Transverse size in electroproduction

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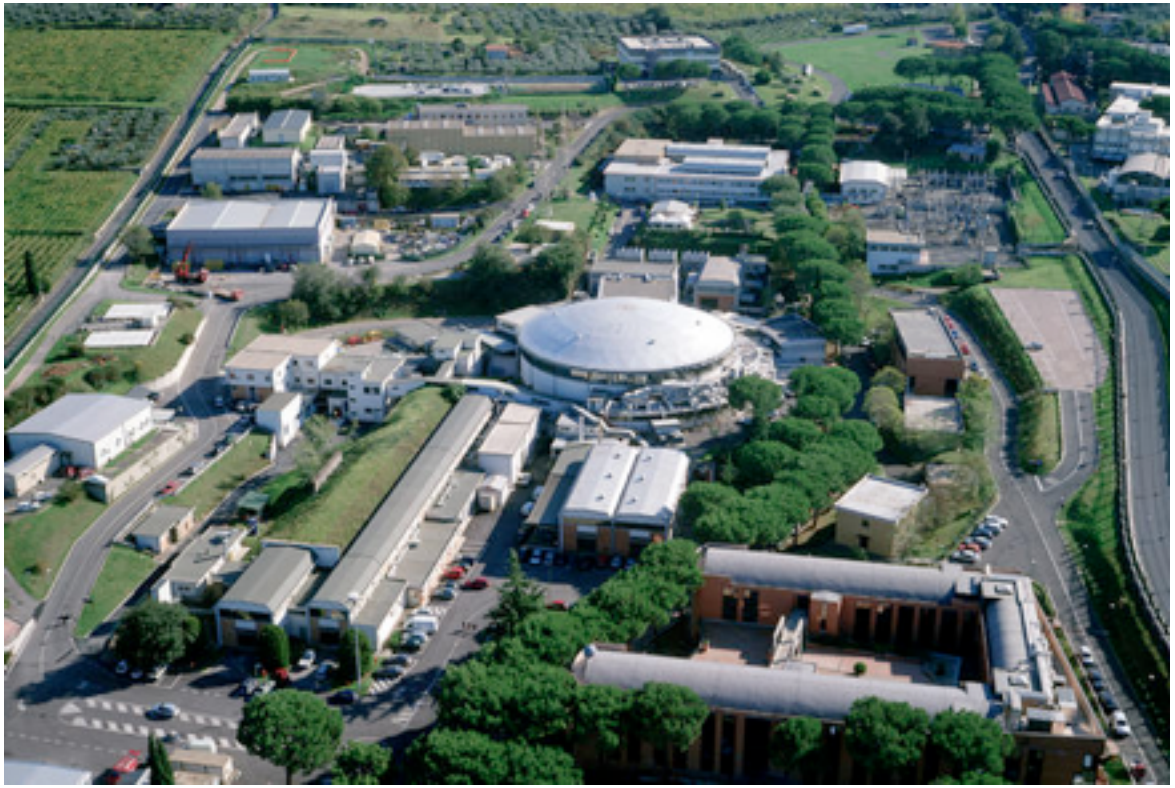
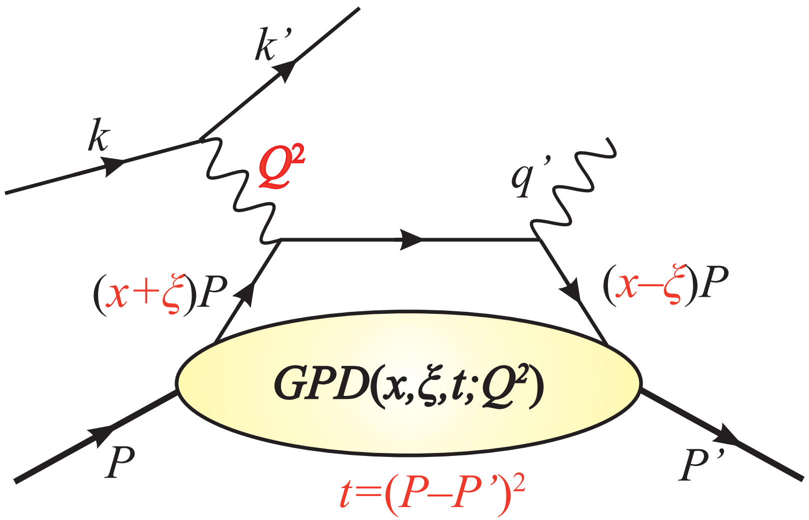


Based on work done with Samu Kurki

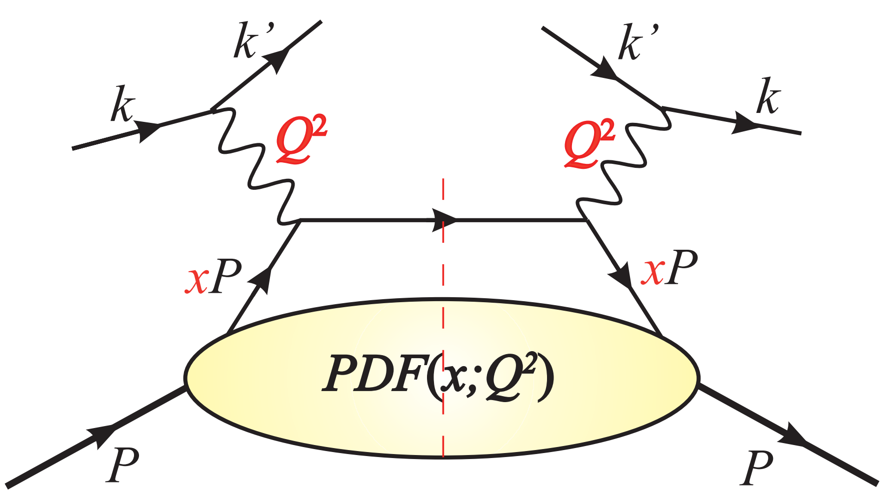
arXiv:0911.3011, arXiv:1101.4810

The two special cases of GPD's



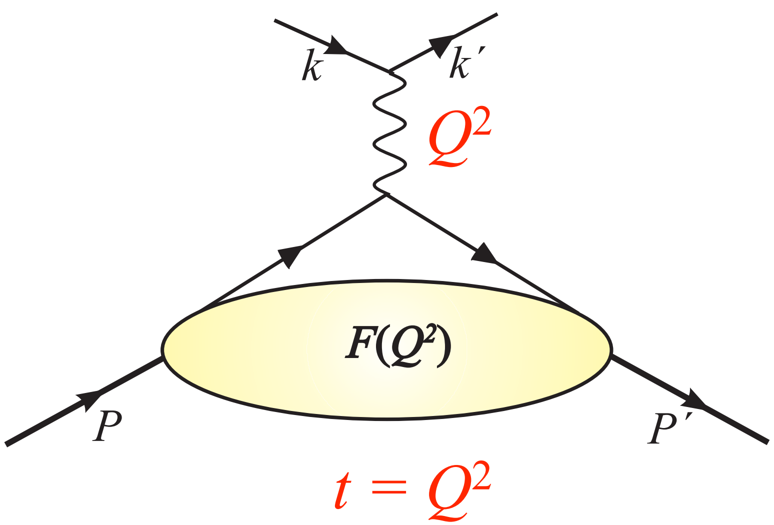
$$t,\ \xi \to 0$$

PDF: DIS, SIDIS,...



$$\int dx$$

Form factors,...



Form Factors ⟹ Quark distributions in b-space

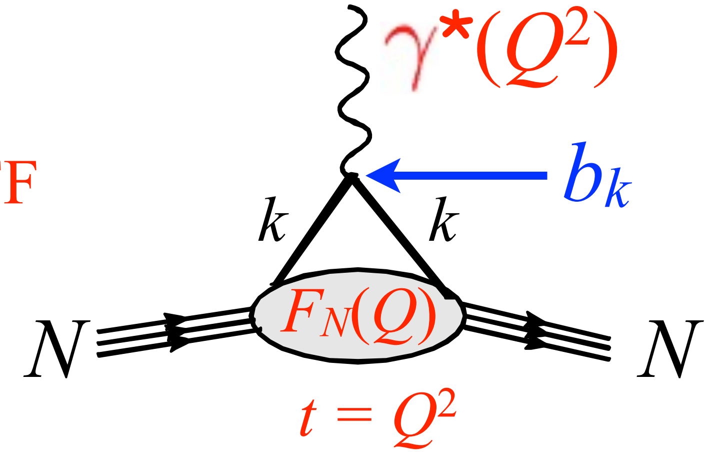
Fourier transforms of form factors give charge densities in impact parameter:

Miller (2007)
Carlson and Vanderhaeghen (2008)

$$\rho_0(\boldsymbol{b}) \equiv \int \frac{d^2\boldsymbol{q}}{(2\pi)^2}\, e^{-i\,\boldsymbol{q}\cdot\boldsymbol{b}}\, \frac{1}{2P^+} \langle P^+, \tfrac{1}{2}\boldsymbol{q}, \lambda|\, J^+(0)\, \left|P^+, -\tfrac{1}{2}\boldsymbol{q}, \lambda\right\rangle$$

$$= \int_0^\infty \frac{dQ}{2\pi}\, Q\, J_0(b\,Q) F_1(Q^2)$$

Dirac FF



PDF's vs. Densities in terms of wave functions

Parton distribution in x_B (neglecting Wilson line):

$$f_{q/N}(x) = \sum_{n,\lambda_i,k} \prod_{i=1}^{n} \Big[\int \frac{dx_i\, d^2\boldsymbol{k}_i}{16\pi^3}\Big] 16\pi^3 \delta(1-\sum_i x_i)\, \delta^{(2)}(\sum_i \boldsymbol{k}_i)$$

$$\times \delta(x-x_k)|\psi_n(x_i, \boldsymbol{k}_i, \lambda_i)|^2$$

Density in impact parameter space:

$$\rho_0(\boldsymbol{b}) = \sum_{n,\lambda_i,k} e_k \Big[\prod_{i=1}^{n} \int dx_i \int 4\pi d^2\boldsymbol{b}_i\Big] \delta(1-\sum_i x_i) \frac{1}{4\pi} \delta^{(2)}(\sum_i x_i \boldsymbol{b}_i)$$

$$\times\; \delta^{(2)}(\boldsymbol{b}-\boldsymbol{b}_k)\, |\psi_n^{\lambda}(x_i, \boldsymbol{b}_i, \lambda_i)|^2$$

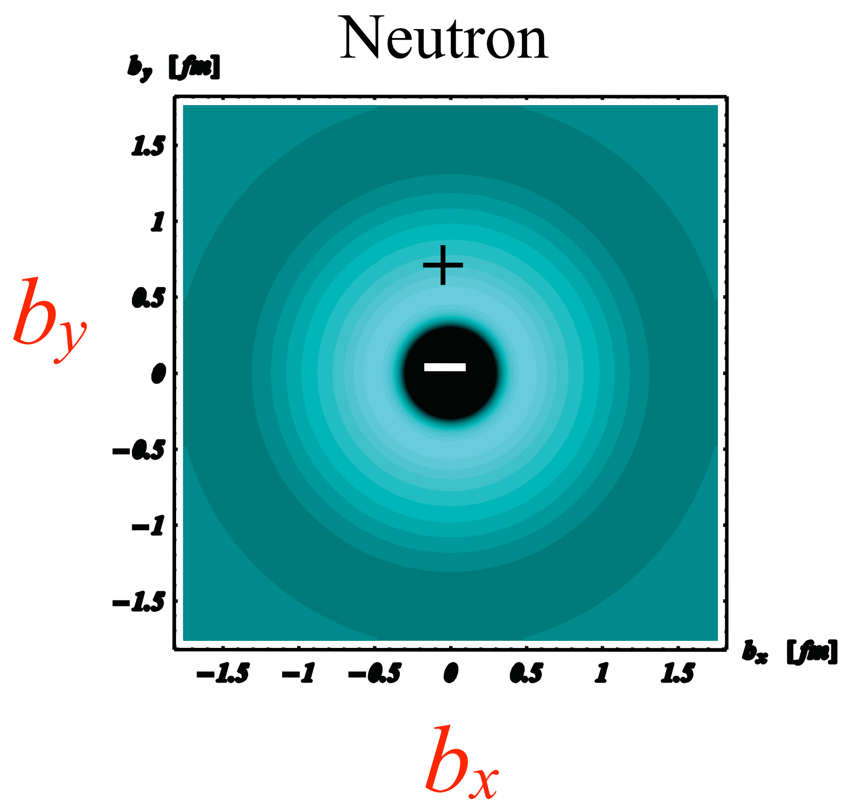
No Wilson line: Fock expansion is "exact"

No "leading twist": Resolution in $b \sim 1/Q_{max}$

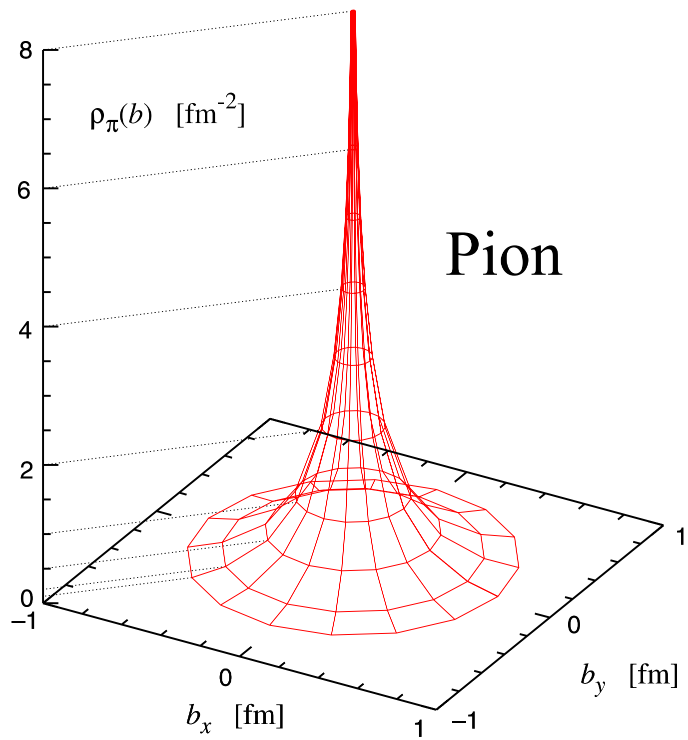
Examples: transverse densities in neutron and pion

Miller (2007)
Carlson and Vanderhaeghen (2008)

Miller, Strikman, Weiss (2010)



$\rho_0(b)$



No model dependence, given $F(Q^2)$

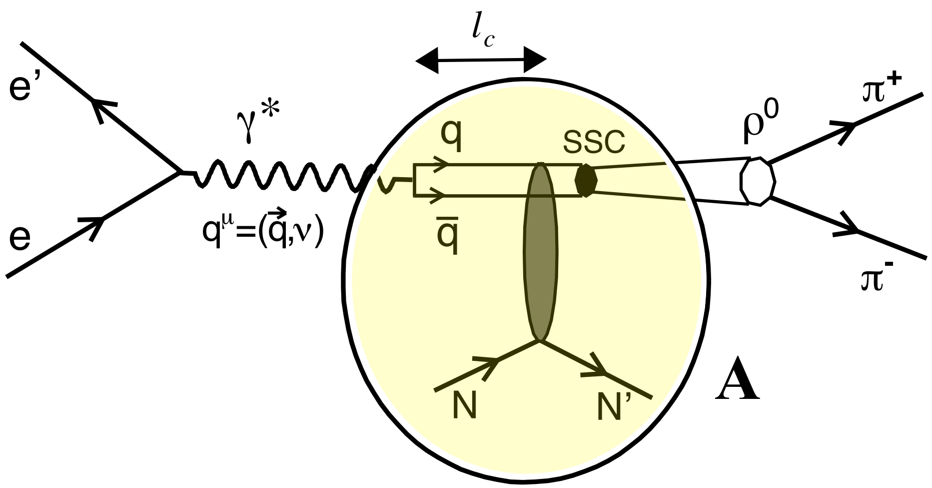
No variables, except target (neutron, pion, ...)

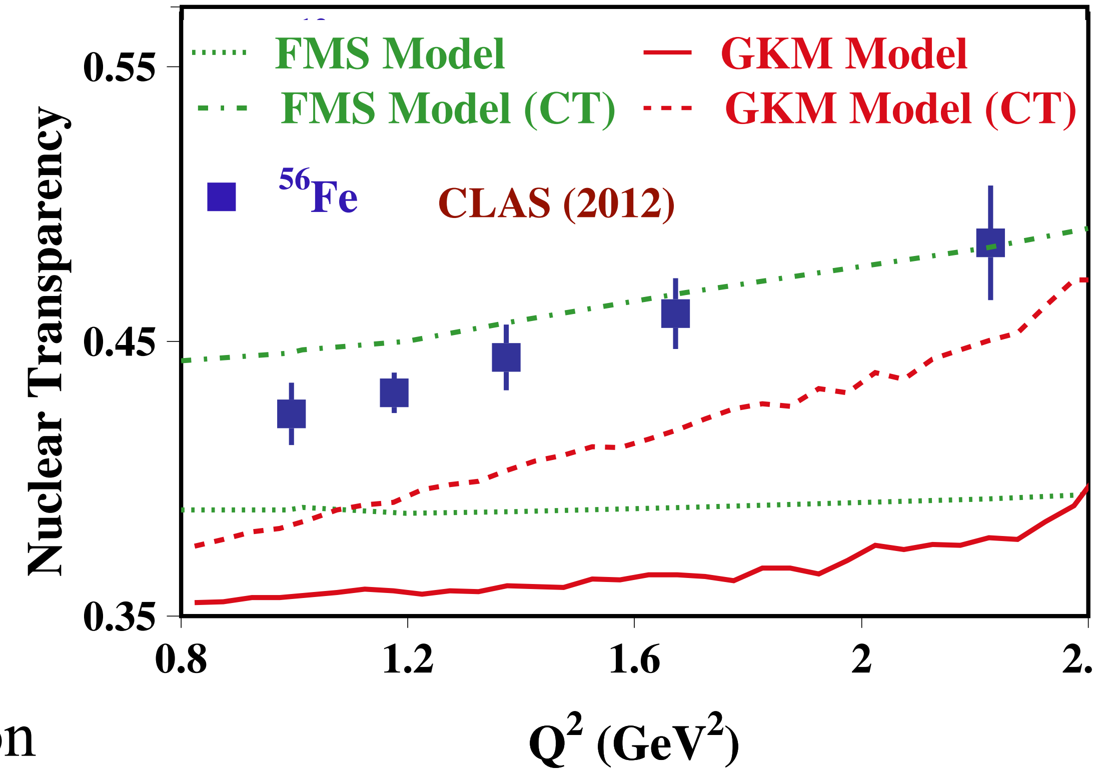
But: Analogous analyses possible for other processes

Compare with Color Transparency

Effective transverse size of hadrons is process-dependent

$$\gamma^*(Q^2)+A \to \rho + A$$



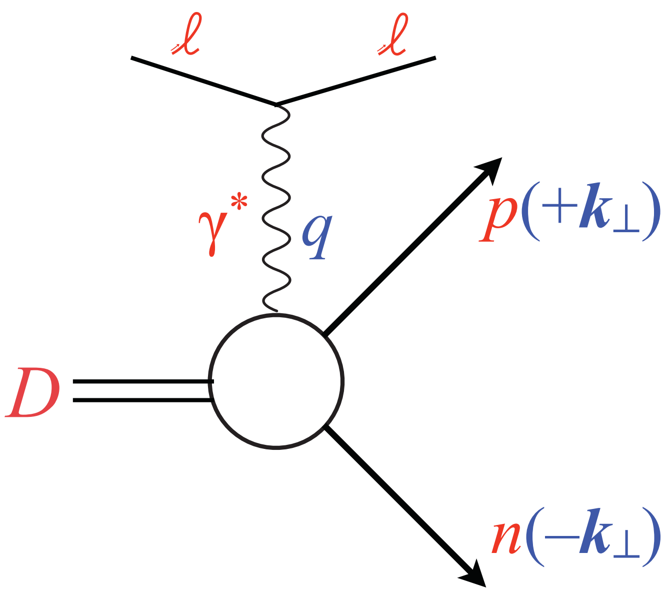


γ^* produces ρ in compact configuration

Transverse size (including gluons) as fn of Q^2 measured by nucleus A

In form factors, Q^2 dependence measures the transverse distribution of quarks

Example: Deuteron disintegration at 90°

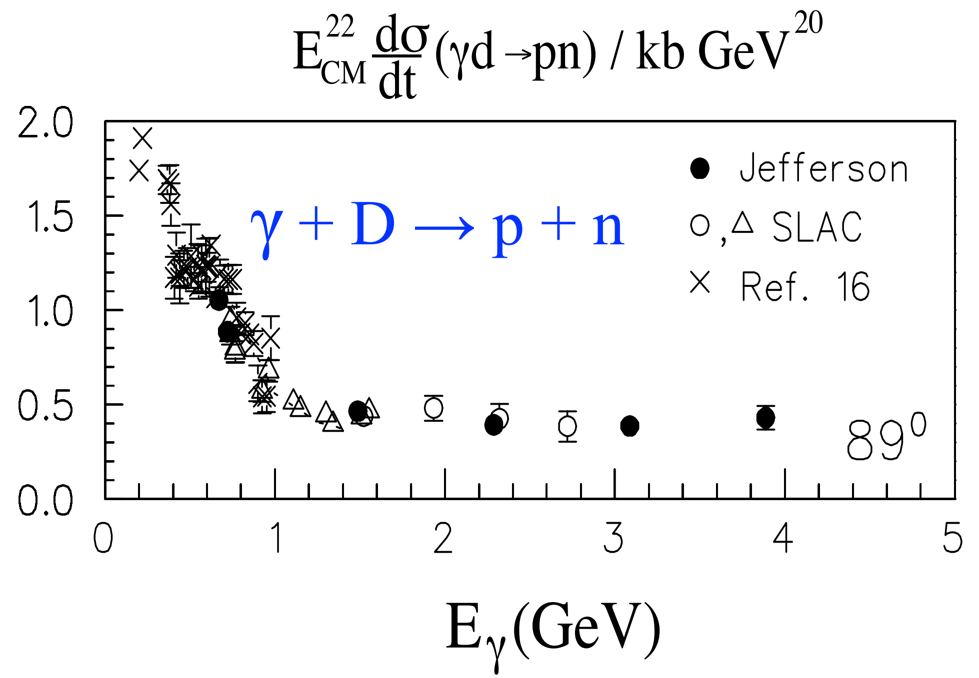


Dimensional scaling:

Only compact deuterons at large $k_\perp$

$\Rightarrow$ Broad distribution in q^2

In photoproduction ($q^2 = 0$):
the 90° cross section shows
dimensional scaling for $E_\gamma \geq 1$ GeV.



C. Bochna et al,
PRL 81 (1998) 4576

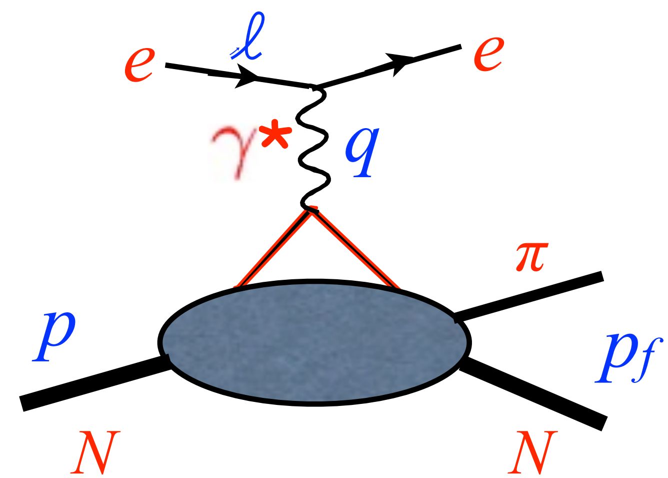
Need to consider electroproduction
to determine the size of the deuteron.

Here: Specify how Fourier transform of $\boldsymbol{q}$ determines the transverse size of deuteron

For any transition $\gamma^* N \to f$

$$\int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \frac{1}{2p^+} \langle f(p_f)|J^+(0)|N(p)\rangle$$

$$\propto \quad \psi_n^{f\,*}(x_i, \boldsymbol{b}_i)\psi_n^N(x_i, \boldsymbol{b}_i) \sum_k e_k \delta^2(\boldsymbol{b}_k - \boldsymbol{b})$$



Note: Wave functions are diagonal in x_i, $\boldsymbol{b}_i$

The wave functions are evaluated at $t+z = 0$, when the γ^* interacts.

The $\boldsymbol{b}$-distribution reflects the transverse size at the LF time of interaction,

and may be studied as a function of the final state f.

Analogous expression for the squared amplitudes (cross sections)

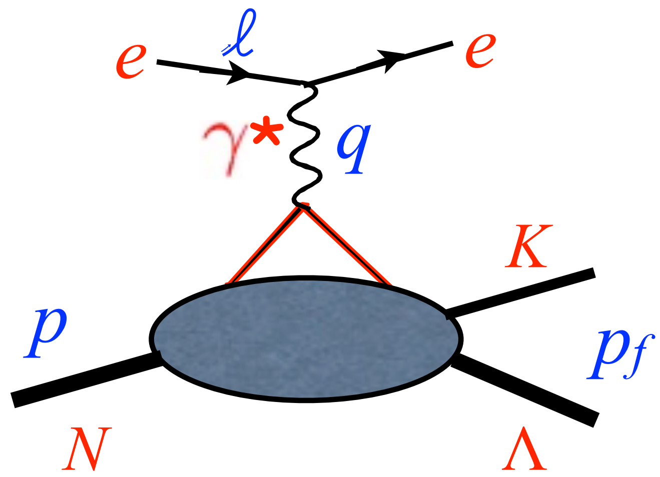
Example: $\gamma^*(q)\ N(p) \to f = K^+(p_1)\ \Lambda(p_2)$

The Fourier transfor over $\boldsymbol{q}$ is done in a specific frame:

$$p = (p^+, p^-, -\tfrac{1}{2}\boldsymbol{q})$$

$$q = (0^+, q^-, \boldsymbol{q})$$

$$p_f = q + p = p_1 + p_2$$



The final state f is characterized by x and $\boldsymbol{k}$, which are independent of q:

$$p_1^+ = x p_f^+ \qquad \boldsymbol{p}_1 = x\boldsymbol{p}_f + \boldsymbol{k}$$

$$p_2^+ = (1-x) p_f^+ \qquad \boldsymbol{p}_2 = (1-x)\boldsymbol{p}_f - \boldsymbol{k}$$

If the γ* couples directly to a strange quark (in the nucleon!), expect to see a narrower b-distribution than for $f = \pi N$.

Fourier transform of the cross section

The $\gamma^*+N \to f$ amplitudes have dynamical phases (resonances,...).

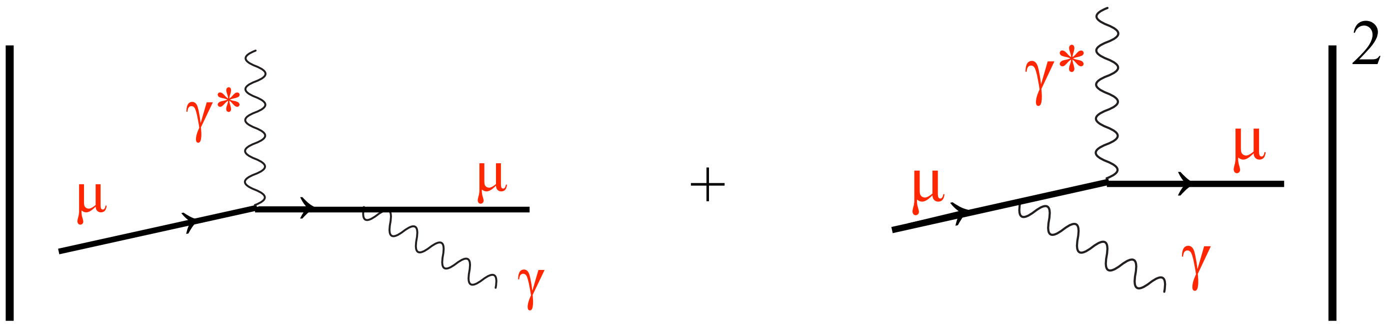
$\Rightarrow$ Calculating their Fourier transforms requires a partial wave analysis.

However, we can Fourier transform the cross section itself.

Then the ***b***-distribution reflects the difference between the impact parameters of the photon vertex in the amplitude and its complex conjugate:

$$\int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \left| \frac{1}{2p^+} \langle f(p_f)|J^+(0)|N(p)\rangle \right|^2 = \int d^2\boldsymbol{b}_q\, \mathcal{A}_{fN}(\boldsymbol{b}_q)\, \mathcal{A}^*_{fN}(\boldsymbol{b}_q - \boldsymbol{b})$$

QED illustration: γ*+ μ → μ + γ



$$\mathcal{S}^{\mu\gamma}(\boldsymbol{b};x,\boldsymbol{k}) = \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \boldsymbol{q}^4 \frac{d\sigma(\ell\mu\to\ell'\mu\gamma)}{d^2\boldsymbol{q}\,dx\,d^2\boldsymbol{k}}$$

$$\mathcal{S}^{\mu\gamma}(\boldsymbol{b};x,\boldsymbol{k}) = 4e^2x\Big\{\frac{\boldsymbol{k}^2/2}{[(1-x)^2m^2+\boldsymbol{k}^2]^2}\delta^{(2)}(\boldsymbol{b}) - \frac{|\boldsymbol{k}|\cos(\phi_b-\phi_k)}{(1-x)^2m^2+\boldsymbol{k}^2}\,\frac{im}{2\pi}\,\frac{\exp\big(-i\frac{\boldsymbol{k}\cdot\boldsymbol{b}}{1-x}\big)}{1-x}K_1(mb)$$

$$+ \frac{1}{4\pi}\frac{\exp\big(-i\frac{\boldsymbol{k}\cdot\boldsymbol{b}}{1-x}\big)}{(1-x)^2}\Big[K_0(mb)-\tfrac{1}{2}mb\,K_1(mb)\Big]\Big\}$$

The 3 terms correspond to 2, 1 and 0 of the γ* interactions occurring on the initial muon. The imaginary part arises from an angular correlation between ***b*** and ***k*** .

Summary

Intuitively, the ***q*** -dependence of a virtual photon interaction gives information about the charge distribution in space.

2-dim. FT's of scattering amplitudes describe charge densities in transverse space at an instant of Light-Front time $x^+ = t + z$

Unlike pdf's, no "leading twist" limit is implied.

The resolution in impact parameter is $\Delta b \sim 1/Q_{max}$

$\Longrightarrow$ Learn new aspects of strong interaction dynamics.

This type of analysis is waiting for an application to real data!

No model dependence – except to estimate finite energy effects.