Analysis of dihadron observables

Aurore Courtoy

IFPA-Université de Liège (Belgium) & LNF-INFN (here)

PSHP, LNF, Frascati

November 12, 2013

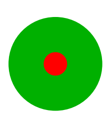
Outline

- Why Dihadron Fragmentation Functions?
- How well do we know DiFFs?
- The very first success: Transverse Target-Spin Asymmetry in two-pion SIDIS
- This year's fav' topic: Higher-twist distributions

Primary goal: Proton structure

- **3 leading-twist PDFs:**

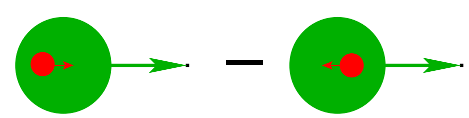
$f_1(x)$



U

unpolarized target

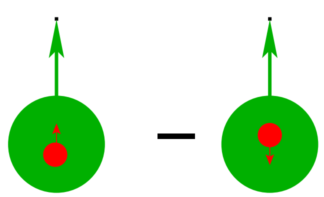
$g_1(x)$



L

longitudinally polarized target

$h_1(x)$



T

transversely polarized target

- ... and a bunch of higher-twist PDFs

Primary goal: Proton structure

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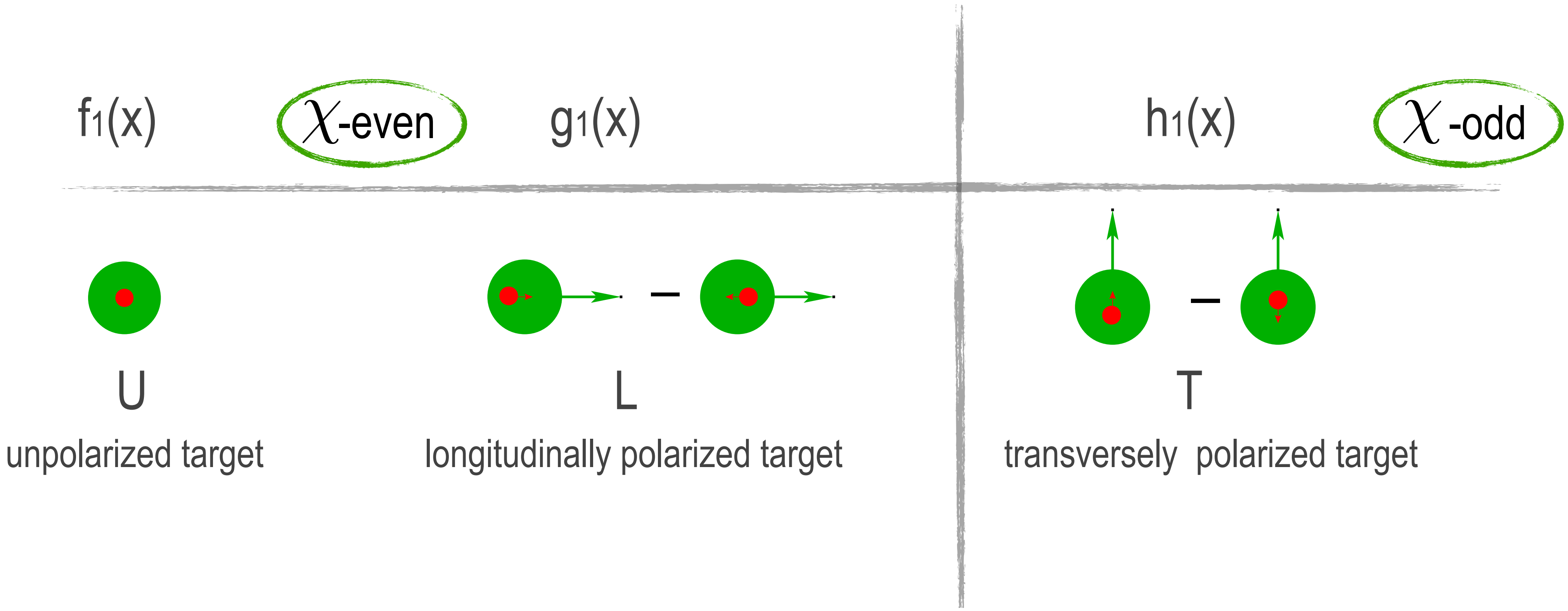
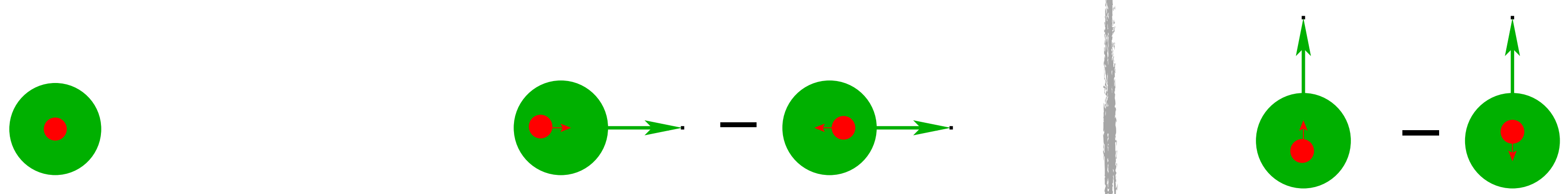


- ... and a bunch of higher-twist PDFs

Primary goal:
Proton structure

- **3 leading-twist PDFs:**

$f_1(x)$ — χ-even — $g_1(x)$ | $h_1(x)$ — χ-odd



U — unpolarized target

L — longitudinally polarized target

T — transversely polarized target

- **3 twist-3 PDFs:**

$e(x)$ $\quad h_L(x)$ $\quad g_T(x)$

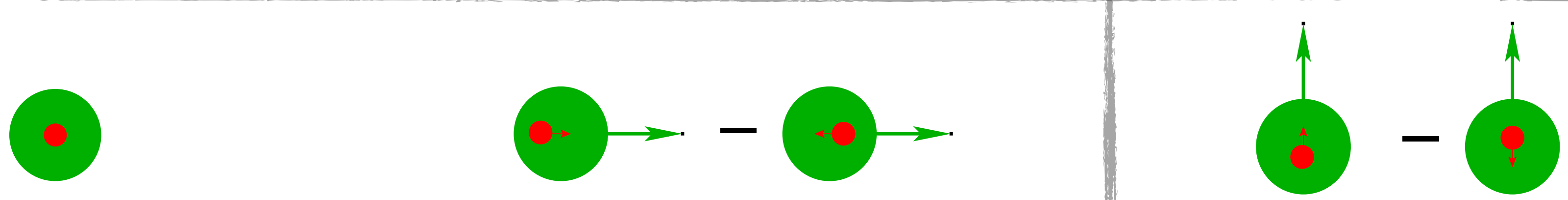
- ... and a bunch of higher-twist PDFs

Primary goal: Proton structure

- **3 leading-twist PDFs:**

$f_1(x)$ (χ-even) $g_1(x)$

$h_1(x)$ (χ-odd)



U
unpolarized target

L
longitudinally polarized target

T
transversely polarized target

- **3 twist-3 PDFs:**

$e(x)$ (χ-odd) $h_L(x)$

$g_T(x)$ (χ-even)

- ... and a bunch of higher-twist PDFs

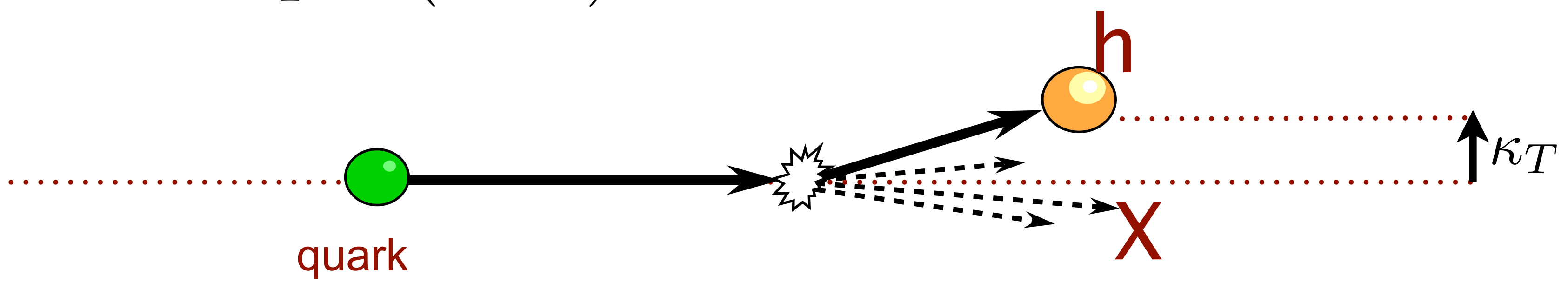
Why Dihadron Fragmentation Functions

Hadronization: fragmentation functions

see Francesca Giordano's talk

Hadronization of the quark into a hadron *h*

$$D_1^{q\rightarrow h}\left(z,\kappa_T^2\right)$$



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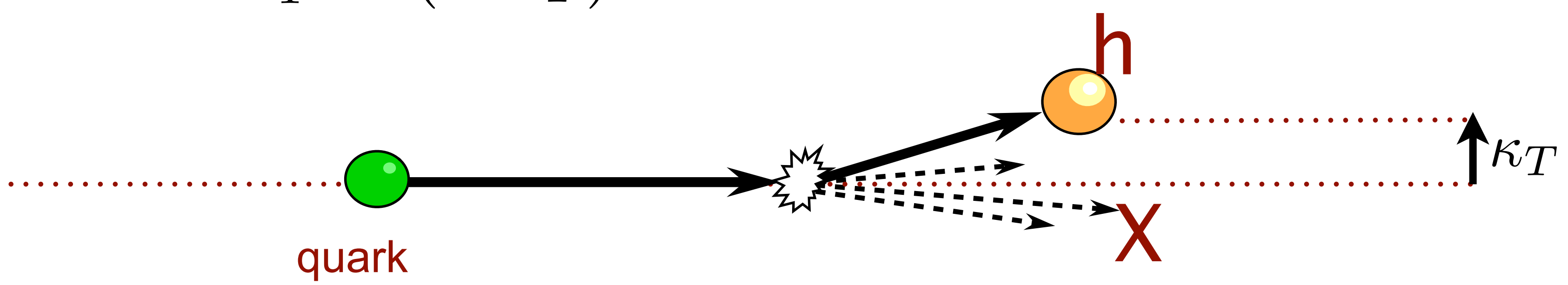

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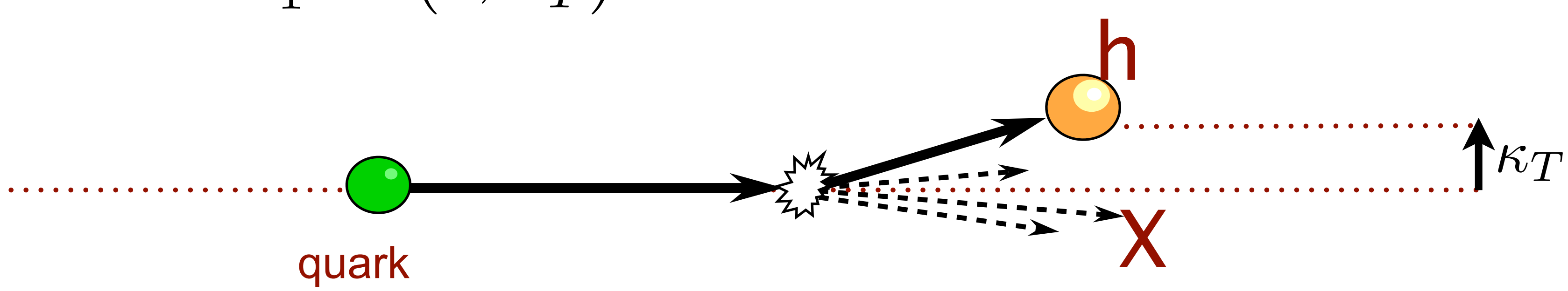
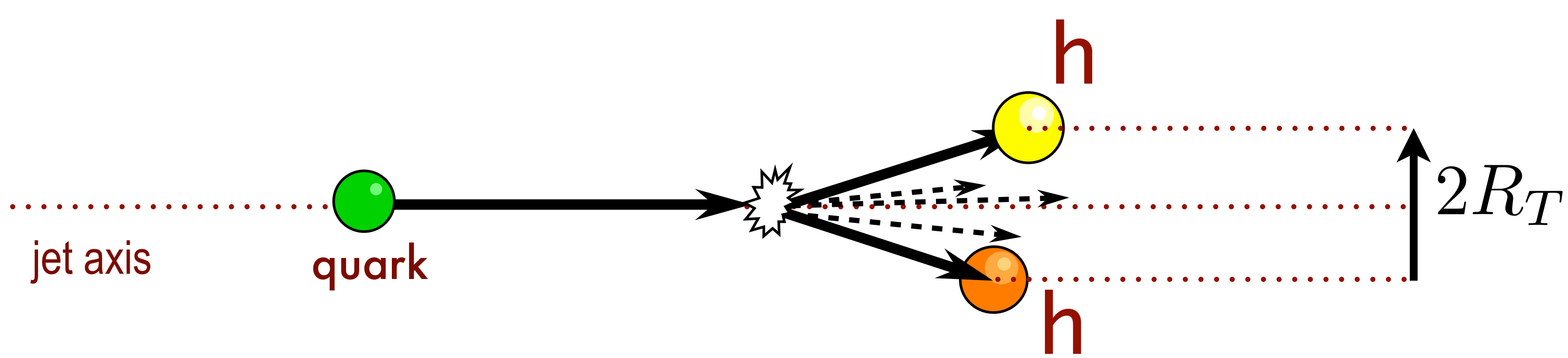
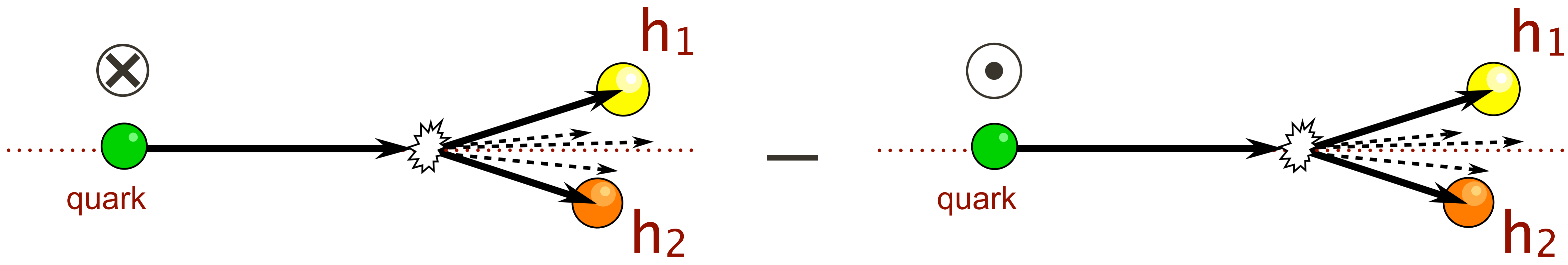


$$D_1^{q \to h_1 h_2}\left(z_1, z_2, R_T^2\right)$$



Interference Fragmentation Functions

$H_1^{\sphericalangle}(z, M_h)$



transverse pol. of the fragm. quark ↔ **angular distribution** of hadron pairs in the transverse plane

Factorization

TMD factorization

$$d\sigma \propto \sum_q \left[\mathrm{PDF}^q \otimes \mathrm{FF}^q\right]\left(x, z, P_{h\perp}^2\right)$$

Collinear factorization

$$d\sigma \propto \sum_q \mathrm{PDF}^q(x) \times \mathrm{DiFF}^q\left(z, M_h\right)$$

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TMD factorization

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TMD PDF vs. collinear PDF

see talks by J.O. Gonzalez H. & M. Radici

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✓ Convenient to use DiFFs!

Collinear factorization

$$d\sigma \propto \sum_q \mathrm{PDF}^q(x) \times \mathrm{DiFF}^q\,(z, M_h)$$

TMD PDF vs. collinear PDF

see talks by J.O. Gonzalez H. & M. Radici

The DiFF family

- **Twist-2**

$$\Delta^{[\gamma^-]} = D_1(z, M_h)$$

$$\Delta^{[i\sigma^{i-}\gamma_5]} = \frac{\epsilon_T^{ij} R_{Tj}}{2M} H_1^{\sphericalangle}(z, M_h)$$

plus 2 k_T dependent FF

- **Twist-3**

Bianconi, Boffi, Jakob & Radici, PRD62

Bacchetta & Radici, PRD69

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Kinematical twist-3 ***Wandzura-Wilzcek approximation***

Dynamical twist-3 ...

- **higher-twists...**

Bianconi, Boffi, Jakob & Radici, PRD62

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$$\tilde{D}^{\sphericalangle}, \tilde{G}^{\sphericalangle}, \tilde{H}, \tilde{E}$$

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$$\tilde{D}^{\sphericalangle}, \tilde{G}^{\sphericalangle}, \tilde{H}, \tilde{E}$$

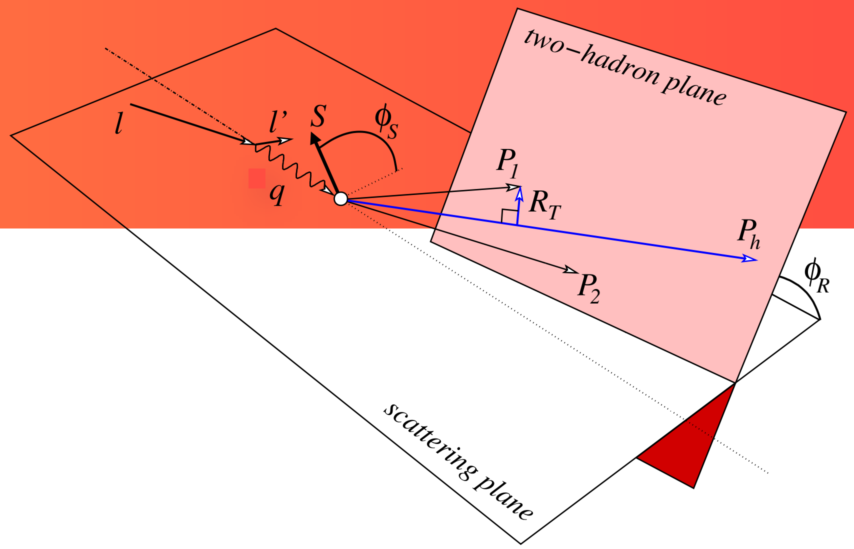
- **higher-twists...**

- **P-odd DiFFs** [Bacchetta, Boer, Radici, in progress??]

Bianconi, Boffi, Jakob & Radici, PRD62

Bacchetta & Radici, PRD69

Two-hadron SIDIS



- **A_{UT}**

$$A_{UT}^{\sin(\phi_R+\phi_S)\,\sin\theta}(x,y,z,M_h;Q) = -\frac{B(y)}{A(y)}\,\frac{|\mathbf{R}|}{M_h}\,\frac{\sum_q e_q^2\, h_1^q(x;Q^2)\, H_{1,sp}^{\sphericalangle\, q}(z,M_h;Q^2)}{\sum_q e_q^2\, f_1^q(x;Q^2)\, D_1^q(z,M_h;Q^2)}$$

Jaffe, Jin, Tiang, PRL 80
Radici, Jakob & Bianconi, PRD65

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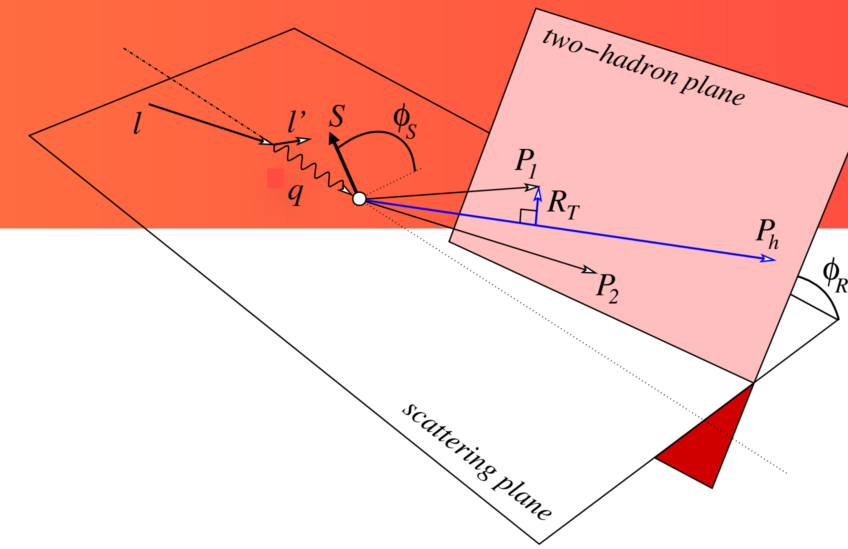

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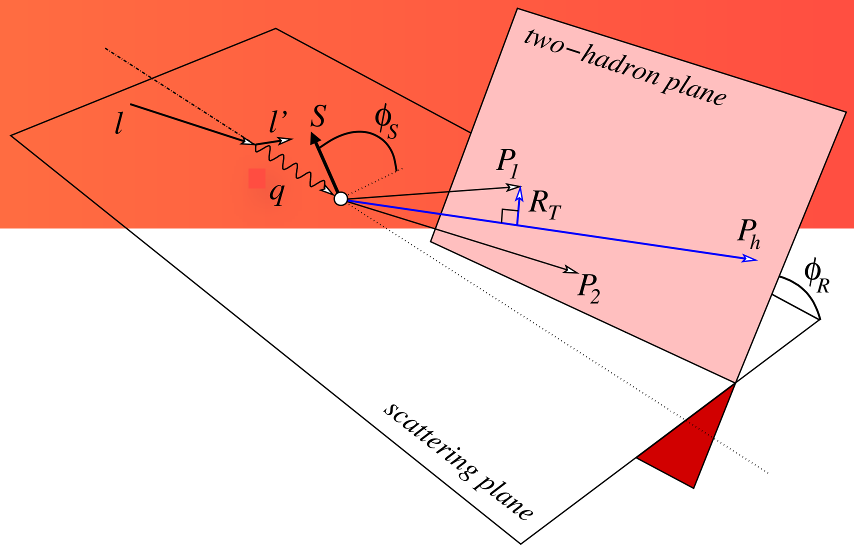

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Jaffe, Jin, Tiang, PRL 80
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Bacchetta & Radici, PRD69

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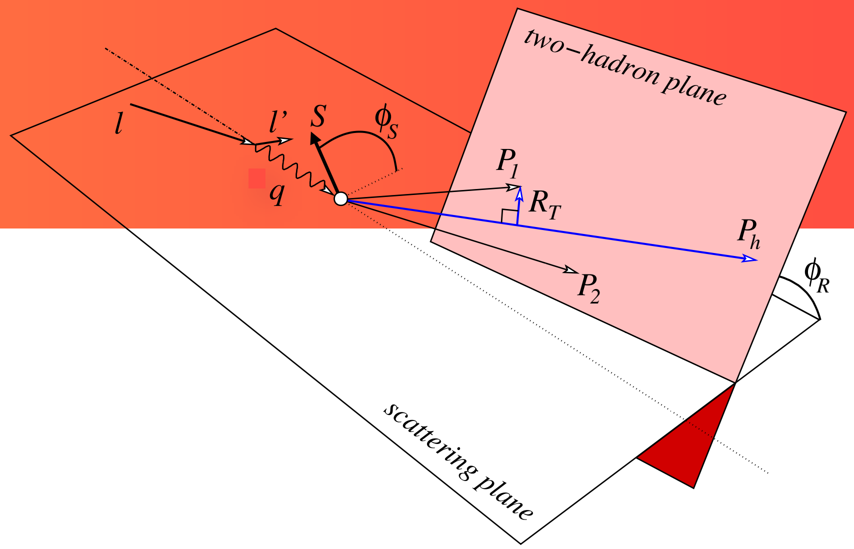

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Stay awake until the last part of the talk !

Bacchetta & Radici, PRD69

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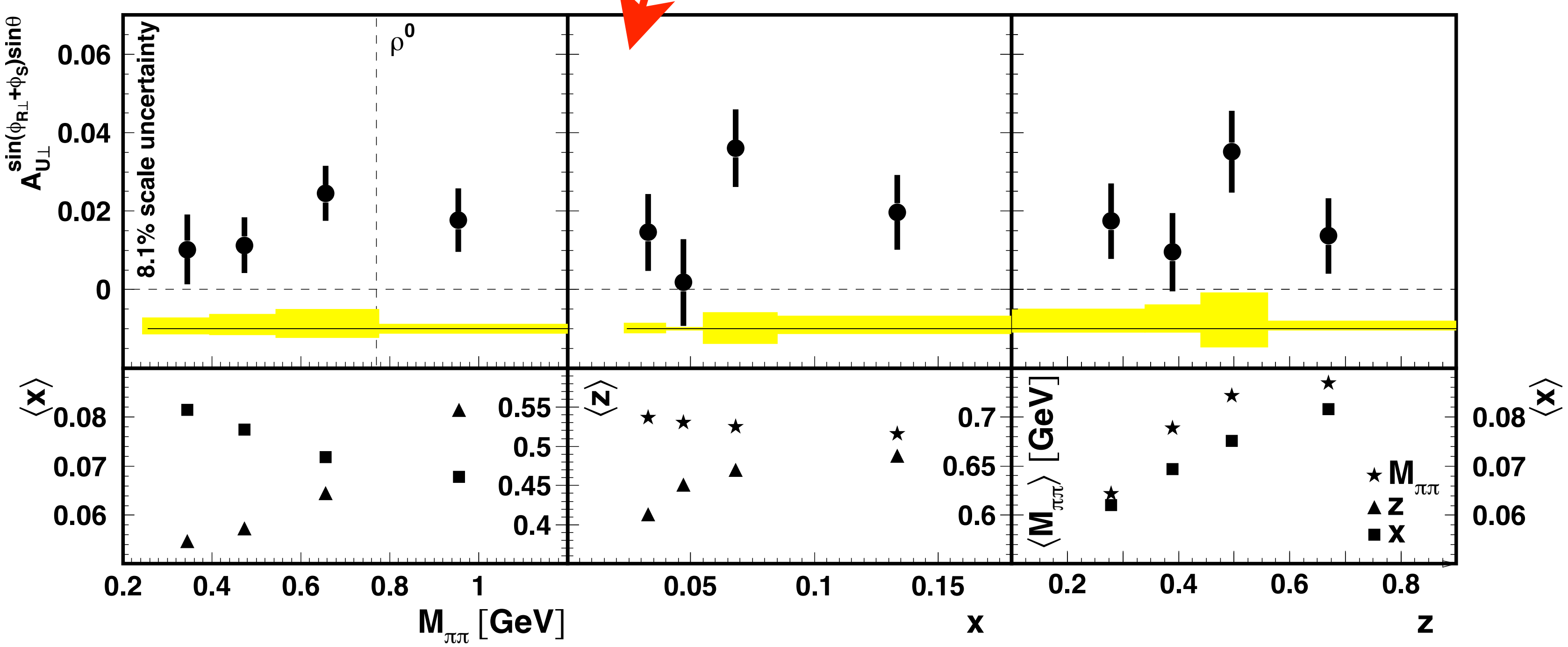
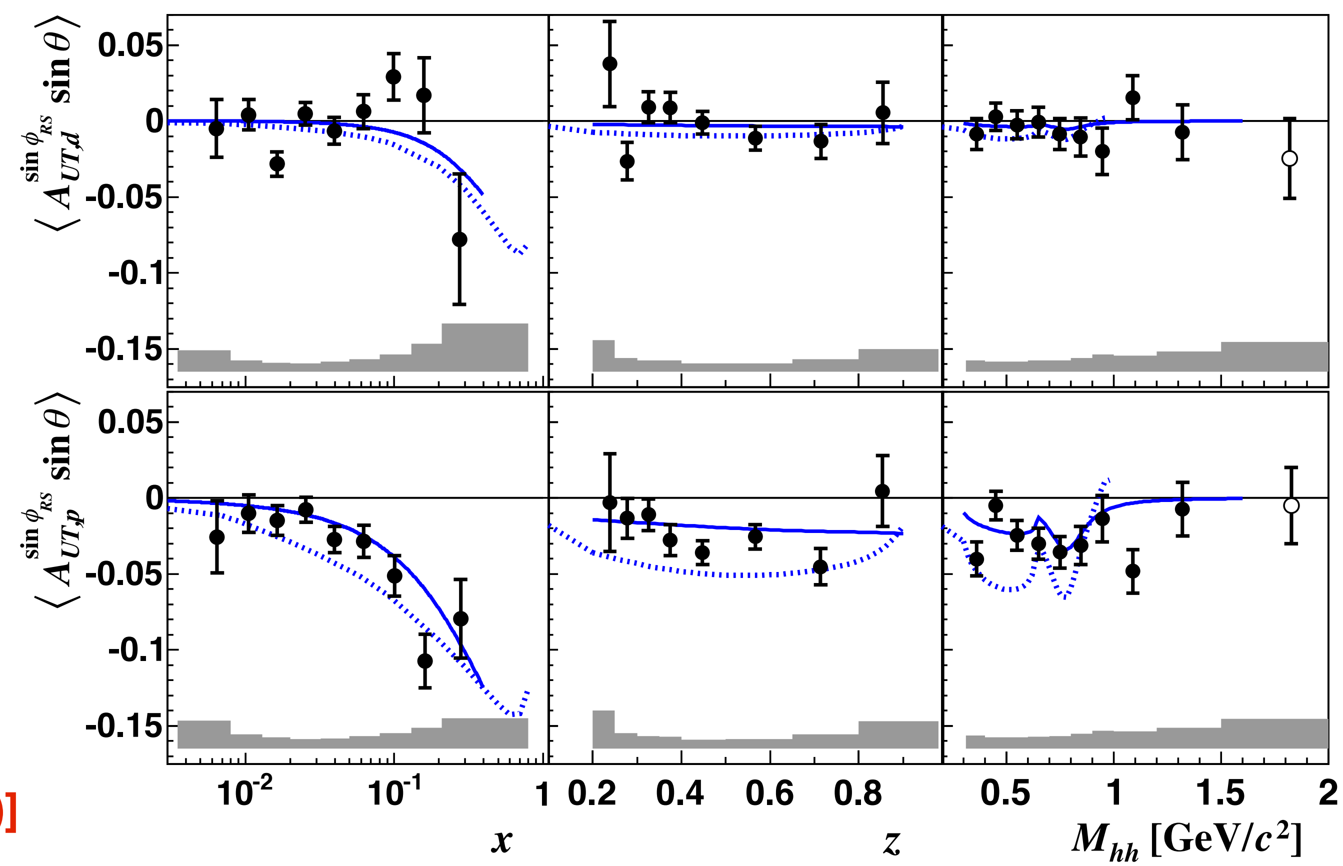
A_{UT} at HERMES and COMPASS

see Christopher Braun's talk



[JHEP 06, 017 (2008)]



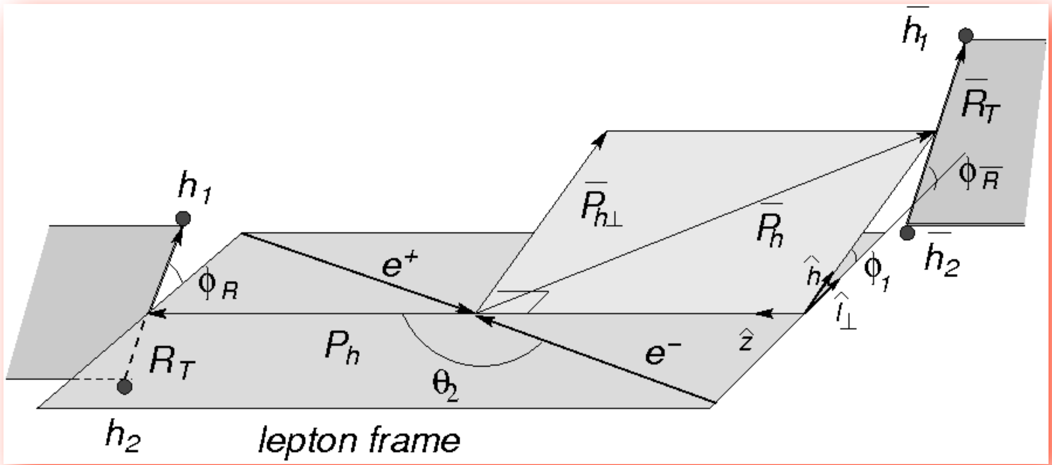
2002-4 Deuteron Data

2007 Proton Data

[Phys. Lett. B 713 (2012)]

Pion pair production in e⁺e⁻ annihilation

- **Artru-Collins asymmetry**



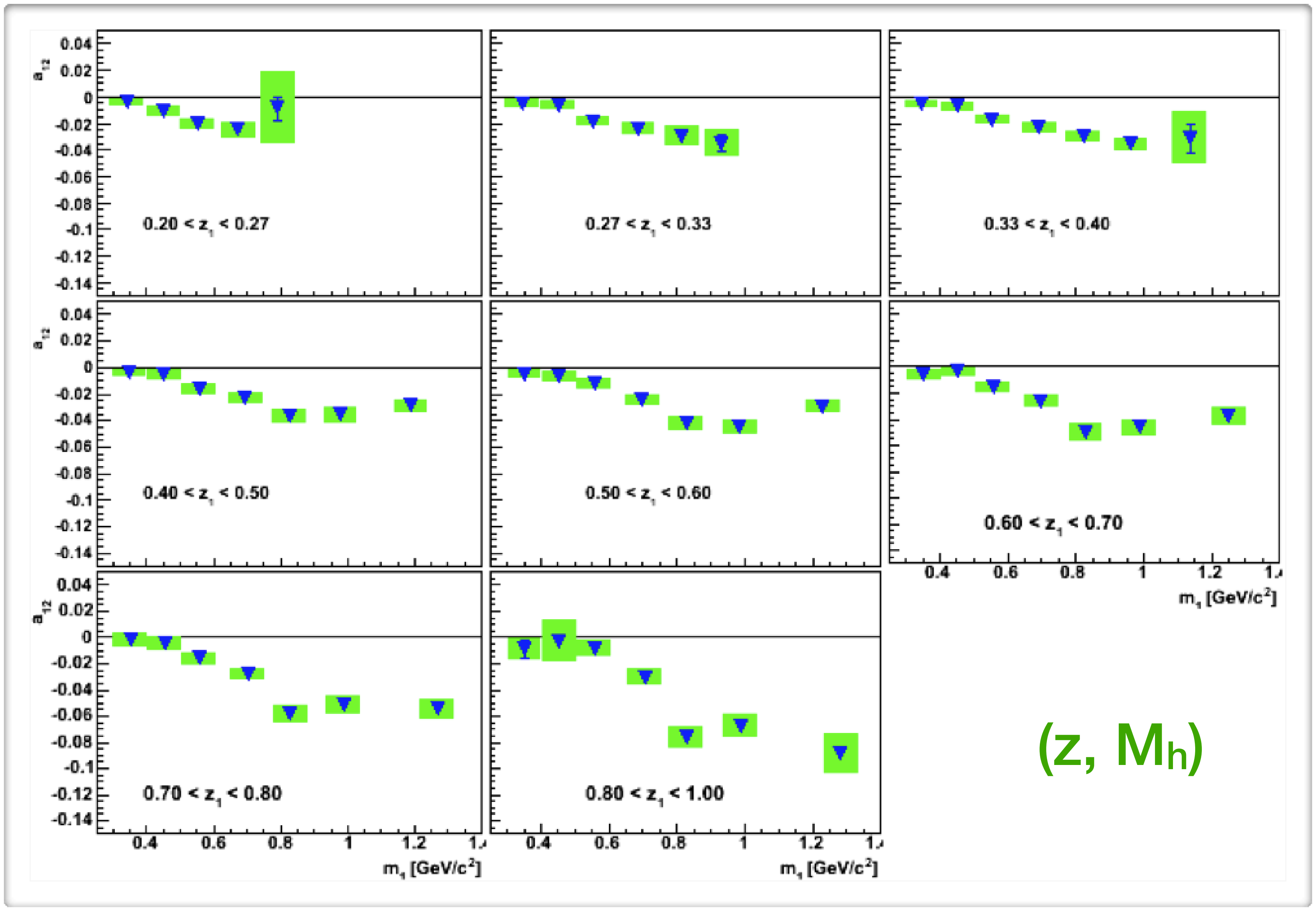
✦ 2 hemispheres
✦ azimuthal modulation between the 2 hemispheres

✦ If we integrate over one hemisphere, we get

$$A^{\cos(\phi_R+\overline{\phi}_R)}\left(z,\ M_h,Q^2\right)=-\frac{\langle\sin^2\theta_2\rangle}{\langle 1+\cos^2\theta_2\rangle}\langle\sin\theta\rangle\langle\sin\overline{\theta}\rangle\frac{|\mathbf{R}|}{M_h}\frac{\sum_q e_q^2\, H_{1,sp}^{\sphericalangle q}(z,M_h;Q^2)\, n_q^{\uparrow}(Q^2)}{\sum_q e_q^2\, D_1^q(z,M_h;Q^2)\, n_q(Q^2)}$$

Boer, Jakob & Radici, PRD67

Artru-Collins asymmetry at Belle



[Phys.Rev.Lett.107 (2011)]

BaBar?
see Isabella Garzia's talk

Pion pair production in pp$^\uparrow$ collision

$$A_N \equiv \frac{d\sigma_{UT}}{d\sigma_{UU}}$$

$$A_N(\eta_C, |\mathbf{P}_{C\perp}|, \cos\theta_C, M_C^2, \phi_{R_C}, \phi_{S_B}) \propto \frac{f_1 \otimes h_1 \otimes H_1^{\sphericalangle}}{f_1 \otimes f_1 \otimes D_1}$$

Bacchetta & Radici, PRD70

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Bacchetta & Radici, PRD70

- $A_N^{\sin\Phi}$ asymmetry @ Phenix

- $A_N^{\sin\Phi}$ asymmetry @ STAR

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Bacchetta & Radici, PRD70

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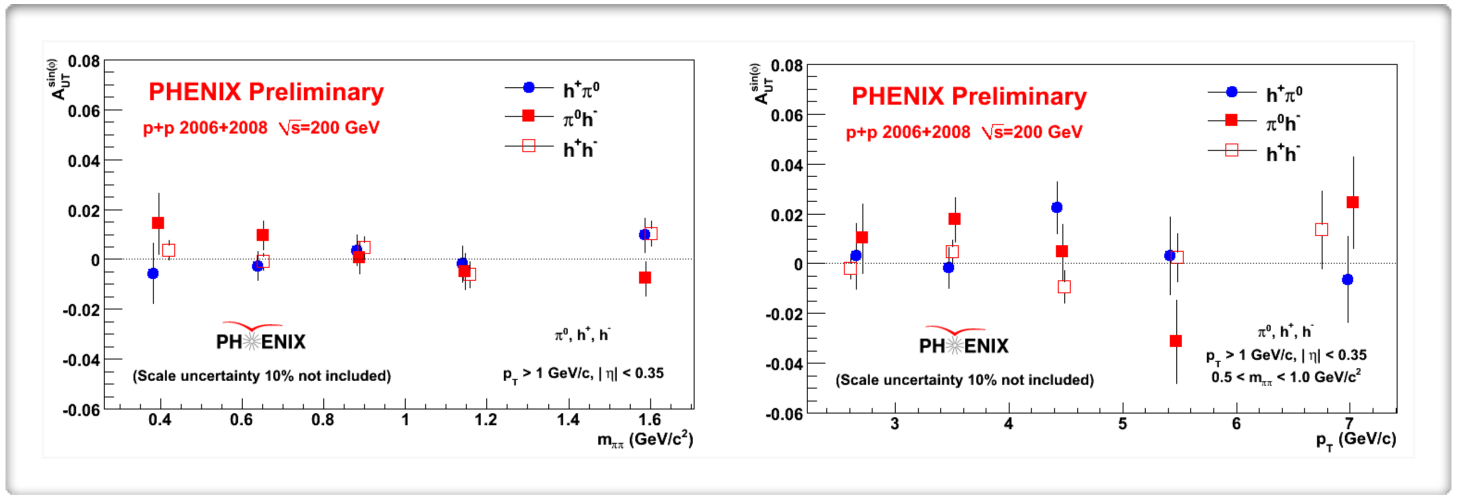
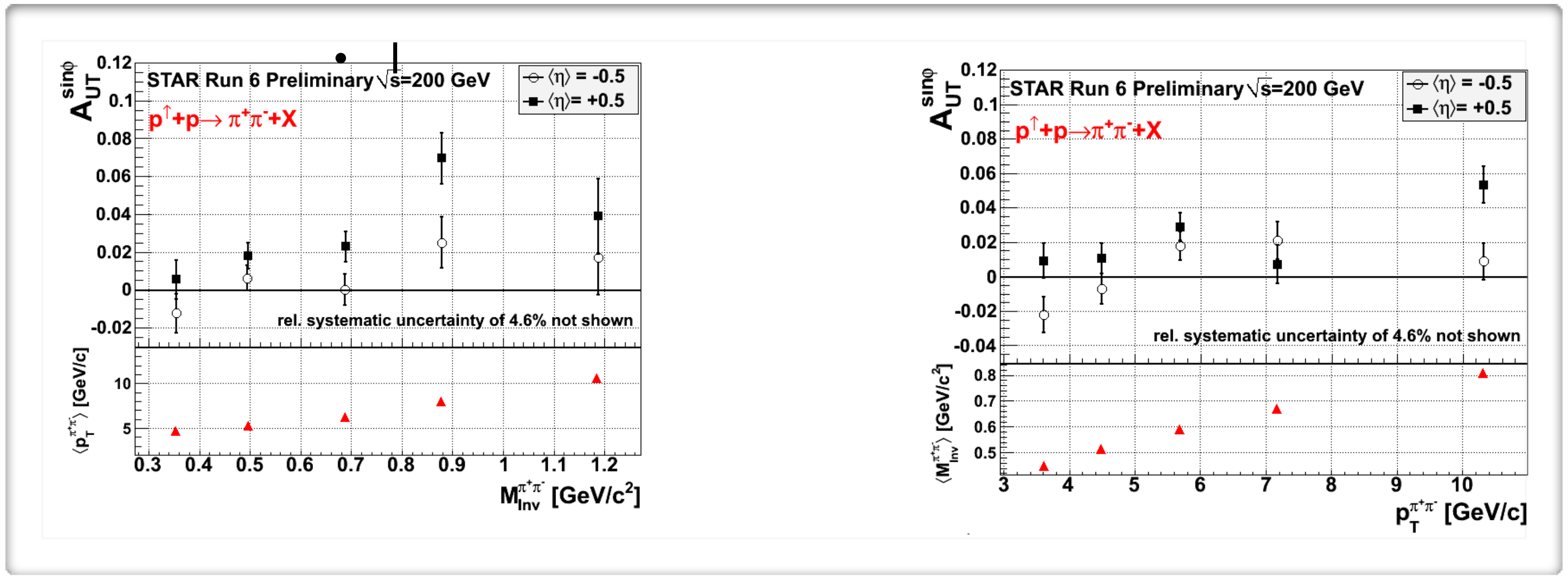


- $A_N^{\sin\Phi}$ asymmetry @ STAR



A. Vossen talk, Evolution workshop, 2012, JLab

What we know about DiFFs

First principles for D_1

Hadronization process:

$q \to \pi^+\pi^- X$

First principles for D_1

Hadronization process:

$q \rightarrow \pi^+\pi^- X$

Related to *MULTIPLICITIES*

see Nour Makke's talk

Nbr of events:

$$\sigma^U(e^+e^-) \equiv \frac{N}{lum} = 4\pi \frac{\alpha^2}{Q^2} \sum_{i=1}^{n_f} e_{q_n}^2$$

Nbr of pion pairs:

$$\sigma^U(e^+e^- \rightarrow \pi^+\pi^- + X) \equiv \frac{n^{\pi^+\pi^-}}{lum} = 4\pi \frac{\alpha^2}{Q^2} \sum_{i=1}^{n_f} e_{q_i}^2 \int dz dM_h \, D_1^{q\rightarrow\pi^+\pi^-}(z, M_h^2)$$

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One-hemisphere differential cross section:

$$\frac{d\sigma}{dz dM_h^2} = \frac{4\pi\alpha^2}{Q^2} \sum_{i=1}^{n_f} e_{q_i}^2 D_1^i(z, M_h^2)$$

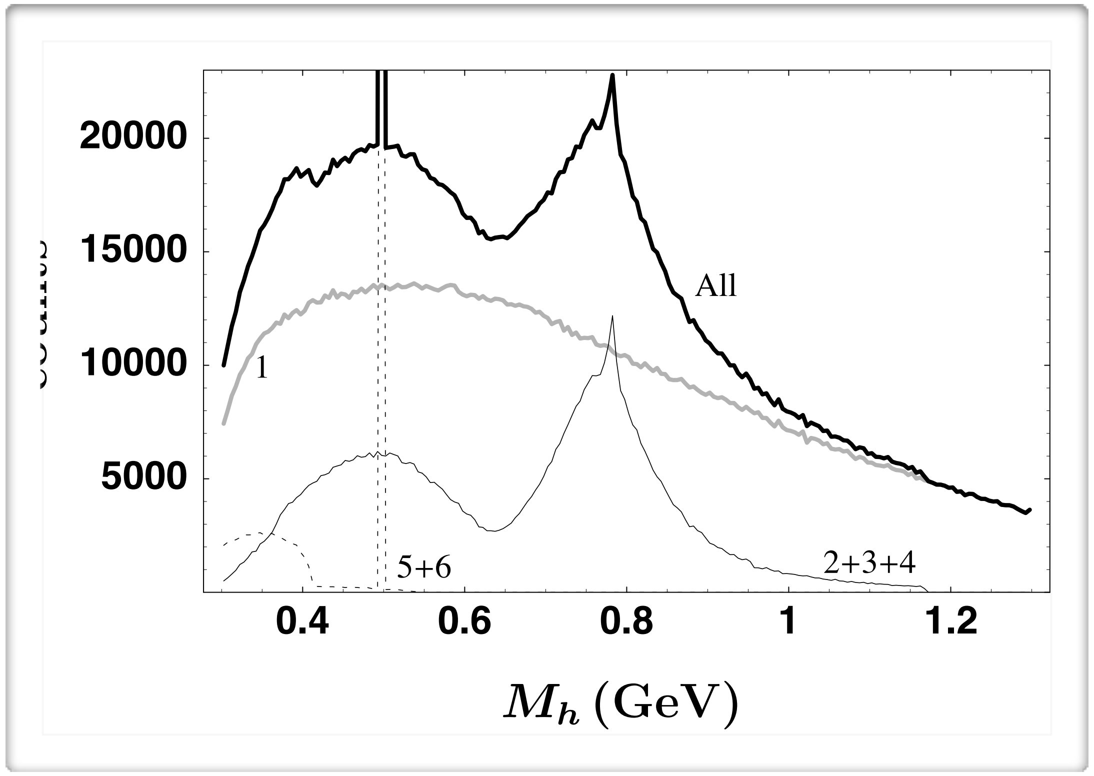
GOOD NEWS:

Bins in z & M_h related to nbr of pion pairs

⇓

Monte Carlo event generator

Spectator model for D_1



Bacchetta & Radici, PRD74

Most prominent channels at $M_h \lesssim 1.8$ GeV

1. Background:

$$q \to \pi^+ \pi^- X_1$$

2. ρ production: $M_h \sim m_\rho = 770$ MeV

$$q \to \rho X_2 \to \pi^+ \pi^- X_2$$

3. ω production: $M_h \sim m_\omega = 782$ MeV

$$q \to \omega X_3 \to \pi^+ \pi^- X_3$$

broad peak at $M_h \sim 500$ MeV

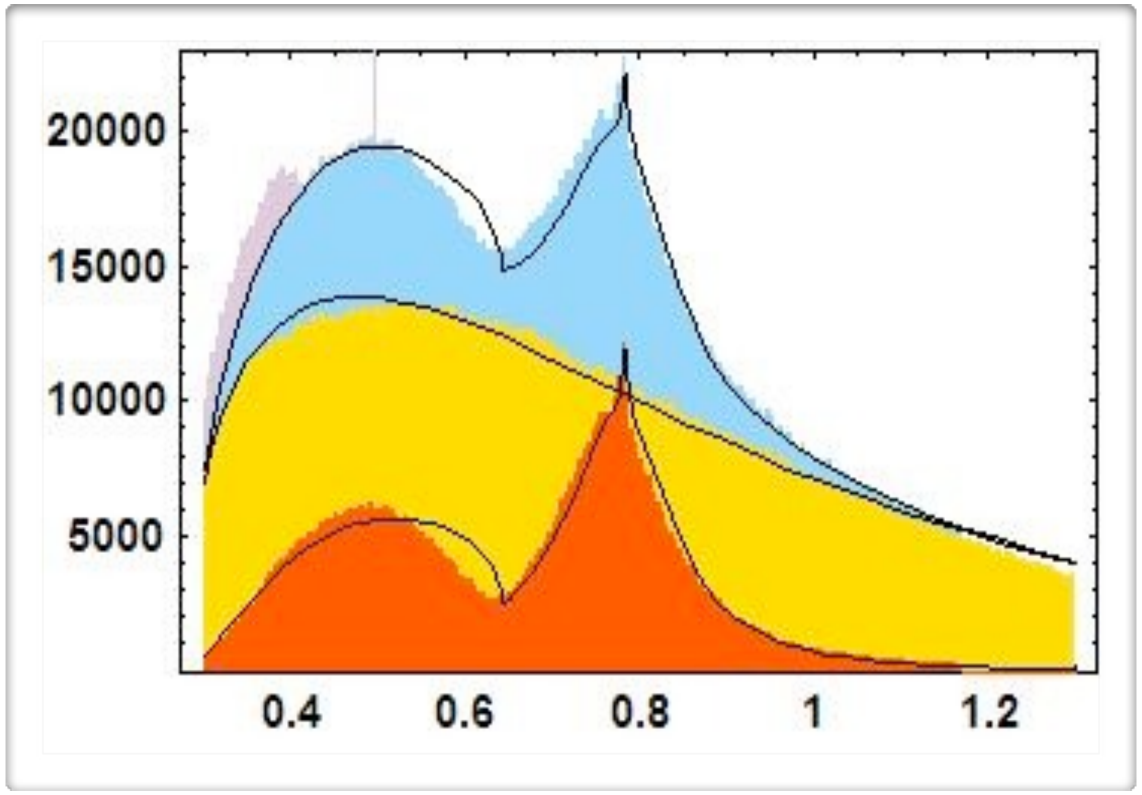
$$q \to \omega \pi^0 X_4' \to \pi^+ \pi^- \pi^0 X_4'$$

undetected π0

4. κ production: $M_h \sim m_K = 497$ MeV

$$q \to K^0 X_6 \to \pi^+ \pi^- X_6$$

Spectator model for D_1



Most prominent channels at $M_h \lesssim 1.8$ GeV

1. **Background:**

$$q \to \pi^+\pi^- X_1$$

2. **ρ production: M_h~m_ρ=770 MeV**

$$q \to \rho X_2 \to \pi^+\pi^- X_2$$

3. **ω production: M_h~m_ω=782 MeV**

$^- X_3$

00 MeV

$\pi^- \pi^0 X'_4$

ected π0

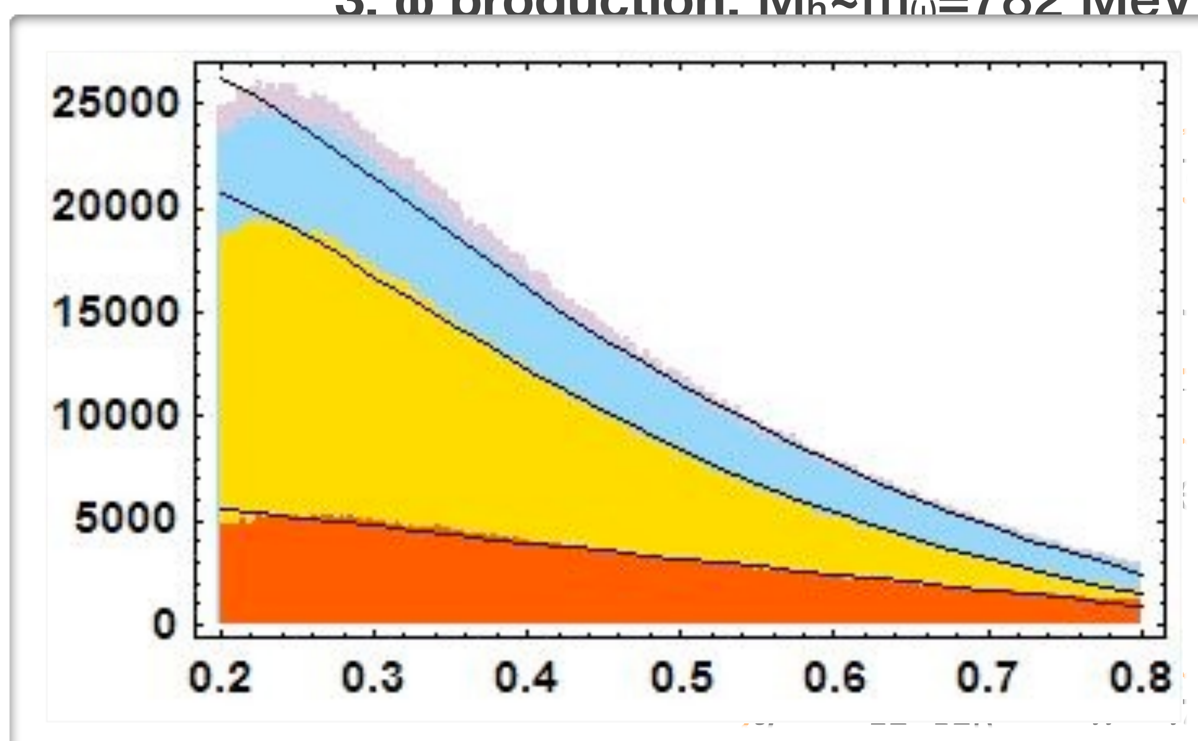
$^- X_6$

Spectator model

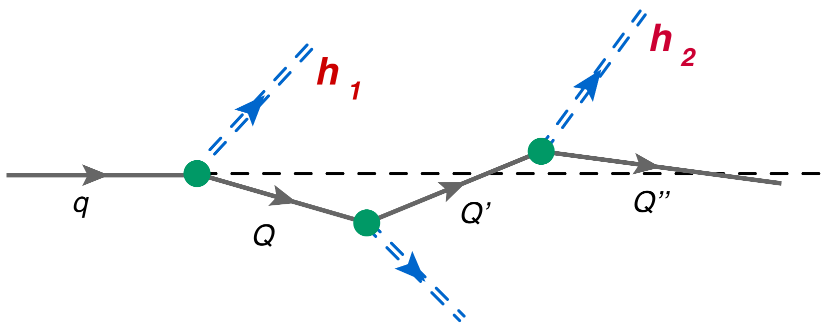
pair produced in relative s-wave

parameter tuned to PYTHIA output for HERMES

Bacchetta & Radici, PRD74

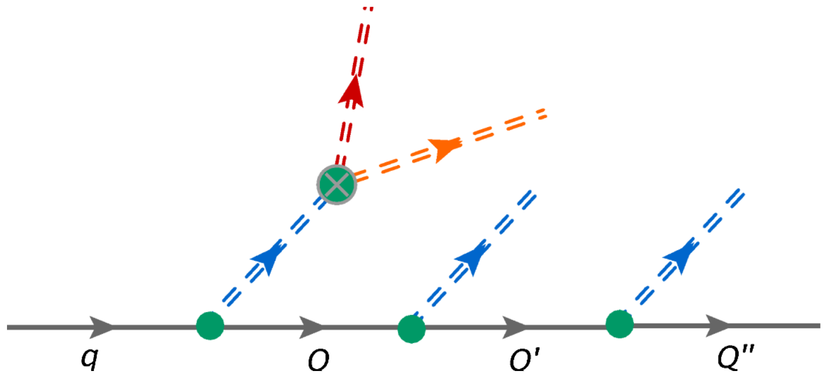


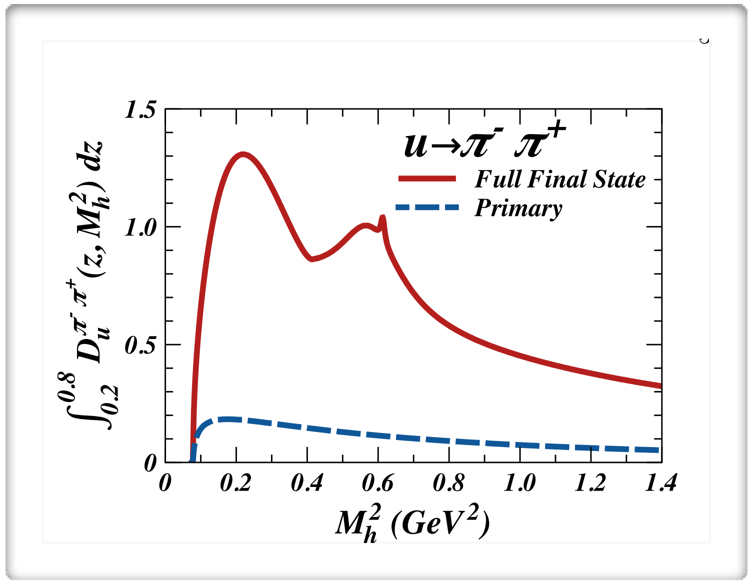
NJL-jet model for D_1

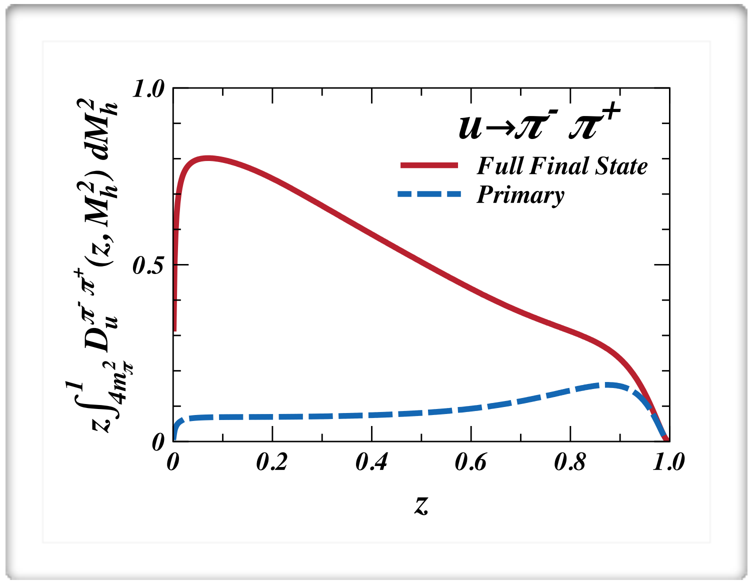


NJL vertices

Vector meson decays





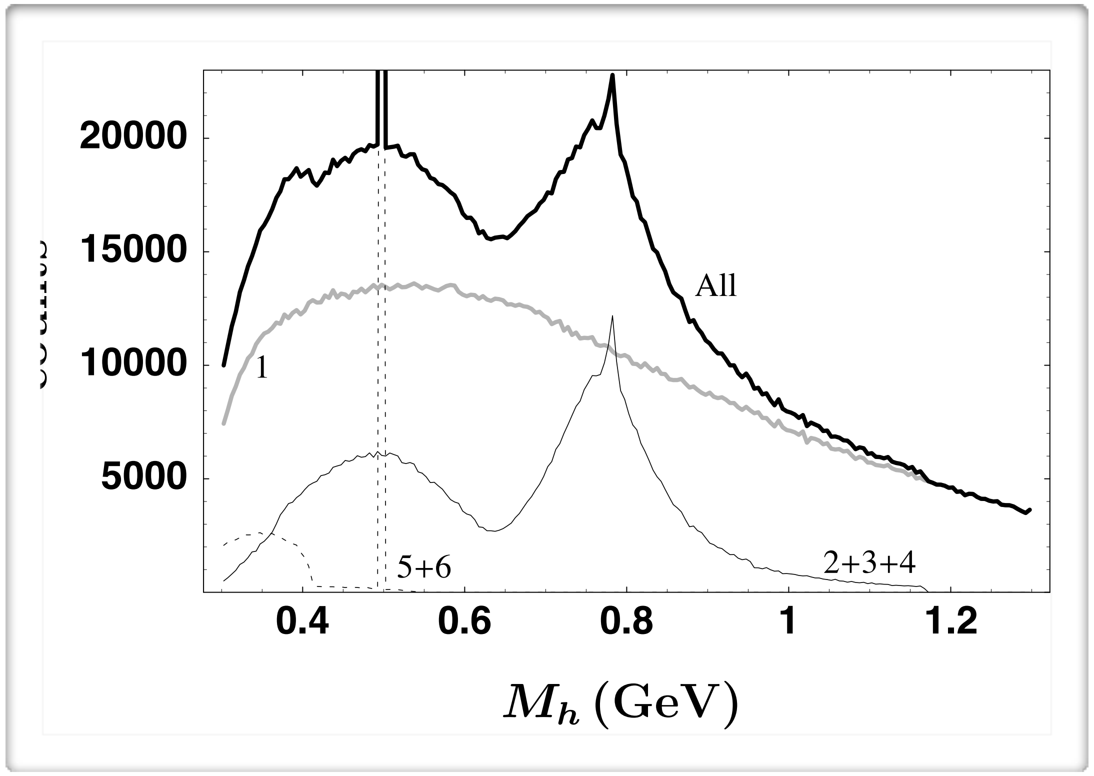


+ d, s

+ $\pi^{\pm}\pi^0$, πK

Casey et al., PRD85
Matevosyan et al., 1310.1917

Parameterization of D_1

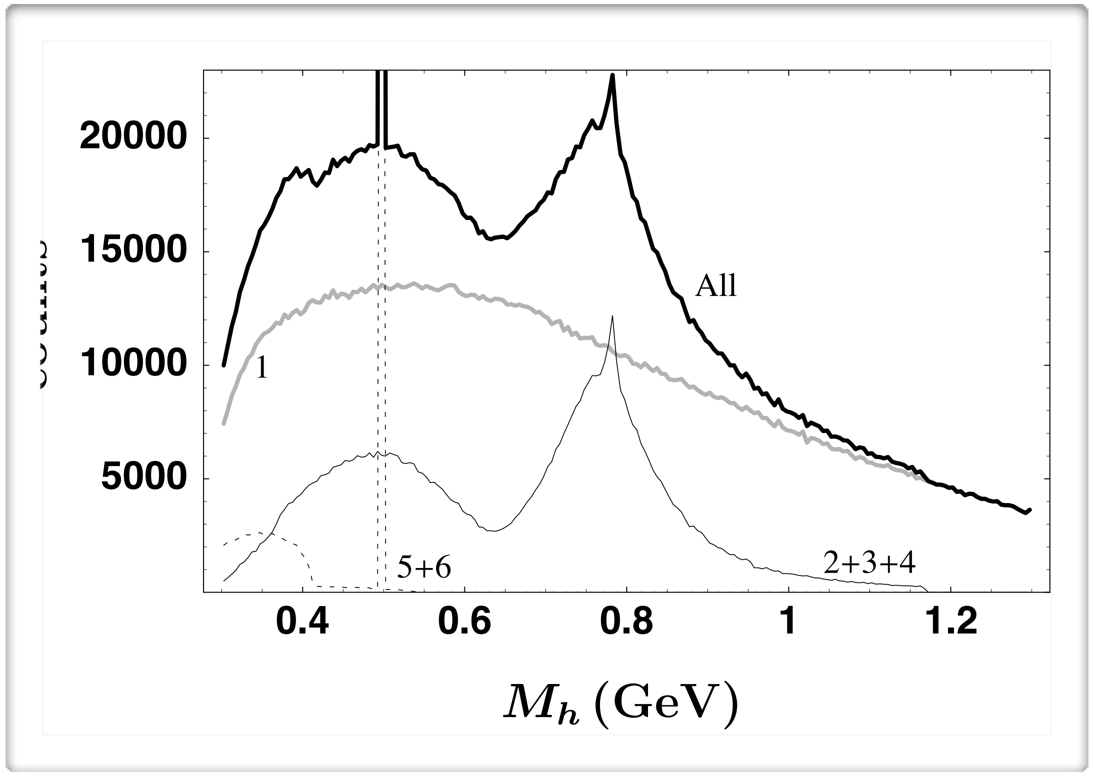


Use PYTHIA output as “data set”

Relevant variables and quantities: (z, M_h)

Standard fitting approach: Functional form in (z, M_h)

Parameterization of D_1



Use PYTHIA output as "data set"

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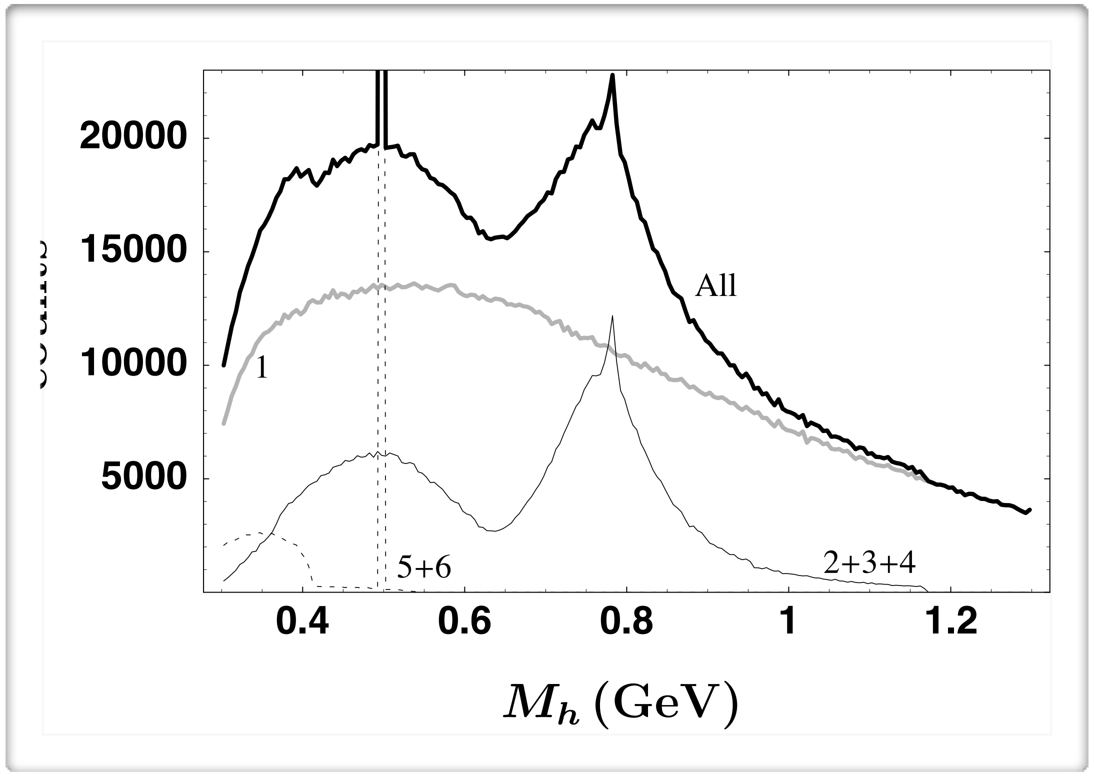
Scale dependence: Belle@100GeV^2 .... but SIDIS@ ~2.5GeV^2

Parameterization of D_1



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Relevant variables and quantities: (z, M_h)

Standard fitting approach: Functional form in (z, M_h)

Scale dependence: Belle@100GeV^2 .... but SIDIS@ ~2.5GeV^2

LO Evolution with unpol. splitting functions & HOPPET:

qq, gq, qg, gg

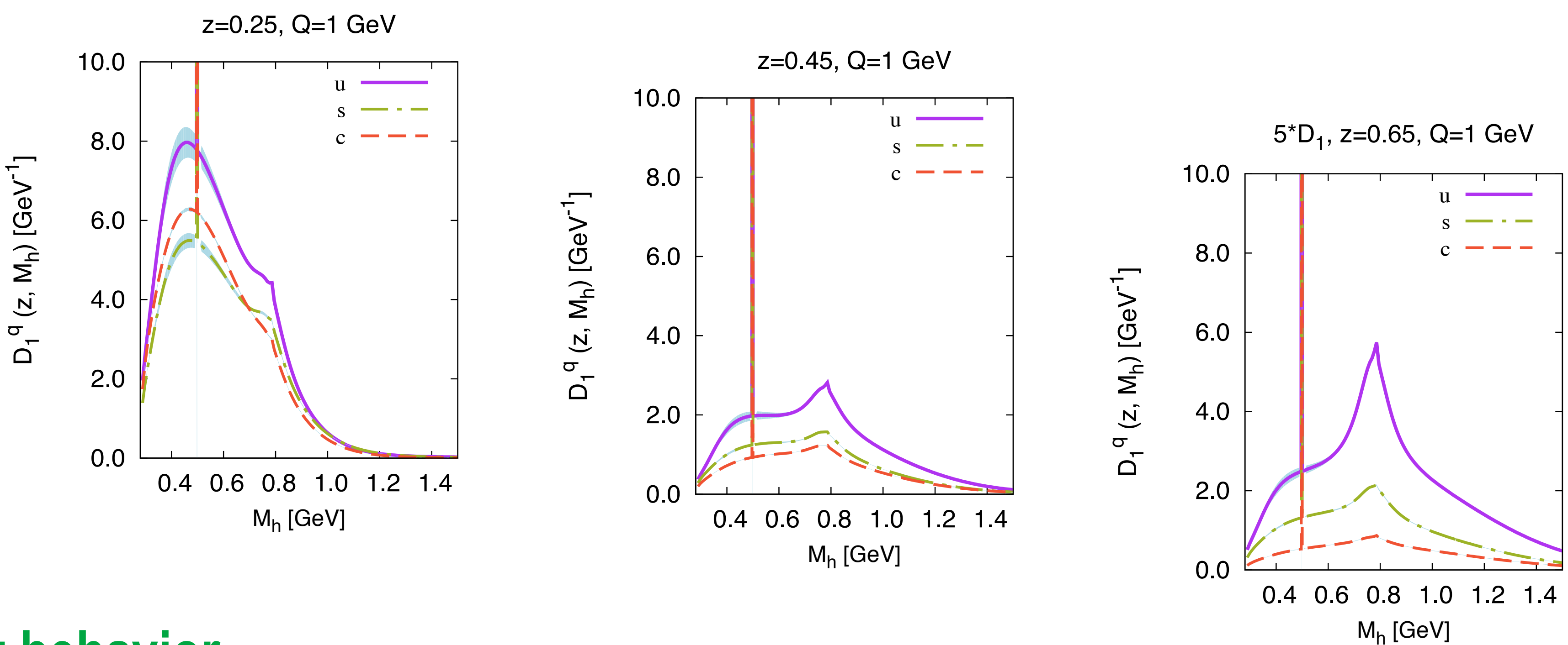
Ceccopieri, Radici & Bacchetta,,PLB 650 (2007) 81-89

1st step: forward evolution

Start fit @ 1GeV : Induce quark & gluon distristributions

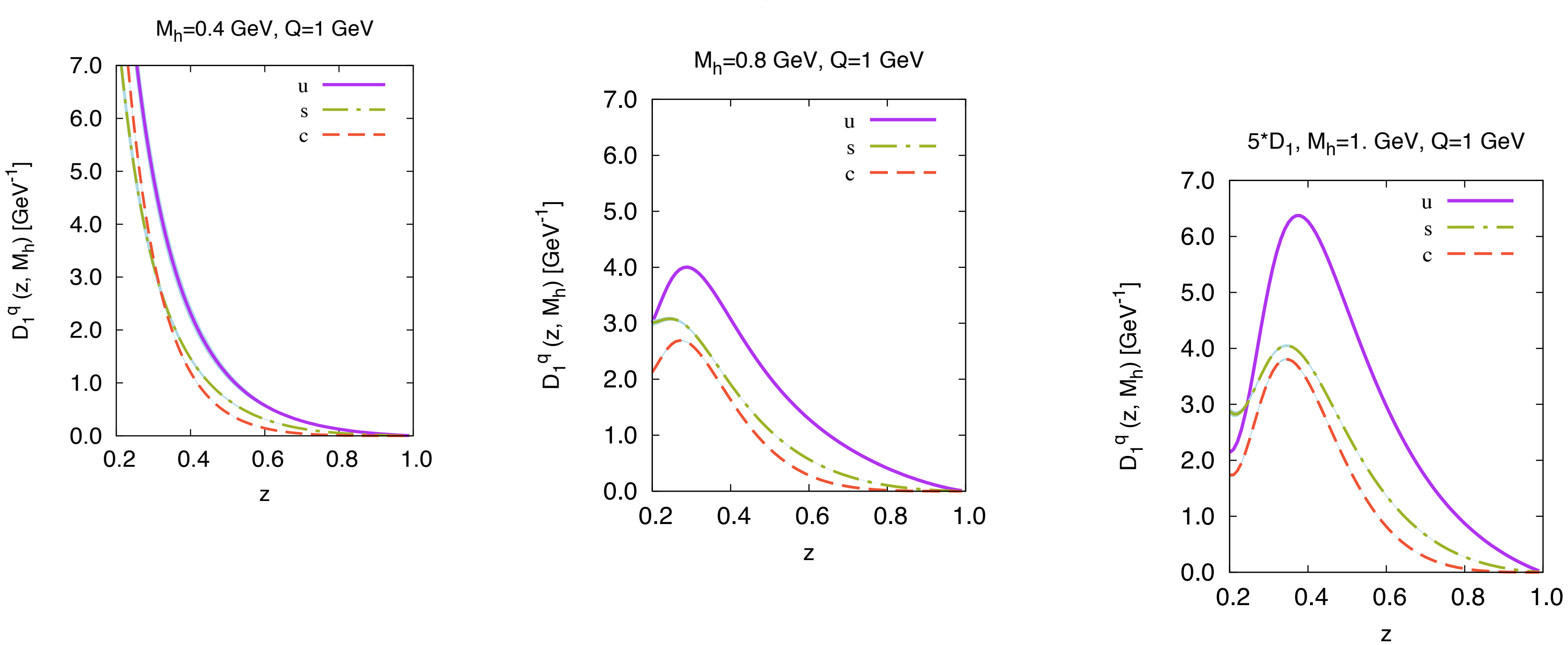
Parameterization of the unpolarized DiFF

M_h behavior



z behavior

error analysis with $\Delta\chi^2=1$



χ^2/dof

ρ: 1.28
ω: 1.68
K_S^0: 1.85
cont: 1.69

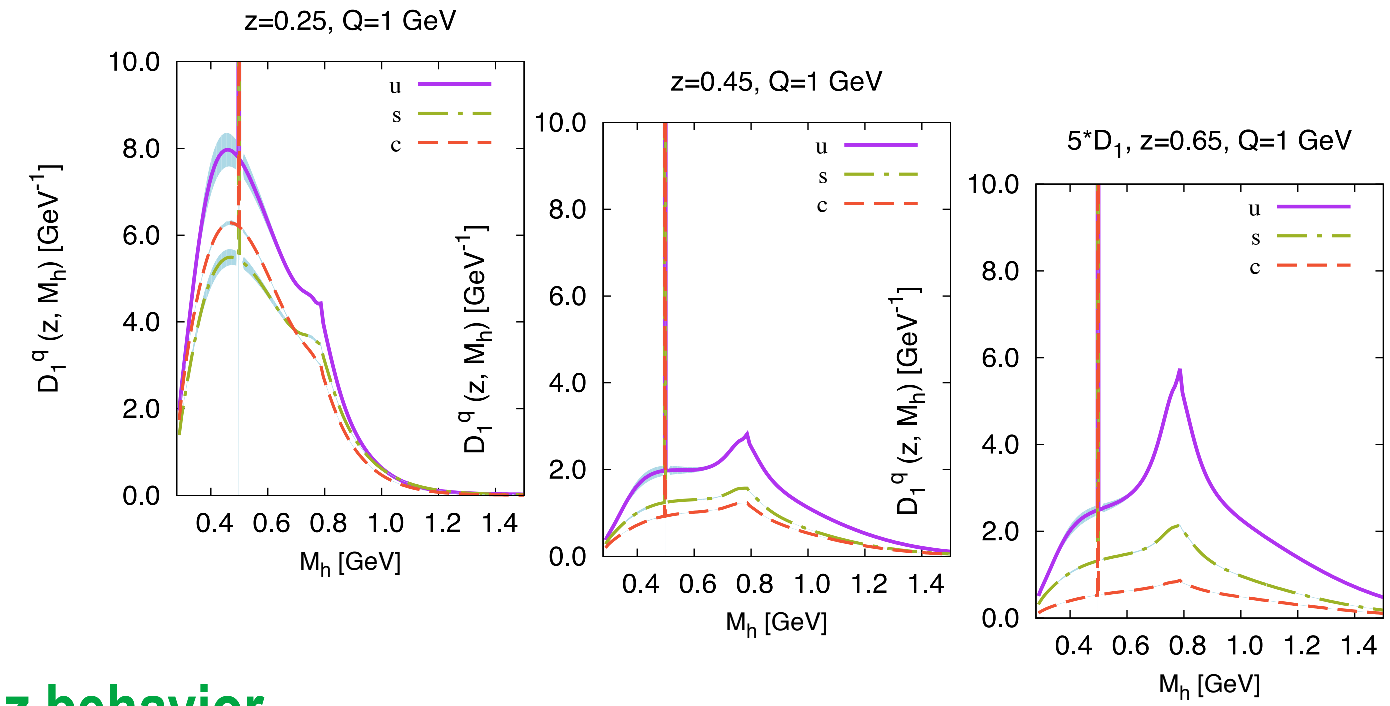
TOT: 1.69

A.C., Bacchetta, Radici & Bianconi, PRD85

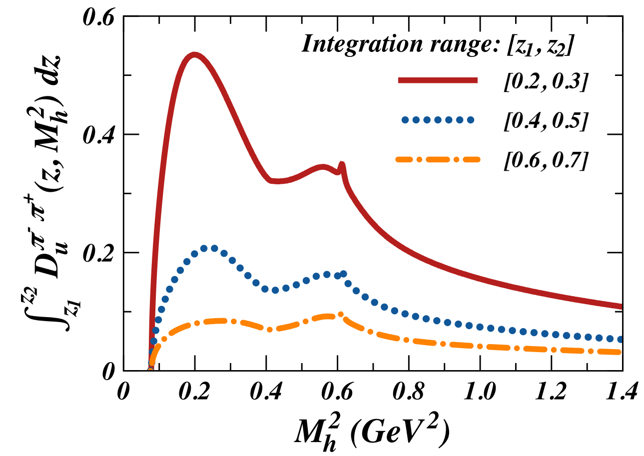
Parameterization of the unpolarized DiFF

M_h behavior

PYTHIA at Belle



NJL-jet based MC event generator



z behavior

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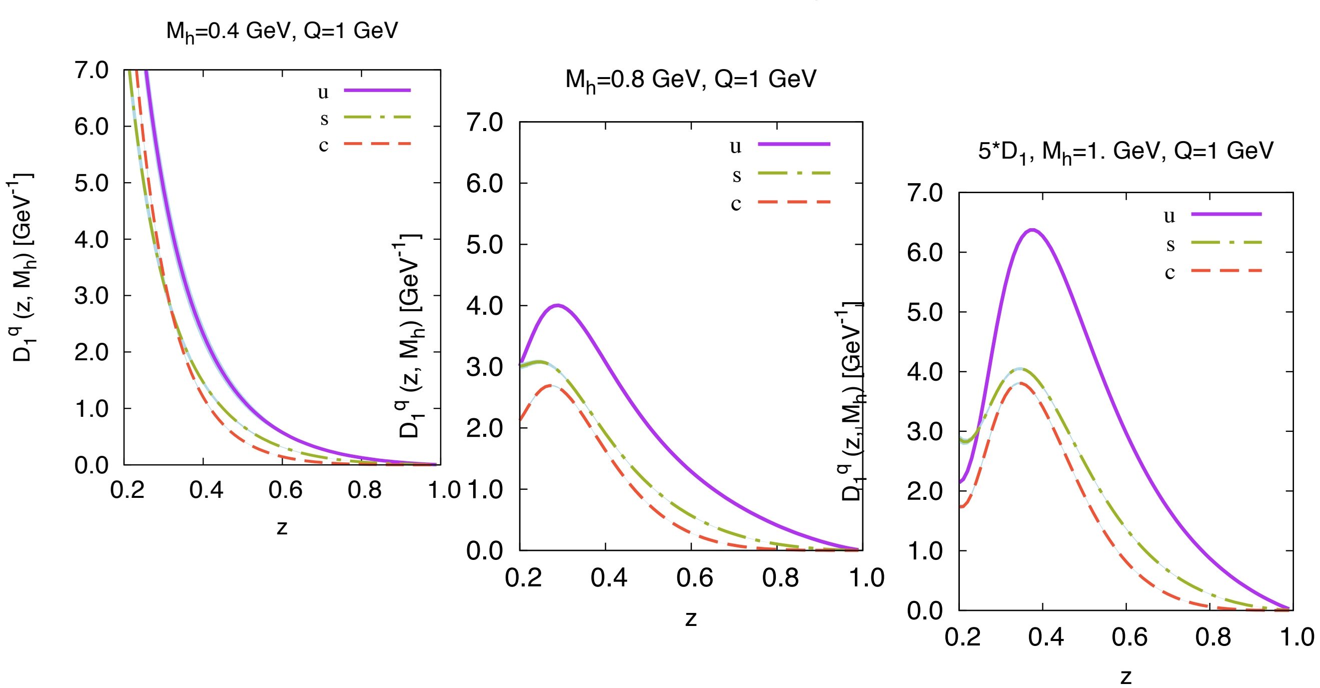
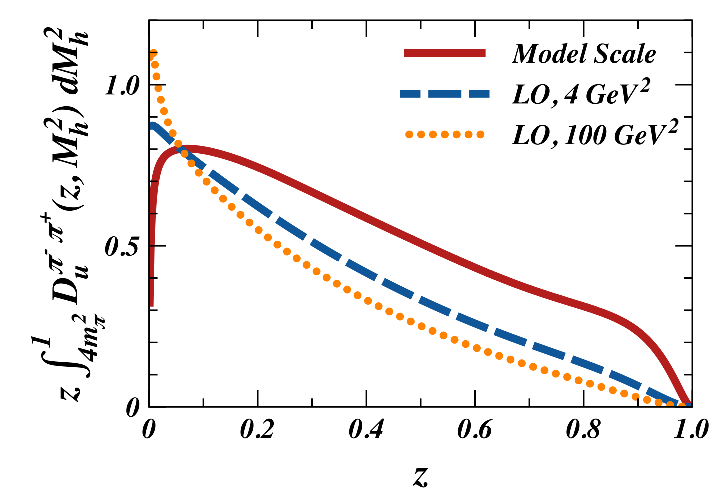
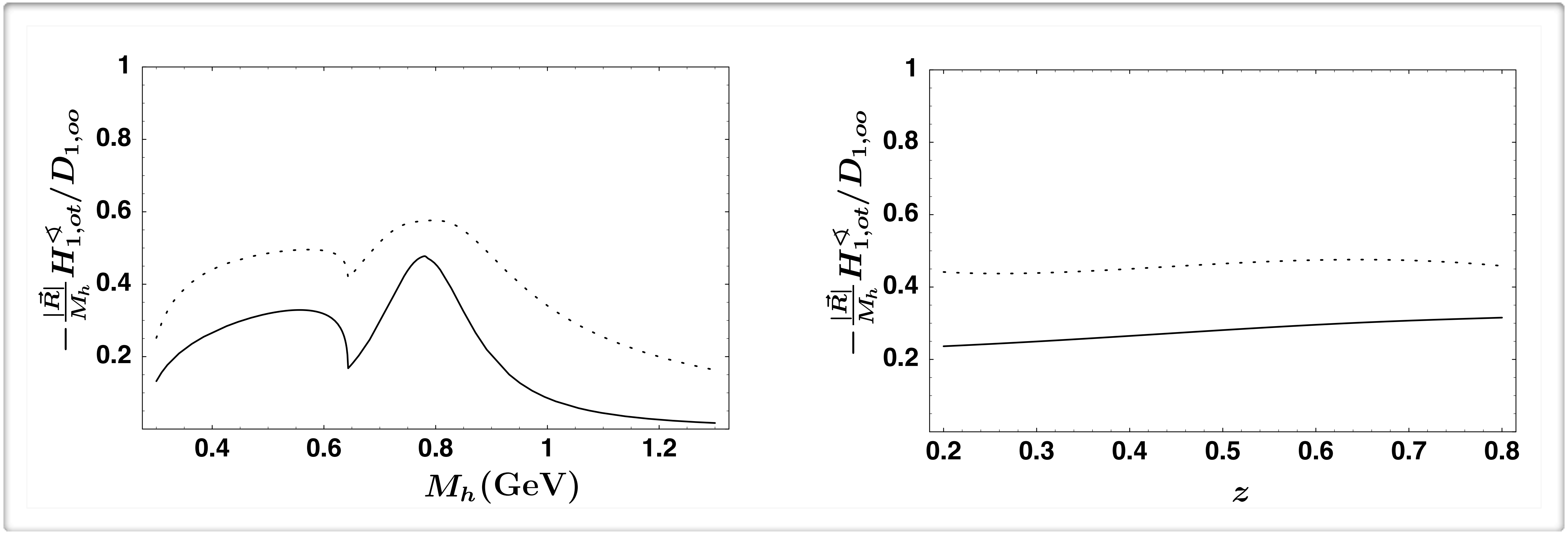




A.C., Bacchetta, Radici & Bianconi,PRD85

H. Matevosyan et al., 1310.1917

Spectator model for $H_1^{\sphericalangle}$



Spectator model for IFF

pair produced in relative p-wave

parameter tuned to PYTHIA output for HERMES

Bacchetta & Radici, PRD74

Parameterization of $H_1^{\sphericalangle}$

Artru-Collins asymmetry@ Belle

$$\frac{|\mathbf{R}|}{M_h} H_{1,sp}^{\sphericalangle u}(z, M_h; Q^2)\, n_u^{\uparrow}(Q^2) \equiv \frac{\langle 1+\cos^2\theta_2\rangle}{\langle \sin^2\theta_2\rangle}\, \frac{9}{5}\, \frac{1}{\langle \sin\theta\rangle\, \langle \sin\bar{\theta}\rangle} \times D(z, M_h; Q^2)\, A(z, M_h; Q^2)$$

we just got them!

LO Evolution with chiral-odd splitting functions & HOPPET:

qq,gg

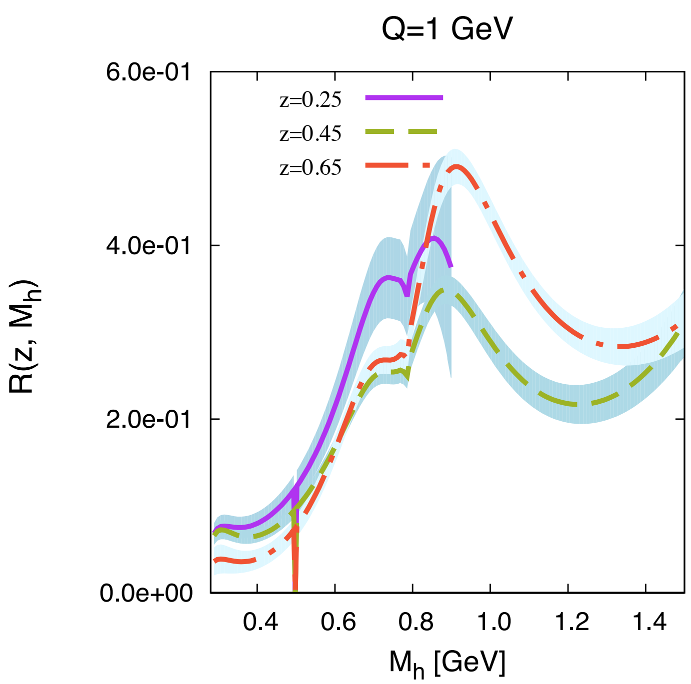
1st step: forward evolution

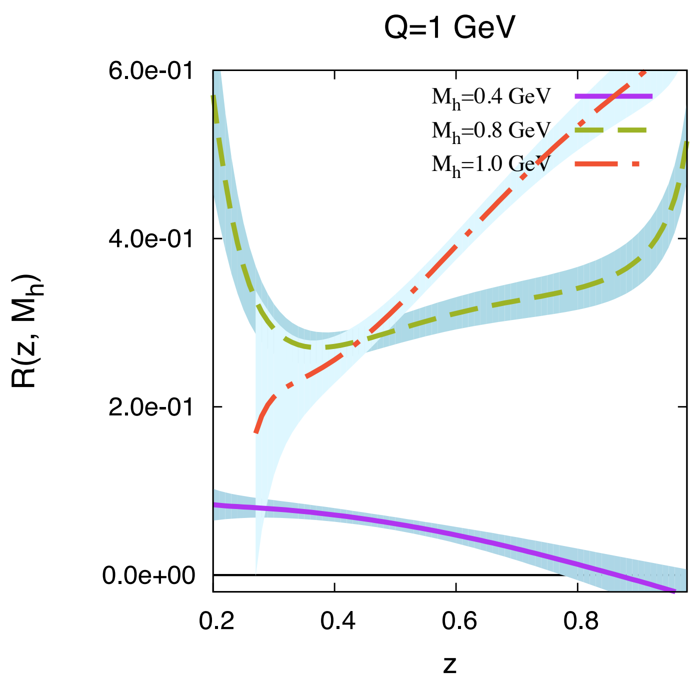
... just like the unpol. case

+ integration over bin ranges

Start fit @ 1GeV : Induce quark distristributions

Parameterization of $H_1^{\sphericalangle}$



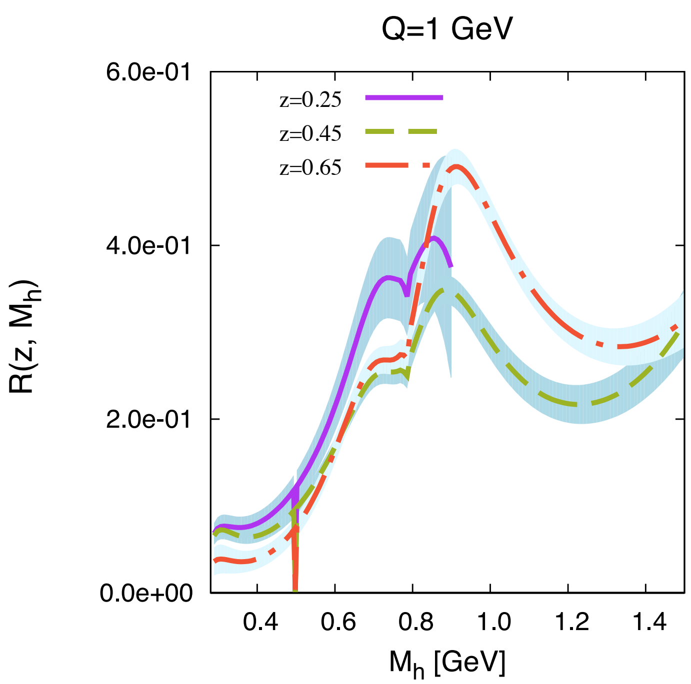


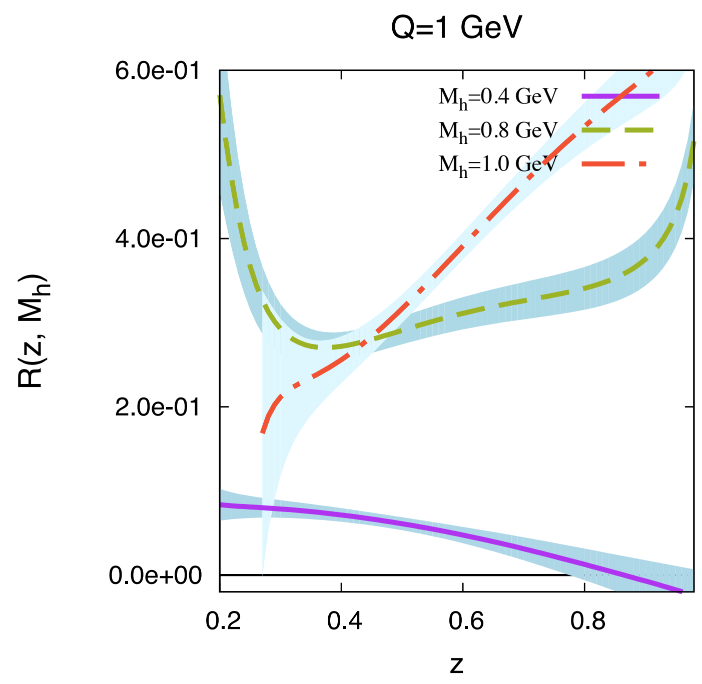
Phenomenological cut $\gamma_h \equiv \dfrac{2M_h}{zQ} \ll 1$

χ^2/dof=0.57

A.C., Bacchetta, Radici & Bianconi,PRD85

Parameterization of $H_1^{\sphericalangle}$





Phenomenological cut $\gamma_h \equiv \frac{2M_h}{zQ} \ll 1$

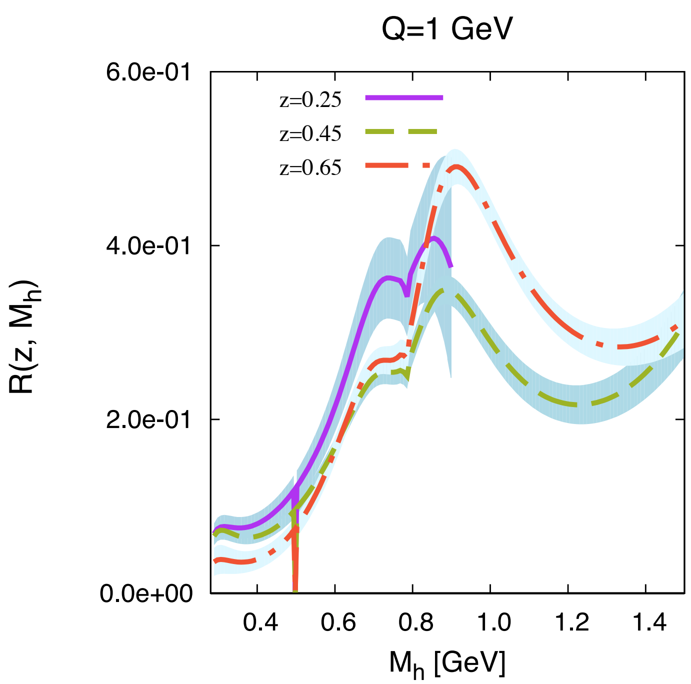
χ^2/dof=0.57

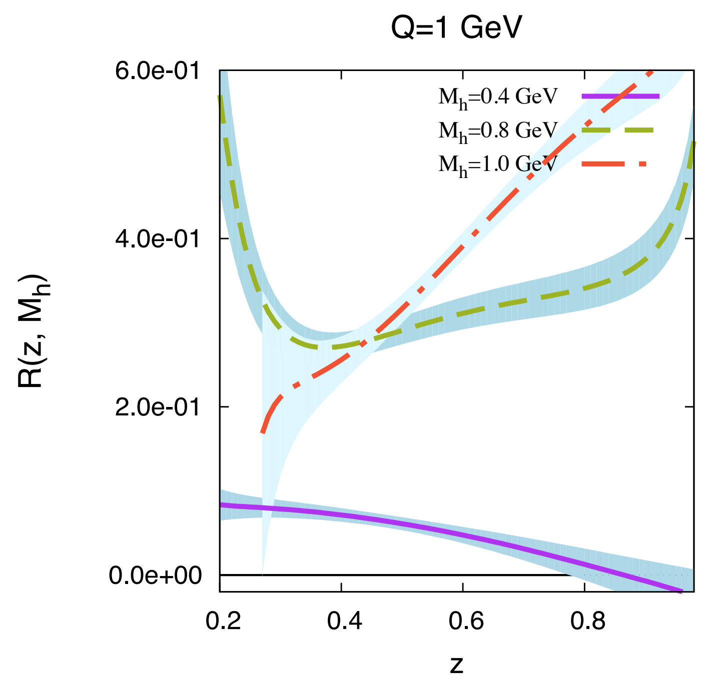
Evolution effects : **From Belle's scale to HERMES and COMPASS'scale**

→ needs analytical expression and gluon DiFF → fits → **[A.C., Bacchetta, Radici & Bianconi, PRD85]**

A.C., Bacchetta, Radici & Bianconi,PRD85

Parameterization of $H_1^{\sphericalangle}$





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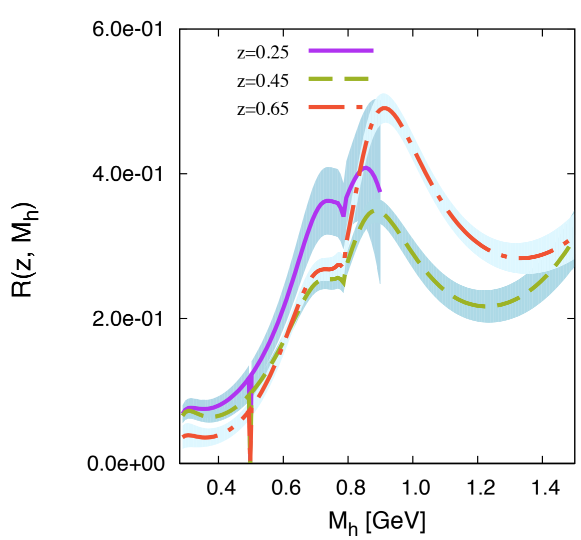
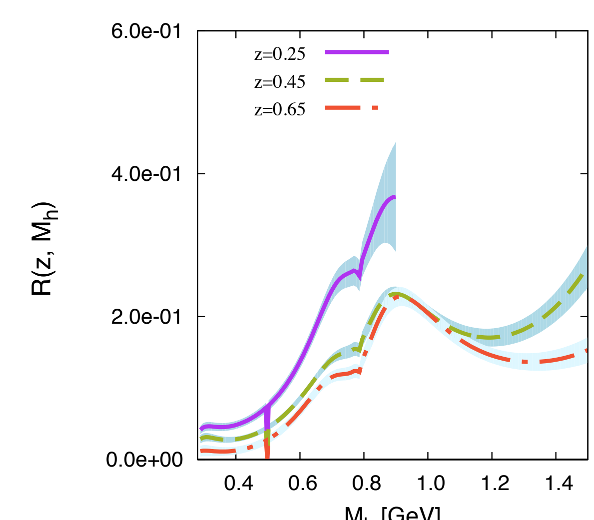
Evolution effects : **From Belle's scale to HERMES and COMPASS'scale**

→ needs analytical expression and gluon DiFF → fits → **[A.C., Bacchetta, Radici & Bianconi, PRD85]**

Q^2=1GeV^2

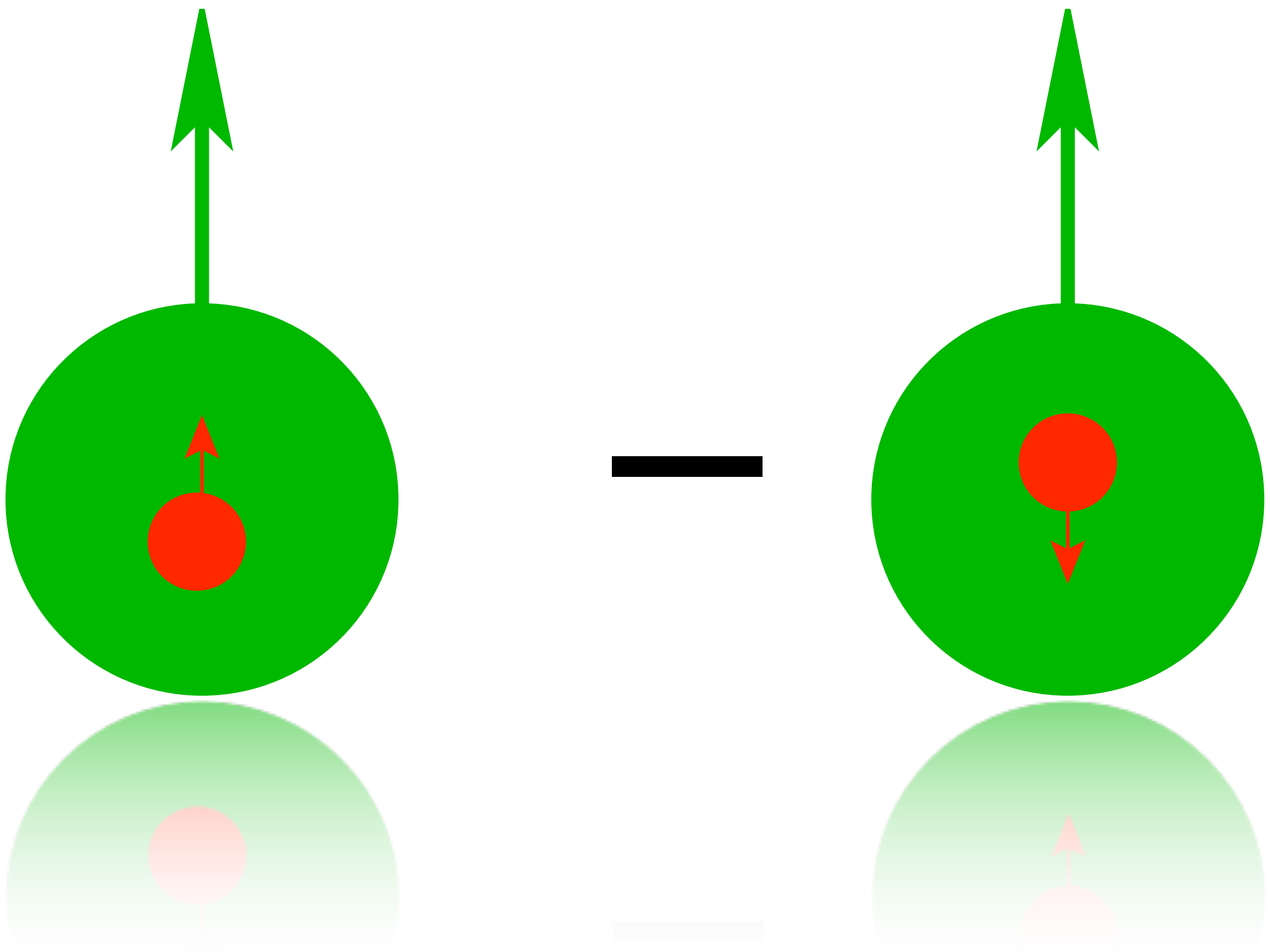


Q^2=100GeV^2



only $H_1^{<}$ evolution on plot

A.C., Bacchetta, Radici & Bianconi,PRD85



The very first success

Transversity at SIDIS

$$A_{\mathrm{DIS}}(x,Q^2) = -C_y \frac{\sum_q e_q^2\, h_1^q(x,Q^2)\, n_q^{\uparrow}(Q^2)}{\sum_q e_q^2\, f_1^q(x,Q^2)\, n_q(Q^2)}$$

Using symmetries for DiFFs:

$$H_1^{\sphericalangle,\,u} = -H_1^{\sphericalangle,\,d} = -\overline{H}_1^{\sphericalangle,\,u} = \overline{H}_1^{\sphericalangle,\,d}$$

$$D_1^u = D_1^d = \overline{D}_1^u = \overline{D}_1^d\,,$$
$$D_1^s = \overline{D}_1^s\,, \quad D_1^c = \overline{D}_1^c$$

Proton

$$x h_1^{u_v}(x,Q^2) - \tfrac{1}{4}\, x h_1^{d_v}(x,Q^2) \propto -A_{\mathrm{DIS}}(x,Q^2)\, \frac{n_u(Q^2)}{n_u^{\uparrow}(Q^2)} \sum_{q=u,d,s} \frac{e_q^2}{e_u^2}\, x f_1^{q+\bar{q}}(x,Q^2)$$

Deuteron

$$x h_1^{u_v}(x,Q^2) + x h_1^{d_v}(x,Q^2) \propto -\frac{5}{3} A_{\mathrm{DIS}}(x,Q^2)\, \frac{n_u(Q^2)}{n_u^{\uparrow}(Q^2)}\, x \left(f_1^{u+\bar{u}} + f_1^{d+\bar{d}} + \frac{2}{5} f_1^{s+\bar{s}} \right)$$

and combinations of both ...

Transversity at SIDIS

$$A_{\mathrm{DIS}}(x,Q^2) = -C_y \frac{\sum_q e_q^2\, h_1^q(x,Q^2)\, n_q^{\uparrow}(Q^2)}{\sum_q e_q^2\, f_1^q(x,Q^2)\, n_q(Q^2)}$$

Using symmetries for DiFFs:

$$H_1^{\sphericalangle,\,u} = -H_1^{\sphericalangle,\,d} = -\overline{H}_1^{\sphericalangle,\,u} = \overline{H}_1^{\sphericalangle,\,d}$$

$$D_1^u = D_1^d = \overline{D}_1^u = \overline{D}_1^d \,;$$
$$D_1^s = \overline{D}_1^s\,, \quad D_1^c = \overline{D}_1^c$$

Proton

$$x h_1^{u_v}(x,Q^2) - \tfrac{1}{4}\, x h_1^{d_v}(x,Q^2) \propto -A_{\mathrm{DIS}}(x,Q^2)\, \frac{n_u(Q^2)}{n_u^{\uparrow}(Q^2)} \sum_{q=u,d,s} \frac{e_q^2}{e_u^2}\, x f_1^{q+\bar{q}}(x,Q^2)$$

Deuteron

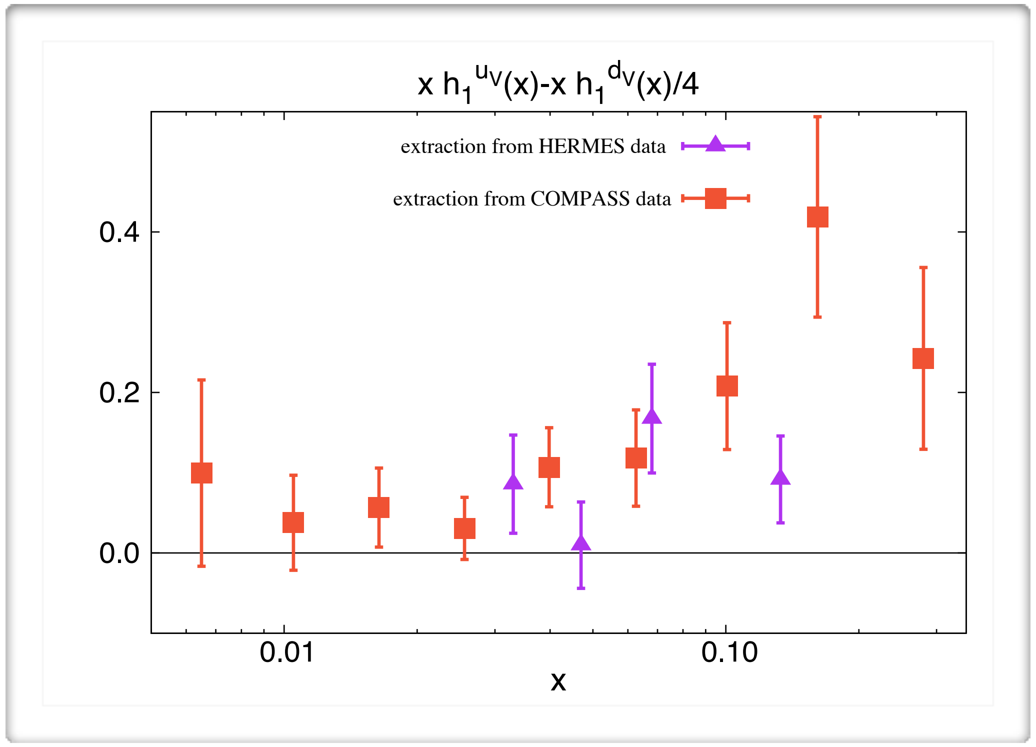
$$x h_1^{u_v}(x,Q^2) + x h_1^{d_v}(x,Q^2) \propto -\frac{5}{3} A_{\mathrm{DIS}}(x,Q^2)\, \frac{n_u(Q^2)}{n_u^{\uparrow}(Q^2)}\, x \left(f_1^{u+\bar{u}} + f_1^{d+\bar{d}} + \frac{2}{5} f_1^{s+\bar{s}} \right)$$

and combinations of both …

We take results for our analysis from pion pair production in e^+e^- annihilation at Belle

Point-by-point transversity

- from HERMES data
- DiFF analysis
  point by point from fit
- [Bacchetta, A.C., Radici, PRL 107]



- from COMPASS data
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- [Bacchetta, A.C., Radici, JHEP 1303]

Point-by-point transversity

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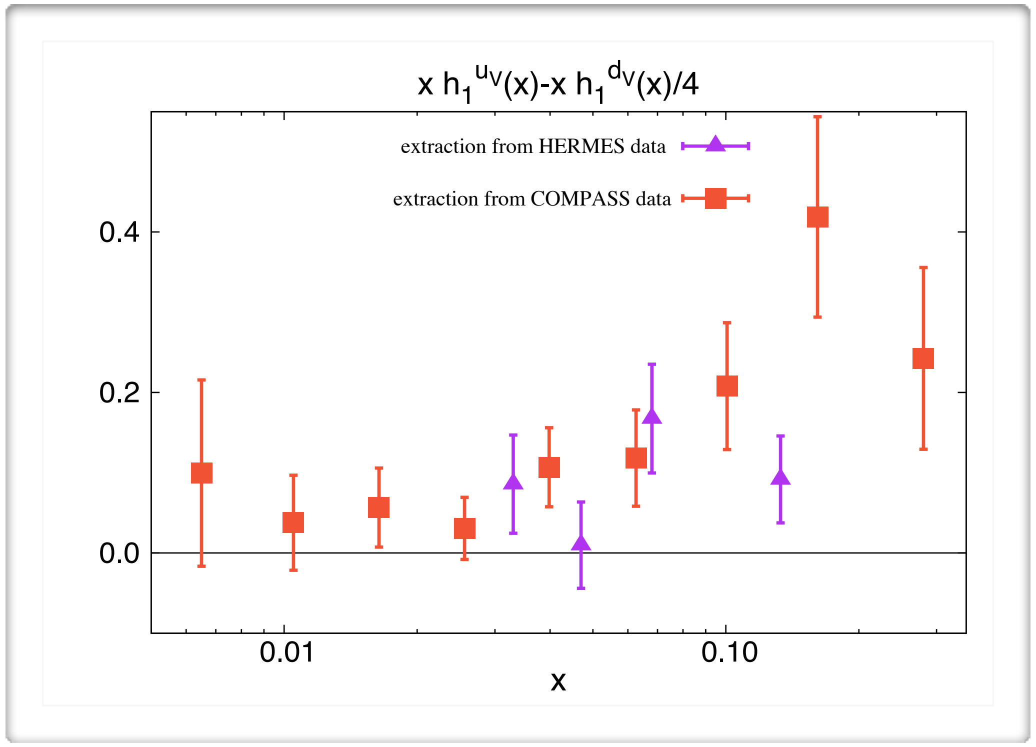

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Ok, but not of practical use!

Point-by-point transversity

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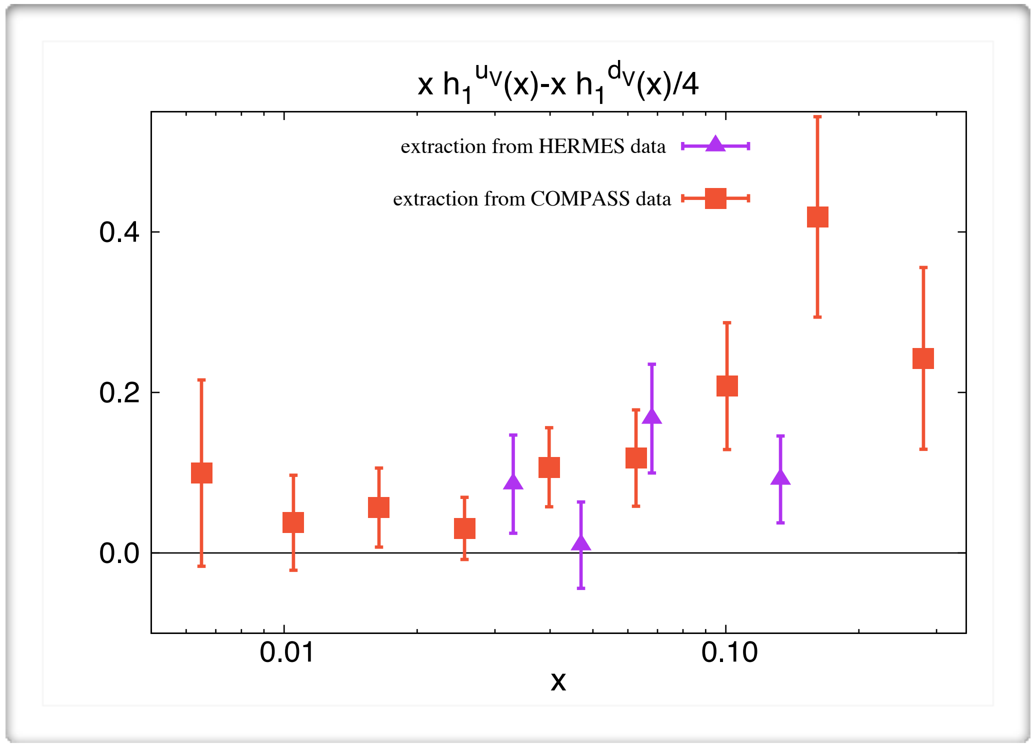

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→ Fit of valence transversity

Point-by-point transversity

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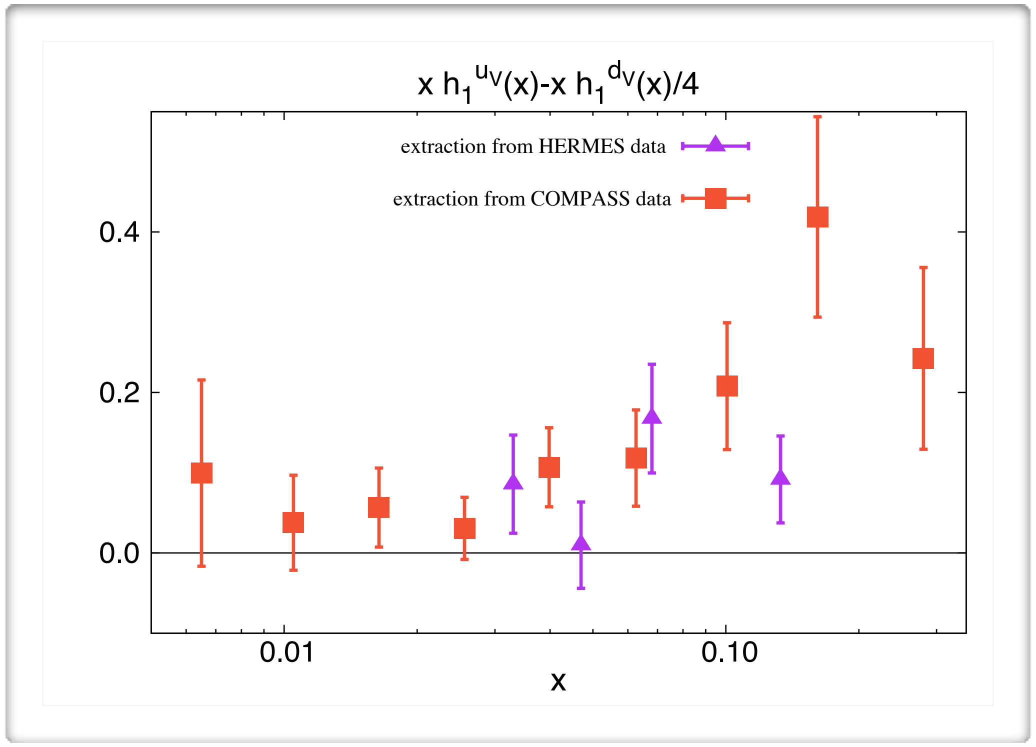
  point by point from fit
- [Bacchetta, A.C., Radici, PRL 107]



- from COMPASS data
- DiFF analysis
  point by point from fit
- [Bacchetta, A.C., Radici, JHEP 1303]

Ok, but not of practical use!

→ Fit of valence transversity

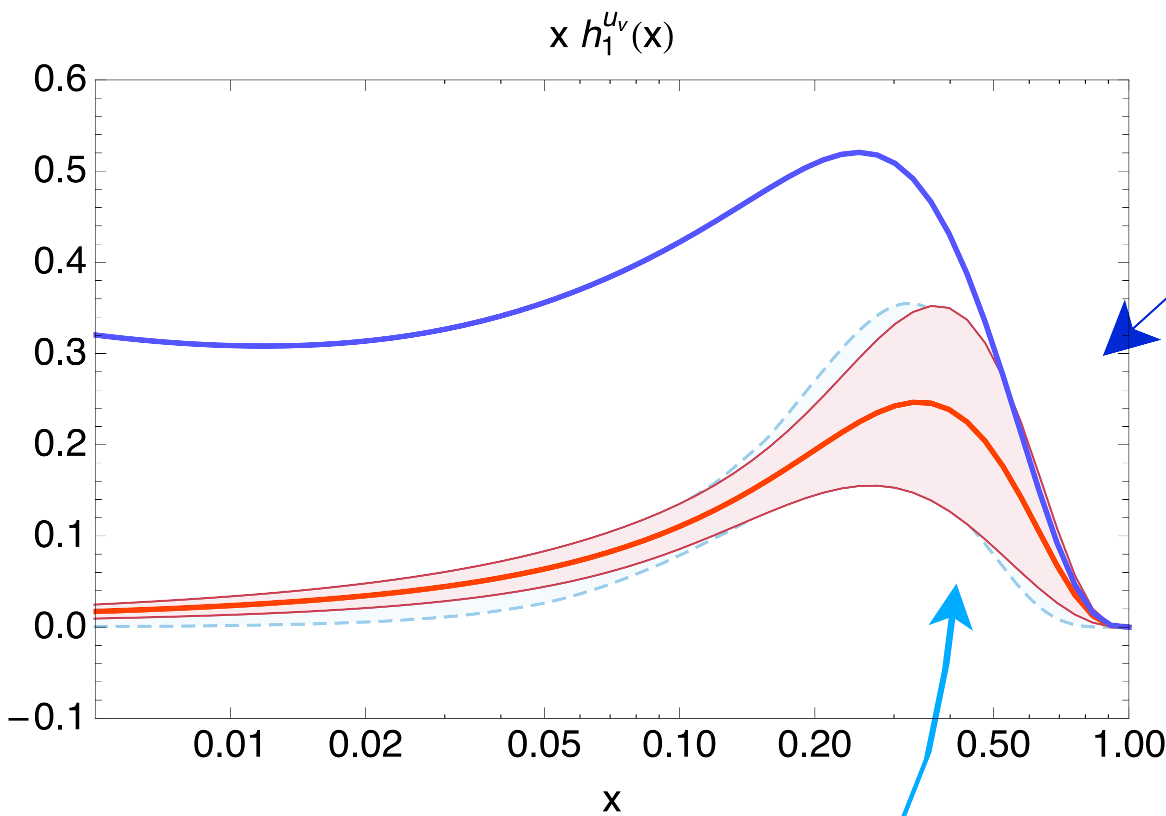
Constraints from first principles

✦ Soffer bound

$$2|h_1^q(x,Q^2)| \leq |f_1^q(x,Q^2) + g_1^q(x,Q^2)| \equiv 2\,\mathrm{SB}^q(x,Q^2)$$

✦ $h_1(x=1)=0$; the parton model predicts $h_1(x=0)=0$ but too restrictive in QCD

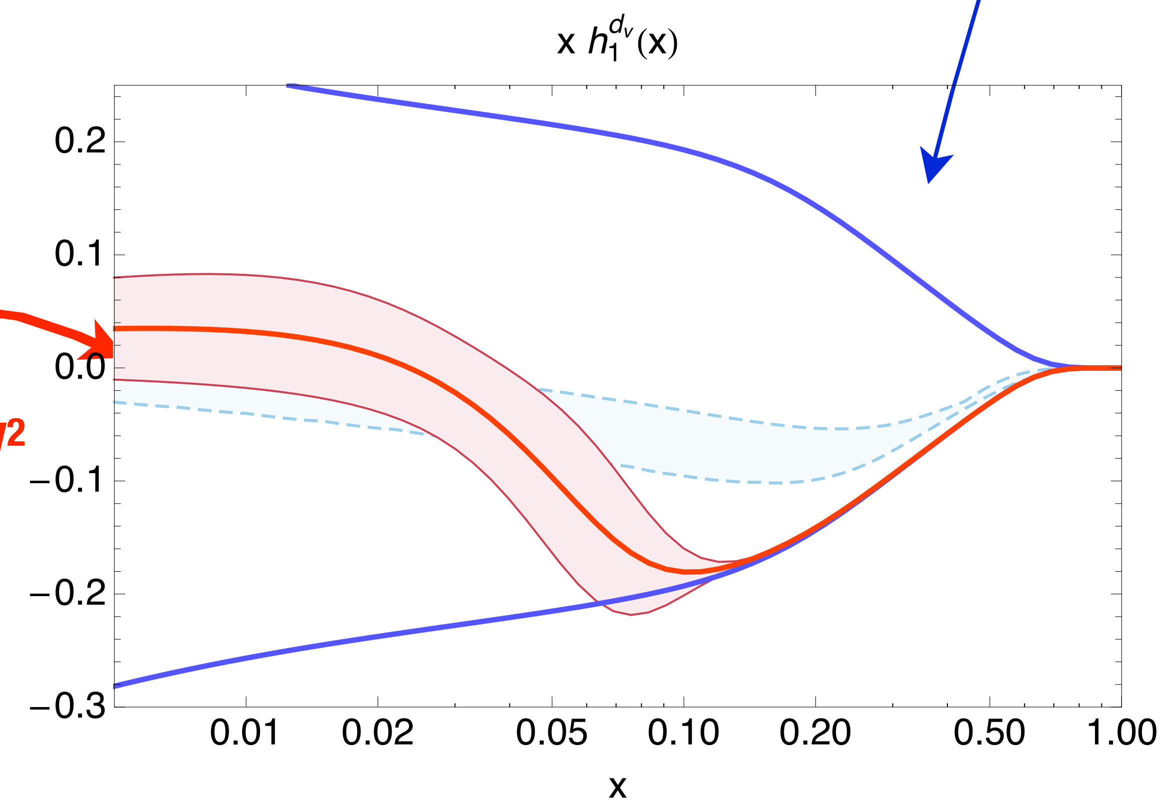
Collinear extraction of h_1



Soffer Bound @ 2.4 GeV^2

Band: Torino 2009 transversity

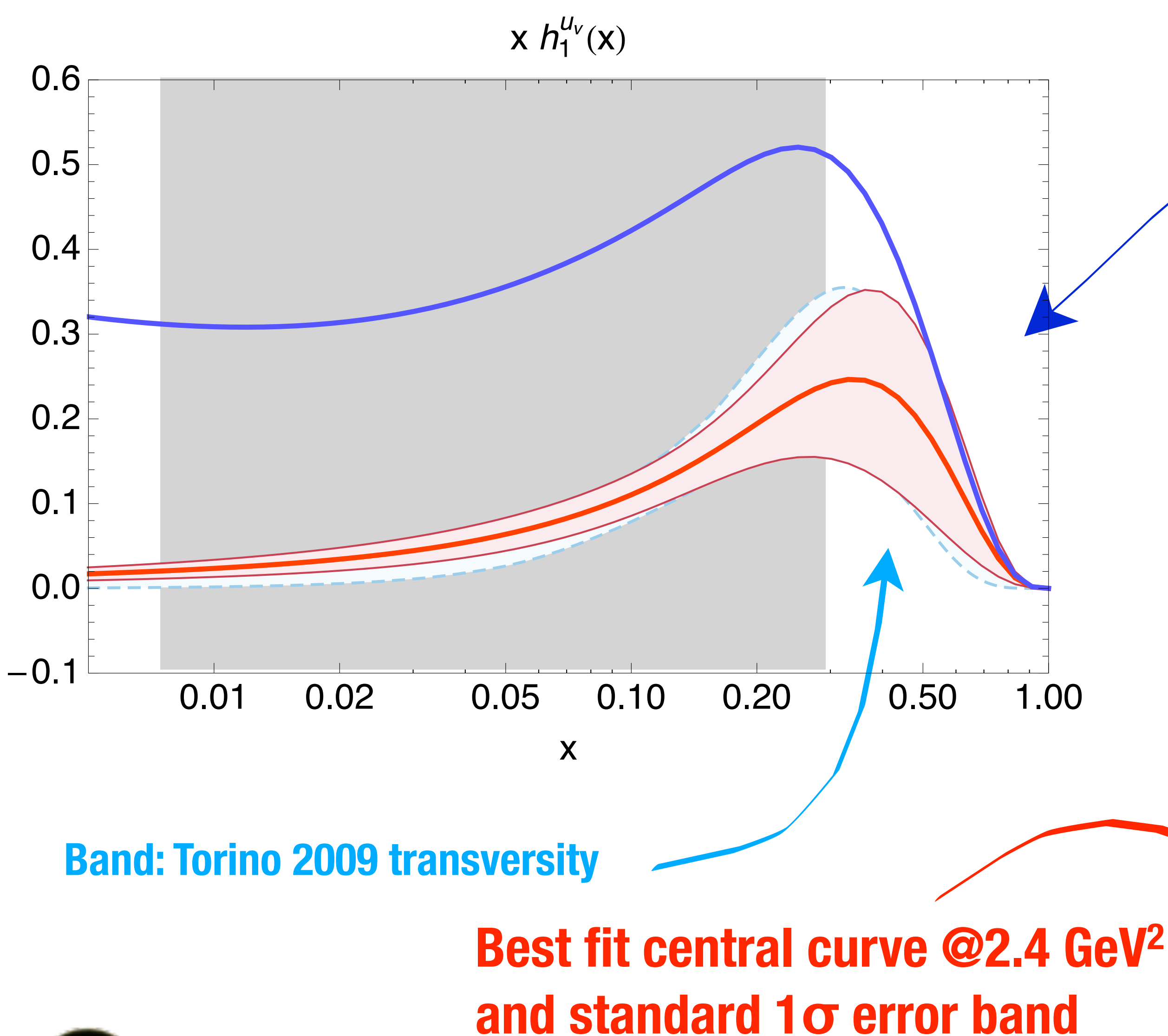
Best fit central curve @2.4 GeV^2
and standard 1σ error band

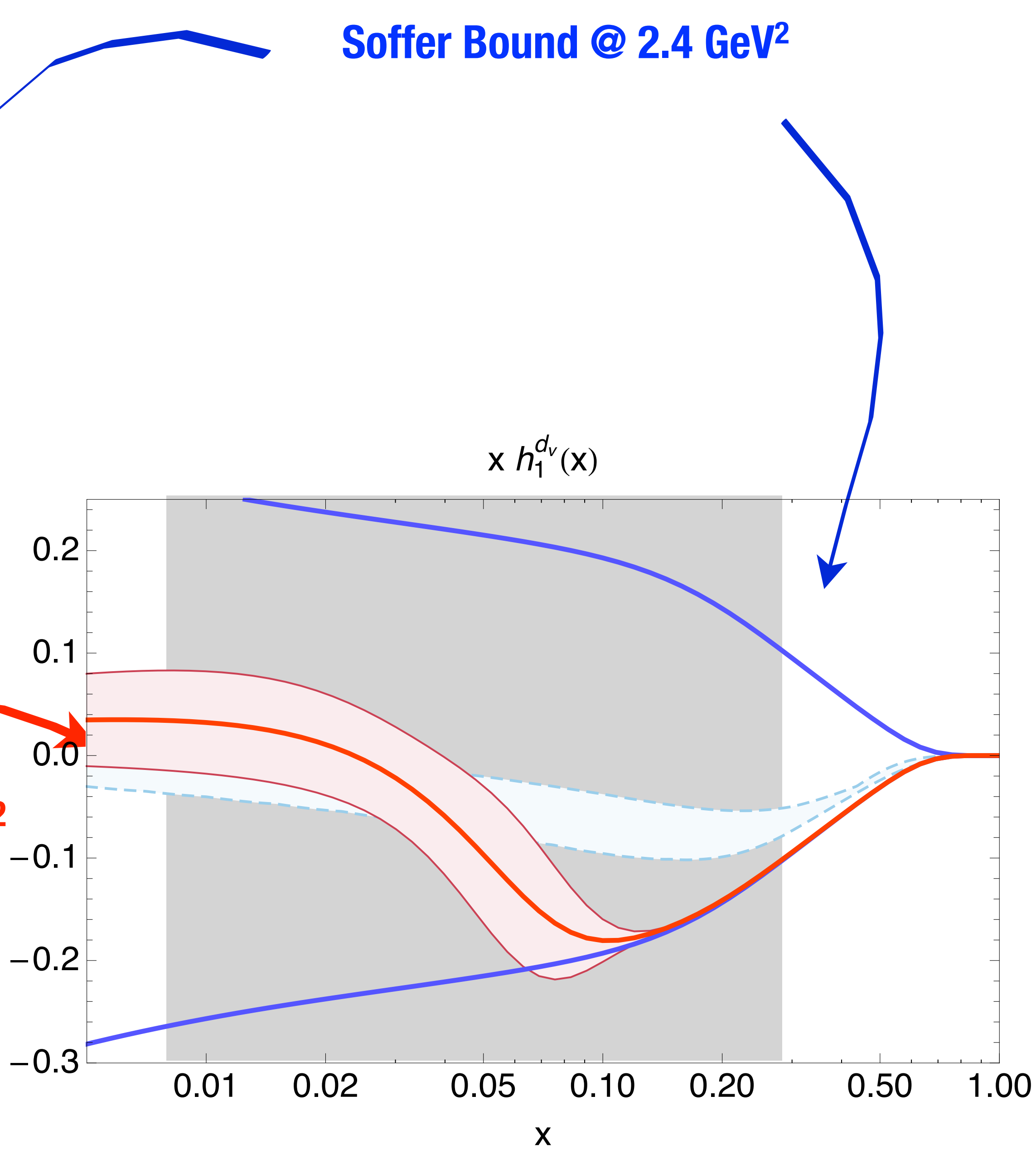


Rigid version

Bacchetta,A.C.& Radici, JHEP1303

Collinear extraction of h_1

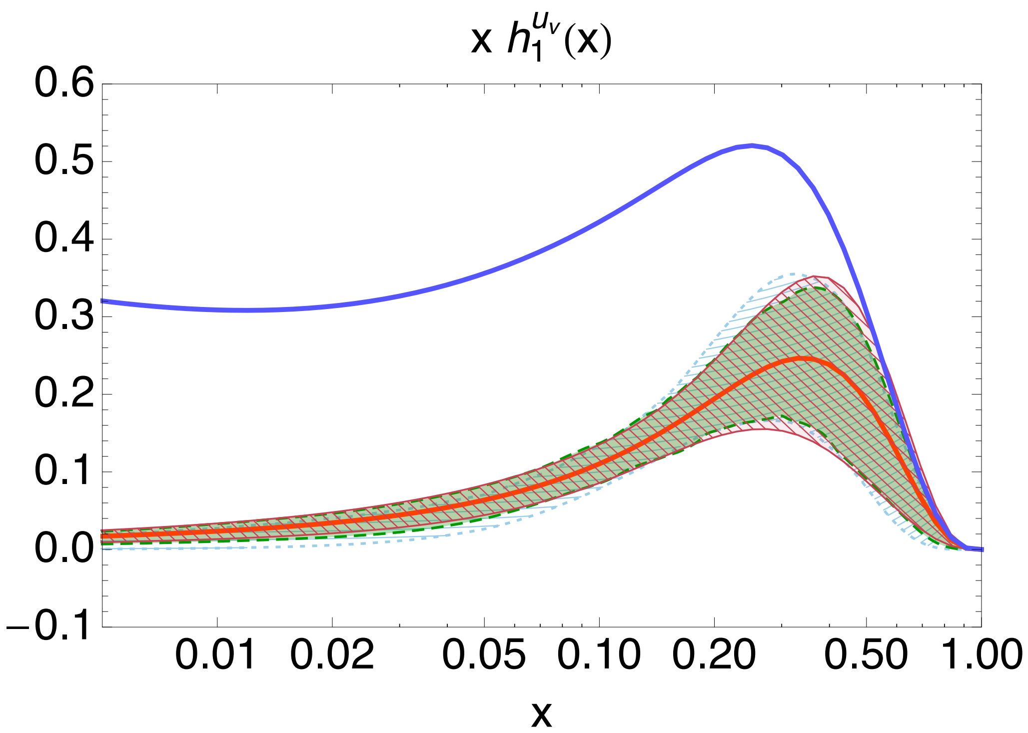




Rigid version

Bacchetta,A.C.& Radici, JHEP1303

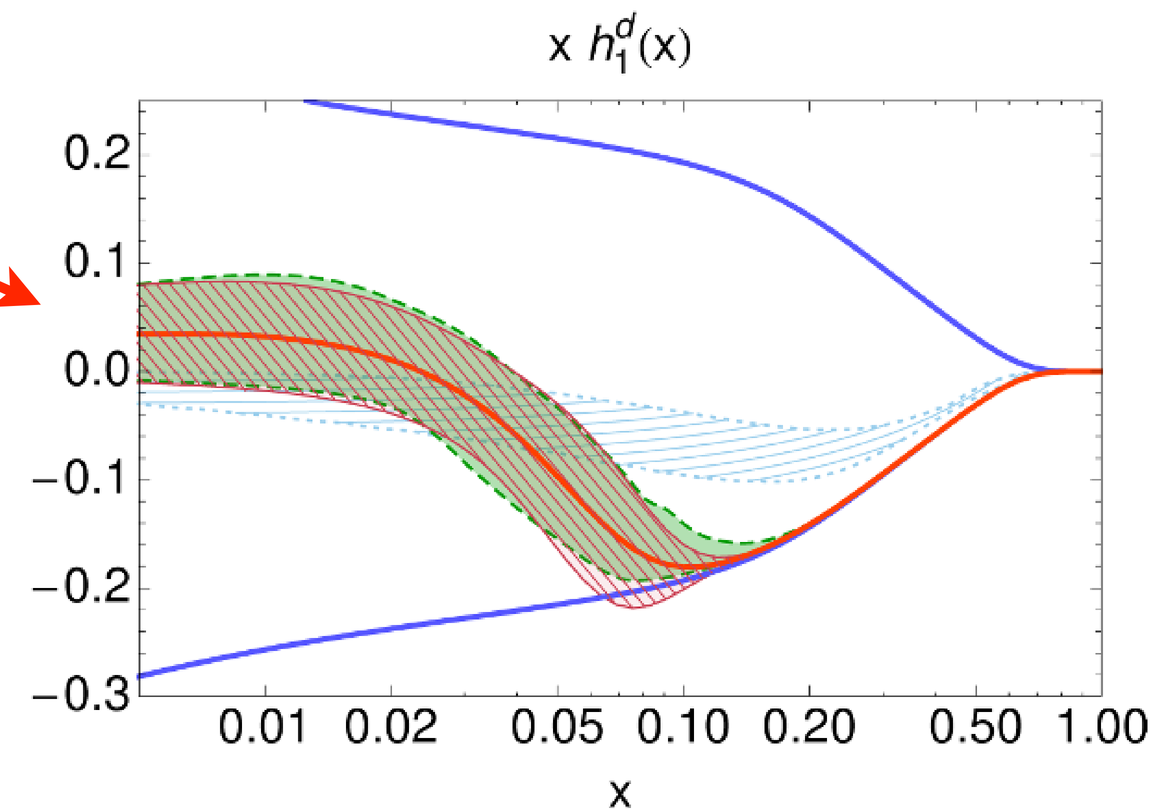
Collinear extraction of h_1



1σ error band from replicas @2.4 GeV^2

see Marco Radici's talk

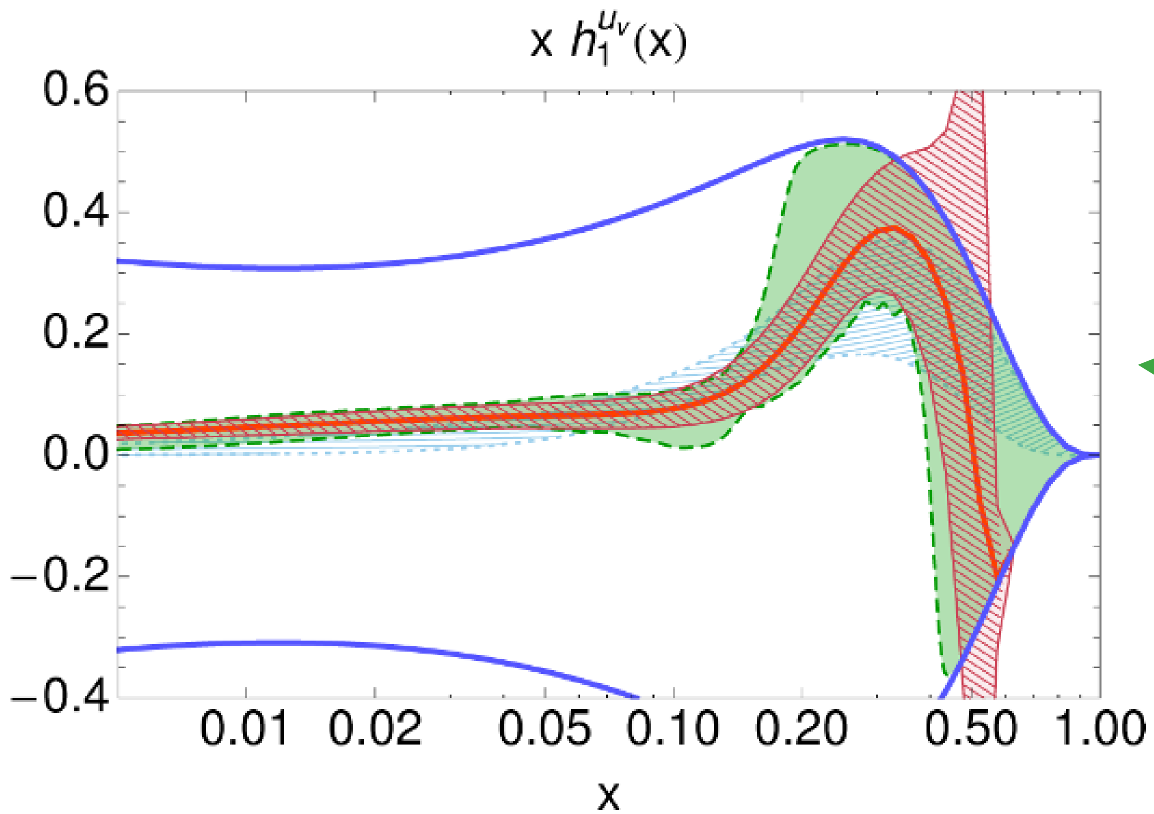
$$\chi^2/d.o.f. \simeq 1.1$$



Best fit central curve @2.4 GeV^2 and standard 1σ error band

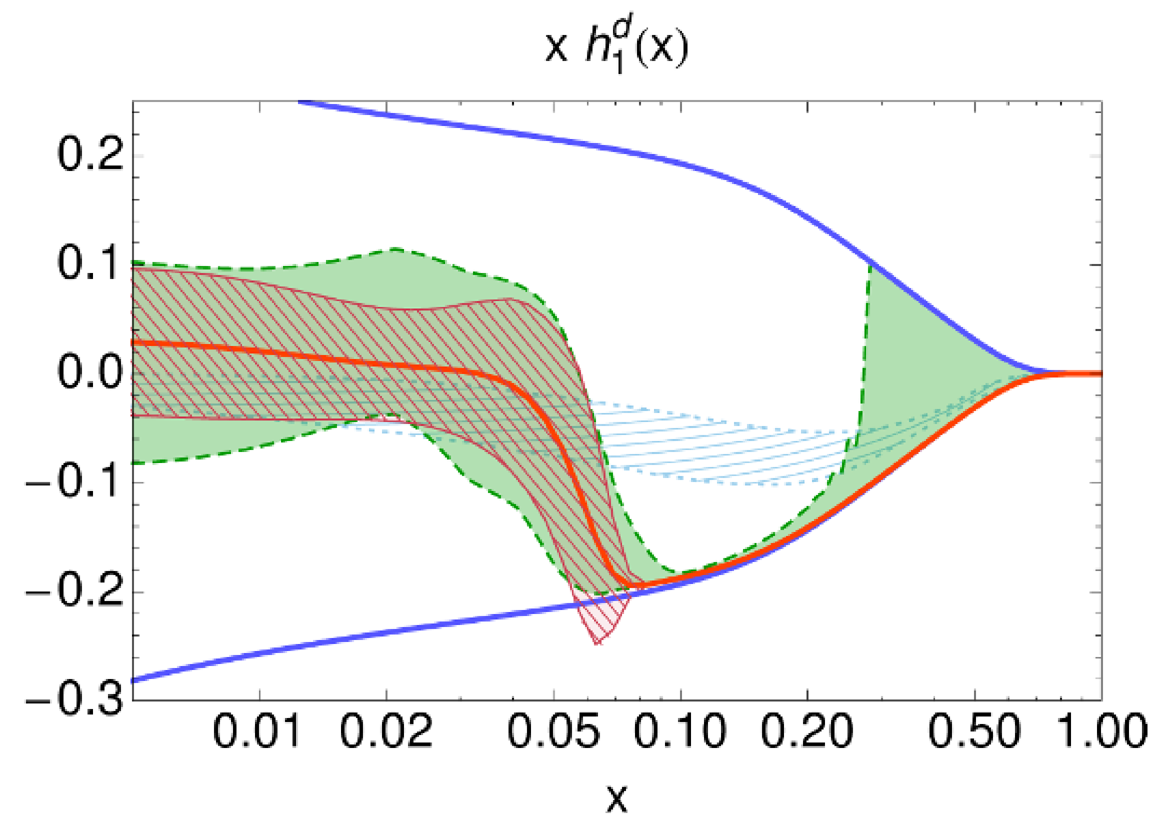
Rigid version

Collinear extraction of h_1



1σ error band from replicas @2.4 GeV²

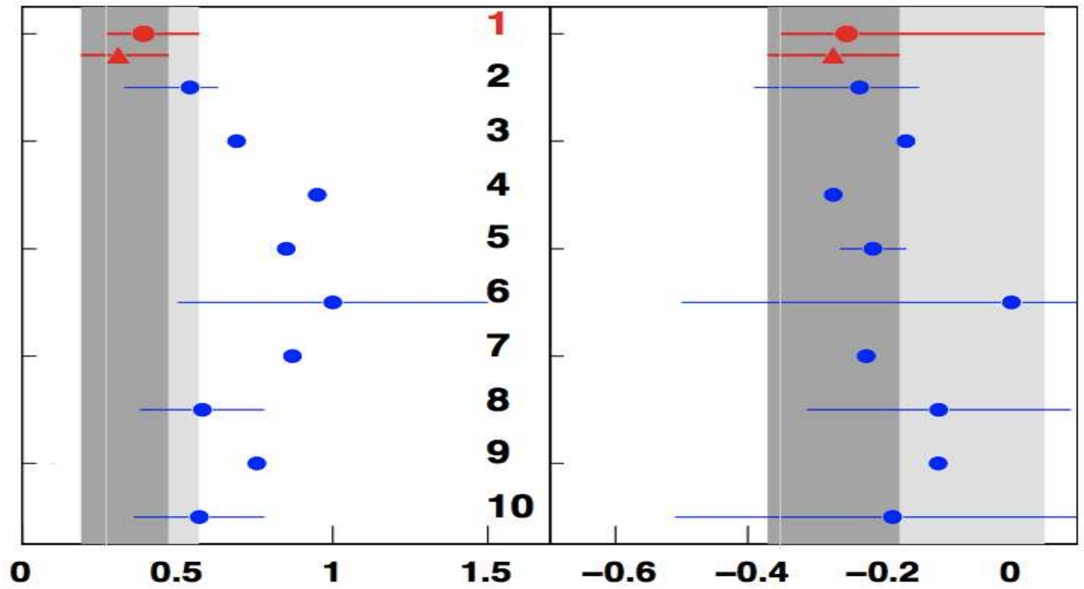
$$\chi^2/d.o.f. \simeq 1.1$$



Best fit central curve @2.4 GeV²
and standard 1σ error band

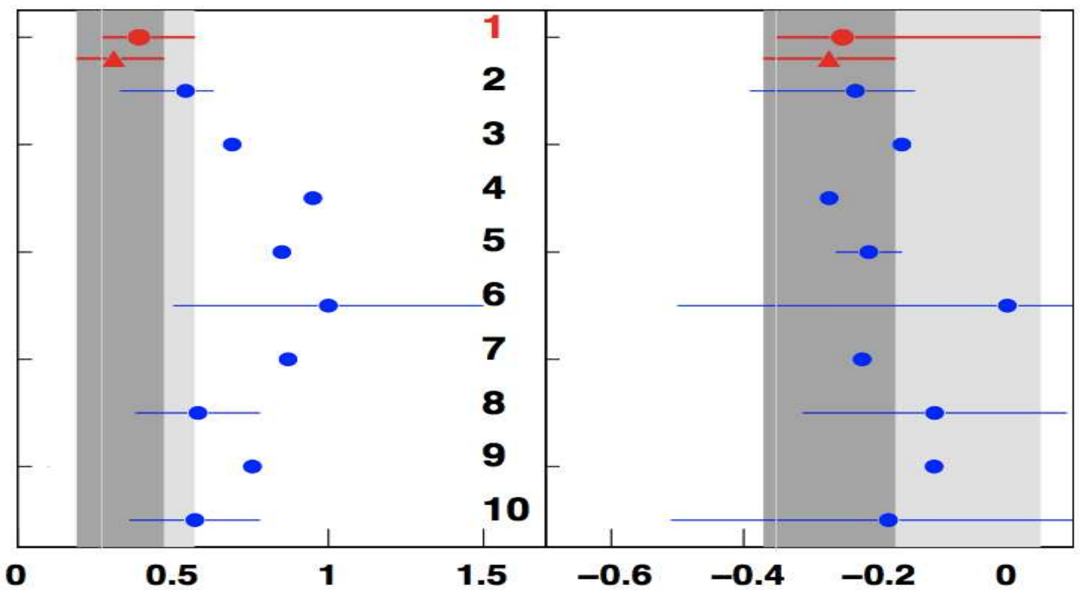
Extra-flexible version

Tensor charge



Torino: Anselmino et al.,(2013)

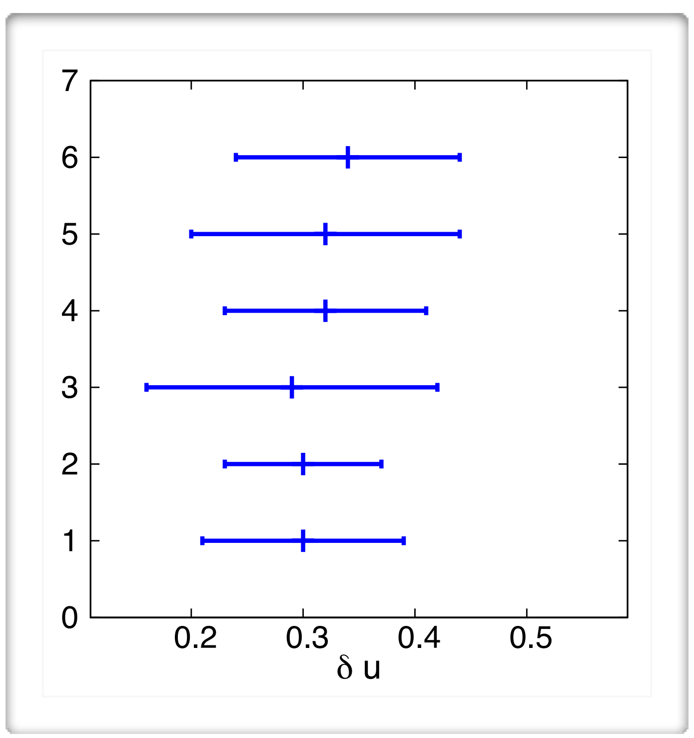
Tensor charge



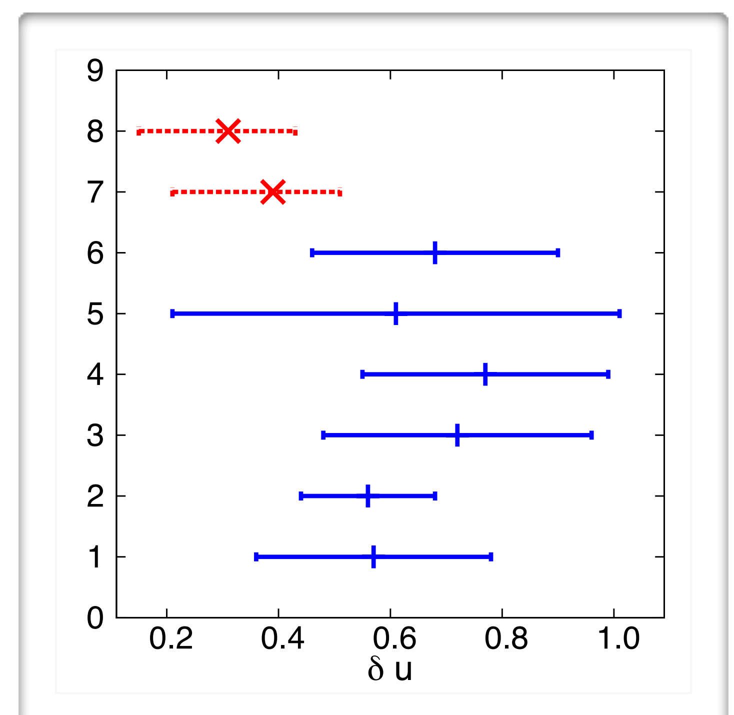
Torino: Anselmino et al.,(2013)

$$\delta q = \int_{6.4\times 10^{-3}}^{0.28} dx\, h_1^{q_v}(x)$$

$$\delta q = \int_{\sim 0}^{1} dx\, h_1^{q_v}(x)$$

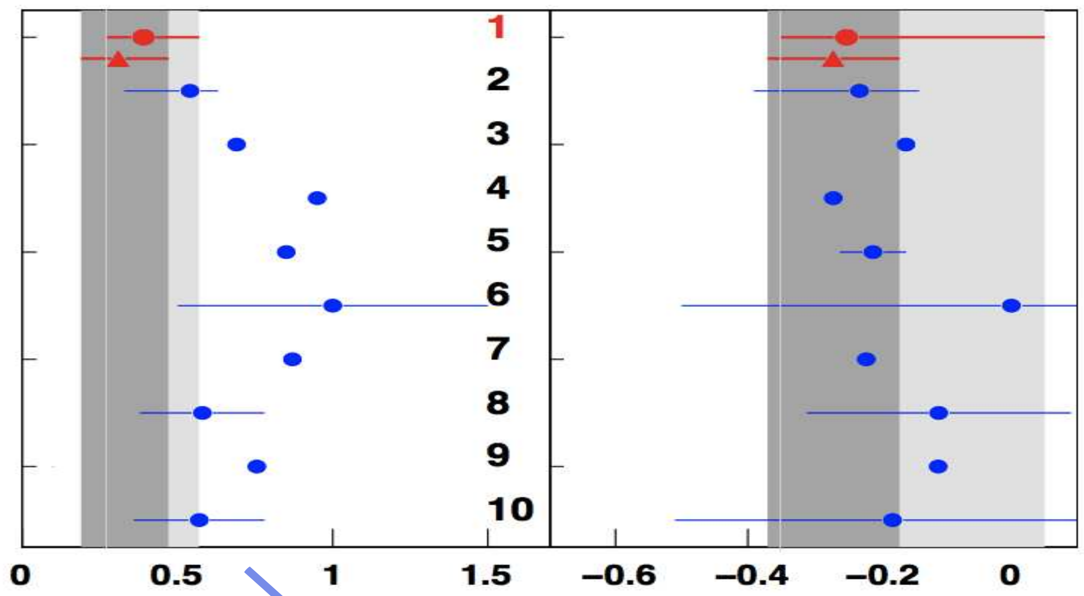


8. fit of A_0
7. fit of A_{12}
6. MC extra flexible
5. standard extra flexible
4. MC flexible
3. standard flexible
2. MC rigid
1. standard rigid



Pavia: Bacchetta et al., JHEP 03 (2012) 119

Tensor charge



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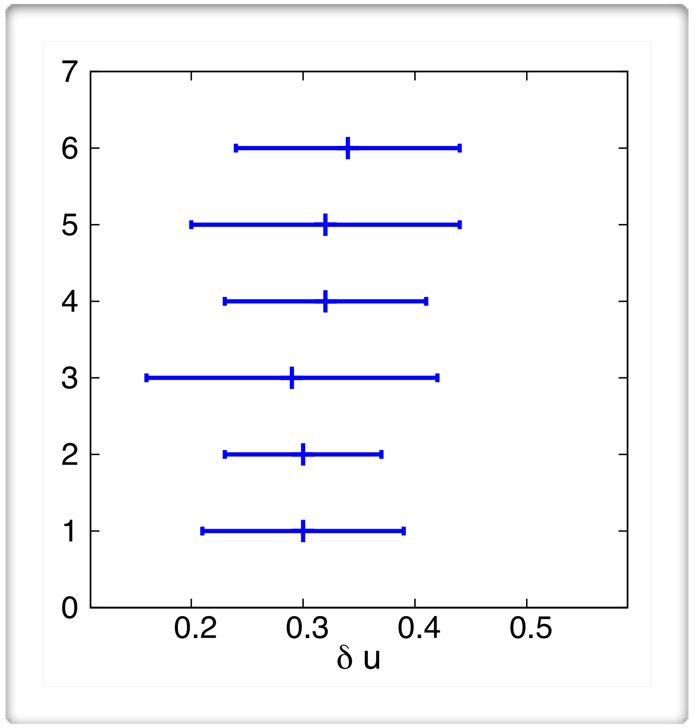

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6. MC extra flexible
5. standard extra flexible
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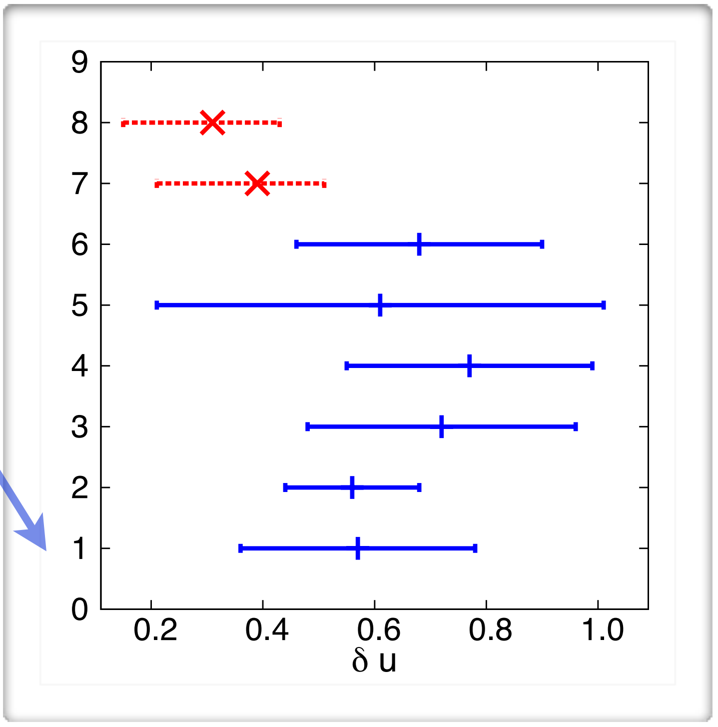
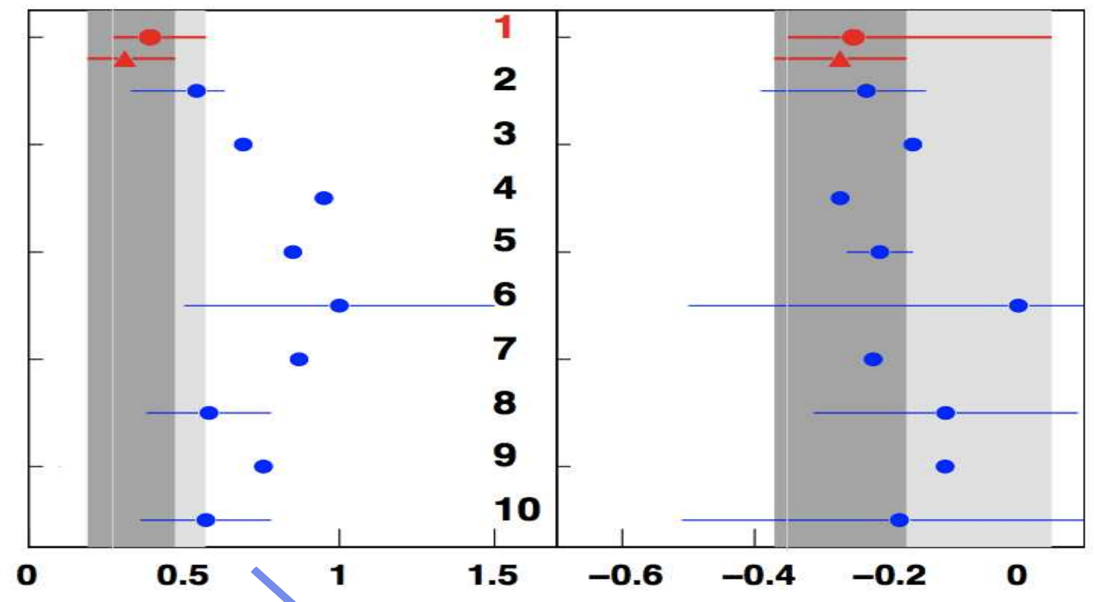
2. MC rigid
1. standard rigid



Pavia: Bacchetta et al., JHEP 03 (2012) 119

Tensor charge

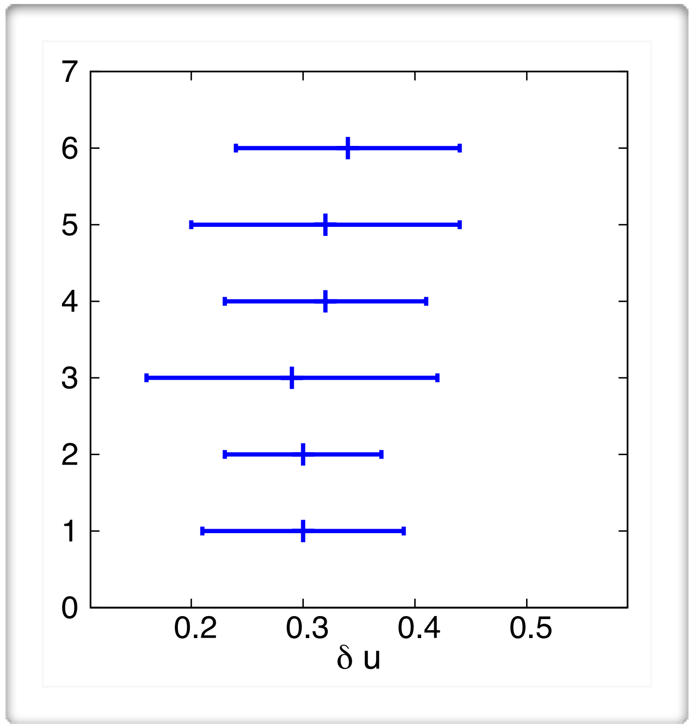


Torino: Anselmino et al.,(2013)

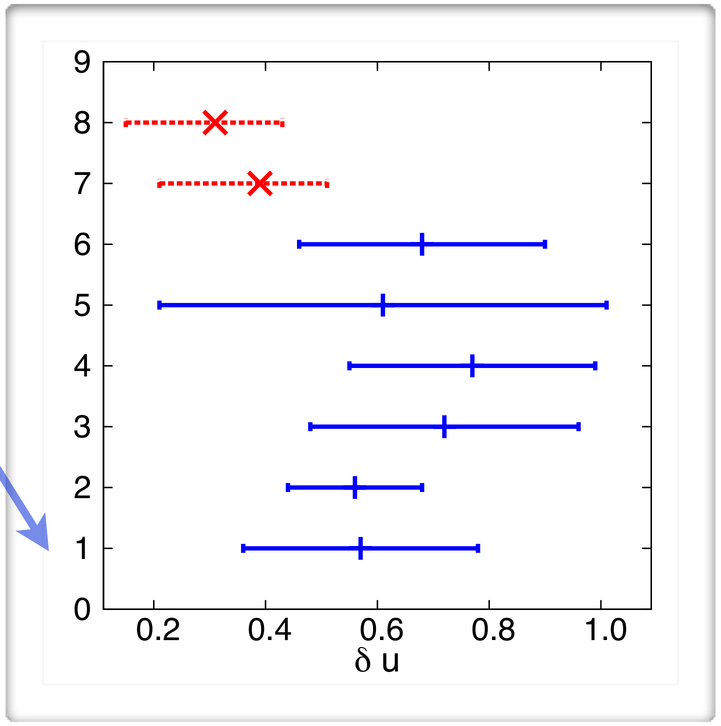
Extrapolation outside the range of data

$$\delta q = \int_{6.4\times 10^{-3}}^{0.28} dx\, h_1^{q_v}(x)$$

$$\delta q = \int_{\sim 0}^{1} dx\, h_1^{q_v}(x)$$



8. fit of A_0
7. fit of A_{12}
6. MC extra flexible
5. standard extra flexible
4. MC flexible
3. standard flexible
2. MC rigid
1. standard rigid



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Future of transversity

- ***Functional Form*** **crucial to standard fitting procedure**
  - ➡ **Highly unconstrained outside data range**
  - ➡ **Important! e.g., for tensor charge**
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Proposal for CLAS12 (A rated & waiting for HDice target to be ready)

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Dihadron Electroproduction in DIS with Transversely Polarized ^{3}He Target at 11 and 8.8 GeV

May 10, 2013

(A Letter of Intent to Jefferson Lab (PAC 40))

Letter of Intent for SoLID

(Proposal to be submit to next PAC)

Higher-twist PDFs

e(x): strange content of the proton

- Pion-nucleon σ term

$$\int_{-1}^{1} dx\, (e^u + e^d)(x) = \frac{1}{2M}\langle P|(\bar{u}u + \bar{d}d)|P\rangle \equiv \frac{\sigma_{\pi N}}{(m_u + m_d)/2}$$

$\sigma_{\pi N}$ =(50-70 MeV)

- related to the strangeness content of the nucleon

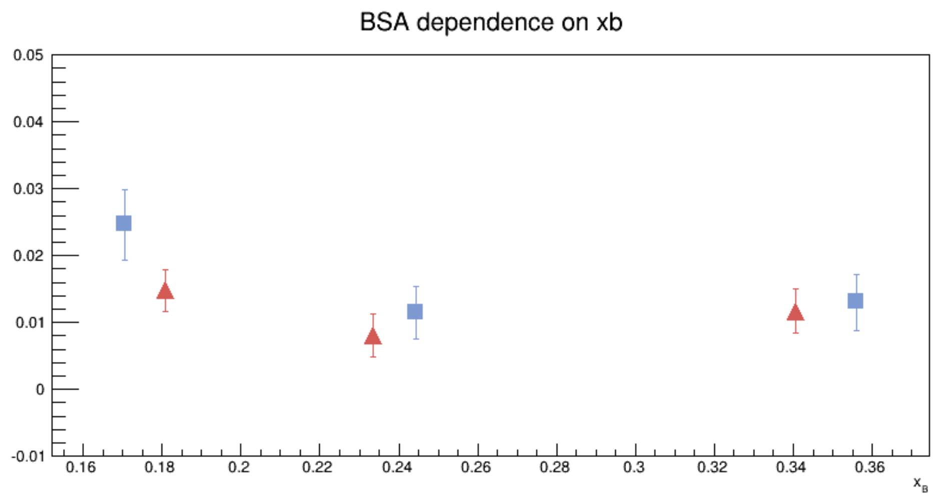
$$y_N = \frac{\langle N|\bar{\psi}_s\psi_s|N\rangle}{\frac{1}{2}\langle N|(\bar{\psi}_u\psi_u + \bar{\psi}_d\psi_d)N\rangle} = 1 - \frac{m}{m_s - m}\frac{M_\Xi + M_\Sigma - 2M_N}{\sigma_{\pi N}}$$

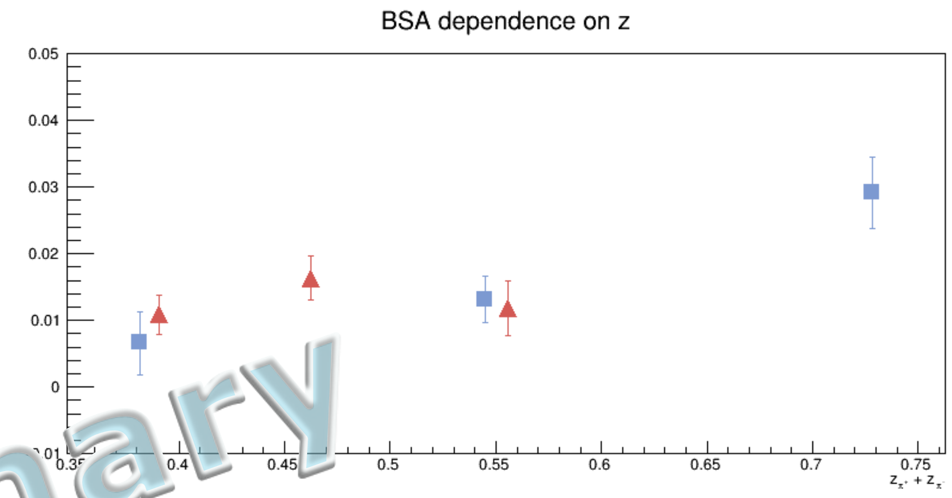
LO ChiPT

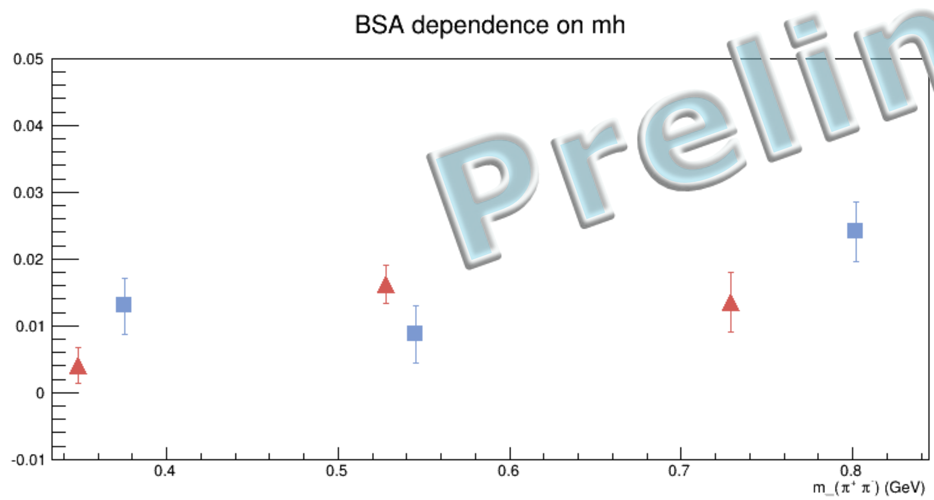
- large strange contribution

- but mass contribution of strange not sensitive to *y*

Higher-twist from experiments







$$\Delta\sigma_{LU} \propto [\mathrm{e(x)}\ H_1^{\sphericalangle q} + \mathrm{f(x)}\ \tilde{G}_1^{\sphericalangle q}] \sin\varphi_R$$

- ❑ **Unpolarized H2 target**
- ❑ **Longitudinally-polarized NH3 target**

Plot from **Silvia Pisano, LNF-INFN – MeNu2013**

$$A_{LU}^{\sin\phi_R \sin\theta}(x,y,z,M_h,Q) = -\frac{W(y)}{A(y)}\ \frac{M}{Q}\ \frac{|\mathbf{R}|}{M_h}\ \frac{\sum_q e_q^2 \left[x e^q(x,Q^2)\, H_{1,sp}^{\sphericalangle,q}(z,M_h,Q^2) + \frac{M_h}{zM} f_1^q(x,Q^2)\, \tilde{G}_{sp}^{\sphericalangle,q}(z,M_h,\right.}{\sum_q e_q^2\, f_1^q(x,Q^2)\, D_{1,ss+pp}^q(z,M_h,Q^2)}$$

Known twist-2 functions **Unknown twist-3 functions**

Higher-twist from experiments

- Analysis of e(x) here at LNF (M. Mirazita, S. Pisano & A.C.)
  - (Second) *extraction* but first in collinear factorization from BSA
- Great experimentalist/theorist collaboration!
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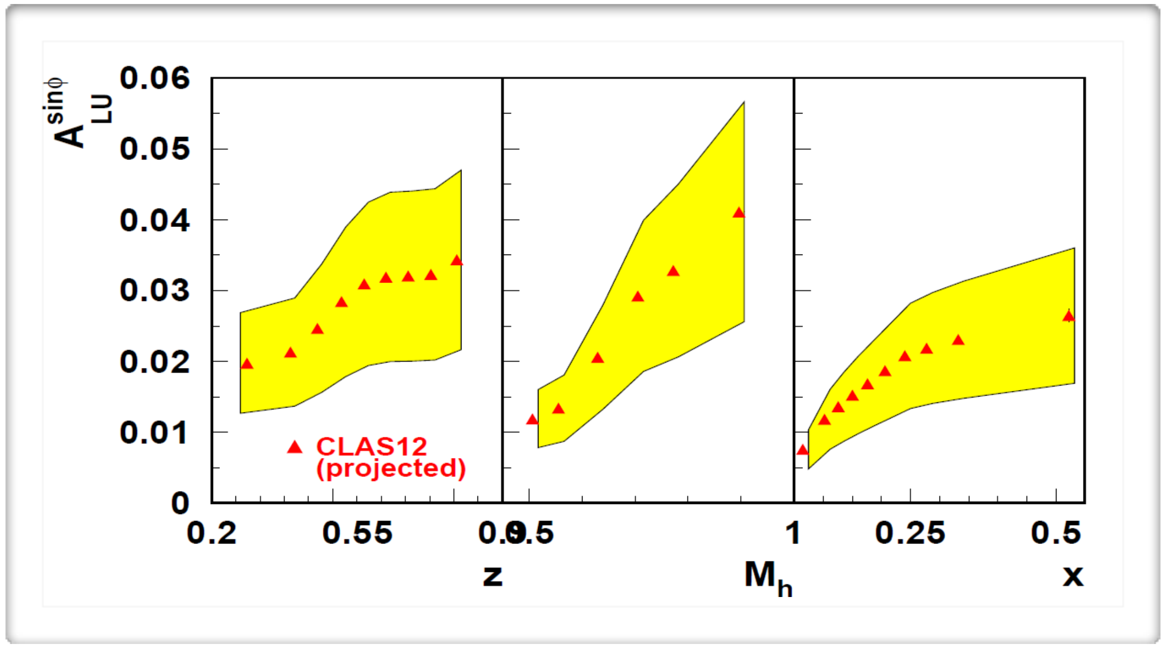
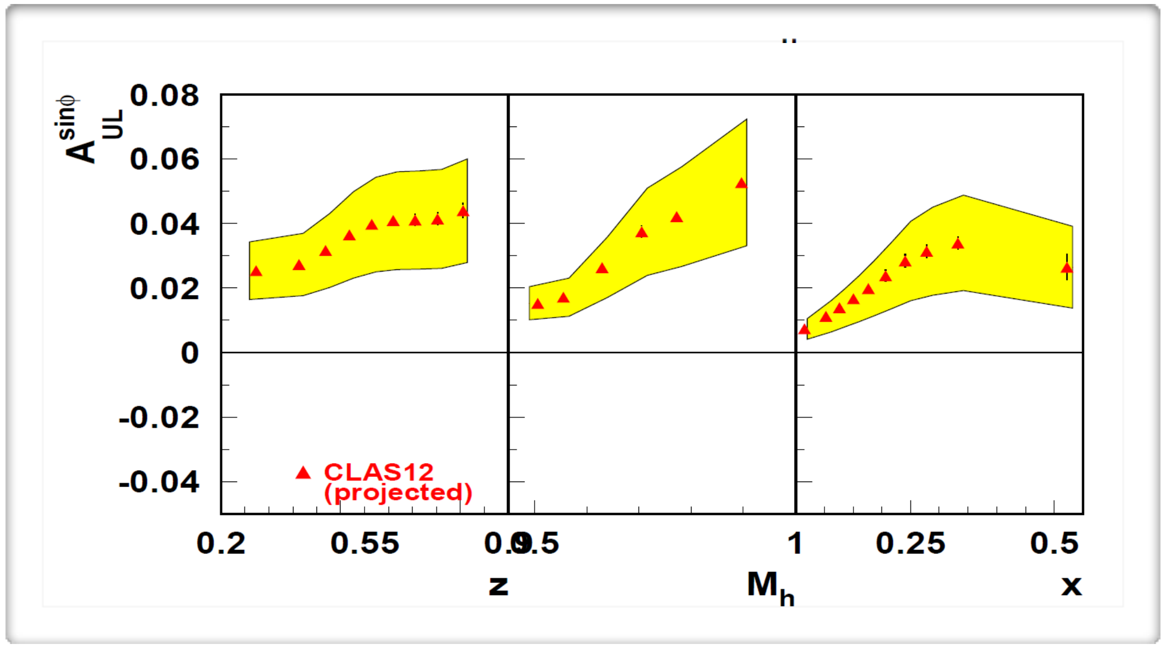
  - (Re)submit a proposal for CLAS@12?
  - Projections based on models for e(x) & $h_L(x)$ for PAC38





More asymmetries?

- Yes, but we need more info on multiplicities for
  - $\pi^{\pm}K^{\mp}$ pairs, $\pi^{\pm}K^{0}$ pairs
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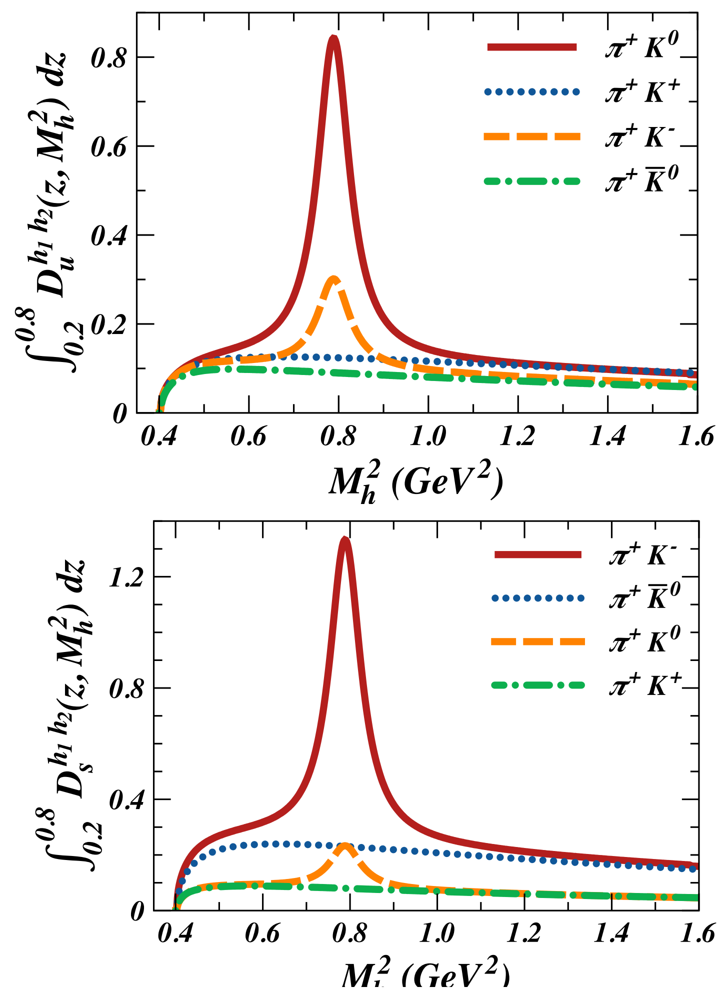

H. Matevosyan et al., 1310.1917

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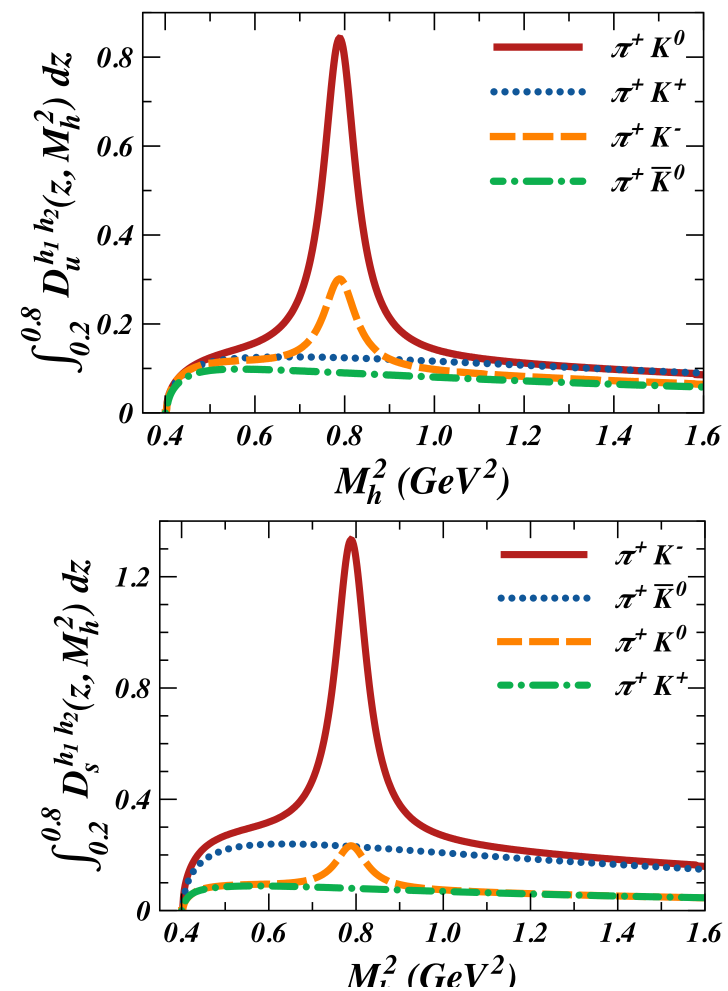

H. Matevosyan et al., 1310.1917

- **Yes**, but CLAS@12 kinematics probably more adapted
- Study di- to single-hadron SIDIS limits.
  - Can we get more *first principles*'based arguments?


Bright Future
Ahead

Back-up Slides

PYTHIA event generator for $q \to (\pi^+\pi^-)\ X$ @ Belle kinematics

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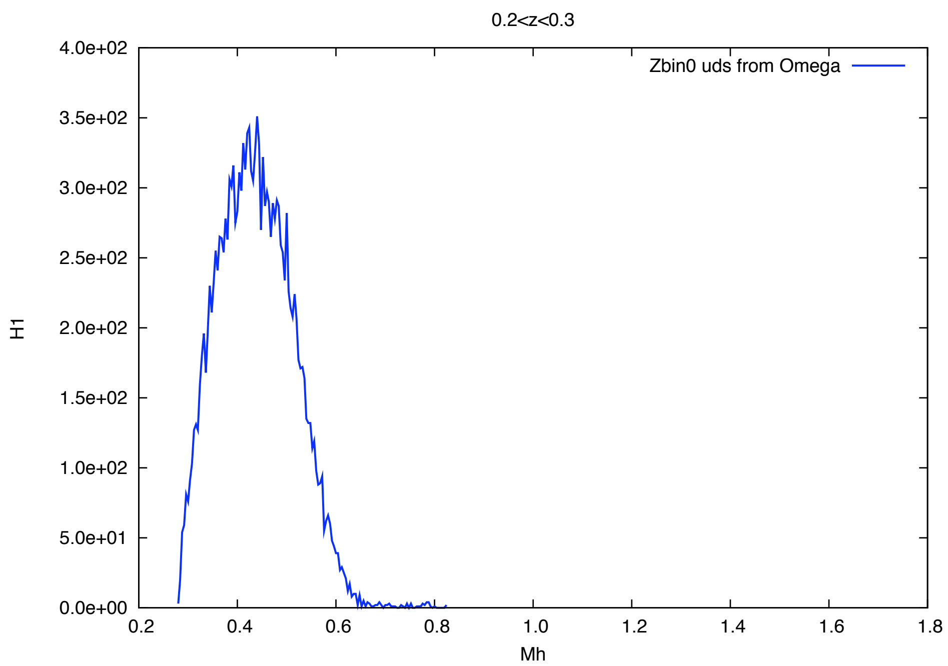

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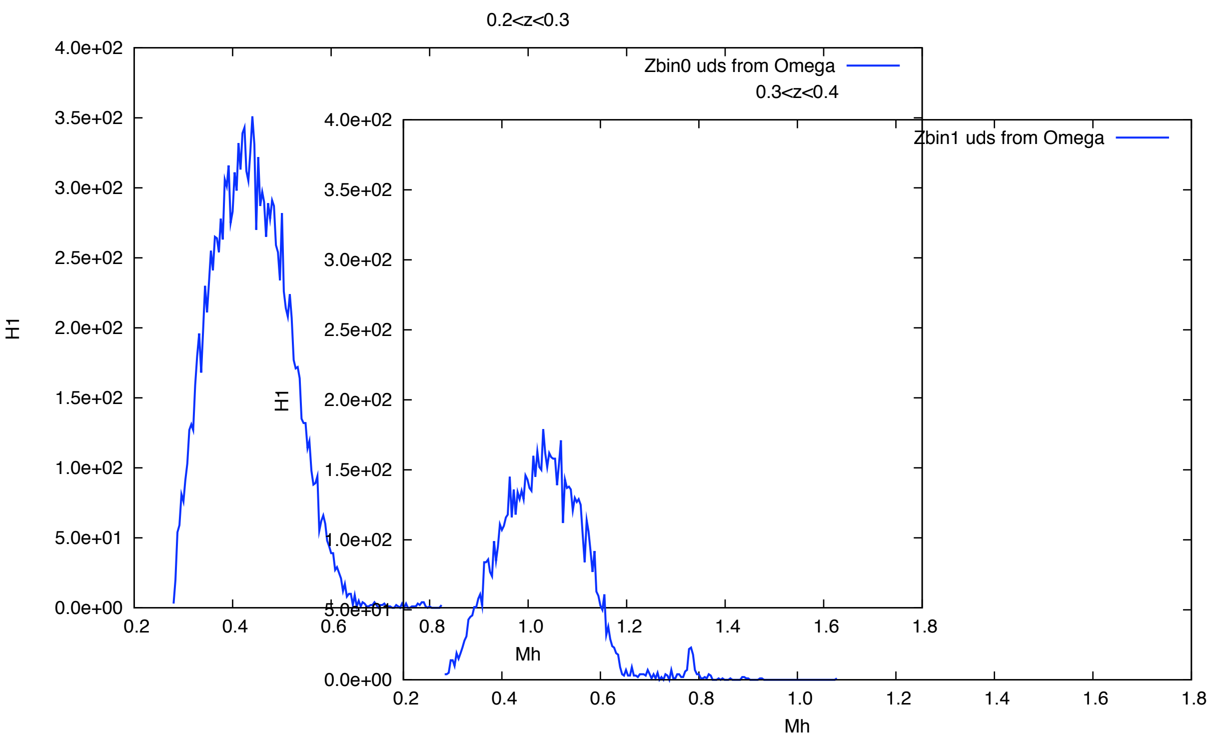

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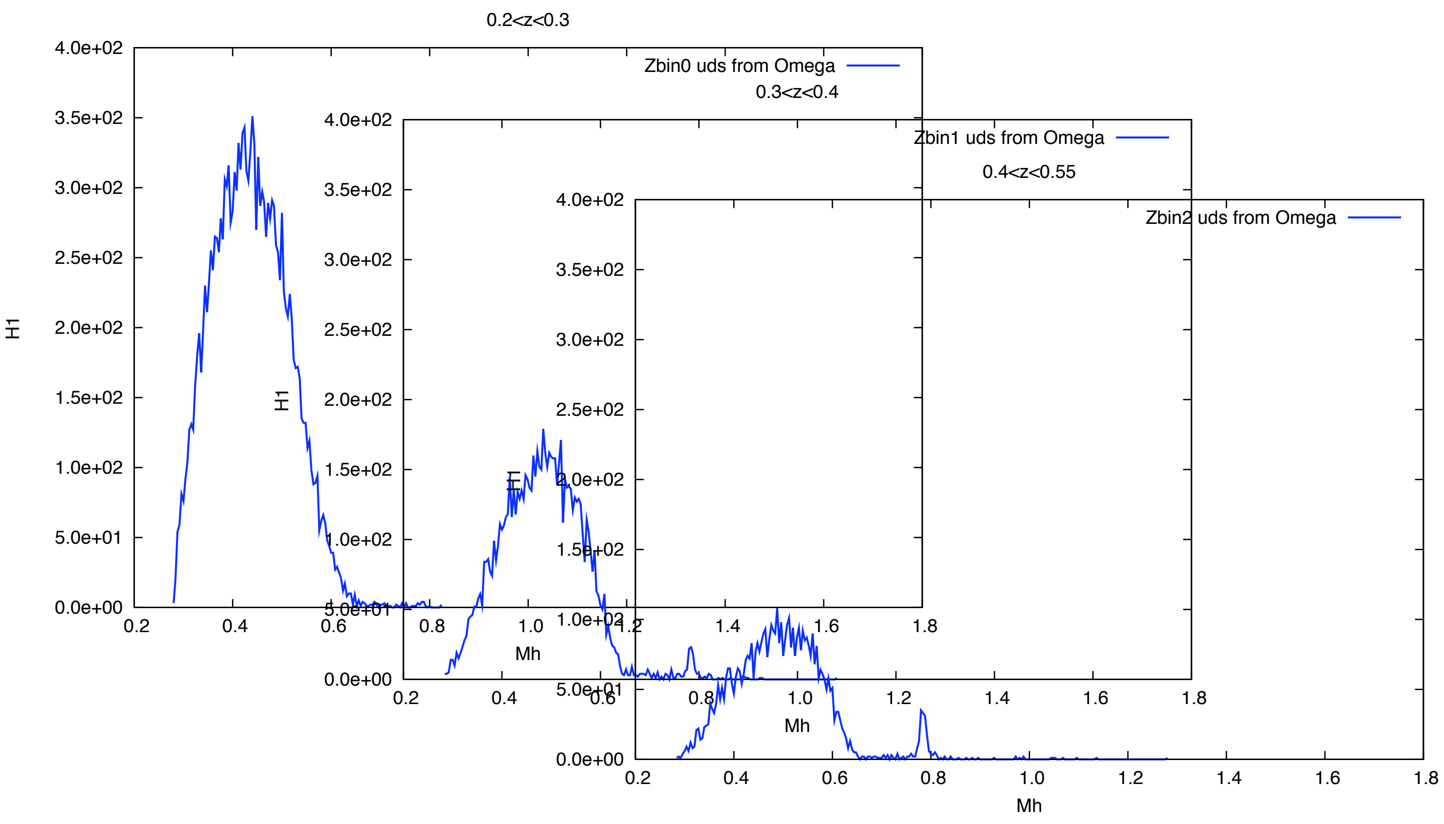

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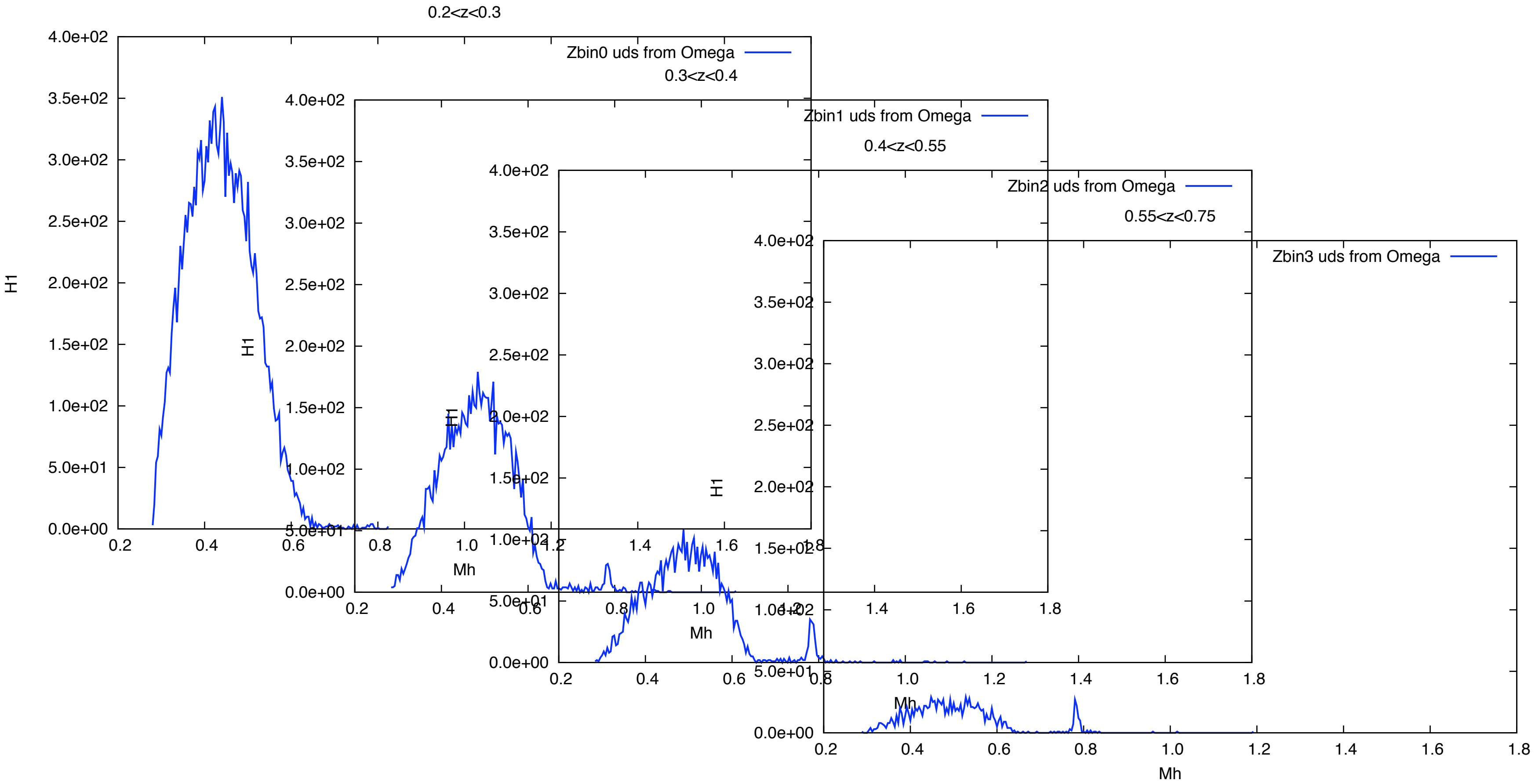

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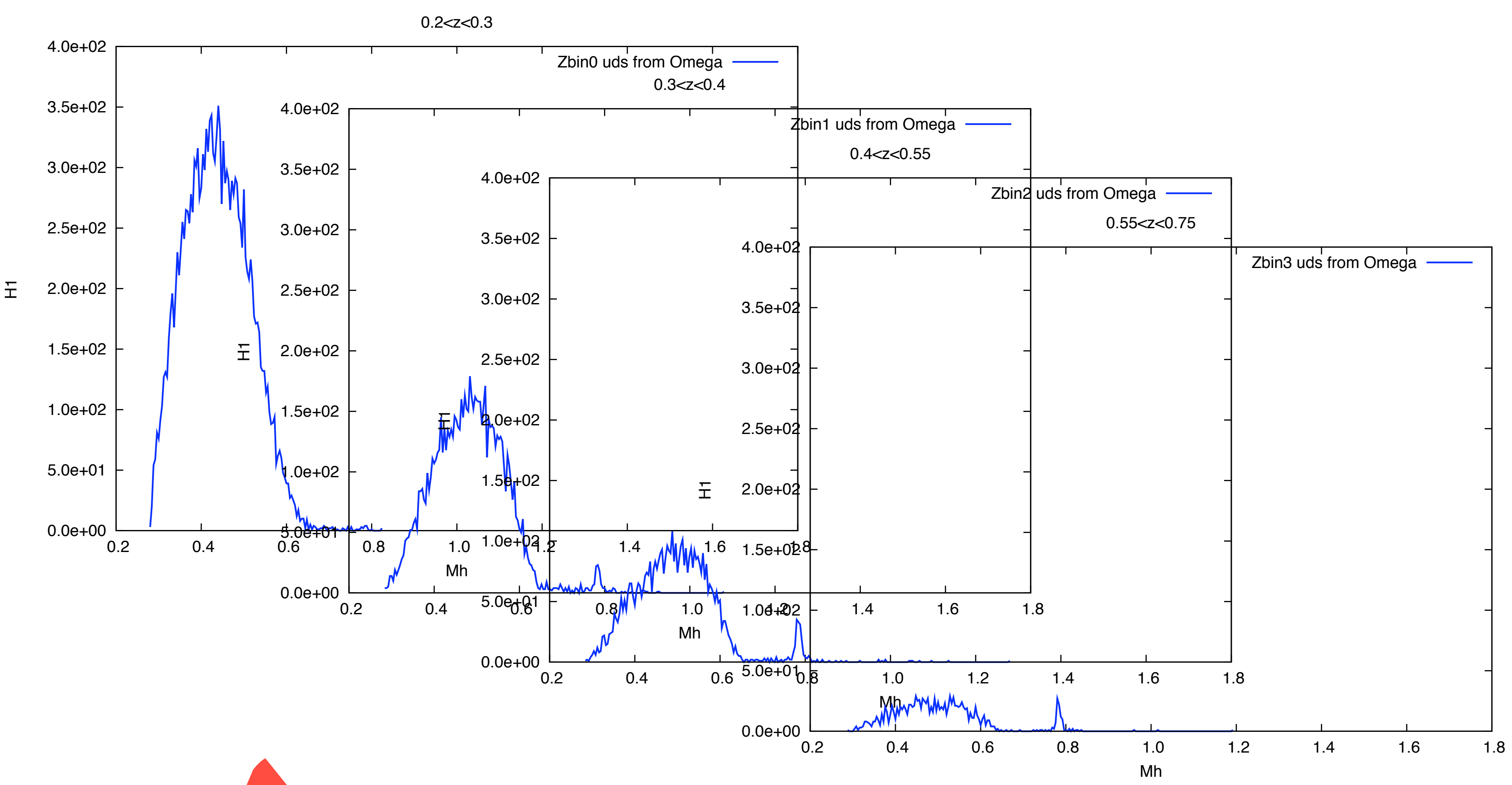

- 9 zbins
- Flavor decomposition
  - up
  - down
  - strange
  - charm
- ρ channel
- ω channels
- K^0 channel
- non resonant contrib.

e.g. uds from ω channels

Pion pair production in pp$^{\uparrow}$ collision

$$A_N \equiv \frac{d\sigma_{UT}}{d\sigma_{UU}}$$

$$d\sigma_{UU} = 2\,|\mathbf{P}_{C\perp}| \sum_{a,b,c,d} \int \frac{dx_a dx_b}{4\pi^2 z_c}\, f_1^a(x_a)\, f_1^b(x_b)\, \frac{d\hat{\sigma}_{ab\to cd}}{d\hat{t}}\, D_{1,oo}(\bar{z}_c, M_C^2),$$

$$d\sigma_{UT} = 2\,|\mathbf{P}_{C\perp}| \sum_{a,b,c,d} \frac{|\mathbf{R}_C|}{M_C}\, |\mathbf{S}_{BT}|\, \sin(\phi_{S_B} - \phi_{R_C}) \int \frac{dx_a dx_b}{16\pi z_c}\, f_1^a(x_a)\, h_1^b(x_b)\, \frac{d\Delta\hat{\sigma}_{ab^\uparrow\to c^\uparrow d}}{d\hat{t}}\, H_{1,ot}^{\sphericalangle c}(\bar{z}_c, M_C^2)$$

Bacchetta & Radici, PRD70

The Functional Form

@ Q_0^2

$$x\, h_1^{q_V}(x) = \tanh\left(x^{1/2}\,(A_q + B_q\,x + C_q\,x^2 + D_q\,x^3)\right)\,\left(x\,\mathrm{SB}^q(x) + x\,\mathrm{SB}^{\bar{q}}(x)\right)$$

1st order polynomial

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2nd order polynomial

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judicious choice for integrability of the transversities

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$$\chi^2/d.o.f. \simeq 1.1$$

no significant change in the X^2/dof in the 3 versions

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Rigid version

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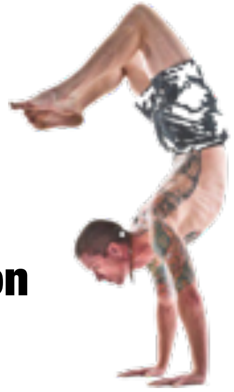

Flexible version

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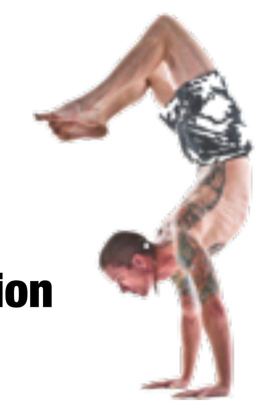

Flexible version

$$\chi^2/d.o.f. \simeq 1.1$$

3rd order polynomial

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Extra-flexible version

no significant change in the χ^2/dof in the 3 versions

Fitting procedure:
(z, M_h) and Q^2-dependence

Relevant variables and quantities: **(z, M_h)**

→ unfactorized functional form → “theo”

Scale dependence: **Belle@100GeV^2** .... **but SIDIS@ ~2.5GeV^2**

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LO Evolution with unpol. splitting functions & HOPPET: Ceccopieri, Radici & Bacchetta,,PLB 650 (2007) 81–89

qq, gq, qg, gg

0th step: backward evolution

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We don't know GLUON distr. a priori !!

Start fit @ 1GeV
Induce q & g distr.

Fitting procedure: chiral-odd DiFF

Binning (z, M_h) of a_{12}@Belle

z-binning : 0.2, 0.27, 0.33, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0
mh-binning : 0.28, 0.4, 0.5, 0.62, 0.77, 0.9, 1.1, 1.5, 2.

1st step: consider only bin $0.5<M_h<1.1$ GeV

$$\chi^2 = \sum_{\text{dof}} \frac{(theo - exp)^2}{err^2}$$

LO Evolution with chiral-odd splitting functions & HOPPET:

qq,gg

1st step: forward evolution

... just like the unpol. case

+ integration over bin ranges

$$H_1^{\sphericalangle\, u}(z, M_h) \propto \sqrt{M_h^2 - 4\, m_\pi^2}\; e^{A\,(1-z) - M_h\, B + C/z}\, \frac{D + F\, z^2 + G\, M_h\, z + H\, M_h^2\, z + J\, \frac{M_h^2}{z}}{(M_h^2 - m_\rho^2)^2 + K^2}$$

Q^2=1GeV^2

χ^2/dof=0.57

From Belle data

Transversity from e p$^{\uparrow}$→e' ($\pi^+\pi^-$) X @ HERMES

$$x h_1^{u_v}(x,Q^2) - \frac{1}{4}\, x h_1^{d_v}(x,Q^2) = -C_y^{-1}\, A_{\rm DIS}(x,Q^2)\, \frac{n_u(Q^2)}{n_u^{\uparrow}(Q^2)} \sum_{q=u,d,s} \frac{e_q^2}{e_u^2}\, x f_1^{q+\bar{q}}(x,Q^2)$$

with 1-to-100 GeV^2 evolution correction: small corrections

HERMES range: -0.251^{-1} (± 9% theo. err.) from BELLE

From Belle data

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with 1-to-100 GeV² evolution correction: small corrections

HERMES range: -0.251⁻¹ (± 9% theo. err.) **from BELLE**

Transversity from e p↑→e' ($\pi^+\pi^-$) X @ COMPASS 2007

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COMPASS range: -0.221⁻¹ (± 5% theo. err.) **from BELLE**

From Belle data

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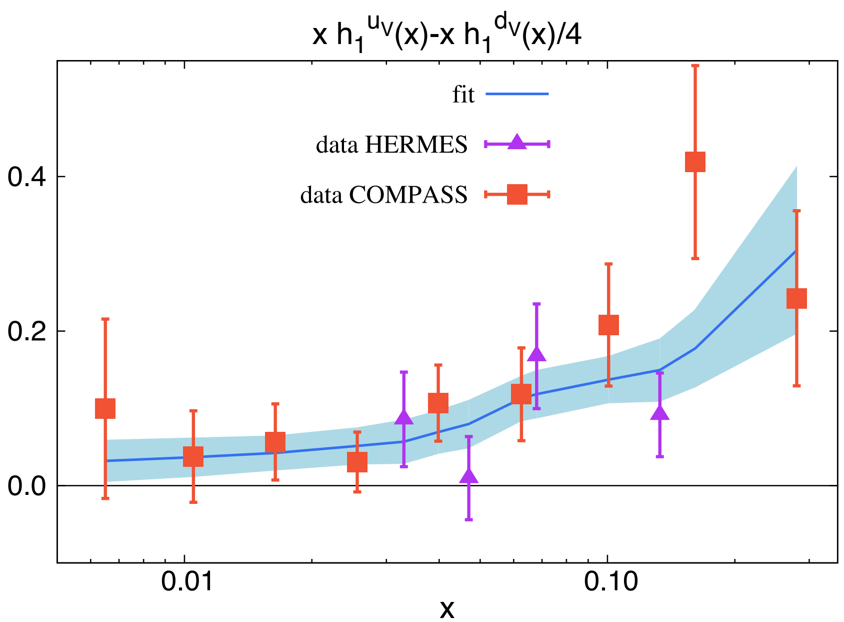
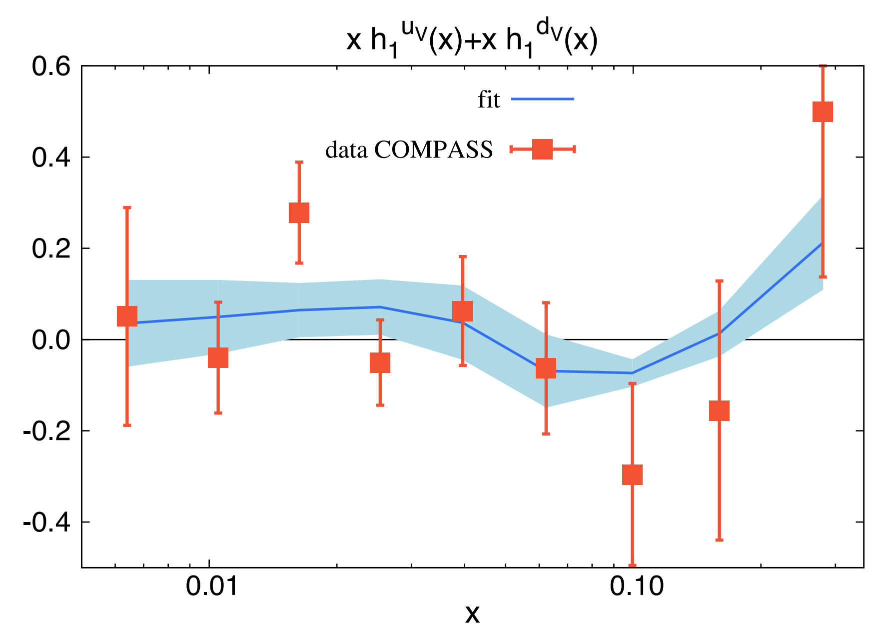
From fit

	HERMES range	COMPASS range
$n_u =$	0.564	0.785
$n_s =$	0.303	0.443
$n_u^\uparrow =$	0.146 ± 0.037	0.163 ± 0.031

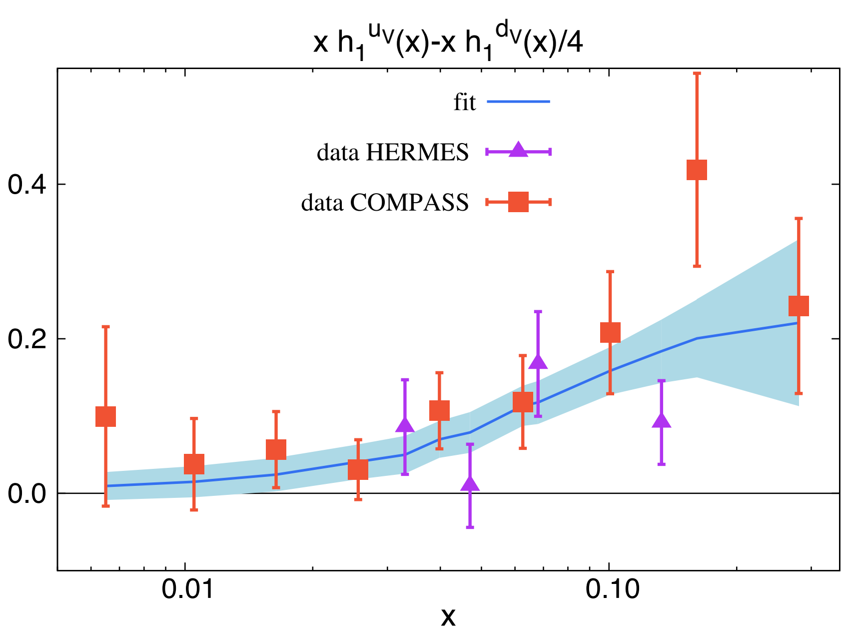
Comparison with extraction

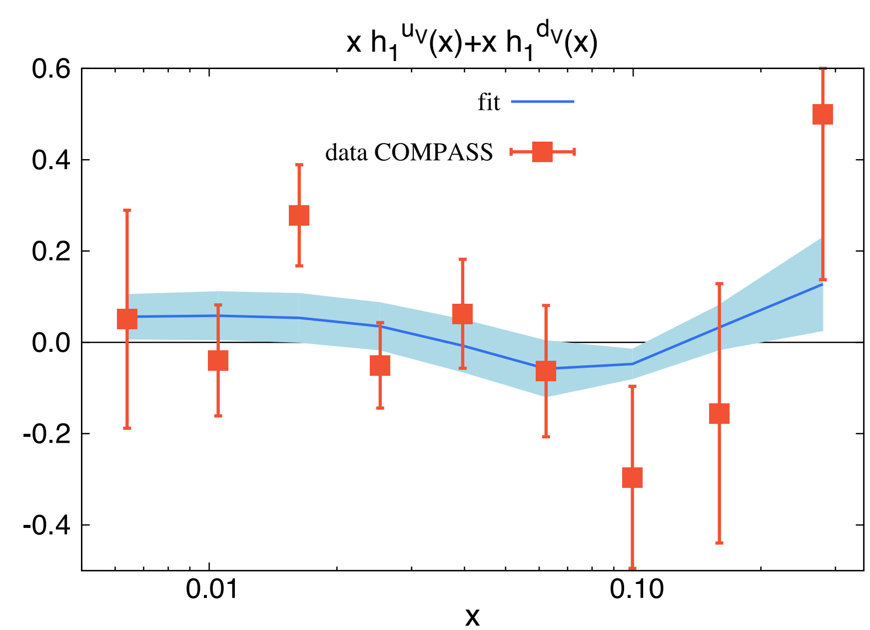
PROTON | **DEUTERON**





flexible functional form





rigid functional form