Extraction of TMDs with global fits

Probing Strangeness in Hard Processes
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Outline

- Update of Transversity and Collins function
- Conclusions I
- Sivers function: TMD evolution, sea Sivers functions
- Conclusions II

Transversity and Collins function

Extraction of transversity & Collins functions

- Azimuthal asymmetry in polarized SIDIS

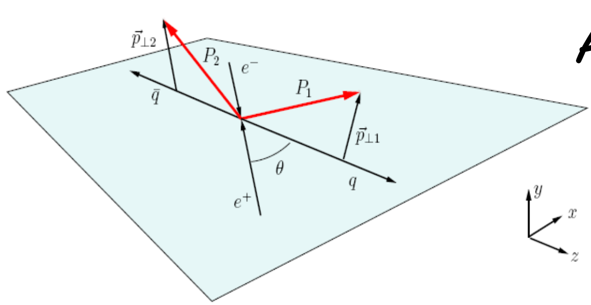
$$\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} = \sum_q h_{1q}(x, \boldsymbol{k}_\perp) \otimes \mathrm{d}\Delta\hat{\sigma}(y, \boldsymbol{k}_\perp) \otimes \Delta^N D_{h/q^\uparrow}(z, \boldsymbol{p}_\perp)$$

Transversity — Collins function

$$A_{UT}^{\sin(\phi+\phi_S)} \equiv 2\, \frac{\int \mathrm{d}\phi\, \mathrm{d}\phi_S\, [\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}]\, \sin(\phi + \phi_S)}{\int \mathrm{d}\phi\, \mathrm{d}\phi_S\, [\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}]}$$

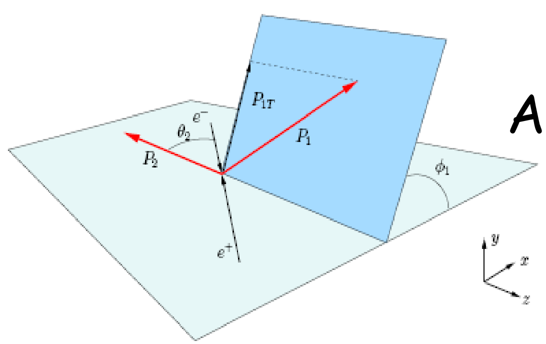
Extraction of transversity & Collins functions

- e^+e^- -> h_1 h_2 X BELLE Data



A_{12} asymmetry → Thrust axis method

$$A(z_1, z_2, \theta, \varphi_1 + \varphi_2) \equiv \frac{1}{\langle d\sigma \rangle} \frac{d\sigma^{e^+e^- \to h_1 h_2 X}}{dz_1\, dz_2\, d\cos\theta\, d(\varphi_1 + \varphi_2)}$$
$$= 1 + \frac{1}{8} \frac{\sin^2\theta}{1 + \cos^2\theta} \cos(\varphi_1 + \varphi_2) \frac{\sum_q e_q^2 \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$



A_0 asymmetry → Hadronic plane method

$$A(z_1, z_2, \theta_2, \phi_1) = 1 + \frac{1}{\pi} \frac{z_1 z_2}{z_1^2 + z_2^2} \frac{\sin^2\theta_2}{1 + \cos^2\theta_2} \cos(2\phi_1) \frac{\sum_q e_q^2 \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$

Extraction of transversity & Collins functions

- To avoid acceptance effects the BELLE Collaboration considered ratio of different combinations of hadron pairs:

$$\frac{\text{Unlike-sign }(\pi^+\pi^- + \pi^-\pi^+)}{\text{Like-sign }(\pi^+\pi^+ + \pi^-\pi^-)} \rightarrow A^{UL} \text{ asymmetry}$$

$$\frac{\text{Unlike-sign }(\pi^+\pi^- + \pi^-\pi^+)}{\text{Charged }(\pi^+\pi^+ + \pi^-\pi^- + \pi^+\pi^- + \pi^-\pi^+)} \rightarrow A^{UC} \text{ asymmetry}$$

- A_{12}^{UL} A_{12}^{UC} A_{0}^{UL} A_{0}^{UC}

Parametrizations

Parametrizations

- Gaussian parametrization of the unpolarized PDF & FF:

- $$f_{q/p}(x, k_\perp) = f_{q/p}(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

- $$D_{h/q}(z, p_\perp) = D_{h/q}(z) \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}$$

[*] $\langle k_\perp^2 \rangle = 0.25 \text{ GeV}^2 \qquad \langle p_\perp^2 \rangle = 0.20 \text{ GeV}^2$

[*] Anselmino et al. Phys. Rev. D71 074006 (2005)

Parametrizations

- Parametrization of Transversity function:

✎ $$\Delta_T q(x, k_\perp) = \frac{1}{2}\mathcal{N}_q^T(x)\left[f_{q/p}(x) + \Delta q(x)\right]\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_T}}{\pi\langle k_\perp^2\rangle_T}$$

Unpolarized PDF

Helicity PDF

$$\mathcal{N}_q^T(x) = N_q^T\, x^{\alpha_q}(1-x)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

N_q^T, α, β free parameters

Parametrizations

- Parametrization of the Collins function:

$$\Delta^N D_{\pi/q^\uparrow}(z,p_\perp) = 2\mathcal{N}_q^C(z)\, h(p_\perp)\, D_{\pi/q}(z,p_\perp)$$

Unpolarized FF

- $\mathcal{N}_q^C(z) = N_q^C\, z^\gamma (1-z)^\delta \dfrac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\gamma \delta^\delta}$

- $h(p_\perp) = \sqrt{2e}\,\frac{p_\perp}{M_h}\, e^{-p_\perp^2/M_h^2}$

N_q^C, γ, δ, M_h free parameters

✔Bound:

$$\Delta^N D_{\pi/q^\uparrow}(z,p_\perp) \le 2\, D_{\pi/q}(z,k_\perp)$$

✔Torino vs Amsterdam notation

$$\Delta^N D_{\pi/q^\uparrow}(z,p_\perp) = \frac{2p_\perp}{zM} H_1^\perp(z,p_\perp)$$

2013 Update of the extraction*

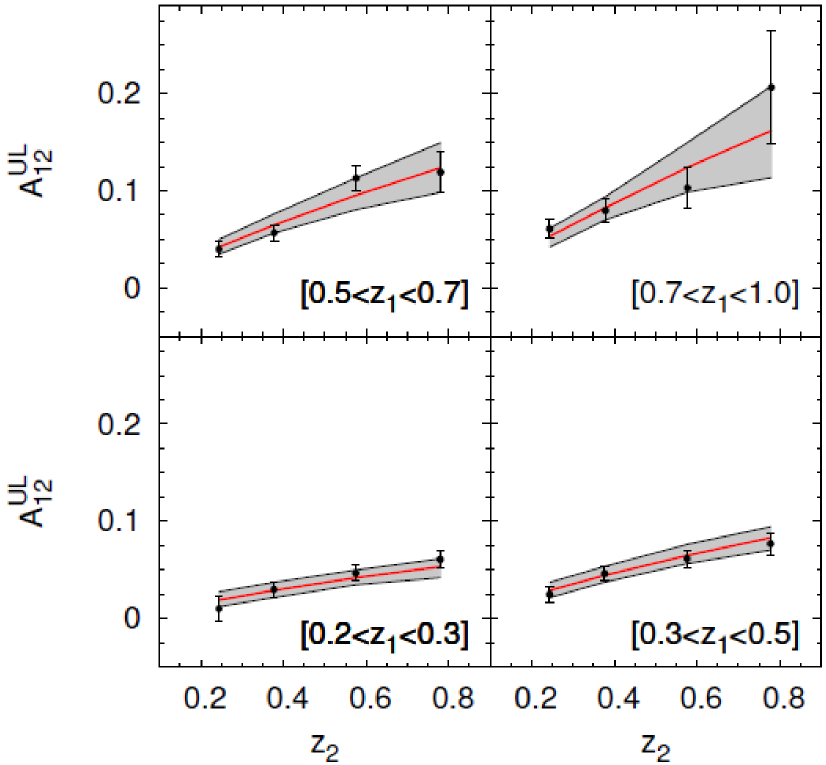
- New analysis (PRD87, 2013):
  - HERMES (2009) π+ π-
  - COMPASS Deuteron (2004) π+ π-
  - COMPASS Proton (2013) π+ π-
  - BELLE A_{12} or A_0 (BELLE ERRATUM 2012, PRD86)

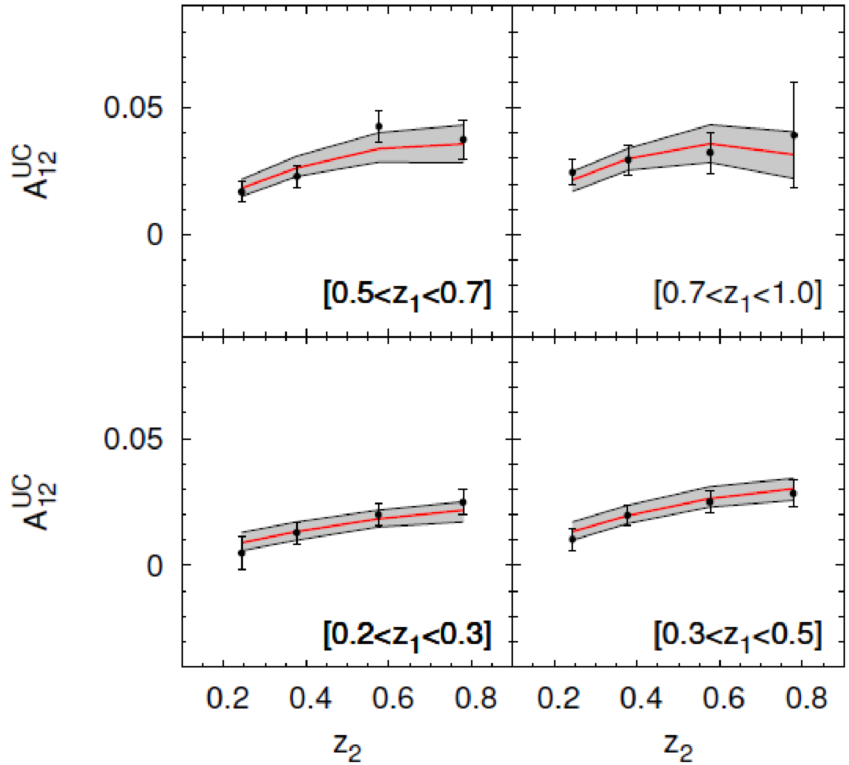
- U and d quarks transversity, favored and disfavored Collins functions
- Two separate fits for A_{12} and A_0 sets
  - FIT I: A_{12} + HERMES+ COMPASS
  - FIT II: A_o + HERMES+ COMPASS

*Anselmino et al. Phys. Rev. D87,094019

Extraction of transversity & Collins functions

- FIT I: A_{12} BELLE data UL & UC +COMPASS+ HERMES

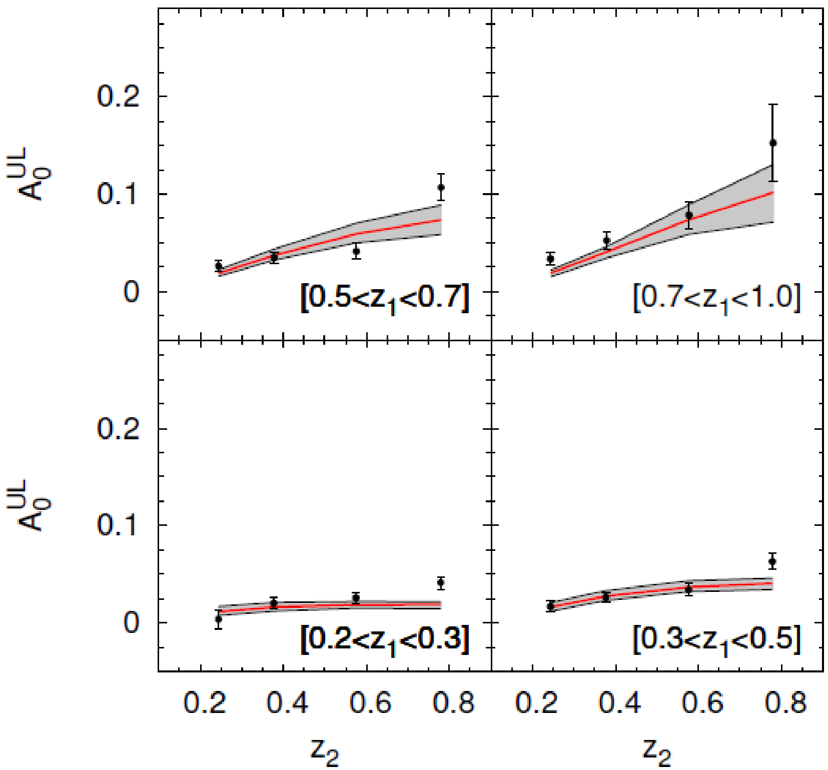


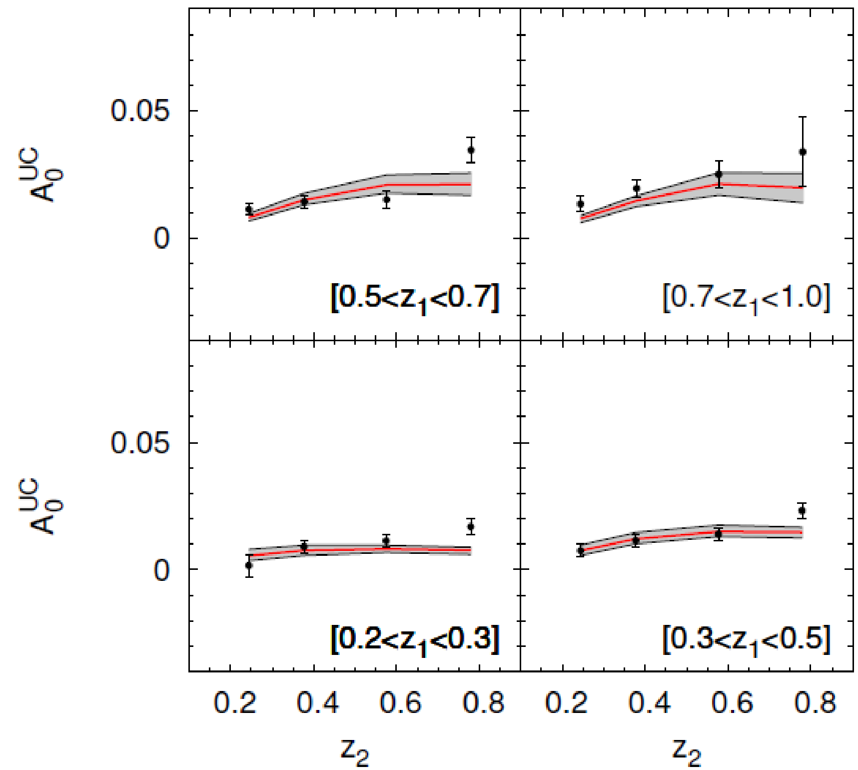


→ Full compatibility between UL and UC, contrary to 2008 BELLE data

Extraction of transversity & Collins functions

- FIT I: A_{12} BELLE data UL & UC +COMPASS+ HERMES

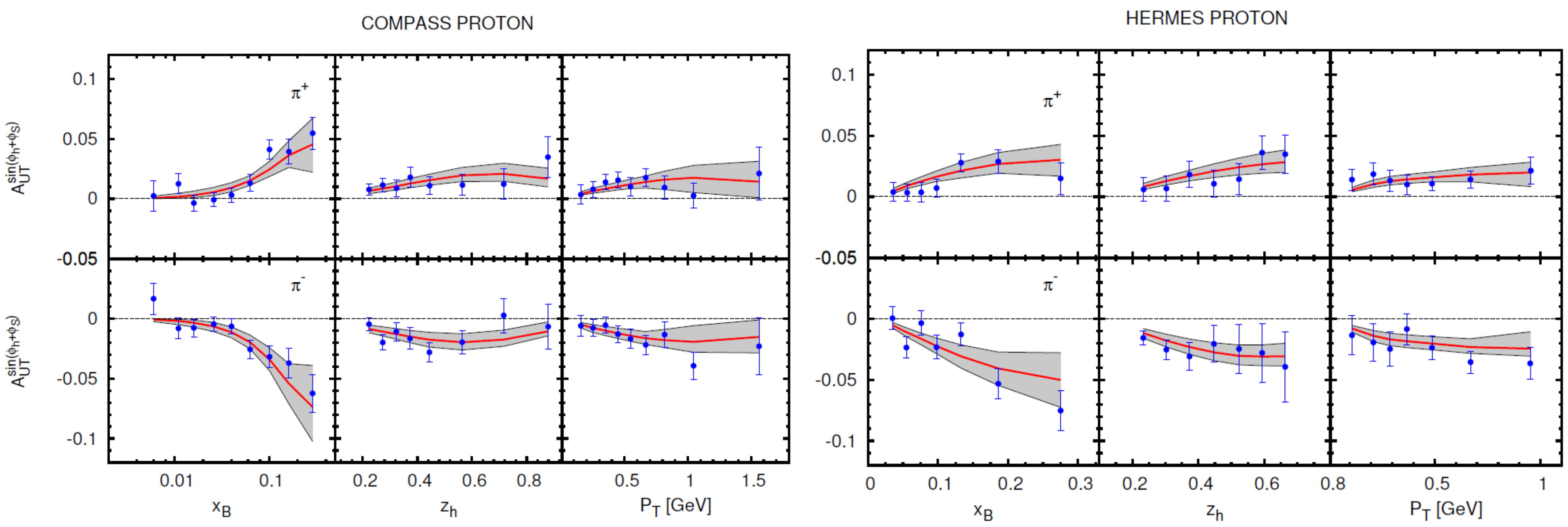




→ Still tension between the two methods A_0 and A_{12}

Extraction of transversity & Collins functions

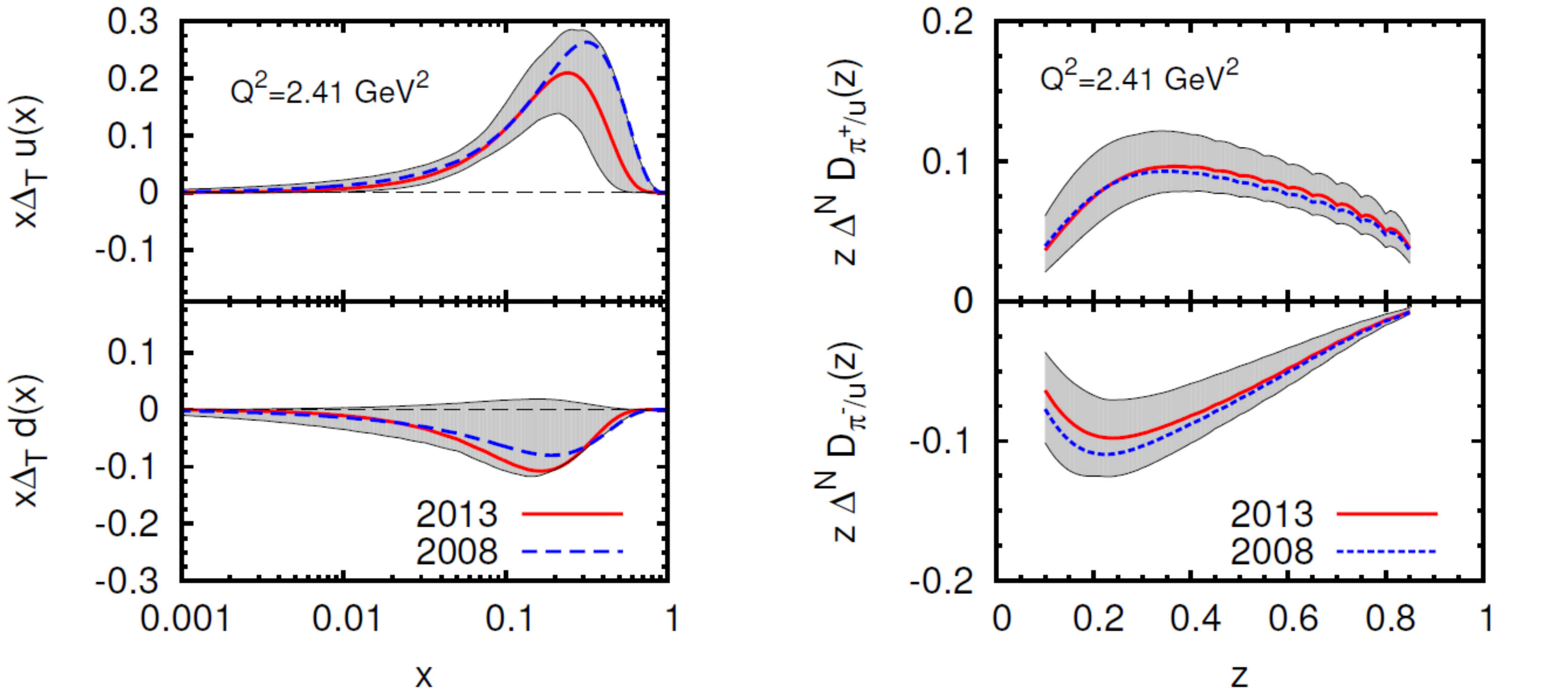
- FIT I: A_{12} BELLE data UL & UC +COMPASS+ HERMES



➔ Similar good description of HERMES and COMPASS

Extraction of transversity & Collins functions

- FIT I: A_{12} BELLE data UL & UC +COMPASS+ HERMES



➔Results similar to 2008 extraction

$N_u^T = 0.46^{+0.20}_{-0.14}$	$N_d^T = -1.00^{+1.17}_{-0.00}$
$\alpha = 1.11^{+0.89}_{-0.66}$	$\beta = 3.64^{+5.80}_{-3.37}$
$N^C_{\rm fav} = 0.49^{+0.20}_{-0.18}$	$N^C_{\rm dis} = -1.00^{+0.38}_{-0.00}$
$\gamma = 1.06^{+0.45}_{-0.32}$	$\delta = 0.07^{+0.42}_{-0.07}$
$M_h^2 = 1.50^{+2.00}_{-1.12}$ GeV^2	

Extraction of transversity & Collins functions

- FIT II: A_0 BELLE data UL & UC +COMPASS+ HERMES ???

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
Standard Parameterization $\chi^2_{\rm d.o.f} = 0.80$	$\chi^2_{\rm tot} = 135$	$\chi^2 = 123$	$\chi^2 = 7$	$\chi^2 = 5$	$\chi^2 = 44$ NO FIT	$\chi^2 = 39$ NO FIT
Standard Parameterization $\chi^2_{\rm d.o.f} = 1.12$	$\chi^2_{\rm tot} = 190$	$\chi^2 = 125$	$\chi^2 = 20$ NO FIT	$\chi^2 = 12$ NO FIT	$\chi^2 = 35$	$\chi^2 = 30$

→ A_0 data cannot be nicely described even if fitted...

Standard parametrization of the Collins function

- Parametrization of the z-dependent part of the Collins function:

$$\Delta^N D_{\pi/q^\uparrow}(z,p_\perp) = 2\mathcal{N}_q^C(z)\, h(p_\perp)\, D_{\pi/q}(z,p_\perp)$$

$$\mathcal{N}_q^C(z) = N_q^C\ z^\gamma (1-z)^\delta\ \frac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\gamma \delta^\delta}$$

Our standard parametrization

- It is equal to 0 at z=0 and z=1

New parametrization of the Collins function

- Let us try to change the parametrization of the z-dependent part of the Collins function:

$$\Delta^N D_{\pi/q^\uparrow}(z, p_\perp) = 2\mathcal{N}_q^C(z)\, h(p_\perp)\, D_{\pi/q}(z, p_\perp)$$

$$\mathcal{N}_q^C(z) = N_q^C\, z\, [(1 - a - b) + a\, z + b\, z^2]$$

NEW Polynomial parametrization

- It is equal to 0 at z=0 and equal to N_q at z=1

Extraction of transversity & Collins functions

- FIT III and IV: Polynomial Parametrization

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
Standard Parameterization $\chi^2_{\rm d.o.f} = 0.80$	$\chi^2_{\rm tot} = 135$	$\chi^2 = 123$	$\chi^2 = 7$	$\chi^2 = 5$	$\chi^2 = 44$ NO FIT	$\chi^2 = 39$ NO FIT
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Polynomial Parameterization $\chi^2_{\rm d.o.f} = 0.81$	$\chi^2_{\rm tot} = 136$	$\chi^2 = 123$	$\chi^2 = 8$	$\chi^2 = 5$	$\chi^2 = 45$ NO FIT	$\chi^2 = 39$ NO FIT
Polynomial Parameterization $\chi^2_{\rm d.o.f} = 1.01$	$\chi^2_{\rm tot} = 171$	$\chi^2 = 141$	$\chi^2 = 44$ NO FIT	$\chi^2 = 27$ NO FIT	$\chi^2 = 15$	$\chi^2 = 15$

Extraction of transversity & Collins functions

- FIT III and IV: Polynomial Parametrization

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- If we fit A_{12} data we get the same description obtained with the std par.
- Almost identical Collins function, again the description of A_0 is not so good

Extraction of transversity & Collins functions

- FIT III and IV: Polynomial Parametrization

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
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→ If we fit A_0 data we can improve their description

→ Still tension with A_{12}

Extraction of transversity & Collins functions

- FIT III and IV: Polynomial Parametrization

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_{0}^{UL} 16 points	A_{0}^{UC} 16 points
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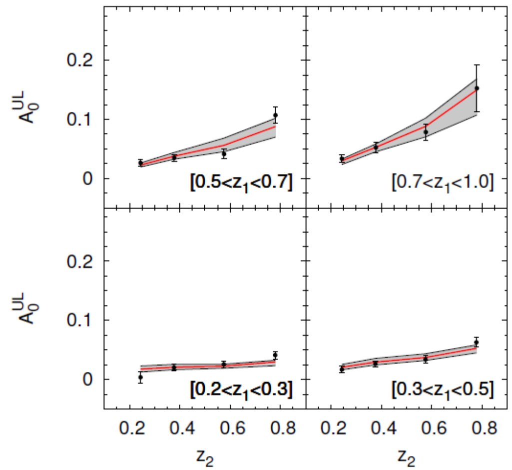
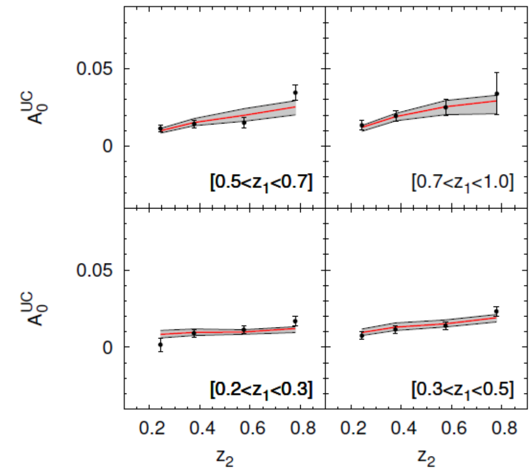
→ If we fit A_0 data we can improve their description

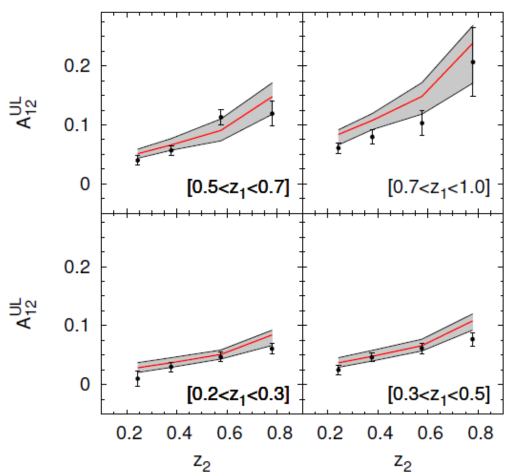
→ Still tension with A_{12}

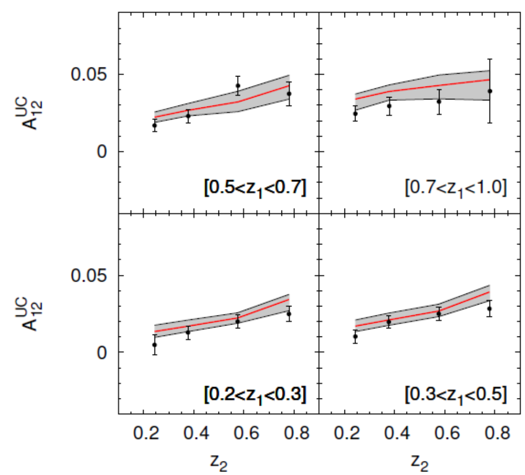
Extraction of transversity & Collins functions

- FIT IV: A_0 BELLE data UL & UC +COMPASS+ HERMES-POLYNOMIAL



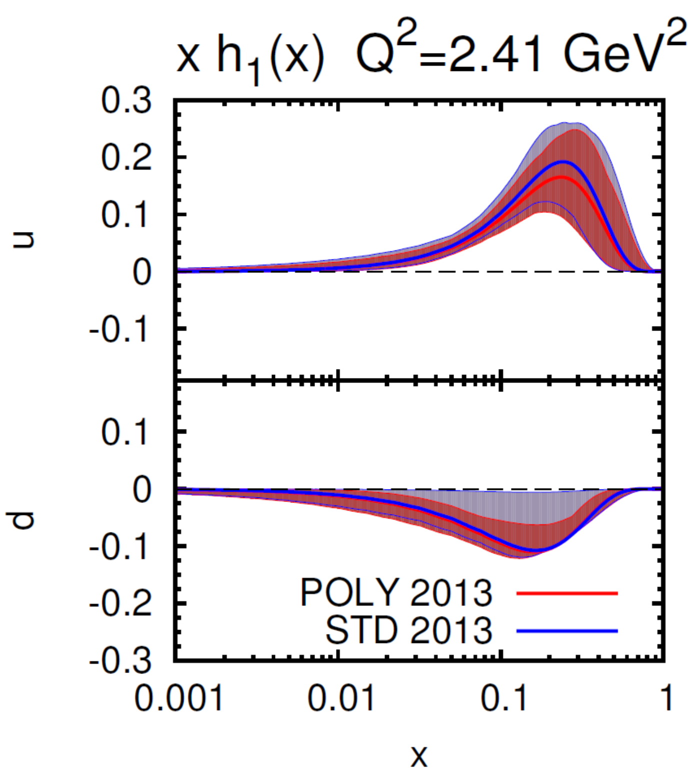


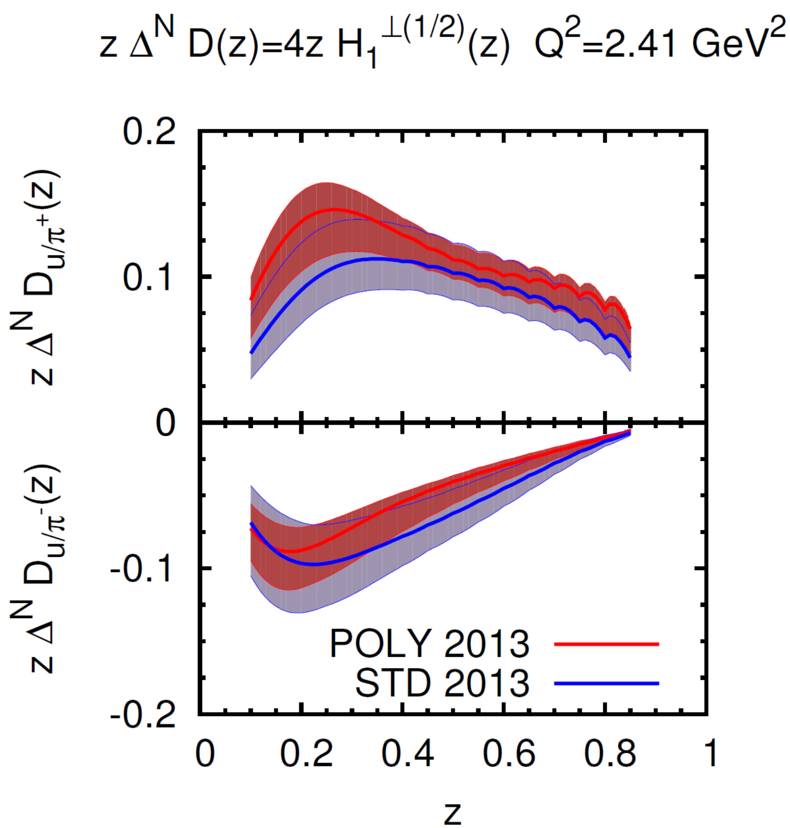




Extraction of transversity & Collins functions

- FIT II vs FIT IV (POLYNOMIAL vs STD; FITTED A_0)

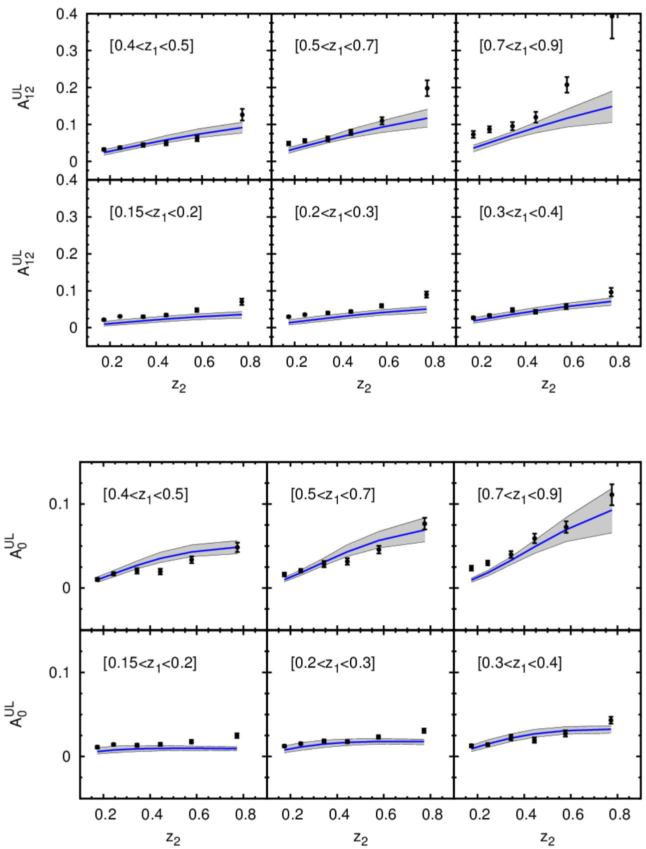




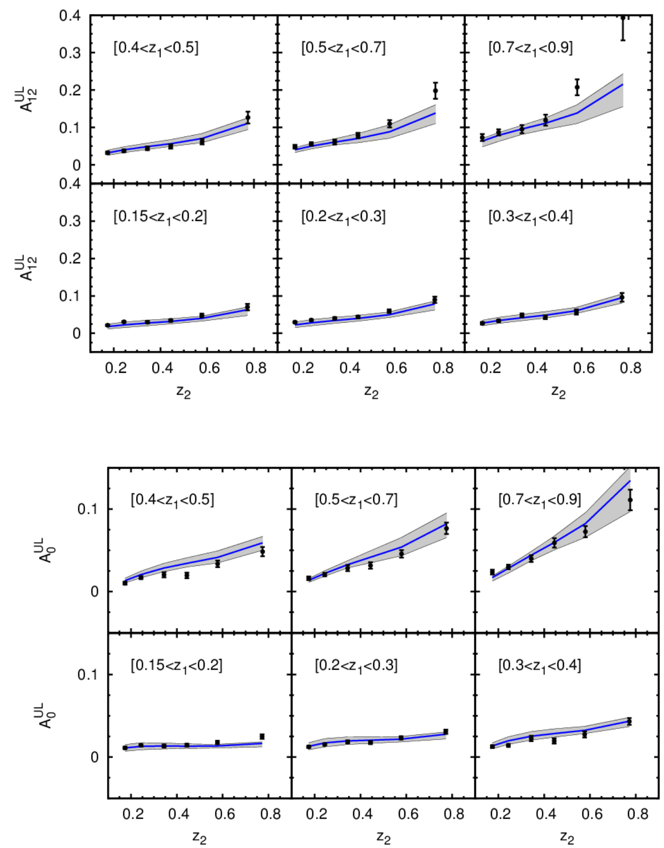
➔ Same transversity

➔ Different Collins functions (but not dramatically different)

BaBar Predictions



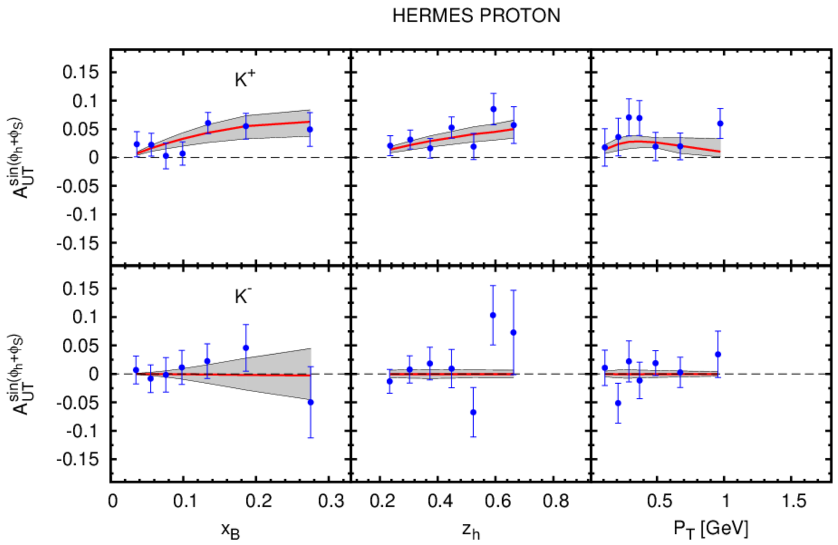
FIT I

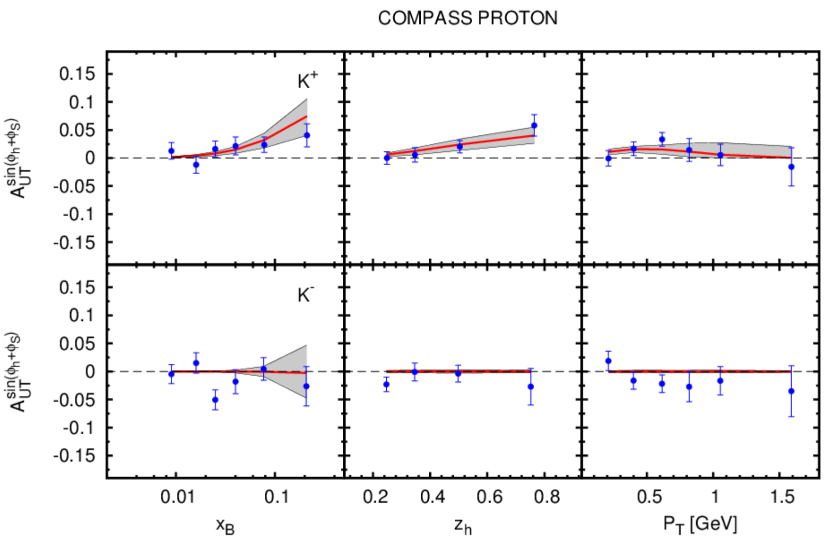


FIT IV

Kaons Collins Function??

- Given a Transversity can we extract information on the Kaons Collins functions??





- Disfavored completely undetermined. Favored large and positive.

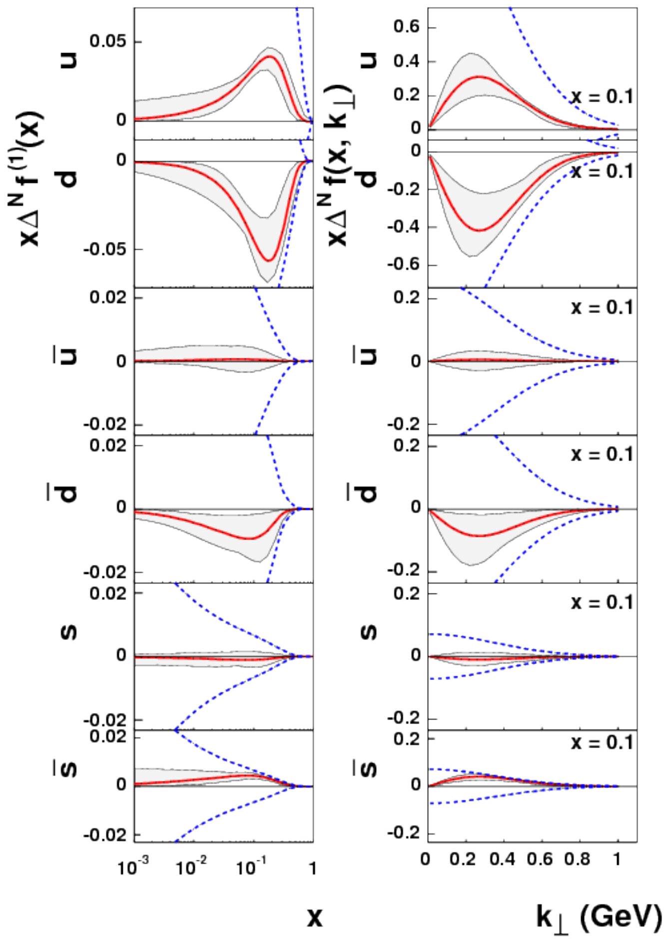
Conclusions

- New extraction of transversity and Collins functions
- UL and UC data fully compatible
- COMPASS & HERMES fully compatible without TMD evolution
- A_{12} analysis in agreement with 2008 analysis
- A_0 is better described if we change parametrization
- Small tension between A_{12} and A_0 sets
- BaBar predictions

The Sivers function from SIDIS data

Sivers function in SIDIS

➢In 2009 we performed a fit of HERMES (2002-5) and COMPASS (Deuteron 2003-4) data on π and K production



✔Valence quark

- $\Delta^N f_{u/p^\uparrow} > 0 \quad \Rightarrow \quad f_{1T}^{\perp u} < 0$
- $\Delta^N f_{d/p^\uparrow} < 0 \quad \Rightarrow \quad f_{1T}^{\perp d} > 0$

✔Sea quarks

- $\Delta^N f_{\bar{s}/p^\uparrow} > 0 \quad \Rightarrow \quad f_{1T}^{\perp \bar{s}} < 0$

➔ $\Delta^N f_q^{(1)}(x) \equiv \int d^2\boldsymbol{k}_\perp \, \frac{k_\perp}{4m_p} \, \Delta^N f_{q/p^\uparrow}(x, k_\perp) = -f_{1T}^{\perp(1)q}(x)$

Anselmino et al. , Eur. Phys. J. A39, 89-100 (2009)

Sivers function in SIDIS

- 2009 extraction: DGLAP evolution (No TMD evolution), No COMPASS proton data
- In 2012 we applied the Collins TMD evolution scheme to the analysis of the new data from HERMES (2009) and from COMPASS (proton target, 2010-11)

Anselmino et al. , Eur. Phys. J. A39, 89-100 (2009)

TMD evolution formalism

- The simplest version of the Collins TMD evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

Corresponding to Eq. 44 of Ref [*] with $\tilde{K}$=0 and : $\mu^2 = \zeta_F = \zeta_D = Q^2$

- *[*]S. M. Aybat, J. C. Collins, J.-W. Qiu and T.C. Rogers, arXiv:1110.6428 [hep-ph]*

TMD evolution formalism

➢ At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

Output function at the scale **Q** in the impact parameter space

Input function at the scale $\mathbf{Q_0}$ in the impact parameter space

Evolution kernel

TMD evolution formalism

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- **Perturbative** part of the evolution kernel

TMD evolution formalism

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- **Perturbative** part of the evolution kernel

$$\widetilde{R}(Q, Q_0, b_T) \equiv \exp\left\{ \ln \frac{Q}{Q_0} \int_{Q_0}^{\mu_b} \frac{\mathrm{d}\mu'}{\mu'} \gamma_K(\mu') + \int_{Q_0}^{Q} \frac{\mathrm{d}\mu}{\mu} \gamma_F\left(\mu, \frac{Q^2}{\mu^2}\right) \right\}$$

TMD evolution formalism

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$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

- **Perturbative** part of the evolution kernel

$$\widetilde{R}(Q, Q_0, b_T) \equiv \exp\left\{\ln\frac{Q}{Q_0}\int_{Q_0}^{\mu_b}\frac{\mathrm{d}\mu'}{\mu'}\gamma_K(\mu') + \int_{Q_0}^{Q}\frac{\mathrm{d}\mu}{\mu}\gamma_F\left(\mu, \frac{Q^2}{\mu^2}\right)\right\}$$

$$\gamma_K(\mu) = \alpha_s(\mu)\frac{2C_F}{\pi}$$

$$\gamma_F(\mu; \frac{Q^2}{\mu^2}) = \alpha_s(\mu)\frac{C_F}{\pi}\left(\frac{3}{2} - \ln\frac{Q^2}{\mu^2}\right)$$

TMD evolution formalism

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$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

- **Perturbative** part of the evolution kernel

$$\widetilde{R}(Q, Q_0, b_T) \equiv \exp\left\{ \ln \frac{Q}{Q_0} \int_{Q_0}^{\mu_b} \frac{d\mu'}{\mu'} \gamma_K(\mu') + \int_{Q_0}^{Q} \frac{d\mu}{\mu} \gamma_F\left(\mu, \frac{Q^2}{\mu^2}\right) \right\}$$

Scale that separates the perturbative region from the non perturbative one

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

- **Perturbative** part of the evolution kernel

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$$\mu_b = \frac{C_1}{b_*(b_T)} \qquad b_*(b_T) \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}} \qquad C_1 = 2e^{-\gamma_E}$$

One of the possible prescription to separate the perturbative region from the non perturbative one

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

- **Non Perturbative** (scale independent) part of the evolution kernel that needs to be empirically modeled

$$g_K(b_T) = \frac{1}{2}\, g_2\, b_T^2$$

Common choice used in the unpolarized DY data analyses in the CSS formalism

Parametrization ot the input functions

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0) \widetilde{R}(Q, Q_0, b_T) \, \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

- Model/parametrization: Different parametrizations here can give very different answers!

- Our approach: Let us apply our standard parametrizations i.e. gaussians factorized among collinear and transverse degree of freedom.
  It is not a unique choice or the best one!

Parametrization of the input functions

➤TMD evolution equations using a gaussian model::

$$\widetilde{f}_{q/p}(x, b_T; Q) = f_{q/p}(x, Q_0)\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{-b_T^2\left(\alpha^2 + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\}$$

$$\widetilde{D}_{h/q}(z, b_T; Q) = \frac{1}{z^2} D_{h/q}(z, Q_0)\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{-b_T^2\left(\beta^2 + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\}$$

$$\widetilde{f}'^{\perp}_{1T}(x, b_T; Q) = -2\,\gamma^2\, f^{\perp}_{1T}(x; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, b_T\ \exp\left\{-b_T^2\left(\gamma^2 + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\}$$

Fit of HERMES and COMPASS SIDIS data

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \frac{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp\, \Delta^N f_{q/p^\uparrow}(x,k_\perp,Q) \sin(\varphi-\phi_S) \frac{d\hat\sigma^{\ell q\to\ell q}}{dQ^2}\; D_q^h(z,p_\perp,Q)\sin(\phi_h-\phi_S)}{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp\, f_{q/p}(x,k_\perp,Q) \frac{d\hat\sigma^{\ell q\to\ell q}}{dQ^2}\; D_q^h(z,p_\perp,Q)}$$

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q_0) = 2\mathcal{N}_q(x) h(k_\perp) \widehat{f}_{q/p}(x,k_\perp;Q_0)$$

$$\mathcal{N}_q(x) = N_q\, x^{\alpha_q}(1-x)^{\beta_q}\, \frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}$$

$$h(k_\perp) = \sqrt{2e}\,\frac{k_\perp}{M_1}\, e^{-k_\perp^2/M_1^2}$$

$$\widehat{f}_{q/p}(x,k_\perp;Q_0) = f_{q/p}(x,Q_0)\,\frac{1}{\pi\langle k_\perp^2\rangle}\, e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$$\widehat{D}_{h/q}(z,p_\perp;Q_0) = D_{h/q}(z,Q_0)\,\frac{1}{\pi\langle p_\perp^2\rangle}\, e^{-p_\perp^2/\langle p_\perp^2\rangle}$$

11 free parameters

N_{u_v}	N_{d_v}	N_s
$N_{\bar u}$	$N_{\bar d}$	$N_{\bar s}$
α_{u_v}	α_{d_v}	α_{sea}
β	M_1 (GeV/c) .	

Fixed parameters

$$\langle k_\perp^2\rangle = 0.25\ \text{GeV}^2$$
$$\langle p_\perp^2\rangle = 0.20\ \text{GeV}^2$$
$$g_2 = 0.68\ \text{GeV}^2$$
$$b_{max} = 0.5\ \text{GeV}^{-1}$$

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$$\mathcal{N}_q(x) = N_q\, x^{\alpha_q}(1-x)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

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$$\widehat{f}_{q/p}(x,k_\perp;Q_0) = f_{q/p}(x,Q_0)\,\frac{1}{\pi\langle k_\perp^2\rangle}\, e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$$\widehat{D}_{h/q}(z,p_\perp;Q_0) = D_{h/q}(z,Q_0)\,\frac{1}{\pi\langle p_\perp^2\rangle}\, e^{-p_\perp^2/\langle p_\perp^2\rangle}$$

11 free parameters

N_{u_v}	N_{d_v}	N_s
$N_{\bar u}$	$N_{\bar d}$	$N_{\bar s}$
α_{u_v}	α_{d_v}	α_{sea}
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$$b_{max} = 0.5\ \text{GeV}^{-1}$$

Fit of HERMES and COMPASS SIDIS data

χ^2 tables

11 free parameters, 261 points

TMD evolution (exact)
$\chi^2_{\rm tot} = 255.8$
$\chi^2_{\rm d.o.f} = 1.02$

DGLAP evolution
$\chi^2_{\rm tot} = 315.6$
$\chi^2_{\rm d.o.f} = 1.26$

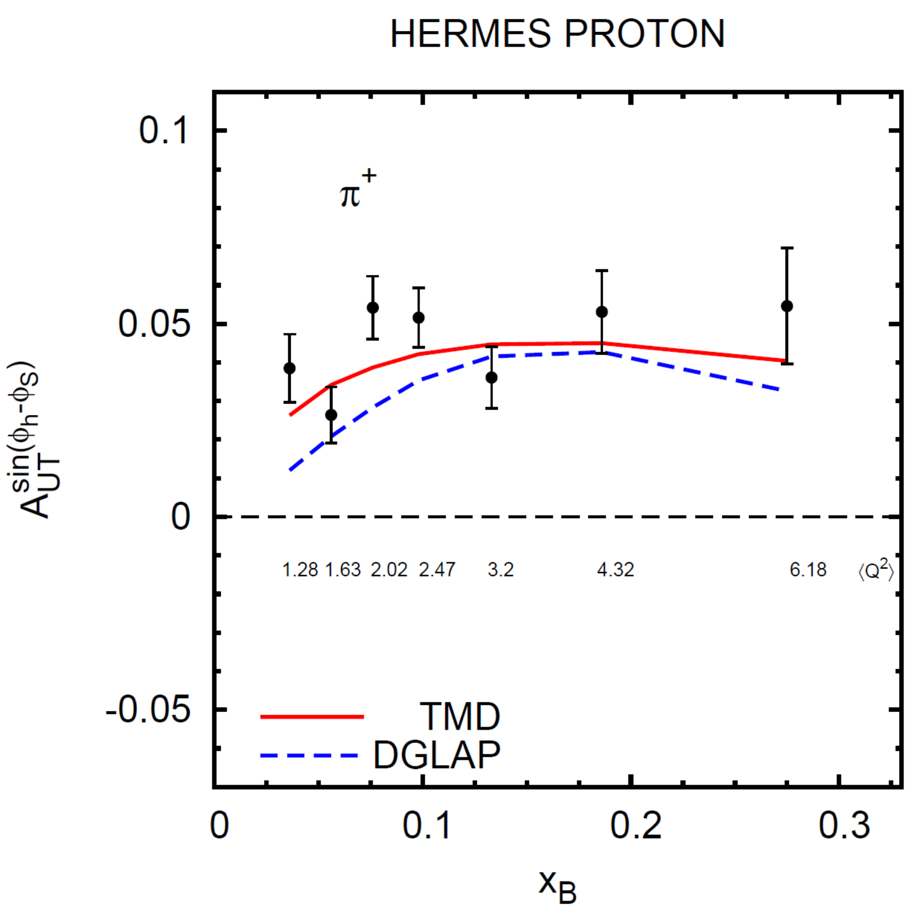
Fit of HERMES and COMPASS SIDIS data

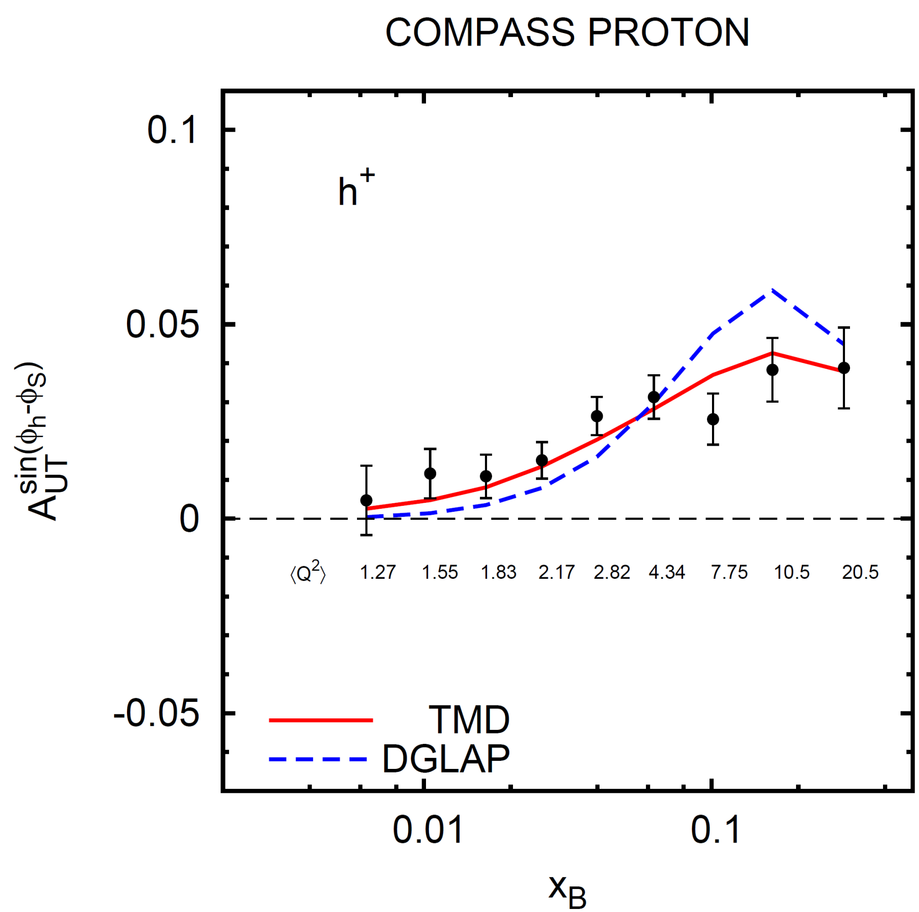
χ^2 tables

11 free parameters, 261 points

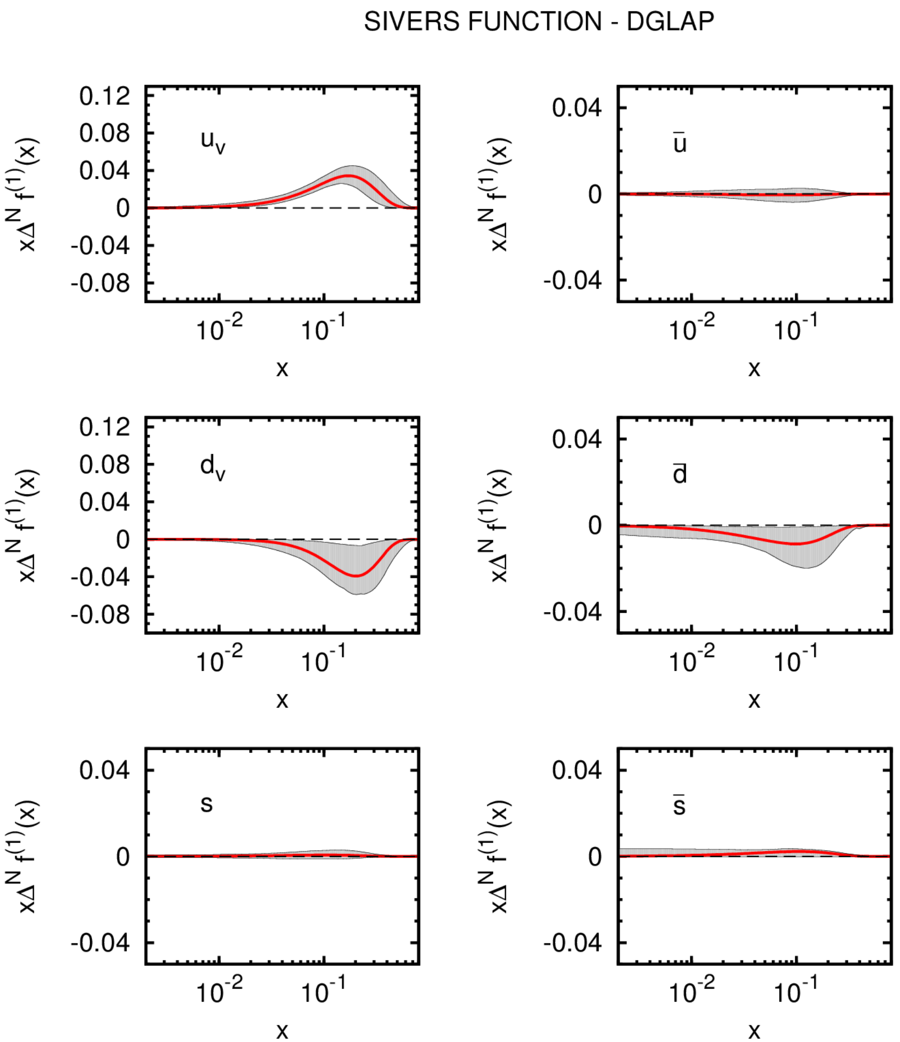
	TMD Evolution (Exact)		DGLAP Evolution
	$\chi^2_{tot} = 255.8$		$\chi^2_{tot} = 315.6$
	$\chi^2_{d.o.f} = 1.02$		$\chi^2_{d.o.f} = 1.26$
HERMES π^+	$\chi^2_x = 10.7$	7 points	$\chi^2_x = 27.5$
	$\chi^2_z = 4.3$		$\chi^2_z = 8.6$
	$\chi^2_{P_T} = 9.1$		$\chi^2_{P_T} = 22.5$
COMPASS h^+	$\chi^2_x = 6.7$	9 points	$\chi^2_x = 29.2$
	$\chi^2_z = 17.8$		$\chi^2_z = 16.6$
	$\chi^2_{P_T} = 12.4$		$\chi^2_{P_T} = 11.8$

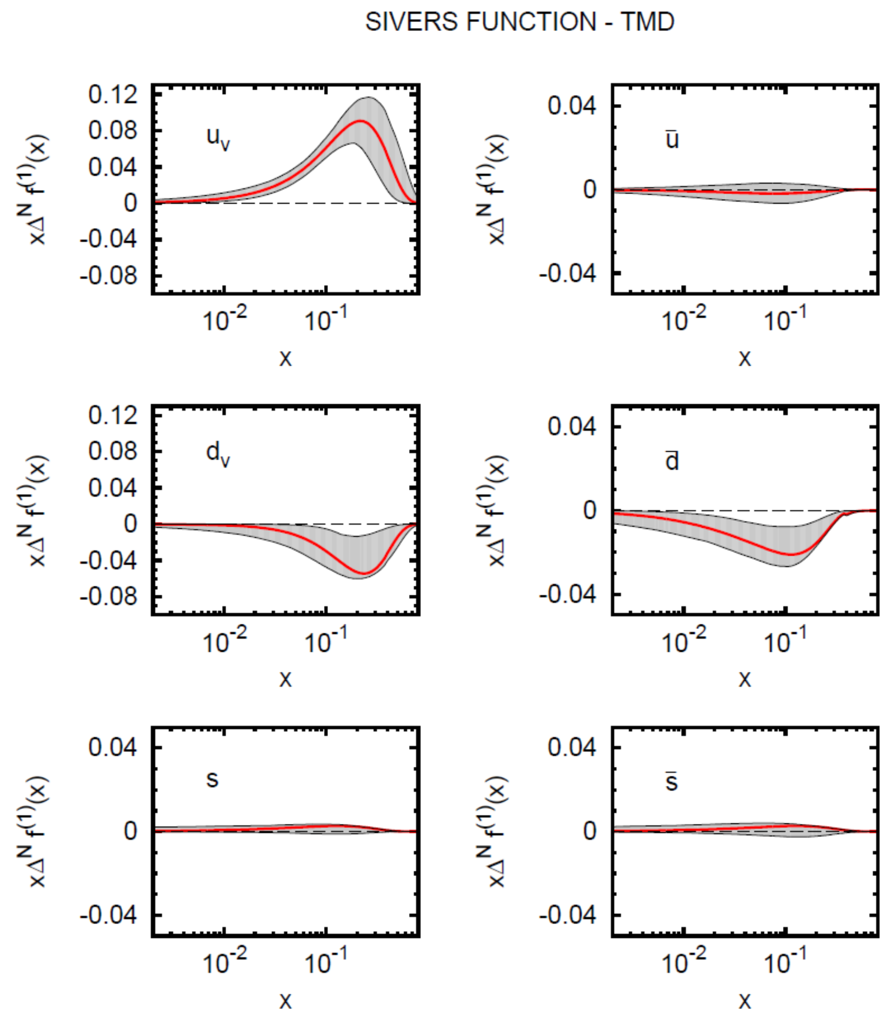
Fit of HERMES and COMPASS SIDIS data

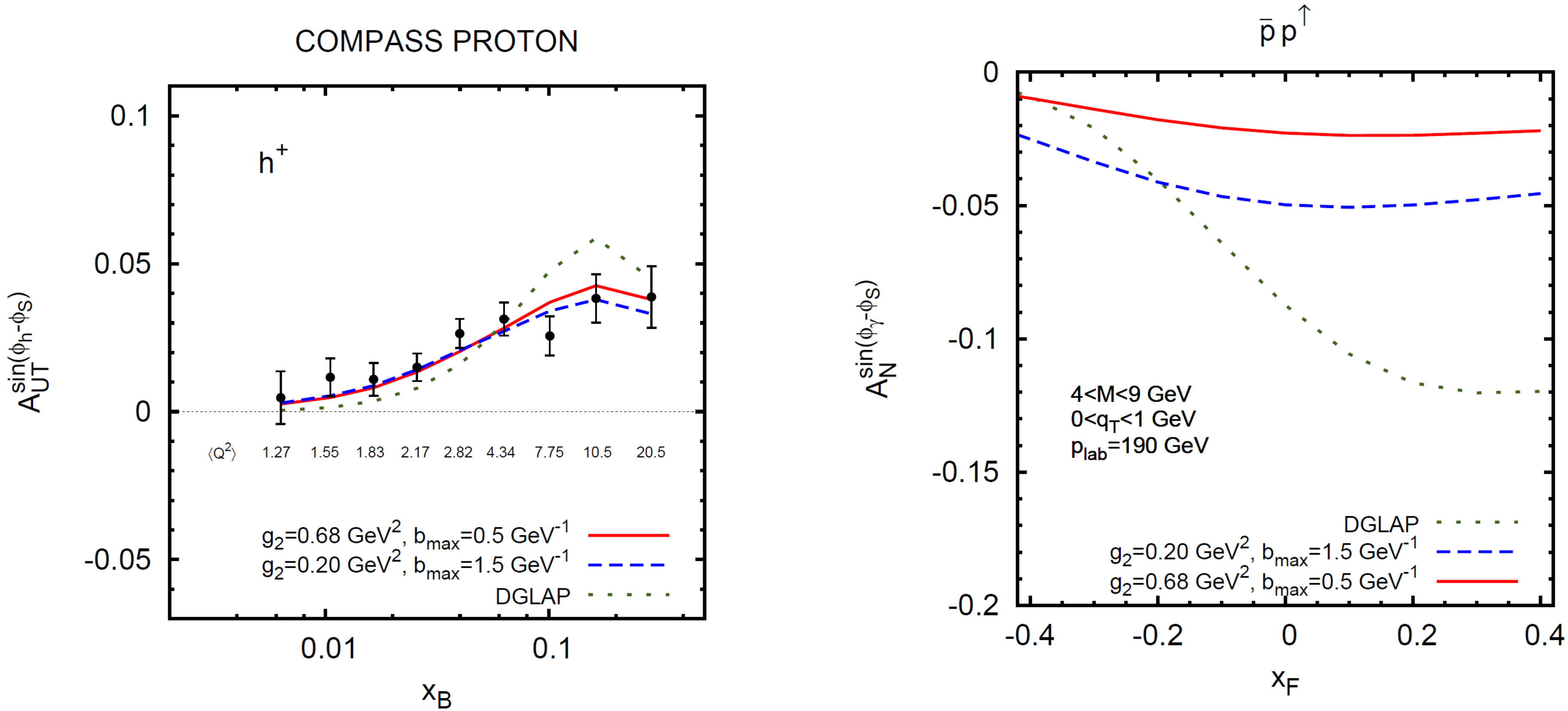




Sivers functions





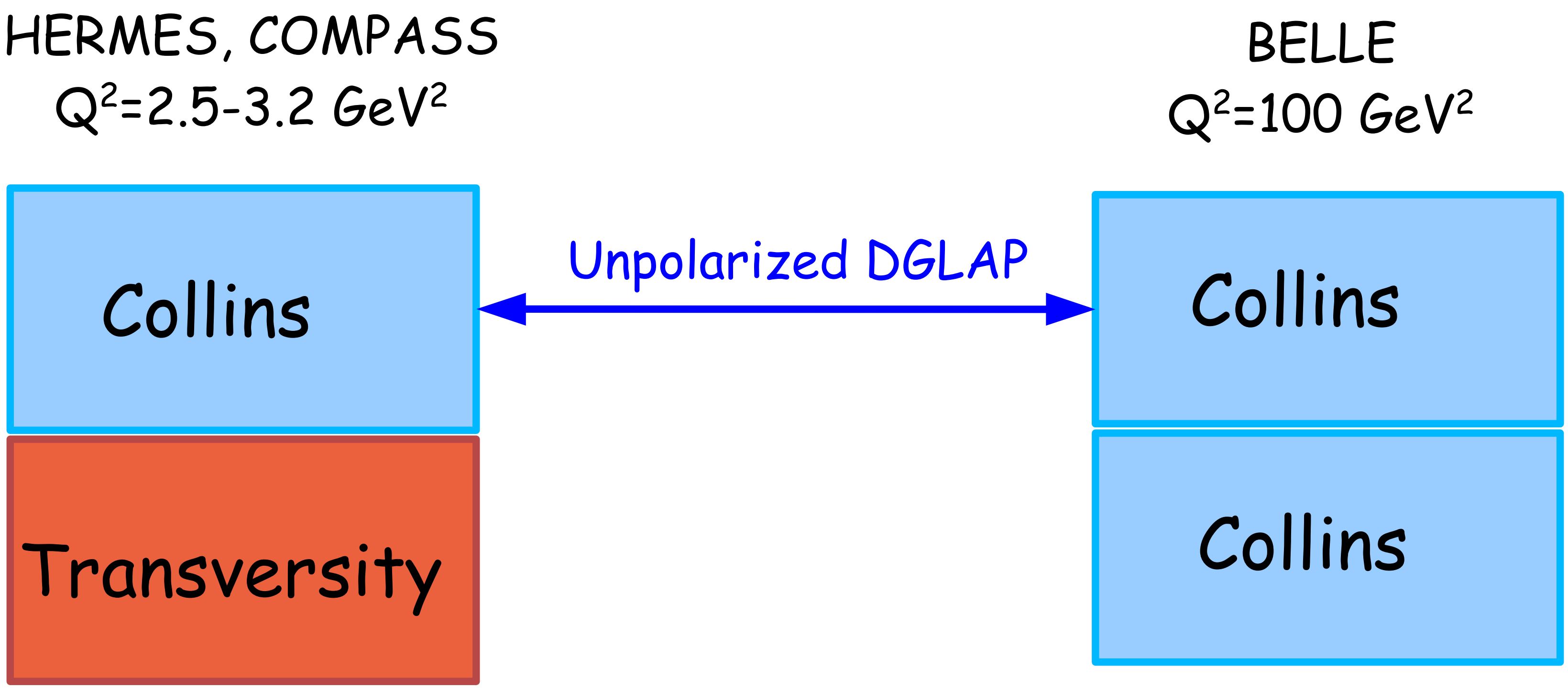

COMPASS PROTON
h+
0.1
0.05
0
-0.05
⟨Q2⟩ 1.27 1.55 1.83 2.17 2.82 4.34 7.75 10.5 20.5
g2=0.68 GeV2, bmax=0.5 GeV-1
g2=0.20 GeV2, bmax=1.5 GeV-1
DGLAP
0.01
0.1
xB
A_UT^sin(φh-φS)
p̄ p↑
0
-0.05
-0.1
-0.15
-0.2
4<M<9 GeV
0<qT<1 GeV
plab=190 GeV
DGLAP
g2=0.20 GeV2, bmax=1.5 GeV-1
g2=0.68 GeV2, bmax=0.5 GeV-1
-0.4
-0.2
0
0.2
0.4
xF
A_N^sin(φγ-φS)

Conclusions II

- Valence Sivers functions are definitively different from zero.
- Sea quark Sivers functions need more precise SIDIS data and/or Drell-Yan data.
- There are indications supporting TMD evolution in SIDIS
- Asymmetry in Drell-Yan are more sensitive to TMD evolution Drell-Yan data are crucial for our understanding of the TMDs

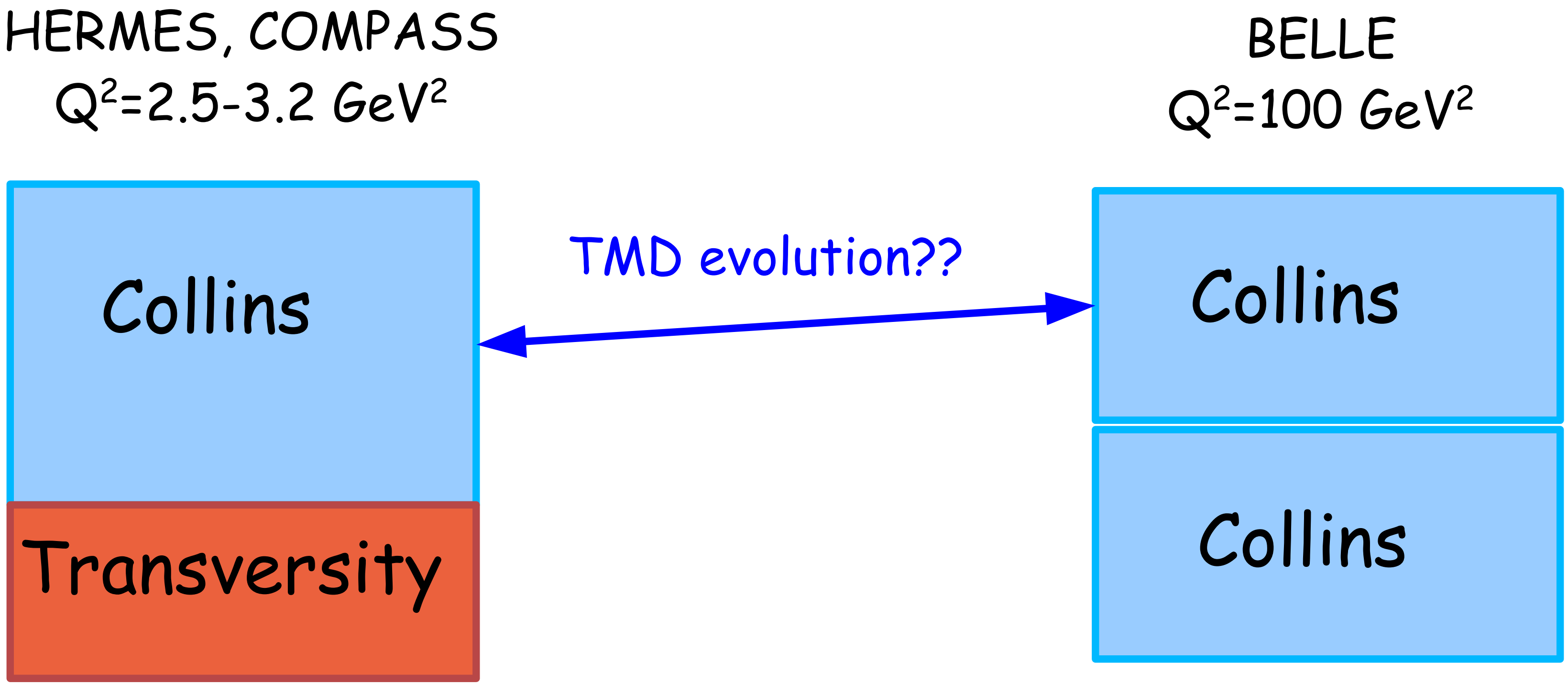
TMD evolution

- TMD evolution for the Collins function is still unknown.
- TMD evolution can suppress the Collins function at large Q^2 (Boer, 2001)



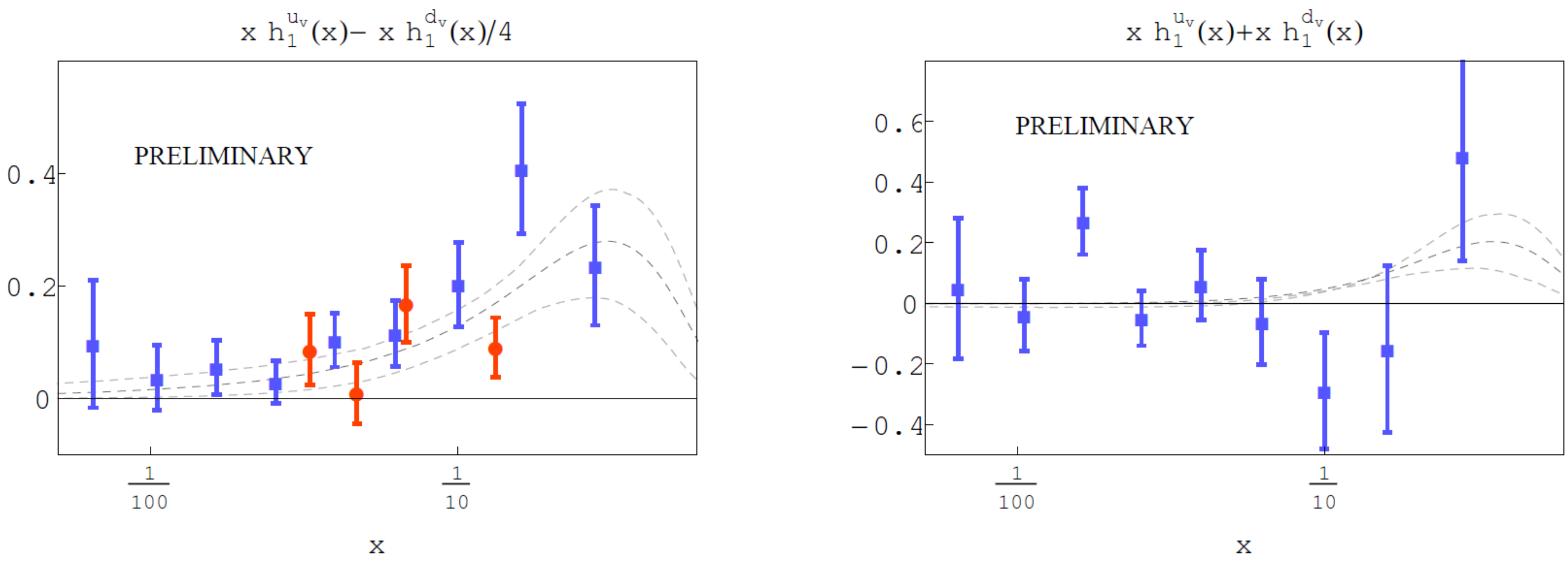
TMD evolution

- TMD evolution for the Collins function is still unknown.
- TMD evolution can suppress the Collins function at large Q^2
  [D. Boer, Nucl. Phys. B603 (2001); Nucl. Phys. B806 (2009)]



The dihadron way: Pavia group extraction

- Comparison Pavia 2012-Torino 2008

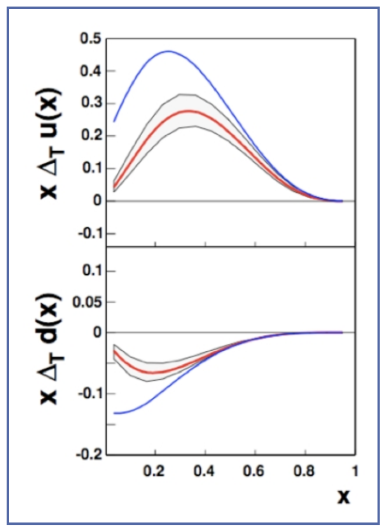


Bacchetta, Courtoy, Radici.,arXiv:1206.1836

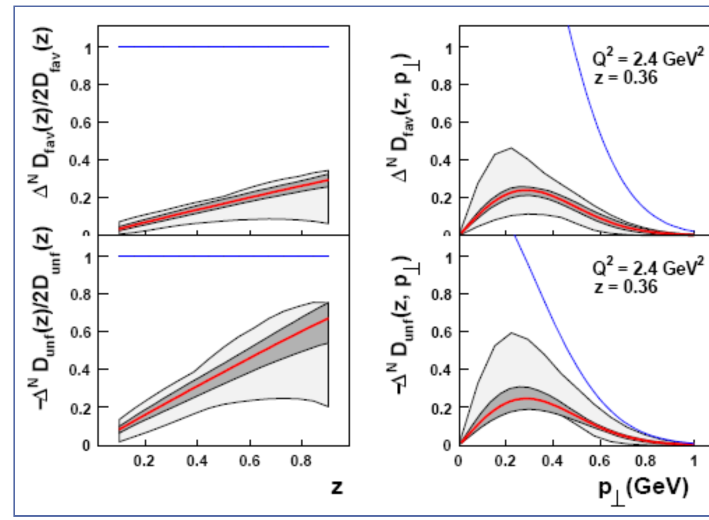
Extraction of transversity & Collins functions

- Simultaneous fit of HERMES, COMPASS D and BELLE (A_{12}^{UL}) data

χ^2_{dof}=1.3



Transversity



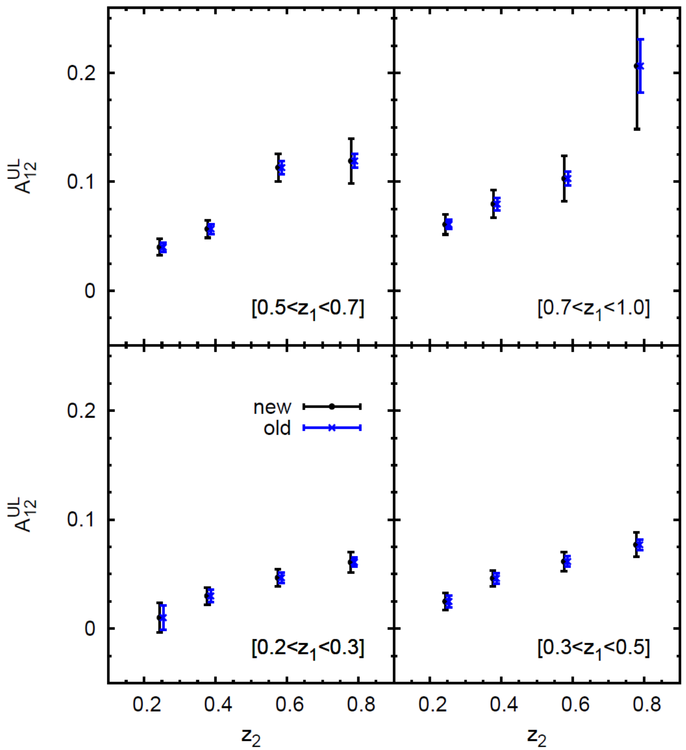
Collins functions

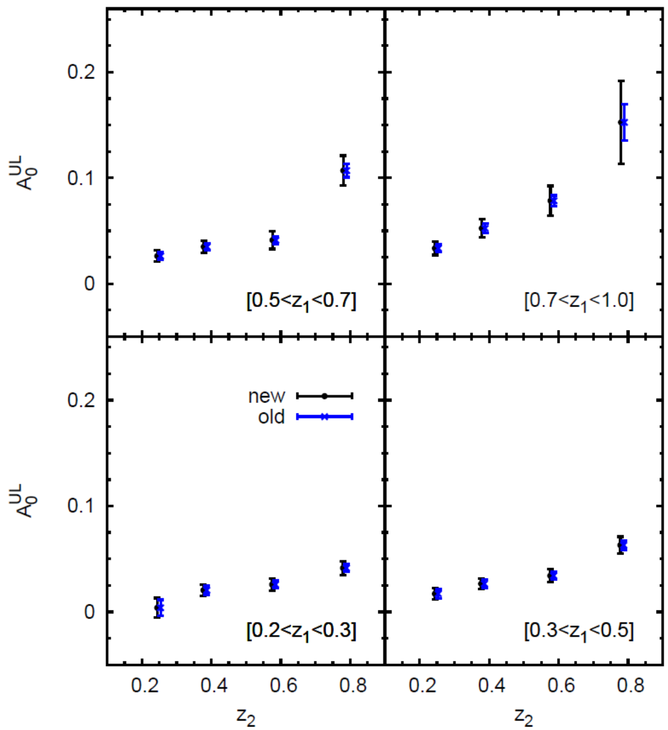
$N_u^T = 0.64 \pm 0.34$	$N_d^T = -1.00 \pm 0.02$
$\alpha = 0.73 \pm 0.51$	$\beta = 0.84 \pm 2.30$
$N_{fav}^C = 0.44 \pm 0.07$	$N_{unf}^C = -1.00 \pm 0.06$
$\gamma = 0.96 \pm 0.08$	$\delta = 0.01 \pm 0.05$
$M_h^2 = 0.91 \pm 0.52 \text{ GeV}^2$	

- Anselmino et. al arXiv: 0812.4366v1

BELLE Erratum

- BELLE Erratum: R. Seidl, PRD 86 (2012) 039905

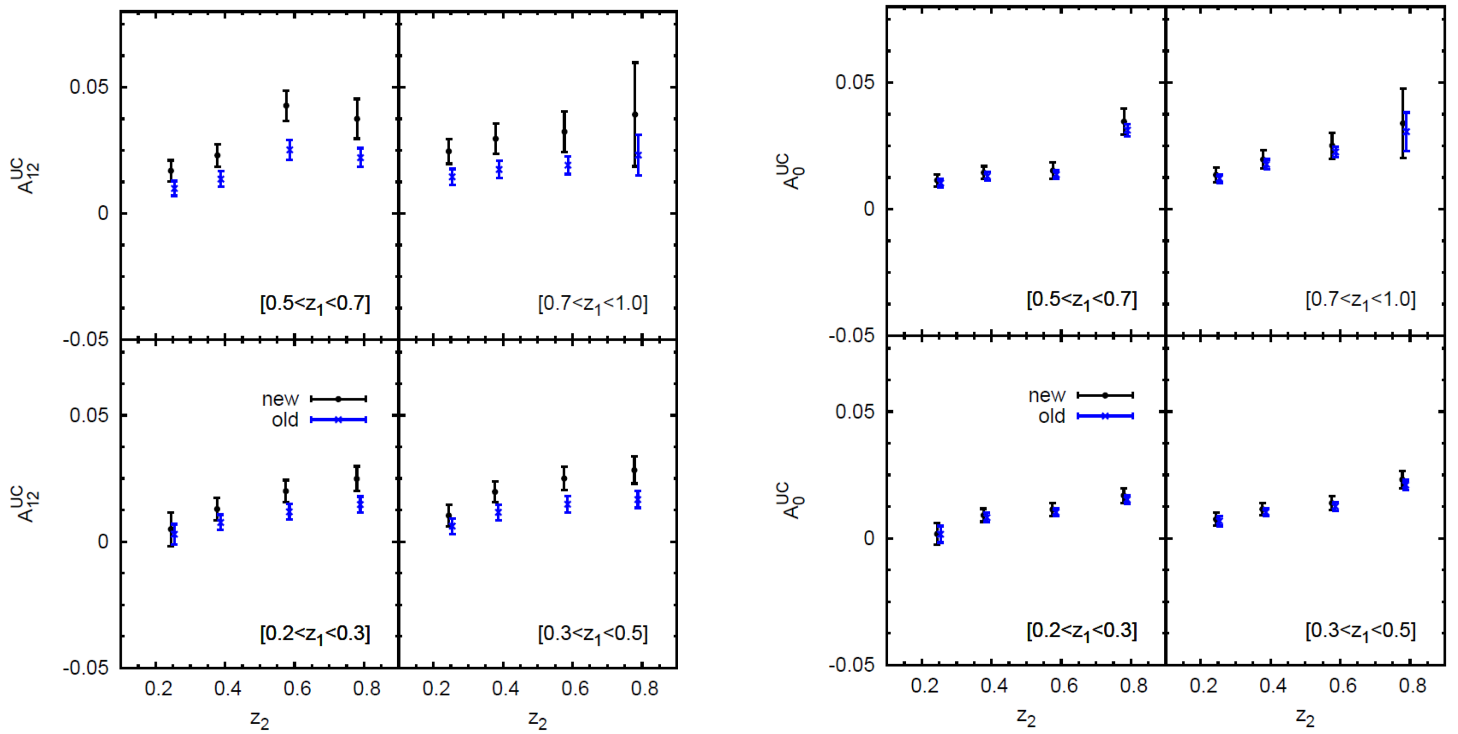




A^{UL}: Same central values, larger errors

BELLE Erratum

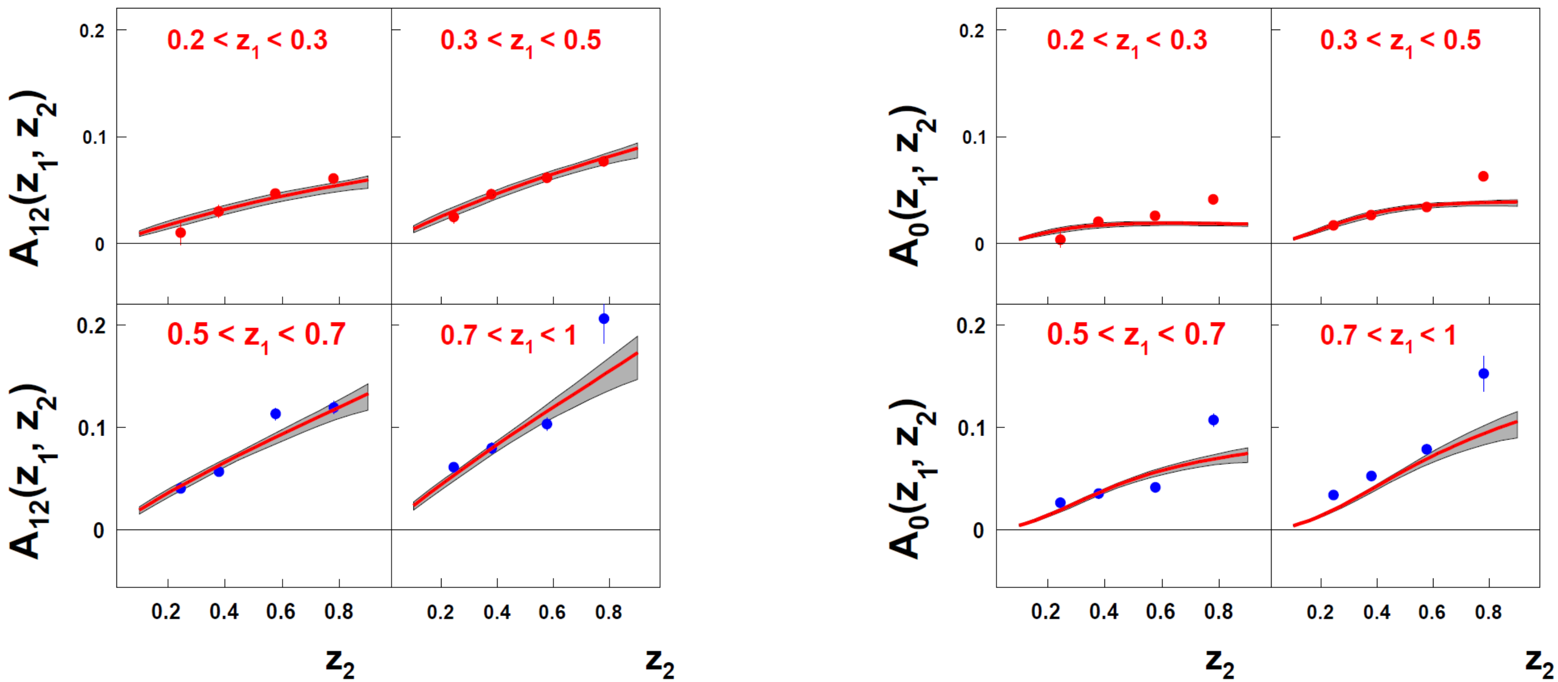
- BELLE Erratum: R. Seidl, PRD 86 (2012) 039905



A^{UC}: Different normalization, larger errors

BELLE Erratum&2008 Extraction

- BELLE Erratum: R. Seidl, PRD 86 (2012) 039905



◇ R. Seidl et al., Phys. Rev. D78

Good news! Previously partial incompatibility between the sets

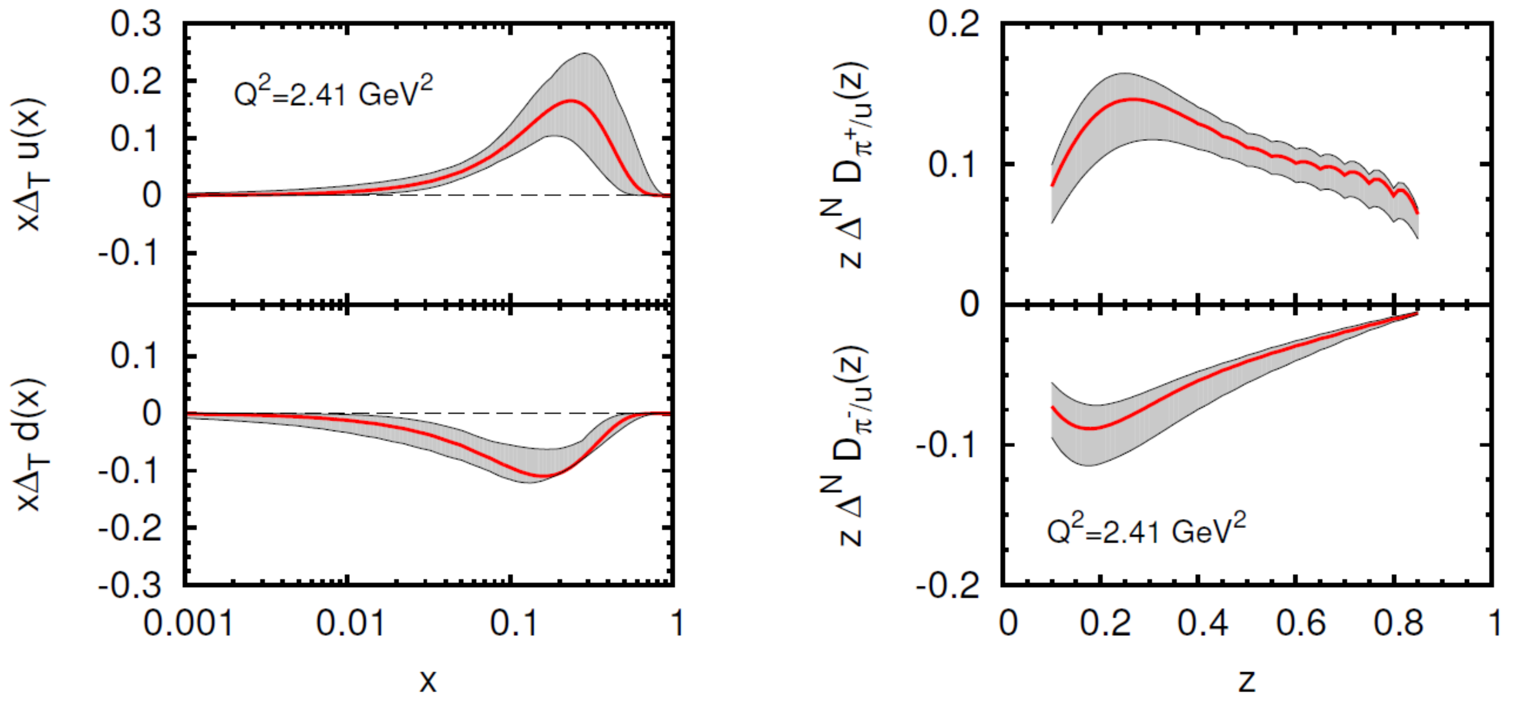
2013 Update of the extraction

- 2008 Extraction:
  - HERMES Preliminary (2002-2005) π+ π-
  - COMPASS Deuteron (2004) π+ π-
  - BELLE A_{12}^{UL} (PRD78, 2008)
- New analysis 2013:
  - HERMES (2009) π+ π-
  - COMPASS Deuteron (2004) π+ π-
  - COMPASS Proton (2013) π+ π-
  - BELLE A_{12} or A_0 (BELLE ERRATUM 2012, PRD86)

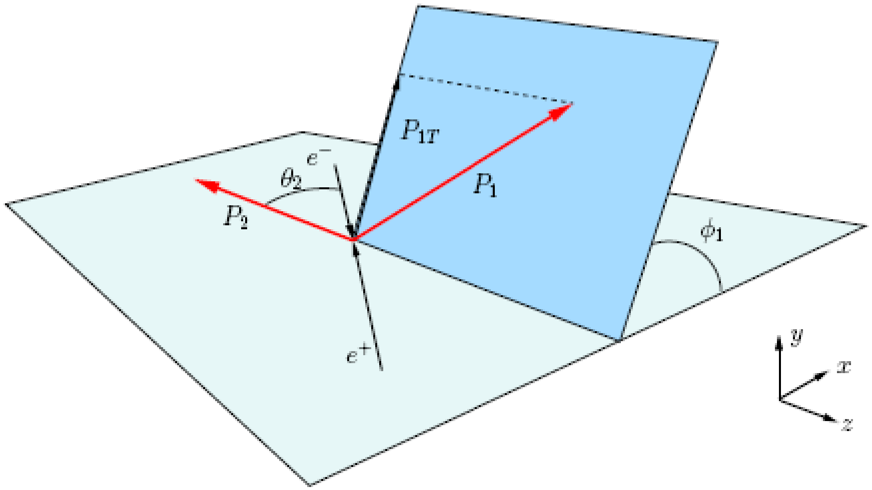
Anselmino et al. Phys. Rev. D87,094019

Extraction of transversity & Collins functions

- FIT IV: A_0 BELLE data UL & UC +COMPASS+ HERMES POLYNOMIAL



Hadronic plane method



where $\phi_q^{h_1}$ is the azimuthal angle of the detected hadron h_1 around the direction of the parent fragmenting quark, q. Technically, $\phi_q^{h_1}$ is the azimuthal angle of $\boldsymbol{p}_{\perp 1}$ in the helicity frame of q. It can be expressed in terms of the integration variables we are using, $\boldsymbol{p}_{\perp 2}$ and P_{1T}. At lowest order in $p_\perp/(z\sqrt{s})$ we have

$$\cos\phi_q^{h_1} = \frac{P_{1T}}{p_{\perp 1}} \cos(\phi_1 - \varphi_2) - \frac{z_1}{z_2}\frac{p_{\perp 2}}{p_{\perp 1}}\,, \qquad (56)$$

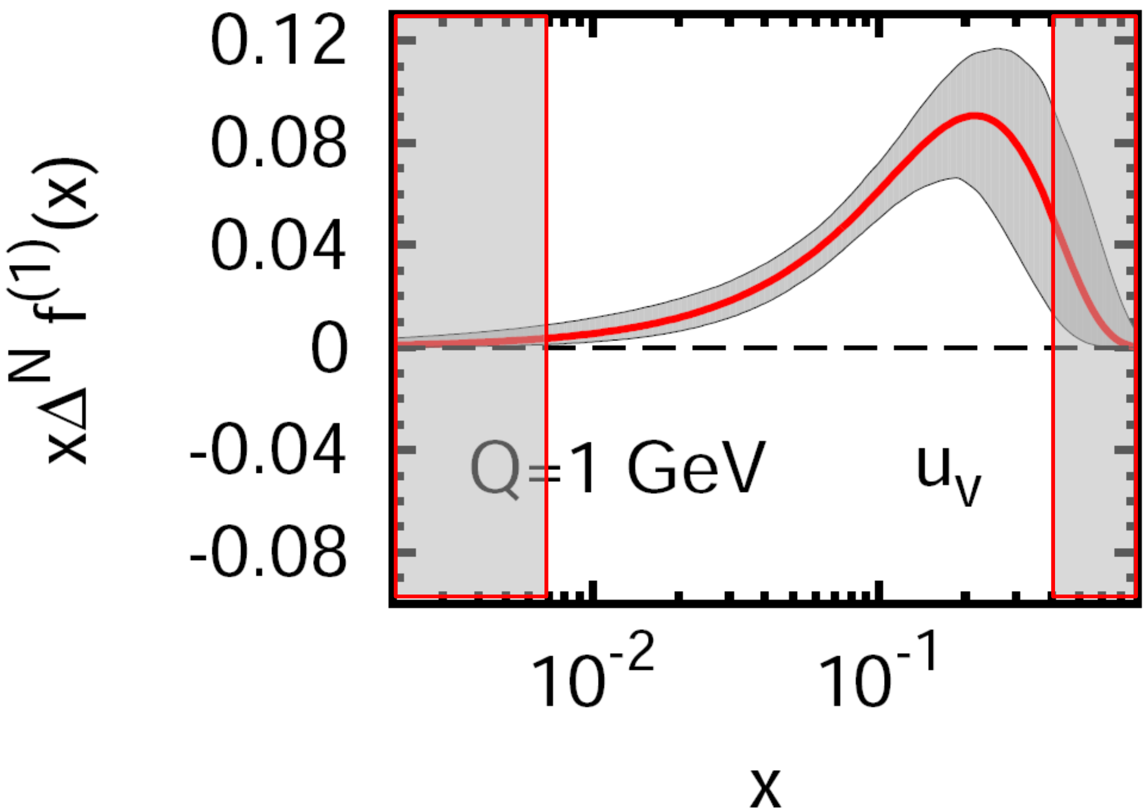
$$\sin\phi_q^{h_1} = \frac{P_{1T}}{p_{\perp 1}} \sin(\phi_1 - \varphi_2)\,. \qquad (57)$$

Sivers function in SIDIS

- In 2012 we applied the Collins TMD evolution scheme* to the analysis of the new data from HERMES (2009) and from COMPASS (proton target, 2010-11)

*
- *J.C. Collins, Foundation of Perturbative QCD, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, No. 32, Cambridge University Press, 2011.*
- *S. M. Aybat and T. C. Rogers, Phys. Rev. D83, 114042 (2011), arXiv:1101.5057 [hep-ph]*
- *S. M. Aybat, J. C. Collins, J.-W. Qiu and T.C. Rogers, arXiv:1110.6428 [hep-ph]*

Sivers functions



Turin standard approach (DGLAP)

➢Unpolarized TMDs are factorized in x and $k_\perp$. Only the collinear part evolves with DGLAP evolution equation. No evolution in the transverse momenta:

$$\widehat{f}_{q/p}(x,k_\perp;Q) = f_{q/p}(x;Q)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}$$

Collinear PDF (DGLAP evolution)

Normalized Gaussian: no evolution

Turin standard approach (DGLAP)

- The Sivers function is factorized in x and $k_\perp$ and proportional to the unpolarized PDF.

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q) = 2\mathcal{N}_q(x)h(k_\perp)\widehat{f}_{q/p}(x,k_\perp;Q)$$
$$= 2\mathcal{N}_q(x)f_{q/p}(x;Q)\sqrt{2e}\frac{k_\perp}{M_1}\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_S}}{\pi\langle k_\perp^2\rangle}$$

Collinear PDF (DGLAP)

$$\mathcal{N}_q(x) = N_q\, x^{\alpha_q}(1-x)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

$$\langle k_\perp^2\rangle_S = \frac{M_1^2\,\langle k_\perp^2\rangle}{M_1^2+\langle k_\perp^2\rangle}$$

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp) = -\frac{2k_\perp}{m_p} f_{1T}^\perp(x,k_\perp)$$

Collins TMD evolution of the Sivers function (PRD85,2012)

$$\tilde{F}'^{\perp f}_{1T}(x, b_T; \mu, \zeta_F) = \tilde{F}'^{\perp f}_{1T}(x, b_T; \mu_0, Q_0^2) \exp\left\{ \ln \frac{\sqrt{\zeta_F}}{Q_0} \tilde{K}(b_*; \mu_b) + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right.$$
$$\left. + \int_{\mu_0}^{\mu_b} \frac{d\mu'}{\mu'} \ln \frac{\sqrt{\zeta_F}}{Q_0} \gamma_K(g(\mu')) - g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{Q_0} \right\}. \quad (44)$$

$$\tilde{F}'^{\perp f}_{1T}(x, b_T; \mu, \zeta_F) = \sum_j \frac{M_p b_T}{2} \int_x^1 \frac{d\hat{x}_1 \, d\hat{x}_2}{\hat{x}_1 \, \hat{x}_2} \tilde{C}^{\text{Sivers}}_{f/j}(\hat{x}_1, \hat{x}_2, b_*; \mu_b^2, \mu_b, g(\mu_b)) \, T_{F\, j/P}(\hat{x}_1, \hat{x}_2, \mu_b)$$
$$\times \exp\left\{ \ln \frac{\sqrt{\zeta_F}}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times \exp\left\{ -g^{\text{Sivers}}_{f/P}(x, b_T) - g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{Q_0} \right\}. \quad (47)$$

Collins TMD evolution of the unpolarized PDF (PRD83,114042,2011)

$$\tilde{F}_{f/P}(x, \mathbf{b}_T; \mu, \zeta_F) = \overbrace{\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b)}^{A}$$
$$\times \overbrace{\exp\left\{\ln\frac{\sqrt{\zeta_F}}{\mu_b}\tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'}\left[\gamma_F(g(\mu'); 1) - \ln\frac{\sqrt{\zeta_F}}{\mu'}\gamma_K(g(\mu'))\right]\right\}}^{B}$$
$$\times \overbrace{\exp\left\{g_{j/P}(x, b_T) + g_K(b_T)\ln\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}}\right\}}^{C}, \quad (26)$$