^{(1)q/A}(x, \boldsymbol{b}_T^2) - \tilde{f}_1^{(1)q/p}(x, \boldsymbol{b}_T^2) \stackrel{\boldsymbol{b}_T^2 \to 0}{\longrightarrow} \Delta p_T^2 \equiv \langle p_T^2 \rangle_A - \langle p_T^2 \rangle_p$$

Converges very slowly, but $\Delta p_T^2$ also converges very slowly to ‘true’ value as function of (experimental or theoretical) cut-off on $p_T$

A study of the link (in)dependence of $p_T$-broadening would be interesting

$$\tilde{f}_1^{(1)q/A[\mathcal{U}]}(x, \boldsymbol{b}_T^2) - \tilde{f}_1^{(1)q/p[\mathcal{U}]}(x, \boldsymbol{b}_T^2) \stackrel{\boldsymbol{b}_T^2 \to 0}{\longrightarrow} \Delta p_T^{2\,[\mathcal{U}]} \equiv \langle p_T^2 \rangle_A^{[\mathcal{U}]} - \langle p_T^2 \rangle_p^{[\mathcal{U}]}$$

A well-defined ratio can also be formed, but as $b_T$ gets smaller the interesting information about the A versus p difference is lost, $(\infty+\Delta)/\infty$:

$$R_\Delta \equiv \frac{\tilde{f}_1^{(1)q/A}(x, \boldsymbol{b}_T^2)}{\tilde{f}_1^{(1)q/p}(x, \boldsymbol{b}_T^2)} \stackrel{\boldsymbol{b}_T^2 \to 0}{\longrightarrow} 1$$

# Conclusions

# Conclusions

- Bessel-weighted asymmetries are well-defined and emphasize TMD region
- Y-term of an asymmetry can be decreasing or increasing, in the latter case best first subtracted or cancelled (like in the cos 2ϕ example)
- Bessel-weighted TMDs, including T-odd ones, are calculable on the lattice, can even tell us about size and shape of Qiu-Sterman function
- The limit $\mathcal{B}_T \to 0$ should be taken with care, divergences and operator mixing can arise
- Average $p_T$ and $p_T$-broadening can be redefined in a useful manner (lattice)

In general, Bessel-weighting offers a more ‘stable’ look at TMDs