Lattice QCD studies of transverse momentum-dependent parton distributions

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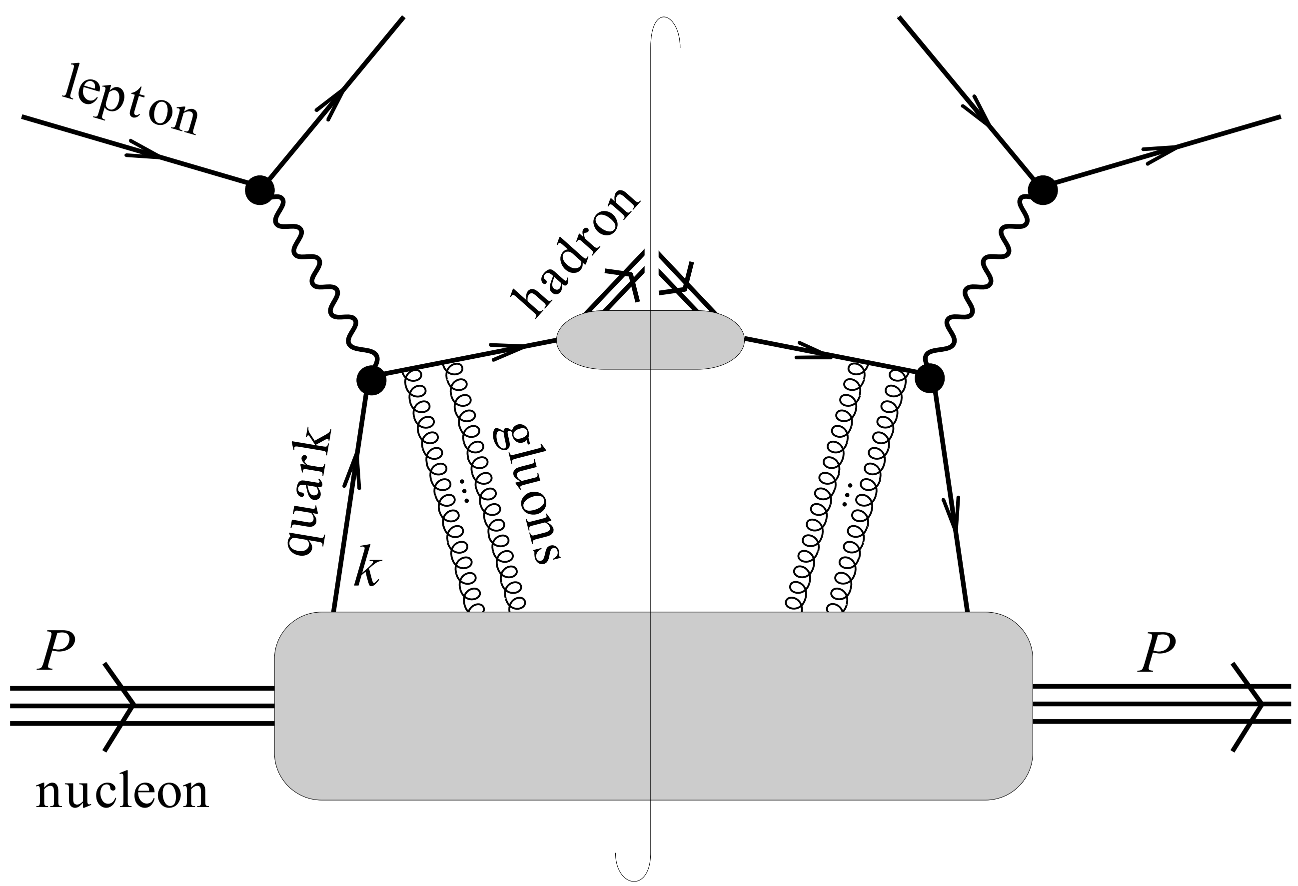
Fundamental TMD correlator

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,\ldots) \equiv \frac{1}{2}\langle P,S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0,\ldots,b]\ q(b)\ |P,S\rangle$$

$$\Phi^{[\Gamma]}(x,k_T,P,S,\ldots) \equiv \int \frac{d^2 b_T}{(2\pi)^2} \int \frac{d(b\cdot P)}{(2\pi)P^+} \exp\left(ix(b\cdot P) - ib_T\cdot k_T\right) \left.\frac{\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,\ldots)}{\widetilde{\mathcal{S}}(b^2,\ldots)}\right|_{b^+=0}$$

- "Soft factor" $\widetilde{\mathcal{S}}$ required to subtract divergences of Wilson line $\mathcal{U}$
- $\widetilde{\mathcal{S}}$ is typically a combination of vacuum expectation values of Wilson line structures
- Here, will consider only ratios in which soft factors cancel

Gauge link structure motivated by SIDIS



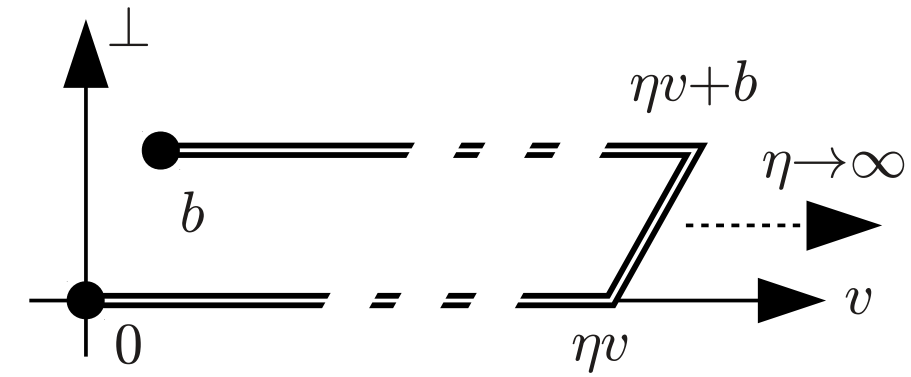
$l + N(P) \longrightarrow l' + h(P_h) + X$

Gauge link structure:

In matrix element $\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots) \equiv$

$\frac{1}{2}\langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \ldots, b]\ q(b)\ |P, S\rangle$

Staple-shaped gauge link $\mathcal{U}[0, \eta v, \eta v + b, b]$



incorporates SIDIS final state effects

Gauge link structure motivated by SIDIS

Staple-shaped links incorporate SIDIS final state effects:

- Gauge link roughly follows direction of ejected quark, (close to) light cone
- Effective resummed description of gluon exchanges between ejected quark and remainder of nucleon in evolving final state
- Beyond tree level: Rapidity divergences suggest taking staple direction slightly off the light cone. Approach of Aybat, Collins, Qiu, Rogers makes v space-like. Parametrize in terms of Collins-Soper parameter

  $$\hat{\zeta} \equiv \frac{P \cdot v}{|P||v|}$$

  Light-like staple for $\hat{\zeta} \to \infty$. Perturbative evolution equations for large $\hat{\zeta}$.
- In this approach, have "modified universality", $f^{\text{T-odd, SIDIS}} = -f^{\text{T-odd, DY}}$ (initial state interactions in DY case). SIDIS: $\eta v \cdot P \to \infty$, DY: $\eta v \cdot P \to -\infty$.

Fundamental TMD correlator

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Decomposition of Φ into TMDs

All leading twist structures:

$$\Phi^{[\gamma^+]} = f_1 - \left[\frac{\epsilon_{ij} k_i S_j}{m_N} f_{1T}^{\perp}\right] \text{odd}$$

$$\Phi^{[\gamma^+ \gamma^5]} = \Lambda g_1 + \frac{k_T \cdot S_T}{m_N} g_{1T}$$

$$\Phi^{[i\sigma^{i+}\gamma^5]} = S_i h_1 + \frac{(2k_i k_j - k_T^2 \delta_{ij}) S_j}{2m_N^2} h_{1T}^{\perp} + \frac{\Lambda k_i}{m_N} h_{1L}^{\perp} + \left[\frac{\epsilon_{ij} k_j}{m_N} h_1^{\perp}\right] \text{odd}$$

TMD Classification

All leading twist structures:

$q \rightarrow$ N $\downarrow$	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1 \quad h_{1T}^\perp$

$\longleftarrow$ Boer-Mulders (T-odd) [row U, column T]

$\uparrow$ Sivers (T-odd) [row T, column U]

Decomposition of $\widetilde{\Phi}$ into TMDs

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \hat{\zeta}, \mu) \equiv \frac{1}{2} \langle P, S | \; \bar{q}(0) \; \Gamma \; \mathcal{U}[0, \eta v, \eta v + b, b] \; q(b) \; | P, S \rangle$$

Decompose in terms of invariant amplitudes; at leading twist,

$$\frac{1}{2P^+} \widetilde{\Phi}^{[\gamma^+]}_{\text{unsubtr.}} = \widetilde{A}_{2B} + i m_N \epsilon_{ij} b_i S_j \widetilde{A}_{12B}$$

$$\frac{1}{2P^+} \widetilde{\Phi}^{[\gamma^+ \gamma^5]}_{\text{unsubtr.}} = -\Lambda \widetilde{A}_{6B} + i[(b \cdot P)\Lambda - m_N (b_T \cdot S_T)] \widetilde{A}_{7B}$$

$$\begin{aligned} \frac{1}{2P^+} \widetilde{\Phi}^{[i\sigma^{i+} \gamma^5]}_{\text{unsubtr.}} = \; & i m_N \epsilon_{ij} b_j \widetilde{A}_{4B} - S_i \widetilde{A}_{9B} \\ & - i m_N \Lambda b_i \widetilde{A}_{10B} + m_N [(b \cdot P)\Lambda - m_N (b_T \cdot S_T)] b_i \widetilde{A}_{11B} \end{aligned}$$

Fourier-transformed TMDs

$$\tilde{f}(x, b_T^2, \ldots) \equiv \int d^2k_T \, \exp(ib_T \cdot k_T) f(x, k_T^2, \ldots)$$

$$\tilde{f}^{(n)}(x, b_T^2, \ldots) \equiv n! \left(-\frac{2}{m_N^2} \partial_{b_T^2} \right)^n \tilde{f}(x, b_T^2, \ldots)$$

In limit $|b_T| \to 0$, recover k_T-moments:

$$\tilde{f}^{(n)}(x, 0, \ldots) \equiv \int d^2k_T \left(\frac{k_T^2}{2m_N^2} \right)^n f(x, k_T^2, \ldots) \equiv f^{(n)}(x)$$

ill-defined for large k_T, so will not attempt to extrapolate to $b_T = 0$, but give results at finite $|b_T|$.

Also, we can only access limited range of $b \cdot P$, so cannot Fourier-transform to obtain x-dependence. For now, consider only first x-moments (accessible at $b \cdot P = 0$):

$$f^{[1]}(k_T^2, \ldots) \equiv \int_{-1}^{1} dx \, f(x, k_T^2, \ldots)$$

Relation between Fourier-transformed TMDs and invariant amplitudes $\widetilde{A}_i$

Invariant amplitudes directly give selected x-integrated TMDs in Fourier (b_T) space (showing just the ones relevant for Sivers, Boer-Mulders shifts), up to soft factors:

$$\tilde{f}_1^{[1](0)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = 2\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

$$\tilde{f}_{1T}^{\perp[1](1)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = -2\widetilde{A}_{12B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

$$\tilde{h}_1^{\perp[1](1)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = 2\widetilde{A}_{4B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

Generalized shifts

Form ratios in which soft factors, (Γ-independent) multiplicative renormalization factors cancel

Sivers shift:

$$\langle k_y \rangle_{TU} \equiv m_N \frac{f_{1T}^{\perp [1](1)}}{f_1^{[1](0)}} = \frac{\int dx \int d^2 k_T \, k_y \Phi^{[\gamma^+]}(x, k_T, S_T = (1,0))}{\int dx \int d^2 k_T \, \Phi^{[\gamma^+]}(x, k_T, S_T = (1,0))}$$

Average transverse momentum of unpolarized ("U") quarks orthogonal to the transverse ("T") spin of nucleon; normalized to the number of valence quarks. "Dipole moment" in $b_T^2 = 0$ limit, "shift".

Issue: k_T-moments in this ratio singular; generalize to ratio of Fourier-transformed TMDs at *nonzero* b_T^2,

$$\langle k_y \rangle_{TU}(b_T^2, \ldots) \equiv m_N \frac{\tilde{f}_{1T}^{\perp [1](1)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)}$$

(remember singular $b_T \to 0$ limit corresponds to taking k_T-moment). "Generalized shift".

Generalized shifts from amplitudes

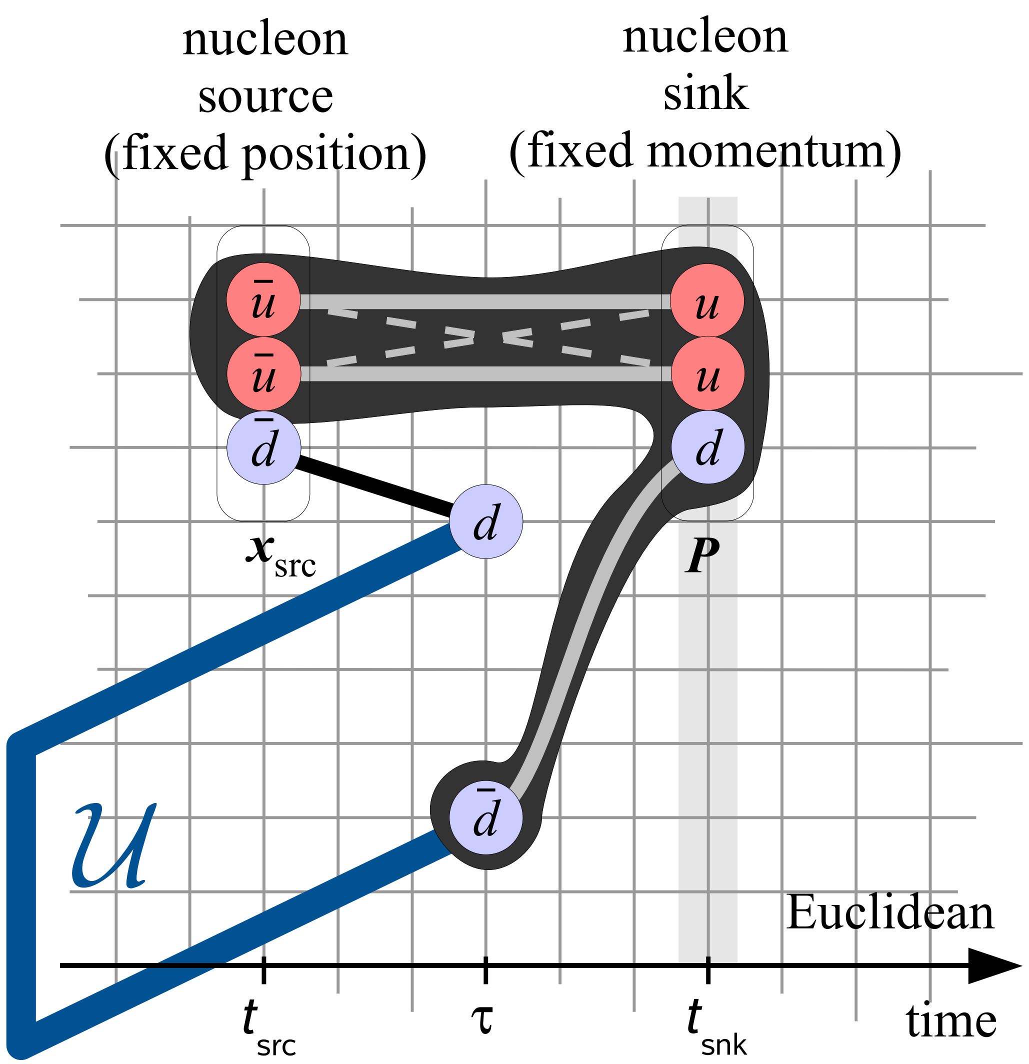
Now, can also express this in terms of invariant amplitudes:

$$\langle k_y \rangle_{TU}(b_T^2, \ldots) \equiv m_N \frac{\tilde{f}_{1T}^{\perp [1](1)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)} = -m_N \frac{\widetilde{A}_{12B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Analogously, Boer-Mulders shift:

$$\langle k_y \rangle_{UT}(b_T^2, \ldots) = m_N \frac{\widetilde{A}_{4B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Lattice setup



- Evaluate directly $\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \hat{\zeta}, \mu)$

  $\equiv \frac{1}{2}\langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \eta v, \eta v + b, b]\ q(b)\ |P, S\rangle$

- Euclidean time: Place entire operator at one time slice, i.e., b, ηv purely spatial
- Since generic b, v space-like, no obstacle to boosting system to such a frame!
- Parametrization of correlator in terms of $\widetilde{A}_i$ invariants permits direct translation of results back to original frame
- Form desired ratios of $\widetilde{A}_i$ invariants
- Extrapolate $\eta \rightarrow \infty$, $\hat{\zeta} \rightarrow \infty$ numerically

Lattice setup

Use three MILC 2+1-flavor gauge ensembles with $a \approx 0.12\,\text{fm}$:

$m_\pi = 369\,\text{MeV}$; $28^3 \times 64$ (nucleon)

$m_\pi = 369\,\text{MeV}$; $20^3 \times 64$ (nucleon)

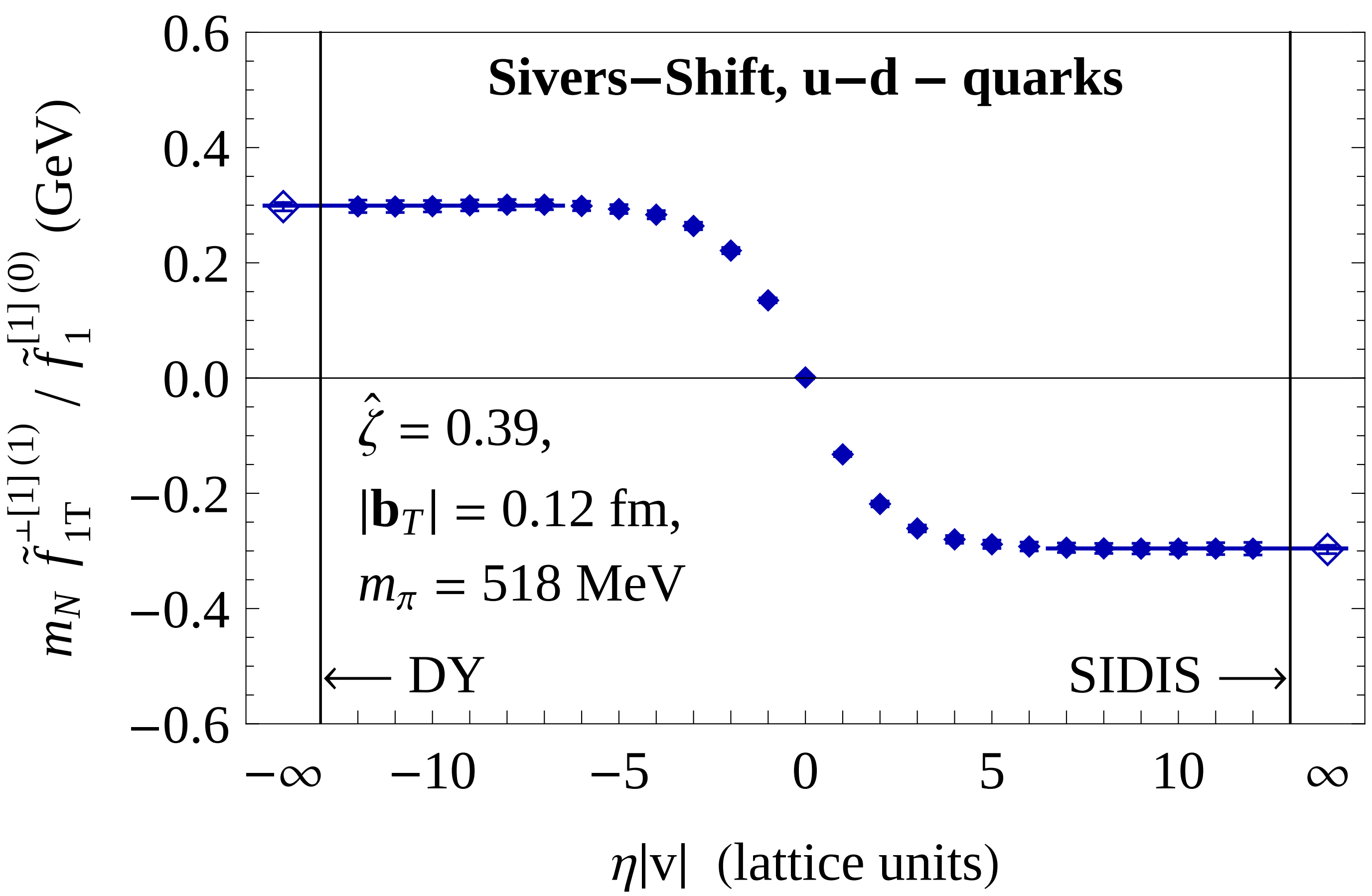
$m_\pi = 518\,\text{MeV}$; $20^3 \times 64$ (nucleon, pion)

Variety of P, b, ηv; note $b \perp P$, $b \perp v$ (lowest x-moment, kinematical choices/constraints)

Nucleon: largest $\hat{\zeta} = 0.78$
Pion: largest $\hat{\zeta} = 2.03$

Results: Sivers shift

Dependence on staple extent; sequence of panels at different $|b_T|$



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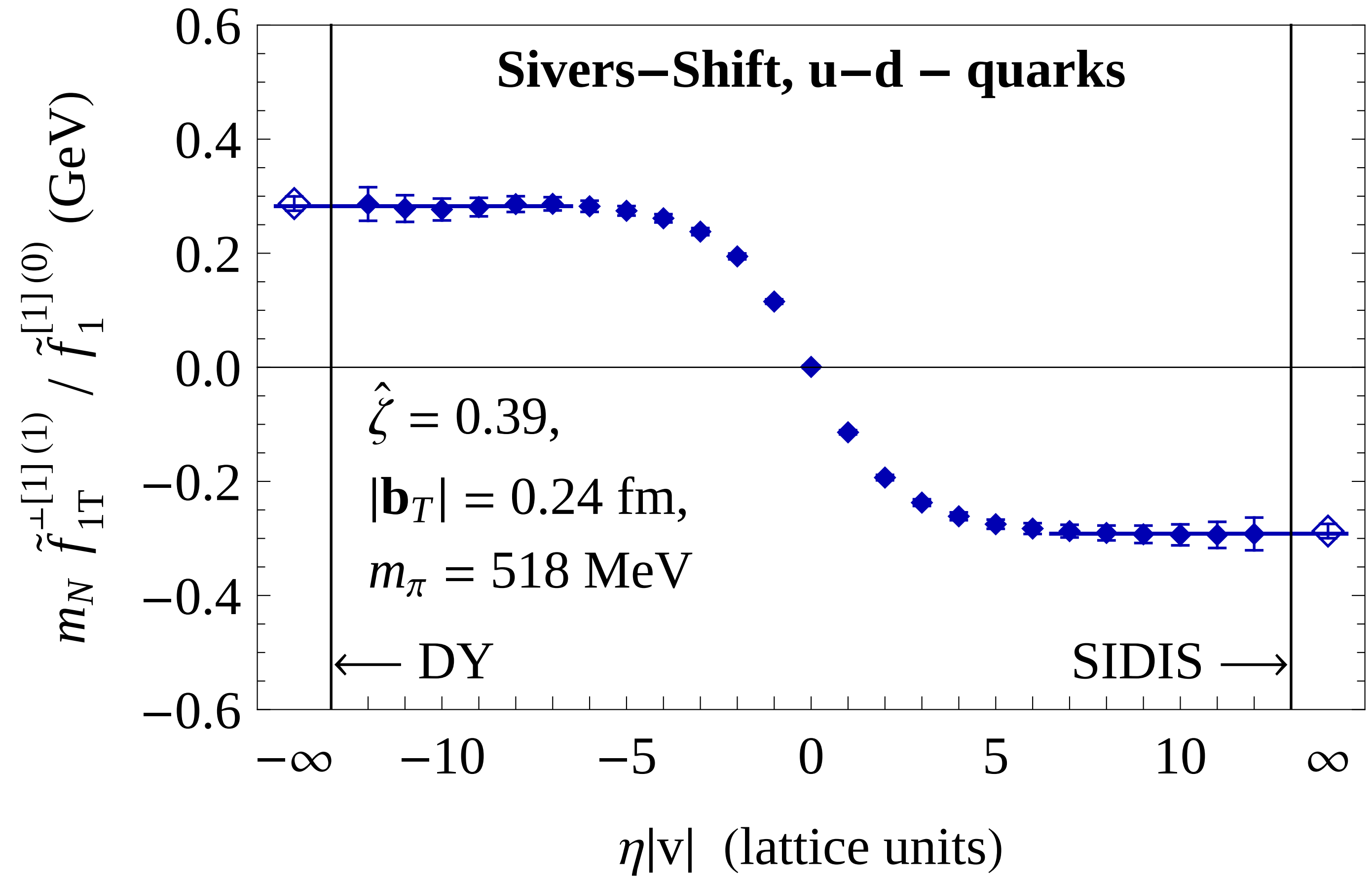

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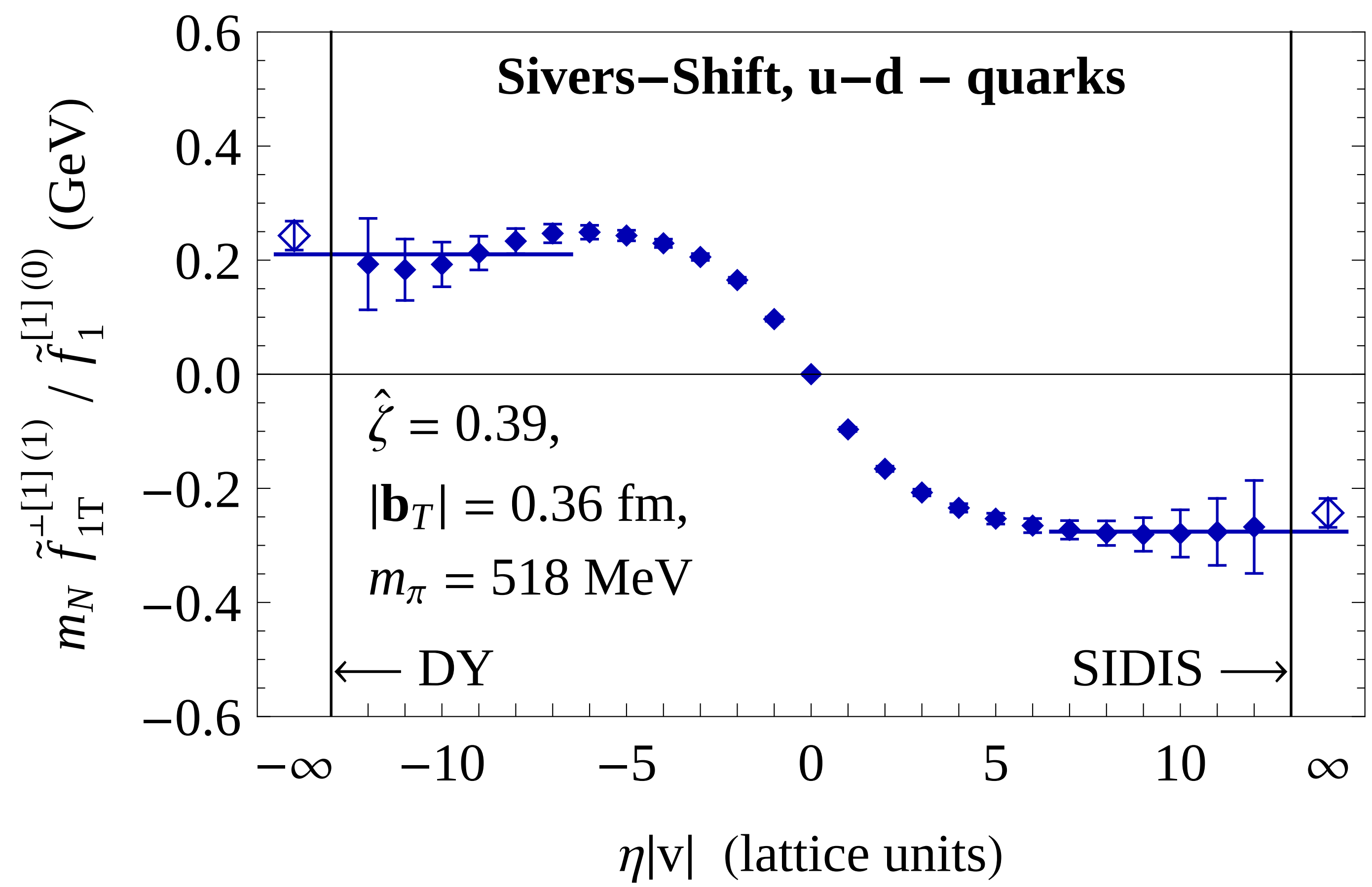

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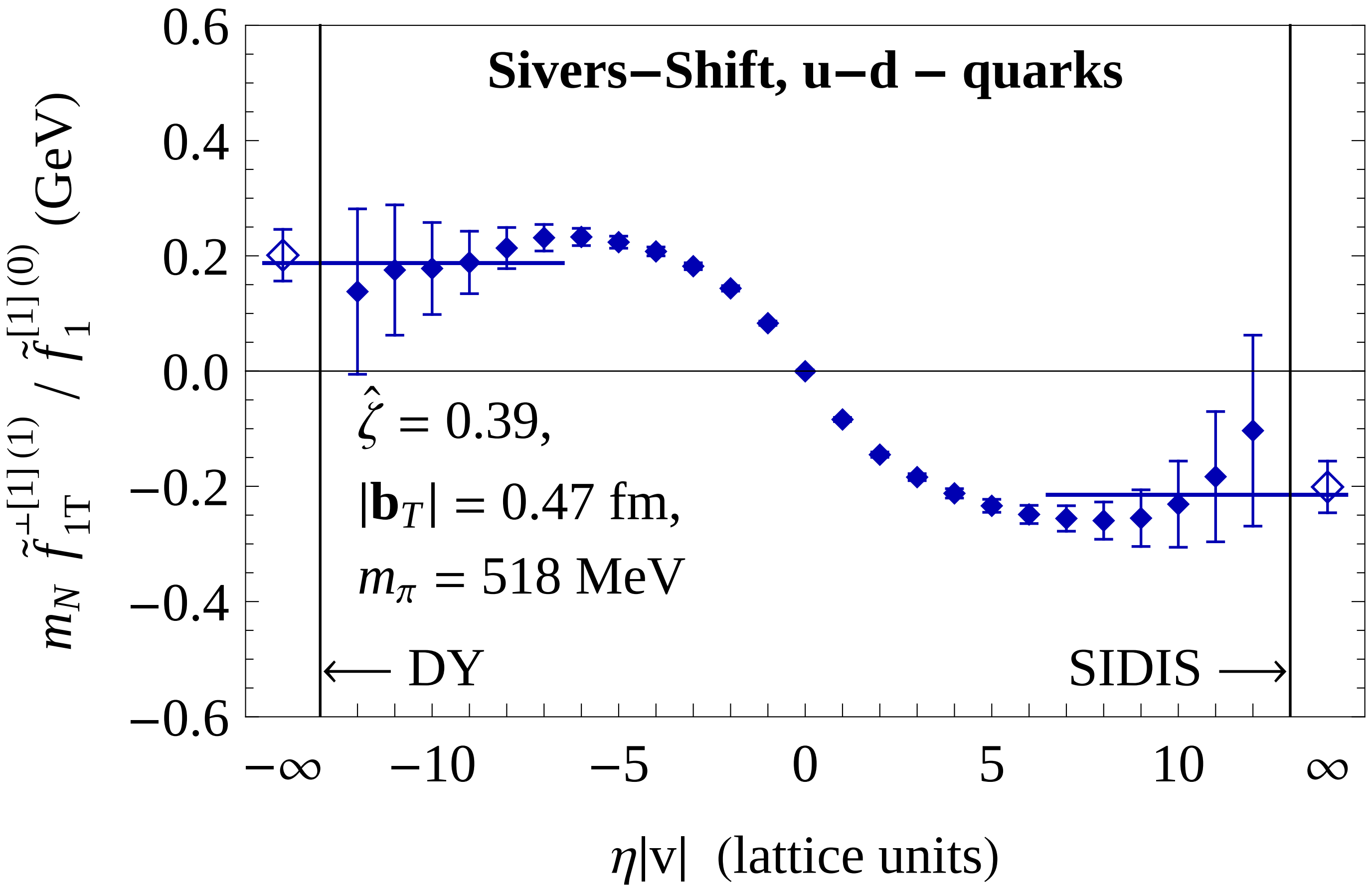
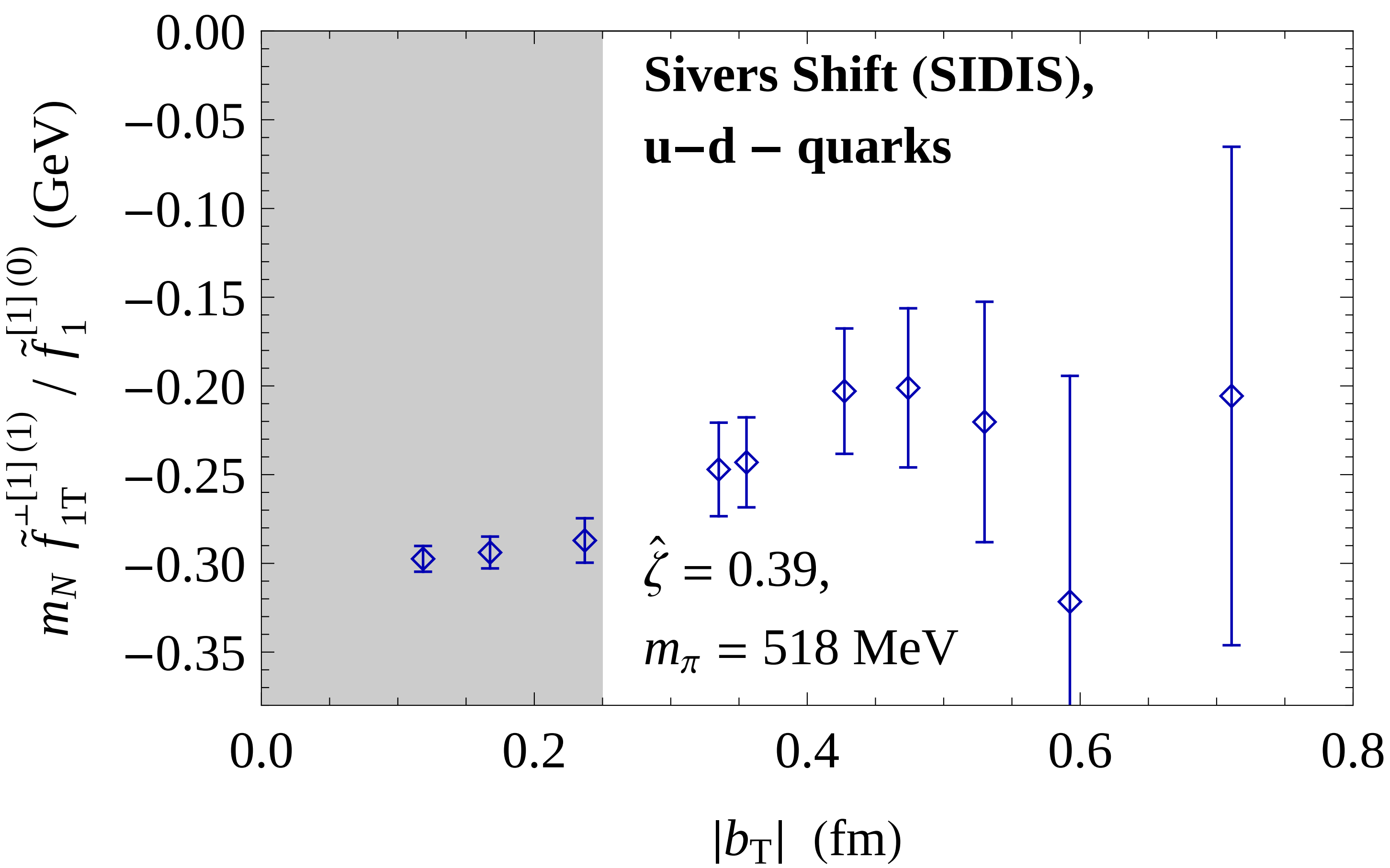


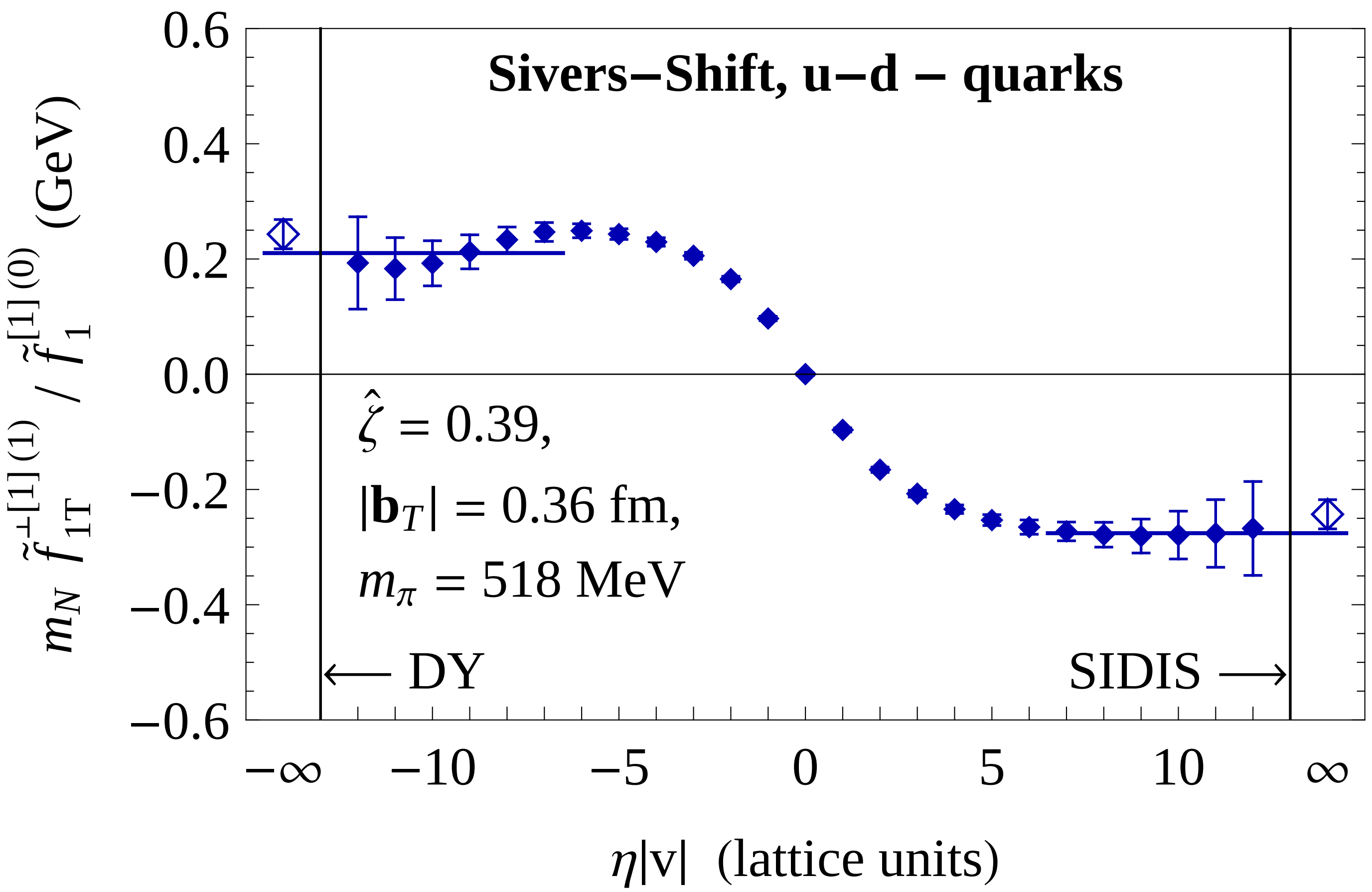
Results: Sivers shift

Dependence of SIDIS limit on $|b_T|$



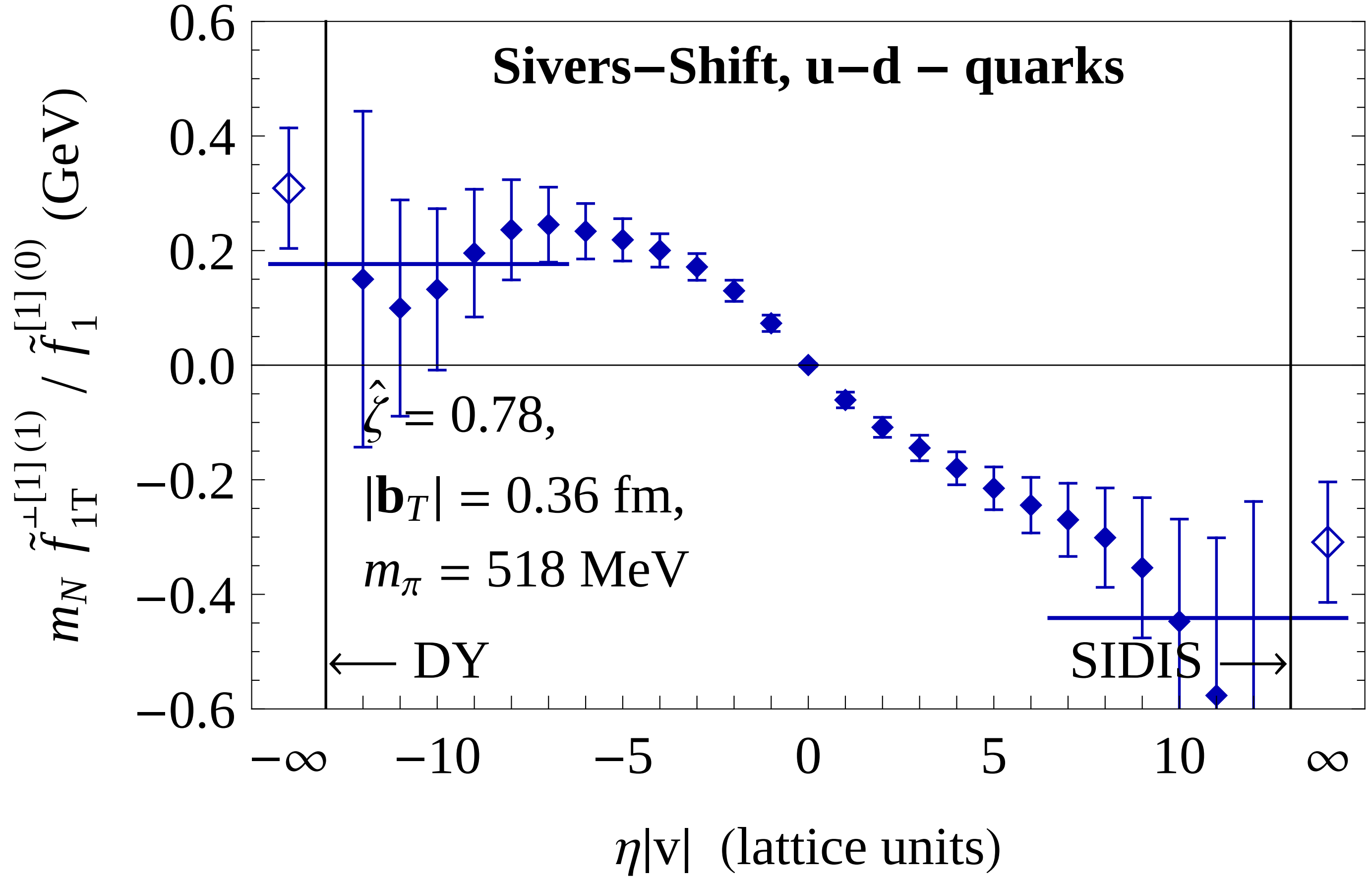
Results: Sivers shift

Dependence on staple extent; sequence of panels at different $\hat{\zeta}$



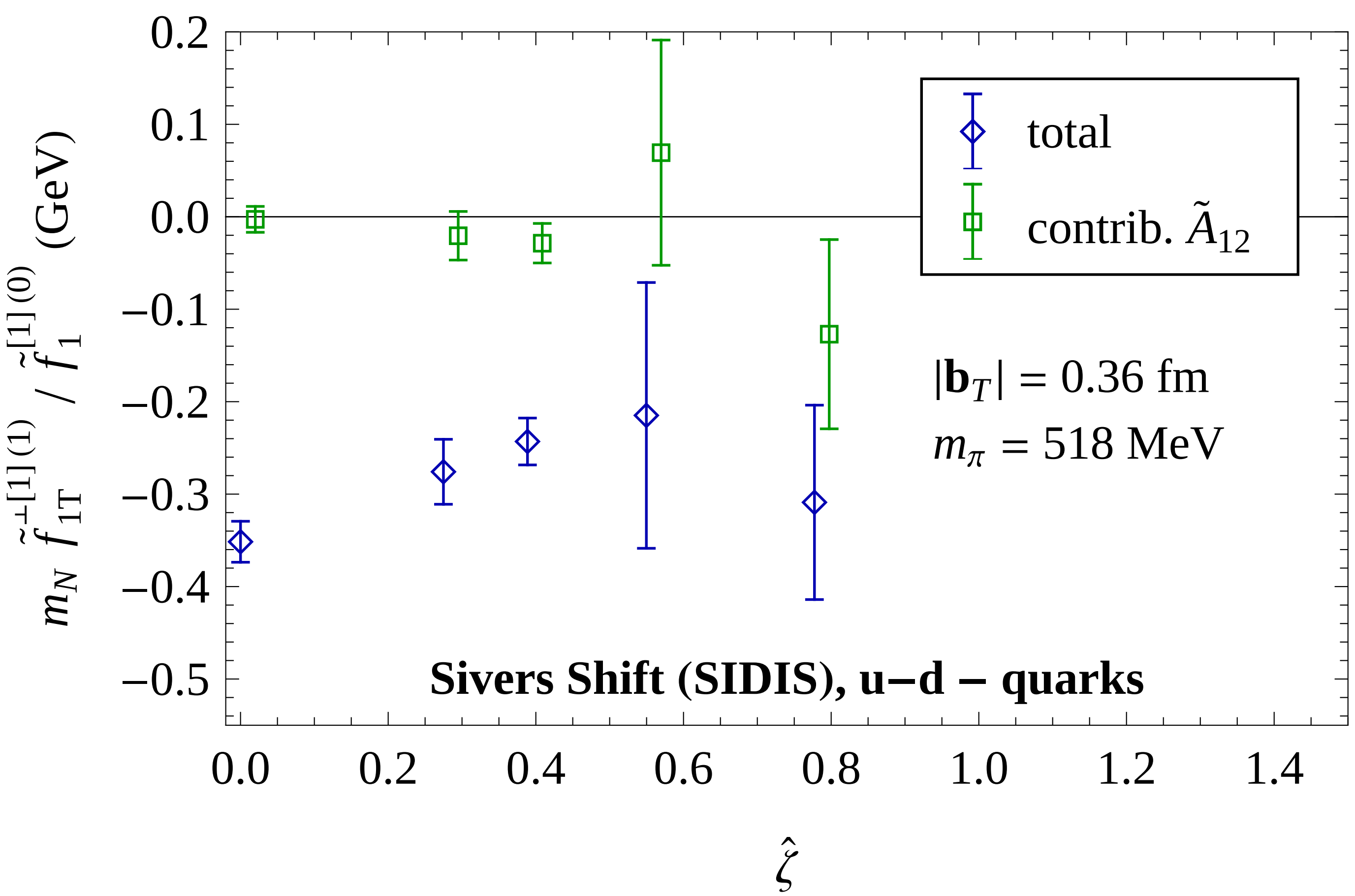
Results: Sivers shift

Dependence on staple extent; sequence of panels at different $\hat{\zeta}$



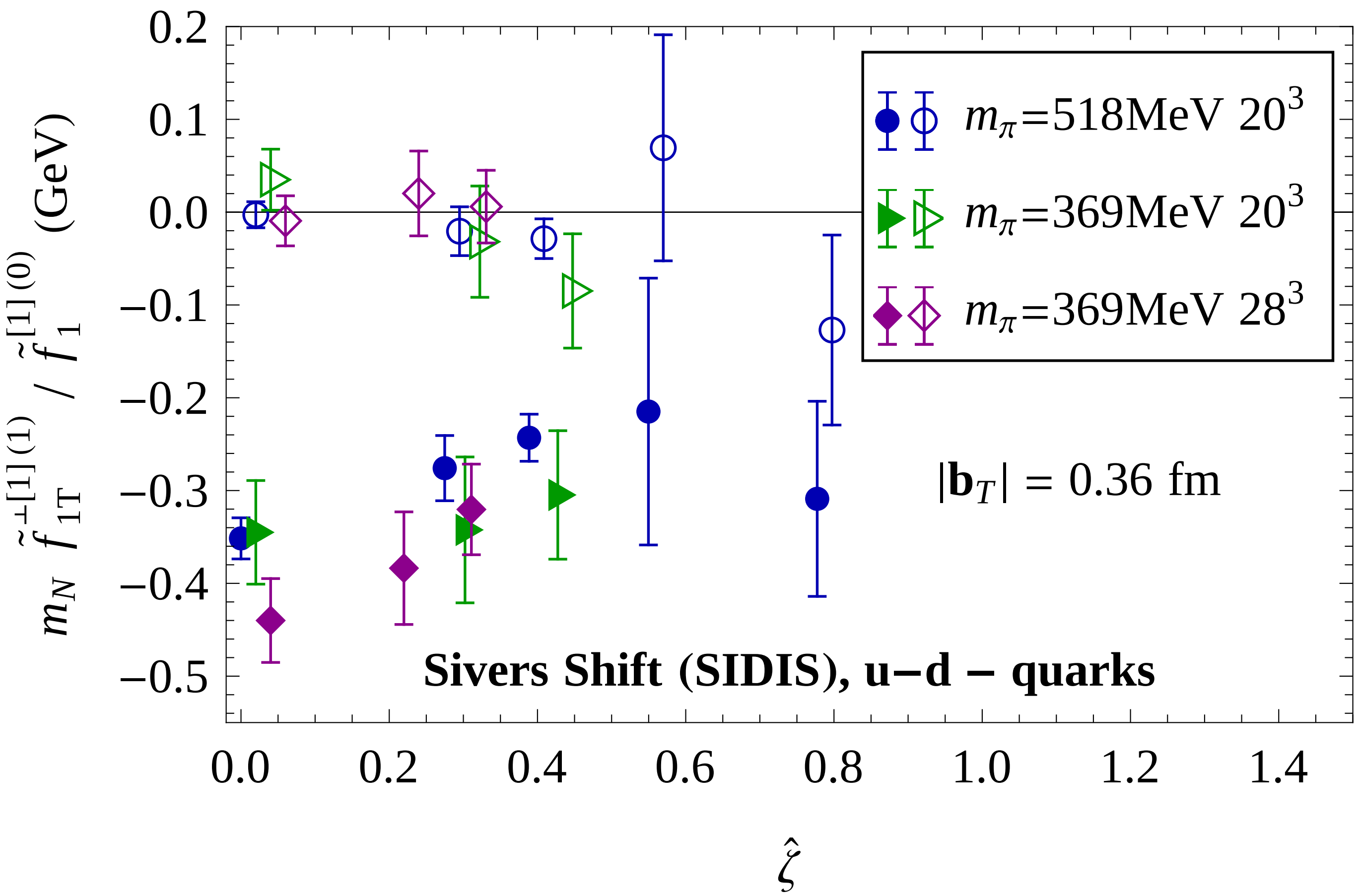
Results: Sivers shift

Dependence of SIDIS limit on $\hat{\zeta}$



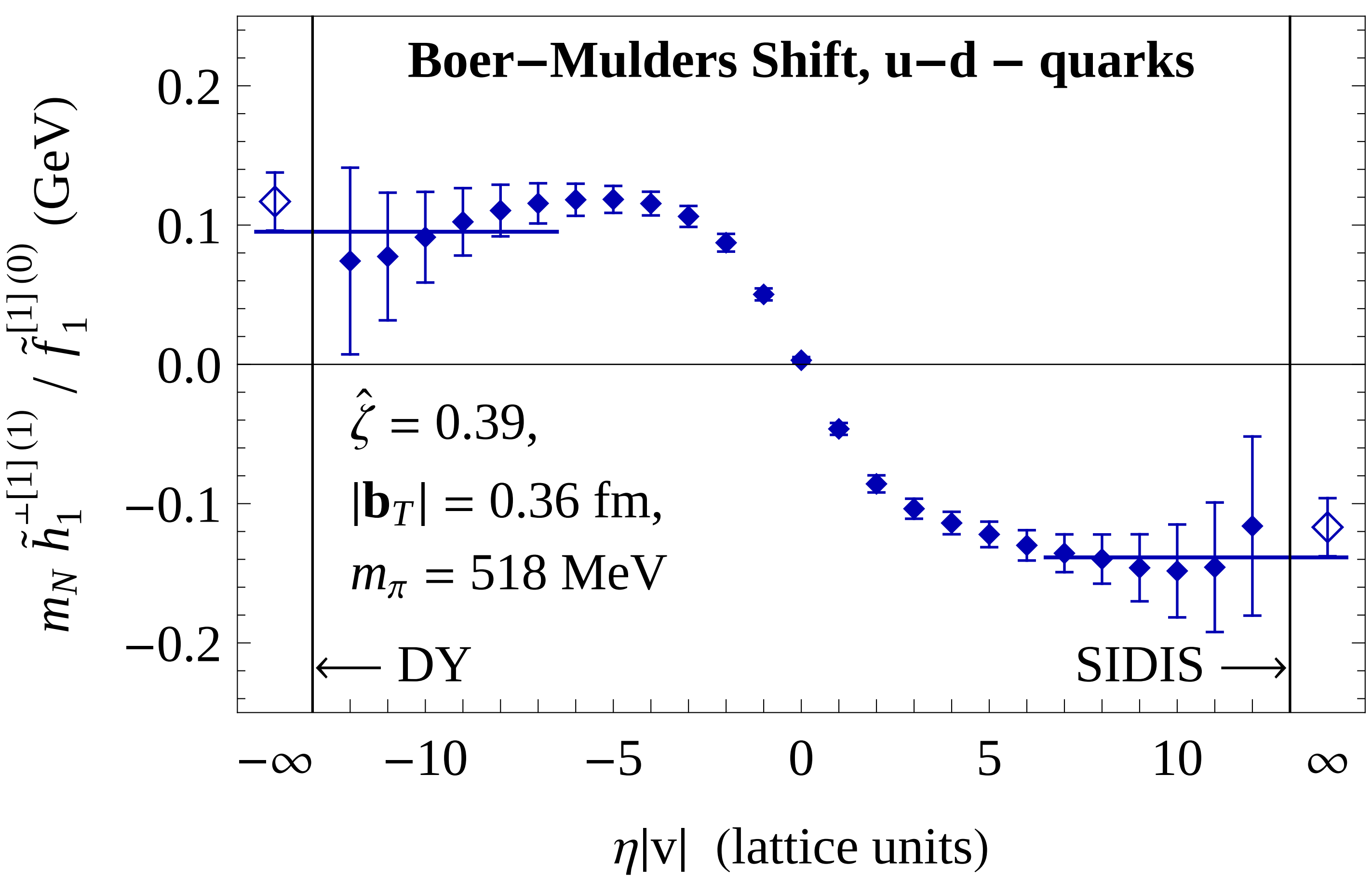
Results: Sivers shift

Dependence of SIDIS limit on $\hat{\zeta}$, all three ensembles



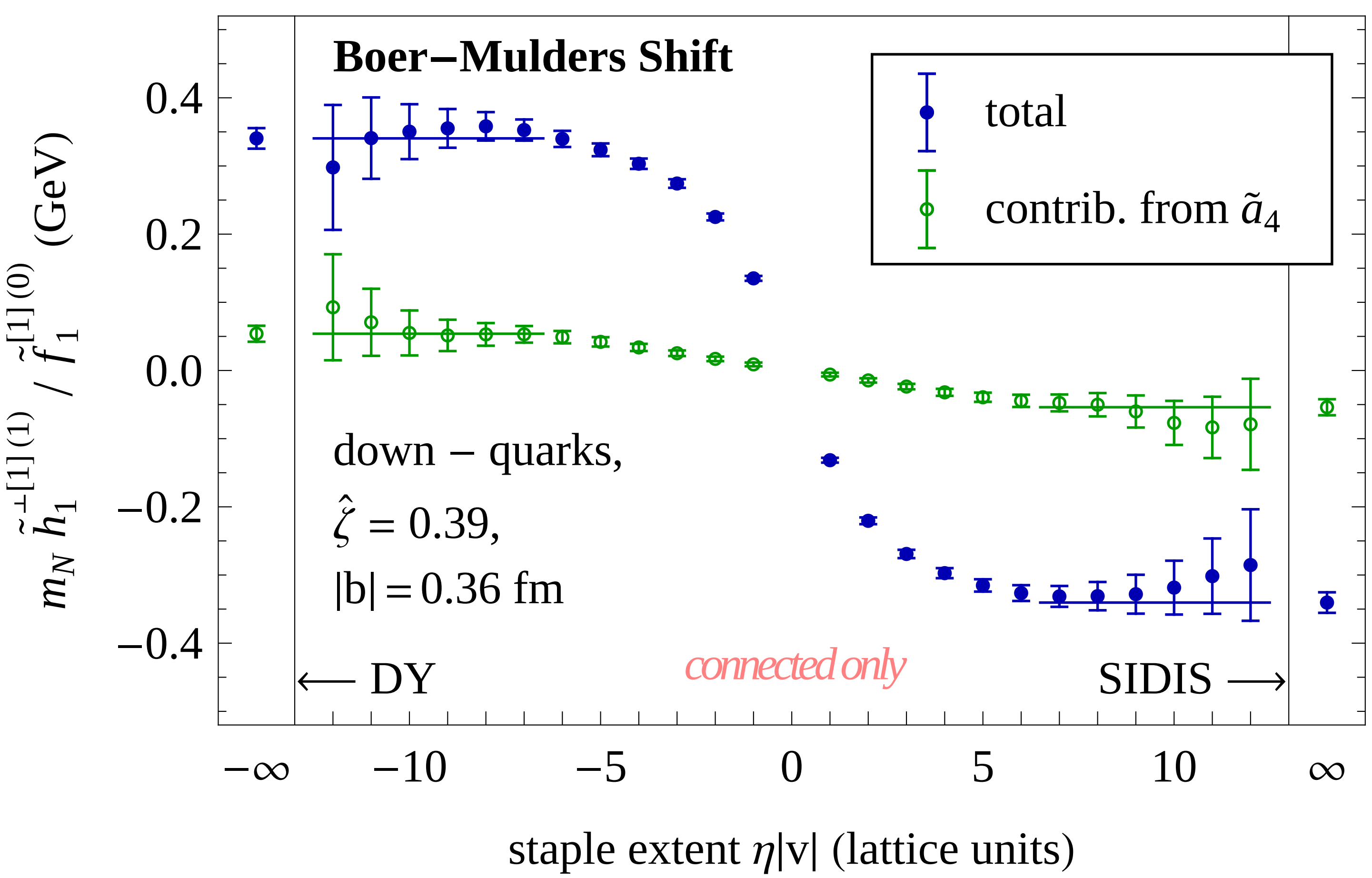
Results: Boer-Mulders shift (nucleon)

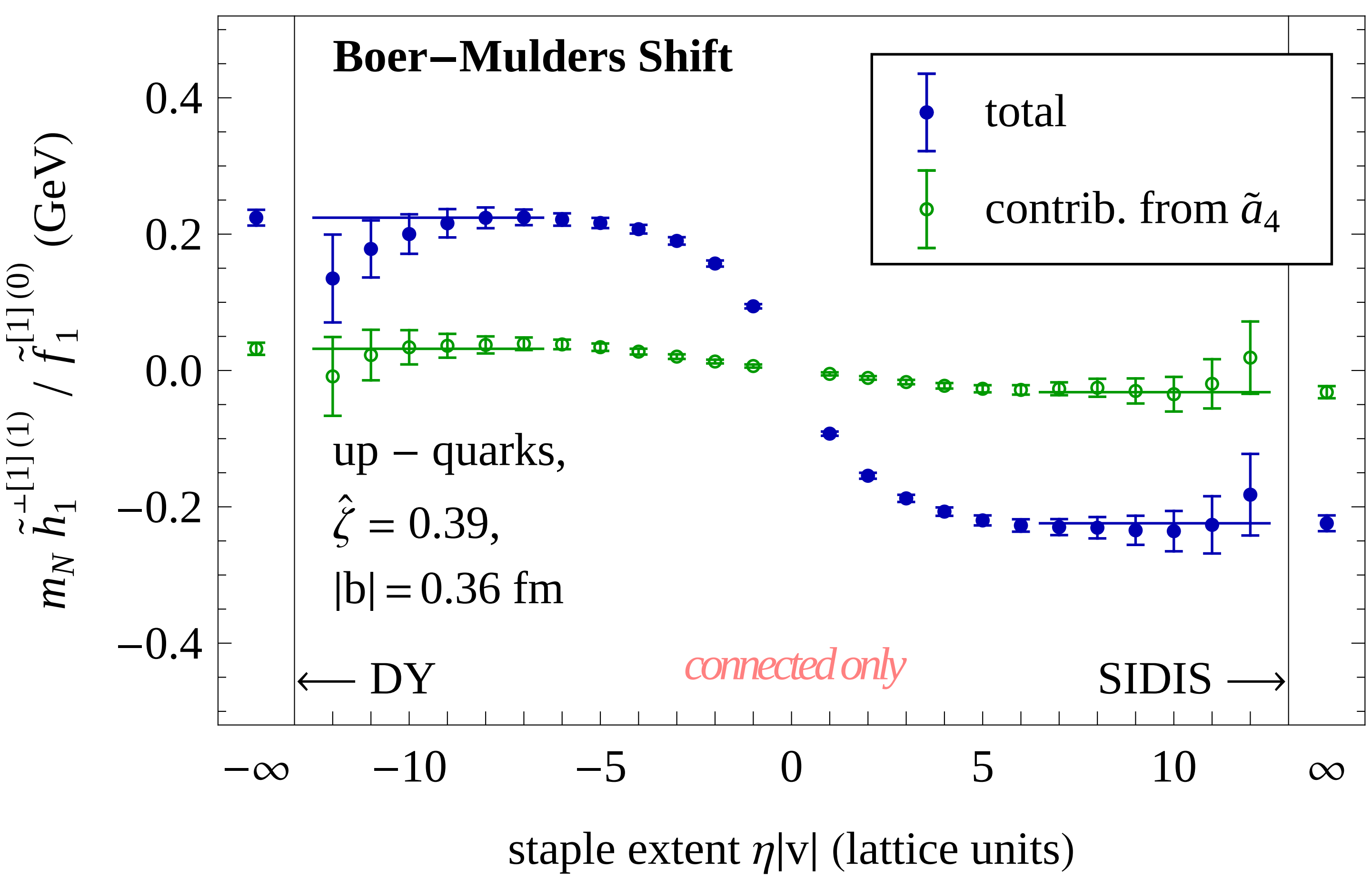
Dependence on staple extent



Results: Boer-Mulders shift (nucleon)

Dependence on staple extent; flavor separated





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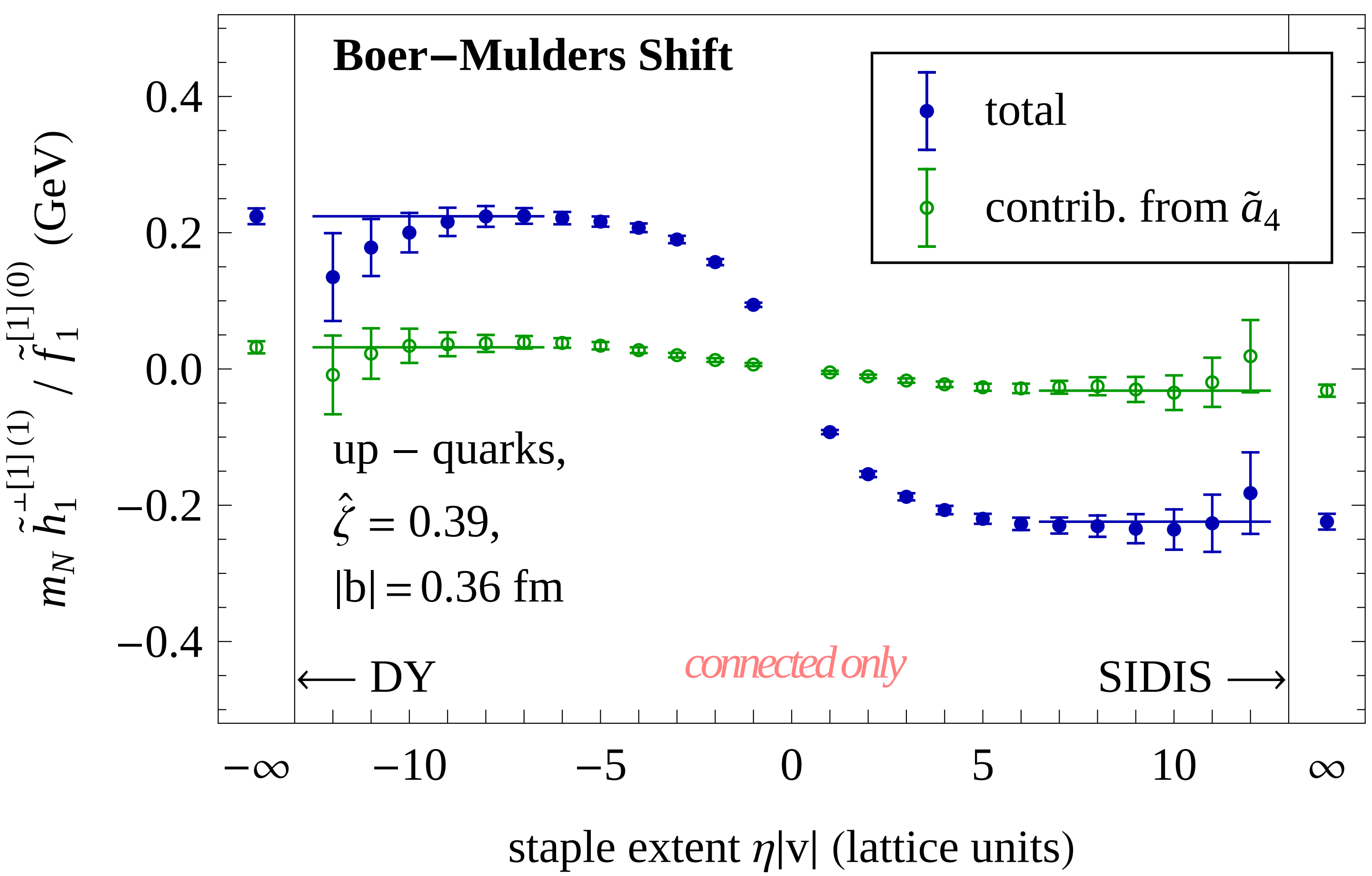
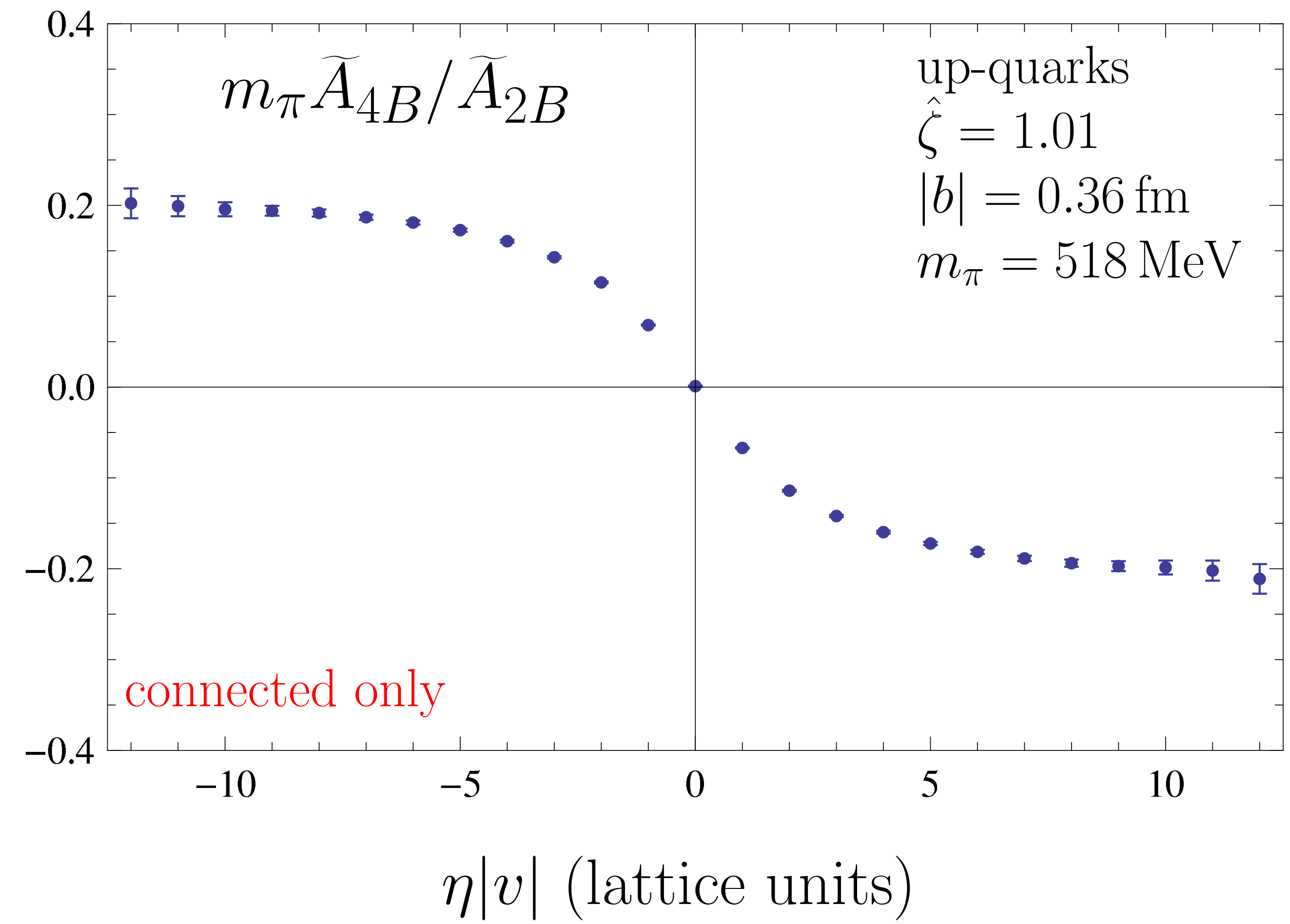


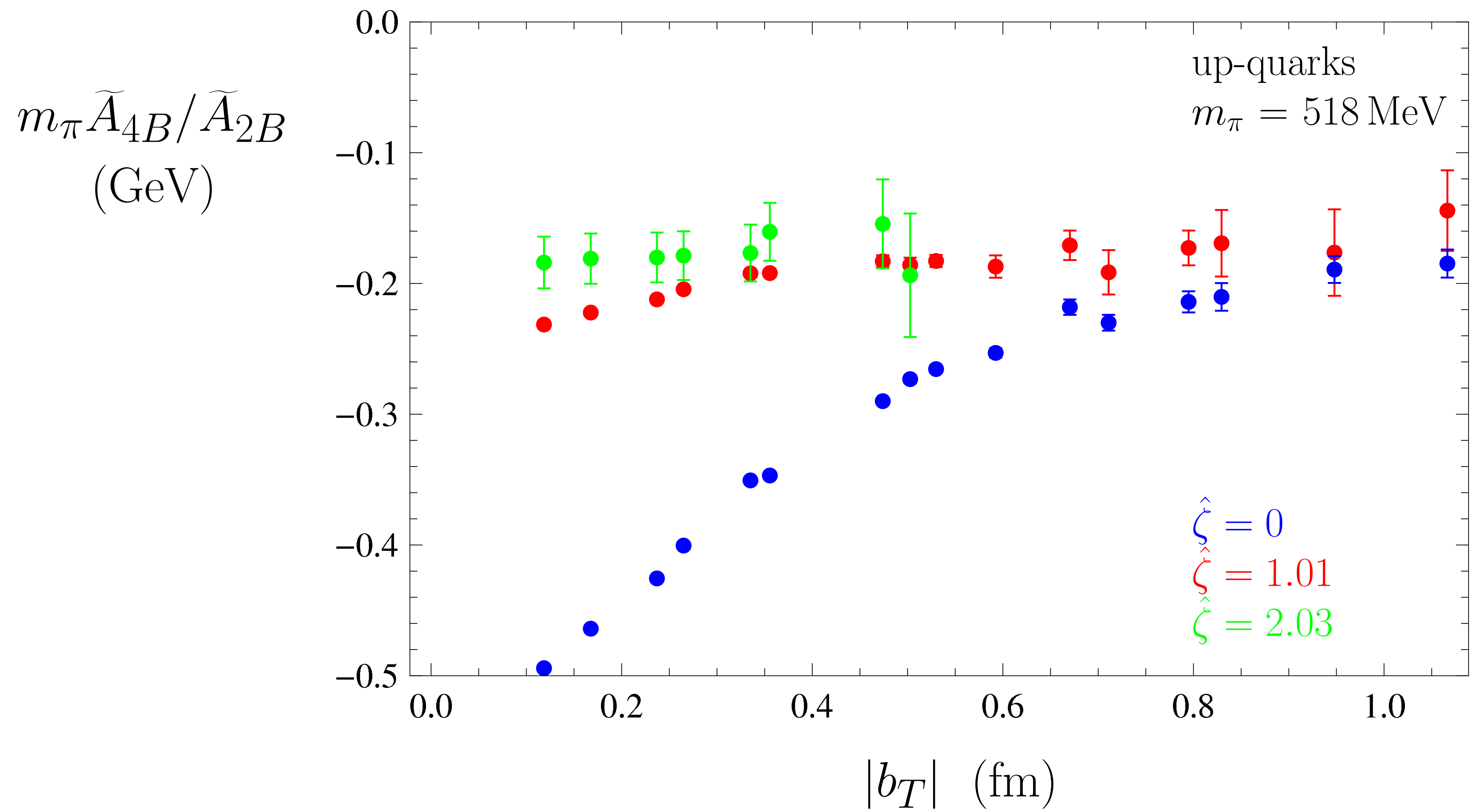
Nucleon



Pion

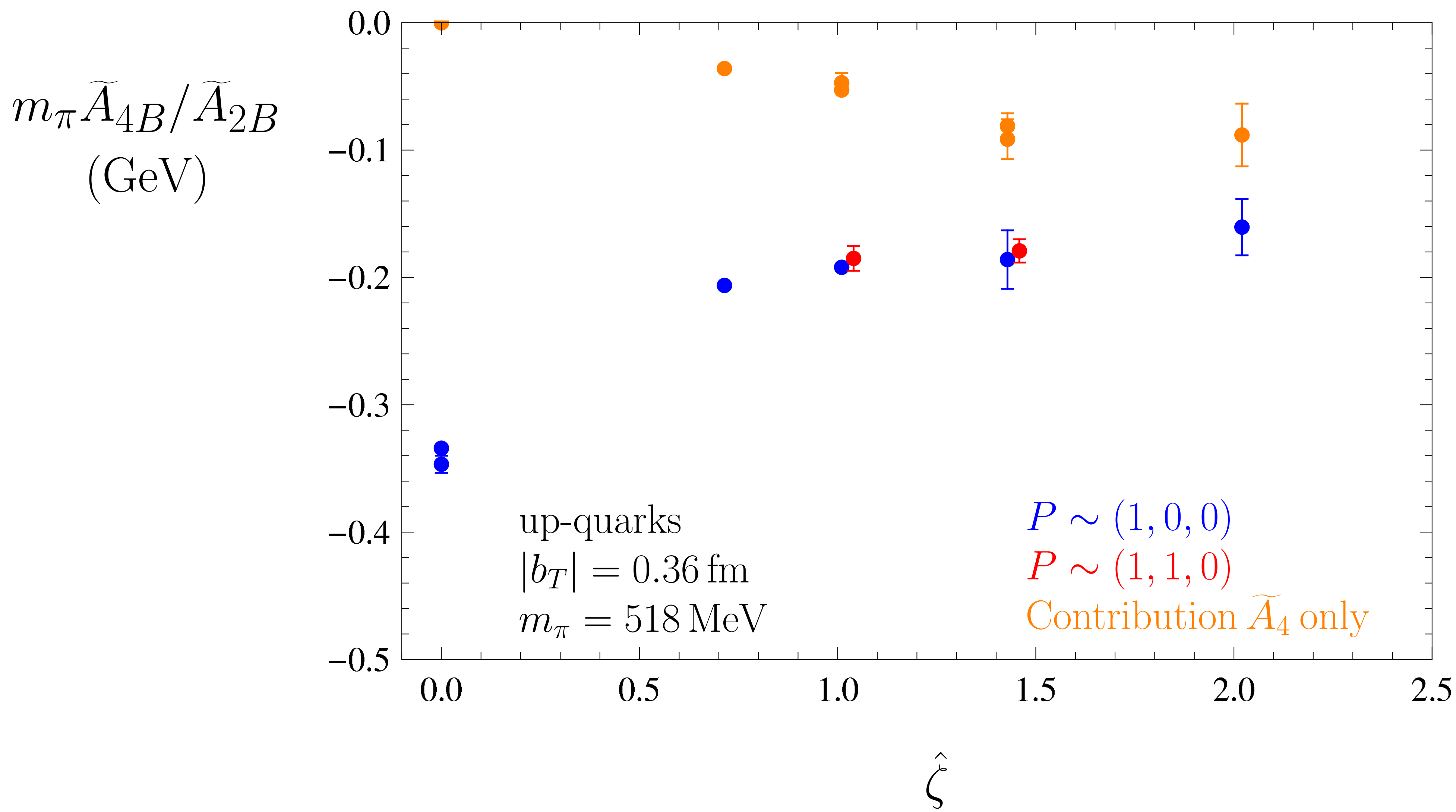
Results: Boer-Mulders shift (pion)

Dependence of SIDIS limit on $|b_T|$



Results: Boer-Mulders shift (pion)

Dependence of SIDIS limit on $\hat{\zeta}$



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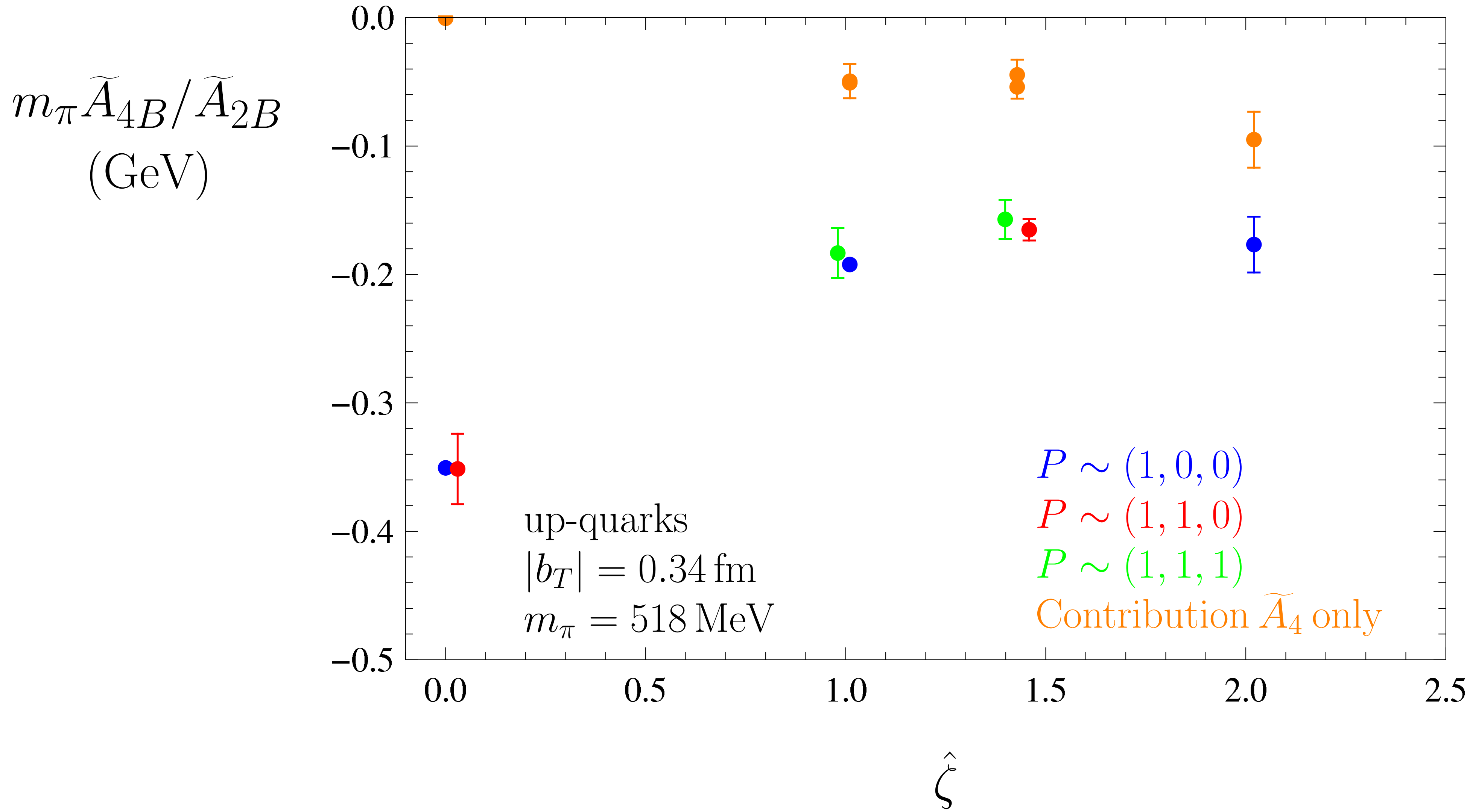
Dependence of SIDIS limit on $\hat{\zeta}$



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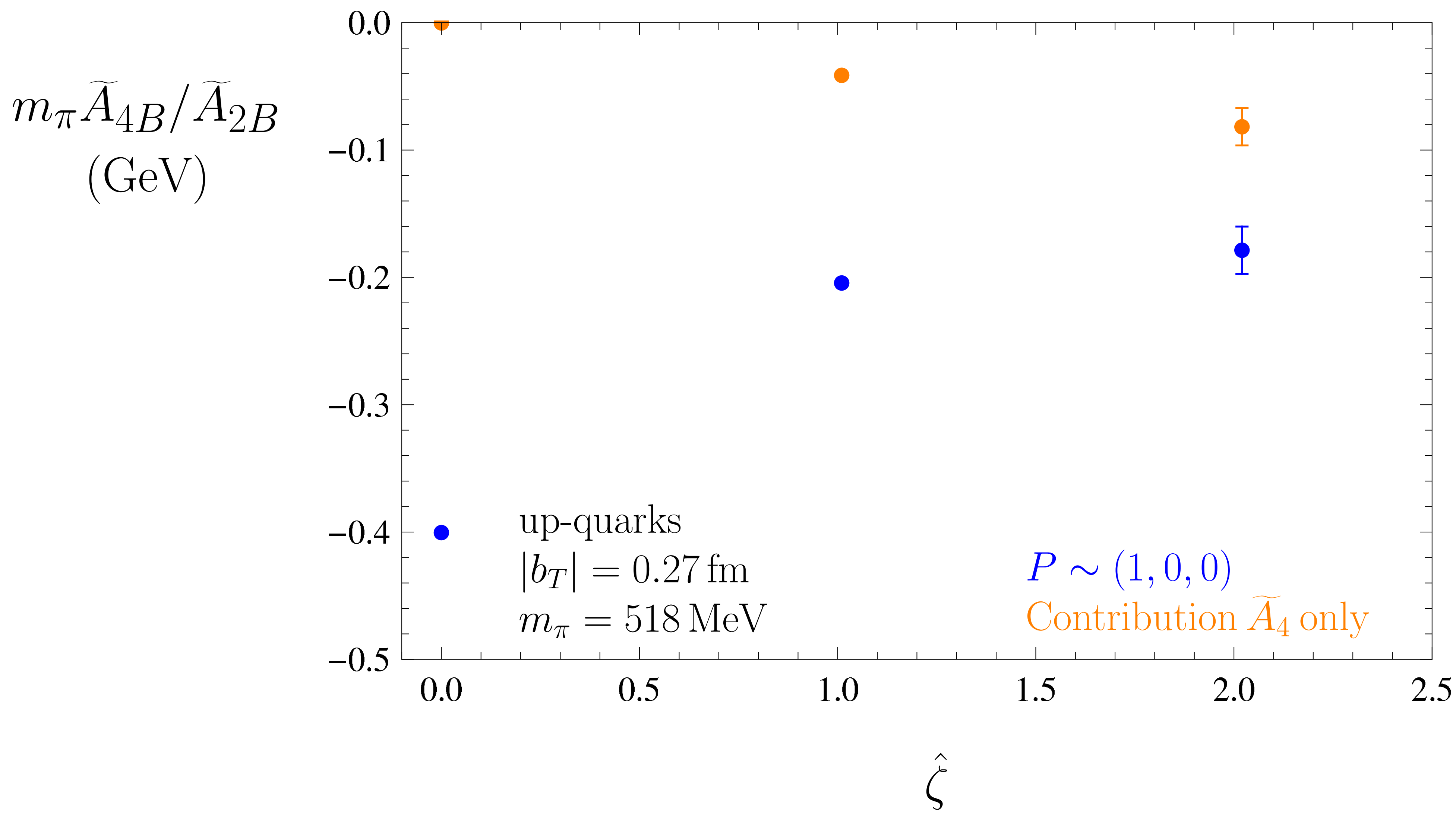


Conclusions

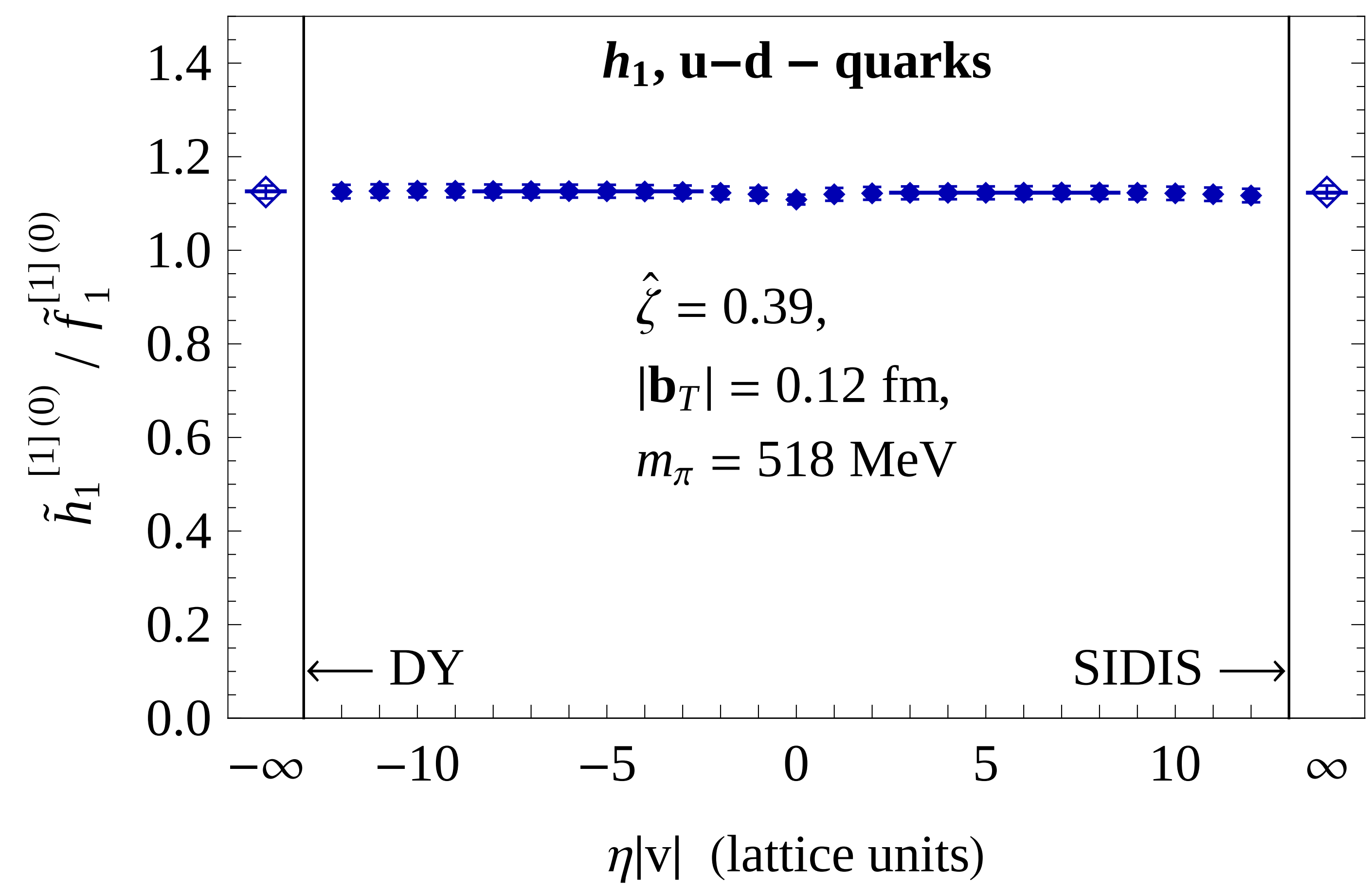
- Study of TMDs using staple-shaped gauge link structures
- Accessed T-odd Sivers, Boer-Mulders observables; SIDIS, DY limits distinguished by sign of $v \cdot P$.
- To avoid soft factors, multiplicative renormalization constants, constructed appropriate ratios of Fourier-transformed TMDs ("shifts").
- v taken off light cone: Dependence on Collins-Soper parameter $\hat{\zeta}$. In addition to $\eta v \to \infty$, need to also consider $\hat{\zeta} \to \infty$.
- $\eta v \to \infty$ seems under good control; plateaux reached at moderate values.
- Significant progress concerning the $\hat{\zeta} \to \infty$ limit in the new pion study compared with earlier nucleon study. Tentative statements concerning light cone limit possible.
- No significant volume dependence, pion mass dependence detected within the limited set of (three) cases considered
- Quantitative correspondence between u-quark Boer-Mulders ratios in proton, π^+ meson.

Outlook

- Cutoff effects and universality: Two nucleon calculations at 300 MeV pion mass; coarse lattice with Wilson fermions (USQCD), fine lattice with domain wall fermions (RBC/UKQCD)
- Chiral regime: Nucleon calculations with domain wall fermions (RBC/UKQCD) at pion masses 180 MeV, 135 MeV.
- Exploration of Wigner functions relevant for quark angular momentum: Staple links (Jaffe-Manohar) vs. straight links (Ji).

Results: Transversity

Dependence on staple extent; sequence of panels at different $|b_T|$



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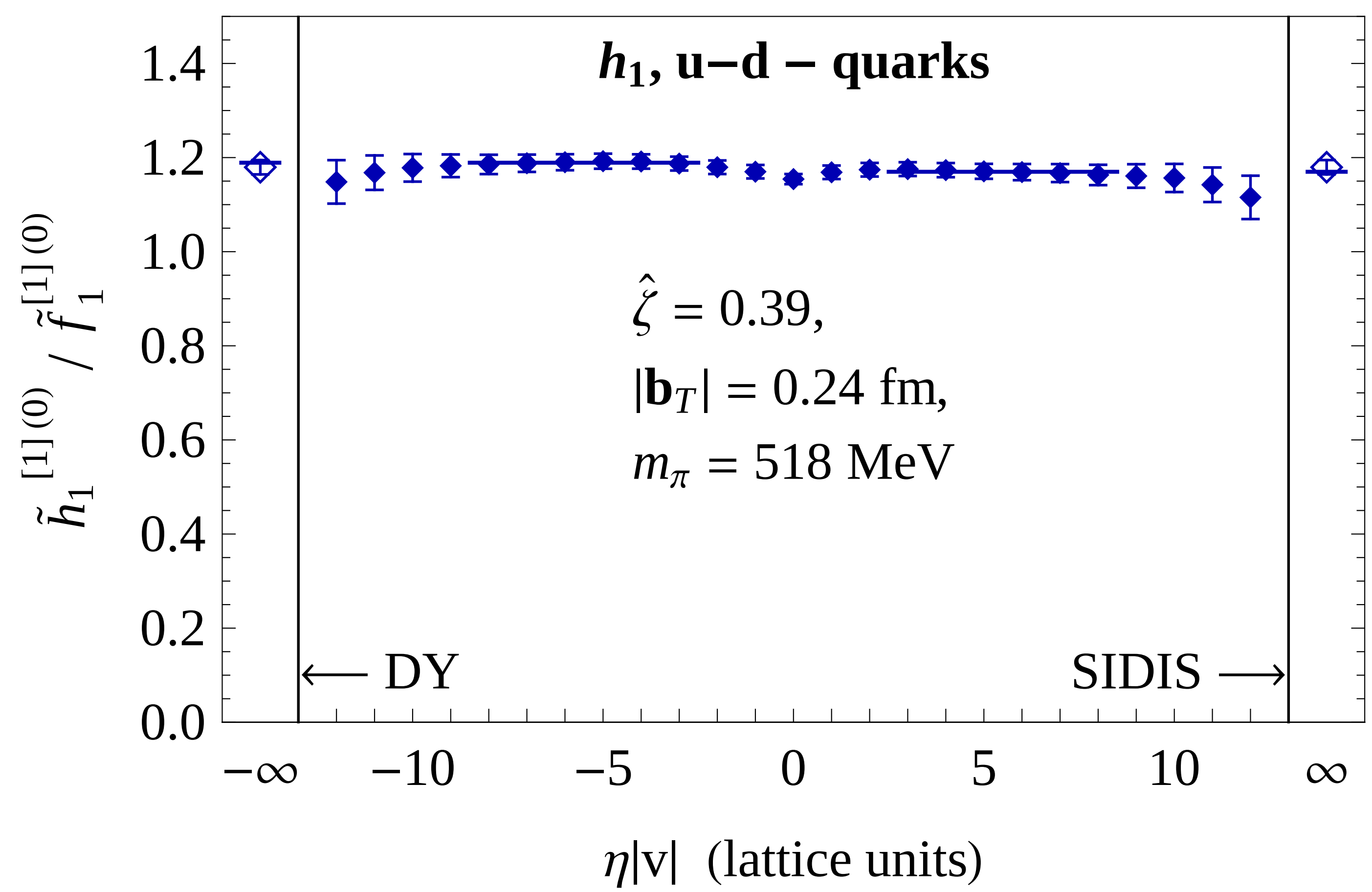

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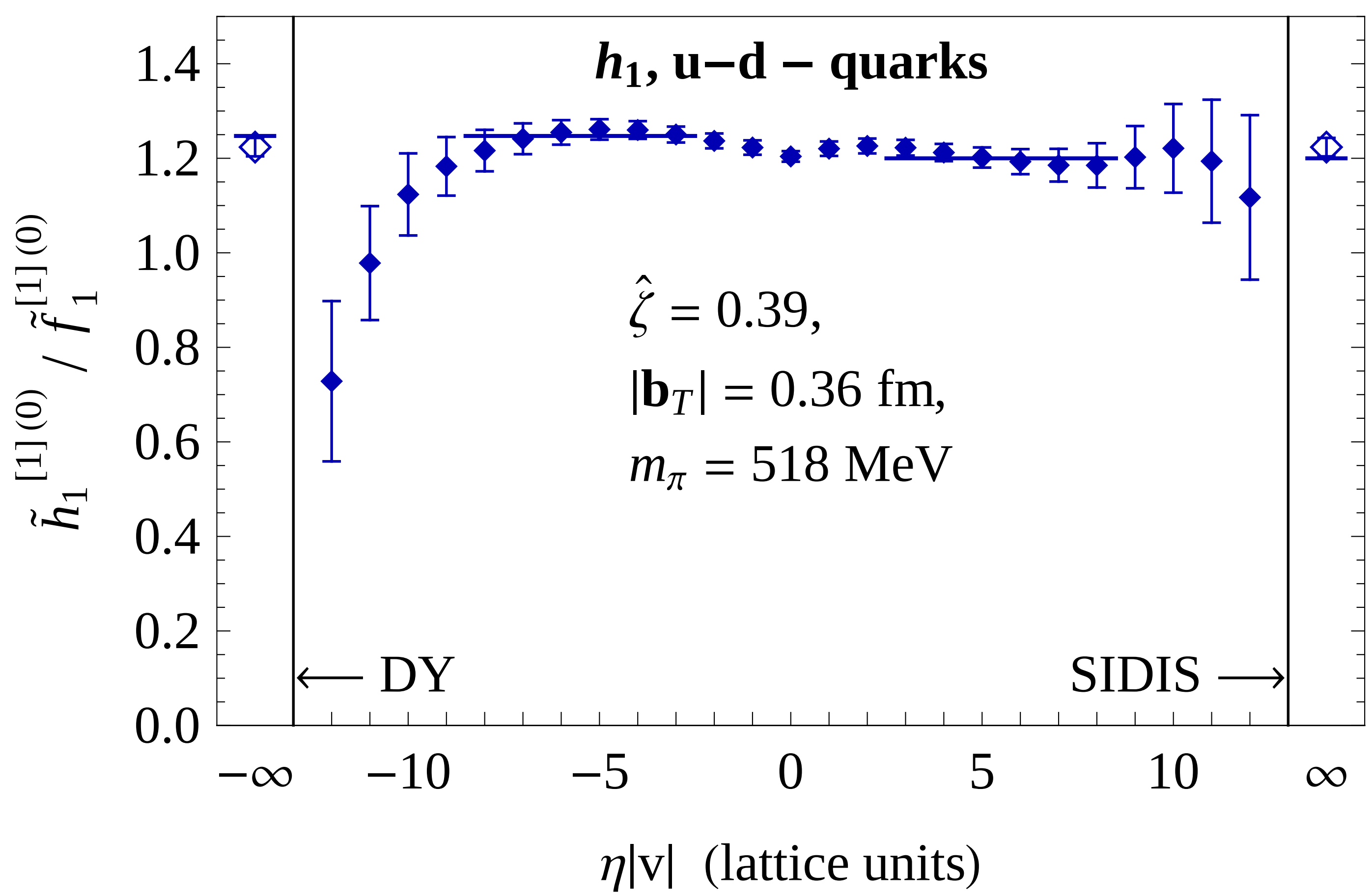
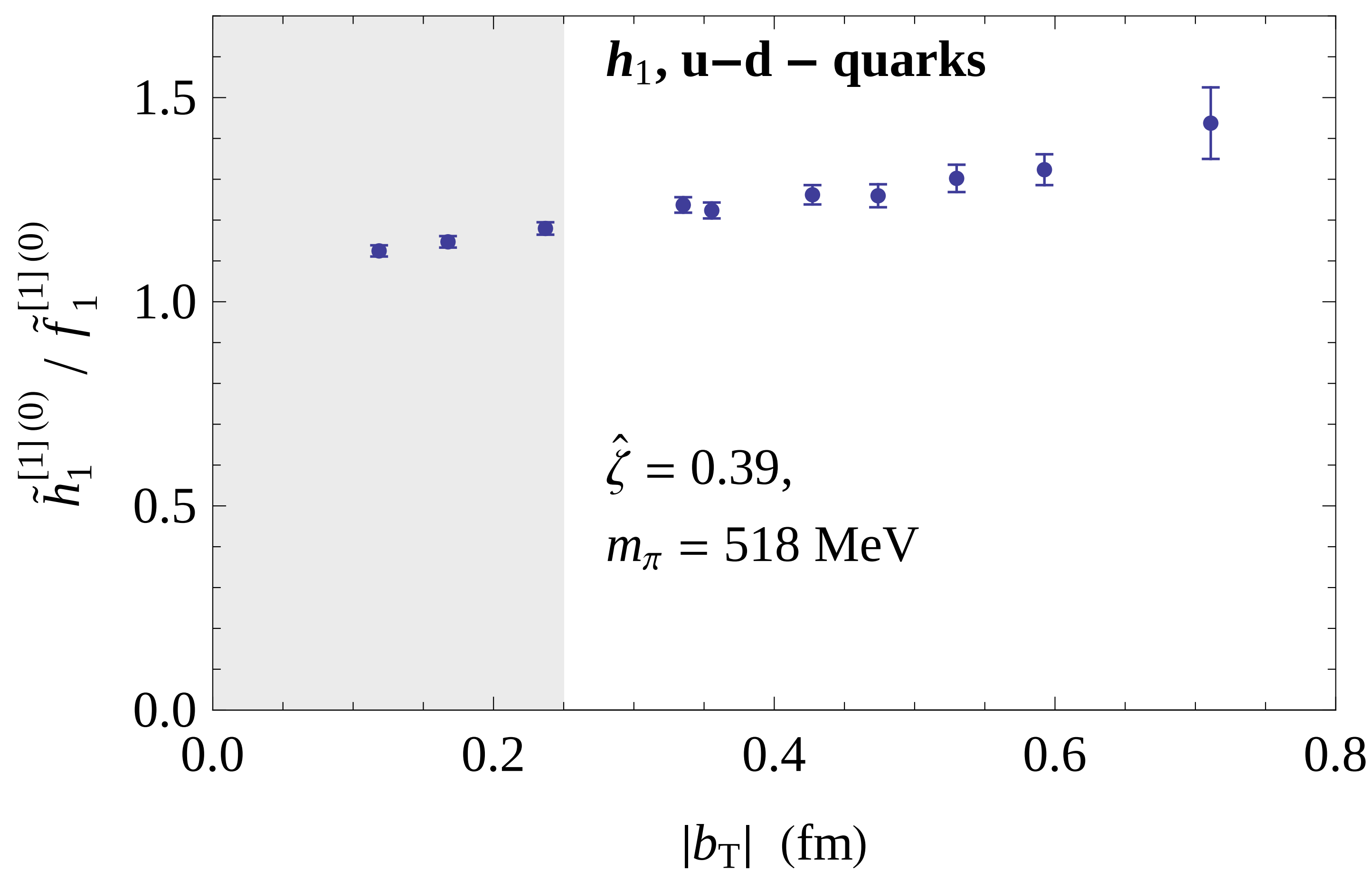


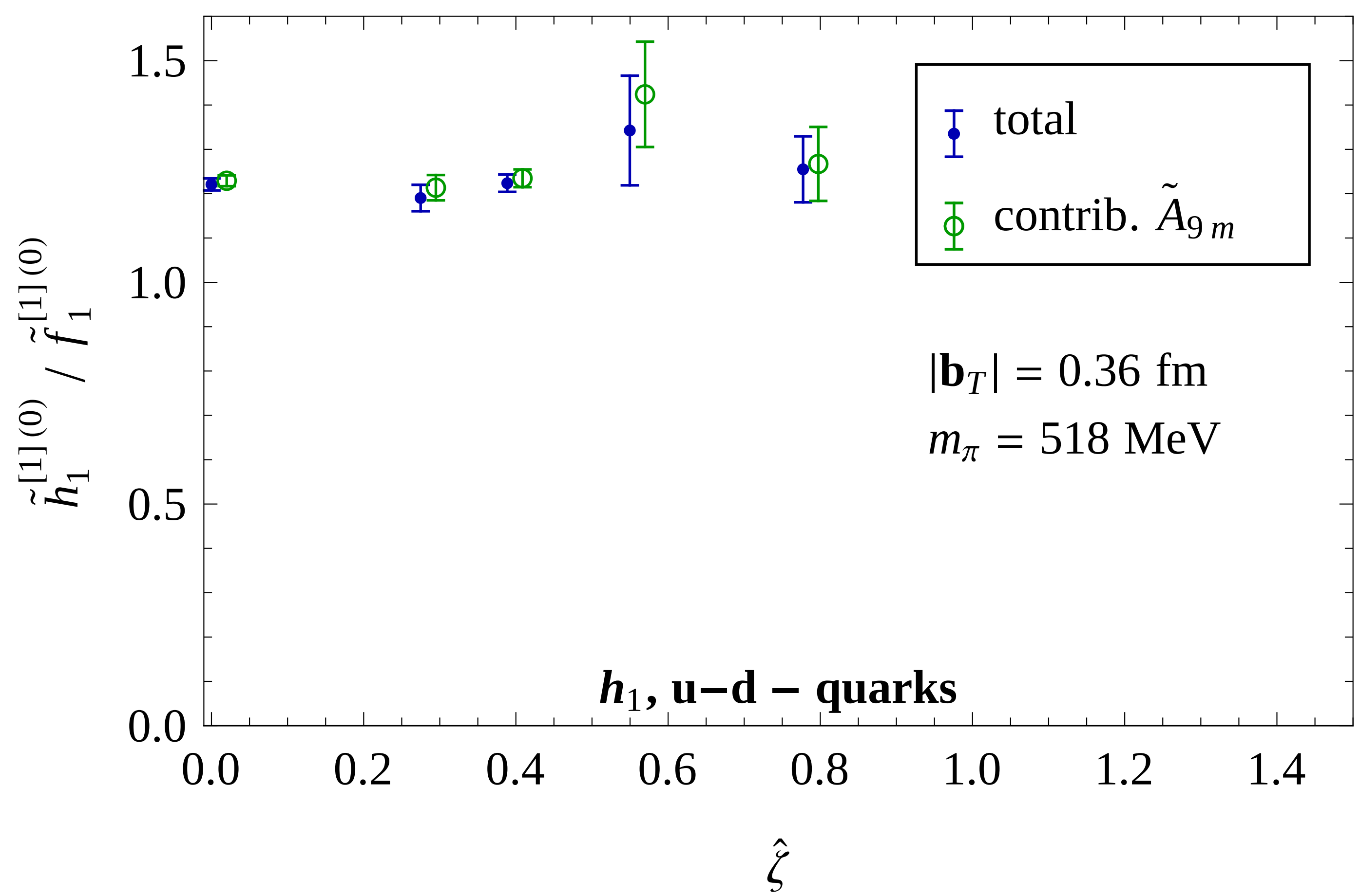
Results: Transversity

Dependence of SIDIS/DY limit on $|b_T|$



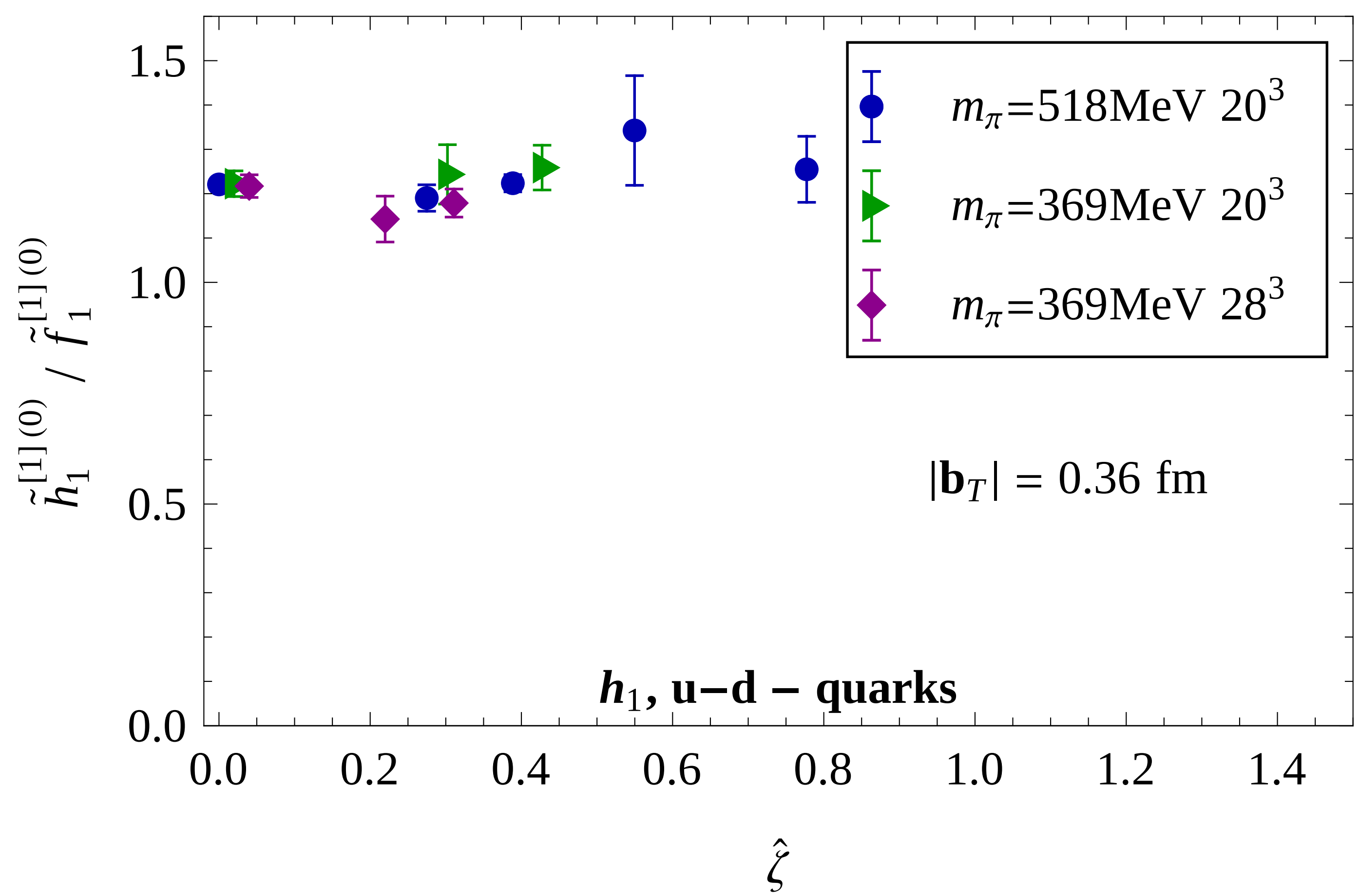
Results: Transversity

Dependence of SIDIS/DY limit on $\hat{\zeta}$



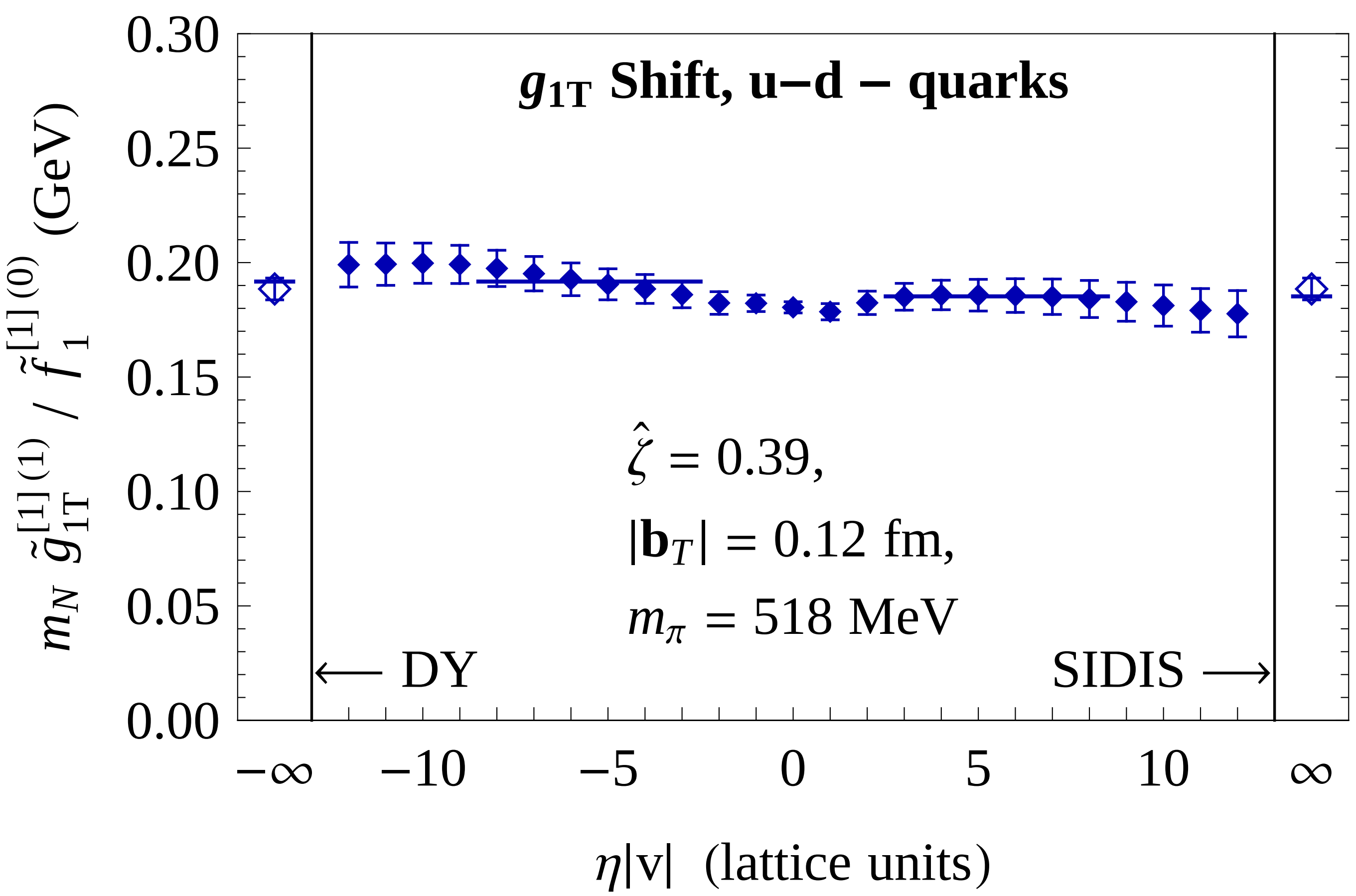
Results: Transversity

Dependence of SIDIS/DY limit on $\hat{\zeta}$, all three ensembles



Results: g_{1T} worm gear shift

Dependence on staple extent; sequence of panels at different $|b_T|$



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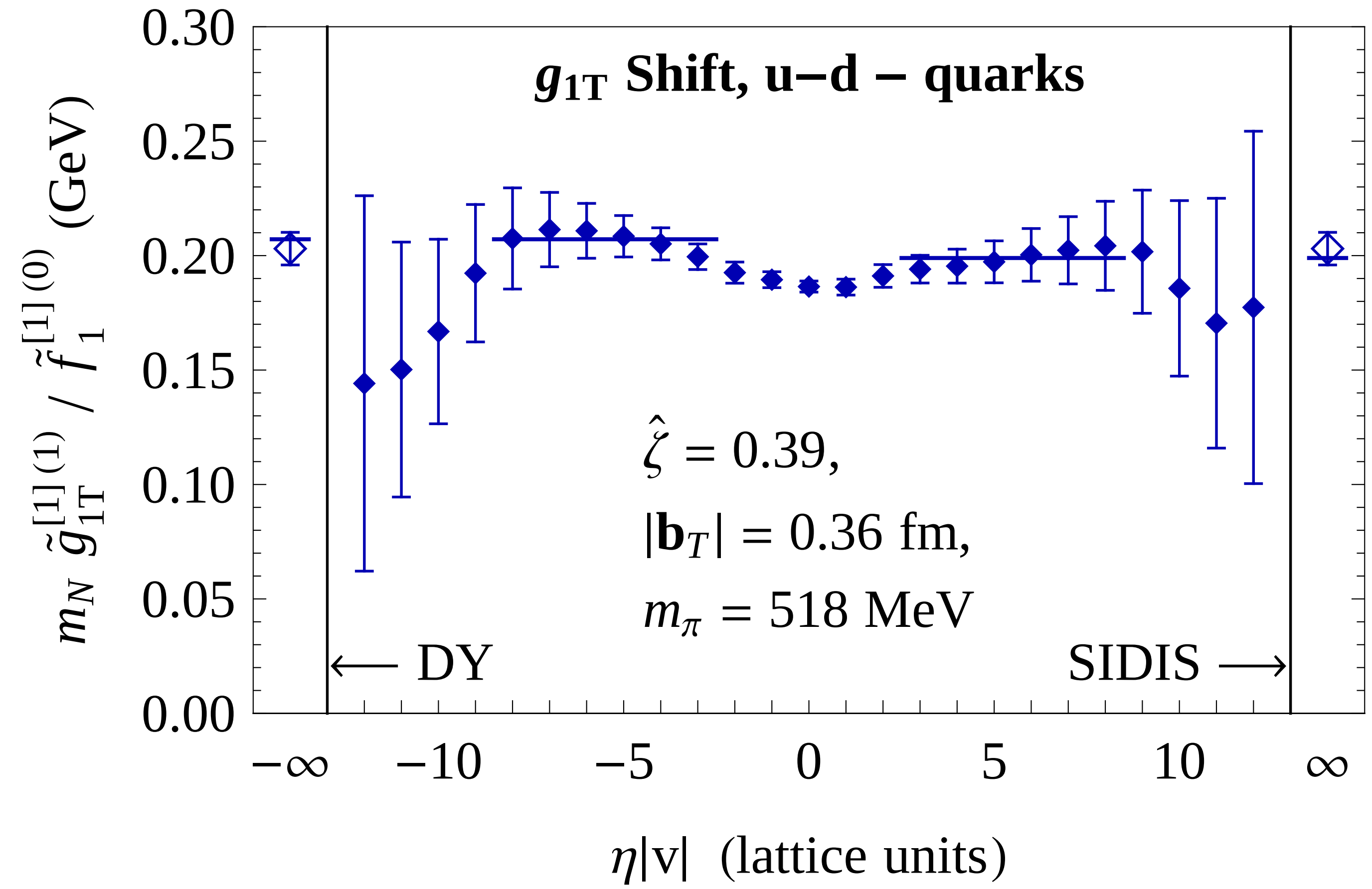

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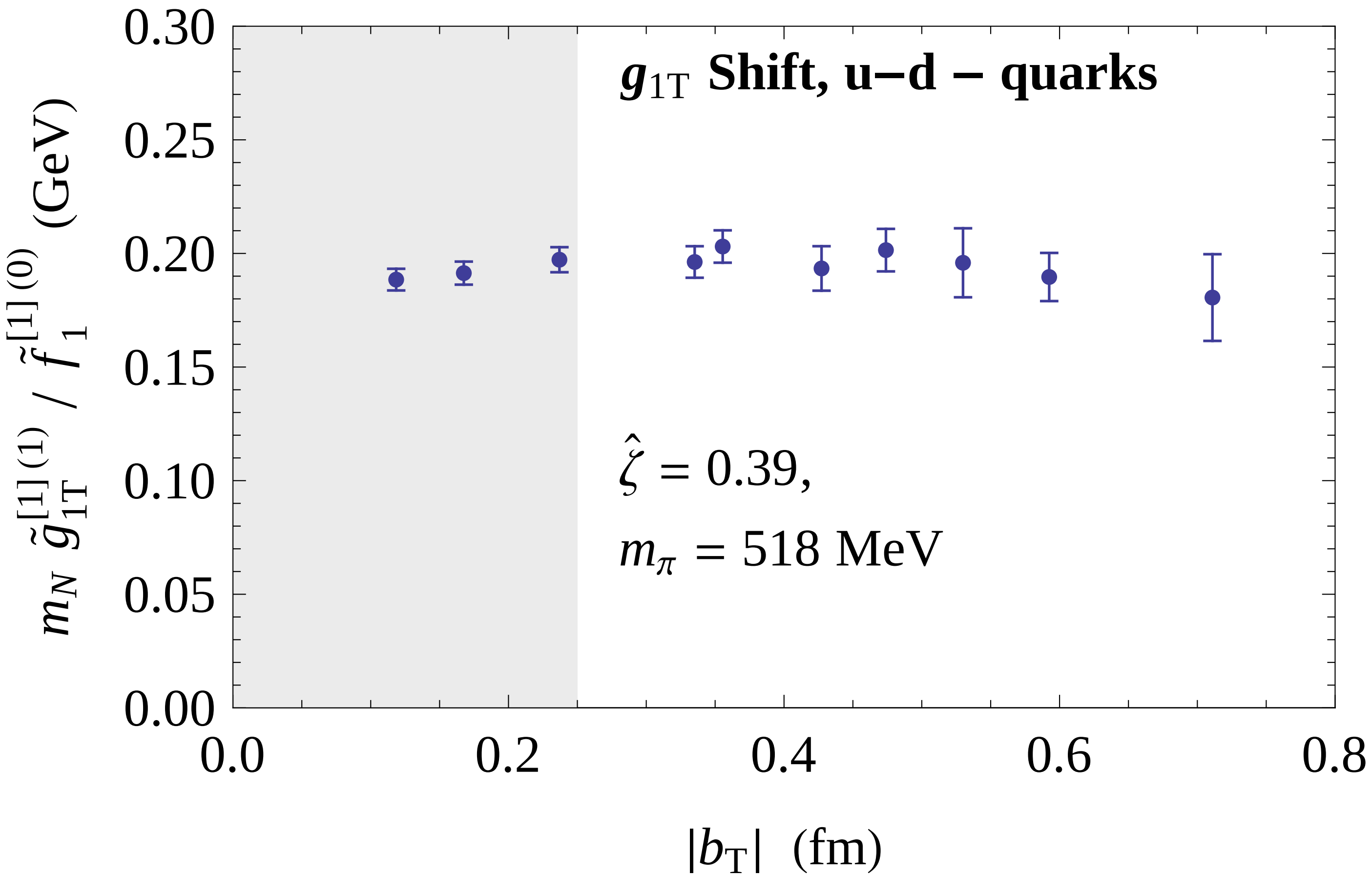

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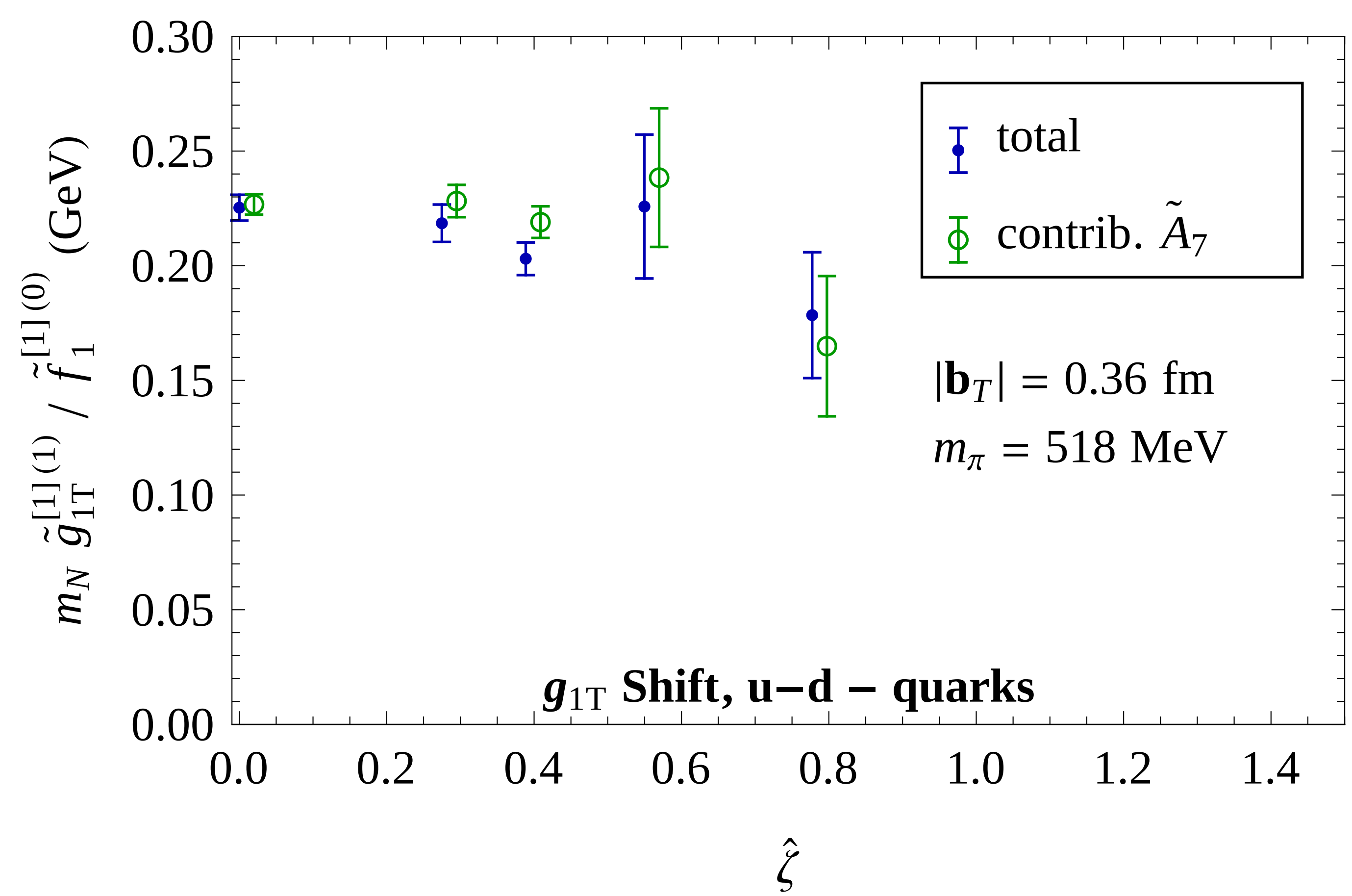

Results: g_{1T} worm gear shift

Dependence of SIDIS/DY limit on $\hat{\zeta}$, all three ensembles