TMD Evolution and Bessel Weighting

INFN
Istituto Nazionale
di Fisica Nucleare
Laboratori Nazionali di Frascati

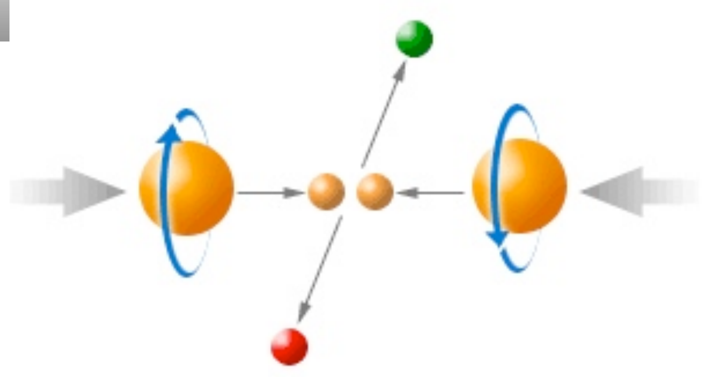
Leonard Gamberg Penn State

Based on Boer, LG,Musch,Prokudin JHEP 2011 and "in progress"

Outline

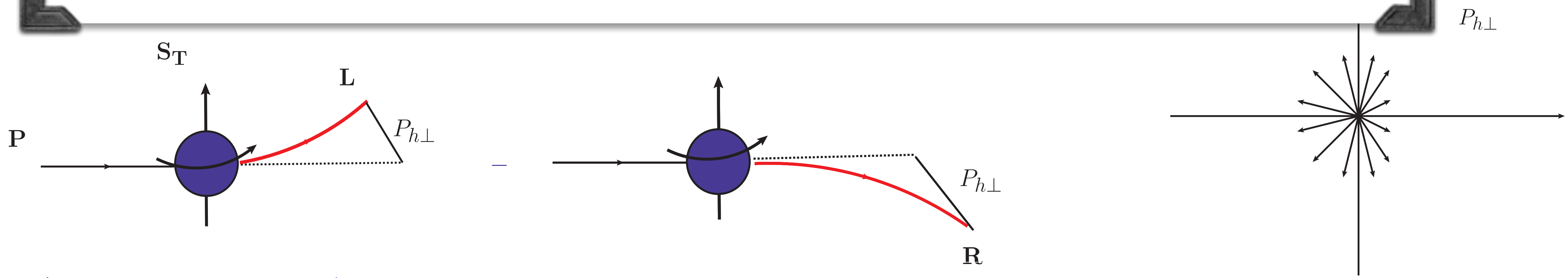
- **Review transverse spin Effects - TSSAs**
- **Summary Elements Factorization-SIDIS**
- Merit of Bessel Weighted Asymmetries (BWA) “S/T” pic of SIDIS
- Fourier Transformed SIDIS cross section & “FT” TMDs
- Cancellation of the Soft, Pert. Sudakov, hard factor in BWA
- JMY & CSS+JC formalism
- Conclusions

Comment



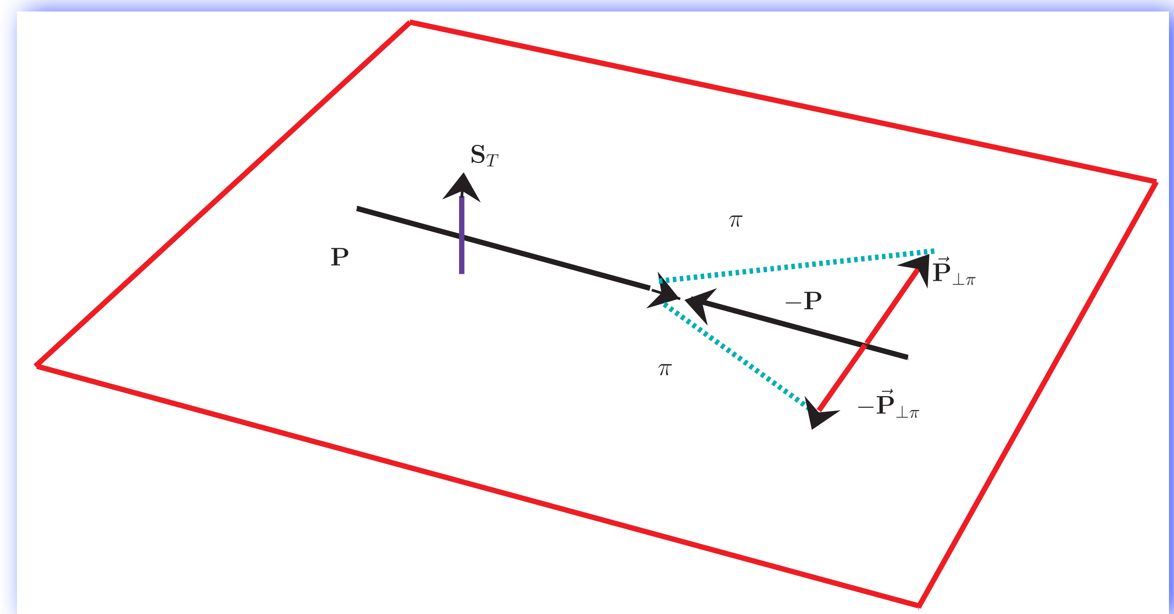
- Single inclusive hadron production in hadronic collisions largest/ oldest observed TSSAs
- From theory view *notoriously* challenging from partonic picture twist-3 power suppressed hard scale (vs. SIDIS, Drell Yan & e^+e^-)
- Why?

What is transverse single spin asymmetry TSSAs



$\sigma^{\downarrow}(x, P_{\perp}) = \sigma^{\uparrow}(x, -P_{\perp})$ Rotational Invariance *"Left-Right" Asymmetry*

$$A_N = \frac{\sigma^{\uparrow}(x, P_{\perp}) - \sigma^{\uparrow}(x, -P_{\perp})}{\sigma^{\uparrow}(x, P_{\perp}) + \sigma^{\uparrow}(x, -P_{\perp})} \equiv \Delta\sigma$$



QCD is Parity Conserving   TSSAs Scattering plane transverse to spin

Naively "T-odd"

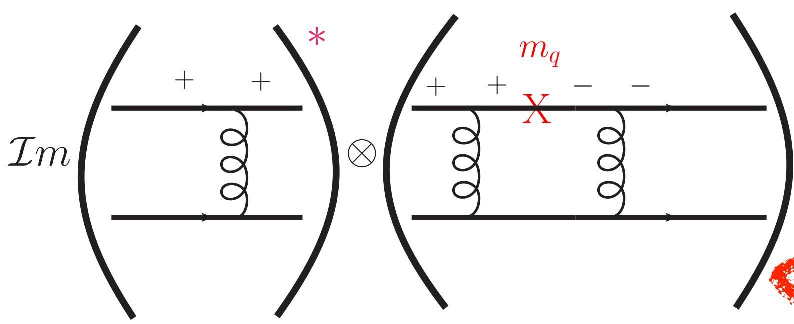
$$\Delta\sigma \sim iS_T \cdot (\mathbf{P} \times P_{\perp}) \otimes (\text{"}T - odd\text{" QCD} - \text{phases})$$

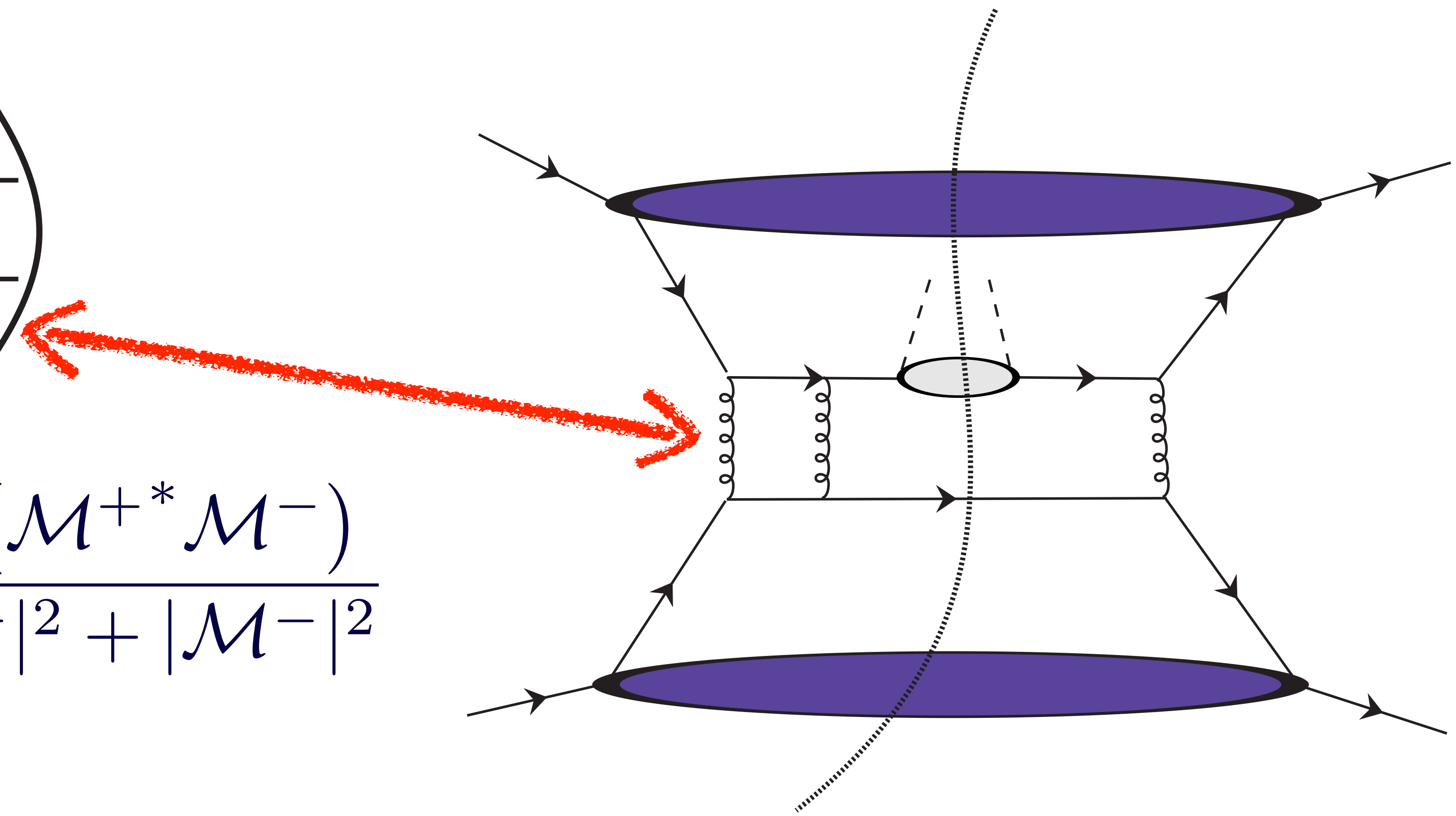
Spin orbit

“QCD Phases” Reaction Mechanism for TSSAs

Collinear picture factorized QCD parton dynamics

$$\Delta\sigma^{pp^{\uparrow}\to\pi X} \sim f_a \otimes f_b \otimes \Delta\hat{\sigma} \otimes D^{q\to\pi}$$





$$\Delta\hat{\sigma} = \frac{\hat{\sigma}^{\uparrow} - \hat{\sigma}^{\downarrow}}{\hat{\sigma}^{\uparrow} + \hat{\sigma}^{\downarrow}} \sim \frac{\mathrm{Im}\left(\mathcal{M}^{+*}\mathcal{M}^{-}\right)}{|\mathcal{M}^{+}|^2 + |\mathcal{M}^{-}|^2}$$

0) Interference of helicity flip and non-flip amps-gen PHASE

1) Relative color phase require higher order correction α_s

2) QCD interactions conserve helicity up to correction
requires breaking of chiral $\mathcal{O}\left(m_q/E_q\right)$

3) Thus, Twist three and trivial in chiral limit

$$\Delta\sigma \propto \frac{m_q}{E}\alpha_s \to 0 \quad \text{chiral limit}$$

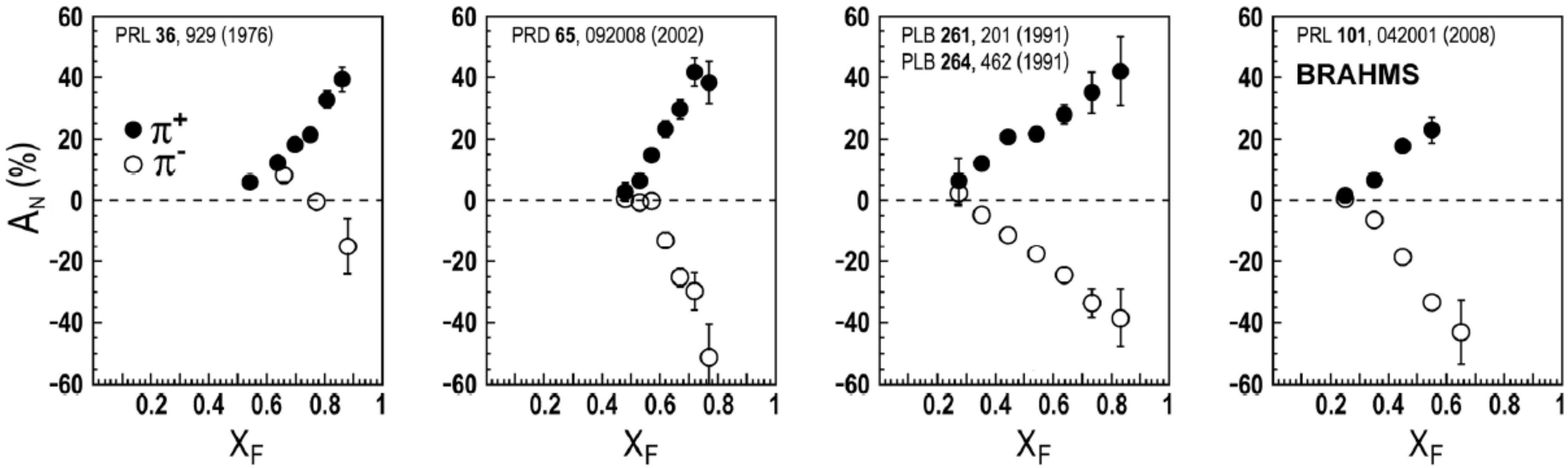
Early theory in striking contrast exp.TSSAs in Inclusive Reactions

Transverse Single-Spin Asymmetries:
From Low to High Energies!

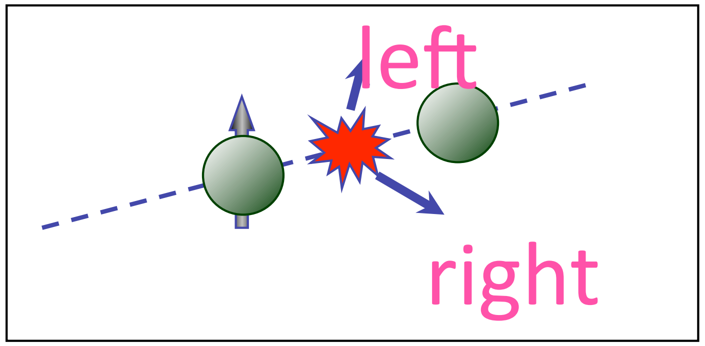
BRAHMS

Pbeam=12 Gev/c ANL √s=4.9 GeV	Pbeam=22 Gev/c BNL √s=6.6 GeV	Pbeam=200 Gev/c FNAL √s=19.4 GeV	RHIC √s=62.4 GeV

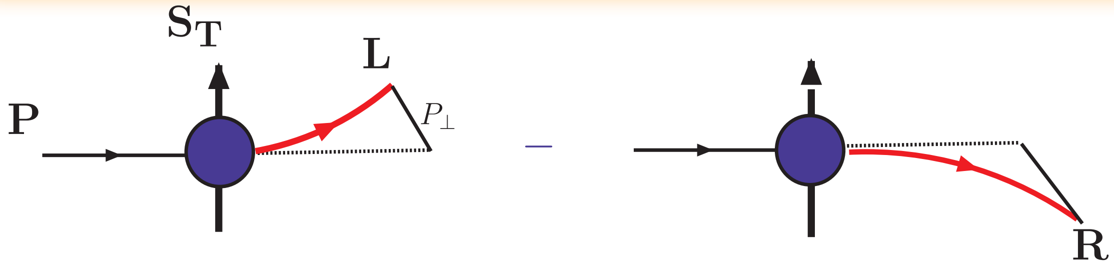
← Fixed target → Collider

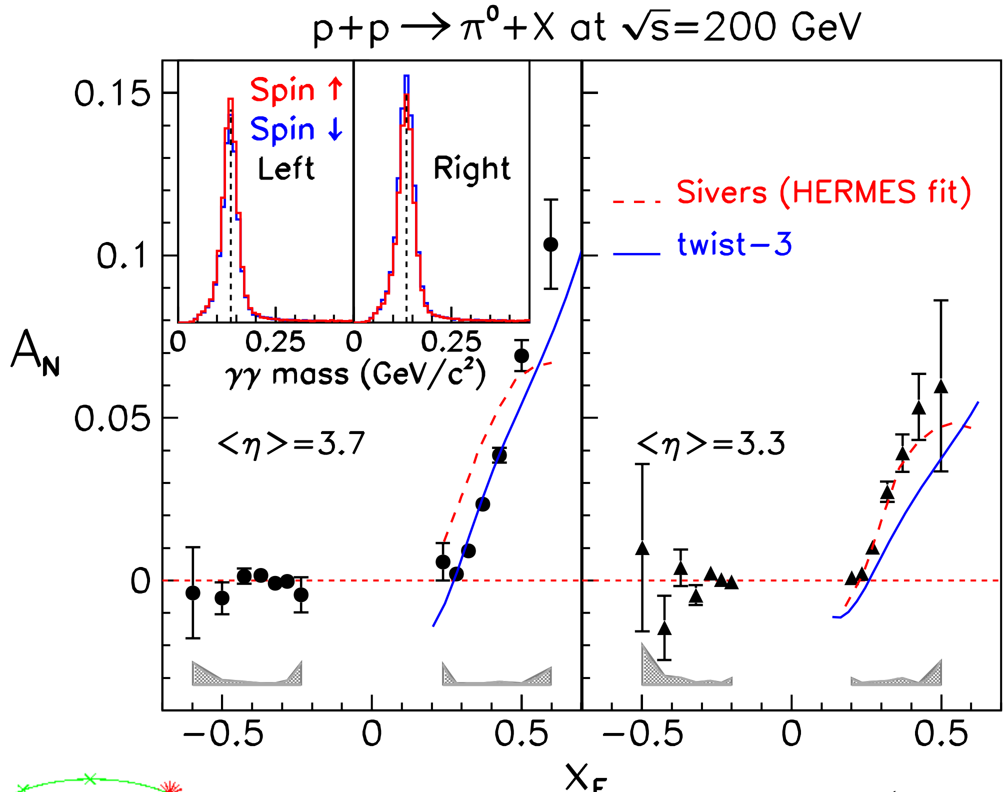


$$x_F = 2p_{long}/\sqrt{s}$$



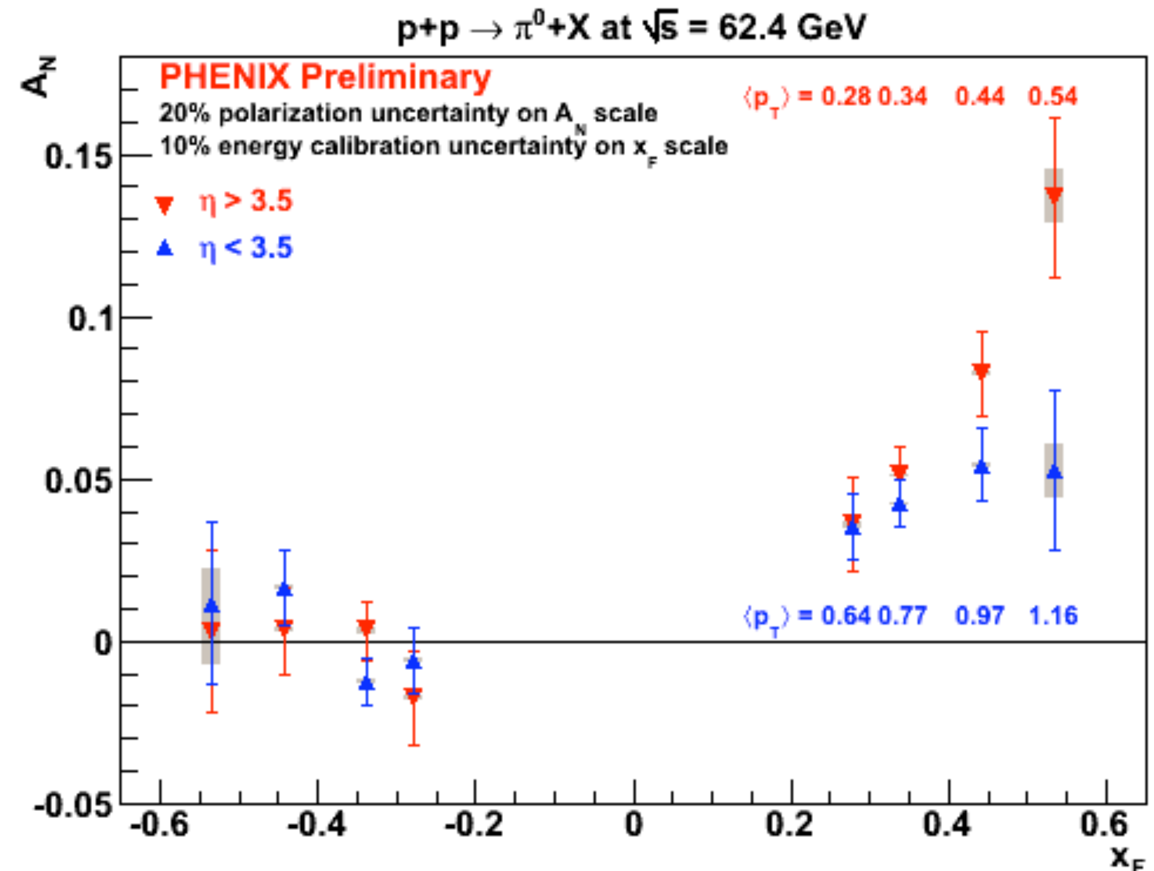
Modern Era Transverse SSA's at $\sqrt{s}$ = 62.4 & 200 GeV at RHIC



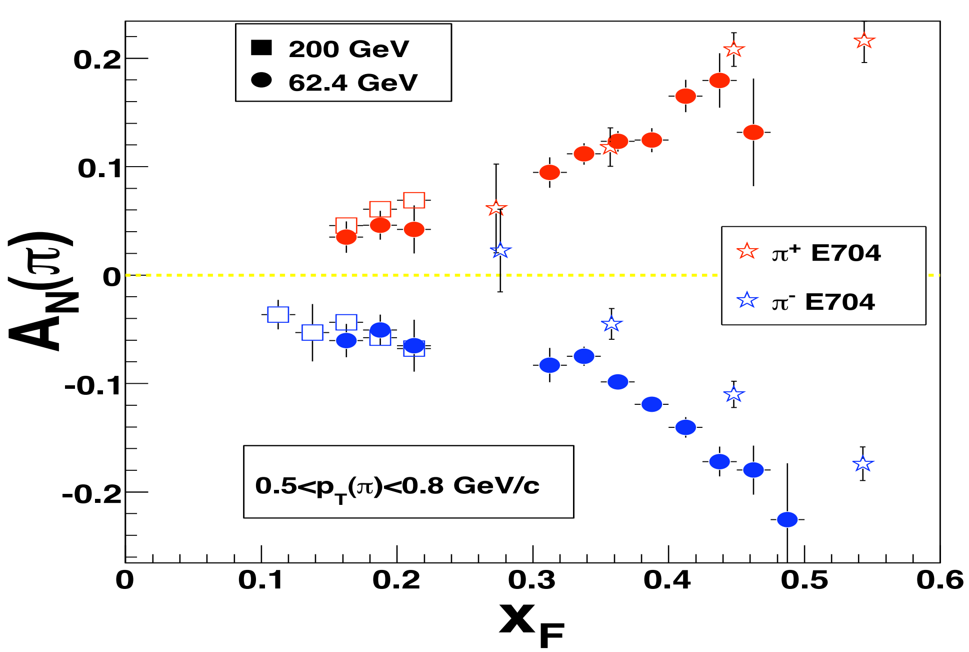


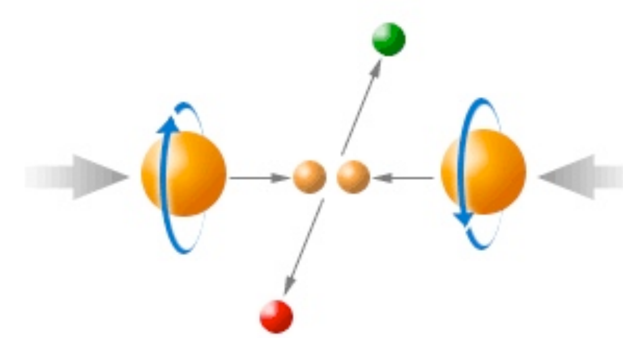
STAR

PRL101, 042001 (2008)



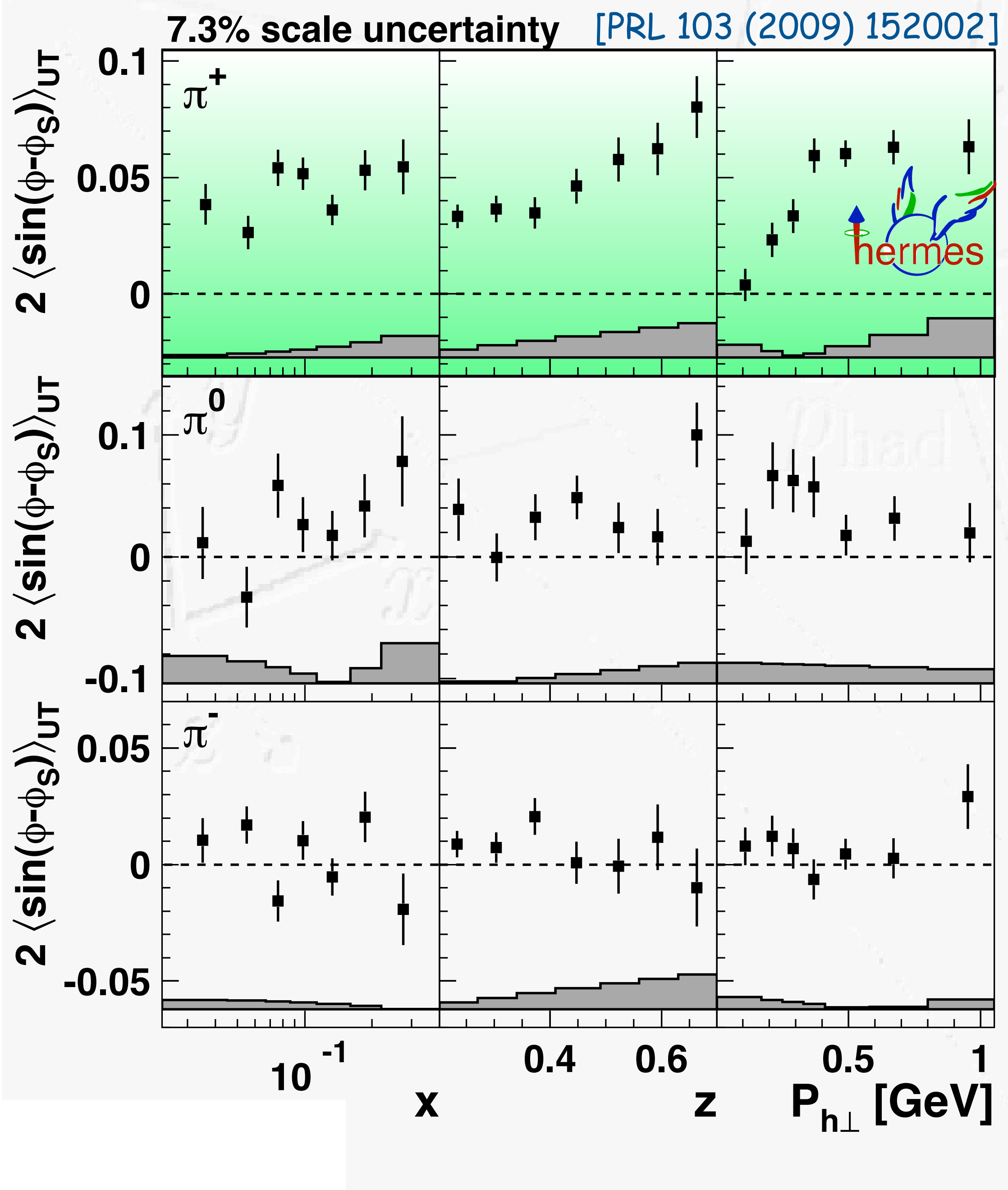
BRAHMS

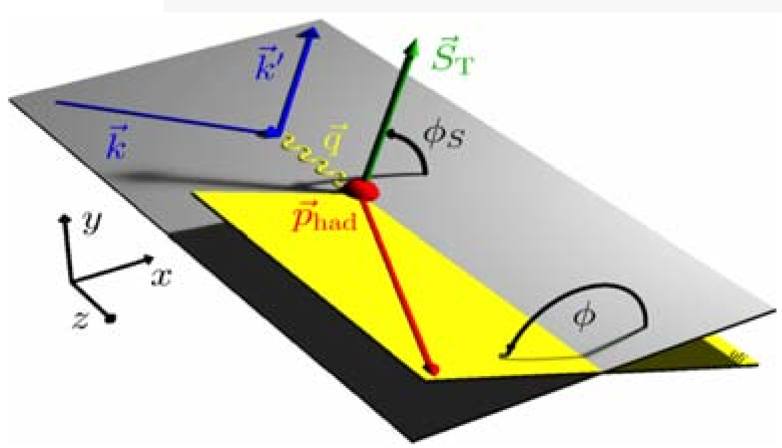


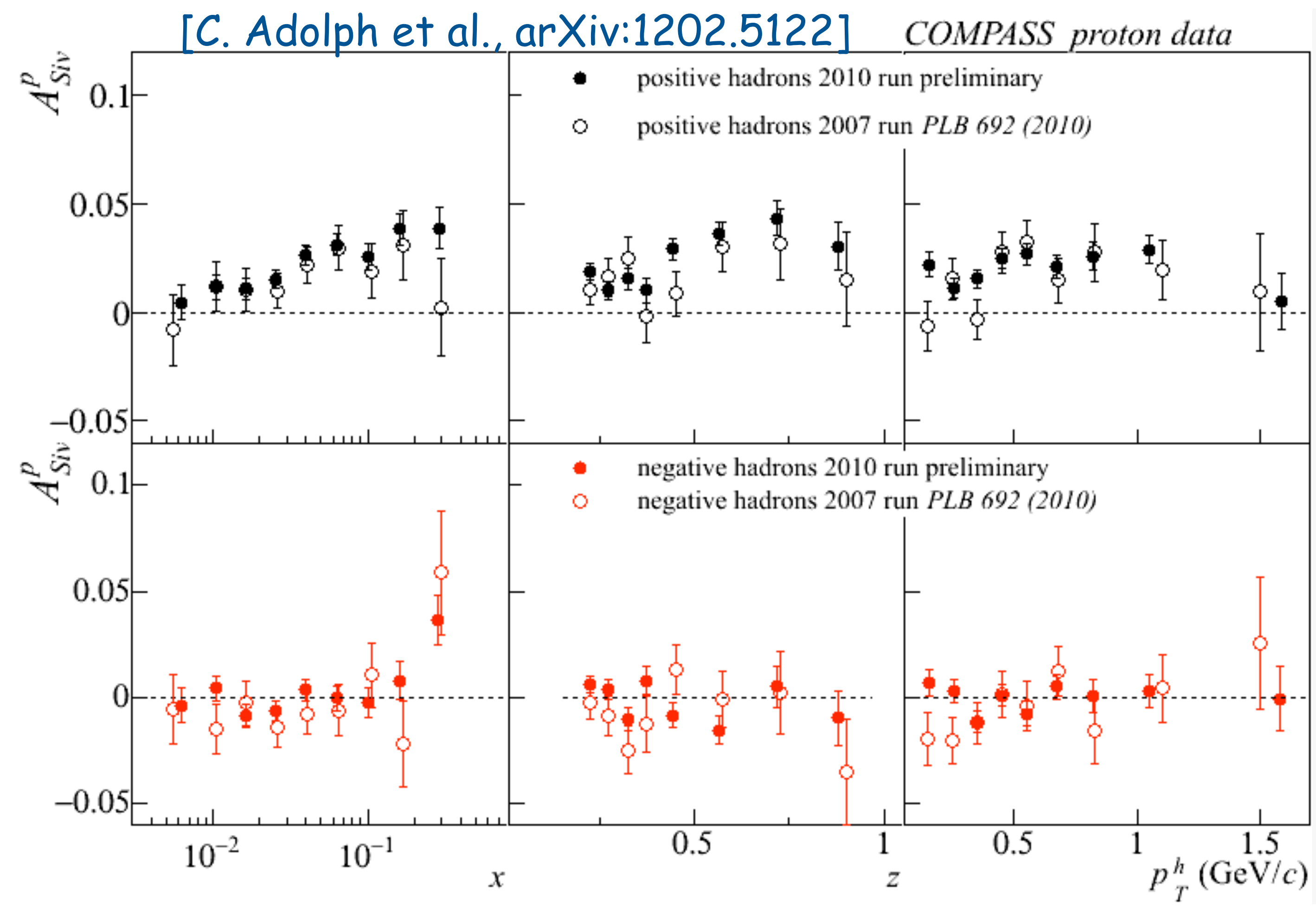


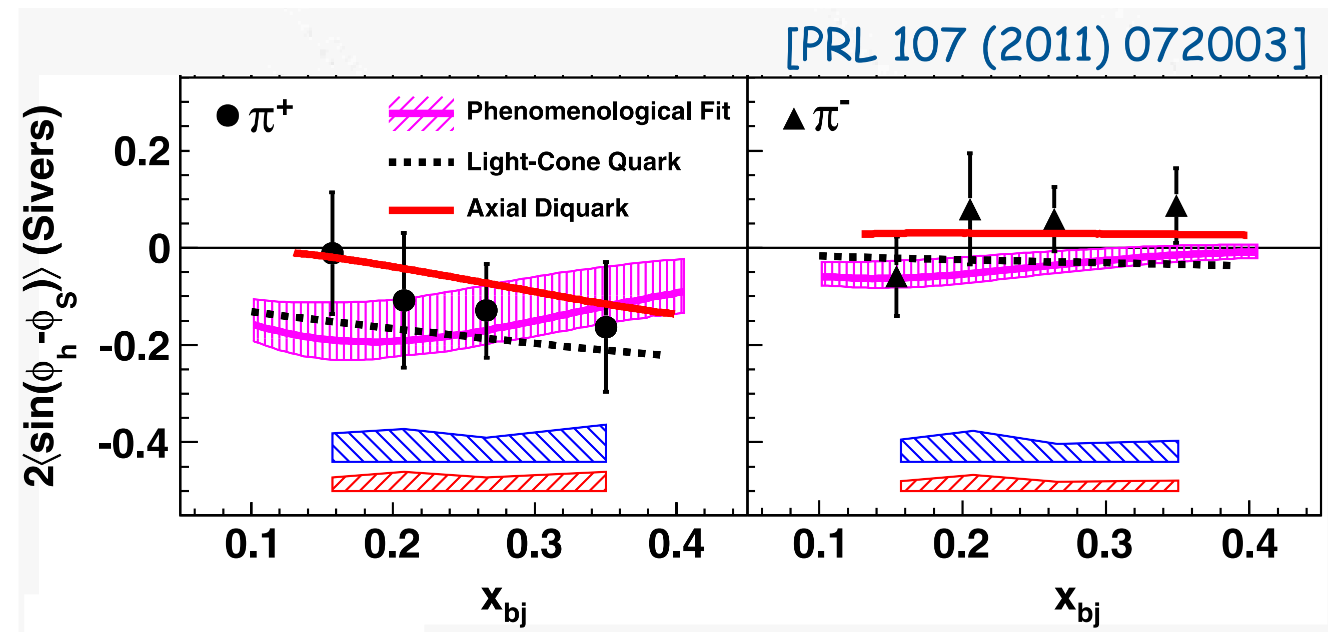
TSSAs in Semi-inclusive Deep Inelastic Scattering

HERMES





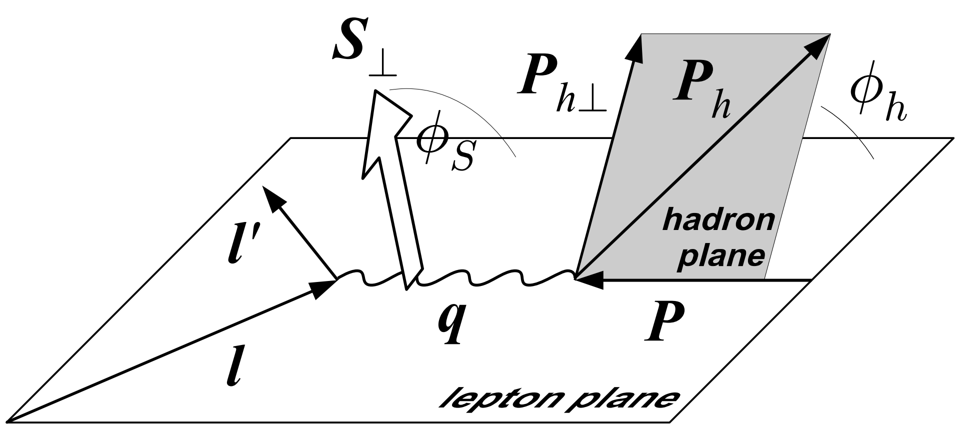


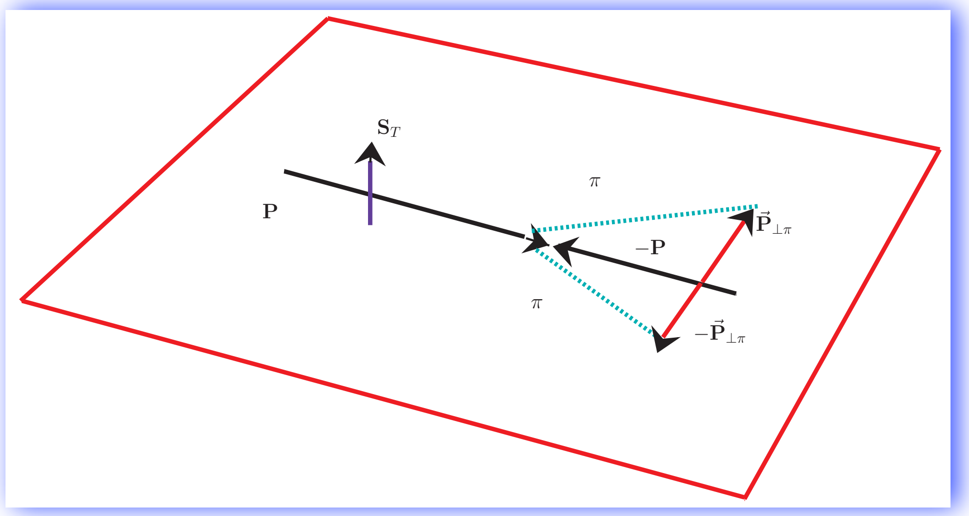


JLAB

N.B. at least 2 methods generate non-trivial TSSA

- Depends on momentum of probe $q^2 = -Q^2$ and momentum of produced hadron $P_{h\perp}$ relative to hadronic scale





- $k_\perp^2 \sim P_{h\perp}^2 \ll Q^2$ two scales-twist 2 TMDs

$$\Delta\sigma(P_h, S) \sim \Delta f^{\perp}_{a/A}(x, k_\perp) \otimes D_{h/c}(z, K_\perp) \otimes \hat{\sigma}_{\text{parton}}$$

- $k_\perp^2 \ll P_{h\perp}^2 \sim Q^2$ twist 3 factorization-ETQSs

$$\Delta\sigma(P_h, S) \sim \frac{1}{Q} \Delta f^{\perp}_{a/A}(x) \otimes f_{b/B}(x) \otimes D_{h/c}(z) \otimes \hat{\sigma}_{\text{parton}}$$

Crucial role of "phases" and Trans polz effects in QCD

- Two scale factorization in terms TMDs twist 2

$$p_T \sim \mathbf{k}_T << \sqrt{Q^2}$$

- Realization that FSI and ISI btwn struck parton and target remnant provide necessary phases that lead to non-vanishing TSSAs

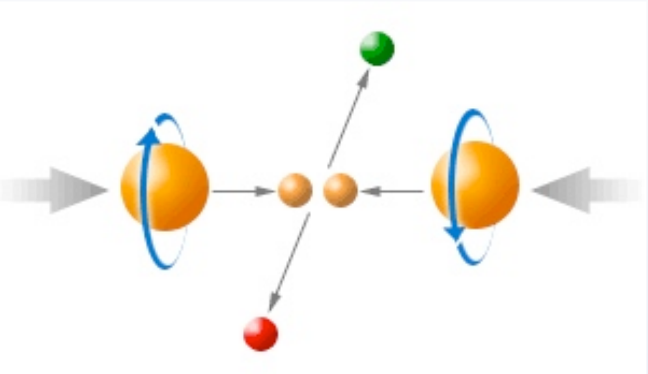
- One large scale factorization in terms twist 3 approach $Q \sim P_T >> \Lambda_{\rm qcd}$

- Phases from interference of two-parton & three-parton scattering amplitudes

- Connection btwn two approaches overlap region for DY and SIDIS Unified picture Ji,Qiu,Vogelsang,Yuan PRL 2006 ...

$$\Lambda_{\rm QCD} << q_T << Q$$

Comments Importance of TMDs in studying partonic content of the nucleon



- Connection w/ twist 2 "TMD" approach
  - Operator level ETQS fnct 1st moment of Sivers

$$gT_F(x,x) = -\int d^2k_T \frac{|k_T^2|}{M} f_{1T}^{\perp}(x, k_T^2) \text{ + "UV" ...}$$

$$= -2M f_{1T}^{\perp(1)}(x)$$

Boer Pijlman Mulders NPB -03

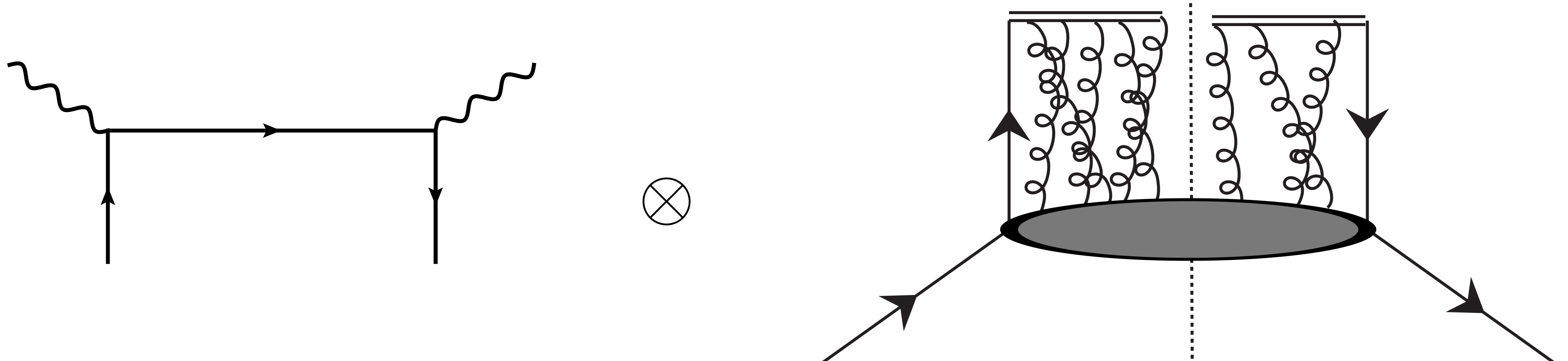
$$\tilde{f}_{1T}^{\perp(1)}(x, |\boldsymbol{b}_T|) = \int d^2p_T \frac{|p_T|}{|\boldsymbol{b}_T| M^2} J_1(|\boldsymbol{b}_T||p_T|) f_{1T}^{\perp}(x, p_T^2)$$

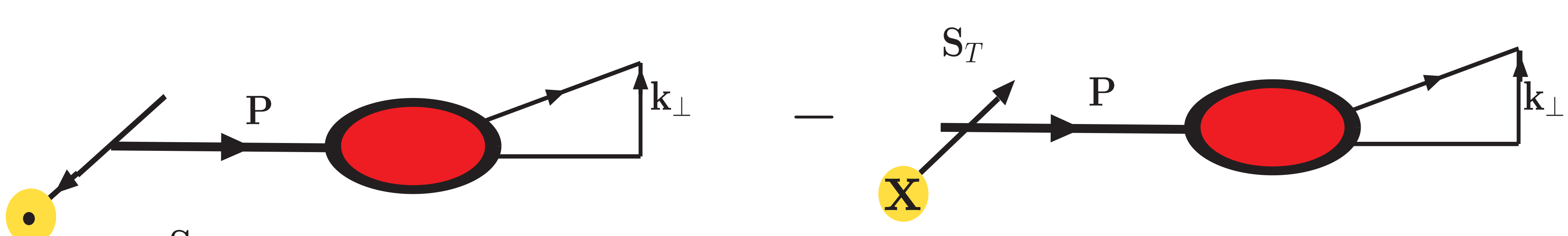
Boer, LG, Musch, Prokudin JHEP-2011--arXiv:1107.529

TSSAs thru "T-odd" non-pertb. **spin-orbit** *correlations....*

Sensitivity to $p_T \sim \mathbf{k}_T << \sqrt{Q^2}$

- Sivers PRD: 1990 TSSA is associated w/ correlation *transverse* spin and momenta in initial state hadron

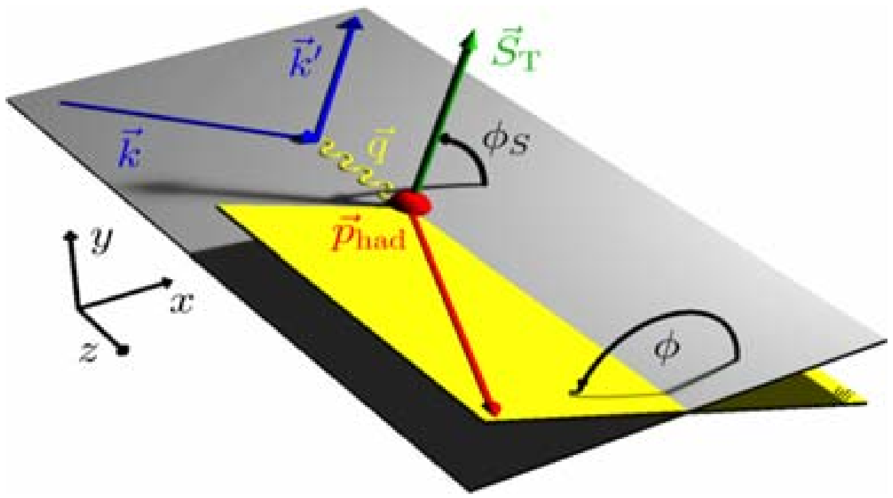




$$\Delta\sigma^{pp^{\uparrow}\to\pi X} \sim D \otimes f \otimes \Delta f^{\perp} \otimes \hat{\sigma}_{Born} \Rightarrow$$

$$\boldsymbol{\Delta f^{\perp}(x, k_{\perp}) = i S_T \cdot (P \times k_{\perp})}\, f_{1T}^{\perp}(x, \boldsymbol{k}_{\perp})$$

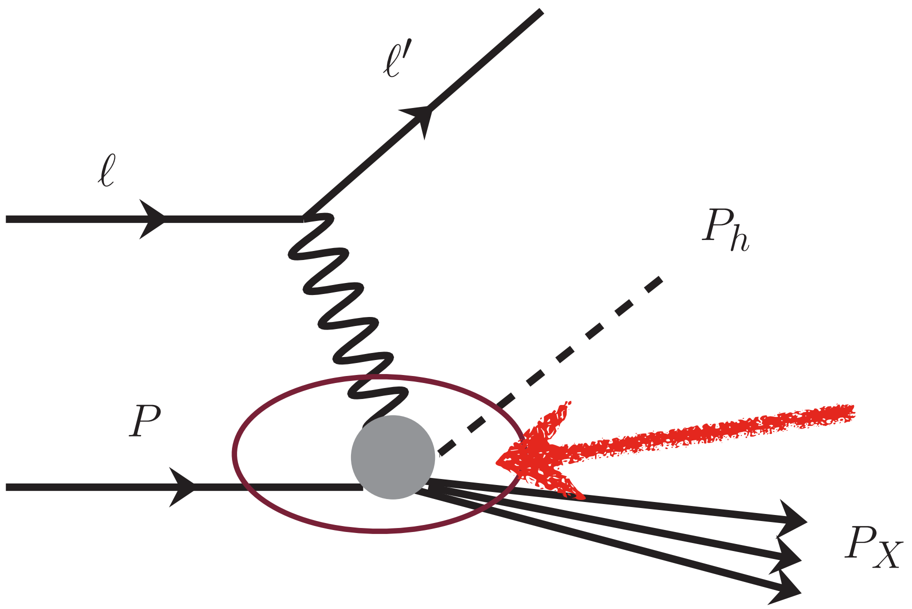
Factorization Parton Model-predicts existence of T-odd PDFs and TSSAs--Boer-Mulders PRD 1998



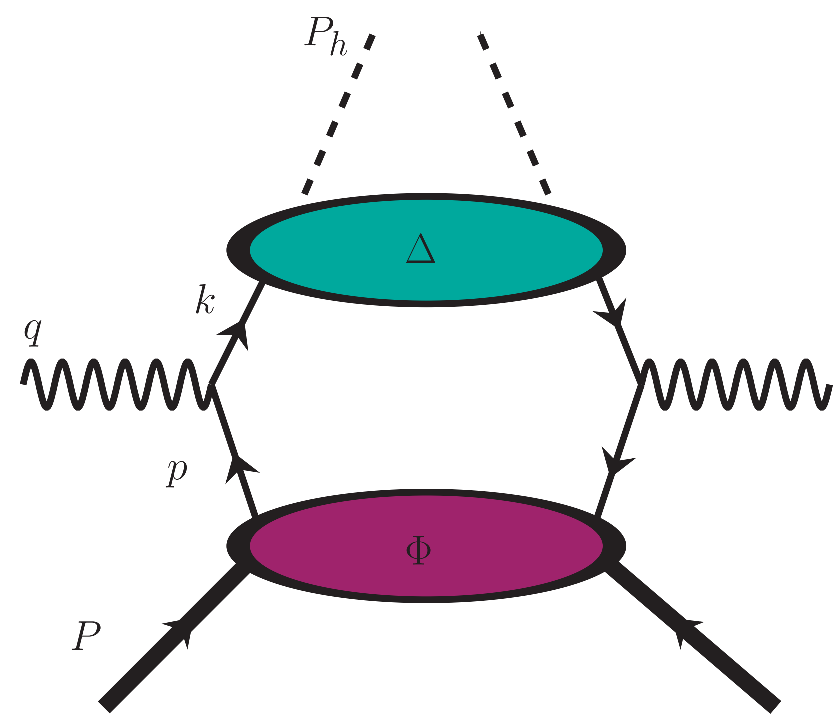
$$x_B = \frac{Q^2}{2P \cdot q}$$

$$z_h = \frac{P \cdot P_h}{P \cdot q} \approx \frac{P_h^-}{q^-}$$

Parton model & DIS kinematics



Factorize



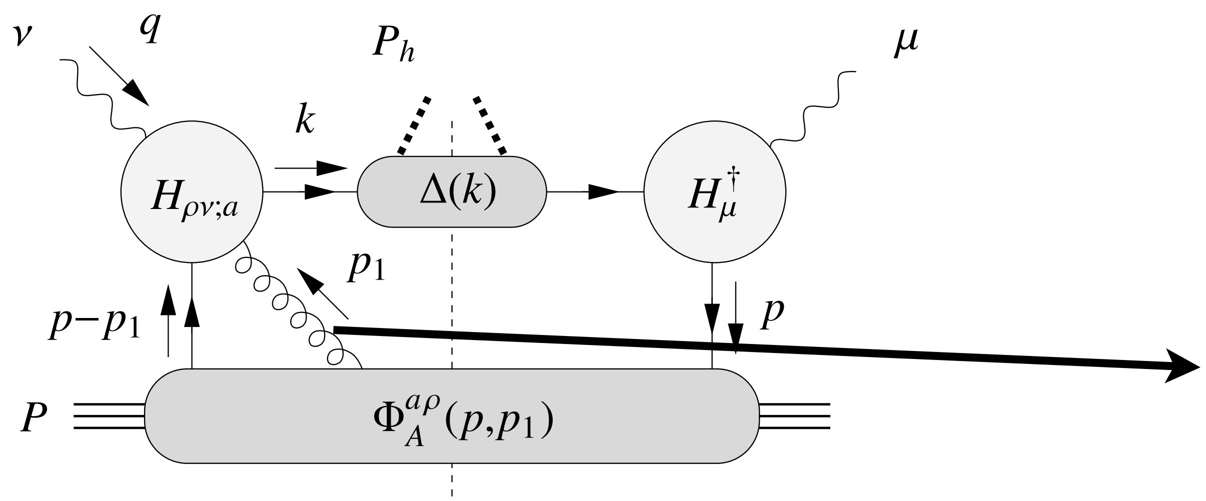
$$\frac{d\sigma}{dx_B\, dy\, d\psi\, dz_h\, d\phi_h\, |\boldsymbol{P}_{h\perp}|\, d|\boldsymbol{P}_{h\perp}|} = \frac{\alpha^2}{x_B y Q^2} \frac{y^2}{(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x_B}\right) L_{\mu\nu} W^{\mu\nu},$$

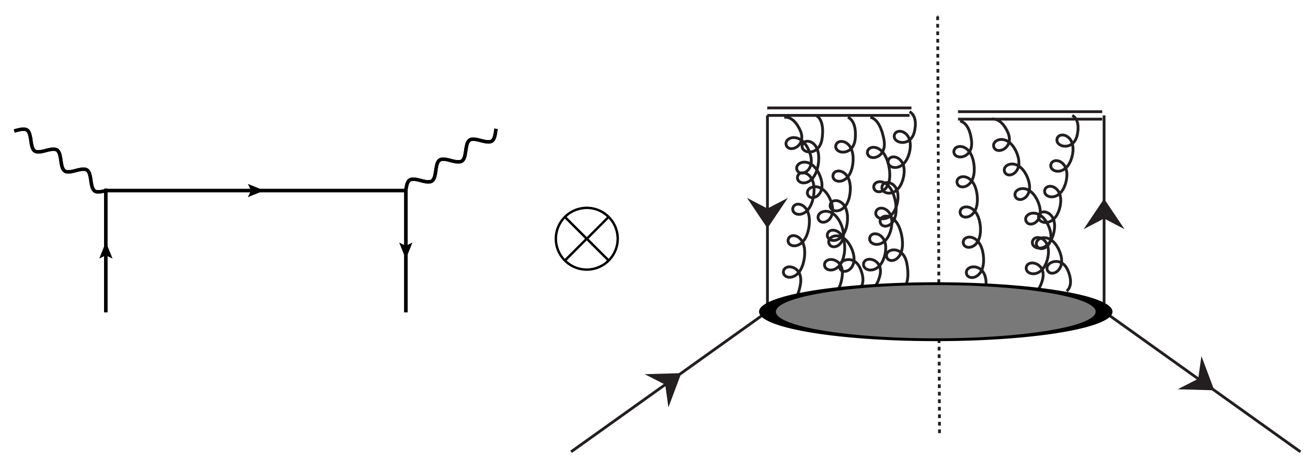
Minimal Requirement for PARTON MDL Factorization

Gauge link determined re-summing leading gluon interactions btwn soft and hard

Efremov,Radyushkin Theor. Math. Phys. 1981,Belitsky, Ji, Yuan NPB 2003,
Boer, Bomhof, Mulders Pijlman, et al. 2003 - 2008- NPB, PLB, PRD

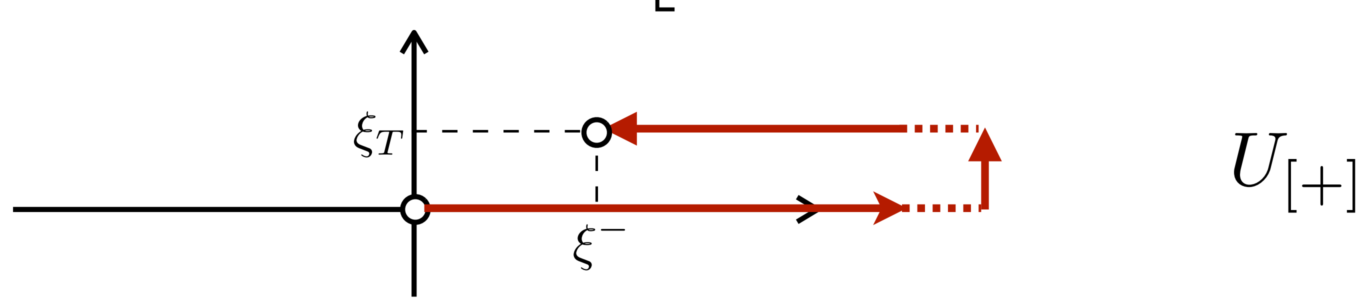
$$\Phi^{[\mathcal{U}[\mathcal{C}]]}(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{2(2\pi)^3} e^{ip\cdot\xi} \langle P|\overline{\psi}(0)\mathcal{U}^{[C]}_{[0,\xi]}\psi(\xi^-, \xi_T)|P\rangle|_{\xi^+=0}$$



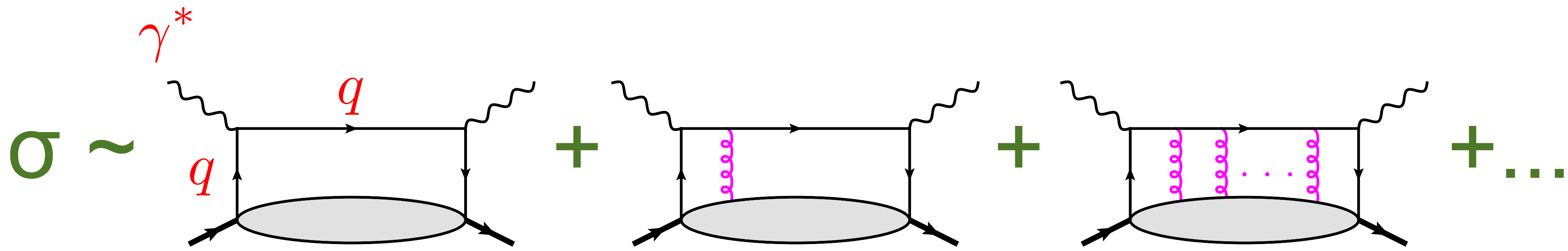


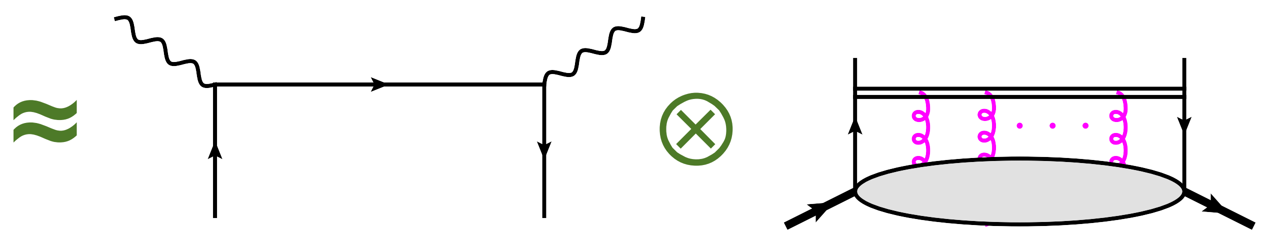
- **The path $[C]$ is fixed by hard subprocess within hadronic process.**

$$W_{\mu\nu}(q, P, S, P_h) = \int d^4p d^4k \delta^4(p + q - k) \mathrm{Tr}\left[\Phi^{\mathcal{U}^{[C]}_{[\infty;\xi]}}(p) H^\dagger_\mu(p, k)\Delta(k)H_\nu(p, k)\right]$$



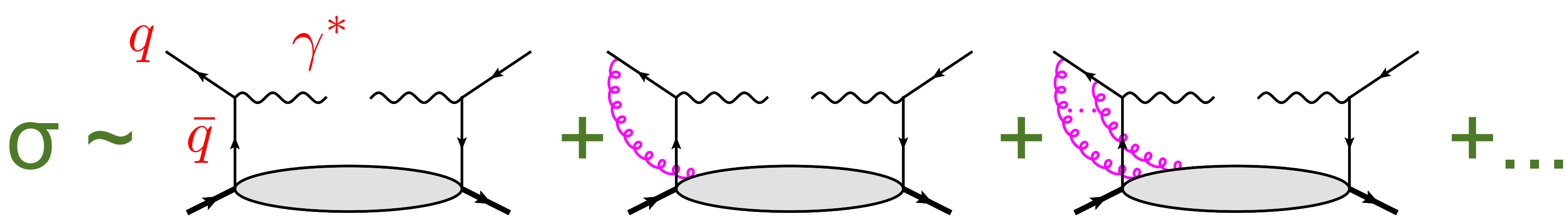
Gauge link determined re-summing leading gluon interactions btwn soft and hard Process Dependence break down of Universality

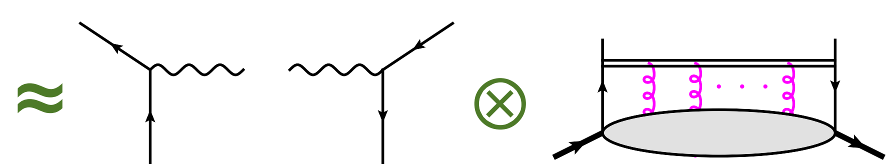




PDFs with SIDIS gauge link

$$\mathcal{P}\, e^{ig\int_y^{\infty} d\lambda\cdot A(\lambda)}$$





PDFs with DY gauge link

$$\mathcal{P}\, e^{ig\int_y^{-\infty} d\lambda\cdot A(\lambda)}$$

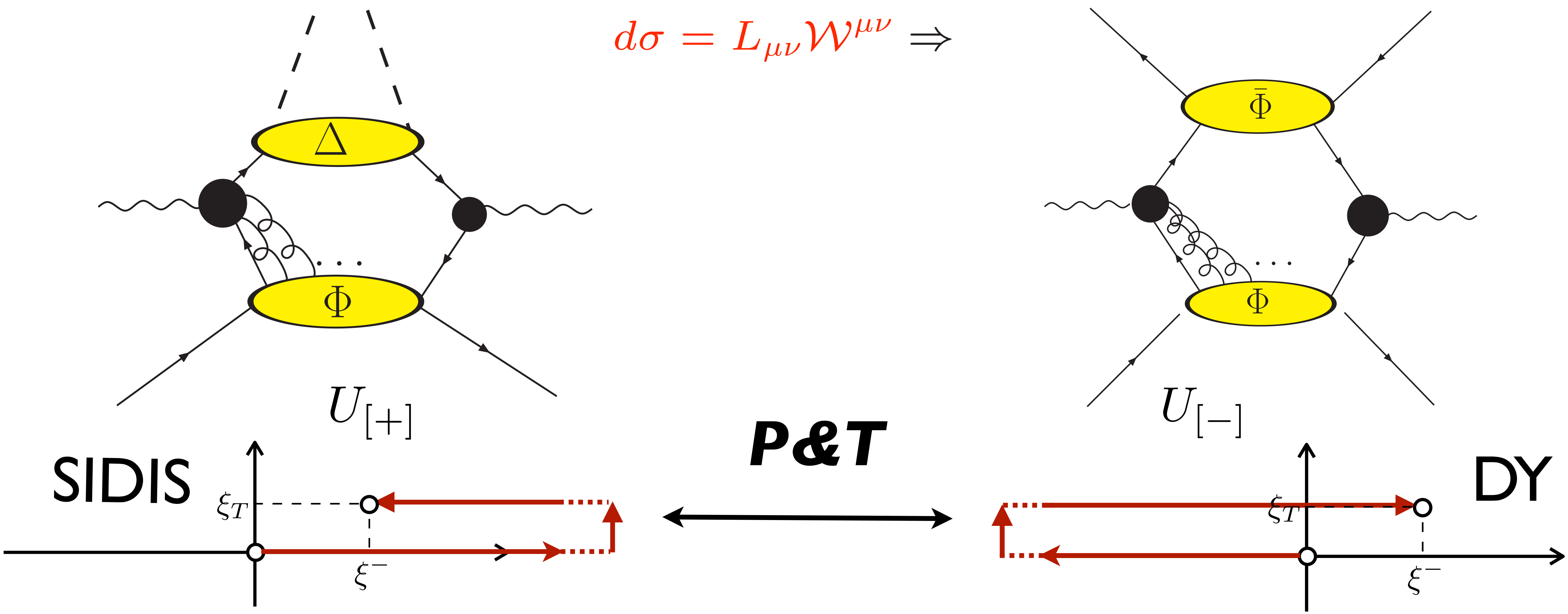
$$f_{1T_{sidis}}^{\perp}(x,k_T) = -f_{1T_{DY}}^{\perp}(x,k_T)$$

"Generalized Universality" Fund. Prediction of QCD Factorization

$$f_{1T}^{\perp}{}_{sidis}(x, k_T) = -f_{1T}^{\perp}{}_{DY}(x, k_T)$$

EIC conjunction with DY exp. E906-Fermi, A_NDY, Compass

Process Dependence, Collins PLB 02, Brodsky et al. NPB 02, Boer Mulders Pijlman Bomhoff 03, 04 ...

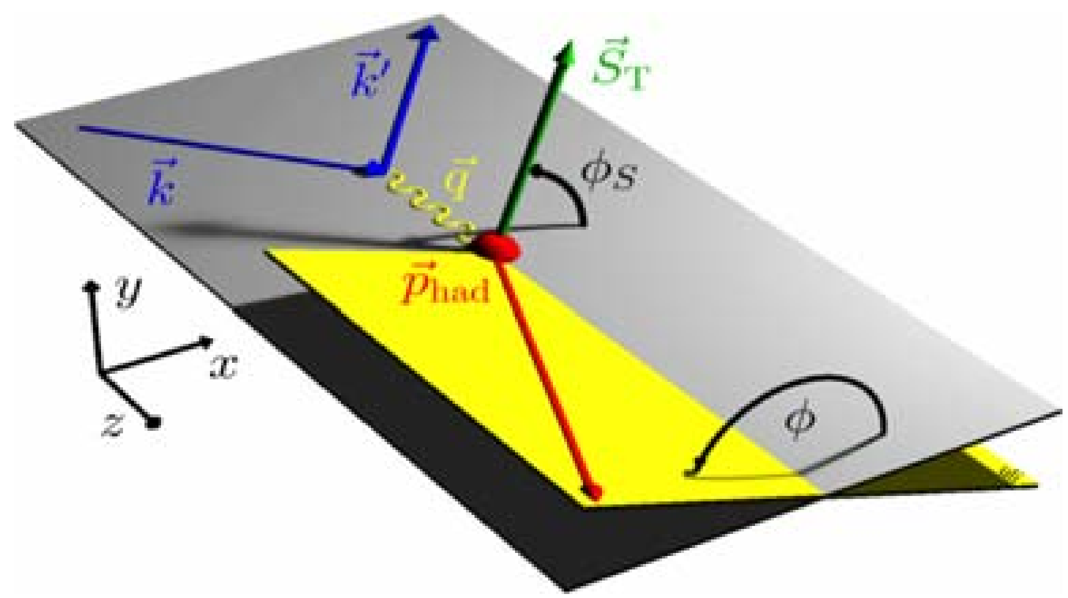


$$\Phi^{[+]*}(x, p_T) = i\gamma^1\gamma^3\Phi^{[-]}(x, p_T) i\gamma^1\gamma^3$$

TSSA in SIDIS-CS expressed thru structure functions

$$\frac{d^6\sigma}{d\Phi} \sim \left\{ F_{UU,T} \cdots + \ldots |S_\perp| \left(\sin(\phi_h - \phi_S) F_{UT,T}^{\sin(\phi_h-\phi_S)} + \sin(\phi_h + \phi_S) \varepsilon F_{UT}^{\sin(\phi_h+\phi_S)} \ldots \right) \ldots \right\}$$

Kotzinian NPB 95,
Mulders Tangermann NPB 96,
Boer & Mulders PRD 97
Bacchetta et al JHEP 08



Spin asymmetry projected from cross section

$$A_{XY}^{\mathcal{F}} \equiv 2 \frac{\int d\phi_h\, d\phi_S\, \mathcal{F}(\phi_h, \phi_S)\ \left(d\sigma^\uparrow - d\sigma^\downarrow\right)}{\int d\phi_h d\phi_S\, (d\sigma^\uparrow + d\sigma^\downarrow)},$$

XY-polarization e.g.

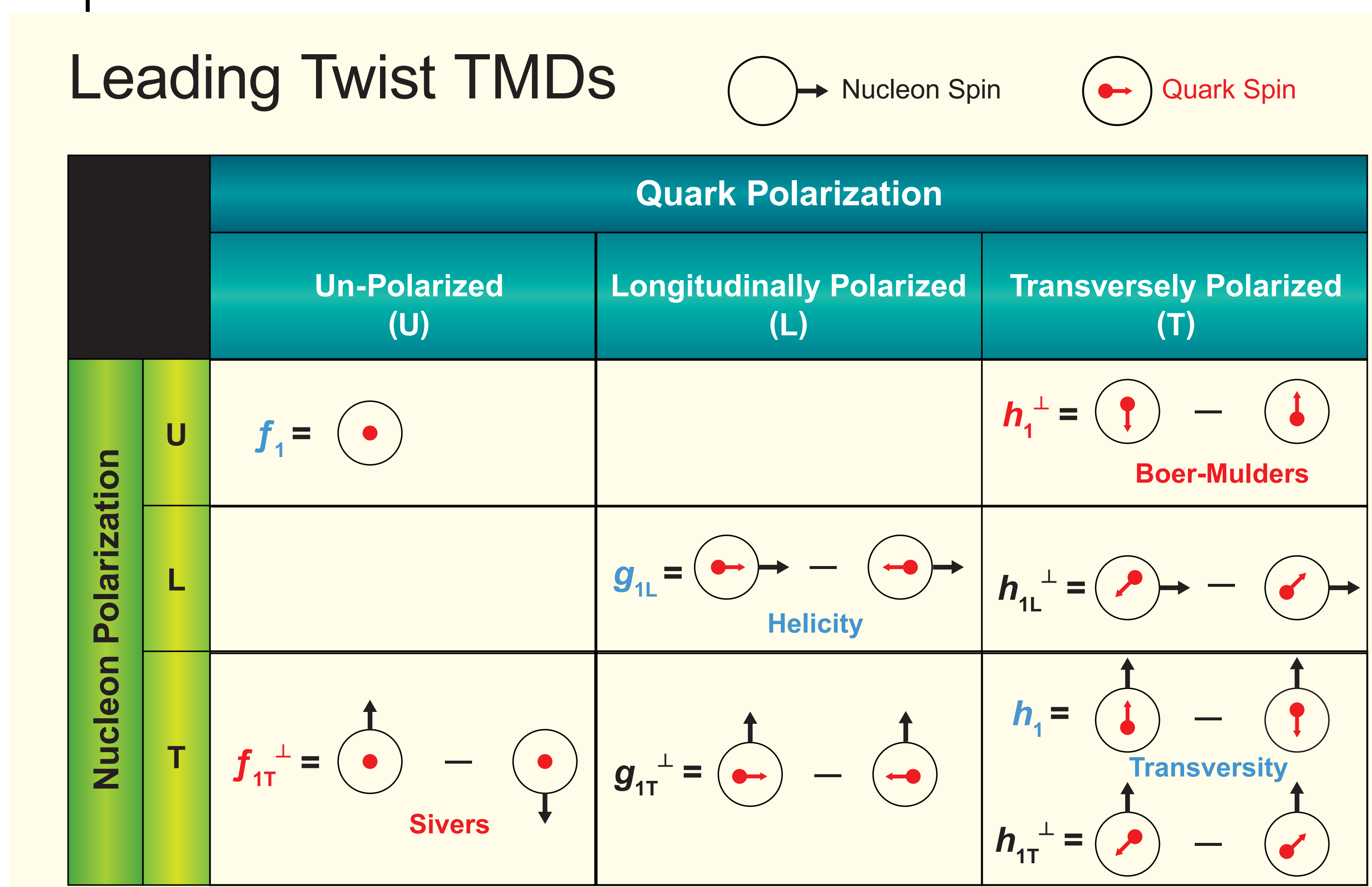
$$\mathcal{F}(\phi_h, \phi_S) = \sin(\phi_h - \phi_S).$$

Partonic picture Structure Functions momentum CONVOLUTION

$$\mathcal{C}\left[w\,f\,D\right] = x\sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\right) w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

$$F_{UU,T} = \mathcal{C}\left[f_1 D_1\right], \qquad F_{UT,T}^{\sin(\phi_h - \phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right],$$

$$F_{UT}^{\sin(\phi_h + \phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right],$$



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$$F_{UU,T} = \mathcal{C}\big[f_1 D_1\big],$$

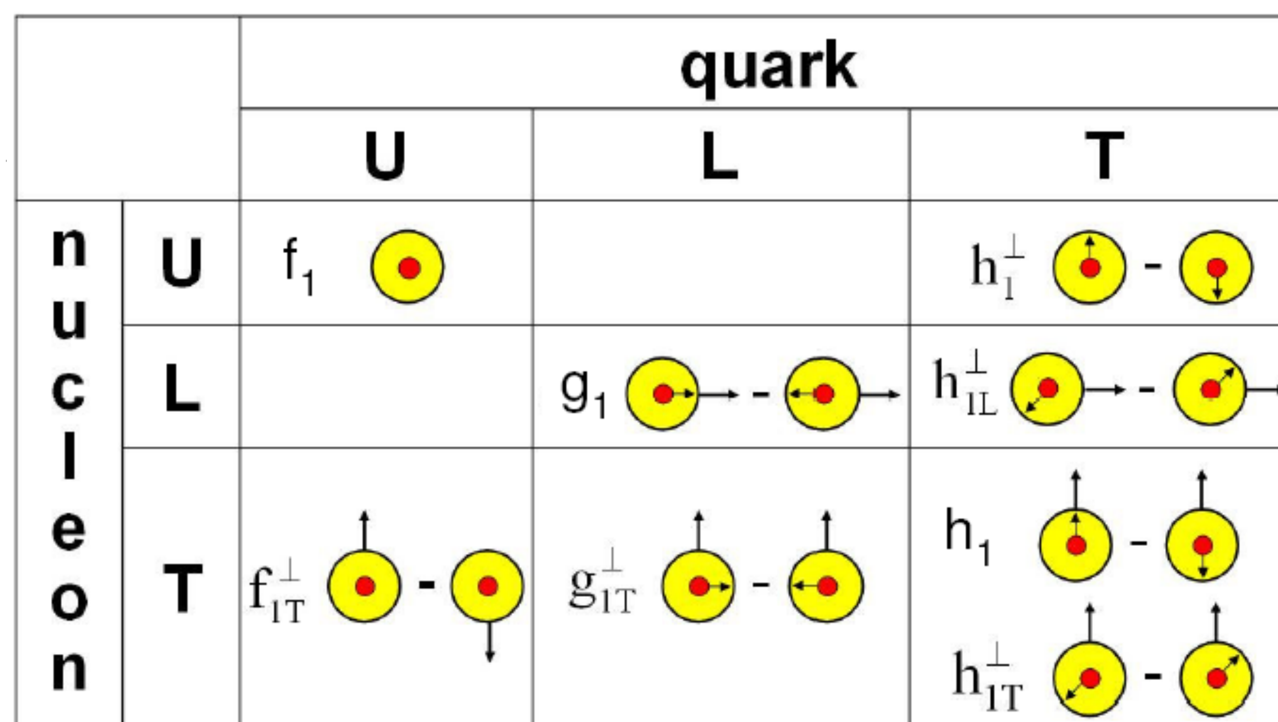
$$F_{LL} = \mathcal{C}\big[g_{1L} D_1\big],$$

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right],$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right],$$

$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_{1L}^{\perp} H_1^{\perp}\right],$$

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_1^{\perp} H_1^{\perp}\right]$$



Weighted asymmetries *Model independent Deconvoltuion* of CS in terms of moments of TMDs

Kotzinian, Mulders PLB 97, Boer, Mulders PRD 98

$$A_{UT,T}^{w_1 \sin(\phi_h - \phi_S)} = 2\,\frac{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}| d\phi_h\, d\phi_S\, w_1(|\boldsymbol{P}_{h\perp}|)\ \sin(\phi_h - \phi_S)\ \{d\sigma(\phi_h, \phi_S) - d\sigma(\phi_h, \phi_S + \pi)\}}{\int d|\boldsymbol{P}_{h\perp}|\, d\phi_h\, |\boldsymbol{P}_{h\perp}| d\phi_S\, w_0(|\boldsymbol{P}_{h\perp}|)\ \{d\sigma(\phi_h, \phi_S) + d\sigma(\phi_h, \phi_S + \pi)\}},$$

e.g. $\mathcal{W}_{\text{Sivers}} = \frac{|\boldsymbol{P}_{h\perp}|}{zM} \sin(\phi_h - \phi_S)$

$$A_{UT}^{\frac{|\boldsymbol{P}_{h\perp}|}{z_h M} \sin(\phi_h - \phi_s)} = -2\,\frac{\sum_a e_a^2\ f_{1T}^{\perp(1)}(x)\ D_1^{a(0)}(z)}{\sum_a e_a^2\ f_1^{a(0)}(x)\ D_1^{a(0)}(z)}$$

Undefined w/o regularization to subtract infinite contribution at large transverse momentum

Bacchetta et al. JHEP 08

Comments

- Propose generalize Bessel Weights-"BW"
- BW procedure has advantages
  - ★ Structure functions become simple product $\mathcal{P}[\quad]$ rather than convolution $\mathcal{C}[\quad]$
  - ★ CS has simple S/T interpretation as a multipole expansion in terms of $\boldsymbol{P}_{h\perp}$ conjugate to b_T $[\mathrm{GeV}^{-1}]$
  - ★ The usefulness of Fourier-Bessel transforms in studying the factorization as well as the scale dependence of transverse momentum dependent cross section has been known for quite sometime.
  - ★ Is the natural language for TMD Evolution
  - ★ Collins Soper (81), Collins, Soper, Sterman NPB 85, Boer NPB 2001, 2009, 2013,Ji,Ma,Yuan (04),Collins-Cambridge University Press(11), Aybat Rogers PRD (11), Abyat, Collins, Qiu, Rogers arXiv(11), Aybat, Prokudin, Rogers PRL (11), Anselminio, Bolglione, Melis PRD (12),

Further Comments

- Introduces a free parameter $\mathcal{B}_T\ [\mathrm{GeV}^{-1}]$ Fourier conjugate to $\boldsymbol{P}_{h\perp}$
- Provides a regularization of infinite contributions at lg. transverse momentum when $\mathcal{B}_T^2$ is non-zero for moments
- Study scale changes in TMD picture, soft factor eliminated from Sivers and ....weighted asymmetries
- Cancellation of perturbative Sudakov Broadening (new)-mentioned by D. Boer NPB 1999, 2007
- Cancellation hard cross section-new observation (new)
- Asymmetry less sensitive scale changes-observable for different scales.... could be useful for EIC

Advantages of Bessel Weighting

1.“Deconvolution”-CS-struct fncts simple product “$\mathcal{P}$“

$$W^{\mu\nu}(\boldsymbol{P}_{h\perp}) \equiv \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2}\, e^{-i\boldsymbol{b}_T\cdot\boldsymbol{P}_{h\perp}}\; \tilde{W}^{\mu\nu}(\boldsymbol{b}_T)\,,$$

$$\tilde{\Phi}_{ij}(x, z\boldsymbol{b}_T) \equiv \int d^2\boldsymbol{p}_T\, e^{iz\boldsymbol{b}_T\cdot\boldsymbol{p}_T}\; \Phi_{ij}(x,\boldsymbol{p}_T)$$

$$\tilde{\Delta}_{ij}(z, \boldsymbol{b}_T) \equiv \int d^2\boldsymbol{K}_T\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{K}_T}\; \Delta_{ij}(z,\boldsymbol{K}_T)$$

$$\frac{d\sigma}{dx_B\, dy\, d\psi\, dz_h\, d\phi_h\, |\boldsymbol{P}_{h\perp}| d|\boldsymbol{P}_{h\perp}|} = \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{-i\boldsymbol{b}_T\cdot\boldsymbol{P}_{h\perp}} \left\{ \frac{\alpha^2}{x_B\, y\, Q^2}\, \frac{y^2}{(1-\varepsilon)} \left(1+\frac{\gamma^2}{2x_B}\right) L_{\mu\nu}\tilde{W}^{\mu\nu} \right\}.$$

$$2M\tilde{W}^{\mu\nu} = \sum_a e_a^2\, \mathrm{Tr}\left(\tilde{\Phi}(x, z\boldsymbol{b}_T)\gamma^\mu \tilde{\Delta}(z,\boldsymbol{b}_T)\gamma^\nu \right).$$

1."Deconvolution"-Sivers struct fnct simple product "$\mathcal{P}$"

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right],$$

"dipole structure"

$$\mathcal{C}\left[w\, f\, D\right] = x \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\right) w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

★ $$F_{UT,T}^{\sin(\phi_h-\phi_S)} = -x_B \sum_a e_a^2 \int \frac{d|\boldsymbol{b}_T|}{(2\pi)} |\boldsymbol{b}_T|^2 J_1(|\boldsymbol{b}_T|\,|\boldsymbol{P}_{h\perp}|) M z\ \tilde{f}_{1T}^{\perp a(1)}(x, z^2\boldsymbol{b}_T^2)\, \tilde{D}_1^a(z, \boldsymbol{b}_T^2).$$

$\tilde{f}_1,\ \tilde{f}_{1T}^{\perp(1)}$, and $\tilde{D}_1$ are Fourier Transf. of TMDs/FFs and finite

- Transversity and Collins

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{C}\left[\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)\left(\boldsymbol{p}_T\cdot\boldsymbol{k}_T\right)+\boldsymbol{p}_T^2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)-4\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)^2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)}{2M^2M_h}\,h_{1T}^{\perp}H_1^{\perp}\right]$$

write out in cylindrical polar--
traceless tensor irreducible
tensor no mixture of Bessels "J_3"

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = x_B\sum_a e_a^2\int\frac{d|\boldsymbol{b}_T|}{(2\pi)}|\boldsymbol{b}_T|^4\,J_3(|\boldsymbol{b}_T|\,|\boldsymbol{P}_{h\perp}|)\,\frac{M^2M_hz^3}{4}\,\tilde{h}_{1T}^{\perp a(2)}(x,z^2\boldsymbol{b}_T^2)\,\tilde{H}_1^{\perp a(1)}(z,\boldsymbol{b}_T^2)\,.$$

Simple product " $\mathcal{P}$ "

★ CS has simpler ***S/T*** interpretation--multipole expansion in terms of $b_T\,[\mathrm{GeV}^{-1}]$ conjugate to $\boldsymbol{P}_{h\perp}$

$$
\begin{aligned}
&\frac{d\sigma}{dx_B\,dy\,d\phi_S\,dz_h\,d\phi_h\,|\boldsymbol{P}_{h\perp}|d|\boldsymbol{P}_{h\perp}|} = \\
&\frac{\alpha^2}{x_B y Q^2}\frac{y^2}{(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)\int\frac{d|\boldsymbol{b}_T|}{(2\pi)}|\boldsymbol{b}_T|\Big\{J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU,T}+\varepsilon\,J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU,L} \\
&+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU}^{\cos\phi_h}+\varepsilon\cos(2\phi_h)\,J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU}^{\cos(2\phi_h)} \\
&+\lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LU}^{\sin\phi_h} \\
&+S_\parallel\Big[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UL}^{\sin2\phi_h}\Big] \\
&+S_\parallel\lambda_e\Big[\sqrt{1-\varepsilon^2}\,J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LL}^{\cos\phi_h}\Big] \\
&+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\left(\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,\mathcal{F}_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\quad+\varepsilon\,\sin(\phi_h+\phi_S)\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin(\phi_h+\phi_S)} \\
&\quad+\varepsilon\,\sin(3\phi_h-\phi_S)\,J_3(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\quad+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin\phi_S} \\
&\quad+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin(2\phi_h-\phi_S)}\Big] \\
&+|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LT}^{\cos(\phi_h-\phi_S)} \\
&\quad+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LT}^{\cos\phi_S} \\
&\quad+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}
\end{aligned}
$$

$$\mathcal{F}_{UT}^{\sin(3\phi_h-\phi_S)} = \frac{1}{4}\mathcal{P}[\tilde{h}_{1T}^{\perp(2)}\;\tilde{H}_1^{\perp(1)}]$$

.)

Structure Functions become

$$
\begin{aligned}
\mathcal{F}_{UU,T} &= \mathcal{P}[\tilde{f}_1^{(0)}\ \tilde{D}_1^{(0)}]\,, \\
\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)} &= -\mathcal{P}[\tilde{f}_{1T}^{\perp(1)}\ \tilde{D}_1^{(0)}]\,, \\
\mathcal{F}_{LL} &= \mathcal{P}[\tilde{g}_{1L}^{(0)}\ \tilde{D}_1^{(0)}]\,, \\
\mathcal{F}_{LT}^{\cos(\phi_h-\phi_s)} &= \mathcal{P}[\tilde{g}_{1T}^{(1)}\ \tilde{D}_1^{(0)}]\,, \\
\mathcal{F}_{UT}^{\sin(\phi_h+\phi_S)} &= \mathcal{P}[\tilde{h}_1^{(0)}\ \tilde{H}_1^{\perp(1)}]\,, \\
\mathcal{F}_{UU}^{\cos(2\phi_h)} &= \mathcal{P}[\tilde{h}_1^{\perp(1)}\ \tilde{H}_1^{\perp(1)}]\,, \\
\mathcal{F}_{UL}^{\sin(2\phi_h)} &= \mathcal{P}[\tilde{h}_{1L}^{\perp(1)}\ \tilde{H}_1^{\perp(1)}]\,, \\
\mathcal{F}_{UT}^{\sin(3\phi_h-\phi_S)} &= \frac{1}{4}\mathcal{P}[\tilde{h}_{1T}^{\perp(2)}\ \tilde{H}_1^{\perp(1)}].
\end{aligned}
$$

$$
\mathcal{P}[\tilde{f}^{(n)}\tilde{D}^{(m)}] \equiv x_B \sum e_a^2\,(zM|\boldsymbol{b}_T|)^n\,(zM_h|\boldsymbol{b}_T|)^m\,\tilde{f}^{a(n)}(x, z^2\boldsymbol{b}_T^2)\,\tilde{D}^{a(m)}(z, \boldsymbol{b}_T^2)\,.
$$

Correlator w/ explicit *spin orbit* correlations

$$
\begin{aligned}
\tilde{\Phi}^{[\gamma^+]}(x,\boldsymbol{b}_T) &= \tilde{f}_1(x,\boldsymbol{b}_T^2) - i\,\epsilon_T^{\rho\sigma} b_{T\rho} S_{T\sigma}\, M\, \tilde{f}_{1T}^{\perp(1)}(x,\boldsymbol{b}_T^2)\,, \\
\tilde{\Phi}^{[\gamma^+\gamma^5]}(x,\boldsymbol{b}_T) &= S_L\,\tilde{g}_{1L}(x,\boldsymbol{b}_T^2) + i\,\boldsymbol{b}_T\cdot\boldsymbol{S}_T M\,\tilde{g}_{1T}^{(1)}(x,\boldsymbol{b}_T^2)\,, \\
\tilde{\Phi}^{[i\sigma^{\alpha+}\gamma^5]}(x,\boldsymbol{b}_T) &= S_T^{\alpha}\,\tilde{h}_1(x,\boldsymbol{b}_T^2) + i\,S_L\, b_T^{\alpha} M\,\tilde{h}_{1L}^{\perp(1)}(x,\boldsymbol{b}_T^2) \\
&\quad + \frac{1}{2}\left(b_T^{\alpha} b_T^{\rho} + \frac{1}{2}\,\boldsymbol{b}_T^2\, g_T^{\alpha\rho}\right) M^2\, S_{T\rho}\tilde{h}_{1T}^{\perp(2)}(x,\boldsymbol{b}_T^2) \\
&\quad - i\,\epsilon_T^{\alpha\rho} b_{T\rho} M \tilde{h}_1^{\perp(1)}(x,\boldsymbol{b}_T^2)\,,
\end{aligned}
$$

TMDs in “config” space--Bessel MOMENTS D. Boer’s talk

a) F.T. SIDIS cross section w/ following Bessel moments

$$\tilde{f}(x, \boldsymbol{b}_T^2) \equiv \int d^2\boldsymbol{p}_T\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{p}_T}\; f(x, \boldsymbol{p}_T^2)$$

$$= 2\pi \int d|\boldsymbol{p}_T||\boldsymbol{p}_T|\; J_0(|\boldsymbol{b}_T||\boldsymbol{p}_T|)\; f^a(x, \boldsymbol{p}_T^2)\ ,$$

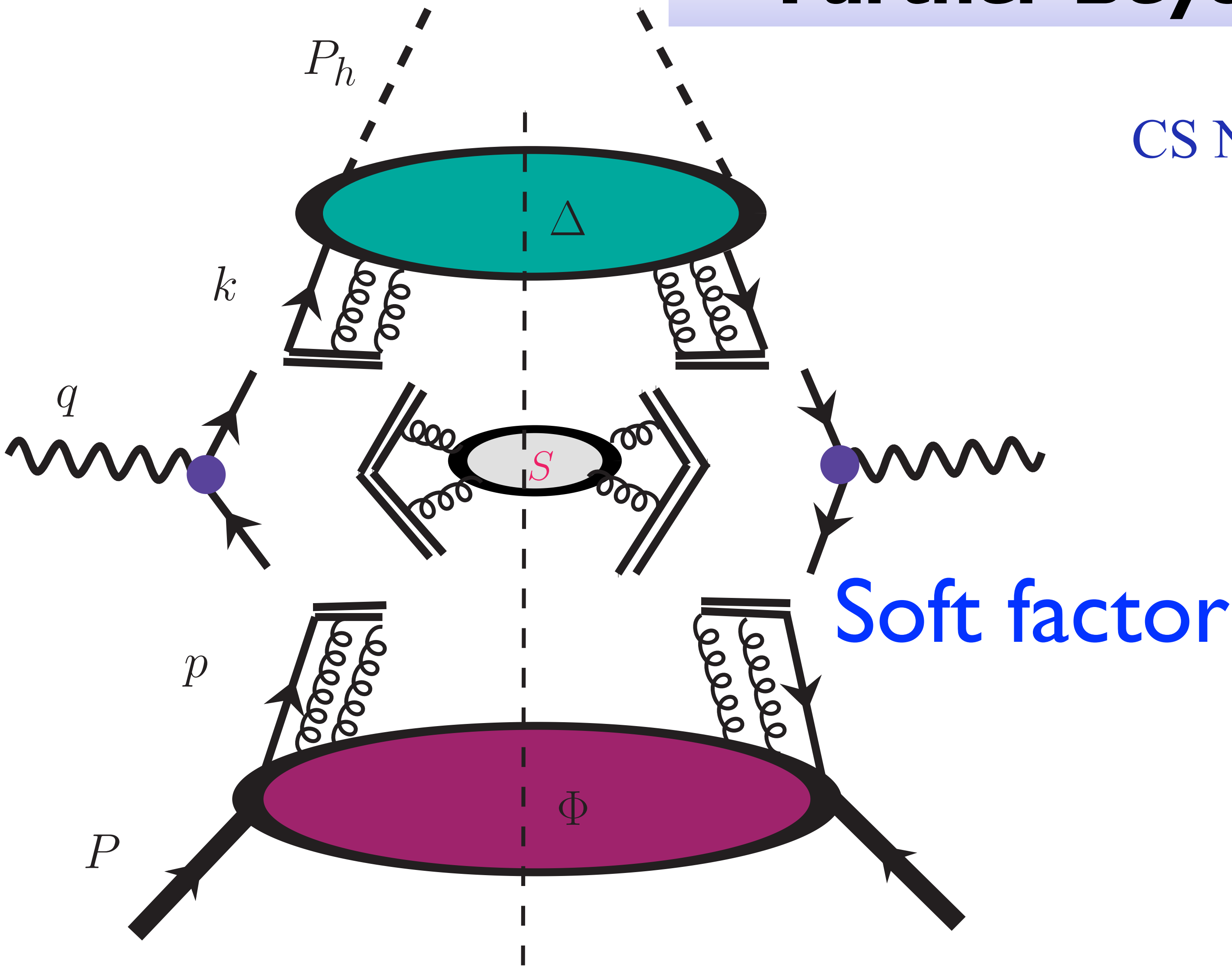
$$\tilde{f}^{(n)}(x, \boldsymbol{b}_T^2) \equiv n!\left(-\frac{2}{M^2}\partial_{\boldsymbol{b}_T^2}\right)^n \tilde{f}(x, \boldsymbol{b}_T^2)$$

$$= \frac{2\pi\; n!}{(M^2)^n}\int d|\boldsymbol{p}_T||\boldsymbol{p}_T|\left(\frac{|\boldsymbol{p}_T|}{|\boldsymbol{b}_T|}\right)^n J_n(|\boldsymbol{b}_T||\boldsymbol{p}_T|)\; f(x, \boldsymbol{p}_T^2)\ ,$$

b) n.b. connection to $\boldsymbol{p}_T$ moments

$$\tilde{f}^{(n)}(x, 0) = \int d^2\boldsymbol{p}_T\left(\frac{\boldsymbol{p}_T^2}{2M^2}\right)^n f(x, \boldsymbol{p}_T^2) \equiv f^{(n)}(x)$$

Further Beyond "tree level" factorization



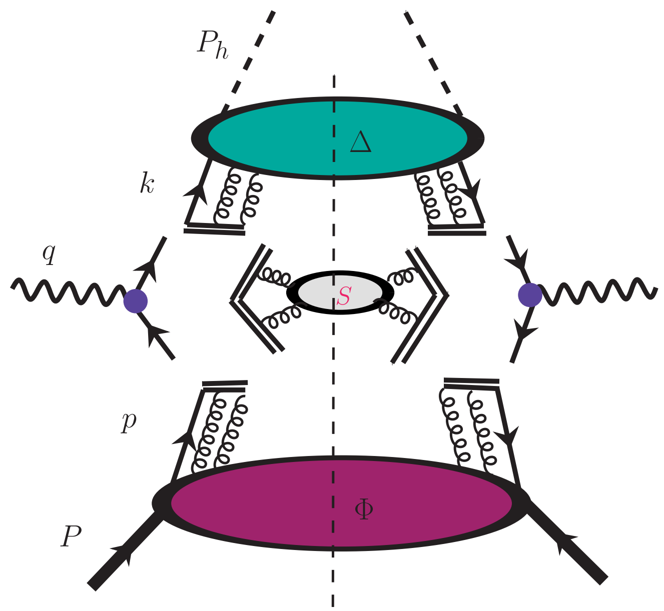
CS NPB 81,CSS NPB 1985 Collins, Hautman PLB 00,
Boer NPB 2001
Idilbi, Ji, Ma, Yuan PRD 05,
Boer NPB 2009
Cherednikov, Karanikas, Stefanis NPB 10,
Collins Oxford Press 2011,
Abyat, Rogers PRD 2011,
Abyat, Collins, Qiu, Rogers PRD 2012 ...
Echevarria,Idilbi, Scimemi JHEP 2012
Boer NPB 2013
Sun & Yuan PRD 2013

- Extra divergences at one loop and higher
- Extra variables needed to regulate
  light-cone, soft & collinear divergences
- Modifies convolution integral introduction of soft & Sudakov factor
- Will show cancels in Bessel weighted asymmetries

Collins Soper NPB 1981, Collins Metz PRL 2004, Ji, Ma, Yuan PRD 2004, Collins 2011, Collins Rogers 2012

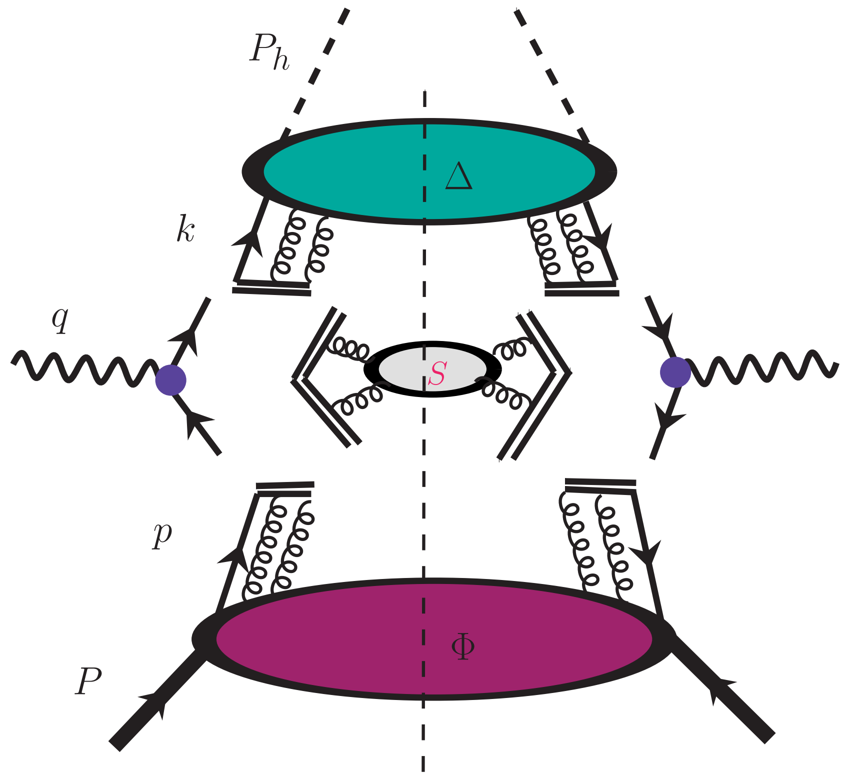
Comments on Soft factor

- Collective effect soft gluons not associated with distribution frag function-factorizes into a matrix of Wilson lines in QCD vacuum
- Subtracts soft divergences from TMD pdf and FF



- Considered to be universal in hard processes
  (Collins Soper 81, ... , Collins & Metz PRL 04, Ji, Ma, Yuan PRD 05)
- At tree level (zeroth order α_s) unity-parton model
- Absent tree level pheno analyses of experimental data
  (e.g. Anselmino et al PRD 05 & 07, Efremov et al PRD 07)
- Potentially, results of analyses can be difficult to compare at different energies **issue for EIC**
- Correct description of energy scale dependence of cross section and asymmetries in TMD picture, soft factor must be included
  (Ji, Ma, Yuan 2004, Collins Oxford Press 2011, Abyat, Collins, Rogers PRD 2011)
- However, possible to consider observables where its affects cancels e.g. weighted asymmetries Boer, LG, Musch, Prokudin JHEP 2011

Momentum space convolution



CS 81, Idilbi, Ji, Ma, Yuan PRD 05 ....

Hard

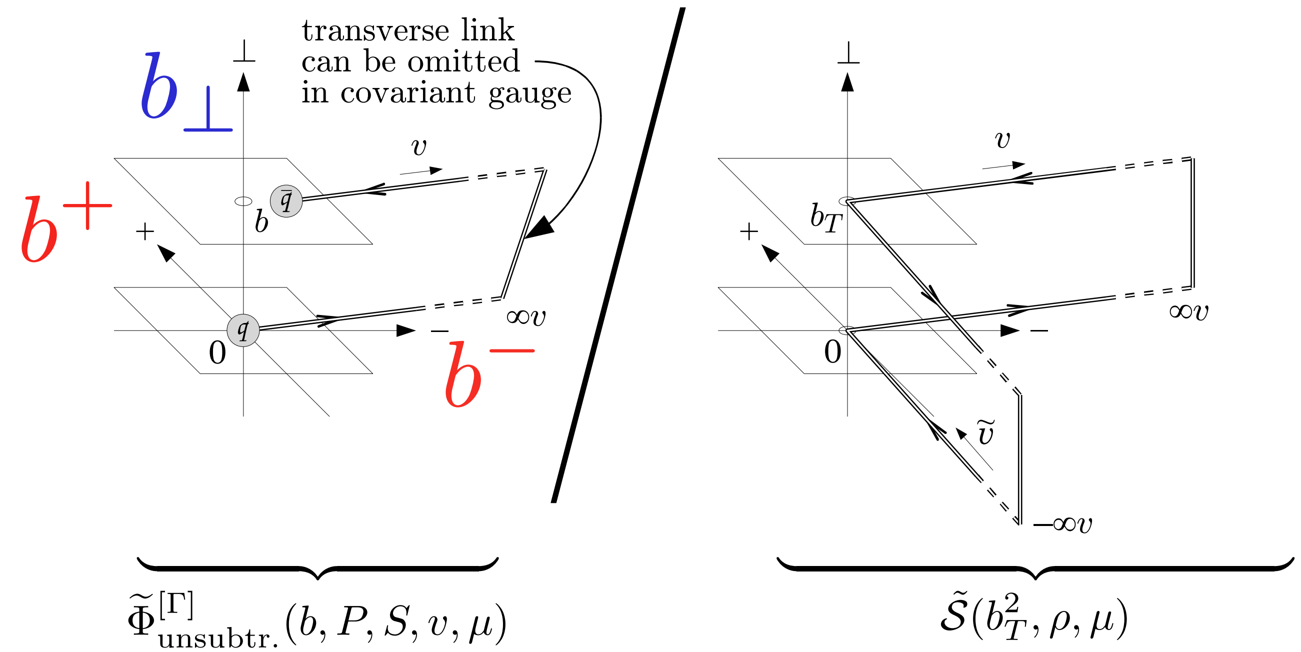
$$\mathcal{C}[H; wfSD] \equiv x_B H(Q^2, \mu^2, \rho) \sum_a e_a^2 \int d^2p_T\, d^2K_T\, d^2\ell_T\, \delta^{(2)}(zp_T + K_T + \ell_T - P_{h\perp})\, w\left(p_T, -\frac{K_T}{z}\right)$$
$$\times f^a(x, p_T^2, \mu^2, x\zeta, \rho)\, S(\ell_T^2, \mu^2, \rho)\, D^a(z, K_T^2, \mu^2, \hat{\zeta}/z, \rho)$$

TMD

Soft

FF

First summarize what we know about correlator off light cone



$v = (v^-, v^+, 0)$ **w/** lightlike directions $n = (1,0,0)$, $\bar{n} = (0,1,0)$.

is slightly off light-cone direction n & $b \cdot v = 0$

Wilson lines starting at infinity running along a direction given by the four-vector v to an endpoint a are denoted $\mathcal{L}_v(\infty; a)$

Direction defined in LI way $\zeta^2 = (2P \cdot v)^2 / v^2$

Direction defined in LI way $\hat{\zeta}^2 = (2P_h \cdot \tilde{v})^2 / \tilde{v}^2$

Angle between v and $\tilde{v}$ $\rho = \sqrt{v^- \tilde{v}^+ / v^+ \tilde{v}^-}$

scales arising from regulating LC div gluon rap. cutoff

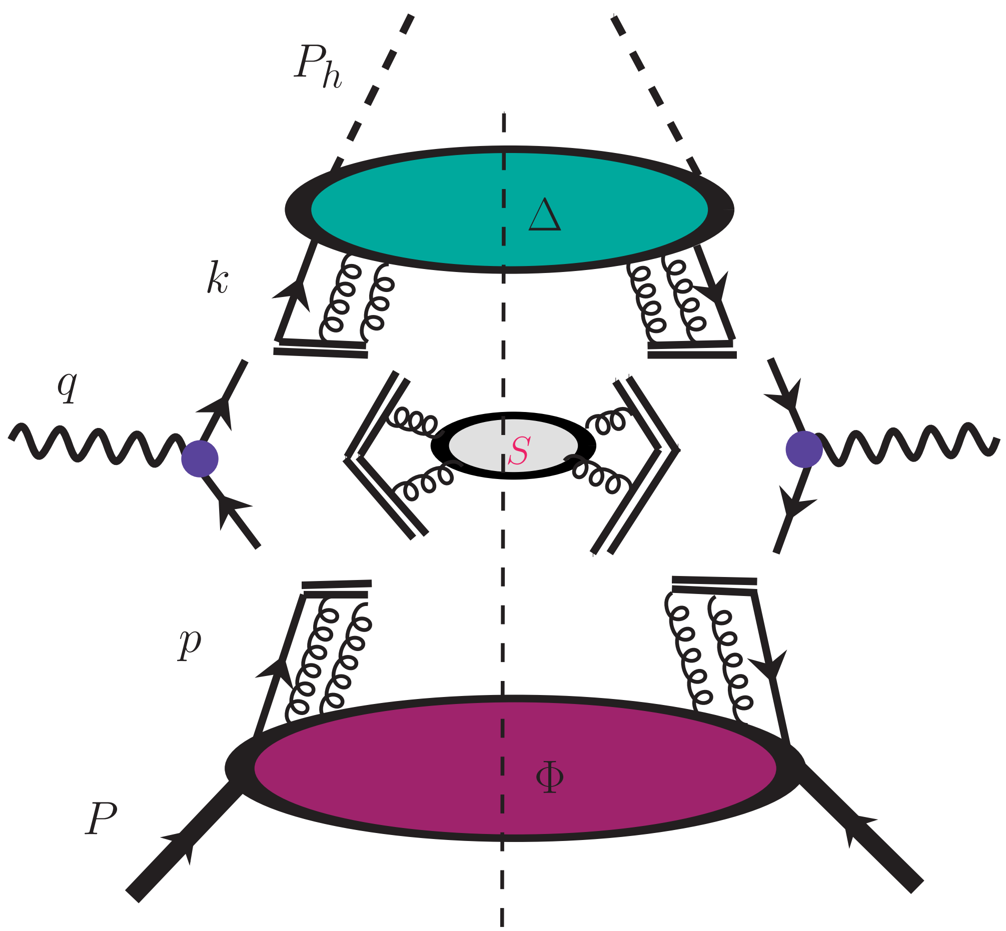
Crucial property of Soft Factor-SIDIS

Soft factor formed from vacuum expt. value of Wilson lines involving both v and $\tilde{v}$ thus depends on relative orientation of directions $\rho = \sqrt{v^- \tilde{v}^+ / v^+ \tilde{v}^-}$

$\tilde{S}^+(\boldsymbol{b}_T, \rho, \mu)$ is invariant under rotations of the $\boldsymbol{b}_T$-vector (provided $b \cdot v = 0$).

Since for TMDS we always consider the case $b^+ = 0$ we have $\mathbf{b}_T^2 = -b^2$

$$\tilde{S}(b^2, \mu^2, \rho) = 1 + \frac{\alpha_s C_F}{2\pi}(2 - \rho^2) \ln\left(\frac{\mu^2 \rho^2}{4}\right)$$



Soft factor deconvoluted in Fourier Bessel rep cross sec.

$\mathcal{P}$ versus $\mathcal{C}$

$$\frac{d\sigma}{dx_B\, dy\, d\phi_S\, dz_h\, d\phi_h\, d|\boldsymbol{P}_{h\perp}|^2} \propto \frac{\alpha^2}{x_B Q^2} \int \frac{d|\boldsymbol{b}_T|}{(2\pi)} |\boldsymbol{b}_T|\, \tilde{S}(\boldsymbol{b}_T^2) \Big\{ \quad \ldots$$

$$+ J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}[\tilde{f}_1\ \tilde{D}_1]$$

Soft factor is

- spin blind
- flavor blind
- factors in $\mathcal{P}$
- Universal

Idilbi,Ji,Ma,Yuan PRD 05

$$+ |\boldsymbol{S}_\perp| \sin(\phi_h - \phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}[\tilde{f}_{1T}^{\perp(1)}\ \tilde{D}_1]$$

$$+\varepsilon \cos(2\phi_h)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}[\tilde{h}_1^{\perp(1)}\ \tilde{H}_1^{\perp(1)}]$$

$$+ \quad \ldots 15 \quad \text{more} \quad \text{structure} \quad \text{functions}$$

Products in terms of " $\boldsymbol{b}_T$ moments "

$$\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)} = H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\mu^2,\rho)\ \tilde{S}^{(+)}(\boldsymbol{b}_T^2,\mu^2,\rho)\ \mathcal{P}[\tilde{f}_{1T}^{(1)}\,\tilde{D}_1^{(0)}] + \tilde{Y}_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\boldsymbol{b}_T^2)\ .$$

$$\mathcal{P}[\tilde{f}^{(n)}\,\tilde{D}^{(m)}] \equiv x_B \sum e_a^2 (zM|\boldsymbol{b}_T|)^n (zM_h|\boldsymbol{b}_T|)^m \tilde{f}^{a(n)}(x, z^2\boldsymbol{b}_T^2, \mu^2, \zeta, \rho)\tilde{D}^{a(m)}(z, \boldsymbol{b}_T^2, \mu^2, \hat{\zeta}, \rho)$$

Bessel Weighting & cancellation of soft factor

Bessel weighting-projecting out Sivers using orthogonality of Bessel Fncts.

$$\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM} = \frac{2\,J_1(|\boldsymbol{P}_{hT}|\mathcal{B}_T)}{zM\mathcal{B}_T}$$

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_S)}(\mathcal{B}_T) =$$

$$2\,\frac{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\,\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_S)\,\left(d\sigma^{\uparrow}-d\sigma^{\downarrow}\right)}{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\;\mathcal{J}_0^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)\;\left(d\sigma^{\uparrow}+d\sigma^{\downarrow}\right)}$$

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) =$$

$$-2\,\frac{\tilde{S}(\mathcal{B}_T^2)\,H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2)\,\sum_a e_a^2\,\tilde{f}_{1T}^{\perp(1)a}(x,z^2\mathcal{B}_T^2)\,\tilde{D}_1^a(z,\mathcal{B}_T^2)}{\tilde{S}(\mathcal{B}_T^2)\,H_{UU,T}(Q^2)\sum_a e_a^2\;\tilde{f}_1^a(x,z^2\mathcal{B}_T^2)\,\tilde{D}_1^a(z,\mathcal{B}_T^2)}$$

Sivers asymmetry with full dependences

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) =$$

$$-2\,\frac{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)\,H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\mu^2,\rho)\,\sum_a e_a^2\,\tilde{f}_{1T}^{\perp(1)a}(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)\,H_{UU,T}(Q^2,\mu^2,\rho)\,\sum_a e_a^2\,\tilde{f}_1^a(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}$$

Circumvents the problem of ill-defined p_T moments

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) \quad =$$

$$-2\,\frac{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)\,H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\mu^2,\rho)\,\sum_a e_a^2\,\tilde{f}_{1T}^{\perp(1)a}(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)\,H_{UU,T}(Q^2,\mu^2,\rho)\,\sum_a e_a^2\,\tilde{f}_1^a(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}$$

Traditional weighted asymmetry recovered but UV divergent

$$\lim_{\mathcal{B}_T\to 0} w_1 = 2J_1(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)/zM\mathcal{B}_T \longrightarrow |\boldsymbol{P}_{h\perp}|/zM$$

$$A_{UT}^{\frac{|\boldsymbol{P}_{h\perp}|}{z_h M}\sin(\phi_h-\phi_s)} = -2\,\frac{\sum_a e_a^2\, f_{1T}^{\perp(1)}(x)\ D_1^{a(0)}(z)}{\sum_a e_a^2\, f_1^{a(0)}(x)\ D_1^{a(0)}(z)}$$

Bacchetta et al. JHEP 08

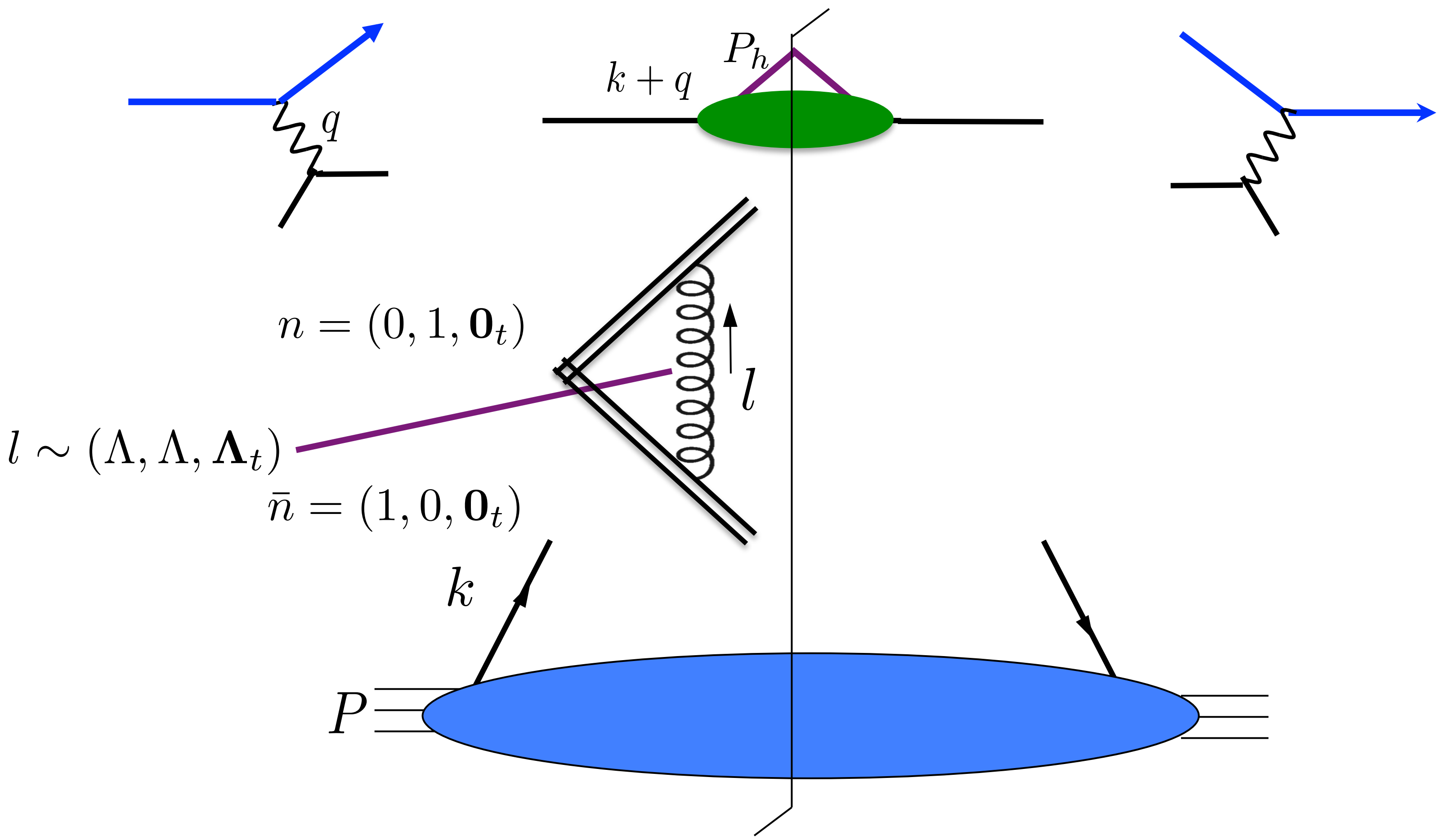
undefined w/o regularization

Is this General ?

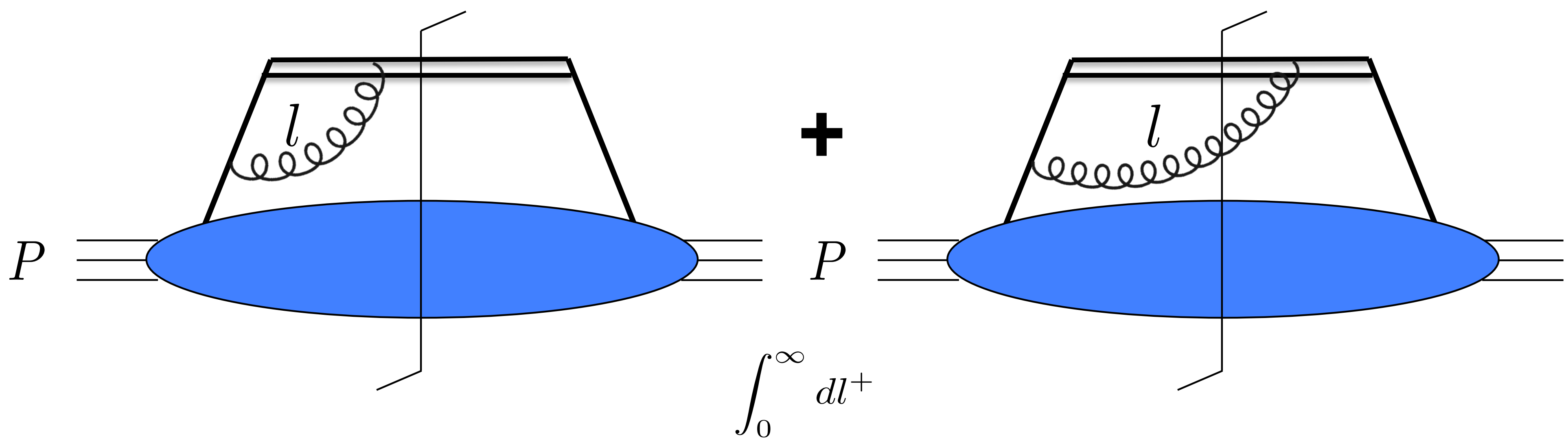
How does this emerge in CSS + JCC 2011

TMD Factorization & treatment of LC divergences- Collins 2011, Aybat & Rogers 2011 PRD

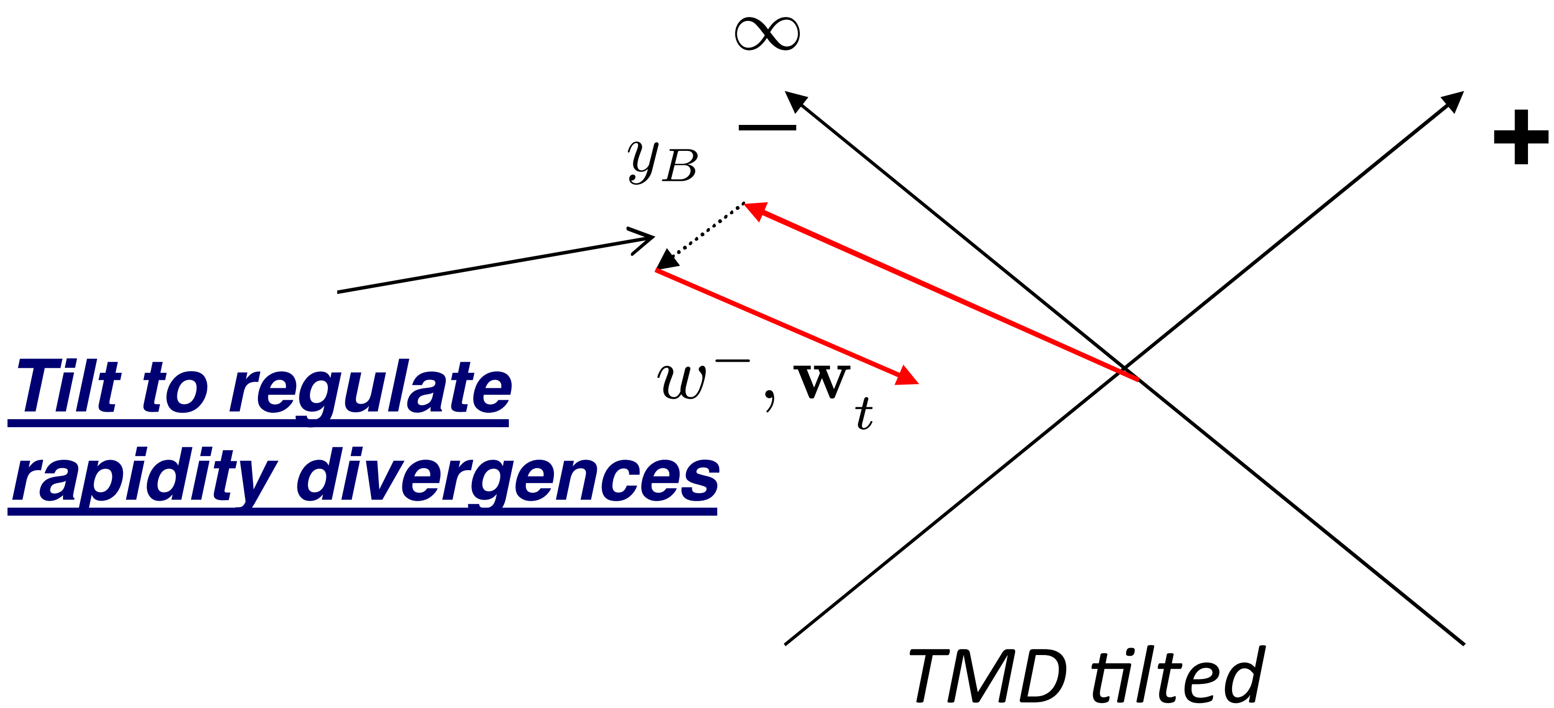
Soft Gluons



Dealing w/ light-cone divergences

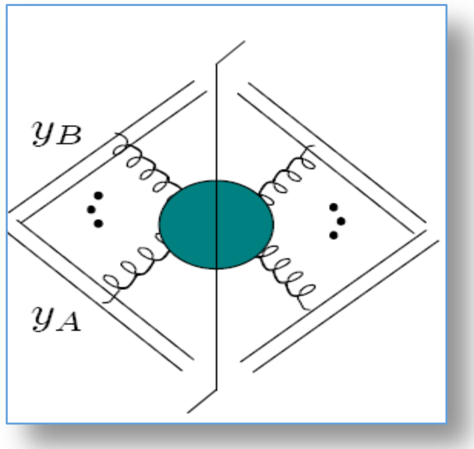


- Divergent contribution at $l^+ = 0$.
- Cancelation in the integral over all l_t.
- What if we don't integrate?


∞
$-$
$+$
y_B
$w^-, \mathbf{w}_t$
Tilt to regulate rapidity divergences
TMD tilted

Again .... Emergence of Soft Factor in CS

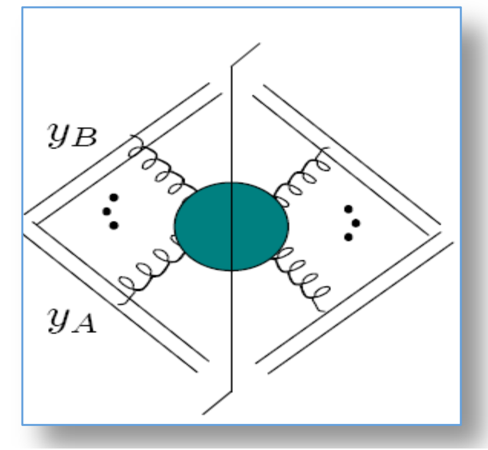
- Lightlike Wilson lines
  - Infinite rapidity QCD radiation in the wrong direction.
  - In soft factor/fragmentation function too.



- Finite rapidity Wilson lines
  - Regulate rapidity of extra gluons.

Emergence of Soft Factor in CS

$$d\sigma = |\mathcal{H}|^2 \, \frac{\tilde{F}_1^{\rm unsub}(y_1 - (-\infty)) \times \tilde{F}_2^{\rm unsub}(+\infty - y_2)}{\tilde{S}(+\infty, -\infty)}$$



! PDFs are still mixed not real factorization !

Separate soft part:

$$d\sigma = |\mathcal{H}|^2 \, \frac{F_1^{\rm unsub}(y_1 - (-\infty))}{\sqrt{\tilde{S}(+\infty, -\infty)}} \times \frac{\tilde{F}_2^{\rm unsub}(+\infty - y_2)}{\sqrt{\tilde{S}(+\infty, -\infty)}}.$$

This is done to both

1) cancel LC divergences and
2) separate "right & left" movers i.e. factorize

Start with only the hard part factorized:

$$d\sigma = |\mathcal{H}|^2 \, \frac{\tilde{F}_1^{\rm unsub}(y_1 - (-\infty)) \times \tilde{F}_2^{\rm unsub}(+\infty - y_2)}{\tilde{S}(+\infty, -\infty)}$$

$$d\sigma = |\mathcal{H}|^2 \, \left\{ F_1^{\rm unsub}(y_1 - (-\infty)) \sqrt{\frac{\tilde{S}(+\infty, y_s)}{\tilde{S}(+\infty, -\infty)\tilde{S}(y_s, -\infty)}} \right\} \times \left\{ \tilde{F}_2^{\rm unsub}(+\infty - y_2) \sqrt{\frac{\tilde{S}(y_s, -\infty)}{\tilde{S}(+\infty, -\infty)\tilde{S}(+\infty, y_s)}} \right\}$$

Separately Well-defined

CS + JCC factorization

$$\mathcal{F}_{UU}(x,z,b,Q^2) = \sum_a \tilde{f}_1^a(x,\boldsymbol{b}^2,\mu^2,\zeta_F)\tilde{D}_1^a(z_h,\boldsymbol{b}^2,\mu^2,\zeta_D)H_{UU}(Q^2,\mu^2)$$

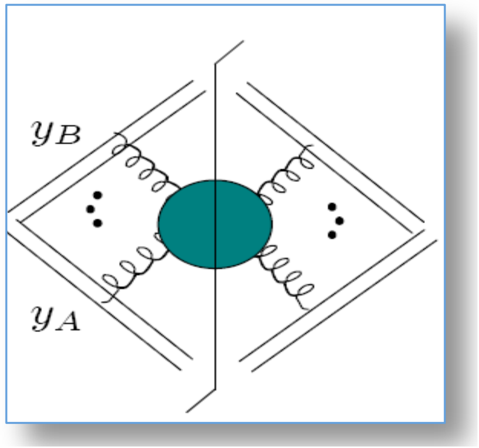
$$\mathcal{F}_{UT}(x,z,b,Q^2) = \sum_a \tilde{f}_{1T}^{\perp\, a(1)}(x,\boldsymbol{b}^2,\zeta_F)\tilde{D}_1^a(z_h,\boldsymbol{b}^2,\zeta_D)H_{UT}(Q^2,\mu^2)$$

$$H_{UT} = H_{UU} = \frac{\alpha_s}{2\pi}C_F\left(3\ln\frac{Q^2}{\mu^2} + \ln^2\frac{Q^2}{\mu^2} + \frac{\pi^2}{2} - 8\right)$$

TMD Evolution…CSS + JCC 2011

Collins-Soper Equation:

$$-\quad \frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$



$$\tilde{K}(b_T; \mu) = \frac{1}{2} \frac{\partial}{\partial y_n} \ln \frac{\tilde{S}(b_T; y_n, -\infty)}{\tilde{S}(b_T; +\infty, y_n)}$$

now effect of
SF in evolution
kernal

along with .... RGE

$$\frac{d\tilde{K}}{d\ln\mu} = -\gamma_K(g(\mu))$$

$$\frac{d\ln\tilde{F}(x,b_T;\mu,\zeta)}{d\ln\mu} = -\gamma_F(g(\mu);\zeta/\mu^2)$$

.... and RGE

Solve this & RGE equation to obtain Evolution kernal

$$\gamma_F(g(\mu);\zeta_F/\mu^2) = \gamma_F(g(\mu);1) - \frac{1}{2}\gamma_K(g(\mu))\ln\frac{\zeta_F}{\mu^2}.$$

Large values of b_T is nonperturbative. Follow CSSS to separate the perturbative and nonperturbative parts of K.. we define

$$\mathbf{b}_* = \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}, \qquad \mu_b = \frac{C_1}{b_*}.$$

$$\tilde{K}(b_T;\mu) = \tilde{K}(b_*;\mu_b) - \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \gamma_K(g(\mu')) - g_K(b_T)$$

Here C_1 is a fixed numerical coefficient and $b_{\max}$ is chosen to keep b_* in the perturbative region. In the fits to unpolarized Drell-Yan, the values chosen were $b_{\max} = 0.5\ \text{GeV}^{-1}$ in [33], and $b_{\max} = 1.5\ \text{GeV}^{-1}$ in [34]. Next we write

$$
\tilde{f}\big(x, b^2; \zeta, Q\big) = \tilde{f}\big(x, b^2; Q_0^2, Q_0\big) \exp\Bigg\{ \ln\frac{\sqrt{\zeta_F}}{Q_0} \tilde{K}(b_*; \mu_b) + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \bigg[\gamma_F(g(\mu'); 1) - \ln\frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \bigg] + \int_{\mu_0}^{\mu_b} \frac{d\mu'}{\mu'} \ln\frac{\sqrt{\zeta_F}}{Q_0} \gamma_K(g(\mu')) - g_K(b_T) \ln\frac{\sqrt{\zeta_F}}{Q_0} \Bigg\}.
$$

& similar for fragmentation function

Note Correlator in b-space

$$\tilde{\Phi}^{[\gamma^+]}(x,\boldsymbol{b}_T) \; = \; \tilde{f}_1(x,\boldsymbol{b}_T^2) - i\,\epsilon_T^{\rho\sigma} b_{T\rho} S_{T\sigma}\, M\,\tilde{f}_{1T}^{\perp(1)}(x,\boldsymbol{b}_T^2)$$

unpolarized and Sivers evolve in same way !!!

CS resummation 81

When $\Lambda^2_{QCD} \ll P^2_{h\perp} \ll Q^2$ get large double logs must re-sum double logs... solution to CS equation CS 81

$$\mathcal{F}_{UT}(x,z,b,Q^2) = \sum_a \tilde{f}_{1T}^{\perp\, a(1)}(x,\boldsymbol{b}^2,\zeta_F)\tilde{D}_1^a(z_h,\boldsymbol{b}^2,\zeta_D)H_{UT}(Q^2,\mu^2)$$

$$\tilde{f}_1^a(x,b^2;\zeta_F,\mu)\,\tilde{D}_1^b(z,b^2;\zeta_D,\mu) = e^{-S(b,Q,Q_0)}\,\tilde{f}_1^a(x,b^2;Q_0^2,Q_0)\,\tilde{D}_1^b(z,b^2;Q_0^2,Q_0)$$

→ See Boer NPB 2013

Leading Q^2 double log

CS NPB 81 Boer NPB 2001, 2009, Idiblim Ji Ma Yuan PRD 2004 ....Bowen, Kang Yuan PRL 2012

Non perturbative Sudakov contribution must be fit Collins & Soper 81

$$W_{UT}(b,Q,x,z) = e^{-S^{pert}(b_*,Q)} e^{-S^{NP}_{UT}(b,Q,x,z)} \qquad b_* = \frac{b}{\sqrt{1+(b/b_{max})^2}}$$

$$e^{-S^{NP}_{UT}}(b,Q,x,z) = \exp\left\{-\left[g_1(x,b)+g_2(z,b)+g_3(b)\ln\left(\frac{Q}{2Q_0}\right)\right]\right\}_{UT}$$

$$\left\{g_i\right\} \to 0 \quad \text{as} \quad \text{b} \to 0 \quad \text{perturbative}$$

Further Cancellation of Sudakov and hard CS

When $\Lambda_{QCD}^2 \ll P_h^2 \ll Q^2$ get large DL and ...

$$\mathcal{A}_{UT}(x,z,b,Q^2) = \frac{\tilde{f}_{1T}^{\perp(1)}(x, z^2\boldsymbol{b}^2, \mu_0^2, Q_0)\tilde{D}_1(z_h, \boldsymbol{b}^2, \mu_0^2, Q_0)\tilde{H}_{UT}(\mu_0^2, Q_0)e^{-S_{\text{hard}}}e^{-S_{UT}^{NP}}}{\tilde{f}_1(x, z^2\boldsymbol{b}^2, \mu_0^2, Q_0)\tilde{D}_1(z_h, \boldsymbol{b}^2, \mu_0^2, Q_0)\tilde{H}_{UU}(\mu_0^2, Q_0)e^{-S_{\text{hard}}}e^{-S_{UU}^{NP}}}$$

$e^{-S(b,Q)}$ = Sudakov

due to re-summation large logs

In prep. Boer, LG, B. Musch, A. Prokudin....

First Attempts

PoS PROCEEDINGS OF SCIENCE

Studies of TMDs with CLAS

M. Aghasyan*
LNF, INFN, Via E. fermi 40, Frascati (RM) 00044, Italy
E-mail: aghasyan@lnf.infn.it

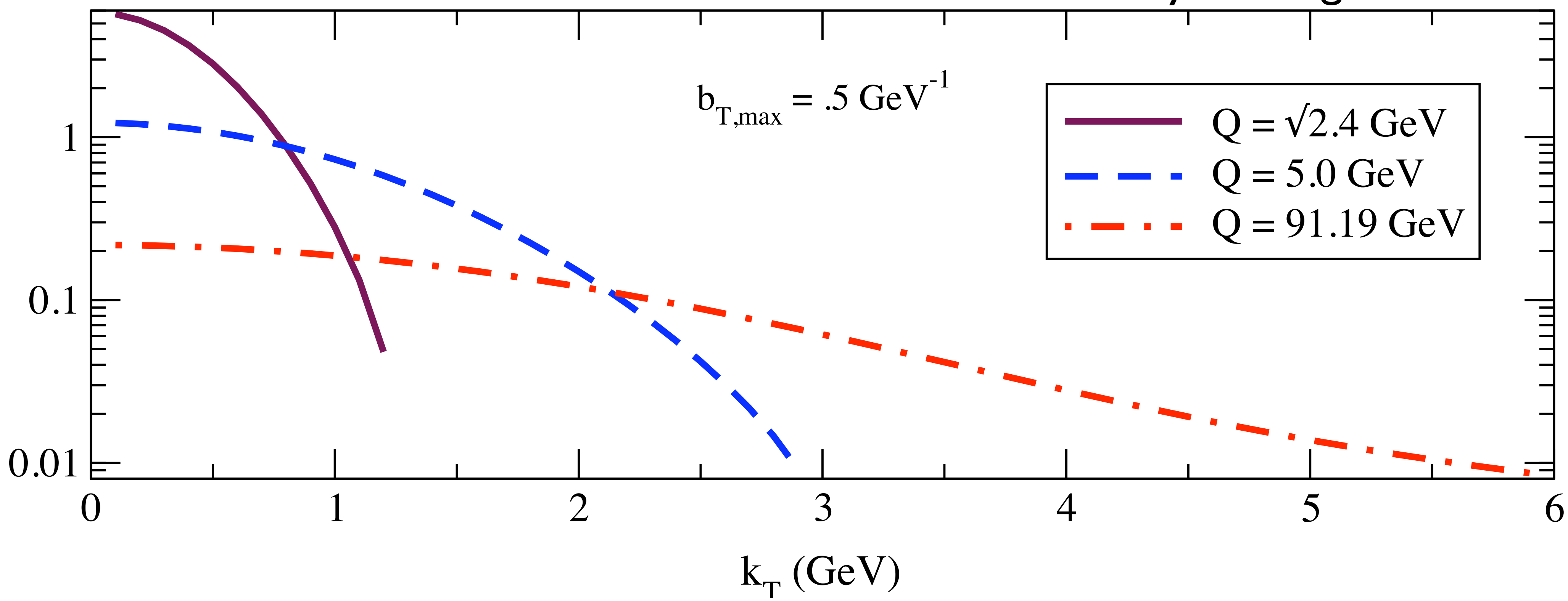
H. Avakian
JLab, 12000 Jeferson Ave, Newport News, VA 23606, USA

Studies of single and double-spin asymmetries in pion electro-production in semi-inclusive deep-inelastic scattering of 5.8 GeV polarized electrons from unpolarized and longitudinally polarized targets at the Thomas Jefferson National Accelerator Facility using CLAS discussed. We present a Bessel-weighting strategy to extract transverse-momentum-dependent parton distribution functions.

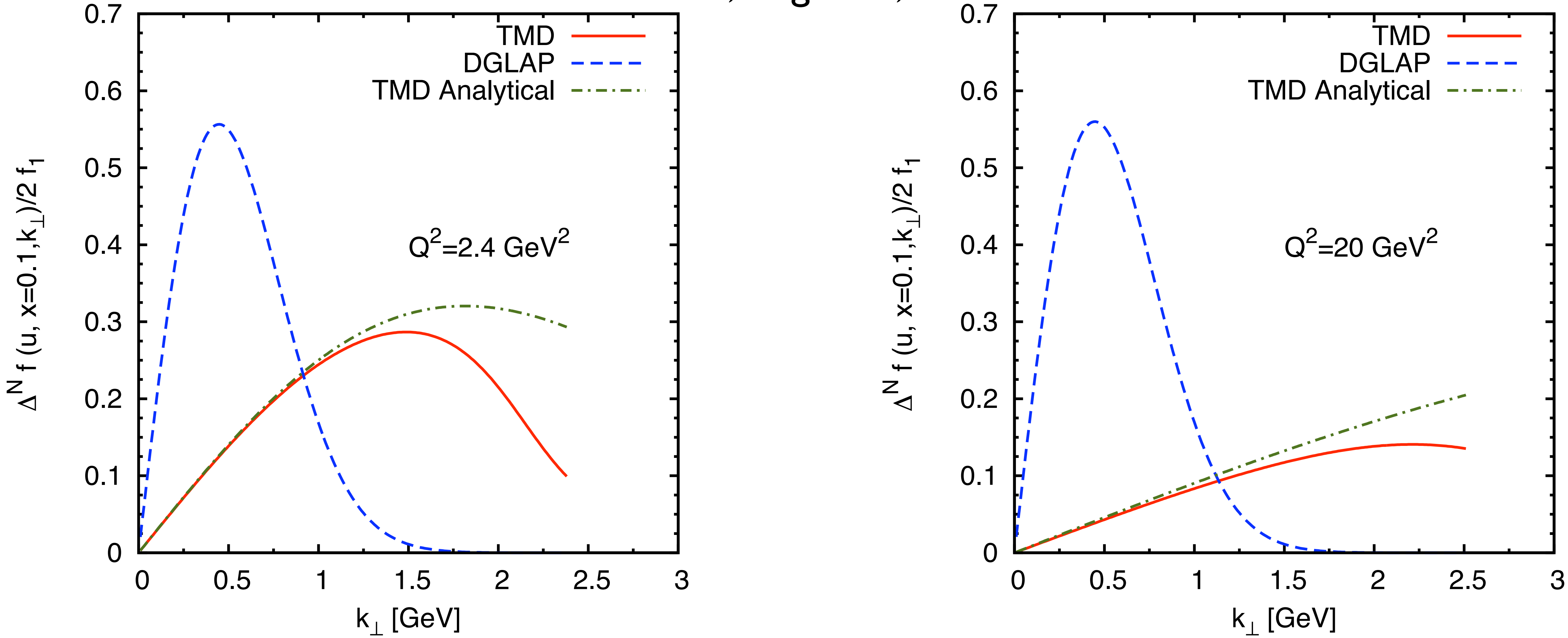
XXI International Workshop on Deep-Inelastic Scattering and Related Subject -DIS2013,
22-26 April 2013
Marseilles,France

Up Quark TMD PDF, x = .09

Aybat Rogers PRD 2011

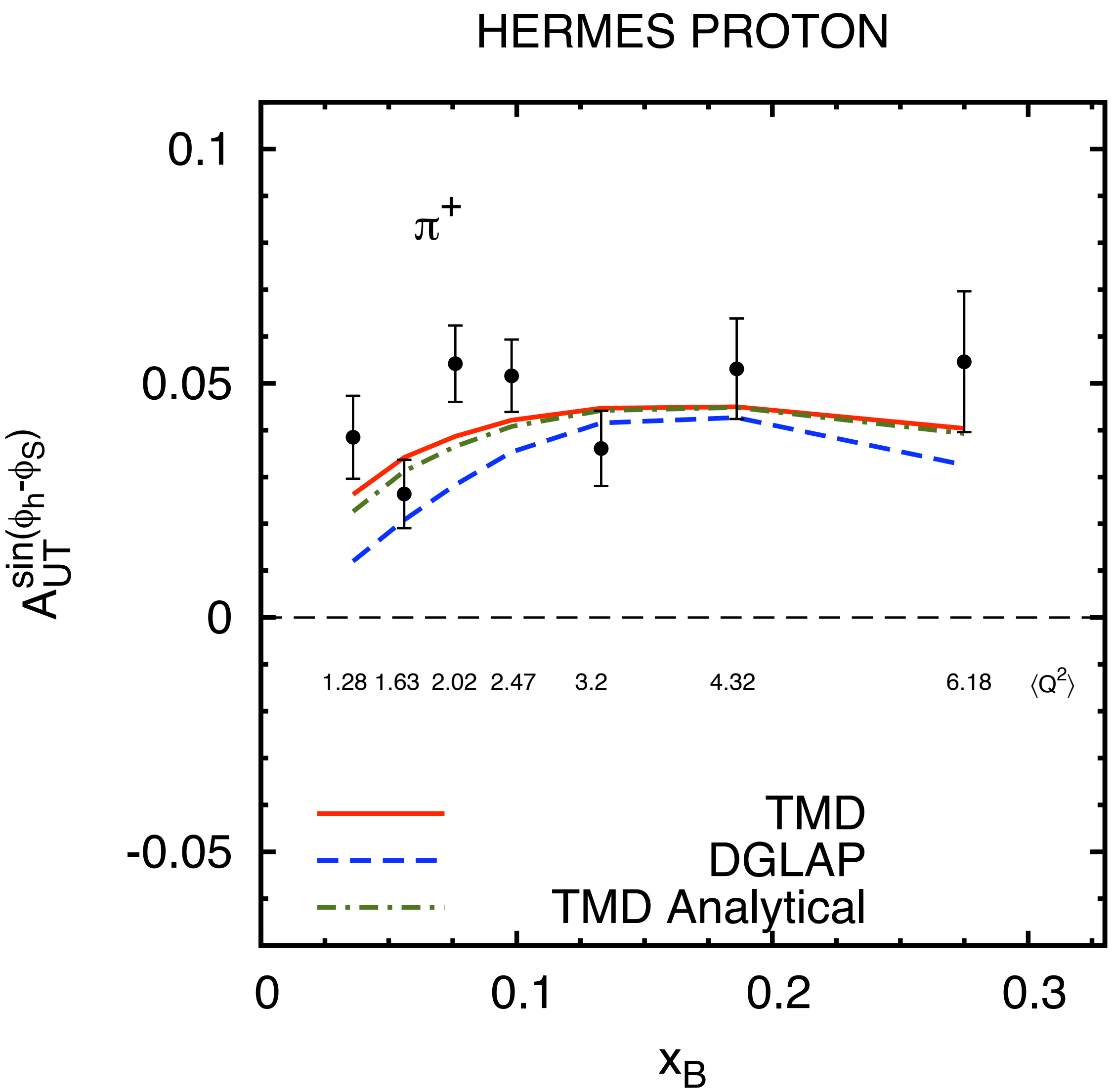


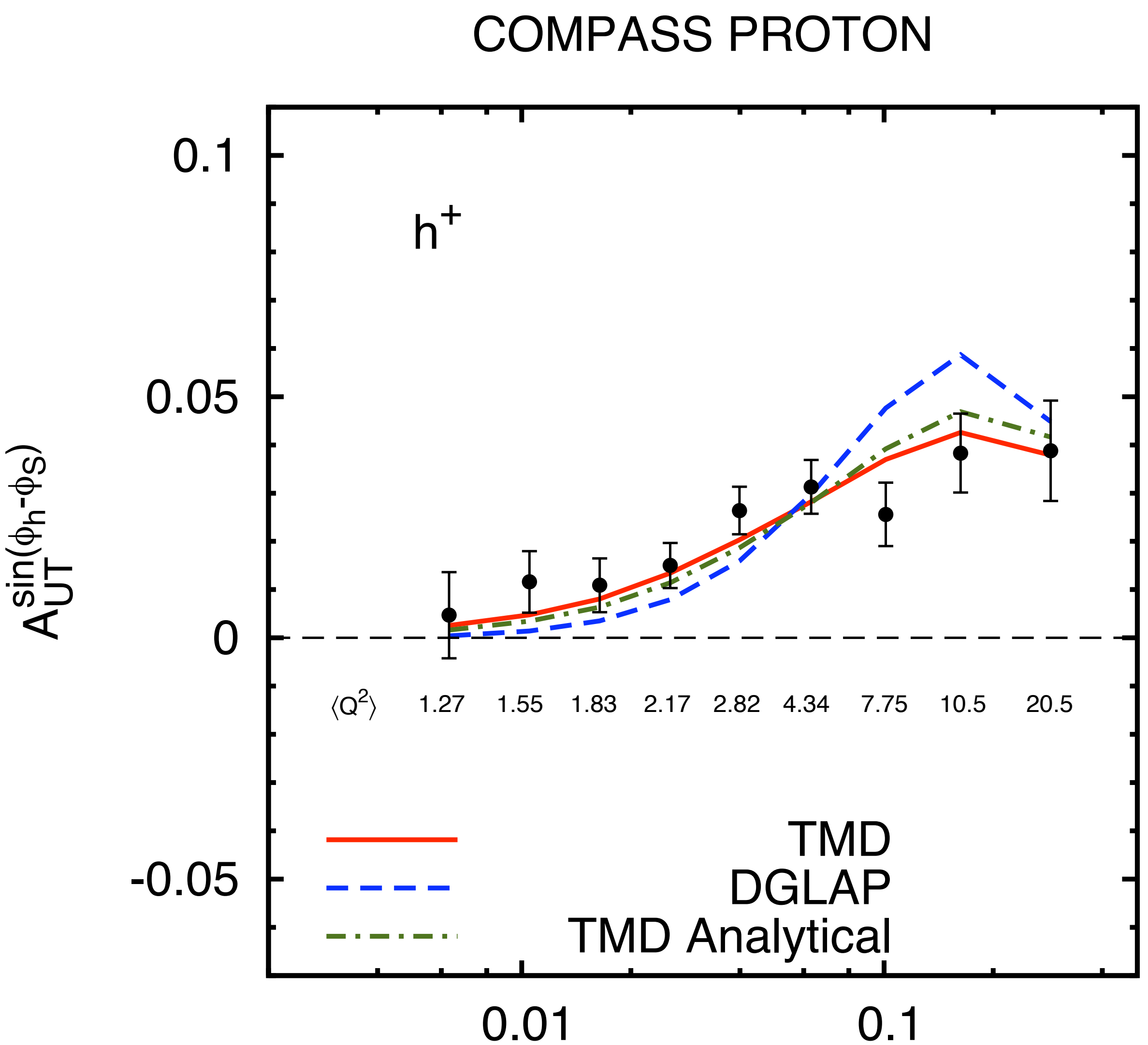
Anselmino, Boglione, Melis arXiv 2012



Anselmino, Boglione, Melis arXiv 2012

The parton-model version of TMD factorization amounts to applying the following approximations to the QCD formula



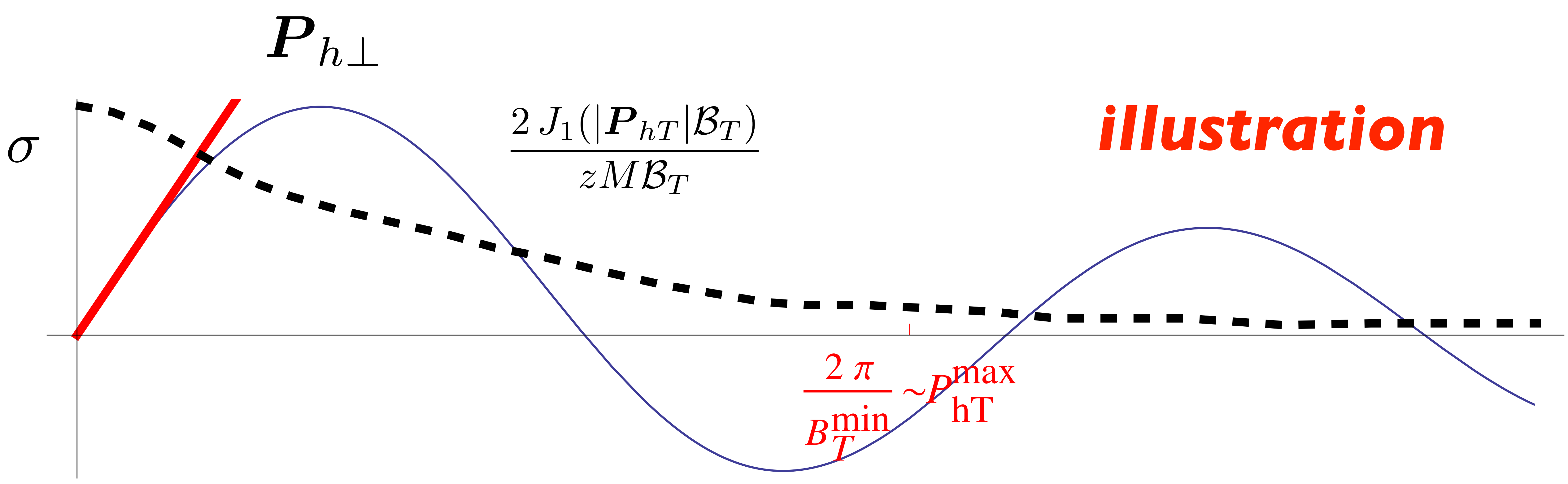


Conclusions-II

- Propose generalized Bessel Weights
- Theoretical weighting procedure-advantages
- Introduces a free parameter $\mathcal{B}_T\,[\mathrm{GeV}^{-1}]$ that is Fourier conjugate to $\boldsymbol{P}_{h\perp}$
- Provides a regularization of infinite contributions at lg. transverse momentum when $\mathcal{B}_T^2$ is non-zero
- Soft, Hard CS, eliminated from weighted asymmetries, Sudakov dpnds coupling of *b* & *Q*
- Possible to compare observables at different scales.... could be useful for an EIC

Extracting TMD contribution to Asymmetries More sensitive to low $\boldsymbol{P}_{h\perp}$ region

$\mathcal{B}_T$ can serve as a lever arm to enhance the low $\boldsymbol{P}_{h\perp}$ description and possibly dampen lg. momentum tail of cross section. We can use it to scan the cross section



Extracting TMD contribution to Asymmetries More sensitive to low $\boldsymbol{P}_{h\perp}$ region

TMD frameworks have been designed to give a good description of the cross section at low transverse momentum, i.e., for $|\boldsymbol{P}_{h\perp}|/z \ll Q$. However, in weighted asymmetries we integrate over the whole range of $|\boldsymbol{P}_{h\perp}|$. The contributions from high $|\boldsymbol{P}_{h\perp}|$ thus lead to theoretical errors in the results if one does not have a description of the cross section that is valid there, even when one restricts to the region $z|\boldsymbol{b}_T| \gg 1/Q$.

What errors do we make by neglecting the Y term and approx the cross section as soley due to TMD contribution.

Bessel weighting gives us a means to estimate these errors

- Resumed term most relevant when $P_{h\perp} << Q$. When $P_{h\perp}$ gets large conventional NLO perturbative contribution is important
- Y term is difference between pert. contribution and asymptotic form of TMD contribution
- The Y term in principle included to eliminate errors, but its $\mathcal{FT}$ expected to be power suppressed in region $\boldsymbol{b}_T >> 1/Q$ since was shown to be power suppressed at small $\boldsymbol{P}_{h\perp}$
- Thus dropping Y means we approximate the full result by the large $\boldsymbol{P}_{h\perp}$ tail of the TMD expression---is this a bad approx?
  Error falls off as $1/\sqrt{\boldsymbol{b}_T^3}$
- In addition extending integrals to arbitrarily large transverse momentum ignores that the physical cross section should vanish above a certain max trans. momentum--what is error?
  Error falls off as $1/\sqrt{\boldsymbol{b}_T^3}$

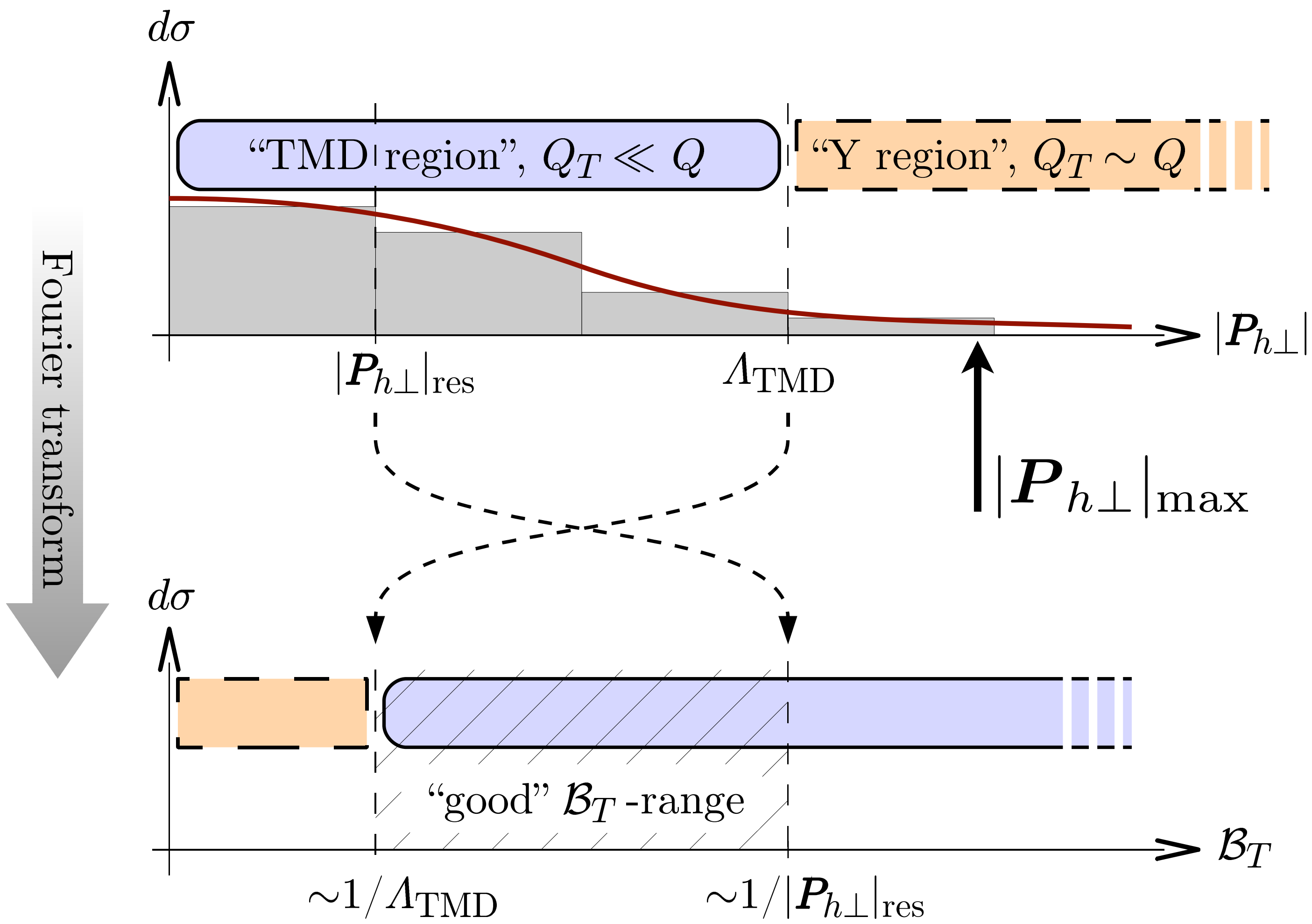
details next two slides

Bound the error in neglecting Y term

Y term signif. btwn scale Λ_{TMD} and $|\boldsymbol{P}_{h\perp}|_{\max}$

$$\tilde{Y}_{XY,Z}^{\sin/\cos(N\phi_h+\ldots)}(Q^2, \boldsymbol{b}_T^2) \approx \int_{\Lambda_{\text{TMD}}}^{|\boldsymbol{P}_{h\perp}|_{\max}} d|\boldsymbol{P}_{h\perp}|\ |\boldsymbol{P}_{h\perp}|\ 2\pi J_N(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\ Y_{XY,Z}^{\sin/\cos(N\phi_h+\ldots)}(Q^2, \boldsymbol{P}_{h\perp}^2)$$

$$\lesssim\ (|\boldsymbol{P}_{h\perp}|_{\max} - \Lambda_{\text{TMD}})\ 2\sqrt{\frac{2\pi}{|\boldsymbol{b}_T|\Lambda_{\text{TMD}}}}\ \left|Y_{XY,Z}^{\sin/\cos(N\phi_h+\ldots)}\right|_{\max} .$$

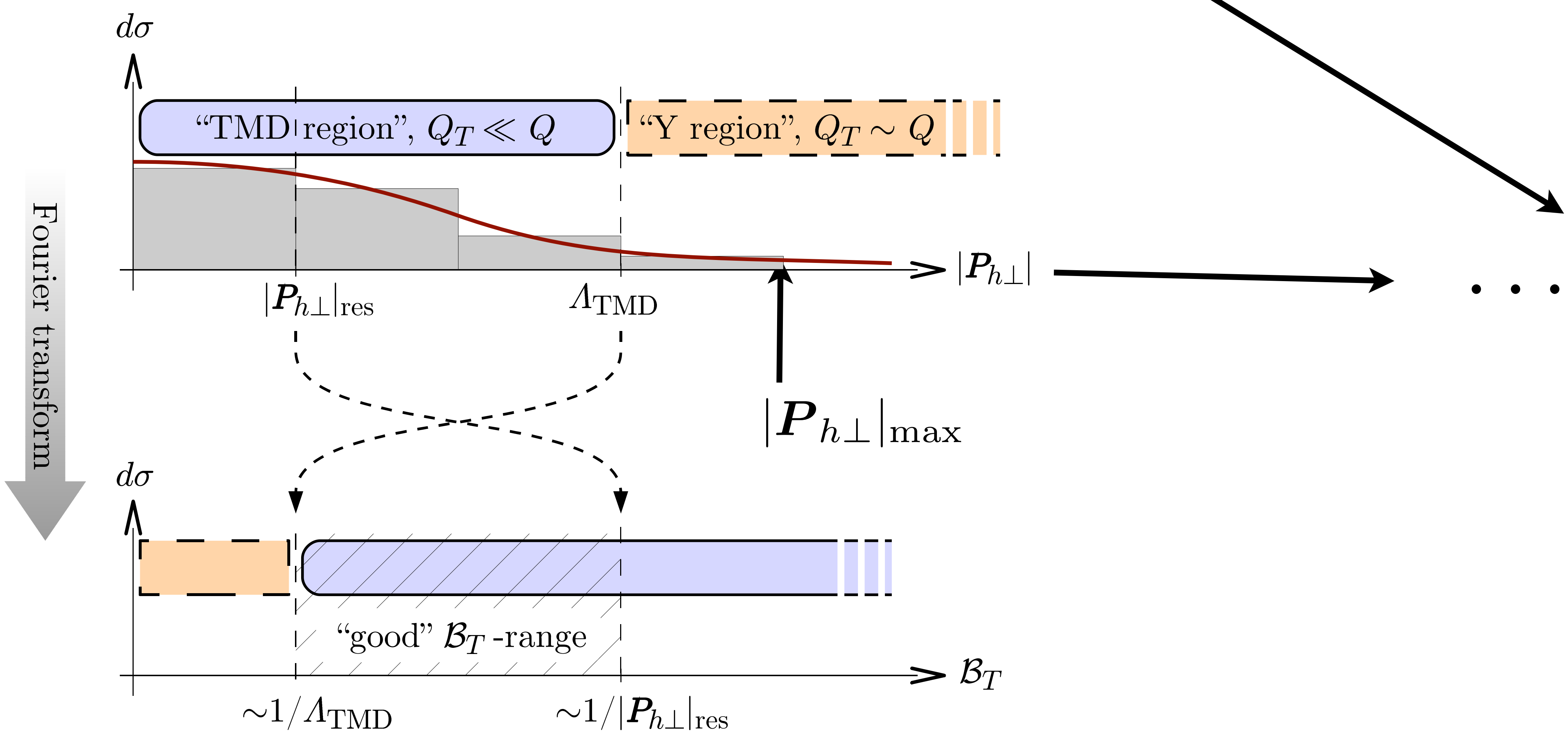


Error in extending TMD expression into perturbative regime

$$\delta\, \mathcal{F}_{XY,Z}^{\sin/\cos(N\phi_h+\dots)}(x, \boldsymbol{b}_T^2)$$

$$\approx \int_{|\boldsymbol{P}_{h\perp}|_{\max}}^{\infty} d|\boldsymbol{P}_{h\perp}|\; |\boldsymbol{P}_{h\perp}|\; 2\pi J_N(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\; [F_{XY,Z}^{\sin/\cos(N\phi_h+\dots)}]_{\text{TMD}}(Q^2, \boldsymbol{P}_{h\perp}^2)$$

$$\lesssim 4\sqrt{\frac{2\pi|\boldsymbol{P}_{h\perp}|_{\max}}{|\boldsymbol{b}_T|^3}}\; \left|[F_{XY,Z}^{\sin/\cos(N\phi_h+\dots)}]_{\text{TMD}}(Q^2, |\boldsymbol{P}_{h\perp}|^2_{\max})\right|\,,$$



Cancellation of Soft Factor on level of the Matrix elements ***(summarize)***

- So far we get ratios of moments of TMDs and FFs that are free/insensitive to soft factor
- It was not necessary to specify explicit def. of TMDs and FFs
- We also analyze ratio of moments of TMDs directly on level of matrix elements of TMDs & FFs
- Again we find cancellation of soft factors in ratio
- Impact for Lattice calculation of moments of TMDS, Musch, Ph. Hagler, M. Engelhardt, J.W. Negele, A. Schafer arXiv 2011