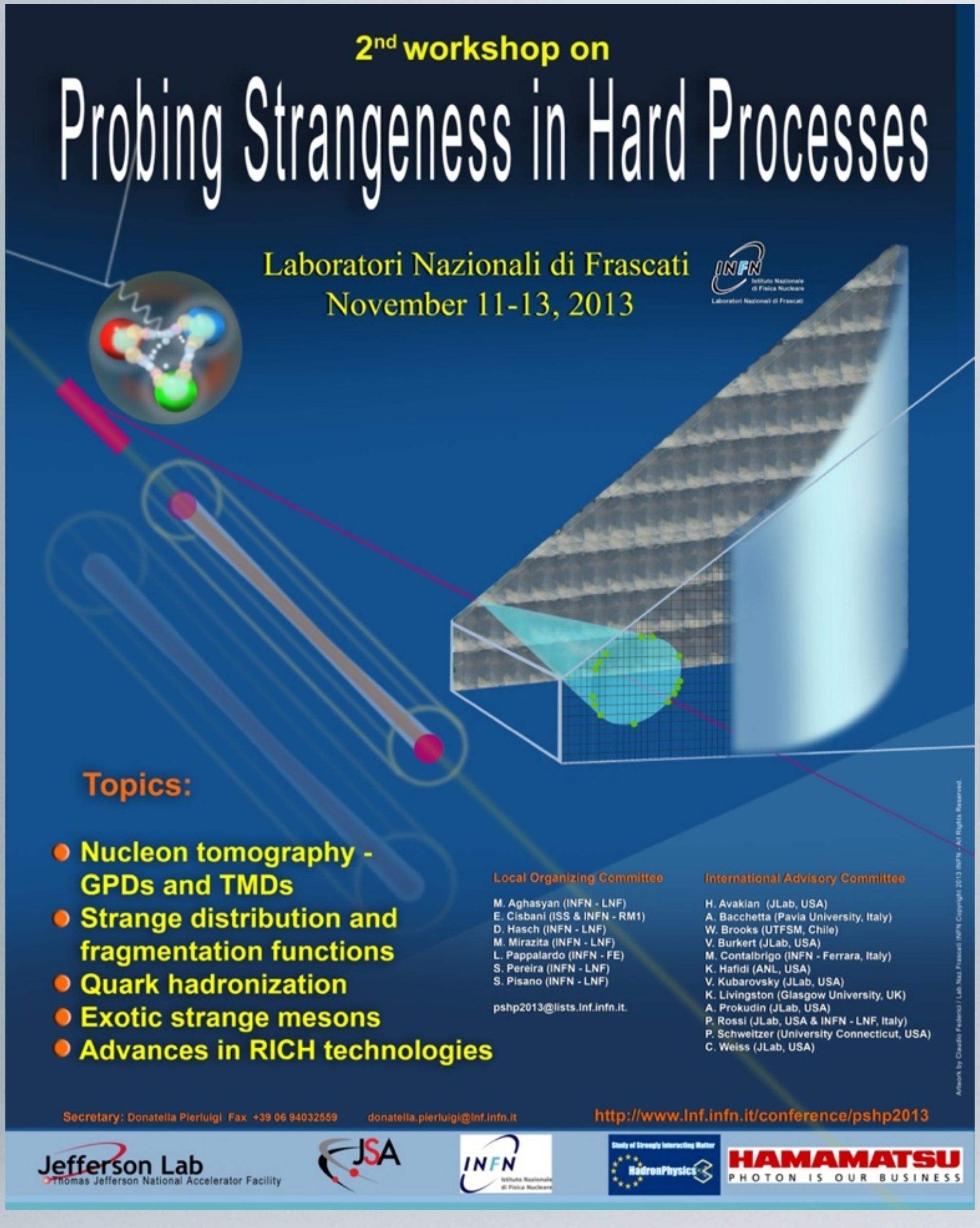
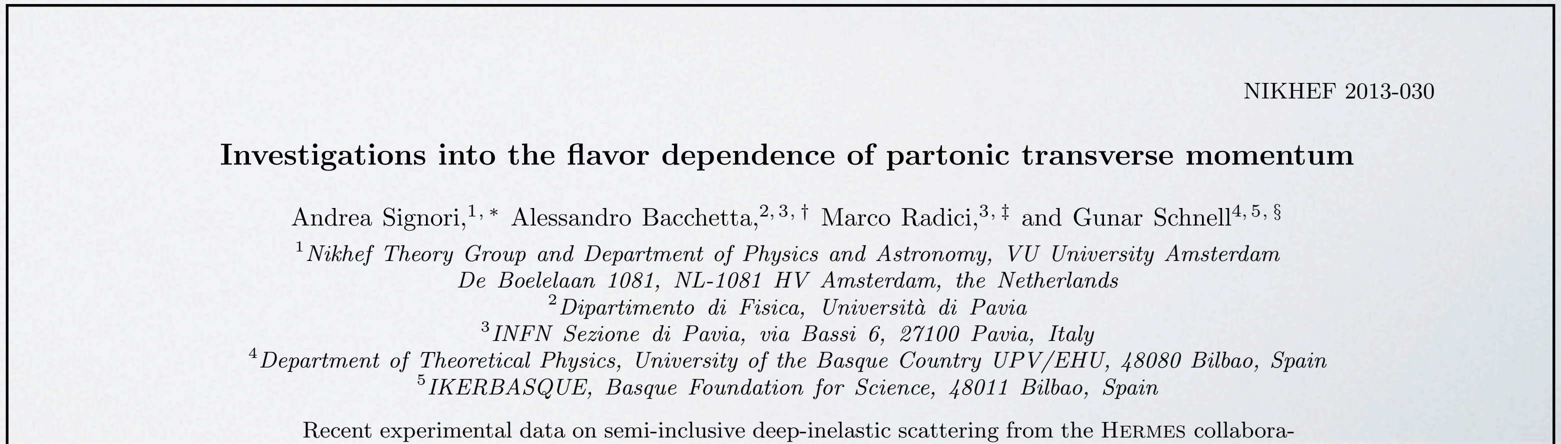


Flavor dependence of partonic transverse momentum

Marco Radici
INFN - Pavia

based on Master Th. A. Signori (now at VU, Amsterdam)
supervisor A. Bacchetta (Univ. Pavia)

preprint (with also G. Schnell)
arXiv:1309.3507 [hep-ph]



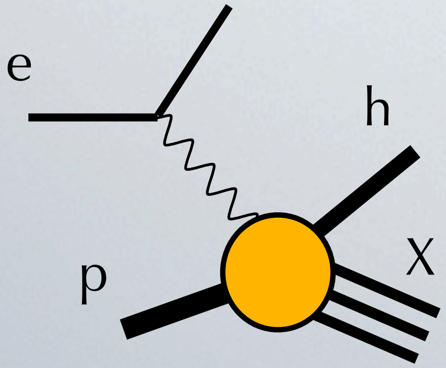
NIKHEF 2013-030

Investigations into the flavor dependence of partonic transverse momentum

Andrea Signori,[1,*] Alessandro Bacchetta,[2,3,†] Marco Radici,[3,‡] and Gunar Schnell[4,5,§]

[1]*Nikhef Theory Group and Department of Physics and Astronomy, VU University Amsterdam*
De Boelelaan 1081, NL-1081 HV Amsterdam, the Netherlands
[2]*Dipartimento di Fisica, Università di Pavia*
[3]*INFN Sezione di Pavia, via Bassi 6, 27100 Pavia, Italy*
[4]*Department of Theoretical Physics, University of the Basque Country UPV/EHU, 48080 Bilbao, Spain*
[5]*IKERBASQUE, Basque Foundation for Science, 48011 Bilbao, Spain*

Recent experimental data on semi-inclusive deep-inelastic scattering from the HERMES collabora-



Semi-Inclusive DIS with unpolarized final hadron "h"

SIDIS cross section @leading twist :

8 **TMD PDF**

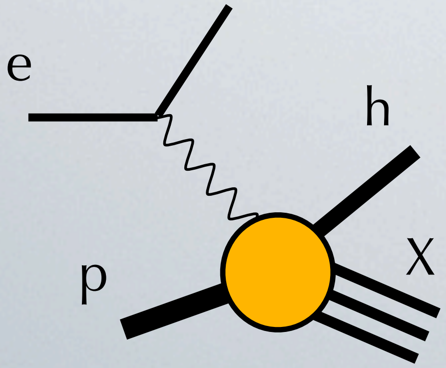
quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

2 **TMD FF**

quark pol.

U	L	T
D_1		$H_1^\perp$



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nucleon pol.

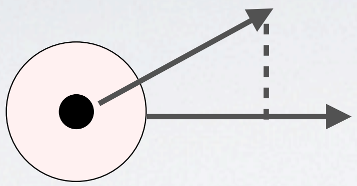
	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

2 TMD FF

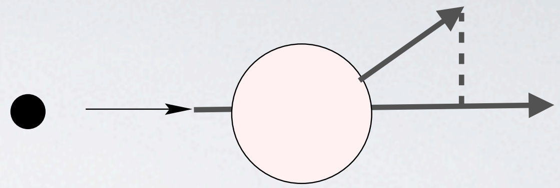
quark pol.

U	L	T
D_1		$H_1^\perp$

$f_1 =$

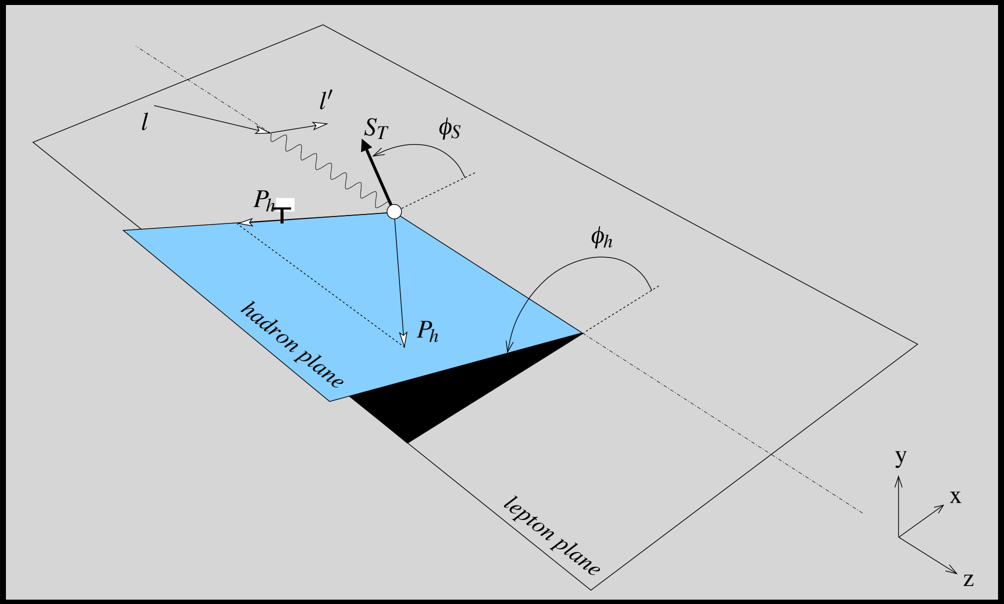


$D_1 =$



only unpolarized objects, but with memory of (poorly known) ⊥ kinematics

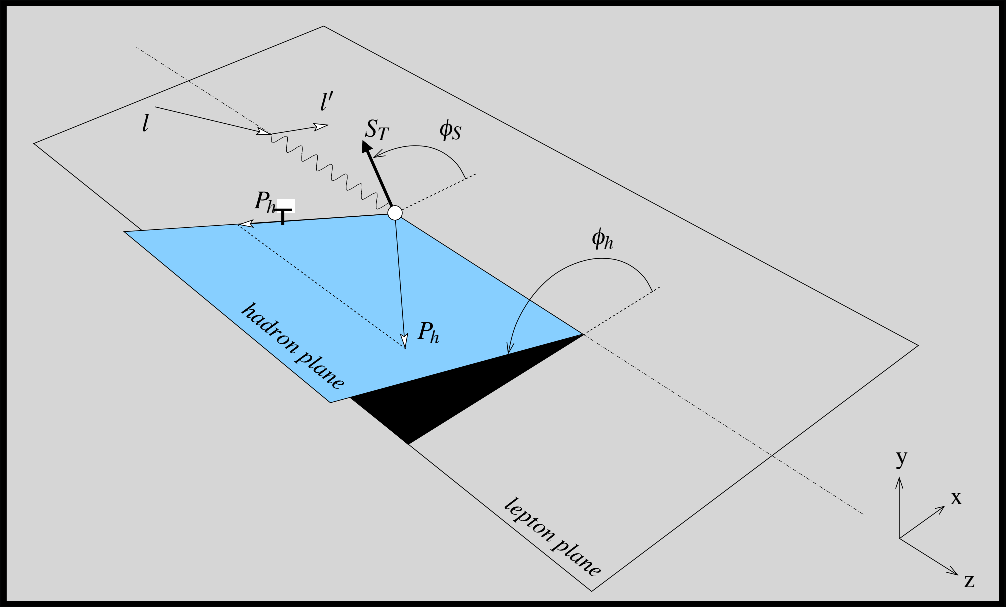
why worrying about the unpolarized cross section ?



spin asymmetry

$$A_{\vec{e}\,\vec{N}}^{f(\phi_h,\phi_S)} \propto \frac{F_{\vec{e}\,\vec{N}}^{f(\phi_h,\phi_S)}}{F_{UU}} \propto \frac{\sum_q e_q^2\ \ \mathrm{TMD\_PDF}^q\ \otimes_w\ \mathrm{TMD\_FF}^q}{\sum_q e_q^2\ \ f_1^q \otimes D_1^q}$$

why worrying about the unpolarized cross section ?



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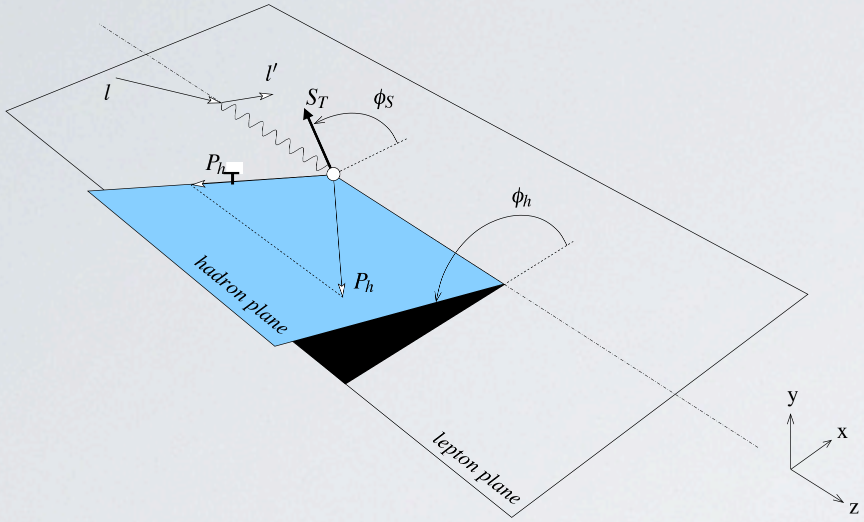
unpolarized TMDs affect spin asymmetries *A*
⇒ they influence the extraction of polarized TMDs

exp. observable : multiplicity

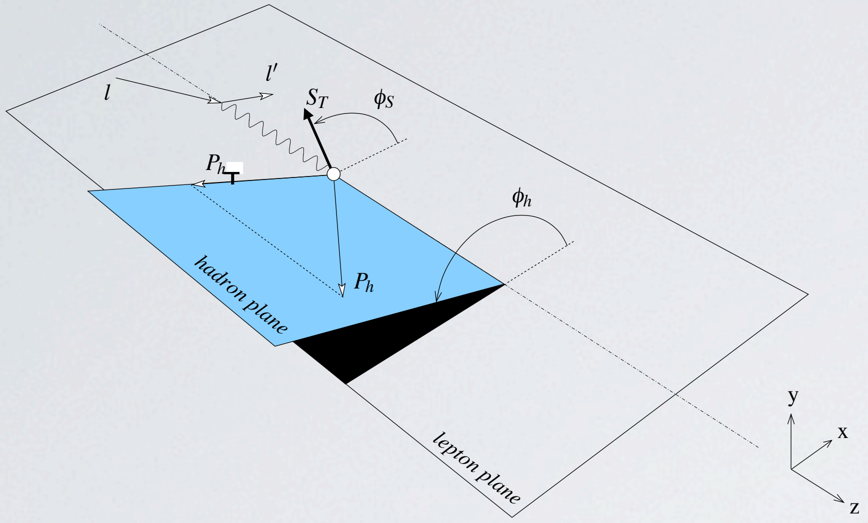


SIDIS process

$$\ell(l) + N(P) \to \ell(l') + h(P_h) + X,$$

$$m_N^h(x, z, \boldsymbol{P}_{hT}^2, Q^2) = \frac{d\sigma_N^h/(dx dz d\boldsymbol{P}_{hT}^2 dQ^2)}{d\sigma_{\rm DIS}/(dx dQ^2)} \approx \frac{\pi\, F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2)}{F_T(x, Q^2)}$$

exp. observable : multiplicity



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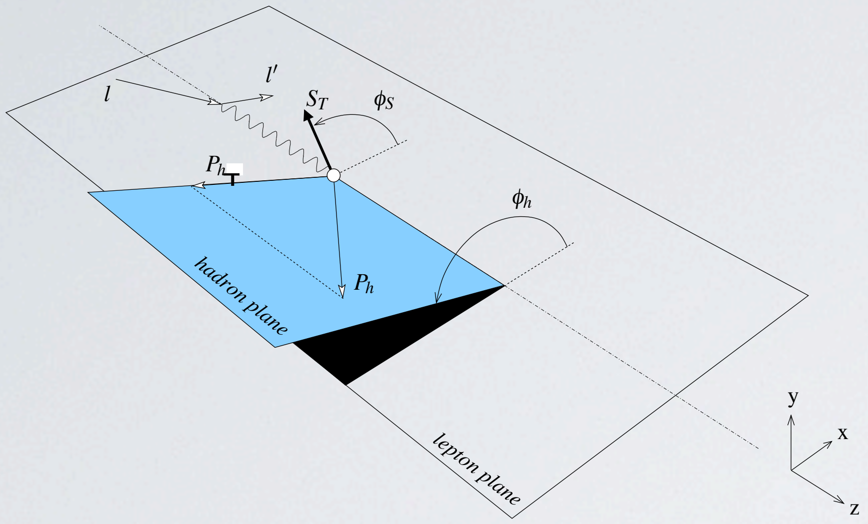
hadron species

$$m_N^h(x, z, \boldsymbol{P}_{hT}^2, Q^2) = \frac{d\sigma_N^h/(dx dz d\boldsymbol{P}_{hT}^2 dQ^2)}{d\sigma_{\mathrm{DIS}}/(dx dQ^2)} \approx \frac{\pi\, F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2)}{F_T(x, Q^2)}$$

target

exp. observable : multiplicity



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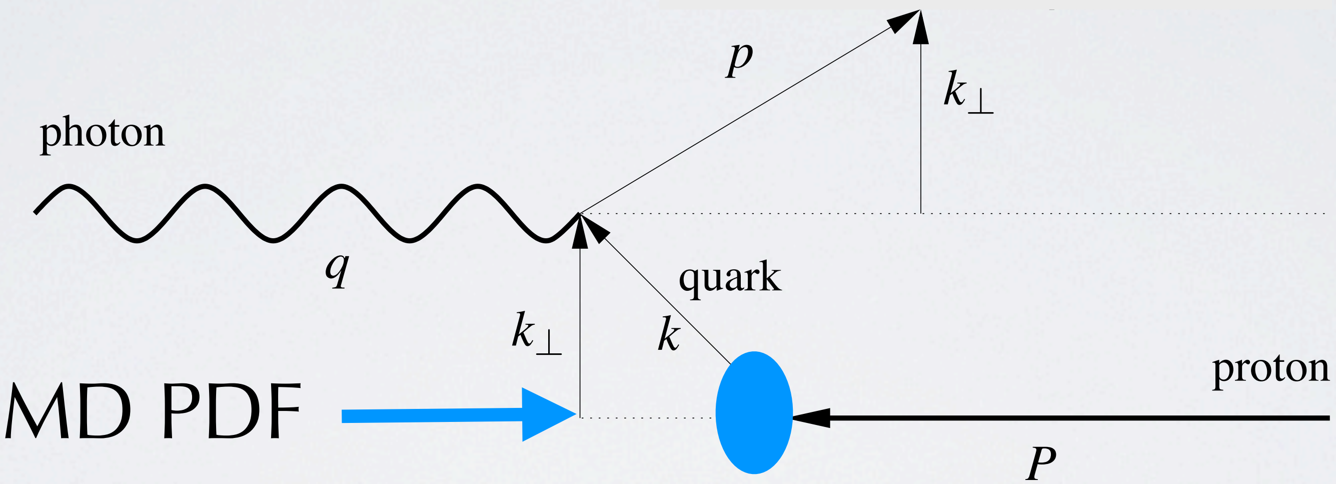
target

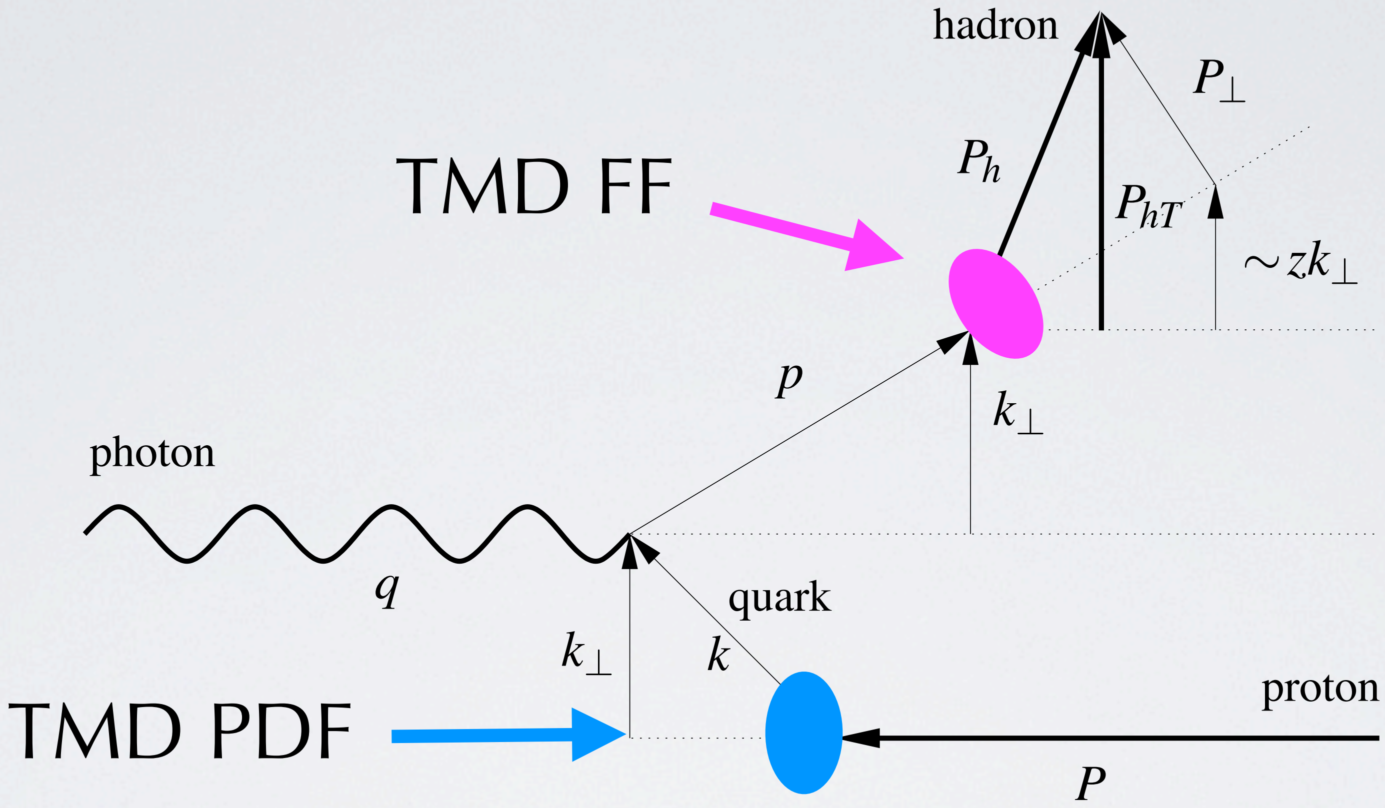
1. M^2, $\boldsymbol{P}_{\rm hT}{}^2 \ll Q^2$: leading twist TMD
2. $O(\alpha_s{}^0)$: parton model
3. Φ_h integrated : acceptance in systematic error

involved transverse momenta



TMD PDF

involved transverse momenta

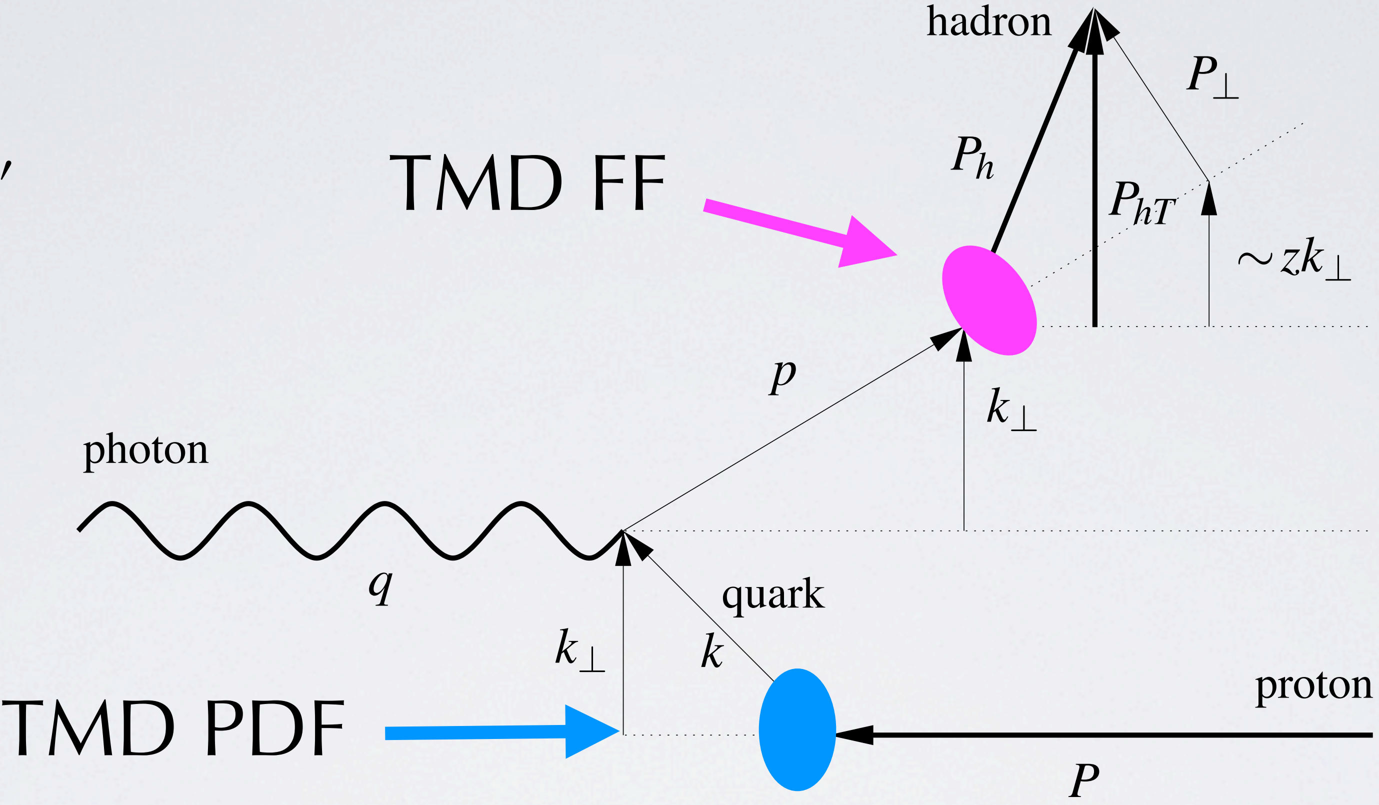


TMD FF

TMD PDF

involved transverse momenta

notation as in
"Seattle convention"
arXiv:1108.1713



TMD FF

TMD PDF

parton model

$$
\begin{aligned}
F_{UU,T}(x,z,\boldsymbol{P}_{hT}^2;Q^2) &= \sum_q e_q^2\, x \int d\boldsymbol{k}_\perp\, d\boldsymbol{P}_\perp\, \delta(z\boldsymbol{k}_\perp + \boldsymbol{P}_\perp - \boldsymbol{P}_{hT})\; f_1^q(x,\boldsymbol{k}_\perp^2,Q^2)\; D_1^{q\to h}(z,\boldsymbol{P}_\perp^2;Q^2) \\
&= \sum_q e_q^2 \left[f_1^q \otimes D_1^{q\to h} \right]
\end{aligned}
$$

usual assumption : flavor independent Gaussian shape for transverse momenta

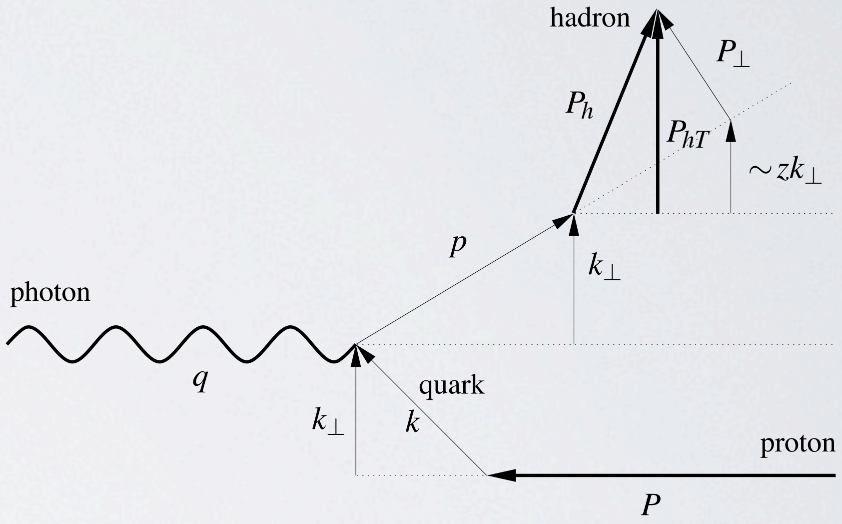
TMD PDF

$$f_1^q(x, \boldsymbol{k}_\perp^2; Q^2) = f_1^q(x; Q^2)\, \frac{e^{-\boldsymbol{k}_\perp^2/\langle \boldsymbol{k}_\perp^2\rangle}}{\pi\langle \boldsymbol{k}_\perp^2\rangle}$$

TMD FF

$$D_1^{q\to h}(z, \boldsymbol{P}_\perp^2; Q^2) = D_1^{q\to h}(z; Q^2)\, \frac{e^{-\boldsymbol{P}_\perp^2/\langle \boldsymbol{P}_\perp^2\rangle}}{\pi\langle \boldsymbol{P}_\perp^2\rangle}$$

$$\langle \boldsymbol{P}_{hT}^2\rangle = z^2\langle \boldsymbol{k}_\perp^2\rangle + \langle \boldsymbol{P}_\perp^2\rangle$$



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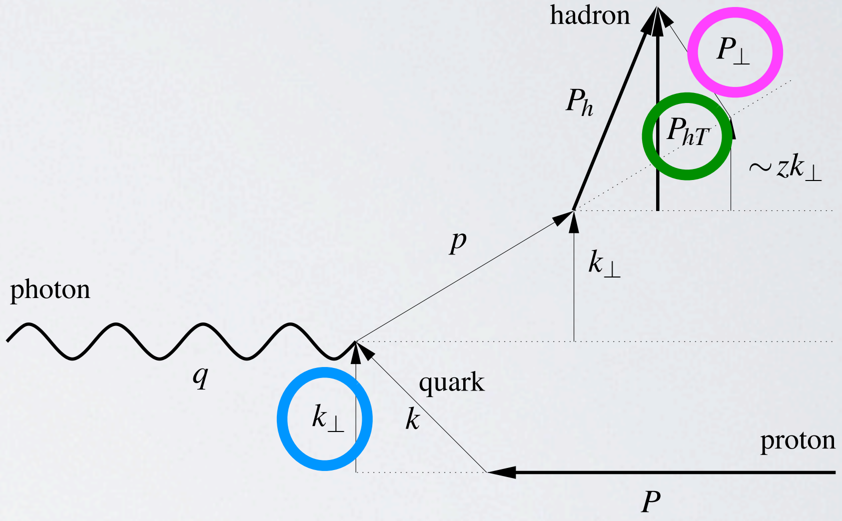
advantage

$$\langle \boldsymbol{P}_{hT}^2 \rangle = z^2 \langle \boldsymbol{k}_\perp^2 \rangle + \langle \boldsymbol{P}_\perp^2 \rangle$$

from observed

get intrinsic



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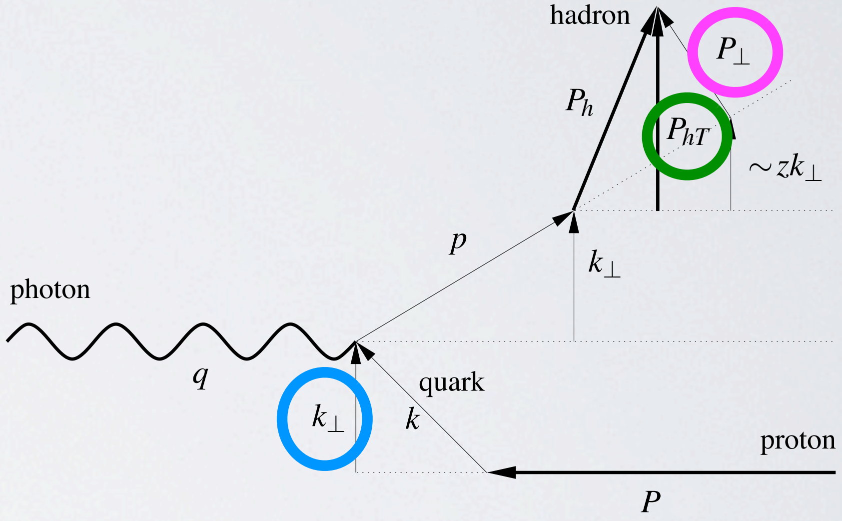
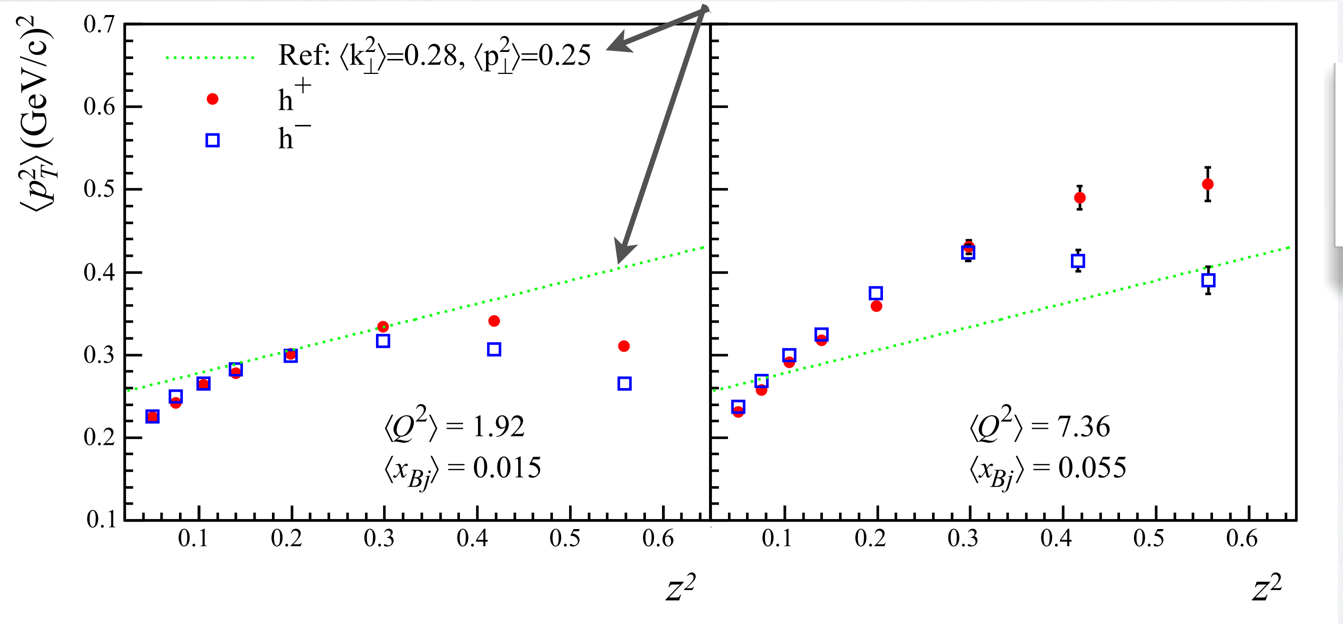


arXiv:1305.7317 [hep-ex]

Anselmino et al, E.P.J. **A31** (07)



- not well supported by data

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from observed

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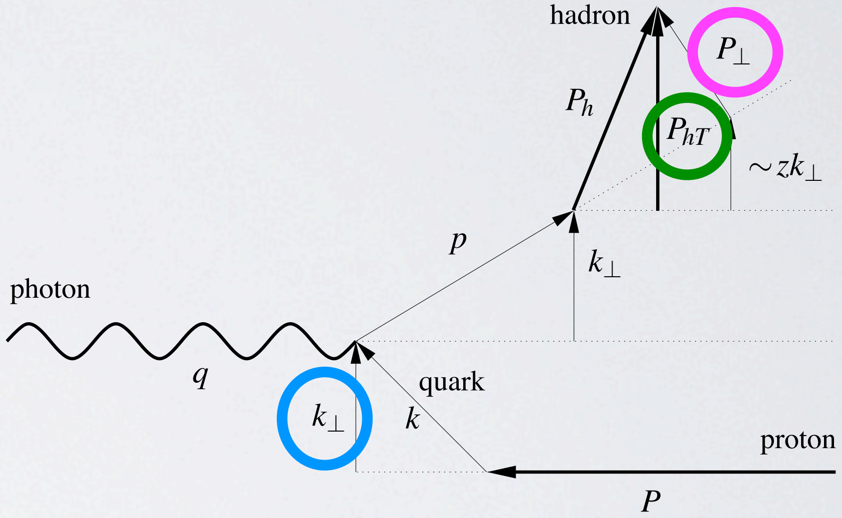
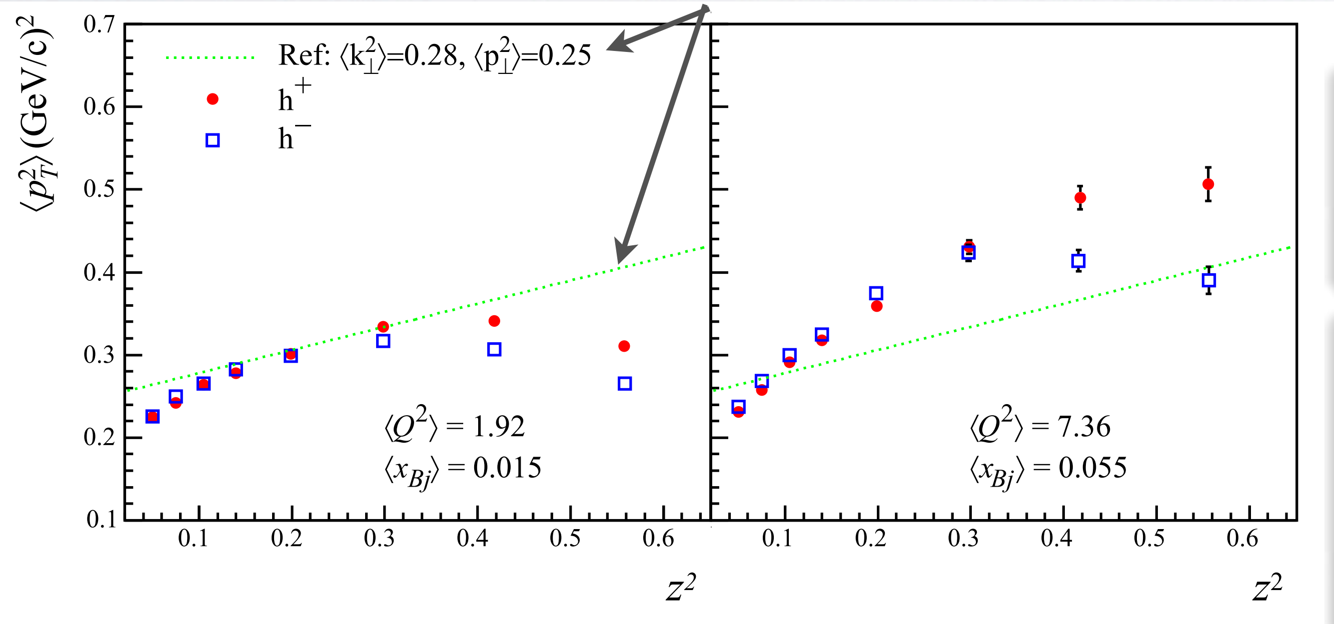


COMPASS arXiv:1305.7317 [hep-ex]



Anselmino et al, E.P.J. **A31** (07)

- not well supported by data

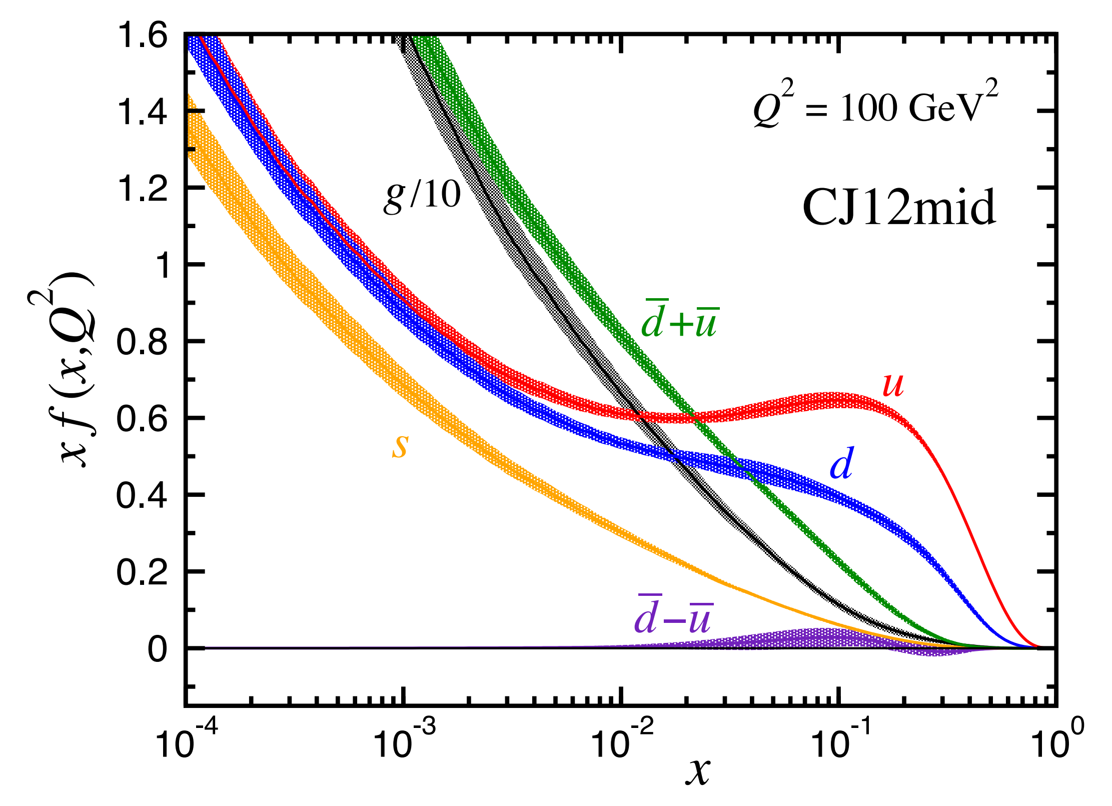
- also hints of **flavor dependence**
  $h^+ \neq h^-$

evidence of flavor dependence from :

unpolarized (collinear) PDFs

example :

Owens, Accardi, Melnitchouk
(CJ12)
P.R. D**87** (13) 094012



similar evidences in

Jimenez-Delgado, Reja (JR09), P. R. D**80** (09) 114011
Alekhin *et al*. (ABKM09), P. R. D**81** (10) 014032
Lai *et al*. (CT10), P. R. D**82** (10) 074024
Alekhin, Blümlein, Moch (ABM11), P. R. D**86** (12) 054009
Ball et al. (NNPDF13), N. P. **B867** (13) 244
....

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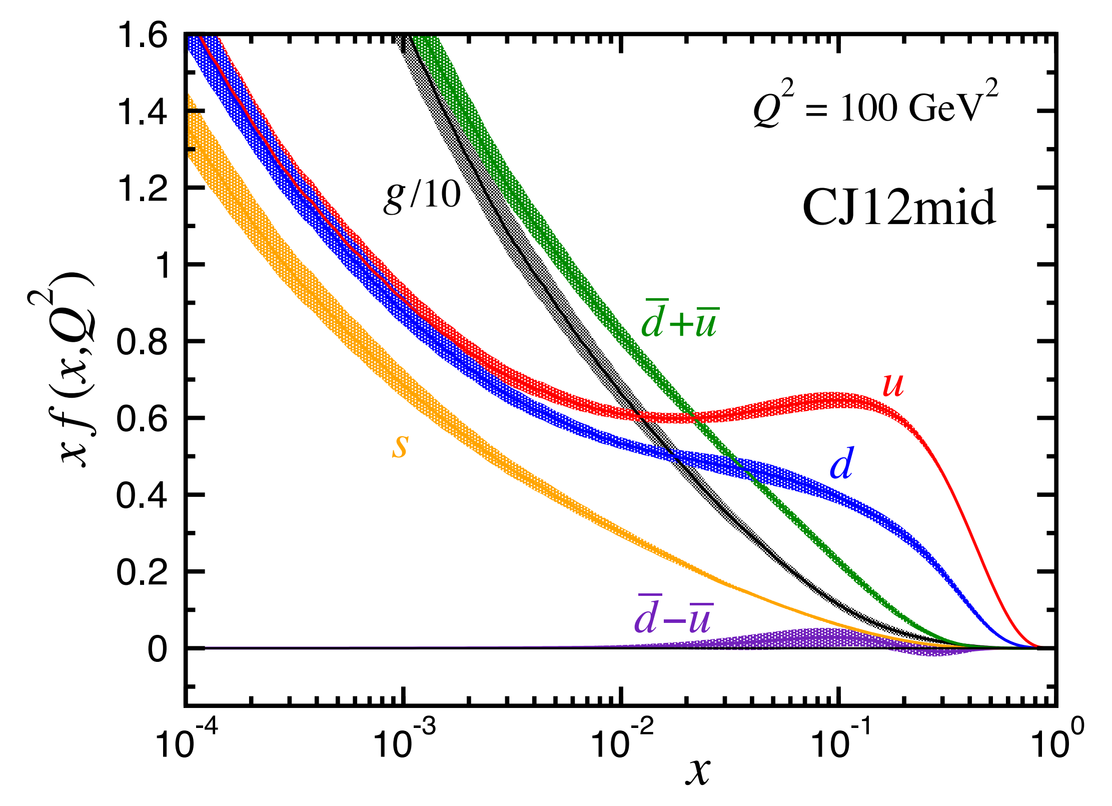

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Ball et al. (NNPDF13), N. P. **B867** (13) 244
....

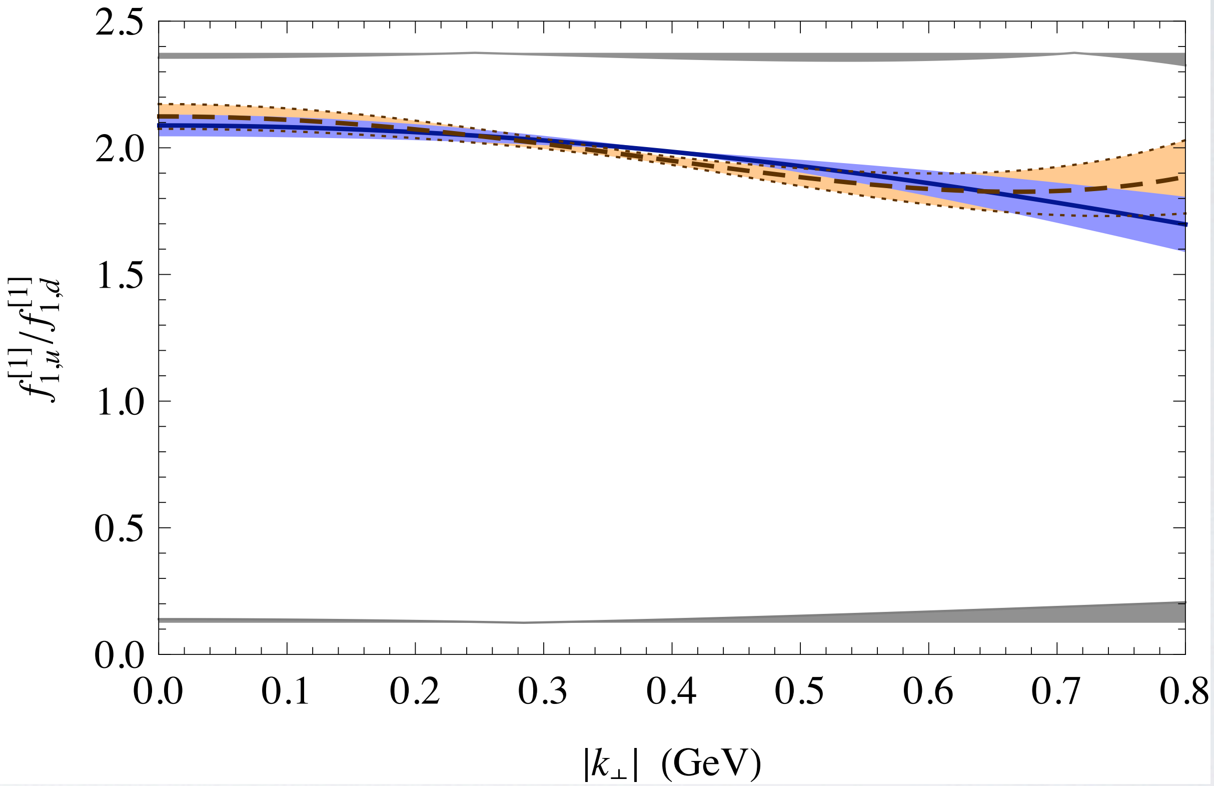
why not for
k$_\perp$ dependence
of TMDs ?

evidence of flavor dependence from :

lattice QCD

valence picture
of proton :
#u / #d = 2

ratio of
number densities
(moments of f_1^q)
depends upon $|\mathbf{k}_\perp|$



Musch *et al.*, P.R. D**83** (11) 094507

evidence of flavor dependence from :

lattice QCD

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of proton :
#u / #d = 2

ratio of
number densities
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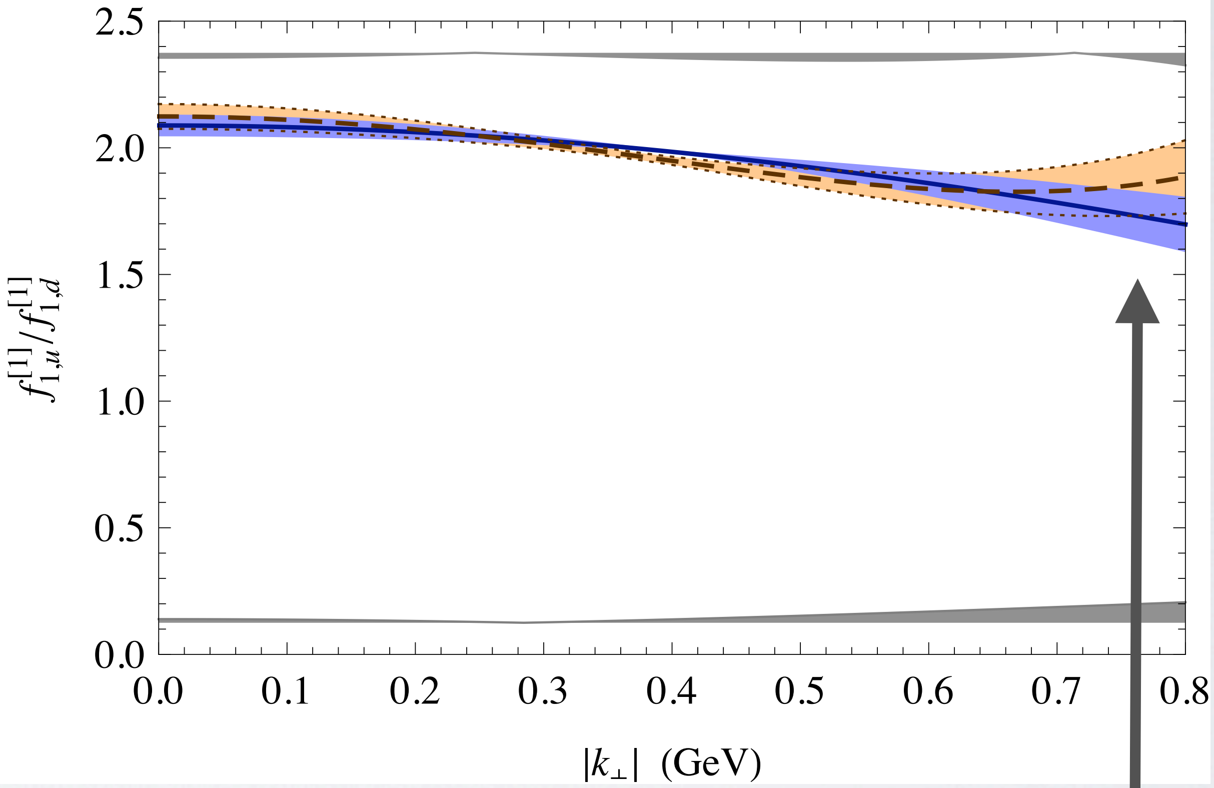


“less” up at large $|\mathbf{k}_\perp|$

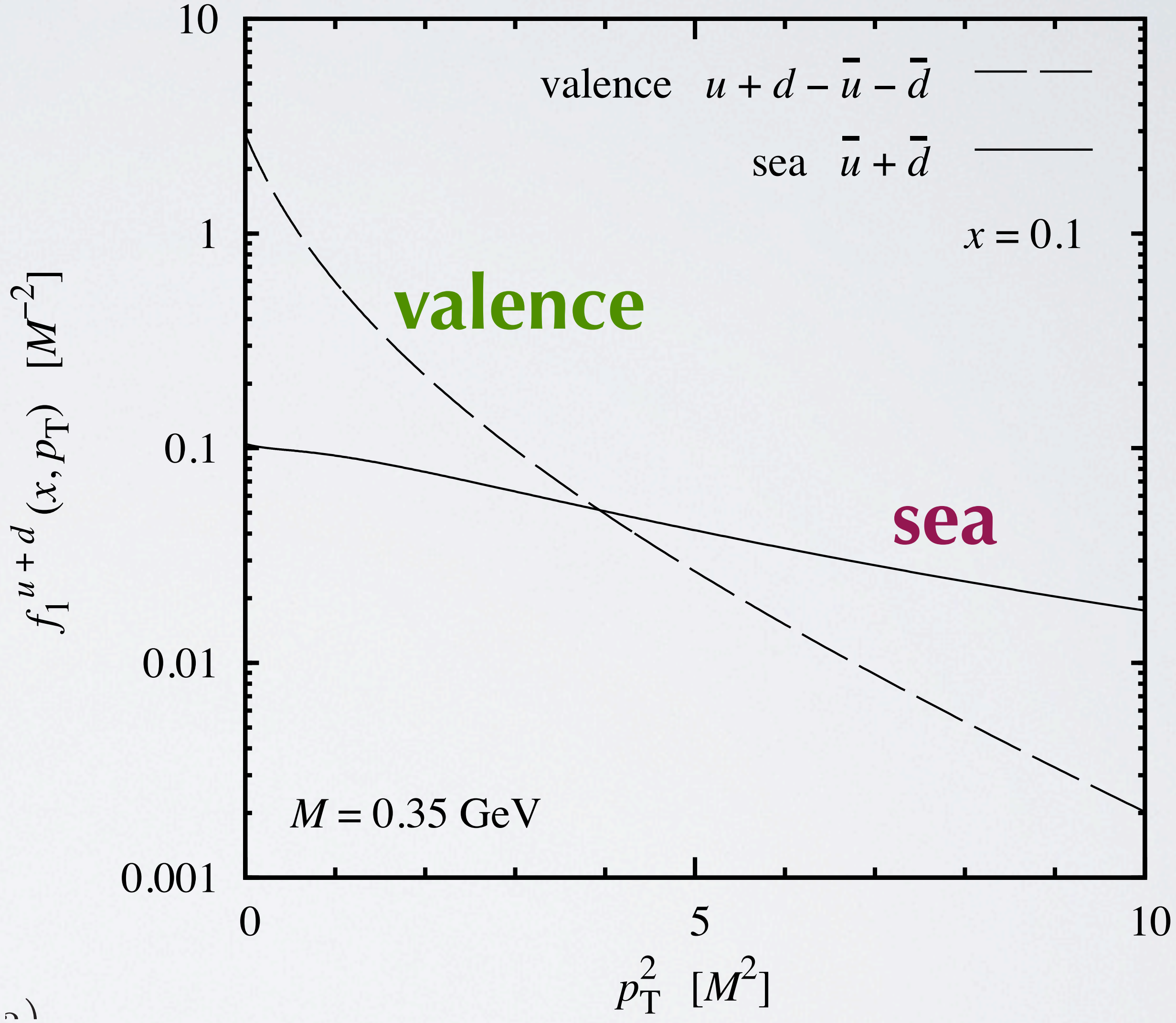
Musch *et al.*, P.R. D**83** (11) 094507

evidence of flavor dependence from :

models of TMD PDFs

example :
chiral quark soliton model

Schweitzer, Strikman, Weiss
JHEP 1301 (13) 163



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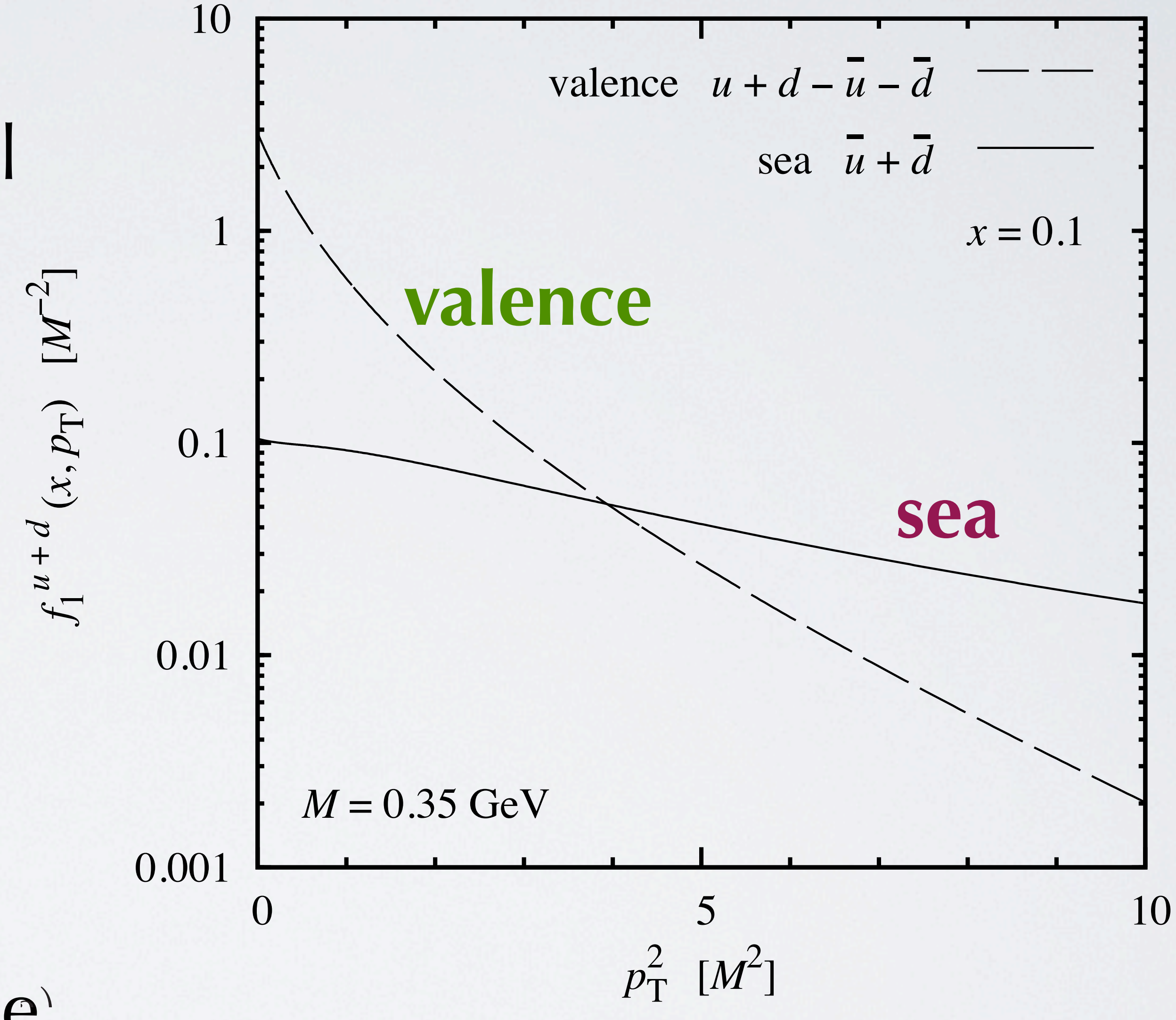
JHEP 1301 (13) 163



similarly in other models like

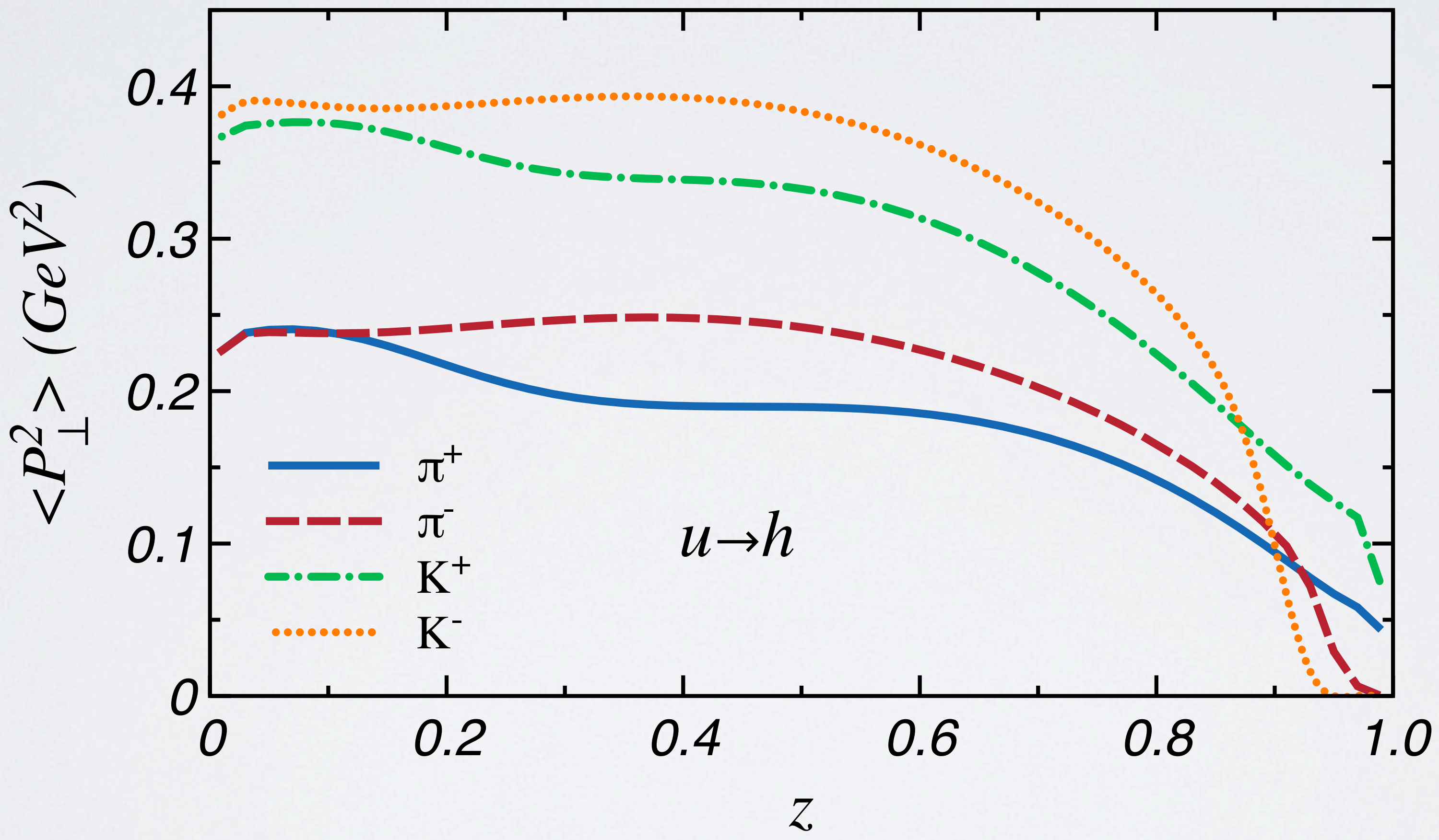
- diquark spectator (Bacchetta, Conti, Radici, P. R. D**78** (08) 074010)
- statistical approach (Bourrely, Buccella, Soffer, P. R. D**83** (11) 074008)
- ……

evidence of flavor dependence from :

models of TMD FFs

example : NJL-jet model

Matevosyan *et al.*,
P. R. D**85** (12) 014021



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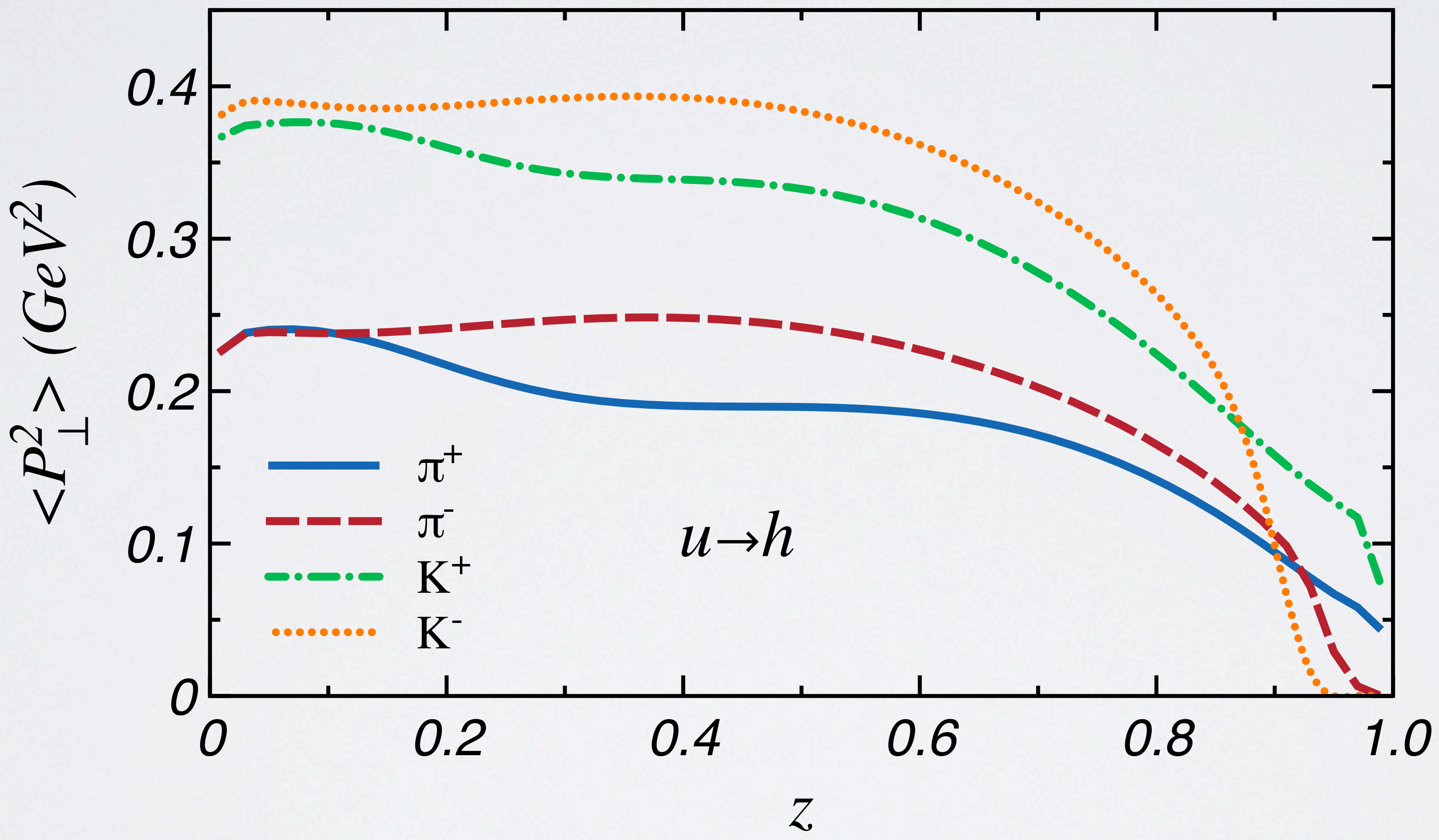
P. R. D**85** (12) 014021



$<\mathbf{P}_{hT}{}^2>$ larger for unfavored / K fragmentation
than for favored π fragmentation

our work :

can we find evidence of
flavor dependence in $\mathbf{k}_\perp$ shape of TMDs
from experimental data on SIDIS ?

our analysis : **flavor dependent** Gaussian shape for transverse momenta

TMD PDF

$$f_1^q(x, \boldsymbol{k}_\perp^2; Q^2) = f_1^q(x; Q^2)\, \frac{e^{-\boldsymbol{k}_\perp^2/\langle \boldsymbol{k}_{\perp,q}^2 \rangle}}{\pi \langle \boldsymbol{k}_{\perp,q}^2 \rangle}$$

TMD FF

$$D_1^{q\to h}(z, \boldsymbol{P}_\perp^2; Q^2) = D_1^{q\to h}(z; Q^2)\, \frac{e^{-\boldsymbol{P}_\perp^2/\langle \boldsymbol{P}_{\perp,q\to h}^2 \rangle}}{\pi \langle \boldsymbol{P}_{\perp,q\to h}^2 \rangle}$$

in the convolution, for each flavor we get a Gaussian with width

$$\langle \boldsymbol{P}_{hT,q}^2 \rangle = z^2 \langle \boldsymbol{k}_{\perp,q}^2 \rangle + \langle \boldsymbol{P}_{\perp,q\to h}^2 \rangle$$

our analysis : **flavor dependent** Gaussian shape for transverse momenta

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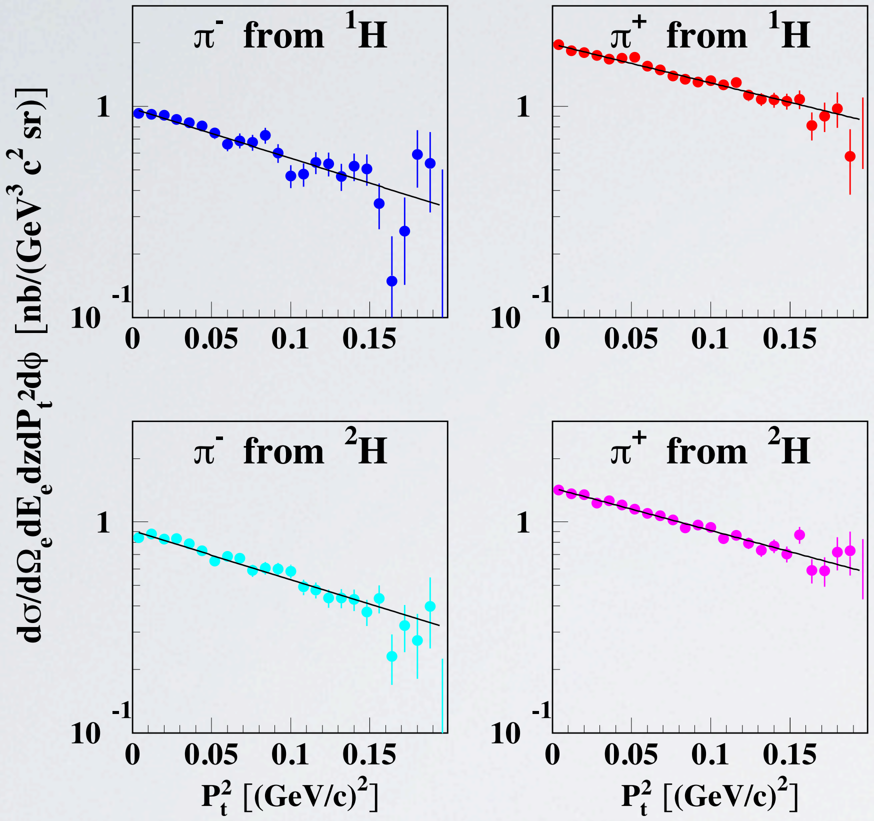
multiplicity

$$m_N^h(x, z, \boldsymbol{P}_{hT}^2; Q^2) = \frac{\pi}{\sum_q e_q^2\, f_1^q(x; Q^2)} \sum_q e_q^2\, f_1^q(x; Q^2)\, D_1^{q\to h}(z; Q^2)\, \frac{e^{-\boldsymbol{P}_{hT}^2/\langle \boldsymbol{P}_{hT,q}^2 \rangle}}{\pi\, \langle \boldsymbol{P}_{hT,q}^2 \rangle}$$

sum of Gaussians $\neq$ Gaussian

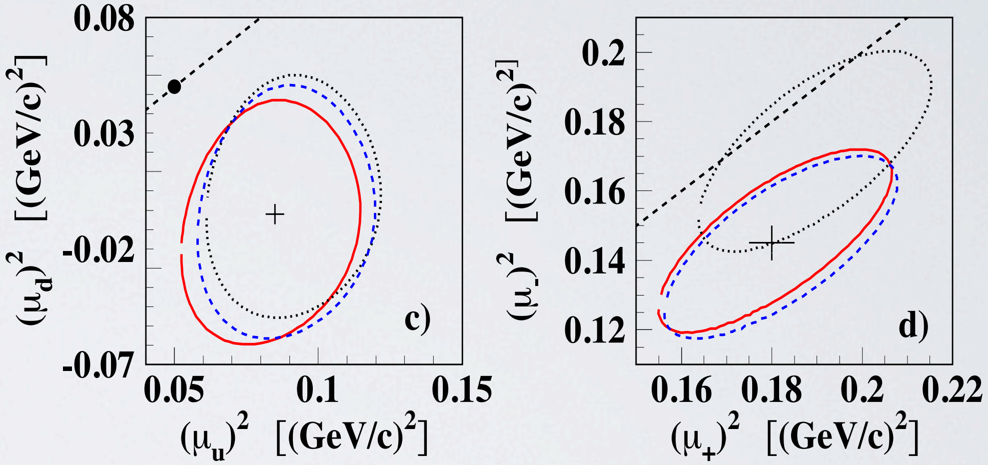
first hints on “$\mathbf{k}_\perp$ flavor dependence”



Jefferson Lab

Asaturyan *et al.* (E00-108),
P. R. C**85** (12) 015202

conclusions



f_1^q

up wider
than down

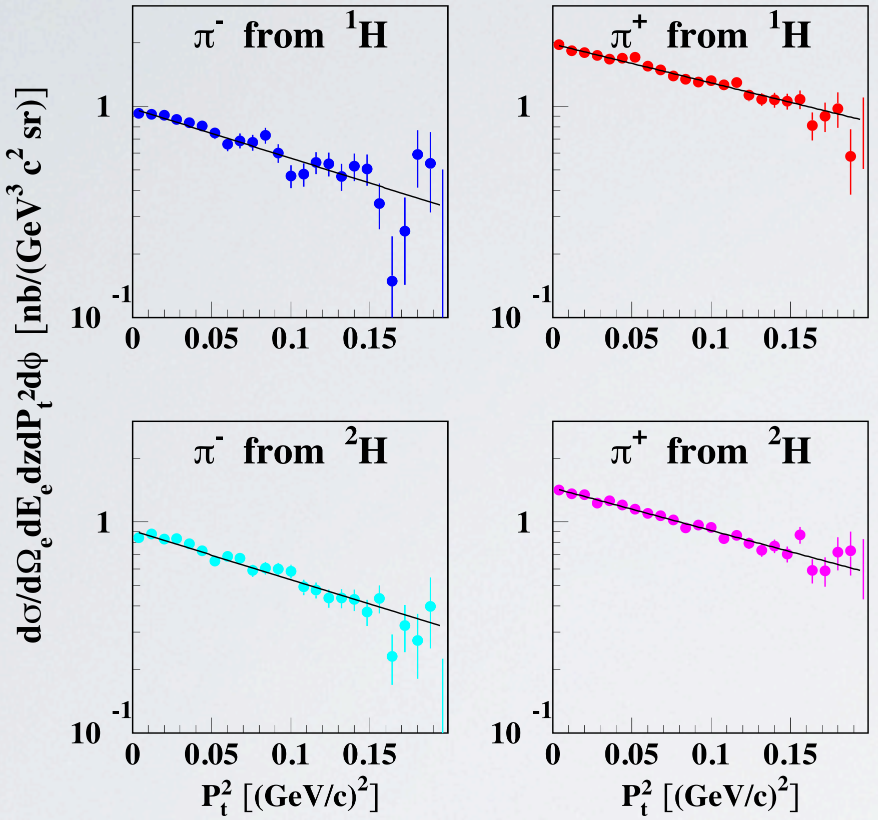
$D_1^{q\to h}$

favored wider
than unfavored

but not a multidimensional analysis :

- no binning in x & z
- no sea contribution
- no K in final state

first hints on "$\mathbf{k}_\perp$ flavor dependence"



Jefferson Lab

Asaturyan *et al.* (E00-108),
P. R. C**85** (12) 015202

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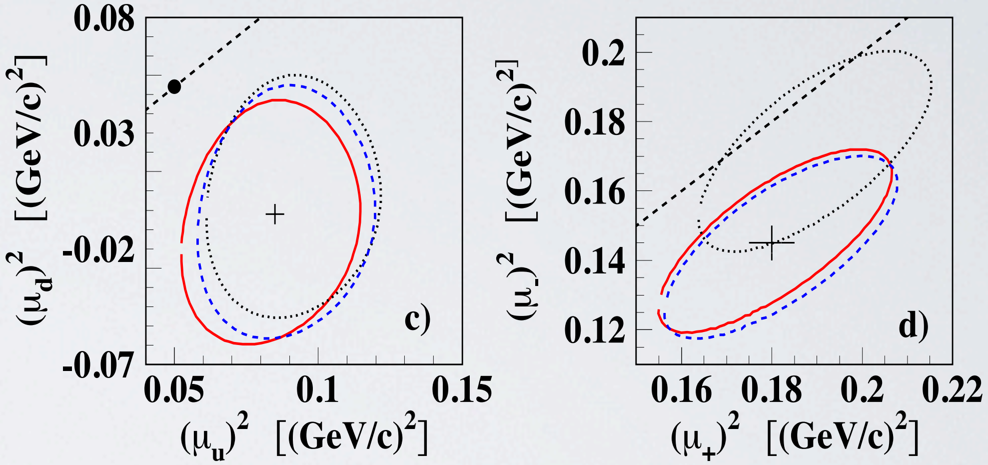

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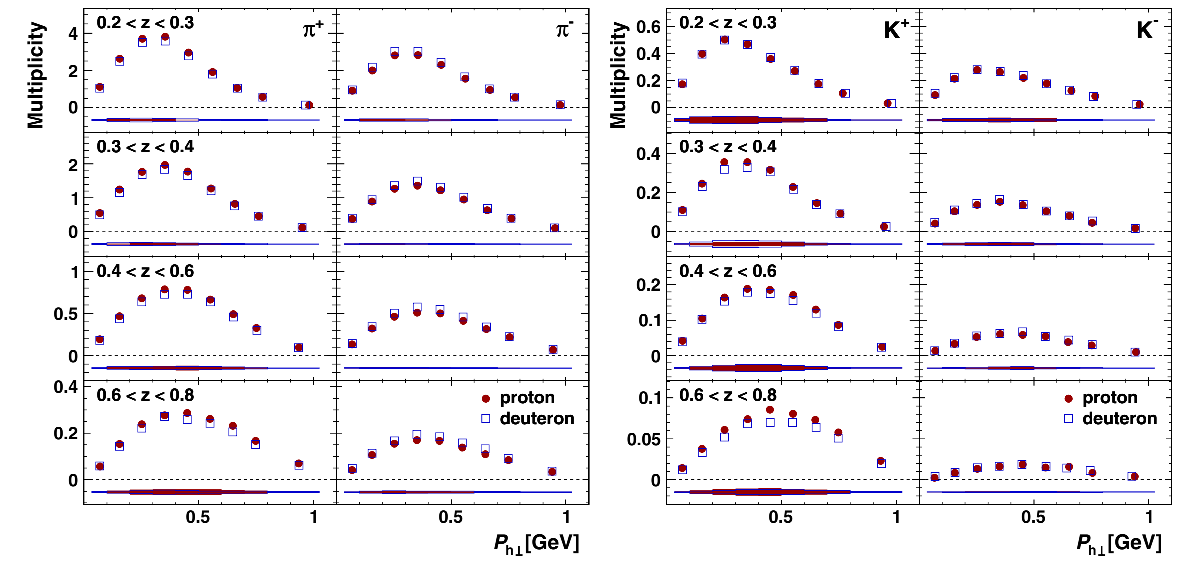
new data coming from JLab
(see Osipenko's talk)

recent data on multiplicities

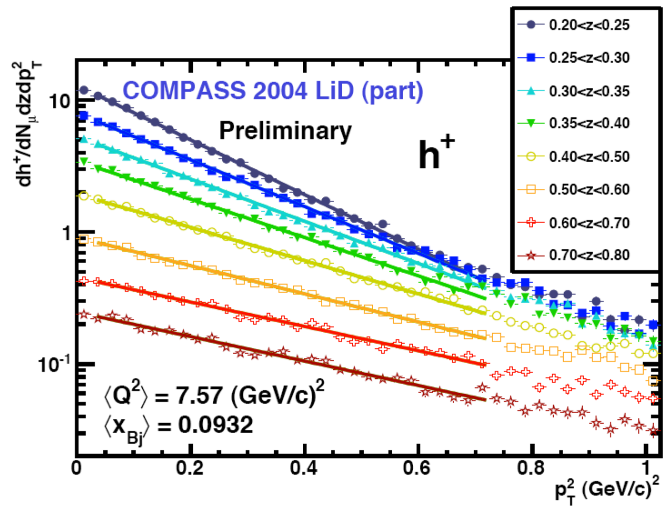
Airapetian *et al.*, P.R. D**87** (13) 074029



- target: proton, deuteron
- final state: π^+, π^-, K^+, K^-

just published!

Adolph *et al.*, E.P.J. **C73** (13) 2531, arXiv:1305.7317

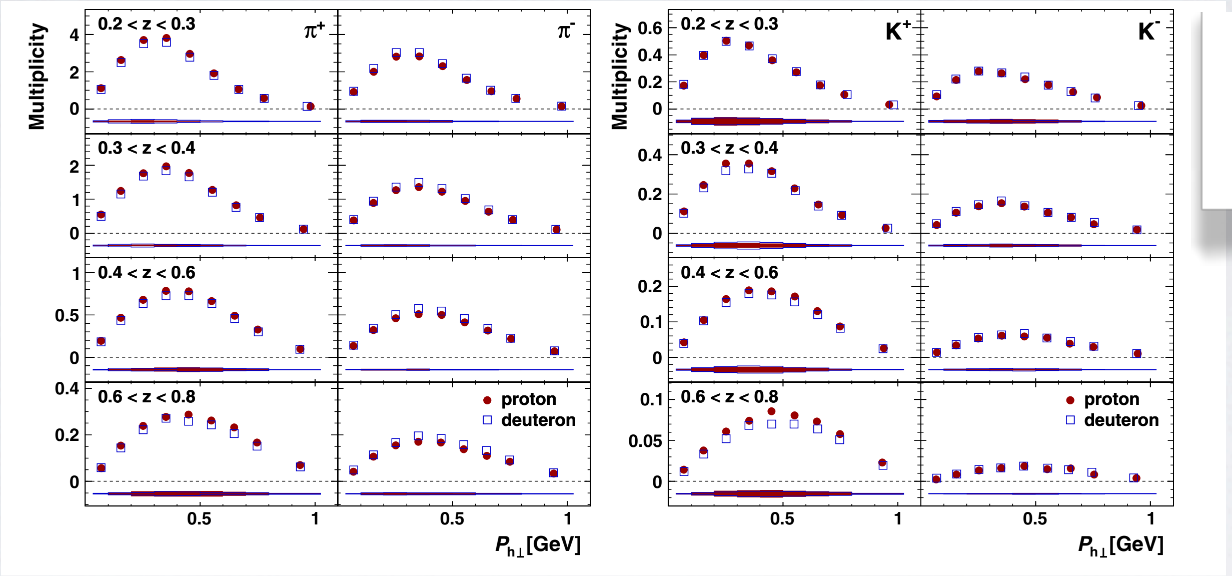


large statistics & kin. coverage, but
- target: deuteron
- final state: h^+, h^- unidentified
(at the time of this work)

now also π^+,π^-,K^+,K^- (see Makke's talk)

recent data on multiplicities

Airapetian *et al.*, P.R. D**87** (13) 074029



ideal for flavor analysis

- target: proton, deuteron
- final state: π^+, π^-, K^+, K^-

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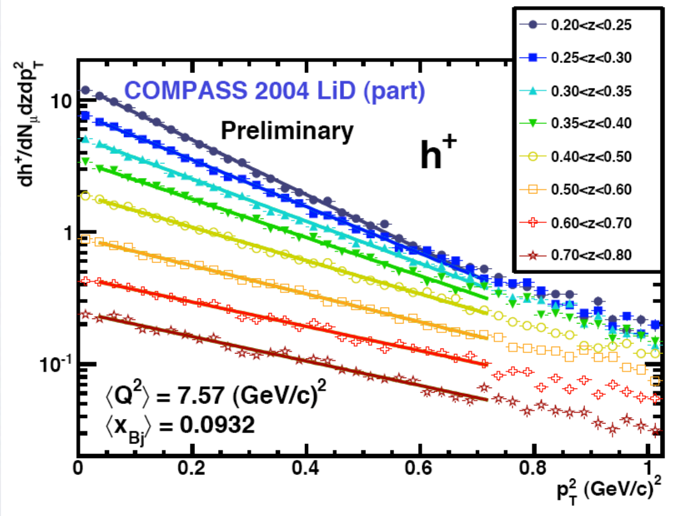

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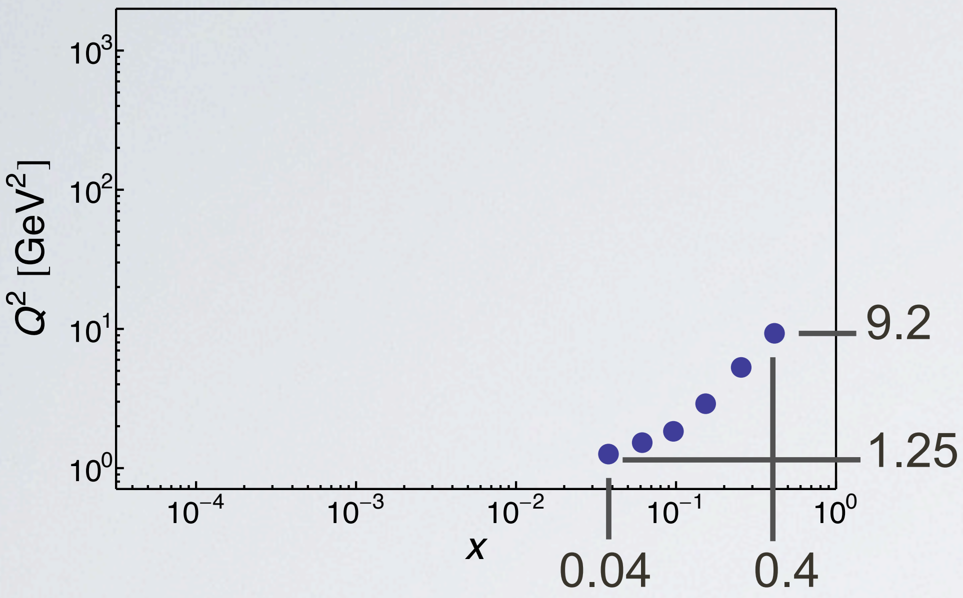
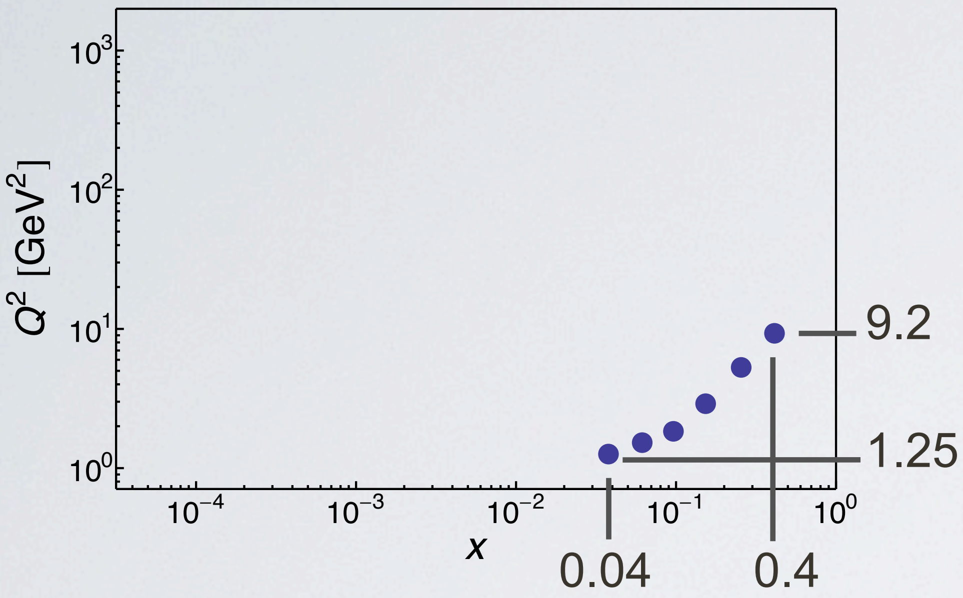
selection of data



limited (x, Q^2) range:	6 bins	x
$0.1 \le z \le 0.9$	8 bins	x
$0.1 \le \lvert \mathbf{P}_{hT} \rvert \le 1$ GeV	7 bins	x
p , D	2 targets	x
π^+, π^-, K^+, K^-	4 final h's	
total	2688 points	

selection of data



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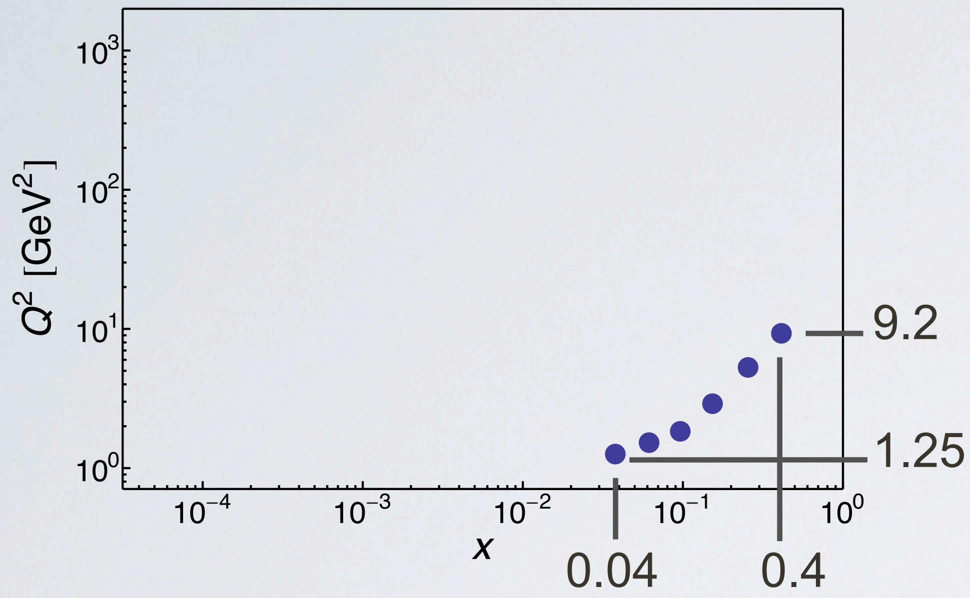
total 2688 points

- TMDs valid for $\mathbf{P}_{hT}^2 \ll Q^2$: cut first bin $Q^2 = 1.4$ GeV^2 (↔ lowest x)
- cut last bin $z = 0.9$ as in DSS (and use VM subtracted set)
- cut $\lvert \mathbf{P}_{hT} \rvert < 0.15$ GeV $\Leftarrow$ problem to be fixed

total analyzed 1538 points $\approx$ 60% of 2688

selection of data



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total analyzed 1538 points ≈ 60% of 2688

limited Q^2 range ⇒ safely neglect evolution everywhere

our analysis : assumptions & parameters

TMD PDF

$$f_1^q(x, \boldsymbol{k}_\perp^2) = f_1^q(x)\Big|_{Q^2=2.4\,\mathrm{GeV}^2} \frac{e^{-\boldsymbol{k}_\perp^2/\langle \boldsymbol{k}_{\perp,q}^2\rangle}}{\pi\langle \boldsymbol{k}_{\perp,q}^2\rangle}$$

MSTW08 LO

Martin *et al*., E.P.J. **C63** (09) 189

TMD FF

$$D_1^{q\to h}(z, \boldsymbol{P}_\perp^2) = D_1^{q\to h}(z)\Big|_{Q^2=2.4\,\mathrm{GeV}^2} \frac{e^{-\boldsymbol{P}_\perp^2/\langle \boldsymbol{P}_{\perp,q\to h}^2\rangle}}{\pi\langle \boldsymbol{P}_{\perp,q\to h}^2\rangle}$$

DSS LO

De Florian *et al*., P.R. D**75** (07) 114010

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DSS LO

De Florian *et al.*, P.R. D**75** (07) 114010

x-dependent width

$$\langle \boldsymbol{k}_{\perp,\,q}^2\rangle(x) = \widehat{\langle \boldsymbol{k}_{\perp,\,q}^2\rangle} \frac{(1-x)^\alpha\, x^\sigma}{(1-\hat{x})^\alpha\, \hat{x}^\sigma}$$

$$\widehat{\langle \boldsymbol{k}_{\perp,\,q}^2\rangle} = \langle \boldsymbol{k}_{\perp,\,q}^2\rangle(\hat{x} = 0.1)$$

5 parameters

uu_v	d_v	sea	α	σ
$\widehat{\langle \boldsymbol{k}_{\perp,\,u_v}^2\rangle}$	$\widehat{\langle \boldsymbol{k}_{\perp,\,d_v}^2\rangle}$	$\widehat{\langle \boldsymbol{k}_{\perp,\,\mathrm{sea}}^2\rangle}$		

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$\langle \widehat{\boldsymbol{k}^2_{\perp,\,u_v}}\rangle$	$\langle \widehat{\boldsymbol{k}^2_{\perp,\,d_v}}\rangle$	$\langle \widehat{\boldsymbol{k}^2_{\perp,\,\mathrm{sea}}}\rangle$	↓	↓
			[0,2]	[-0.3,0.1]

randomly chosen in

(⇔ loosely bound)

our analysis : assumptions & parameters

TMD PDF

$$f_1^q(x,\boldsymbol{k}_\perp^2) = f_1^q(x)\Big|_{Q^2=2.4\,\mathrm{GeV}^2}\ \frac{e^{-\boldsymbol{k}_\perp^2/\langle \boldsymbol{k}_{\perp,q}^2\rangle}}{\pi\langle \boldsymbol{k}_{\perp,q}^2\rangle}$$

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$$D_1^{q\to h}(z,\boldsymbol{P}_\perp^2) = D_1^{q\to h}(z)\Big|_{Q^2=2.4\,\mathrm{GeV}^2}\ \frac{e^{-\boldsymbol{P}_\perp^2/\langle \boldsymbol{P}_{\perp,q\to h}^2\rangle}}{\pi\langle \boldsymbol{P}_{\perp,q\to h}^2\rangle}$$

DSS LO

De Florian *et al.*, P.R. D**75** (07) 114010

z-dependent width

$$\langle \boldsymbol{P}_{\perp,\,q\to h}^2\rangle(z) = \langle \widehat{\boldsymbol{P}_{\perp,\,q\to h}^2}\rangle\ \frac{(z^\beta+\delta)\,(1-z)^\gamma}{(\hat{z}^\beta+\delta)\,(1-\hat{z})^\gamma}$$

$$\langle \widehat{\boldsymbol{P}_{\perp,\,q\to h}^2}\rangle = \langle \boldsymbol{P}_{\perp,\,q\to h}^2\rangle(\hat{z}\ =\ 0.5)$$

7 parameters

u→π$^+$, d→π$^-$ d̄→π$^+$, ū→π$^-$	u→K$^+$ ū→K$^-$	s→K$^+$ s̄→K$^-$	other	β, δ, γ
$\langle \widehat{\boldsymbol{P}_{\perp,\,\mathrm{fav}}^2}\rangle$	$\langle \widehat{\boldsymbol{P}_{\perp,\,uK}^2}\rangle$	$\langle \widehat{\boldsymbol{P}_{\perp,\,sK}^2}\rangle$	$\langle \widehat{\boldsymbol{P}_{\perp,\,\mathrm{unfav}}^2}\rangle$	

[0.125, 0.25]

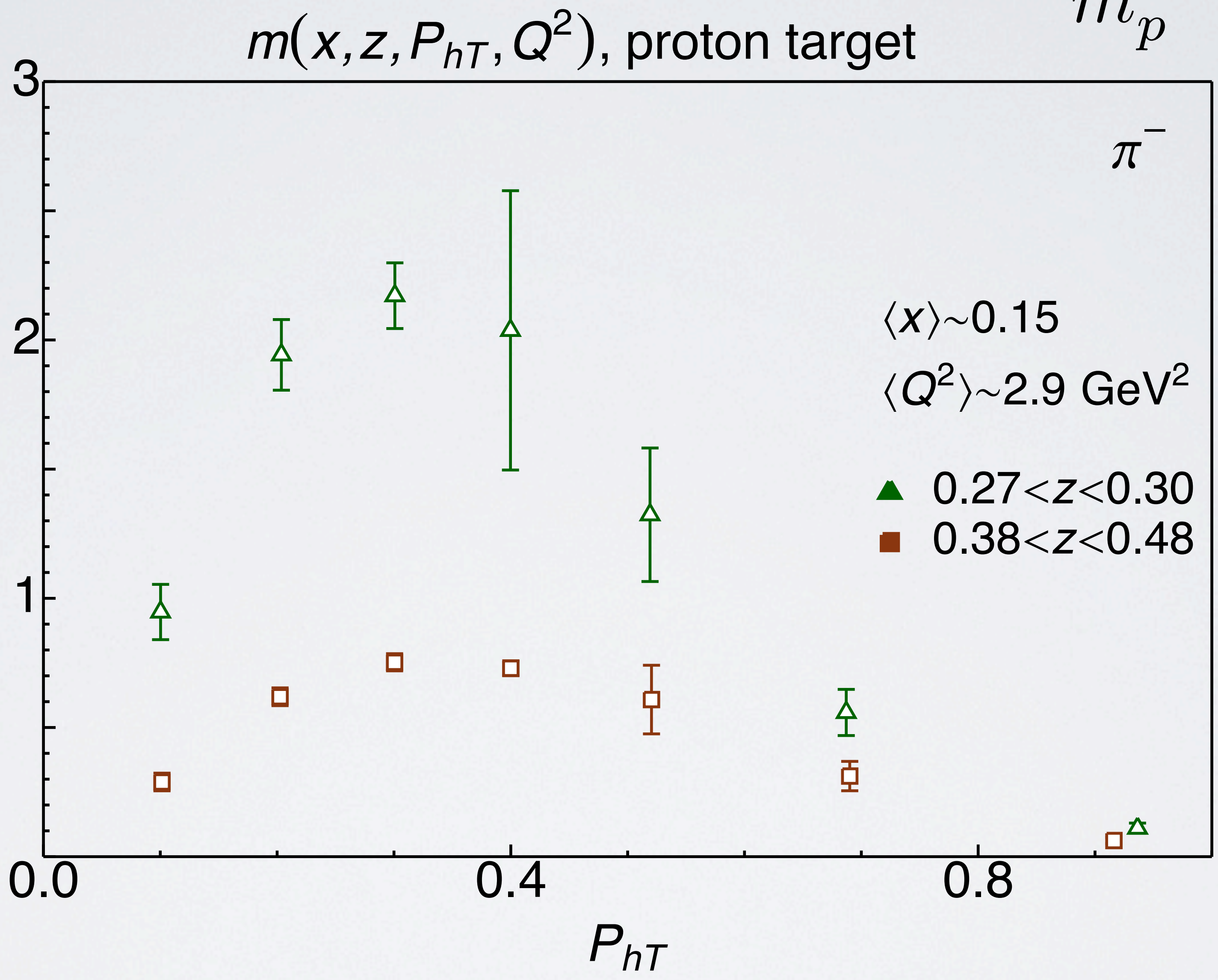
ditto

inspired by NNPDF
(see Nocera's talk)

our fitting procedure

used in transversity extraction
(see Aurore's talk)

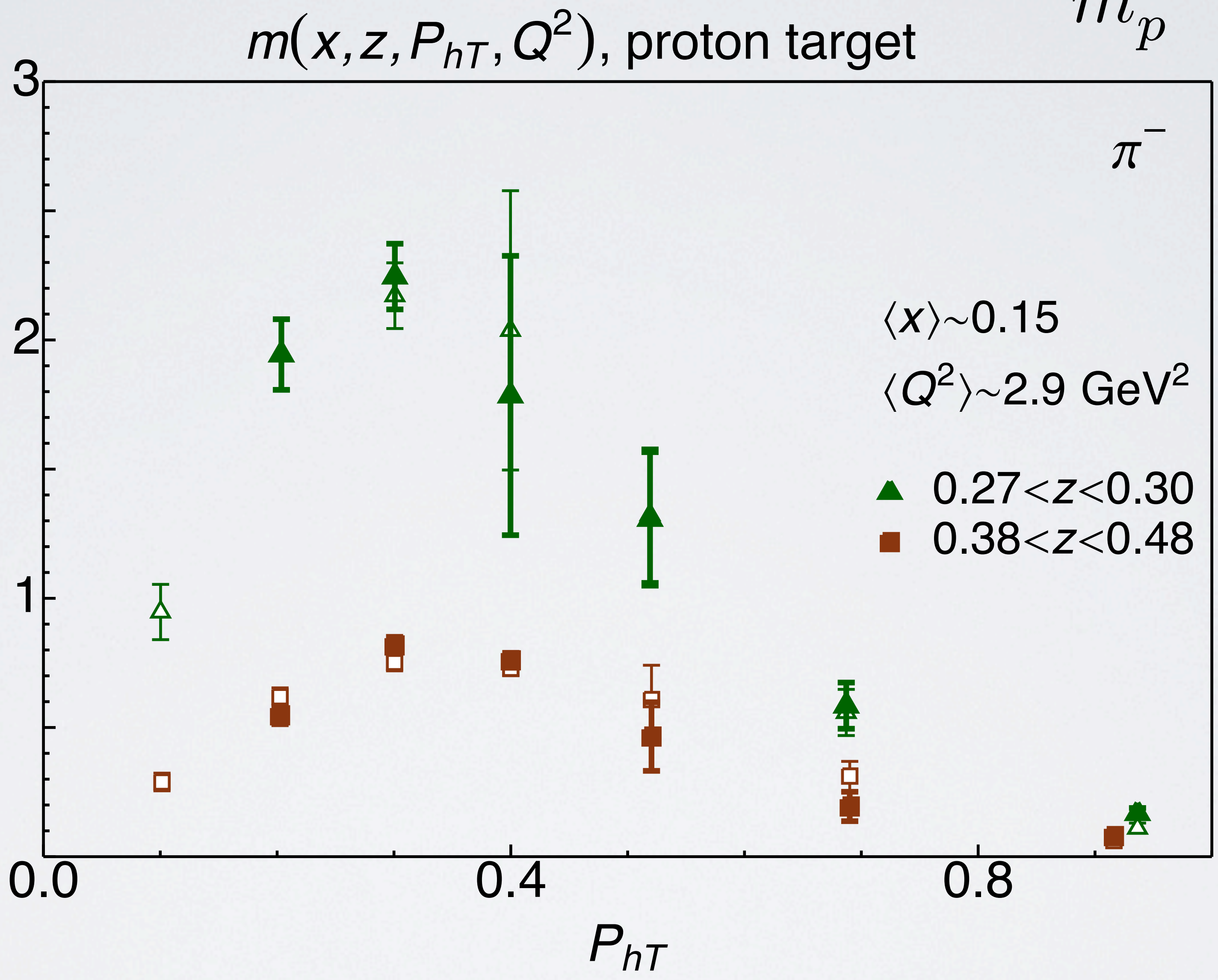
$m_p^{\pi^-}$



sample of original data

our fitting procedure

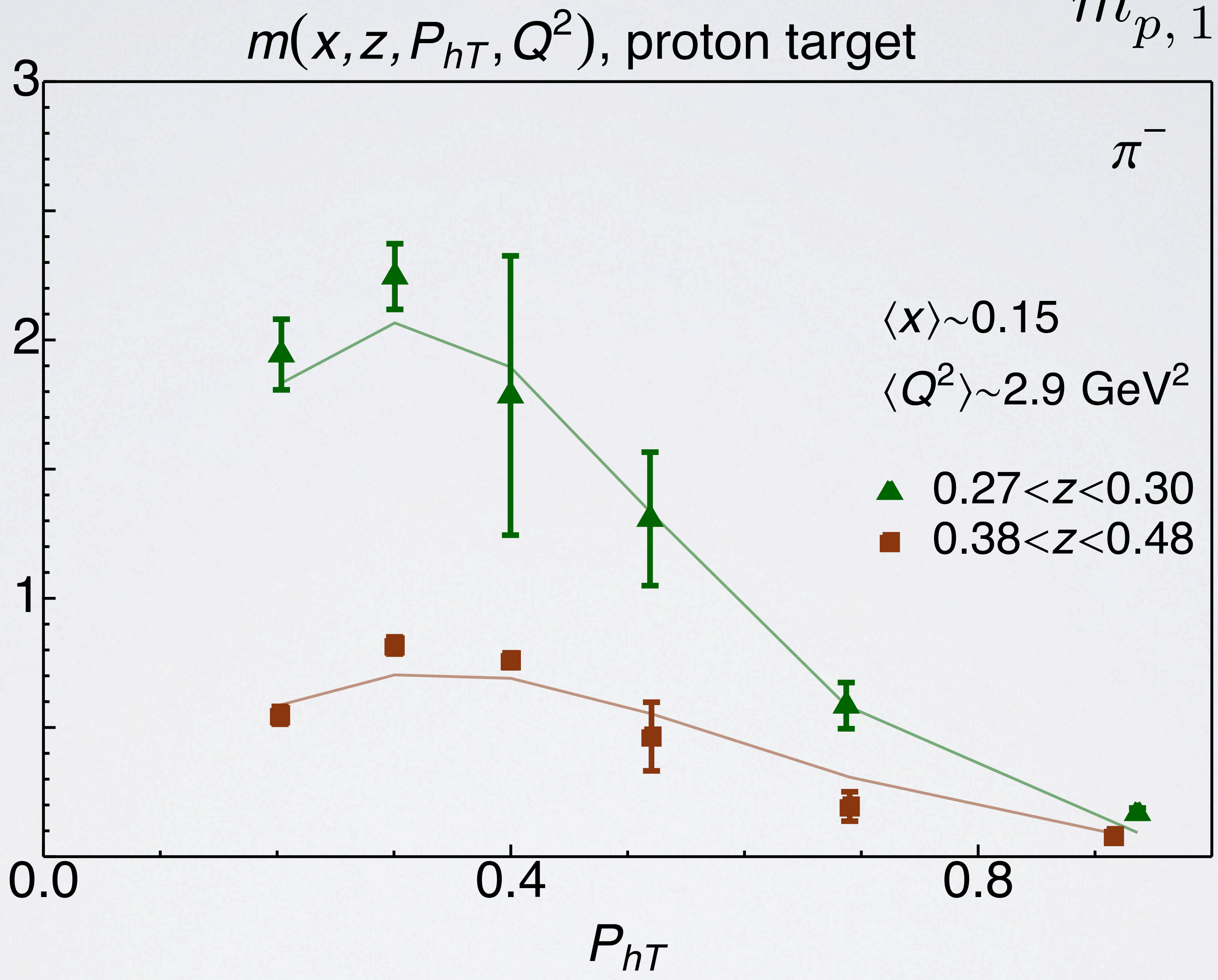
$m_p^{\pi^-}$



data are replicated with Gaussian noise
(within exp. variance)

our fitting procedure

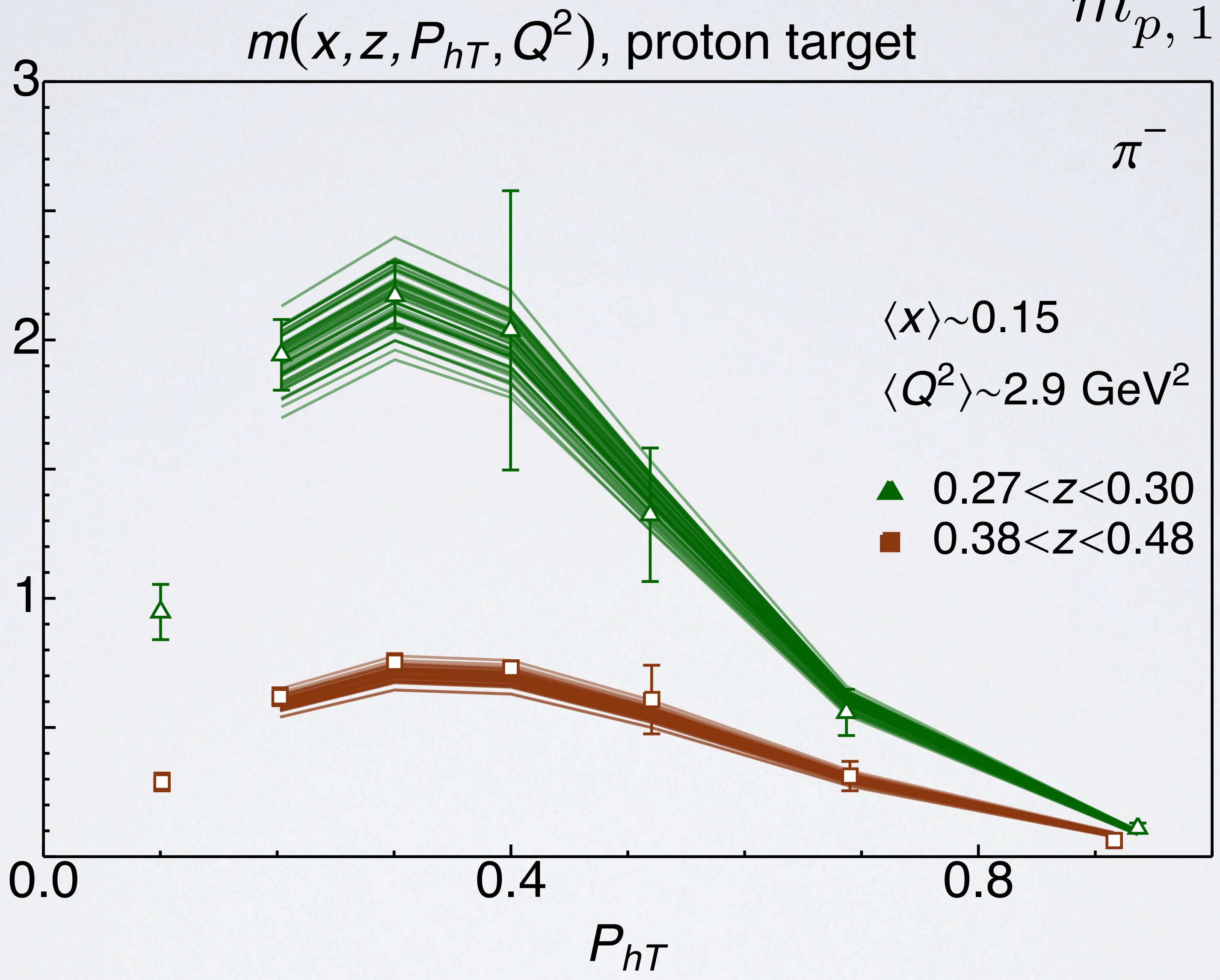
$m_{p,\,1}^{\pi^-}$



fit the replicated data

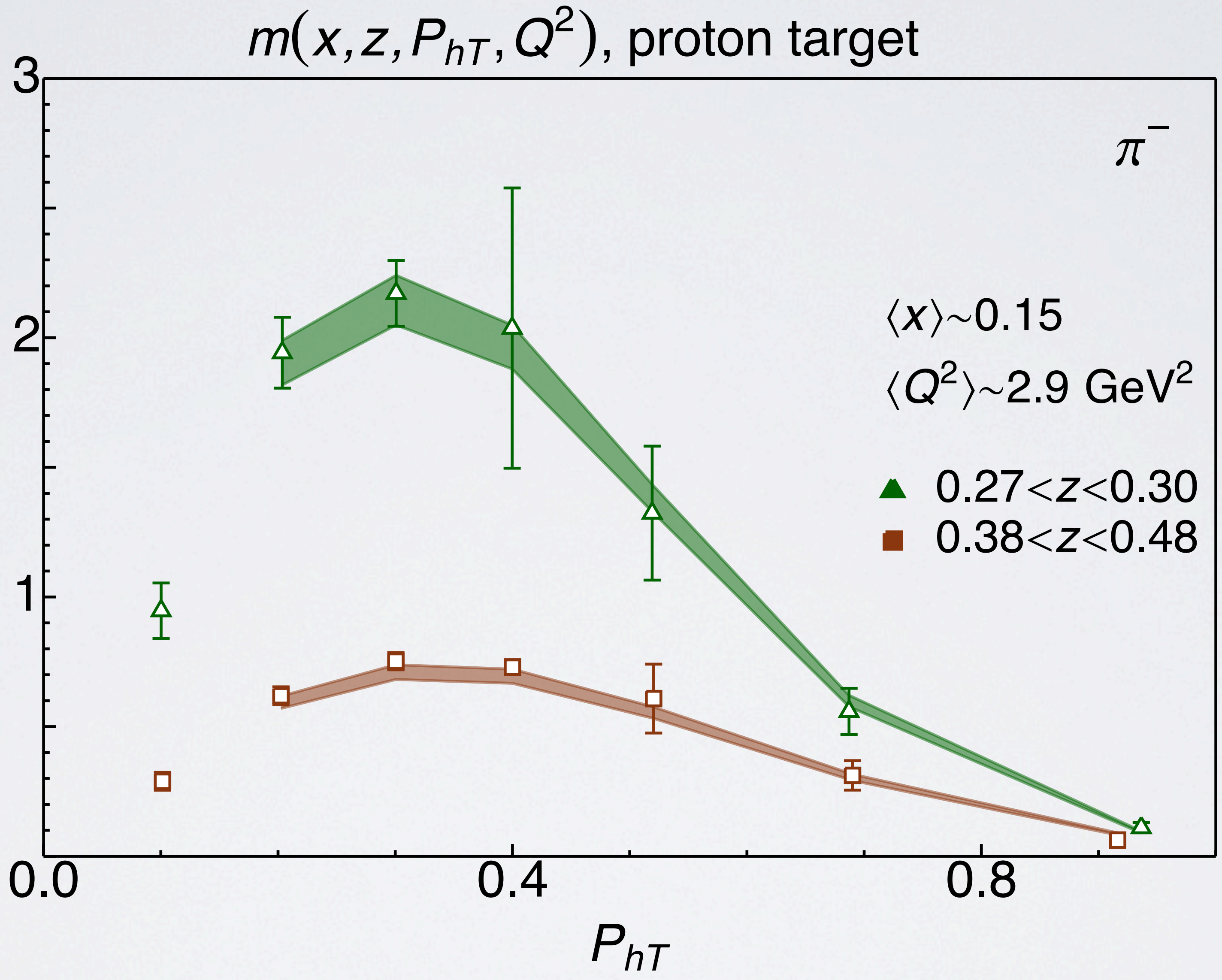
our fitting procedure

$m^{\pi^-}_{p,\,1-200}$



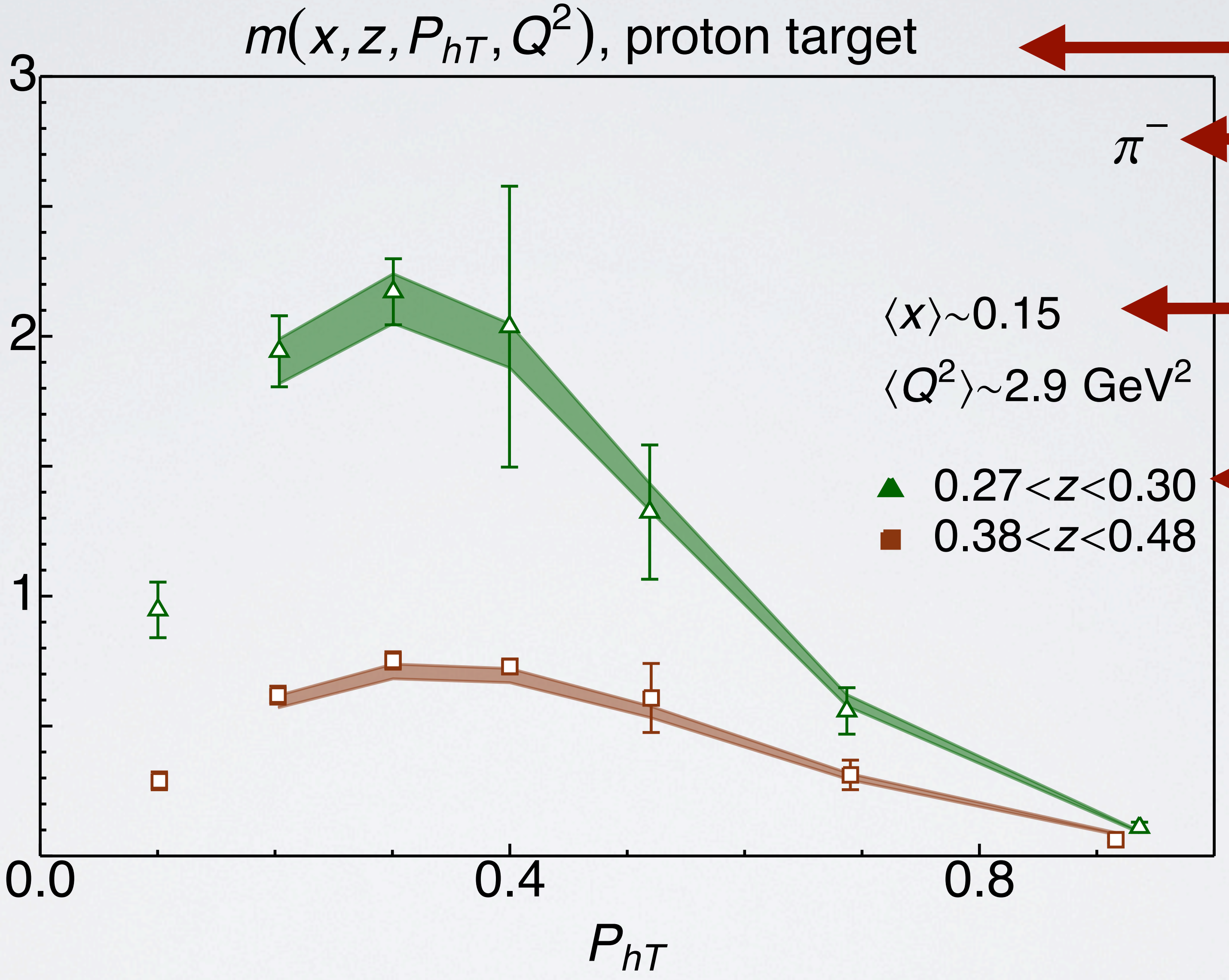
procedure repeated 200 times
(until reproduce mean and std. deviation of original data)

our fitting procedure



for each point, a central 68% confidence interval is identified
(distribution is not necessarily Gaussian)

our fitting procedure



× 2 targets

× 4 final h's

× 5 x-bins

× 7 z-bins

for each point, a central 68% confidence interval is identified
(distribution is not necessarily Gaussian)

quality of the fit

proton target global χ^2 / d.o.f. = 1.63 ± 0.12

no flavor dep. 1.72 ± 0.11

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π^-

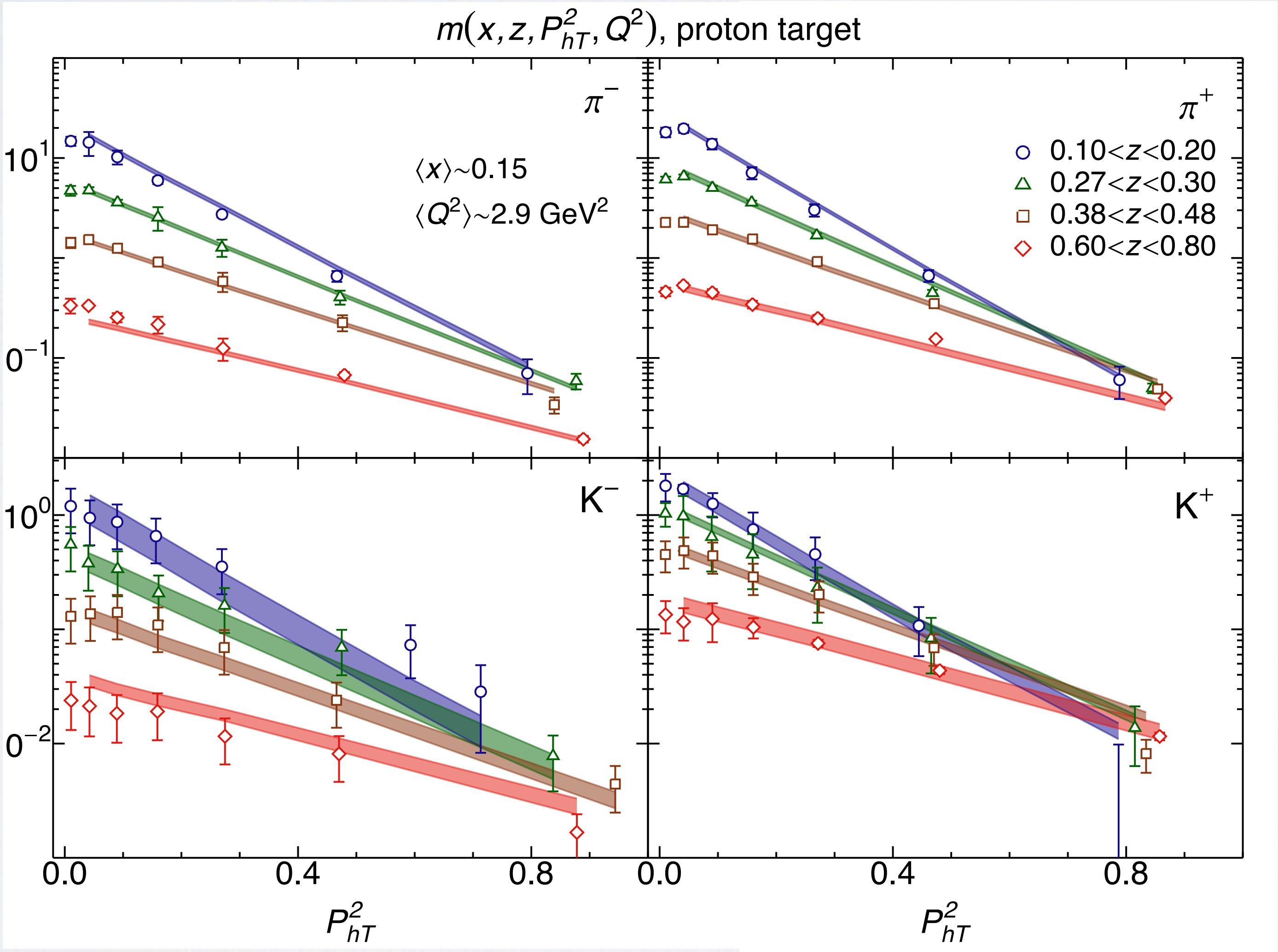
1.80 ± 0.27

1.83 ± 0.25

K^-

0.78 ± 0.15

0.87 ± 0.16



π^+

2.64 ± 0.21

2.89 ± 0.23

K^+

0.46 ± 0.07

0.43 ± 0.07

for more details, see arXiv:1309.3507 [hep-ph]

Results – Scenario : **no** flavor dep.

TMD FF

$$\begin{tikzcd}
\langle\widehat{\boldsymbol{P}^2_{\perp,\,\mathrm{fav}}}\rangle \arrow[rd] & \\
\langle\widehat{\boldsymbol{P}^2_{\perp,\,uK}}\rangle \arrow[r] & \langle\widehat{\boldsymbol{P}^2_{\perp}}\rangle \\
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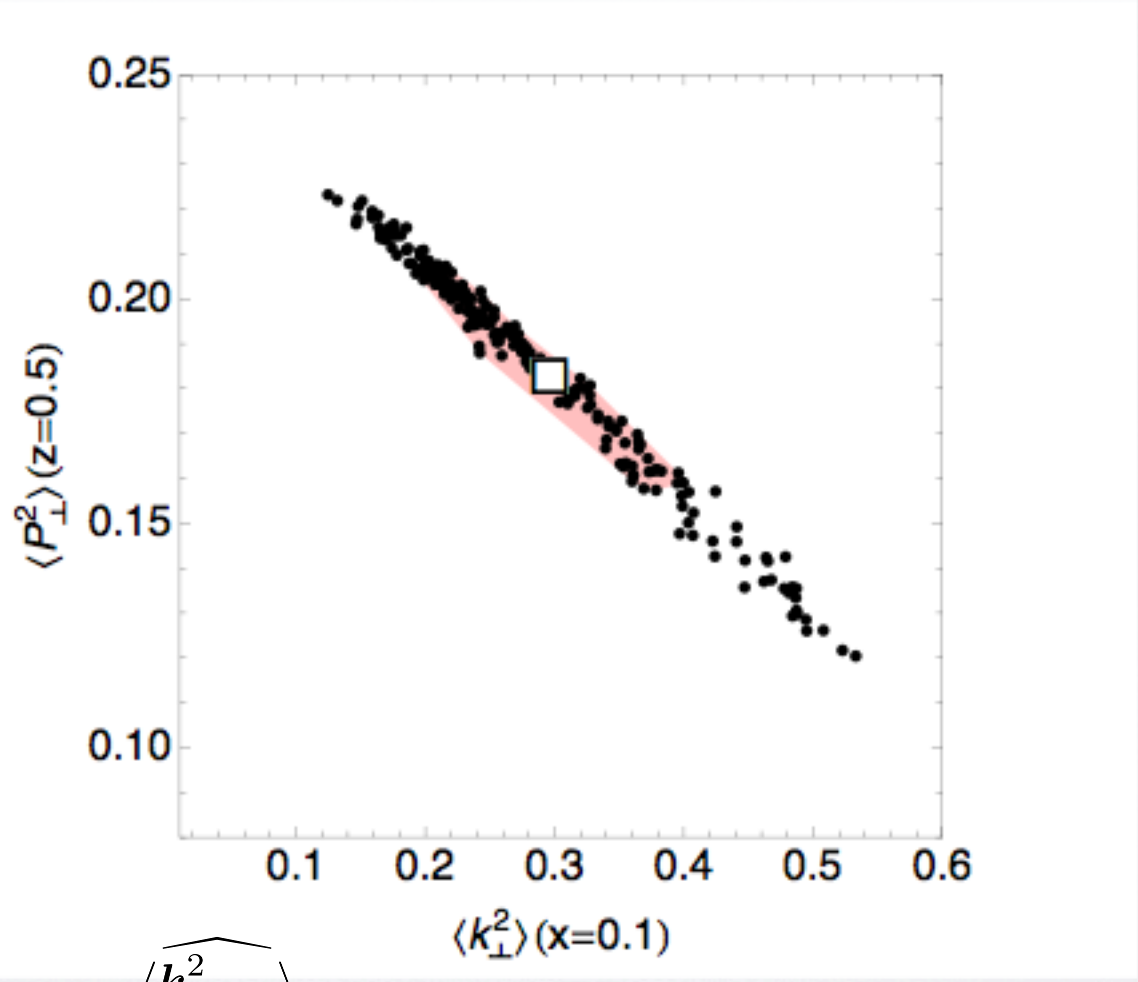
\end{tikzcd}$$



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TMD PDF

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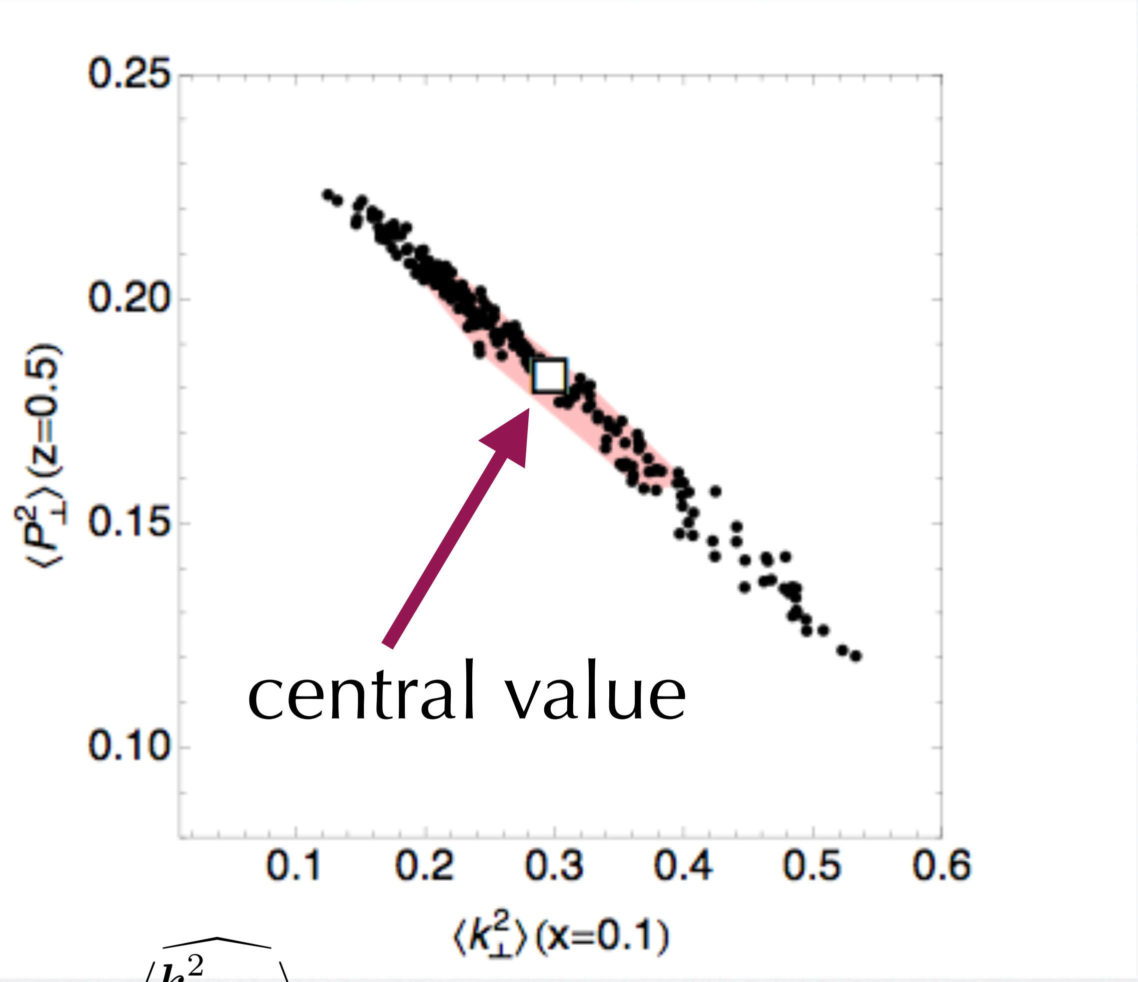

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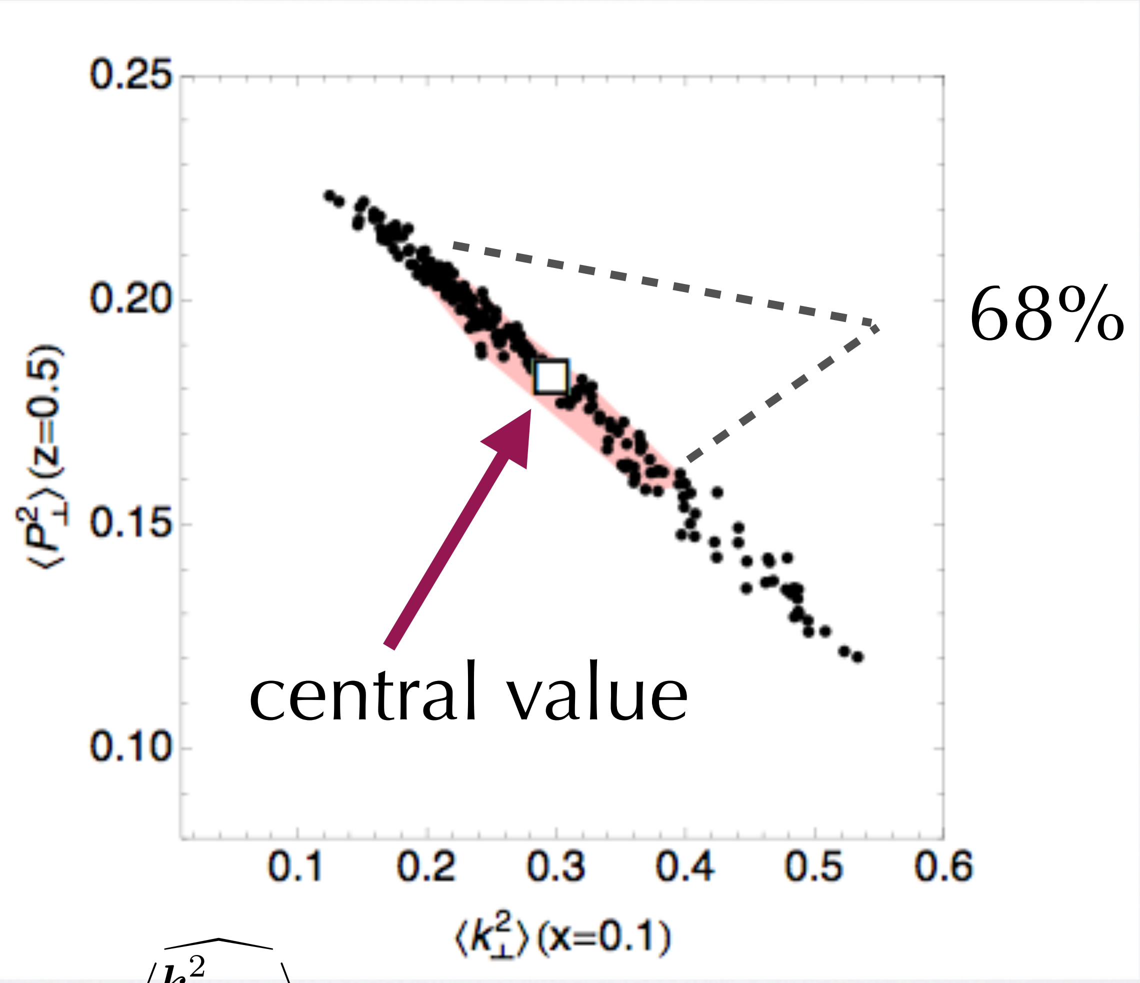

68% confidence band

central value

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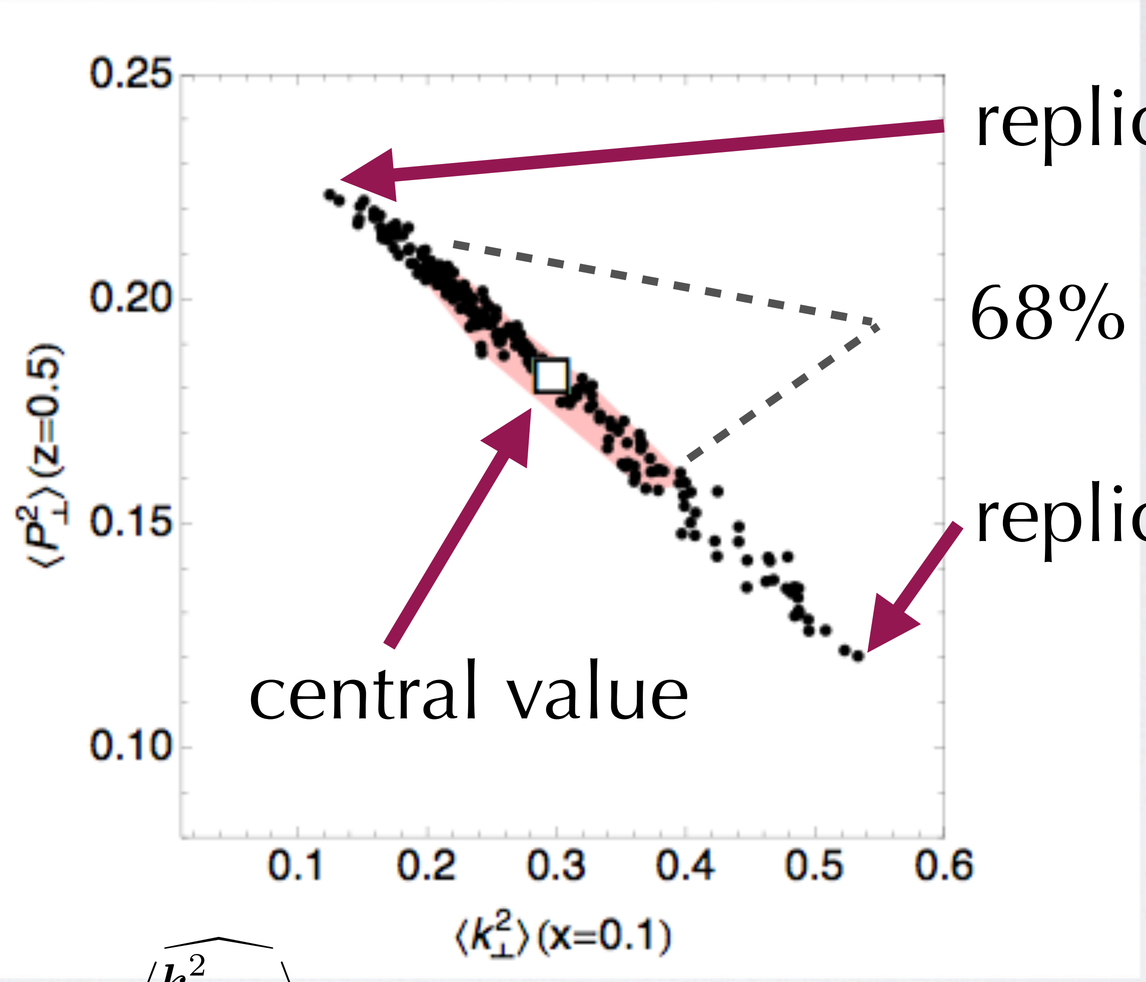

replica 51 χ^2/dof = 1.57

68% confidence band

replica 33 χ^2/dof = 1.90

central value

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\widehat{\langle \boldsymbol{k}^2_{\perp,\,u_v}\rangle} \arrow[dr] & \\
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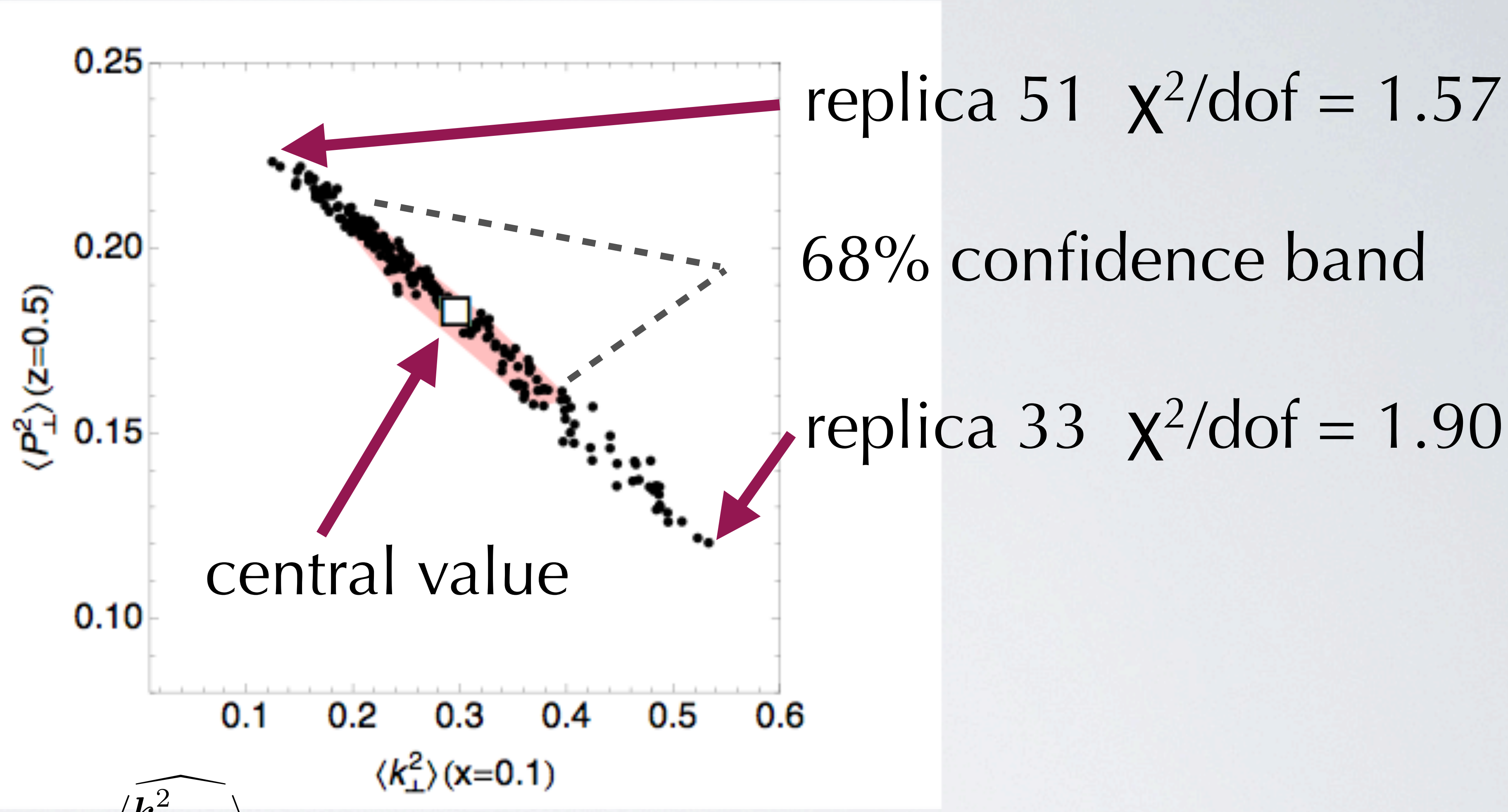

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\widehat{\langle \boldsymbol{k}^2_{\perp,\,\mathrm{sea}}\rangle} \arrow[ur] &
\end{tikzcd}$$

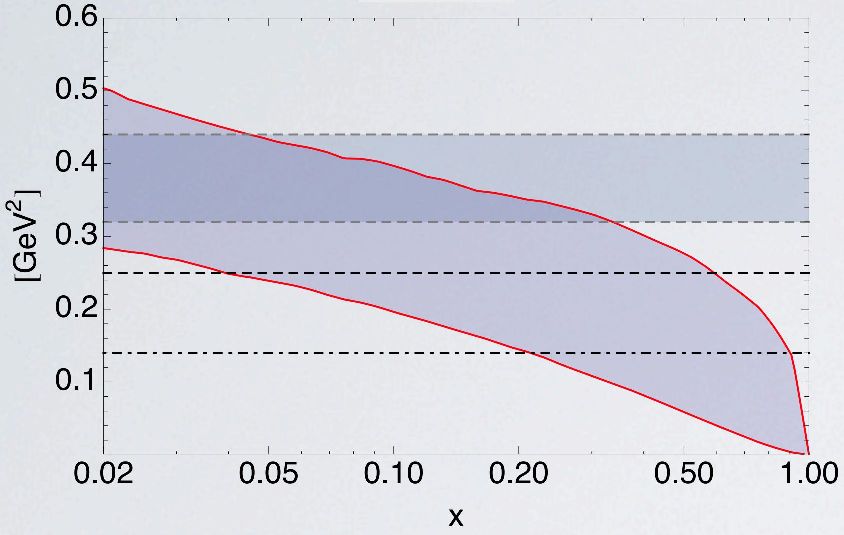
TMD PDF

strong anticorrelation between **distribution** and **fragmentation**

anticorrelation and 68% band

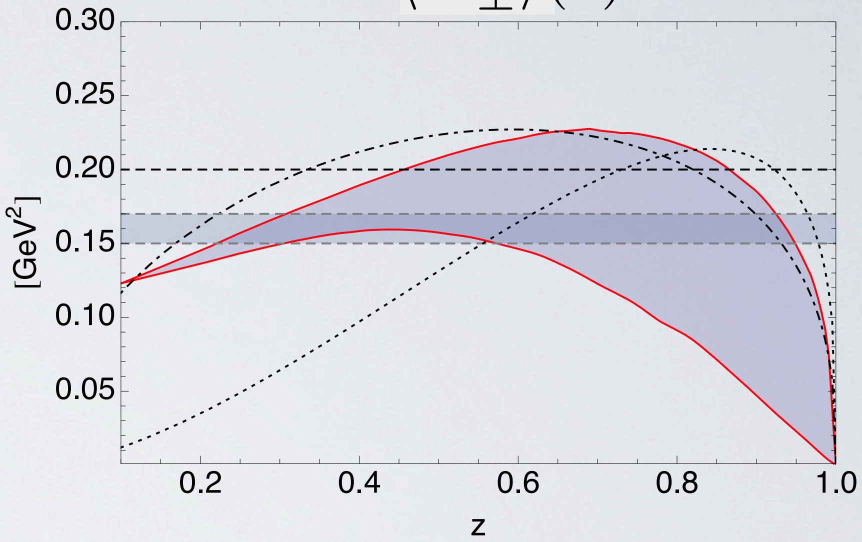
TMD PDF

$\langle \boldsymbol{k}_\perp^2 \rangle (x)$



TMD FF

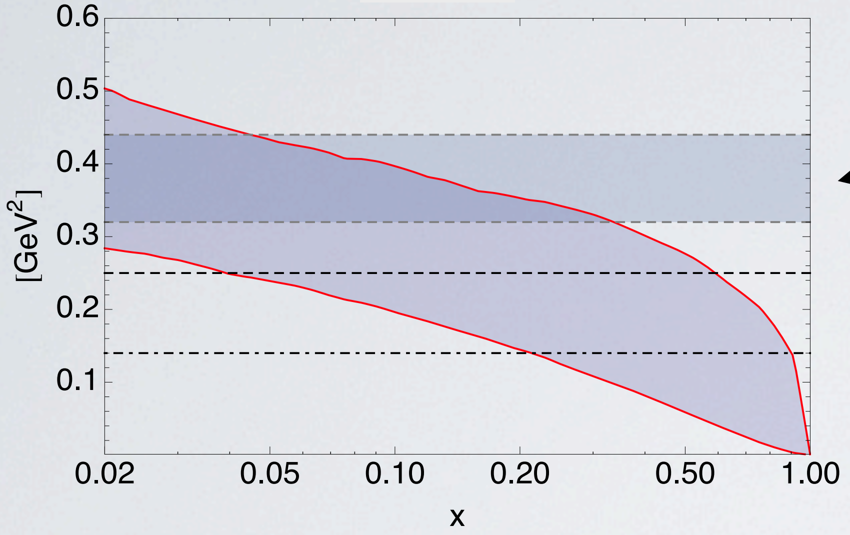
$\langle \boldsymbol{P}_\perp^2 \rangle (z)$



anticorrelation and 68% band

TMD PDF

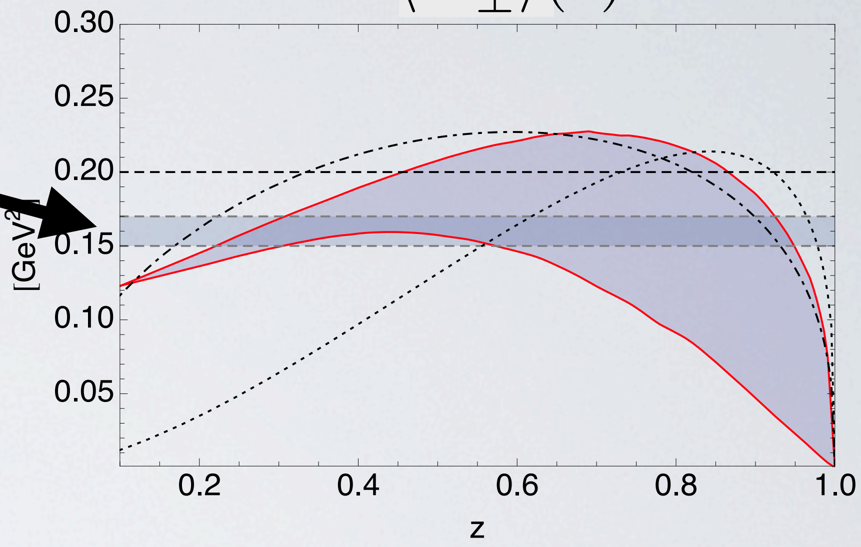
$\langle \boldsymbol{k}_\perp^2 \rangle(x)$



Schweitzer *et al.*
P.R. D**81** (10) 094019

TMD FF

$\langle \boldsymbol{P}_\perp^2 \rangle(z)$



anticorrelation and 68% band

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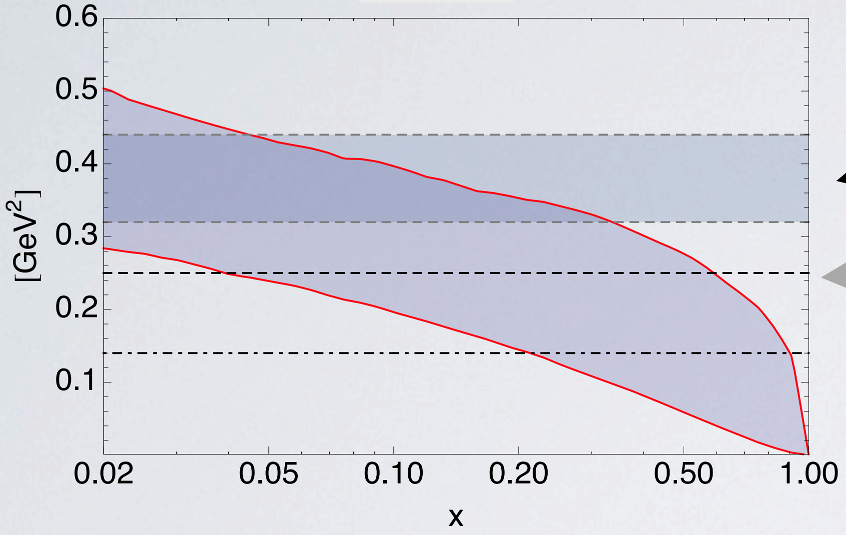
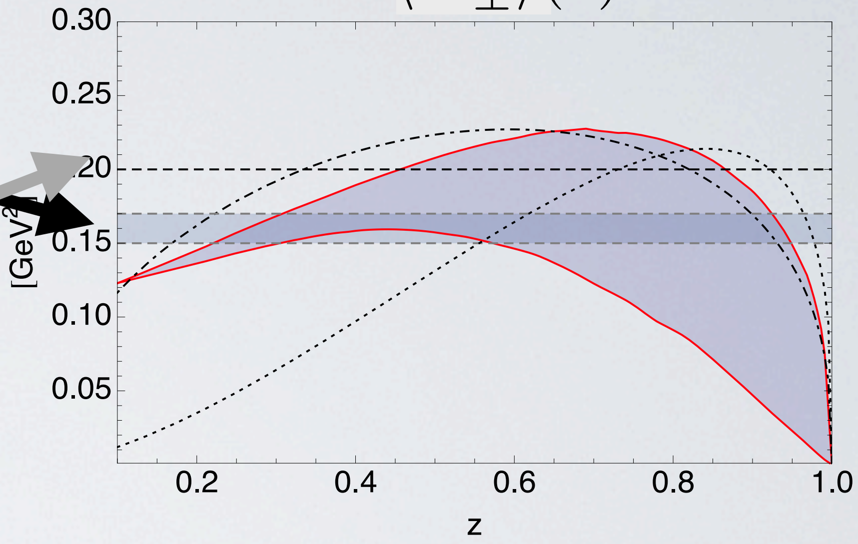


Schweitzer *et al.*
P.R. D**81** (10) 094019

Anselmino *et al.*
P.R. D**71** (05) 074006

TMD FF

$\langle \boldsymbol{P}_\perp^2 \rangle(z)$



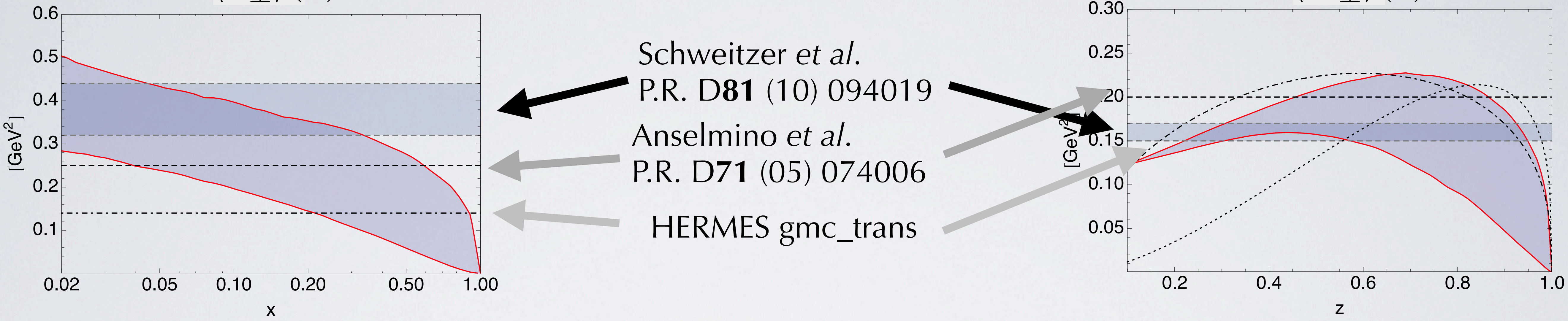
anticorrelation and 68% band

TMD PDF

$\langle \boldsymbol{k}_\perp^2 \rangle(x)$

TMD FF

$\langle \boldsymbol{P}_\perp^2 \rangle(z)$



Schweitzer *et al.*
P.R. D**81** (10) 094019

Anselmino *et al.*
P.R. D**71** (05) 074006

HERMES gmc_trans

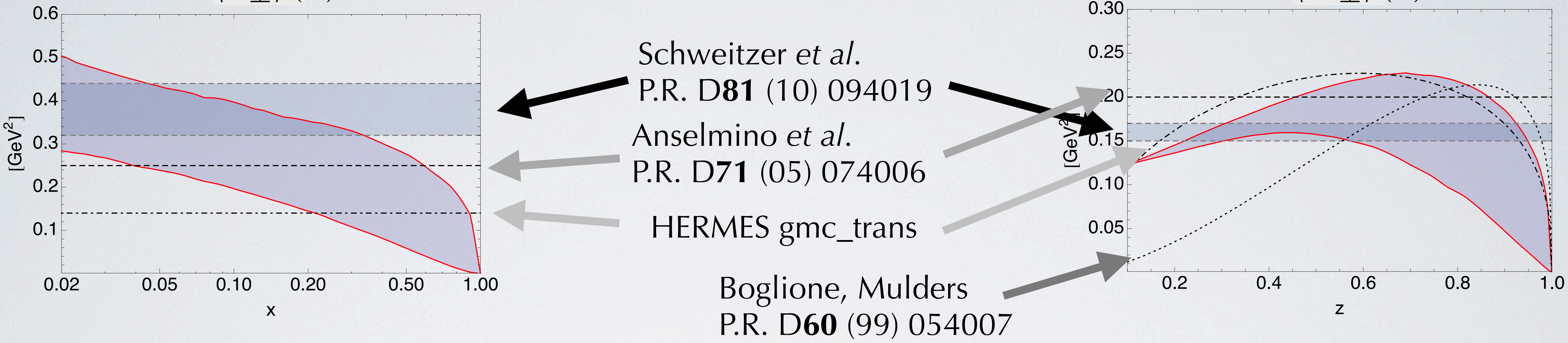
anticorrelation and 68% band

TMD PDF

$\langle \boldsymbol{k}_\perp^2 \rangle(x)$

TMD FF

$\langle \boldsymbol{P}_\perp^2 \rangle(z)$



Schweitzer *et al.*
P.R. D**81** (10) 094019

Anselmino *et al.*
P.R. D**71** (05) 074006

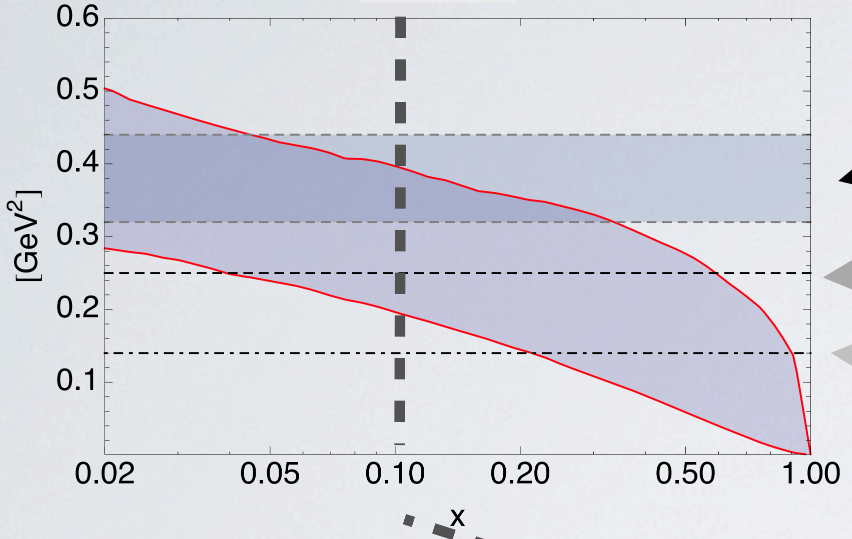
HERMES gmc_trans

Boglione, Mulders
P.R. D**60** (99) 054007

anticorrelation and 68% band

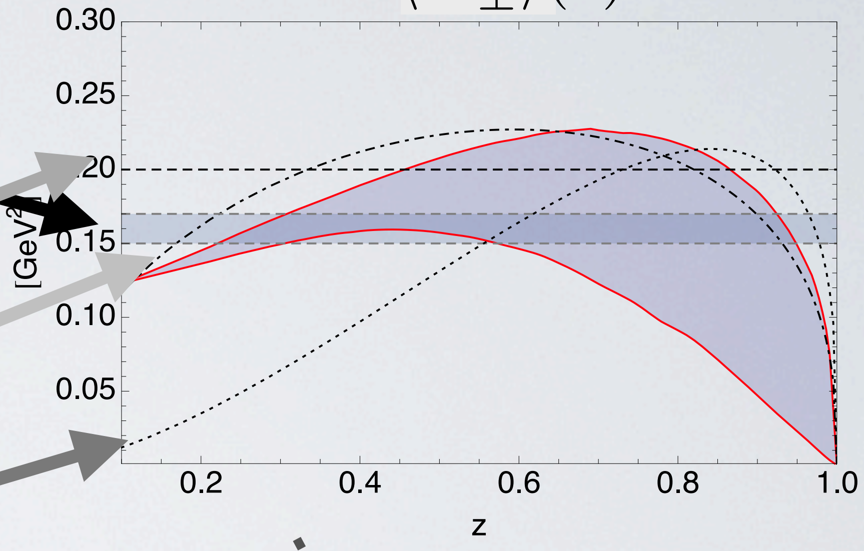
TMD PDF

$\langle \boldsymbol{k}_\perp^2 \rangle(x)$



TMD FF

$\langle \boldsymbol{P}_\perp^2 \rangle(z)$



Schweitzer *et al.*
P.R. D**81** (10) 094019

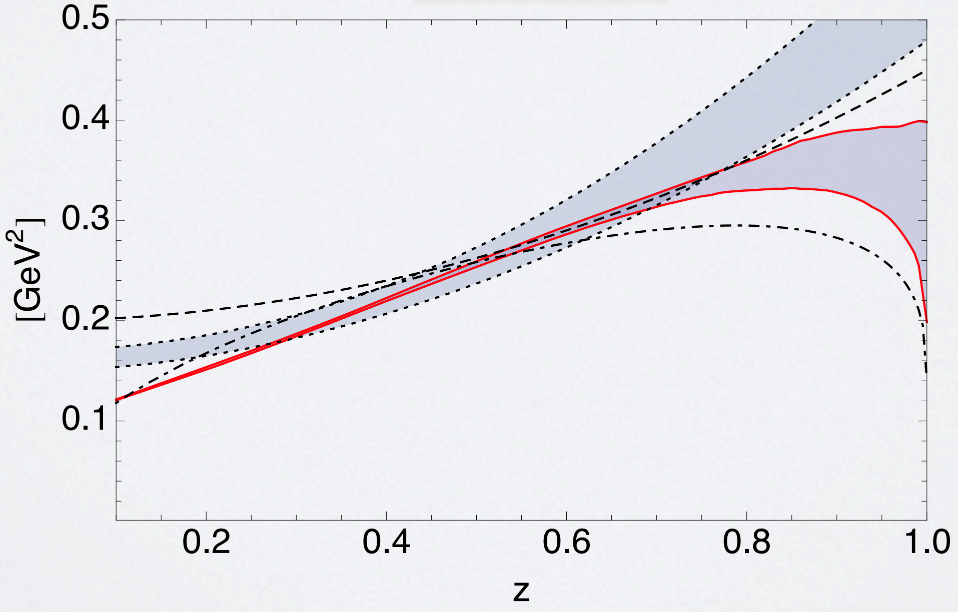
Anselmino *et al.*
P.R. D**71** (05) 074006

HERMES gmc_trans

Boglione, Mulders
P.R. D**60** (99) 054007

$$\langle \boldsymbol{P}_{hT}^2 \rangle(x=0.1, z) = z^2 \langle \boldsymbol{k}_\perp^2 \rangle(x=0.1) + \langle \boldsymbol{P}_\perp^2 \rangle(z)$$

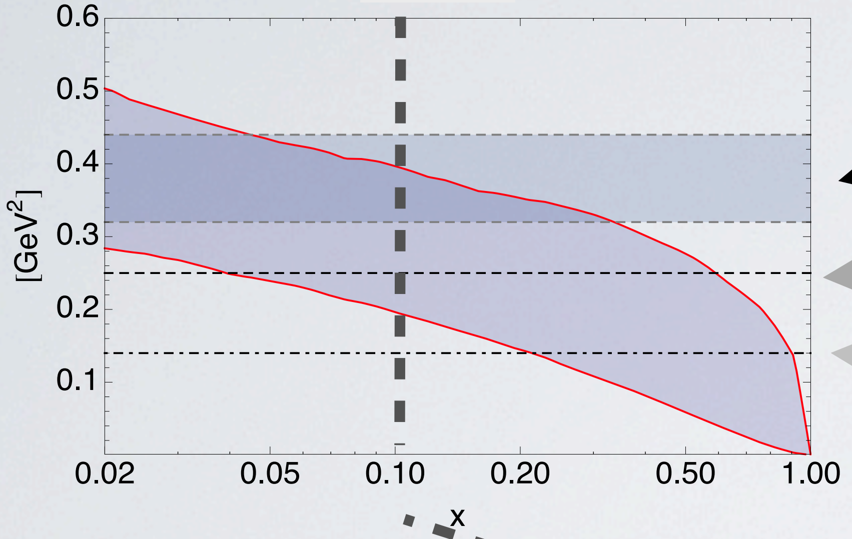
observed



anticorrelation and 68% band

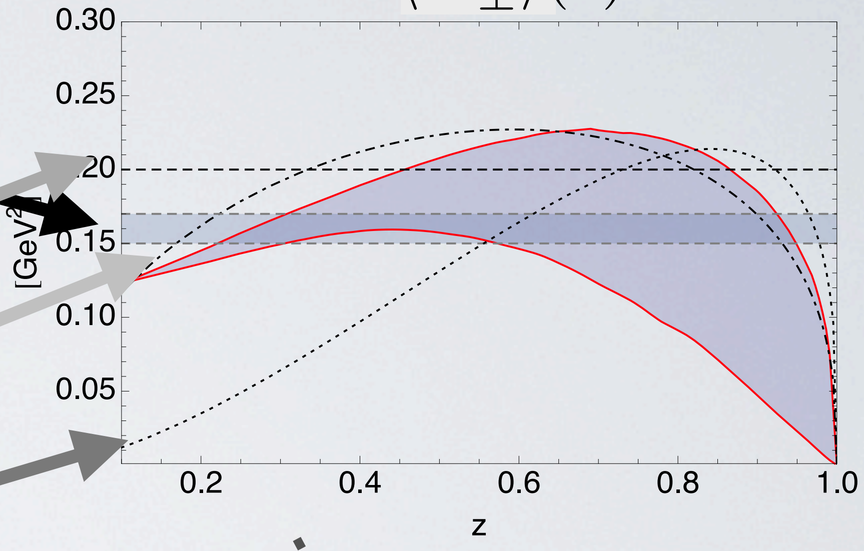
TMD PDF

$\langle \boldsymbol{k}_\perp^2\rangle(x)$



TMD FF

$\langle \boldsymbol{P}_\perp^2\rangle(z)$



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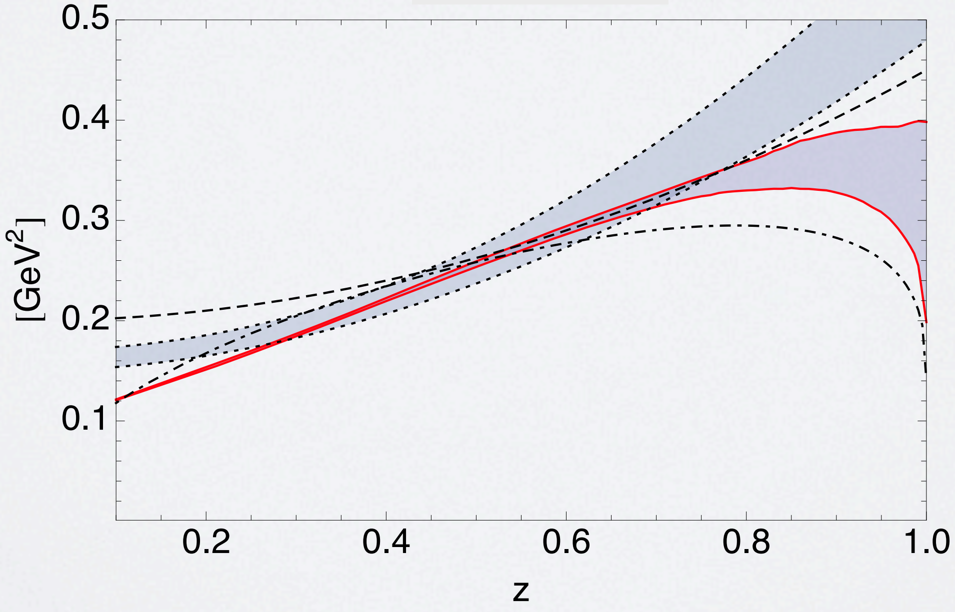
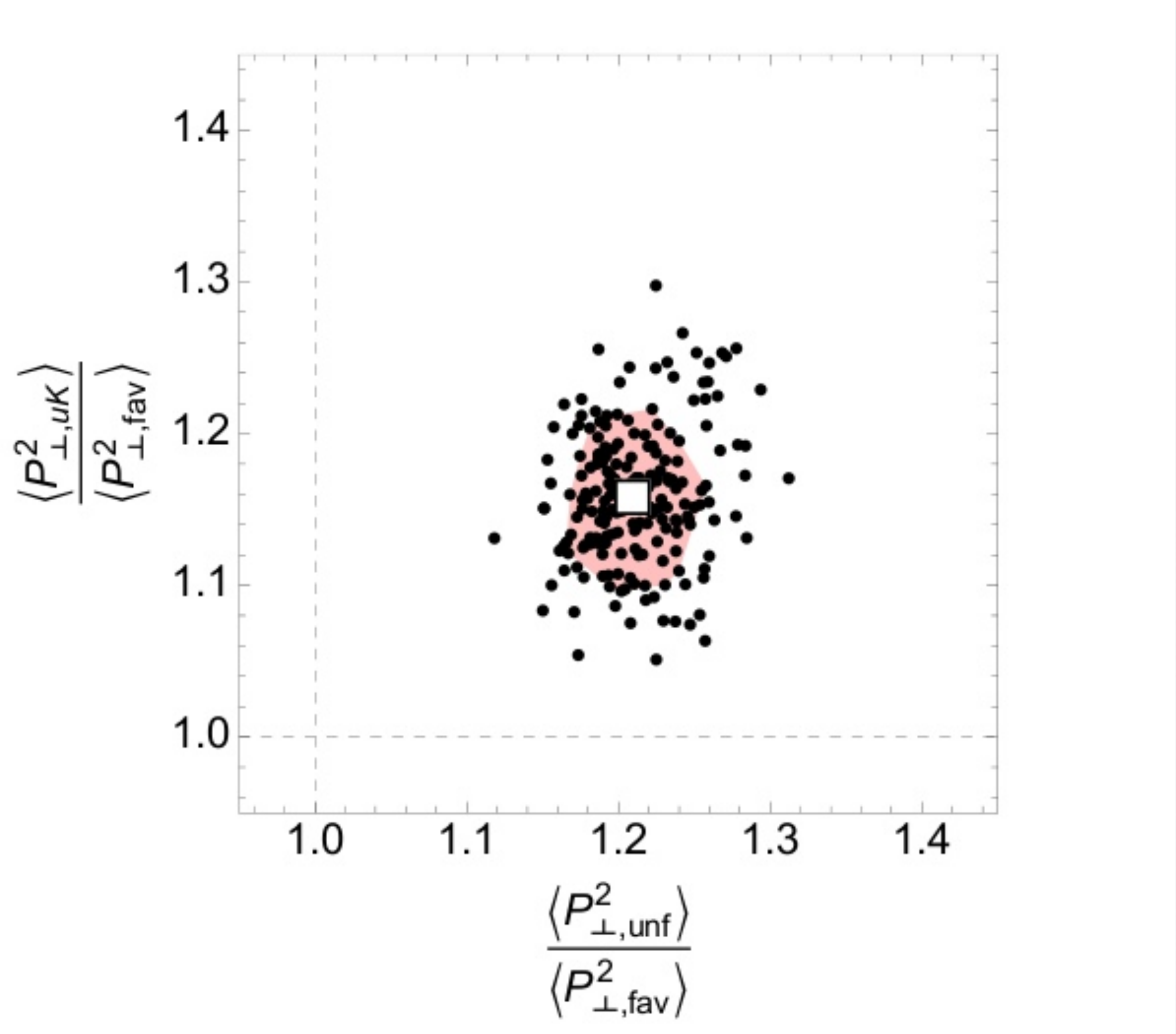
observed

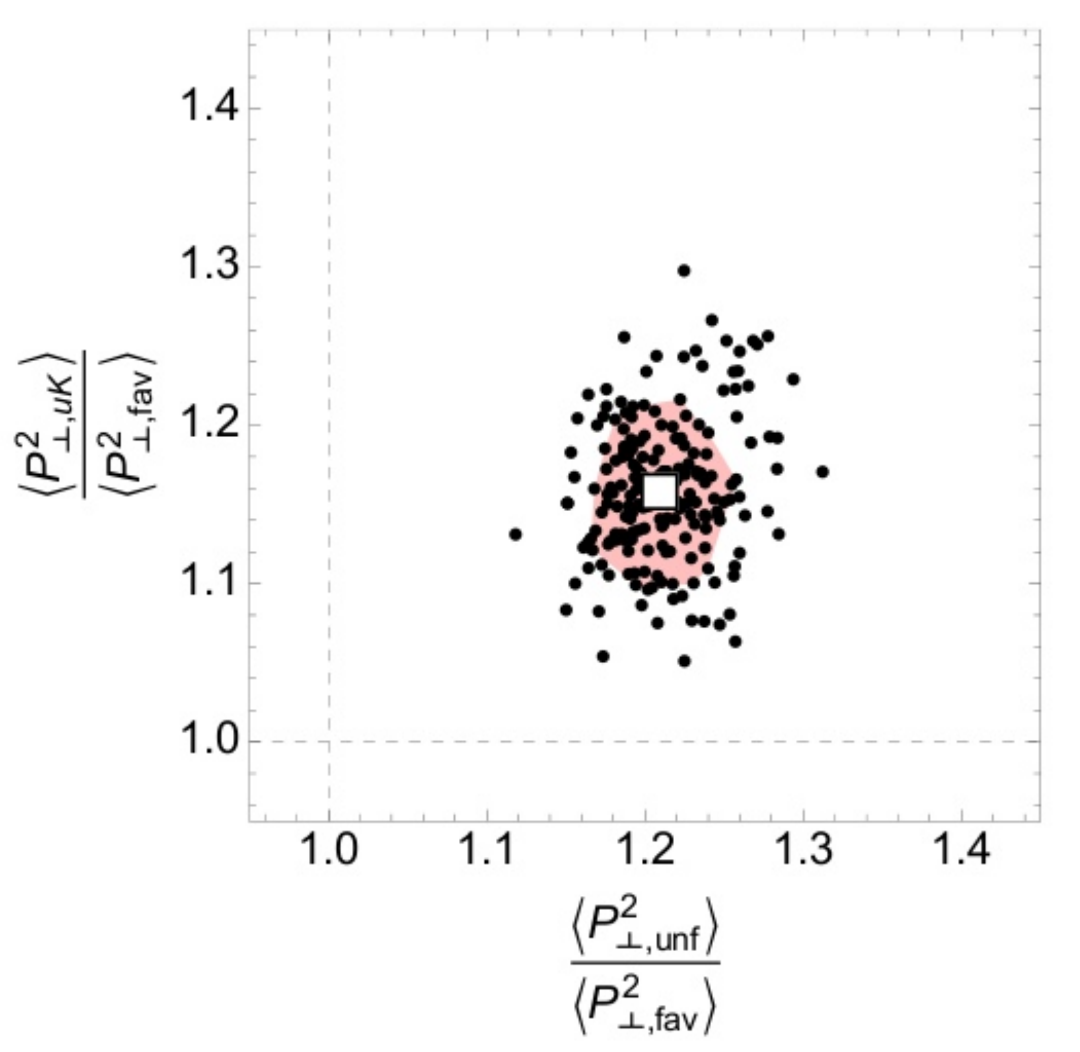


several {**k**$_\perp$,**P**$_\perp$}
give
same **P**$_{\text{hT}}$

Results – Scenario : flavor dep. in **TMD FF**



Results – Scenario : flavor dep. in **TMD FF**



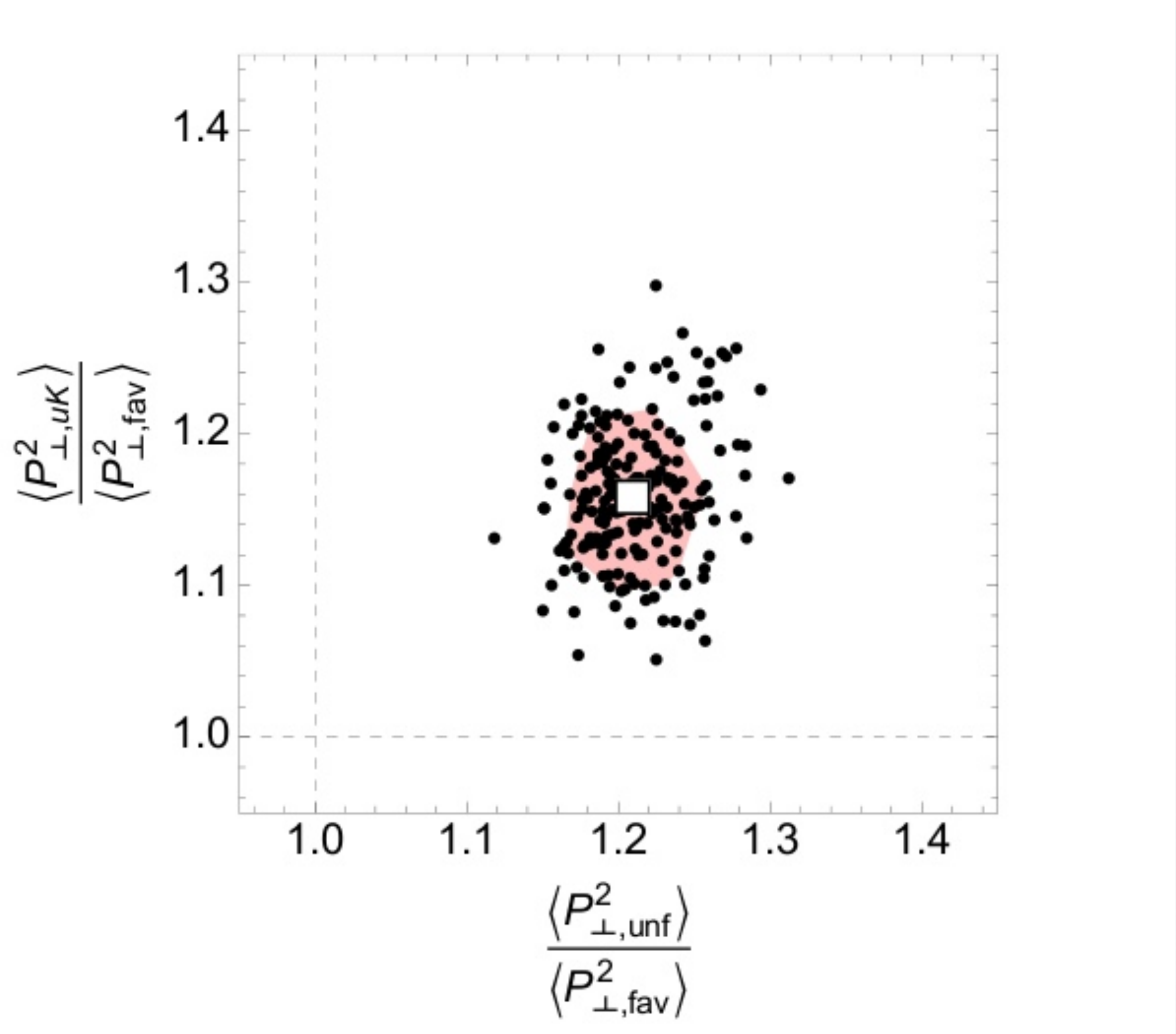
q→π favored width < unfavored

Results – Scenario : flavor dep. in **TMD FF**

q→π favored width

<

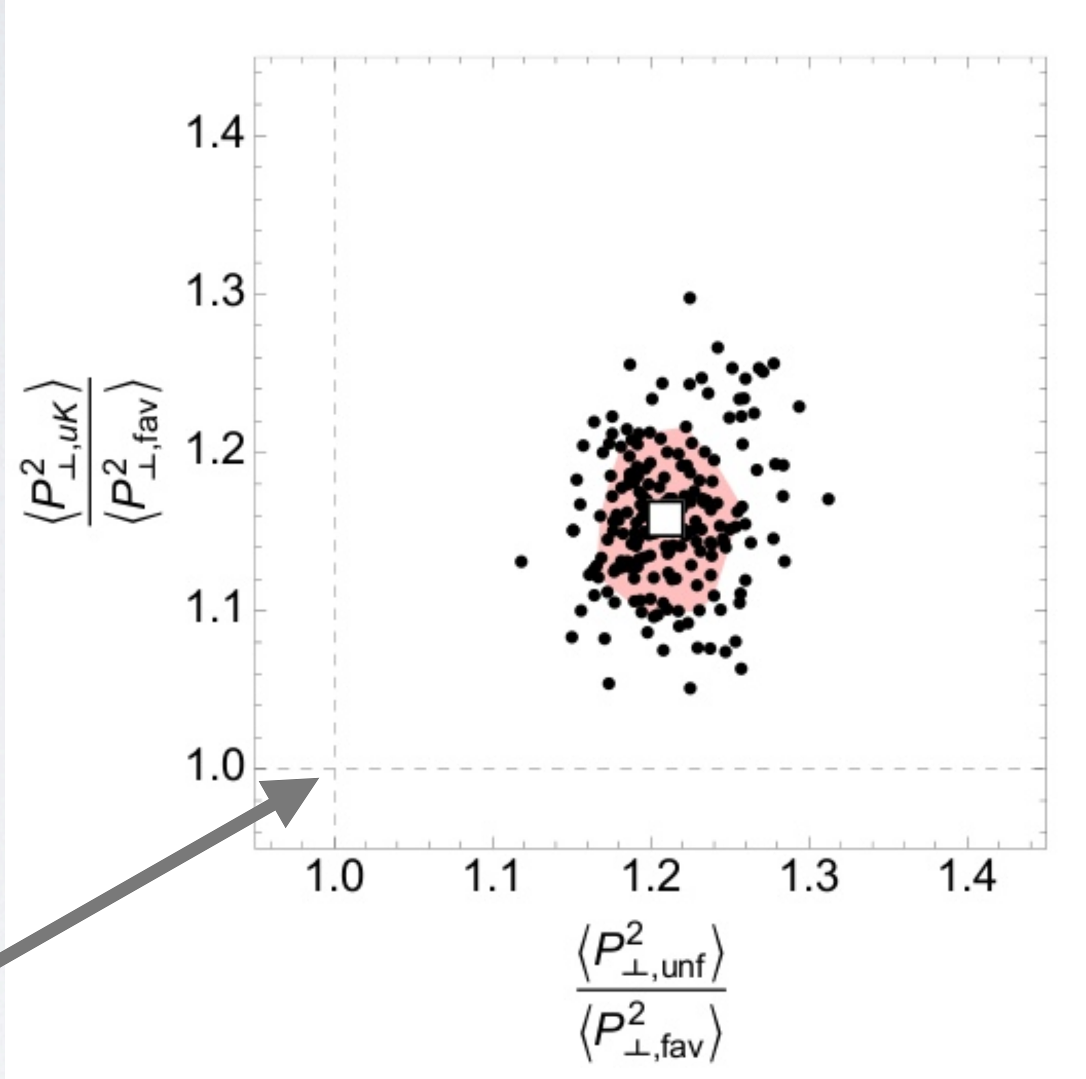
q→K favored width



q→π favored width < unfavored

Results – Scenario : flavor dep. in **TMD FF**

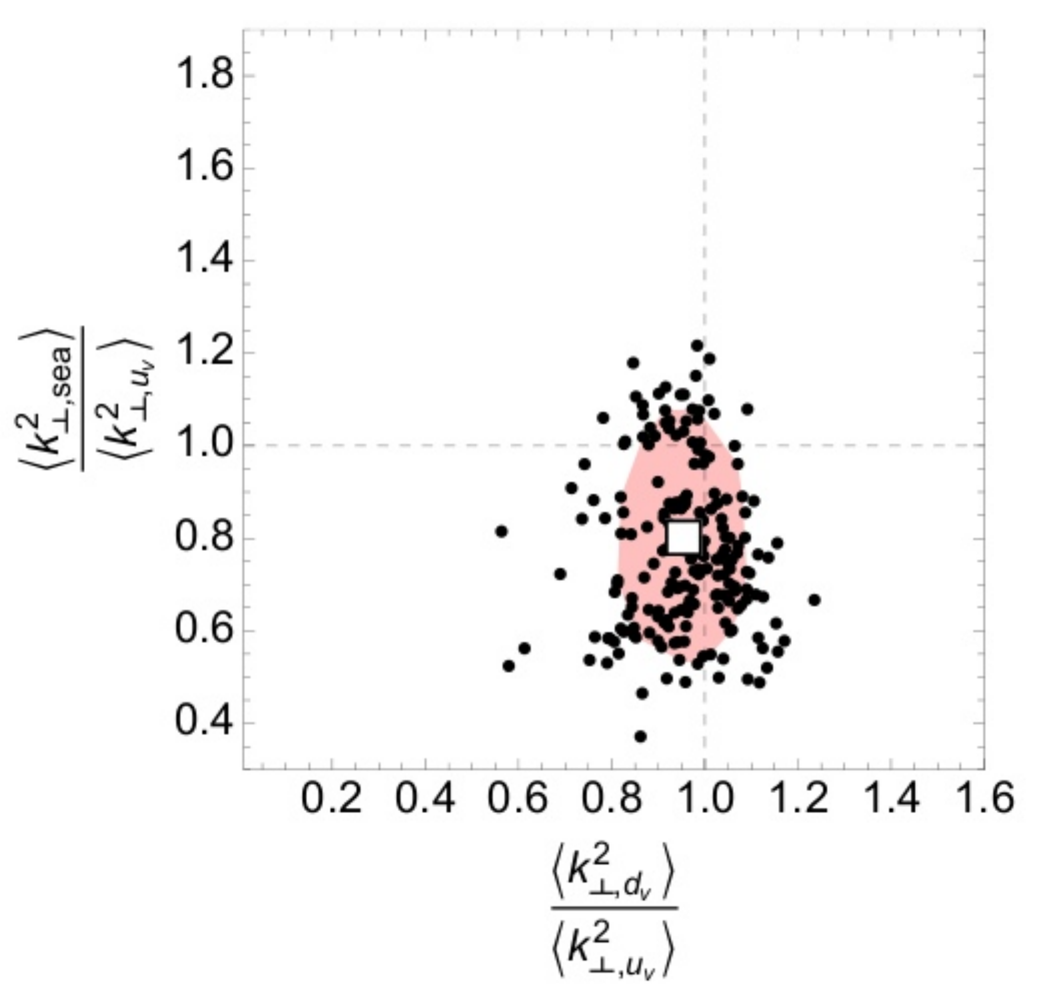
q→π favored width
<
q→K favored width

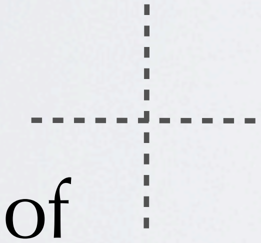


point of
no flavor dep.

q→π favored width < unfavored

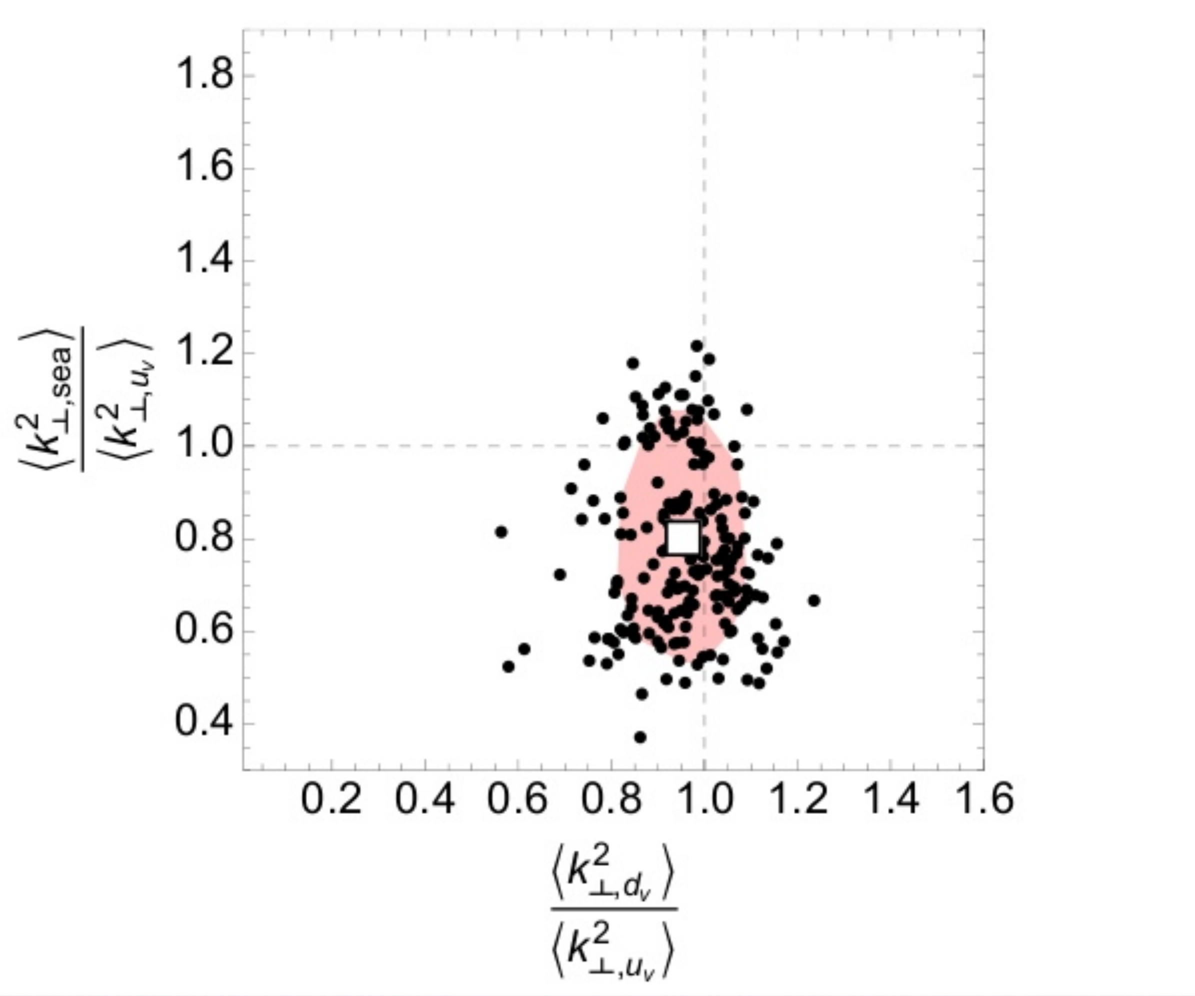
Results – Scenario : **TMD PDF** and **no** final K

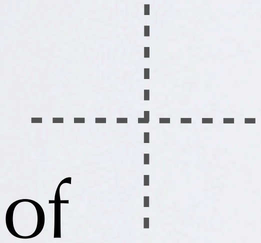




point of
no flavor dep.

Results – Scenario : **TMD PDF** and **no** final K





point of
no flavor dep.

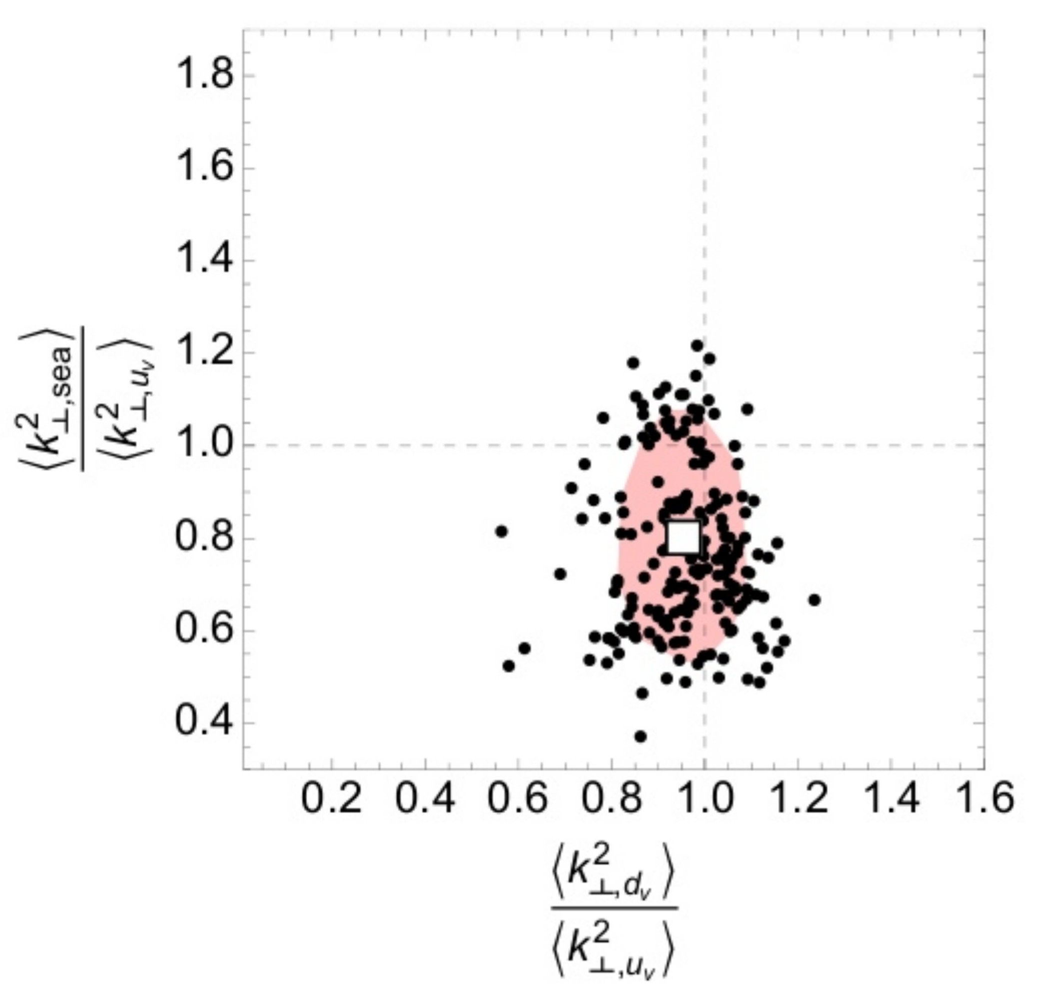
d_v width ~ (mostly) u_v width

Results – Scenario : **TMD PDF** and **no** final K

sea width

< (mostly)

u_v width



point of
no flavor dep.

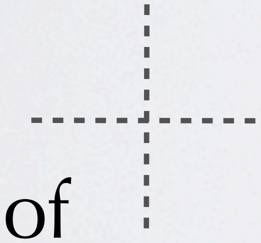
d_v width ~ (mostly) u_v width

Results – Scenario : **TMD PDF** full analysis

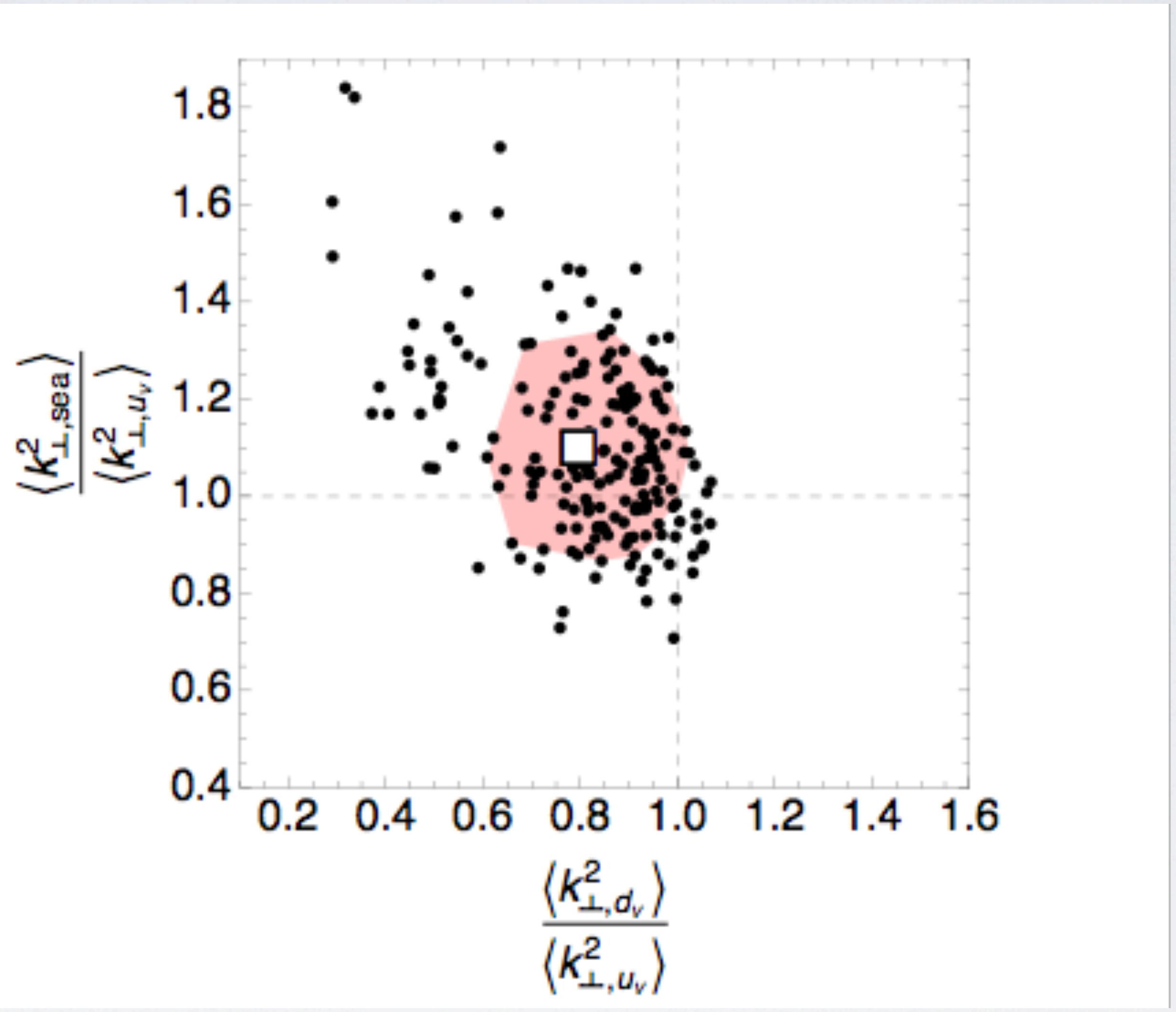
sea width

> (mostly)

u_v width



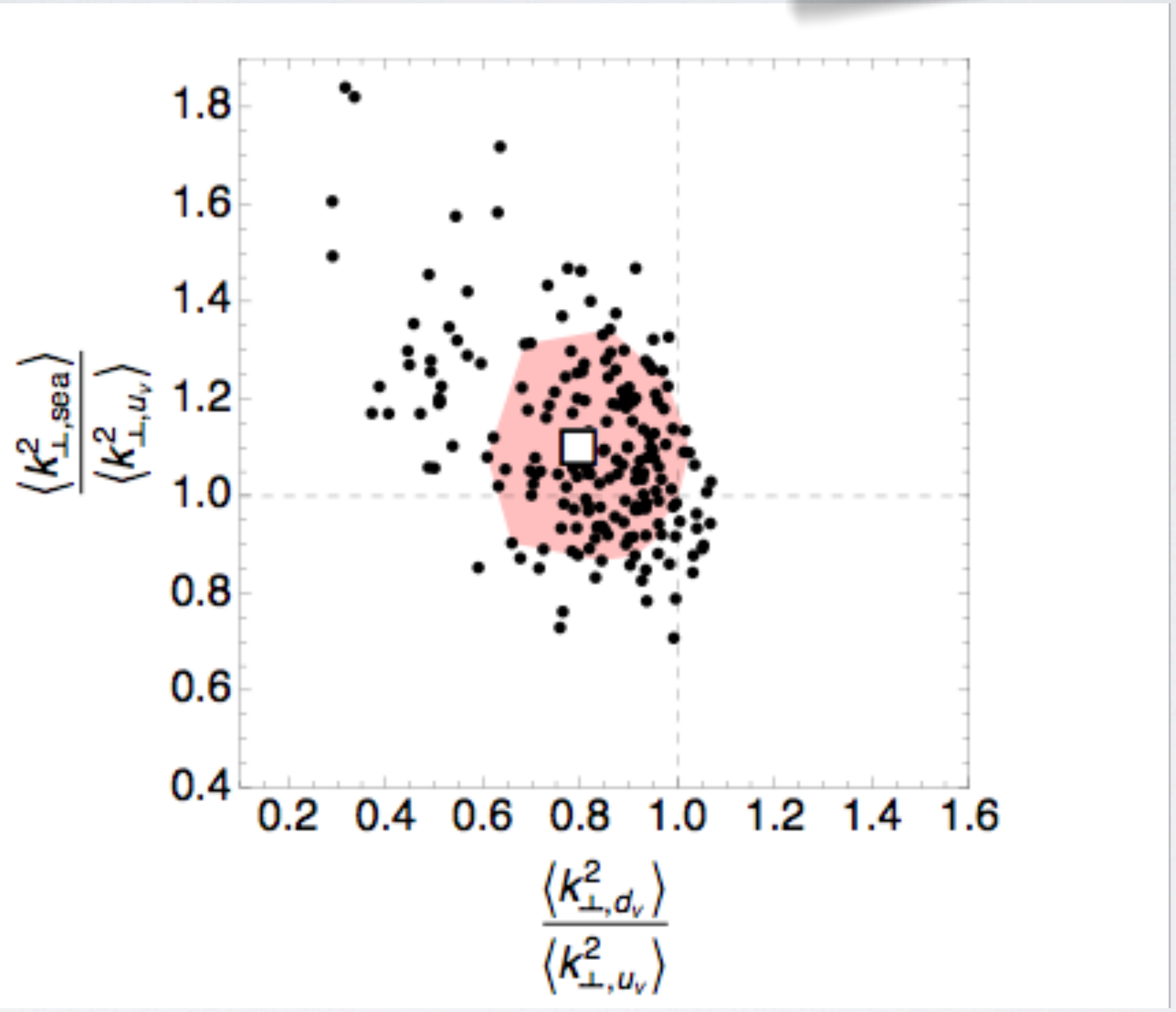
point of
no flavor dep.



d_v width **<** (mostly) u_v width

Results – Scenario : **TMD PDF** full analysis

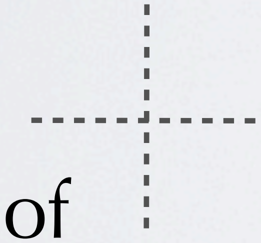
$s, \bar{s}$ are important



sea width

\> (mostly)

u_v width



point of
no flavor dep.

d_v width < (mostly) u_v width

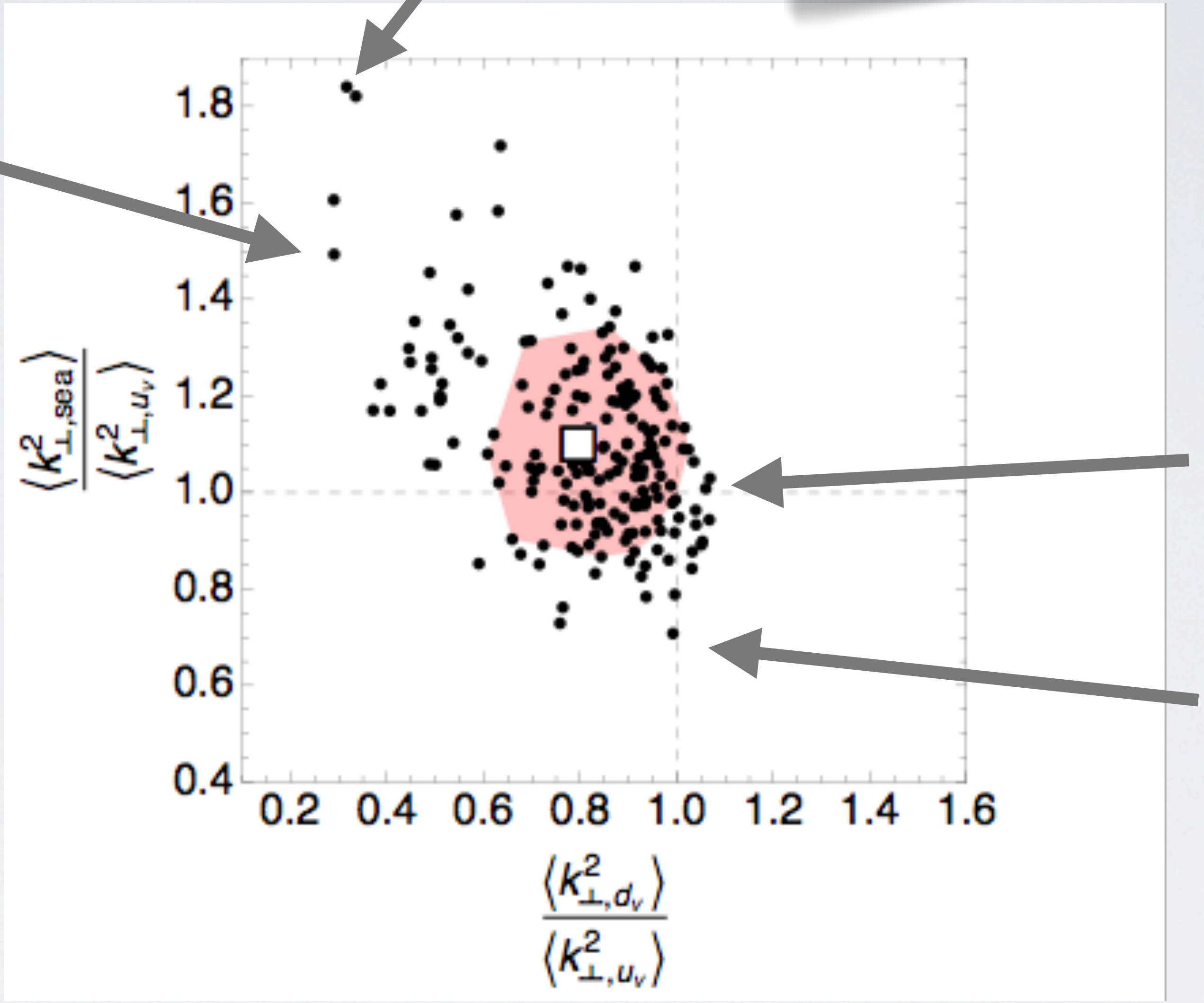
Results – Scenario : **TMD PDF** full analysis

s, $\bar{s}$ are important

replica 149
χ^2/dof = 1.87

replica 73
χ^2/dof = 1.70

sea width
> (mostly)
u_v width



replica 130
χ^2/dof = 1.77

replica 186
χ^2/dof = 1.38

point of
no flavor dep.

d_v width < (mostly) u_v width

Conclusions

1. fitting SIDIS multiplicities from HERMES,
   first experimental exploration of flavor dependence
   in TMD PDF and TMD FF

2. clear & stable indication in TMD FF that
   “q→π favored” width < “unfavored” & “q→K favored”

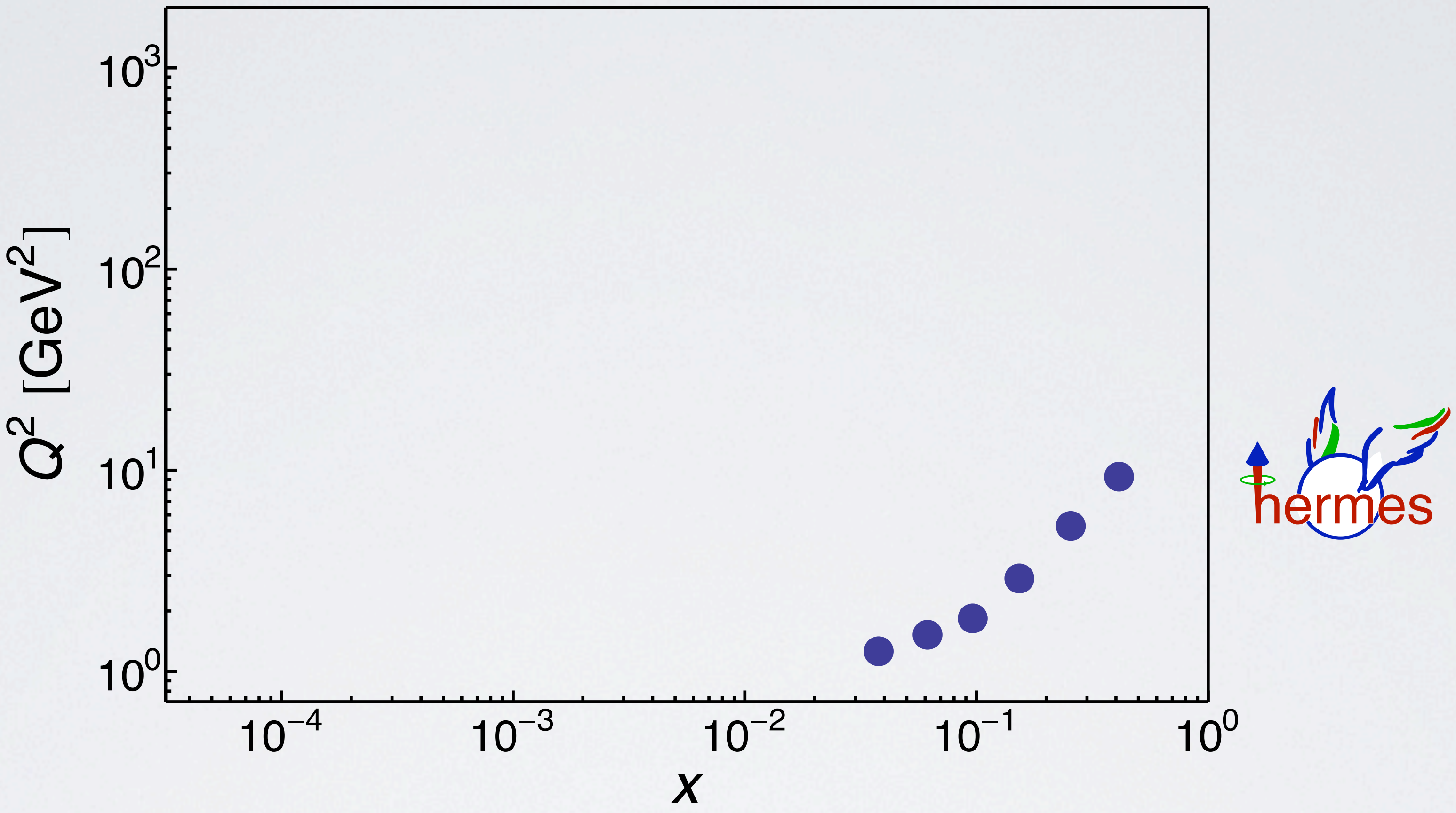
3. tendency in TMD PDF to d_v width < u_v width < sea width

4. no K in final state : sea width < $d_v \sim u_v$ width
   ⇒ importance of strange

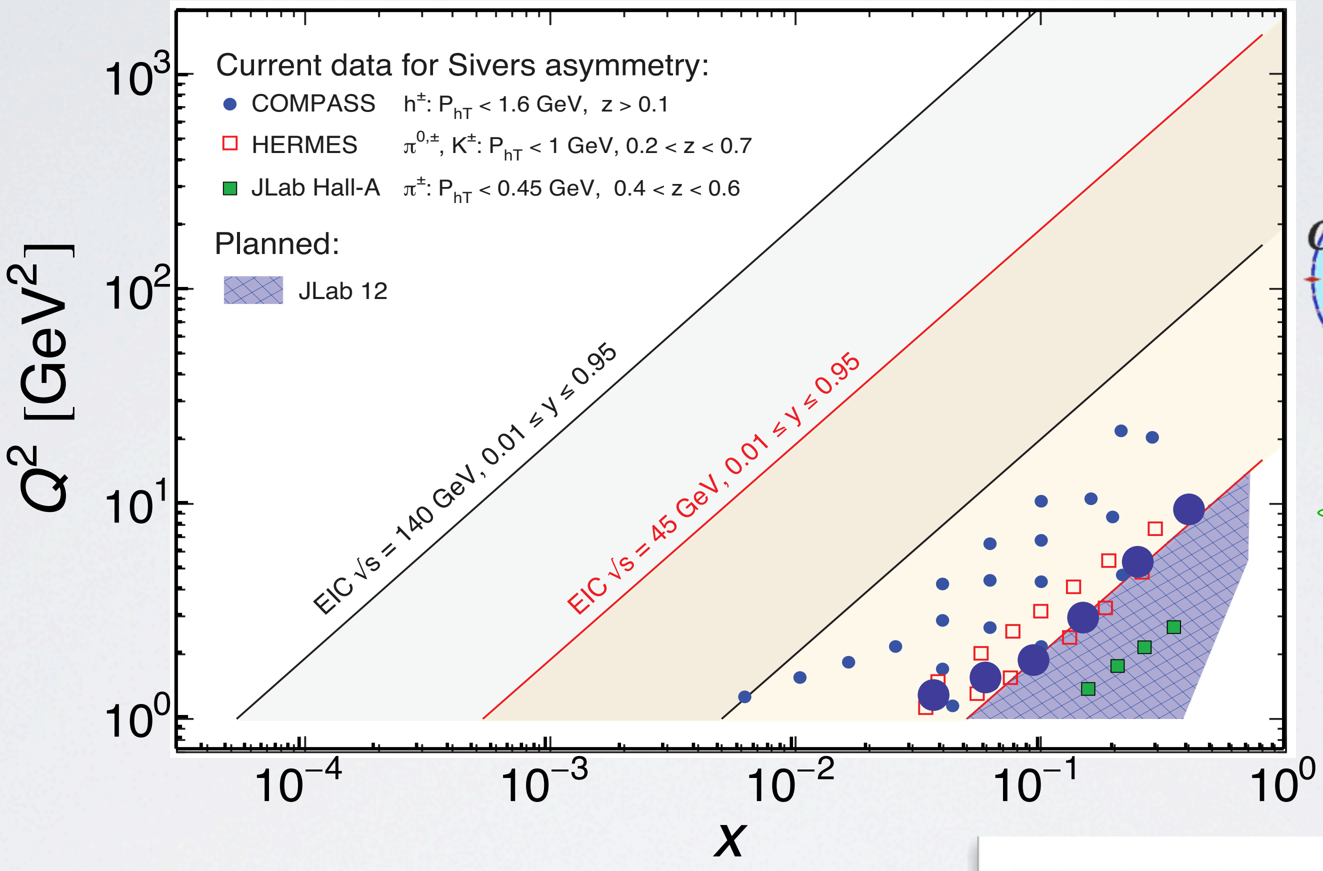
5. flavor-independent fit performs worse but not ruled out
   strong anticorrelation: many intrinsic $\{\mathbf{k}_\perp,\mathbf{P}_\perp\}$ give same $\mathbf{P}_{hT}$

Future

Future



Future

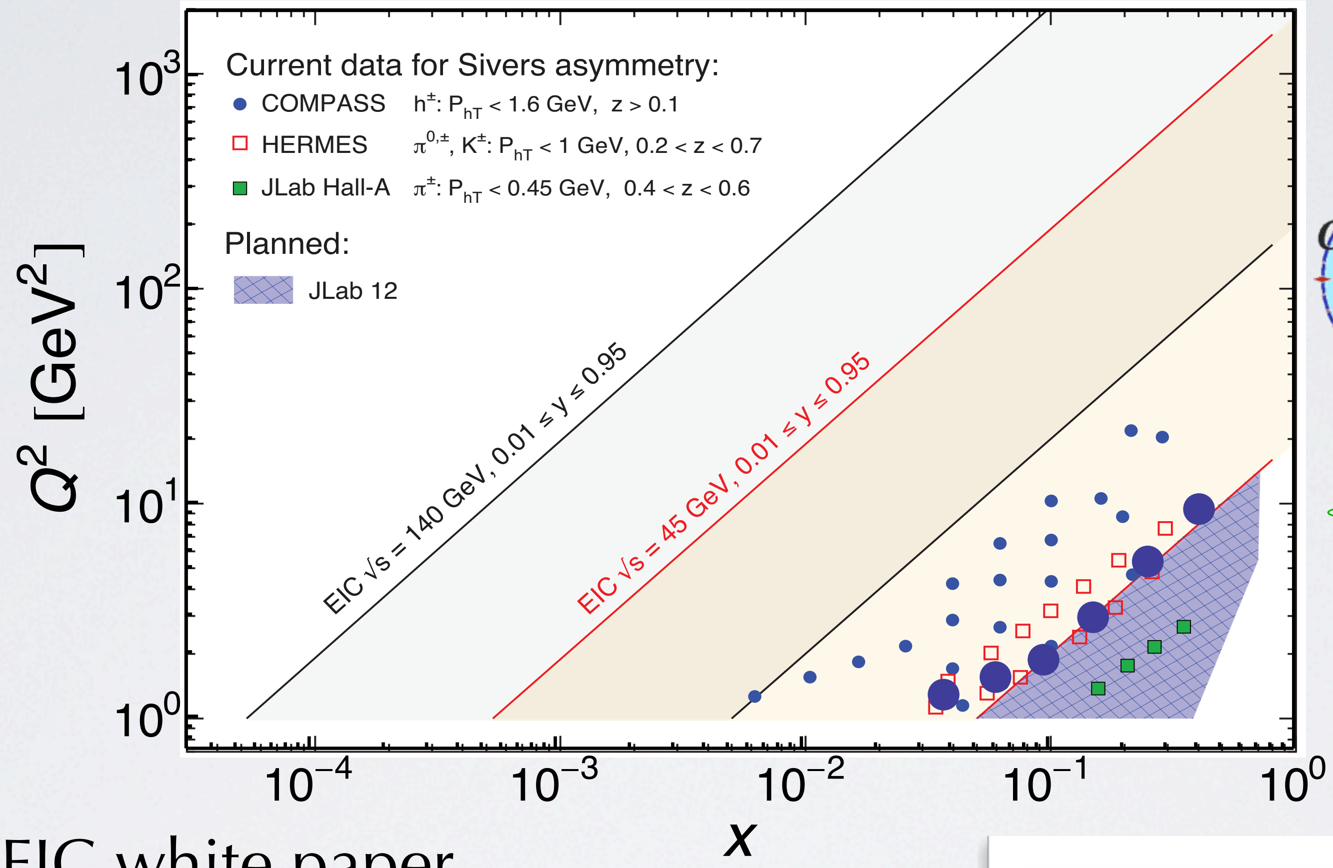


near
future

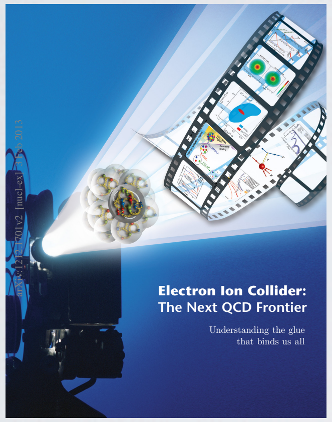
- enlarge (x,Q^2) range

Future

far future



near future



Electron Ion Collider:
The Next QCD Frontier

Understanding the glue
that binds us all

EIC white paper
arXiv:1212.1701v2 [nucl-ex]

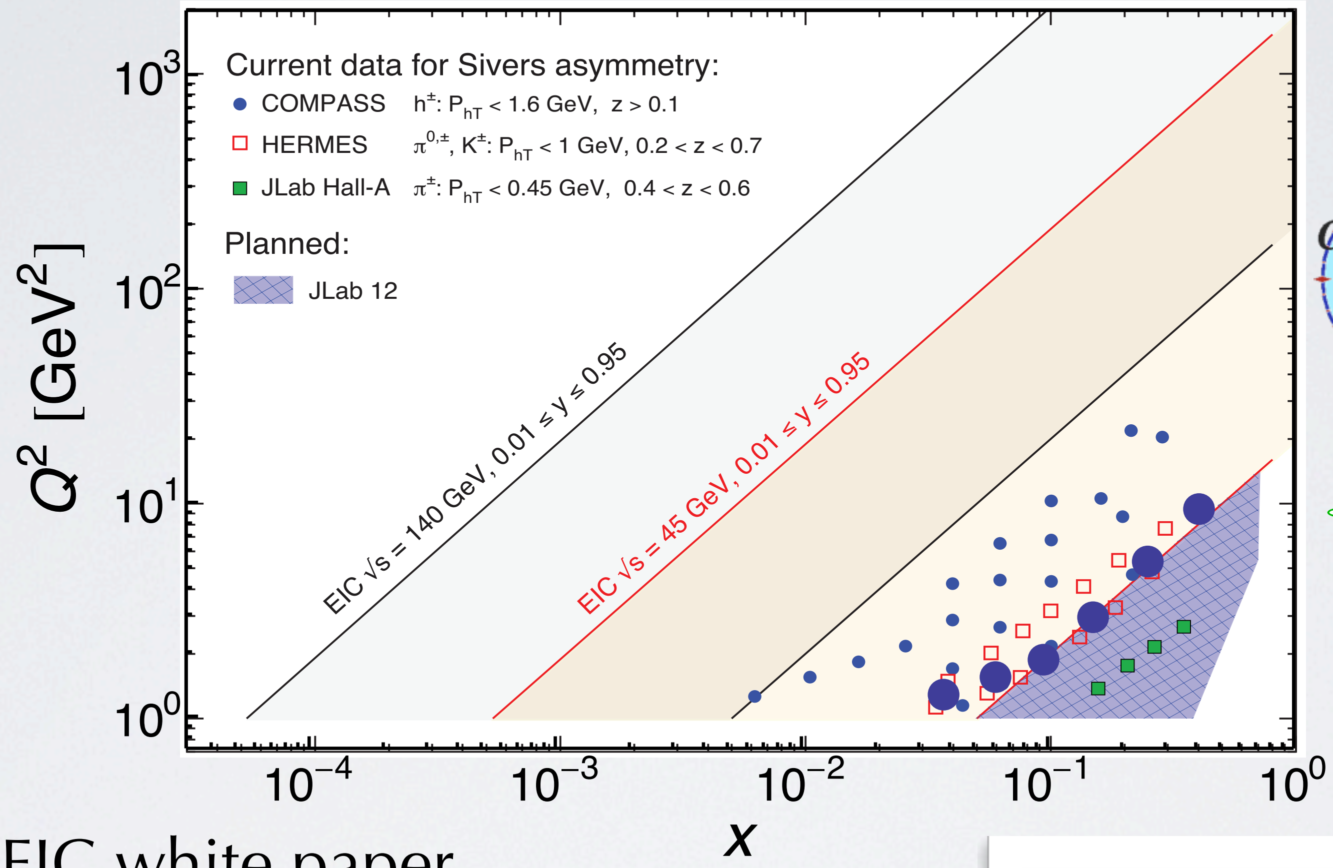
- enlarge (x,Q^2) range

TMD FF $(z, \mathbf{P}_{hT}^2; Q^2)$

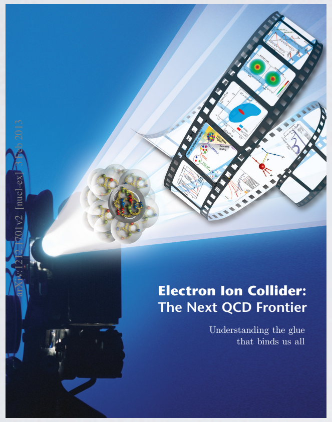
Drell-Yan...

Future

far future



near future



EIC white paper
arXiv:1212.1701v2 [nucl-ex]

- enlarge (x,Q^2) range

TMD FF (z, $\mathbf{P}_{hT}^2$; Q^2)

- uncorrelated x(z) & Q^2 bins
- different targets & final hadrons

Drell-Yan...