

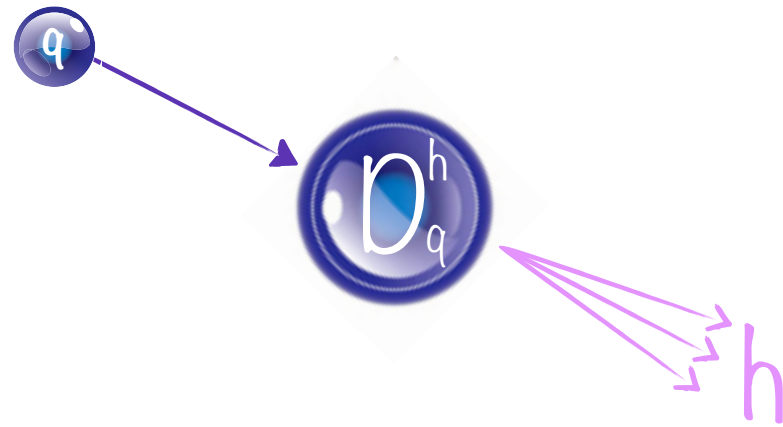
# PRECISE PION AND KAON CROSS SECTIONS @ BELLE

PSHP2013, November 11-13, 2013, Frascati

Francesca Giordano, Martin Leitgab

for the BELLE collaboration

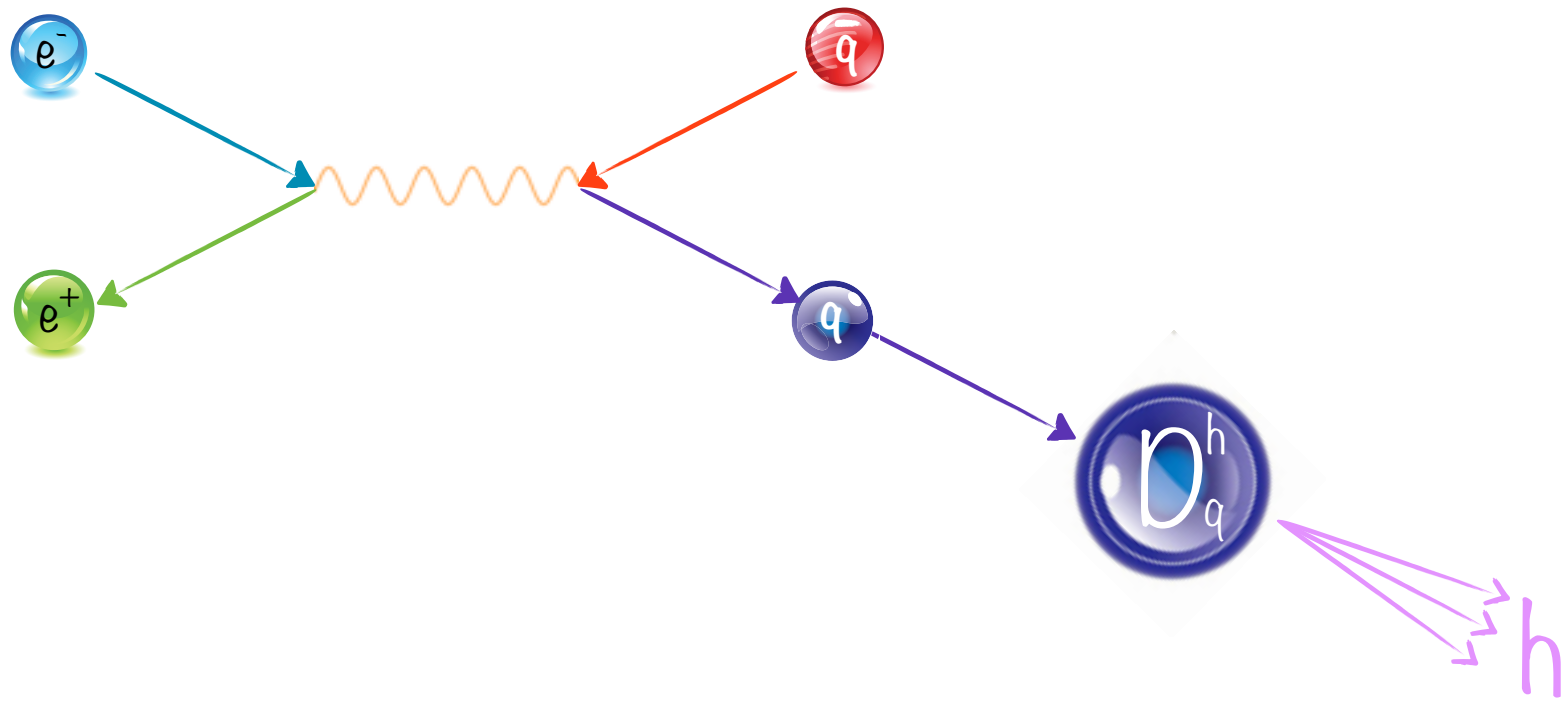
# Fragmentation process or how do the hadrons get formed?



- Fragmentation function describes the process of hadronization of a parton
- Strictly related to quark confinement



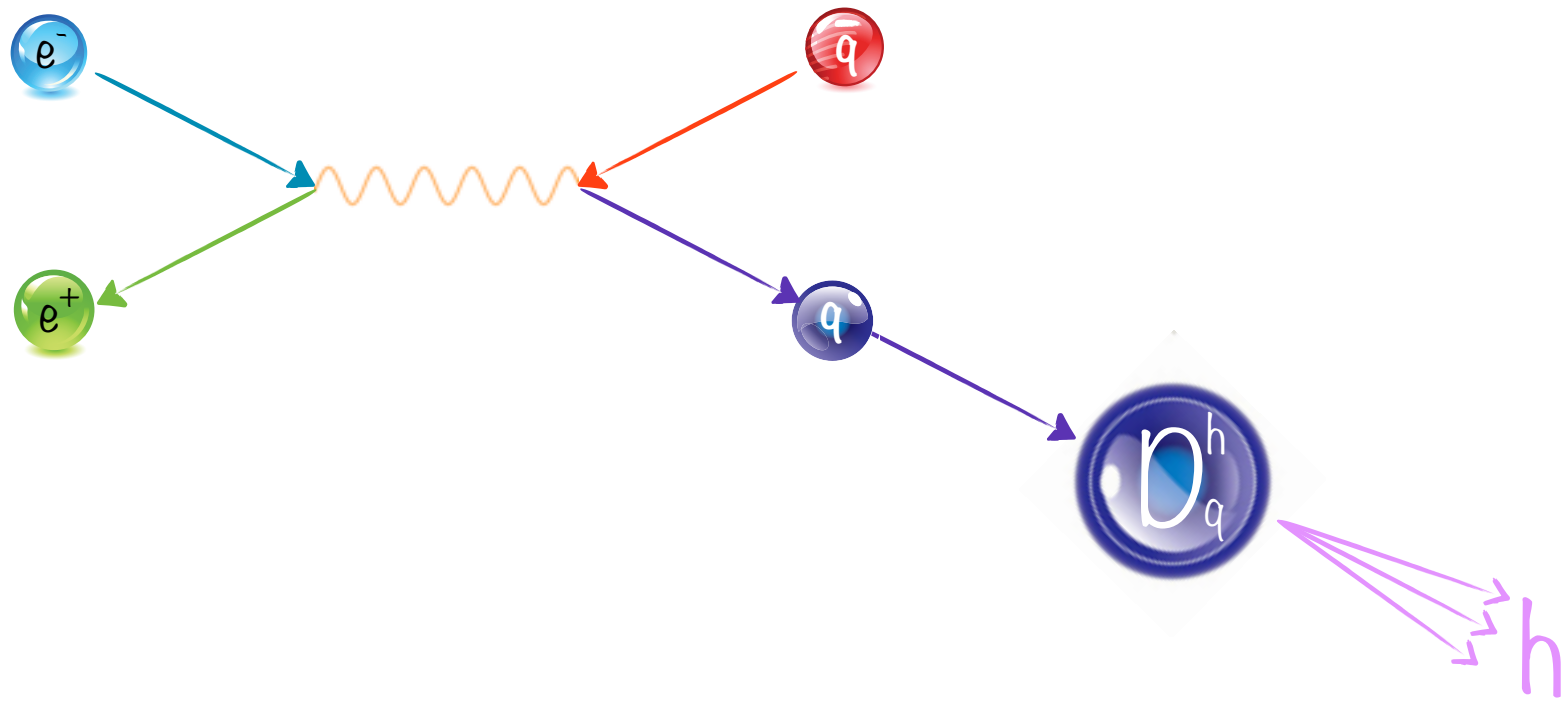
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- Universal: can be used to study the nucleon structure when combined with SIDIS and hadronic reactions data

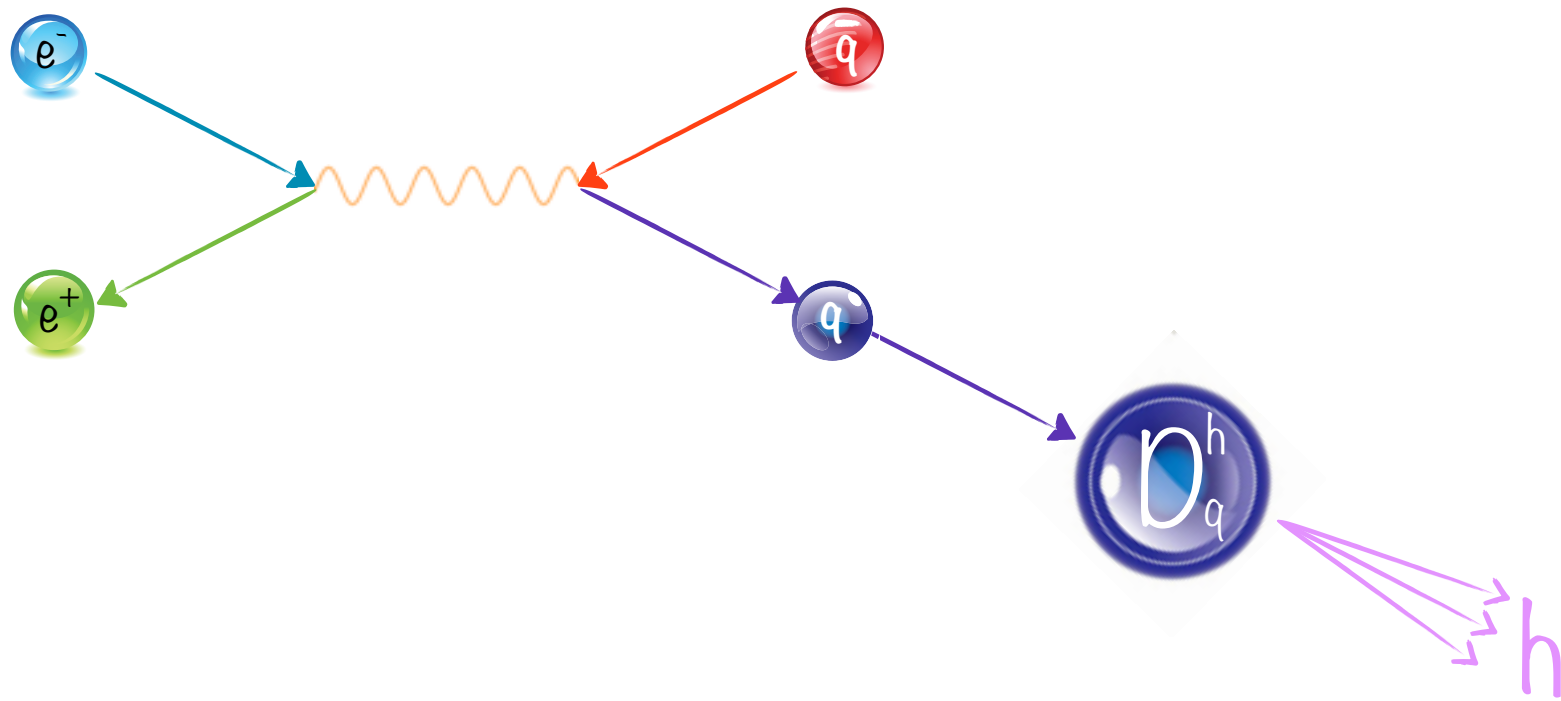
$$A_{LL}^h = \frac{\sigma_{\Rightarrow}^{\rightarrow} - \sigma_{\Leftarrow}^{\rightarrow}}{\sigma_{\Leftarrow}^{\rightarrow} + \sigma_{\Leftarrow}^{\leftarrow}} \propto D_q^h D_g^h$$

$$A_{UT}^h = \frac{\sigma^{\uparrow\uparrow} - \sigma^{\uparrow\downarrow}}{\sigma^{\uparrow\uparrow} + \sigma^{\uparrow\downarrow}} \propto D_q^h$$





# Fragmentation process or how do the hadrons get formed?



$$\frac{d\sigma_{h^\pm}}{dz} \begin{cases} \xrightarrow{\text{LO, NLO}} D_q^h D_{\bar{q}}^h \\ \xrightarrow{\text{NLO}} D_g^h \end{cases}$$

- Fragmentation function describes the process of hadronization of a parton
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- Universal: can be used to study the nucleon structure when combined with SIDIS and hadronic reactions data

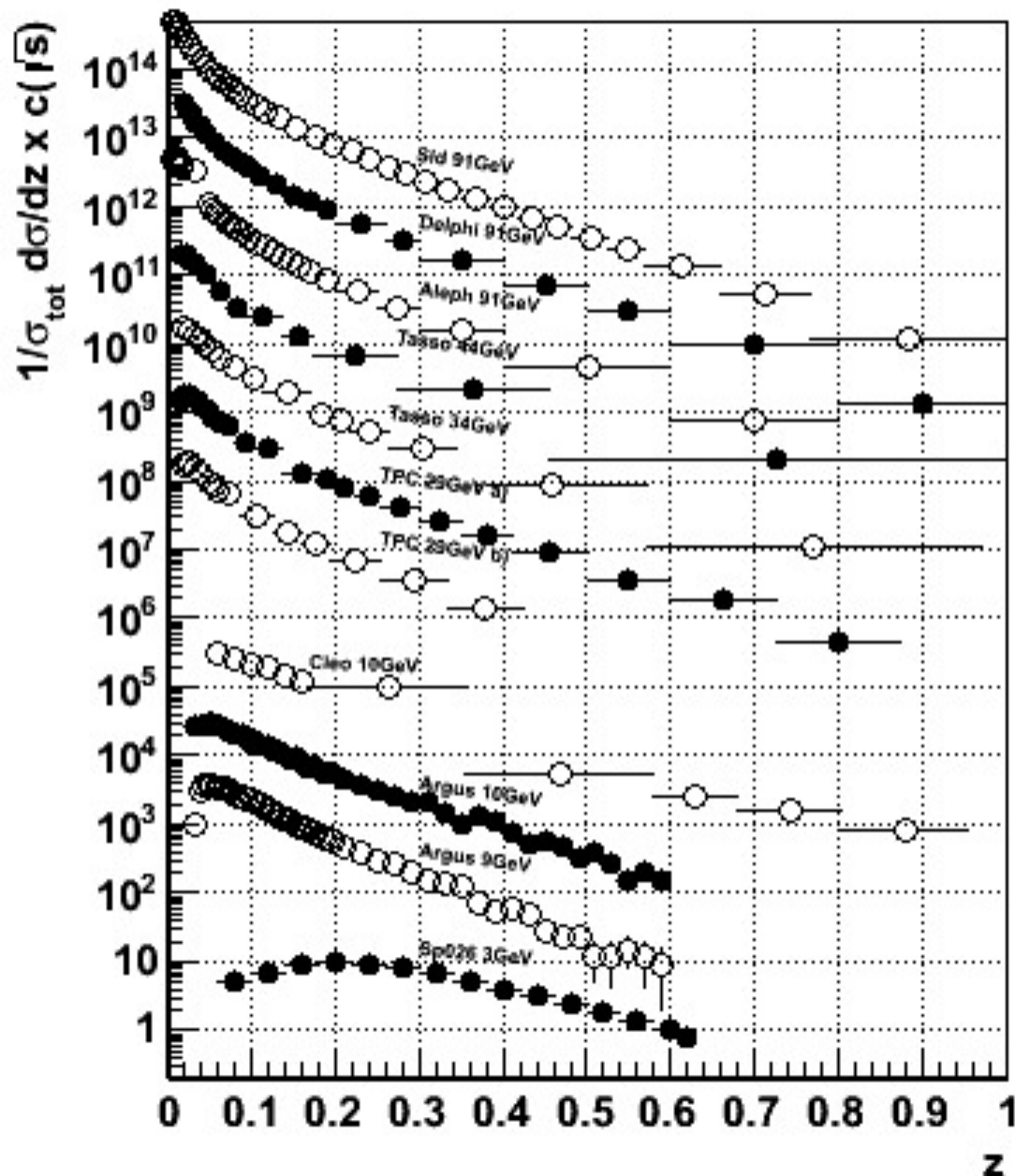
$$A_{LL}^h = \frac{\sigma_{\Rightarrow\Rightarrow} - \sigma_{\Leftarrow\Leftarrow}}{\sigma_{\Leftarrow\Rightarrow} + \sigma_{\Rightarrow\Leftarrow}} \propto D_q^h D_g^h$$

$$A_{UT}^h = \frac{\sigma^{\uparrow\uparrow} - \sigma^{\uparrow\downarrow}}{\sigma^{\uparrow\uparrow} + \sigma^{\uparrow\downarrow}} \propto D_q^h$$



# $e^+e^-$ data

World Data (Sel.) for  $e^+e^- \rightarrow \pi^{+-} + X$ , Multiplicities



2007: First unpolarized FF extraction  
with estimated uncertainties!

**Hirai, Kumano, Nagai, Sudoh**  
**Phys. Rev. D 75, 094009 (2007)**

Global analyses:

$e^+e^-$ , SIDIS, pp: (including uncertainties)

**de Florian, Sassot, Stratmann**  
**Phys. Rev. D 75, 114010 (2007) and**  
**Phys. Rev D 76, 074033 (2007)**

**Epele, Llubaroff, Sassot, Stratmann**  
**arXiv:1209.3240 [hep-ph]**

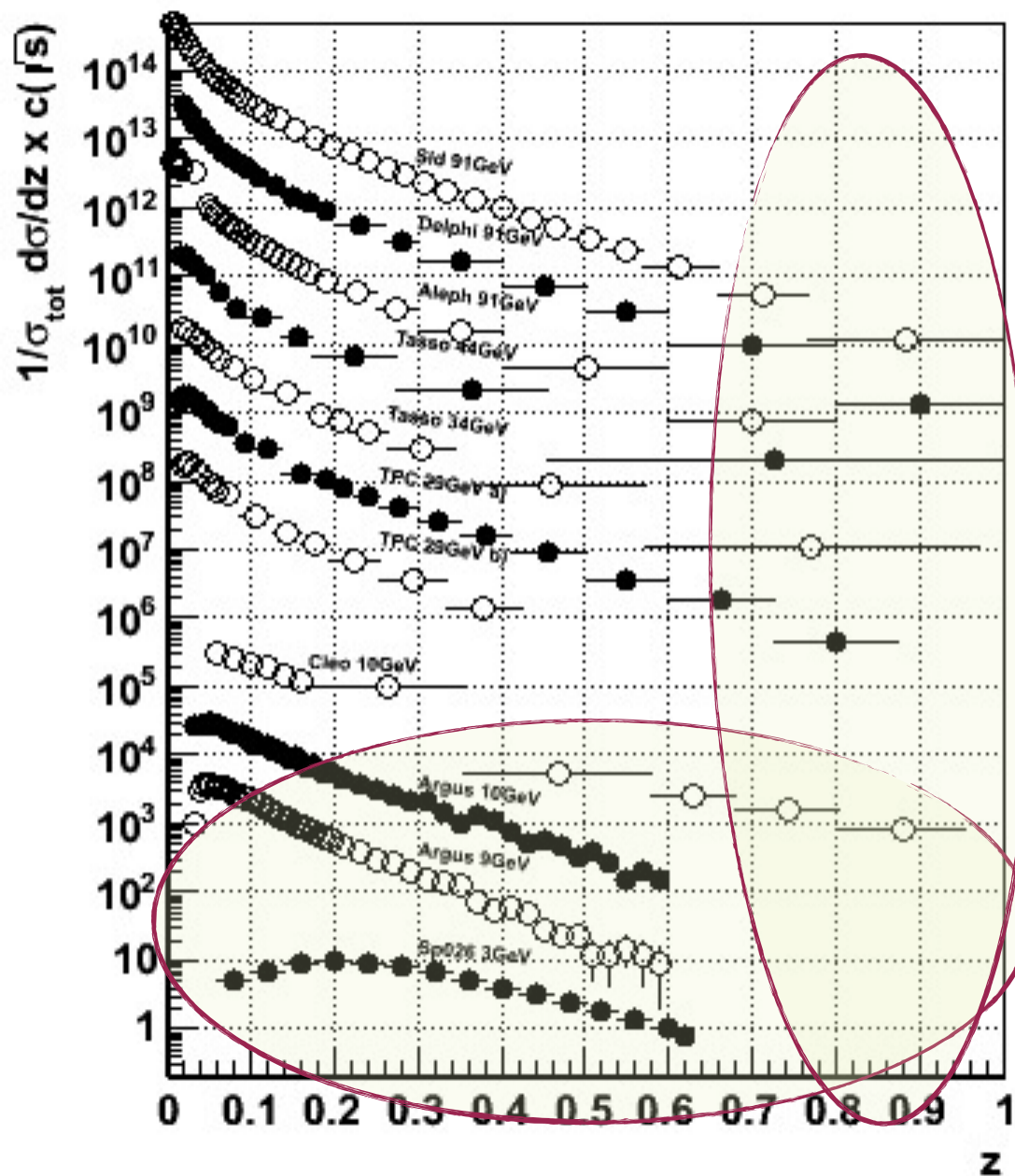
$e^+e^-$ , pp:

**Albino, Kniehl, Kramer**  
**Nucl. Phys. B 803, 42 (2008)**

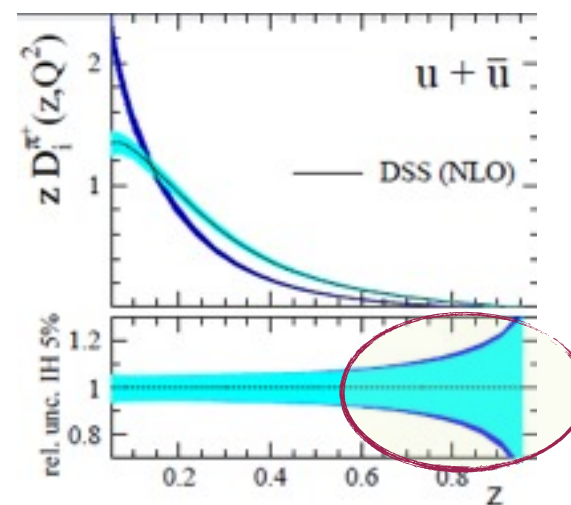


# $e^+e^-$ data

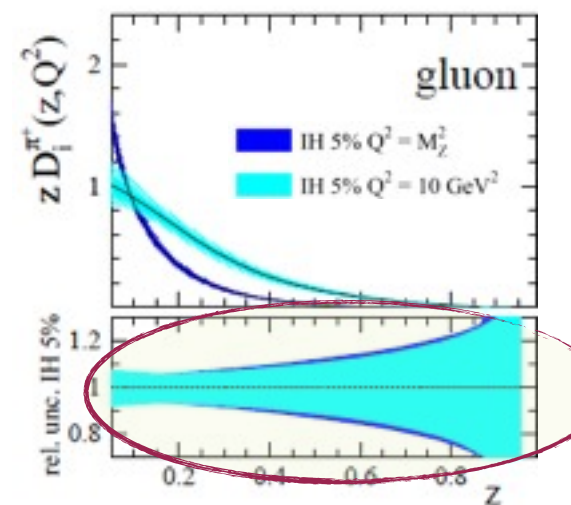
World Data (Sel.) for  $e^+e^- \rightarrow \pi^{+-} + X$ , Multiplicities



Epele, Llubaroff, Sassot, Stratmann  
arXiv:1209.3240 [hep-ph]



Few data at high  $z$



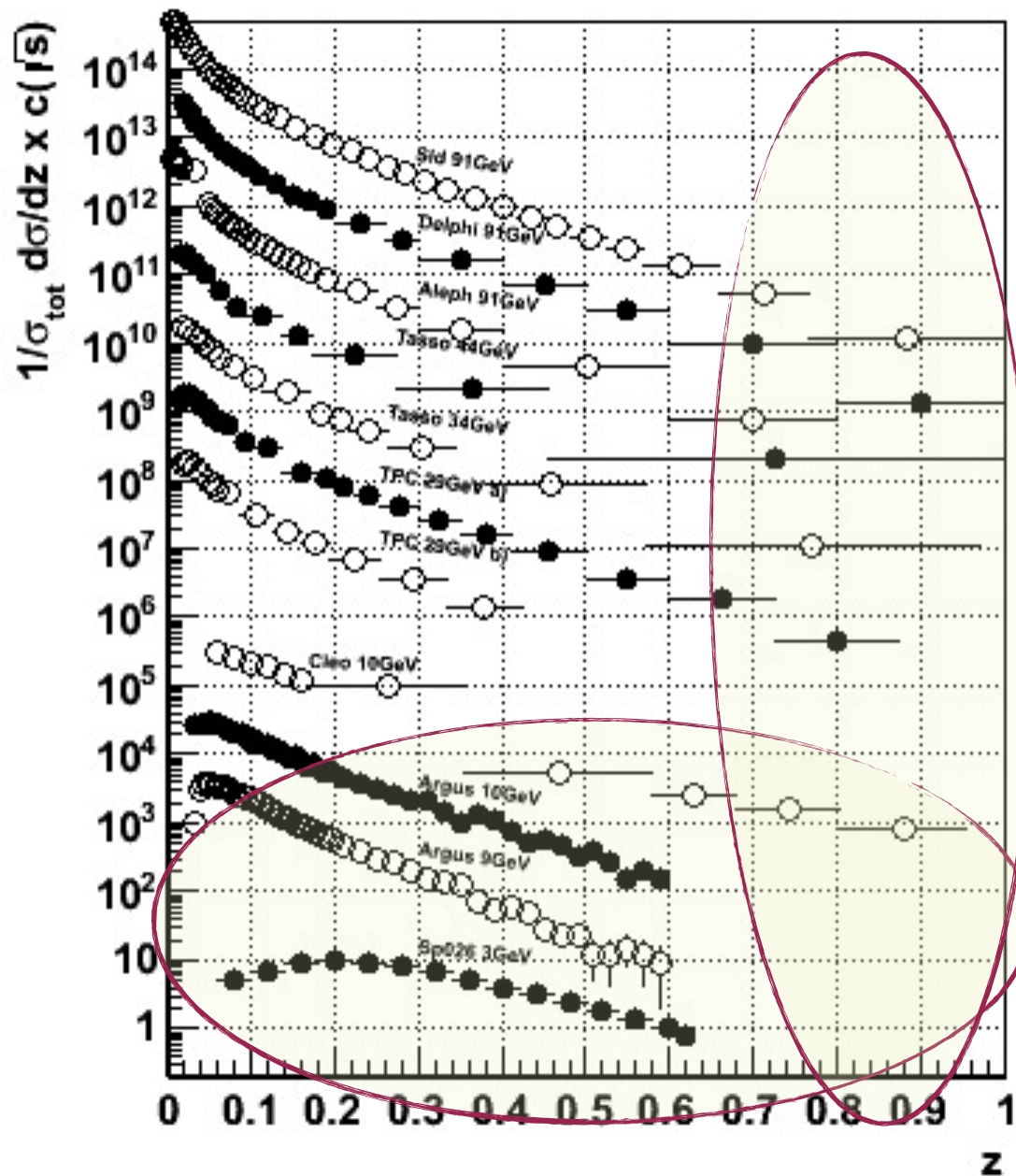
Few data at low energy



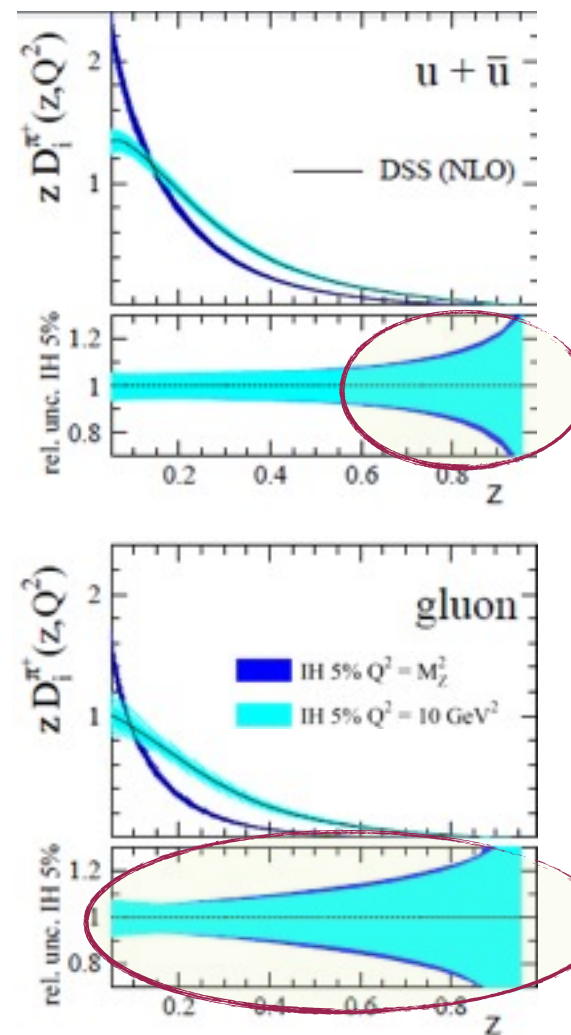


# $e^+e^-$ data

World Data (Sel.) for  $e^+e^- \rightarrow \pi^{+-} + X$ , Multiplicities



Epele, Llubaroff, Sassot, Stratmann  
[arXiv:1209.3240 \[hep-ph\]](https://arxiv.org/abs/1209.3240)



Few data at high z



Few data at low energy





# BELLE @ KEKB

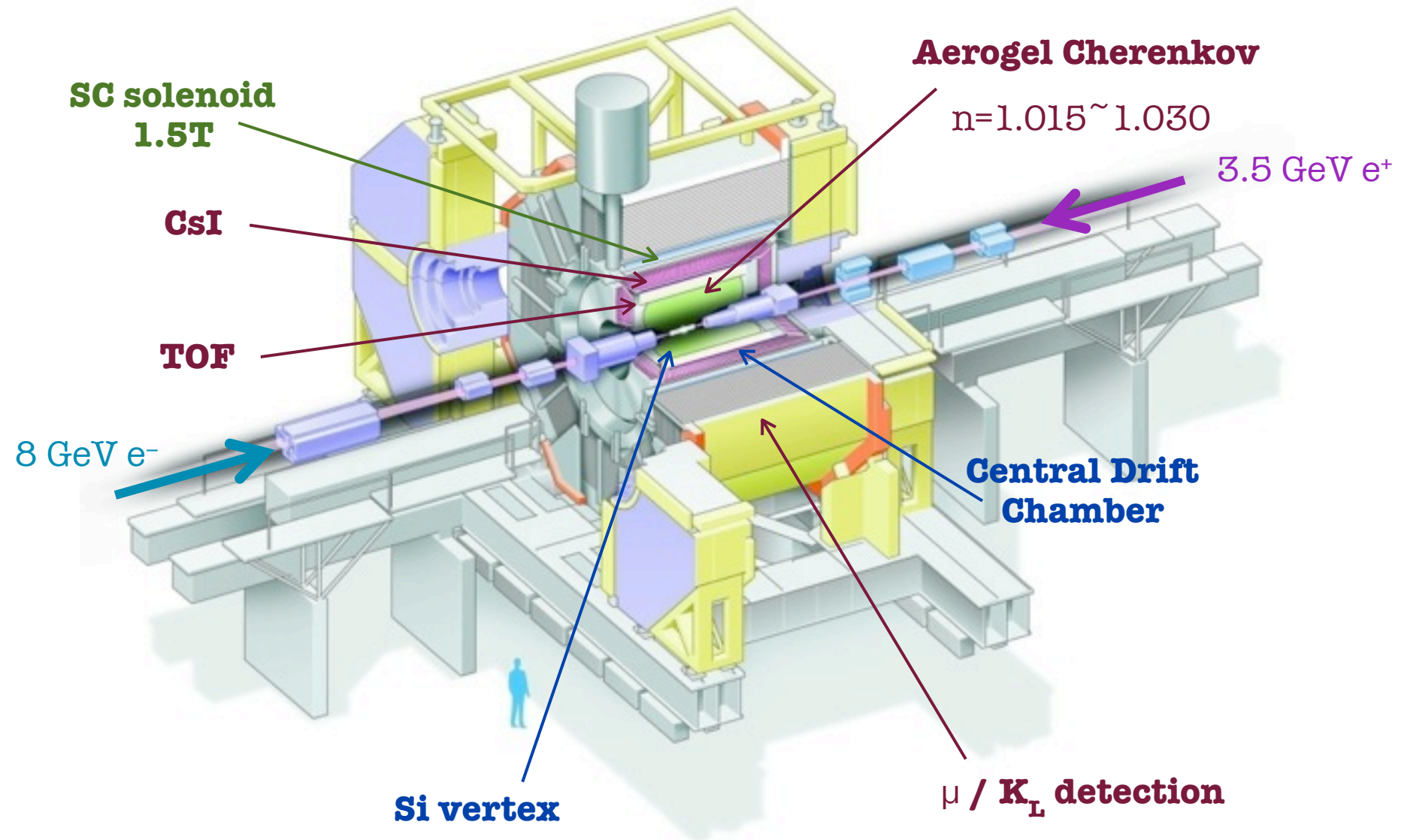
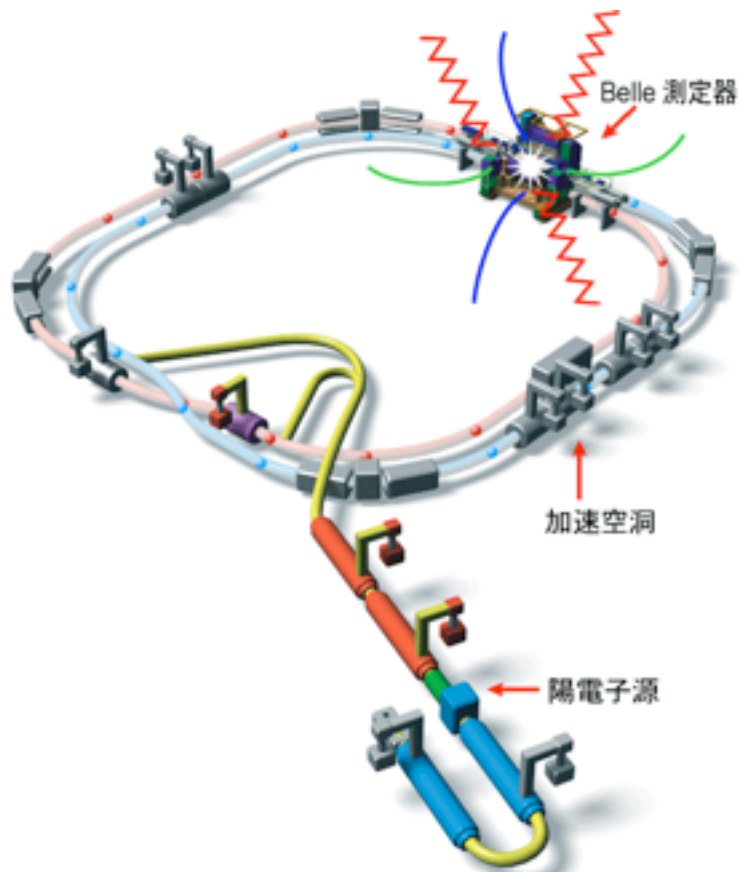


Asymmetric  
 $e^+$  (3.5 GeV)  $e^-$  (8 GeV) collider





# BELLE @ KEKB



Asymmetric  $e^+ e^-$  collider

On resonance:  $\sqrt{s} = 10.58 \text{ GeV}$  ( $e^+ e^- \rightarrow Y(4S) \rightarrow B\bar{B}$ )

Off resonance  $\sqrt{s} = 10.52 \text{ GeV}$  ( $e^+ e^- \rightarrow q\bar{q}$  ( $q=u,d,s,c$ ))

$\sim 220 \cdot 10^6$  events

Good tracking  $\Theta [17^\circ; 150^\circ]$

Good PID:  $\epsilon(\pi) \gtrsim 90\%$

$\epsilon(K) \gtrsim 85\%$



# Cross sections extraction

$$i = \pi, K$$

$$\frac{d\sigma_i}{dz} = \frac{1}{L_{tot}} \epsilon_{joint}^i(z) \epsilon_{ISR/FSR}^i(z) S_{zz_m}^{-1} \epsilon_{impu}^i(z_m) P_{ij}^{-1} N^{j,raw}(z_m)$$



# Cross sections extraction

$$i = \pi, K$$

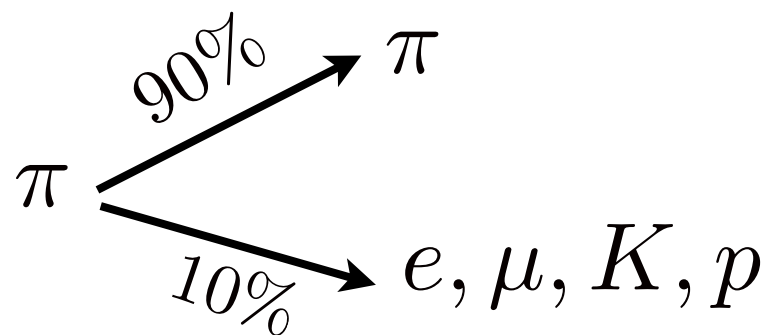
$$\frac{d\sigma_i}{dz} = \frac{1}{L_{tot}} \epsilon_{joint}^i(z) \epsilon_{ISR/FSR}^i(z) S_{zz_m}^{-1} \epsilon_{impu}^i(z_m) P_{ij}^{-1} N^{j,raw}(z_m)$$

Perfect PID  $\Leftrightarrow j = i$

$$j = e, \mu, \pi, K, p$$

**BUT!!**

$$\epsilon(\pi) \gtrsim 90\% \quad \epsilon(K) \gtrsim 85\%$$





# Cross sections extraction

$$i = \pi, K$$

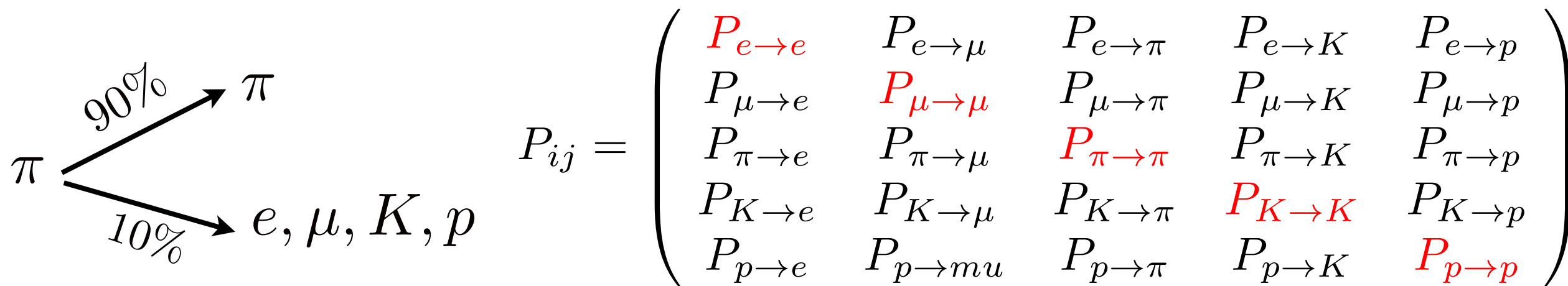
$$\frac{d\sigma_i}{dz} = \frac{1}{L_{tot}} \epsilon_{joint}^i(z) \epsilon_{ISR/FSR}^i(z) S_{zz_m}^{-1} \epsilon_{impu}^i(z_m) P_{ij}^{-1} N^{j,raw}(z_m)$$

Perfect PID  $\Leftrightarrow j = i$   $N^{j,raw} = P_{ij} N^i$

$$j = e, \mu, \pi, K, p$$

**BUT!!**

$$\epsilon(\pi) \gtrsim 90\% \quad \epsilon(K) \gtrsim 85\%$$



# Cross sections extraction

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$$\frac{d\sigma_i}{dz} = \frac{1}{L_{tot}} \epsilon_{joint}^i(z) \epsilon_{ISR/FSR}^i(z) S_{zz_m}^{-1} \epsilon_{impu}^i(z_m) P_{ij}^{-1} N^{j,raw}(z_m)$$

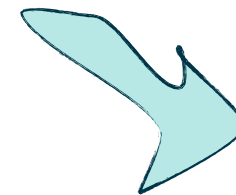
Perfect PID  $\Leftrightarrow j = i$

$$N^{j,raw} = P_{ij} N^i$$

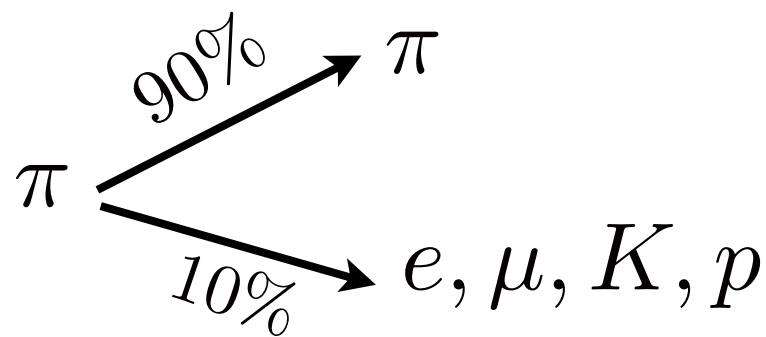
$$j = e, \mu, \pi, K, p$$

**BUT!!**

$\epsilon(\pi) \gtrsim 90\%$   $\epsilon(K) \gtrsim 85\%$



$$N^i = P_{ij}^{-1} N^{j,raw}$$

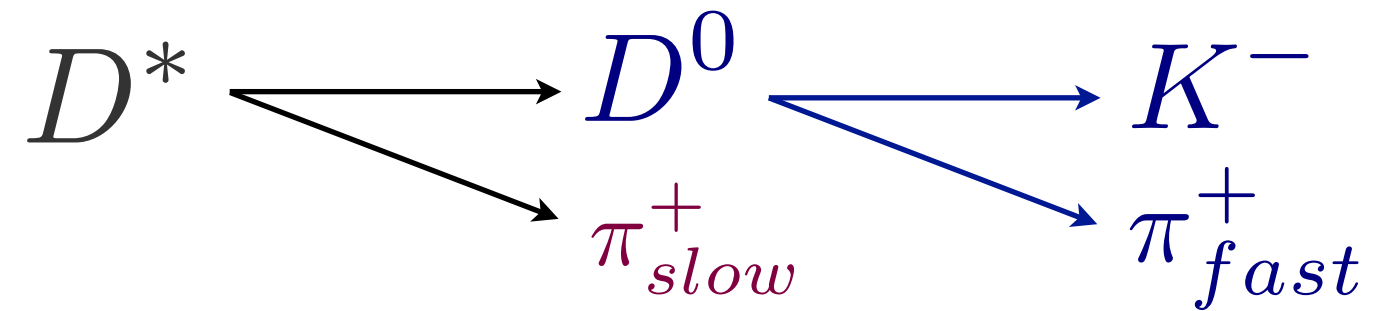


$$P_{ij} = \begin{pmatrix} P_{e \rightarrow e} & P_{e \rightarrow \mu} & P_{e \rightarrow \pi} & P_{e \rightarrow K} & P_{e \rightarrow p} \\ P_{\mu \rightarrow e} & P_{\mu \rightarrow \mu} & P_{\mu \rightarrow \pi} & P_{\mu \rightarrow K} & P_{\mu \rightarrow p} \\ P_{\pi \rightarrow e} & P_{\pi \rightarrow \mu} & P_{\pi \rightarrow \pi} & P_{\pi \rightarrow K} & P_{\pi \rightarrow p} \\ P_{K \rightarrow e} & P_{K \rightarrow \mu} & P_{K \rightarrow \pi} & P_{K \rightarrow K} & P_{K \rightarrow p} \\ P_{p \rightarrow e} & P_{p \rightarrow \mu} & P_{p \rightarrow \pi} & P_{p \rightarrow K} & P_{p \rightarrow p} \end{pmatrix}$$



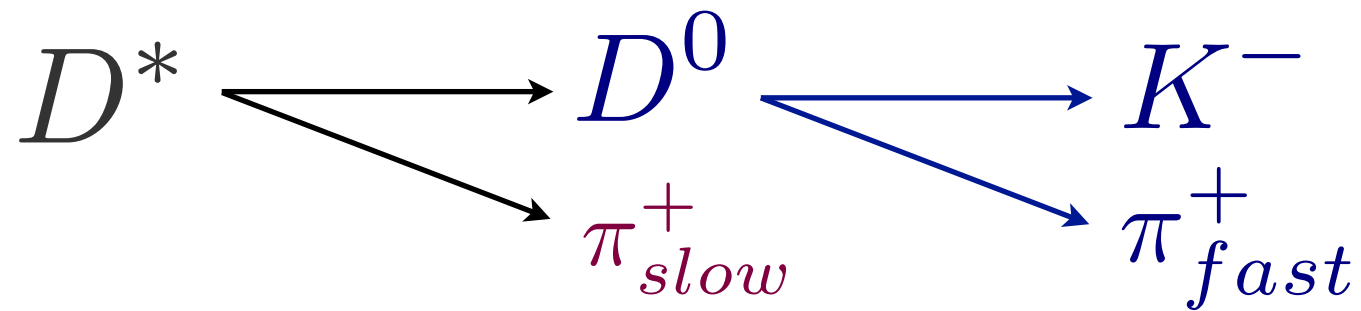
# How to determine the $P_{ij}$ ?

From data!



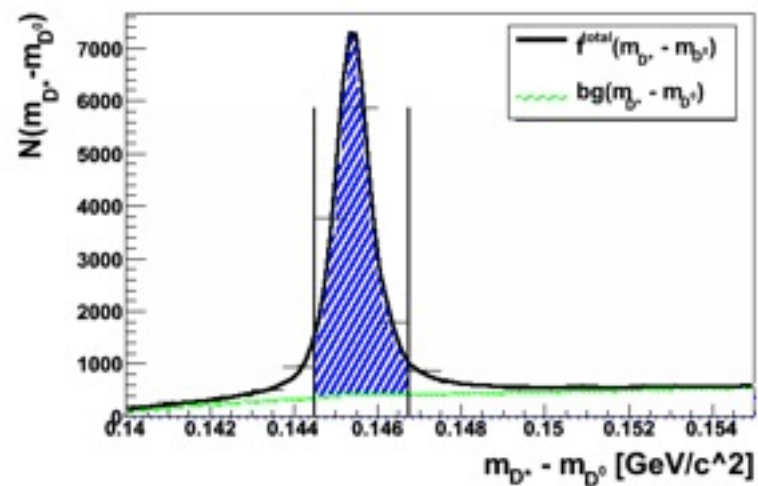
# How to determine the $P_{ij}$ ?

From data!



$$P_{K^- \rightarrow \pi^-}$$

$$m_D^* - m_D^0$$

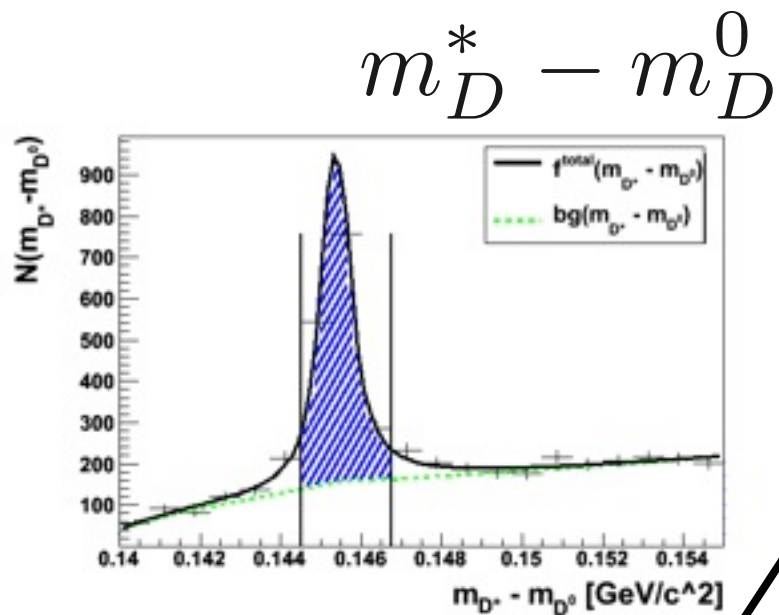
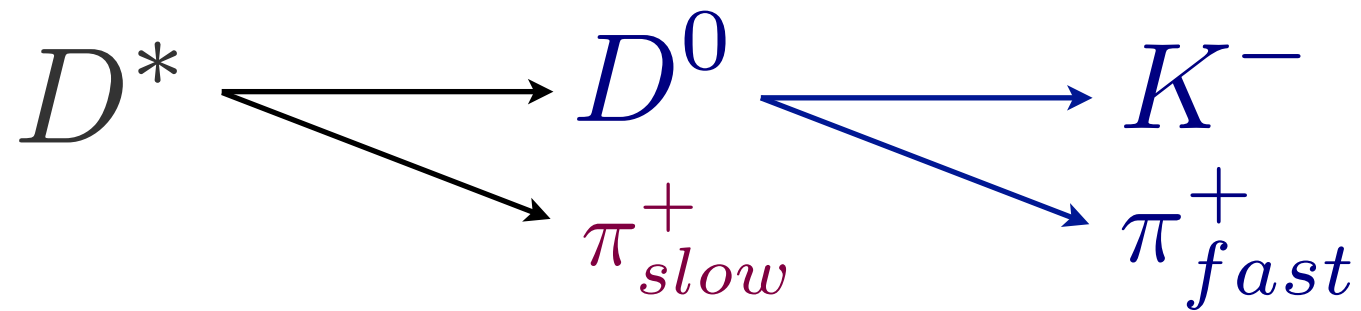


Negative hadron =  $K^-$   
(no PID likelihood used)



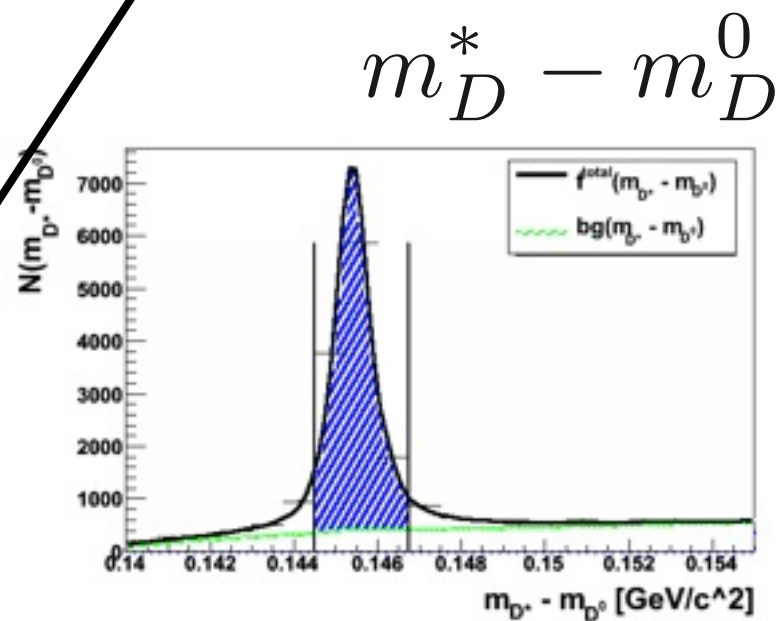
# How to determine the $P_{ij}$ ?

From data!



Negative hadron identified as  $\pi^-$

$$P_{K^- \rightarrow \pi^-}$$

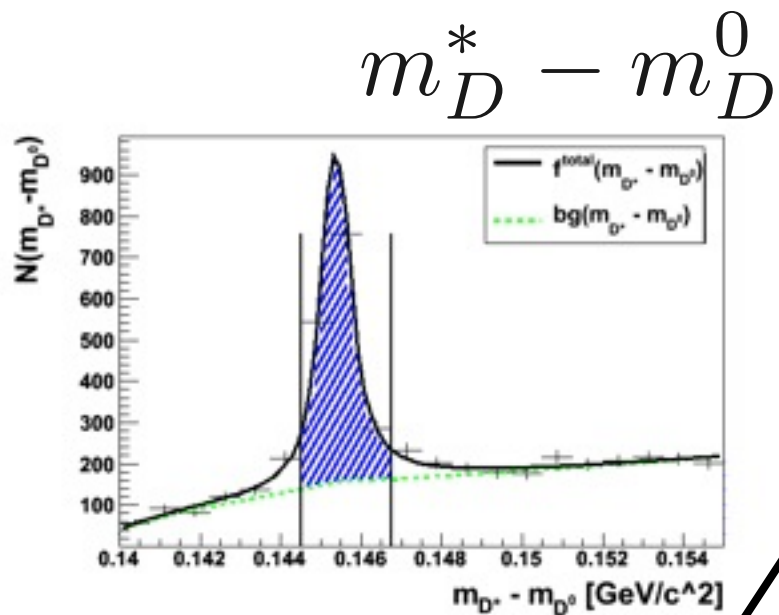
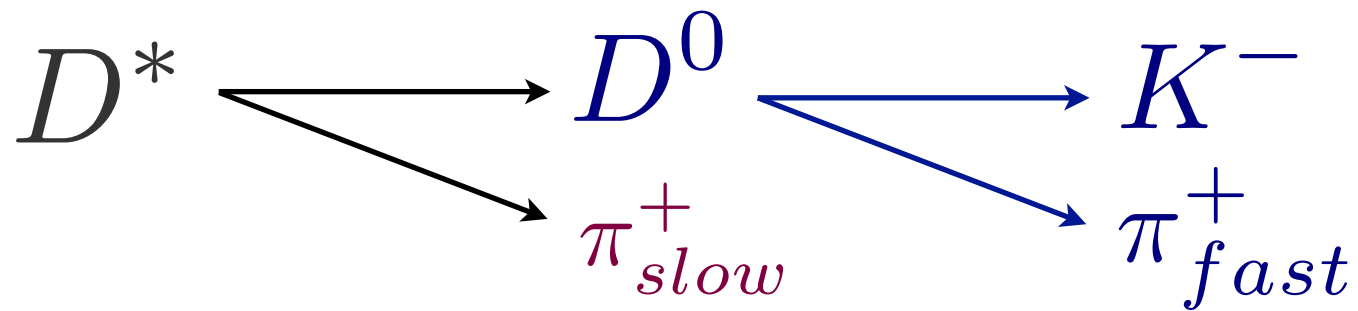


Negative hadron =  $K^-$   
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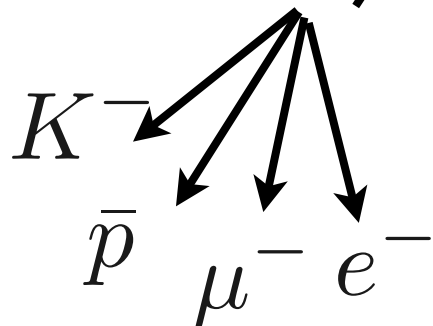


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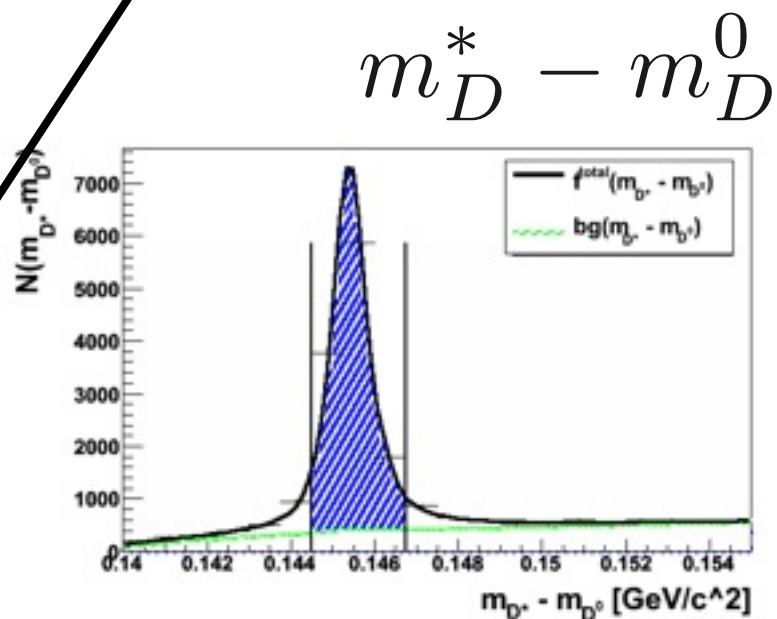
From data!



Negative hadron identified as  $\pi^-$



$$P_{K^- \rightarrow \pi^-}$$



Negative hadron =  $K^-$   
(no PID likelihood used)

$$P_{K^- \rightarrow K^-}$$

$$P_{K^- \rightarrow \bar{p}}$$

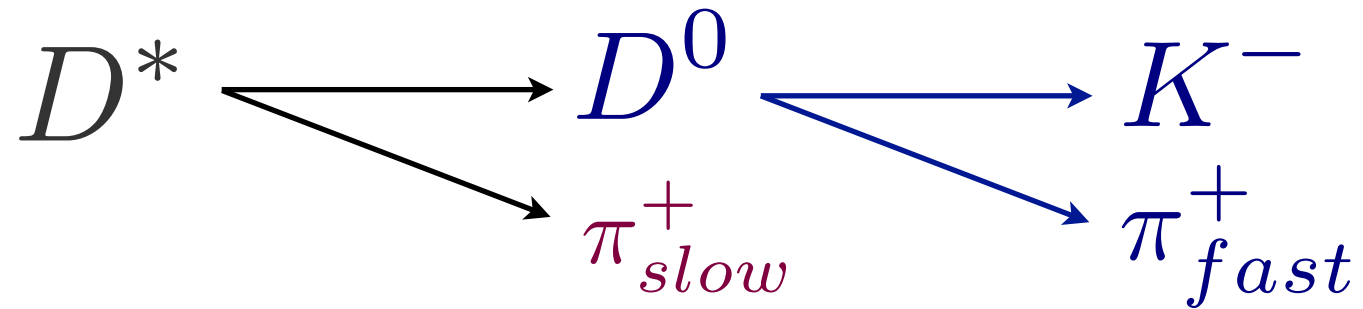
$$P_{K^- \rightarrow \mu^-}$$

$$P_{K^- \rightarrow e^-}$$



# How to determine the $P_{ij}$ ?

From data!



$\mathbf{P}_{\pi, K \rightarrow j}$  from  $D^*$  decay

$\mathbf{P}_{\pi, p \rightarrow j}$  from  $\Lambda$  decay

$\mathbf{P}_{e, \mu \rightarrow j}$  from  $J/\psi$  decay

$$\begin{pmatrix}
 P_{e \rightarrow e} & P_{e \rightarrow \mu} & P_{e \rightarrow \pi} & P_{e \rightarrow K} & P_{e \rightarrow p} \\
 P_{\mu \rightarrow e} & P_{\mu \rightarrow \mu} & P_{\mu \rightarrow \pi} & P_{\mu \rightarrow K} & P_{\mu \rightarrow p} \\
 P_{\pi \rightarrow e} & P_{\pi \rightarrow \mu} & P_{\pi \rightarrow \pi} & P_{\pi \rightarrow K} & P_{\pi \rightarrow p} \\
 P_{K \rightarrow e} & P_{K \rightarrow \mu} & P_{K \rightarrow \pi} & P_{K \rightarrow K} & P_{K \rightarrow p} \\
 P_{p \rightarrow e} & P_{p \rightarrow \mu} & P_{p \rightarrow \pi} & P_{p \rightarrow K} & P_{p \rightarrow p}
 \end{pmatrix}$$

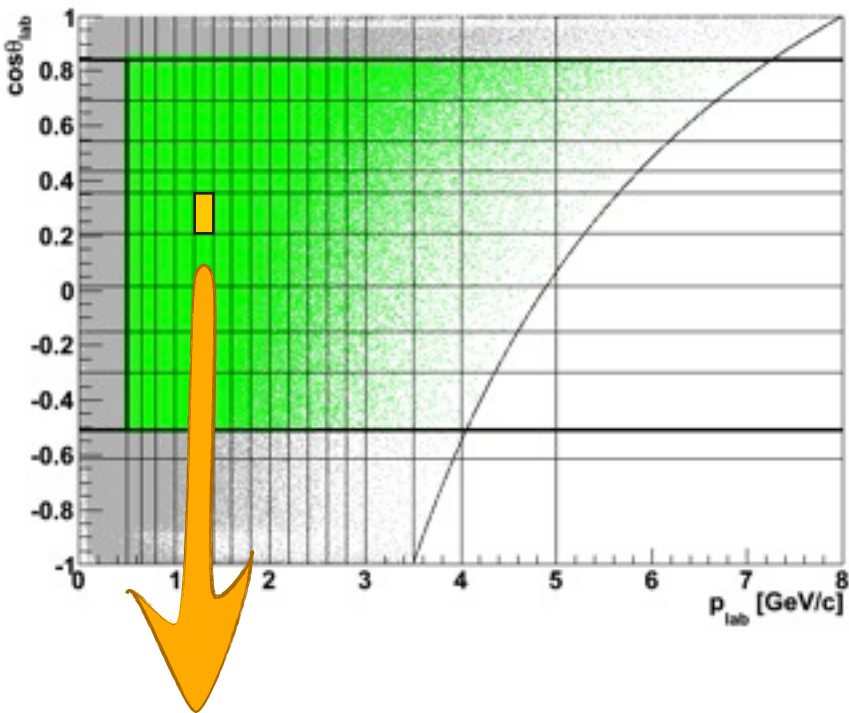
$$\begin{aligned}
 &P_{K^- \rightarrow \pi^-} \\
 &P_{K^- \rightarrow K^-} \\
 &P_{K^- \rightarrow \bar{p}} \\
 &P_{K^- \rightarrow \mu^-} \\
 &P_{K^- \rightarrow e^-}
 \end{aligned}$$





# 2D correction

Detector performance depends on momentum and scattering angle!

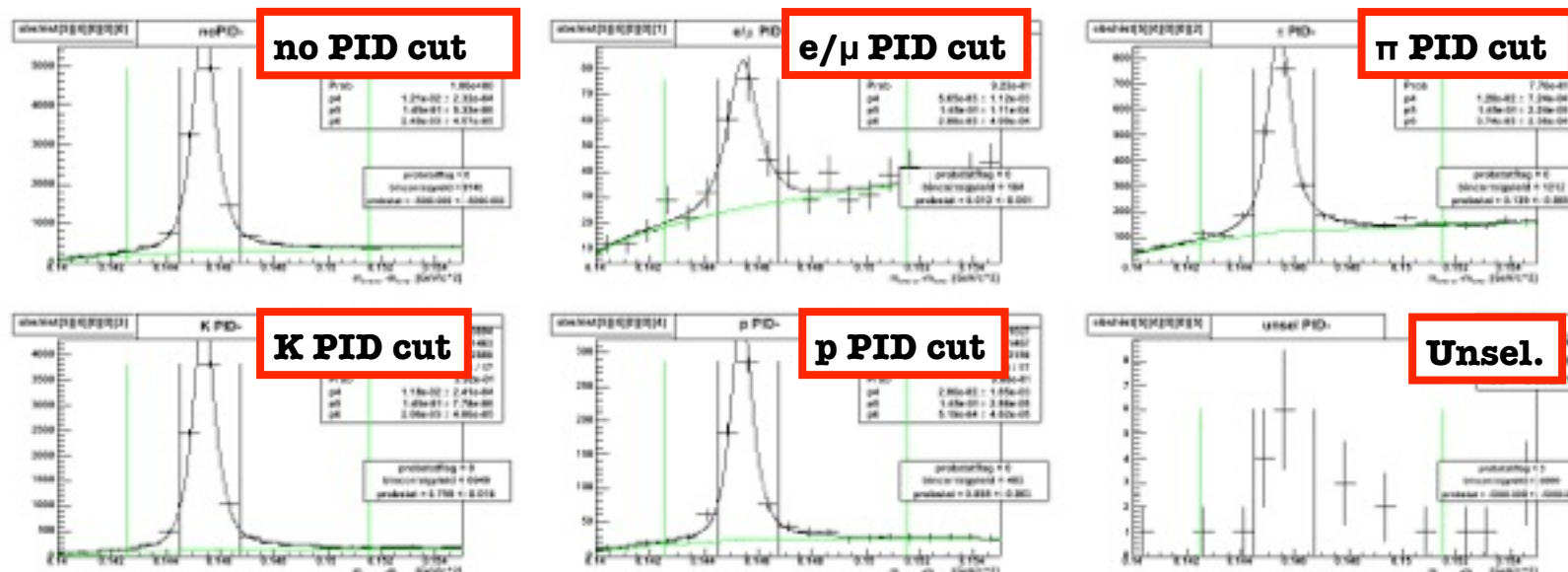


$$P_{ij} \Rightarrow P_{ij}(p, \theta)$$

Bin #	$\cos\theta_{\text{lab}}$ bin ranges
0	[-0.511,-0.300)
1	[-0.300,-0.152)
2	[-0.512,0.017)
3	[0.017,0.209)
4	[0.209,0.355)
5	[0.355,0.435)
6	[0.435,0.542)
7	[0.542,0.692)
8	[0.692,0.842)

Bin #	$p_{\text{lab}}$ [GeV/c] bin ranges
0	[0.5,0.65)
1	[0.65,0.8)
2	[0.8,1.0)
3	[1.0,1.2)
4	[1.2,1.4)
5	[1.4,1.6)
6	[1.6,1.8)
7	[1.8,2.0)
8	[2.0,2.2)
9	[2.2,2.4)
10	[2.4,2.6)
11	[2.6,2.8)
12	[2.8,3.0)
13	[3.0,3.5)
14	[3.5,4.0)
15	[4.0,5.0)
16	[5.0,8.0)

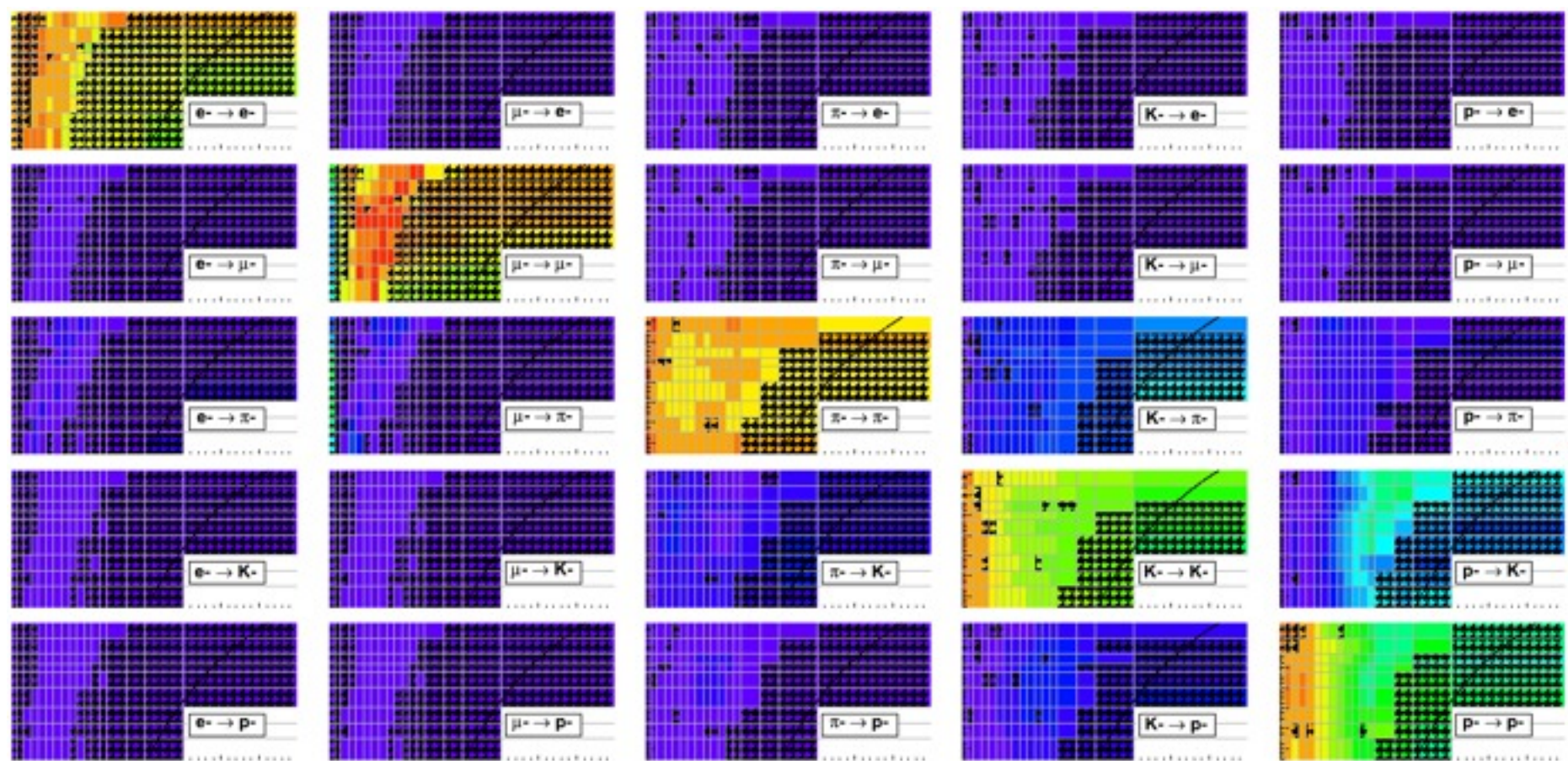
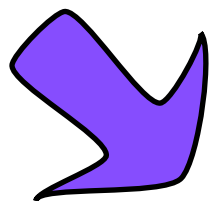
K from  $D^*$  decay for  $p_{\text{lab}}$  in [1.4,1.6) and  $\cos\theta_{\text{lab}}$  in [0.209,0.355)





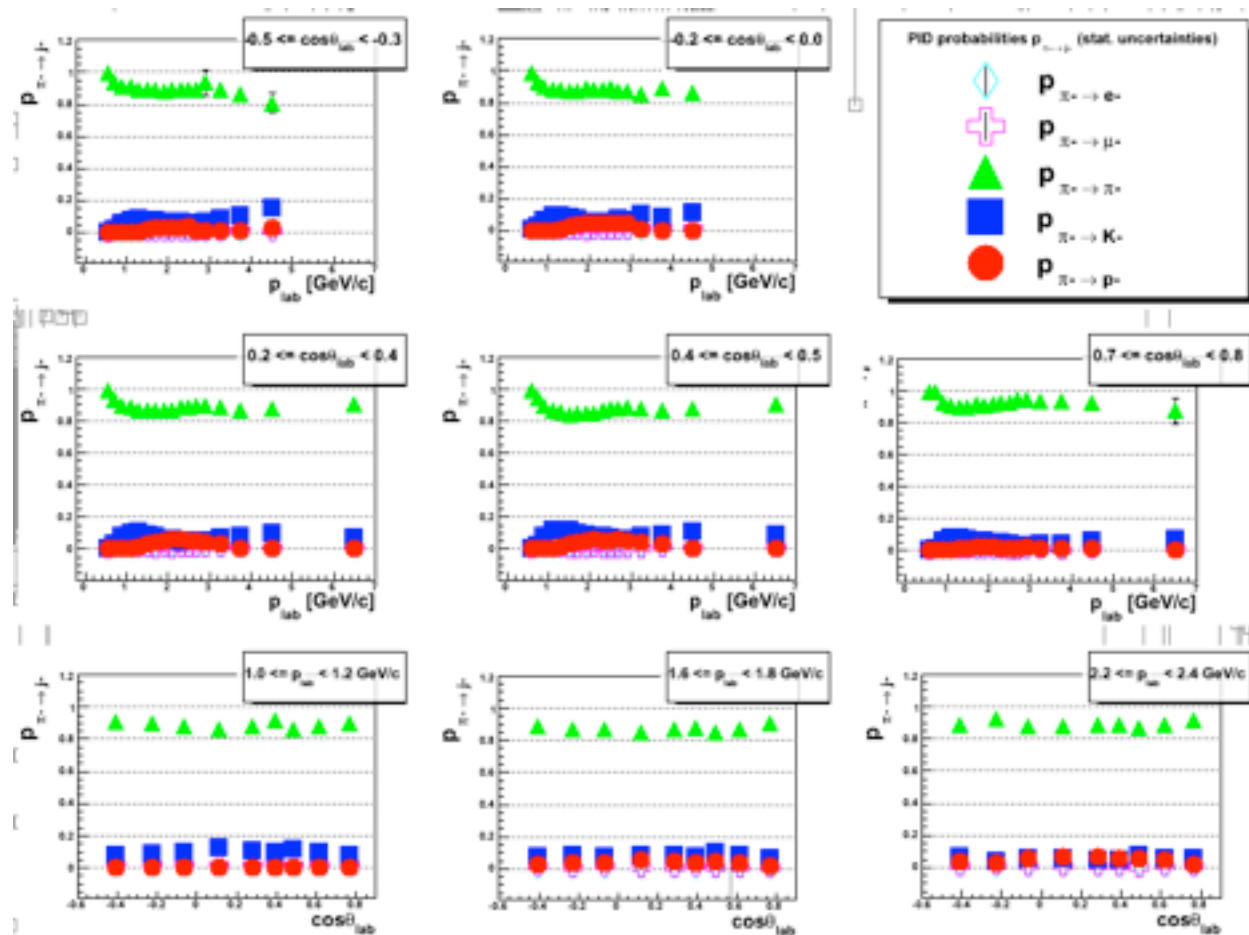
# 2D correction

$$\begin{pmatrix} P_{e \rightarrow e} & P_{e \rightarrow \mu} & P_{e \rightarrow \pi} & P_{e \rightarrow K} & P_{e \rightarrow p} \\ P_{\mu \rightarrow e} & P_{\mu \rightarrow \mu} & P_{\mu \rightarrow \pi} & P_{\mu \rightarrow K} & P_{\mu \rightarrow p} \\ P_{\pi \rightarrow e} & P_{\pi \rightarrow \mu} & P_{\pi \rightarrow \pi} & P_{\pi \rightarrow K} & P_{\pi \rightarrow p} \\ P_{K \rightarrow e} & P_{K \rightarrow \mu} & P_{K \rightarrow \pi} & P_{K \rightarrow K} & P_{K \rightarrow p} \\ P_{p \rightarrow e} & P_{p \rightarrow \mu} & P_{p \rightarrow \pi} & P_{p \rightarrow K} & P_{p \rightarrow p} \end{pmatrix}$$

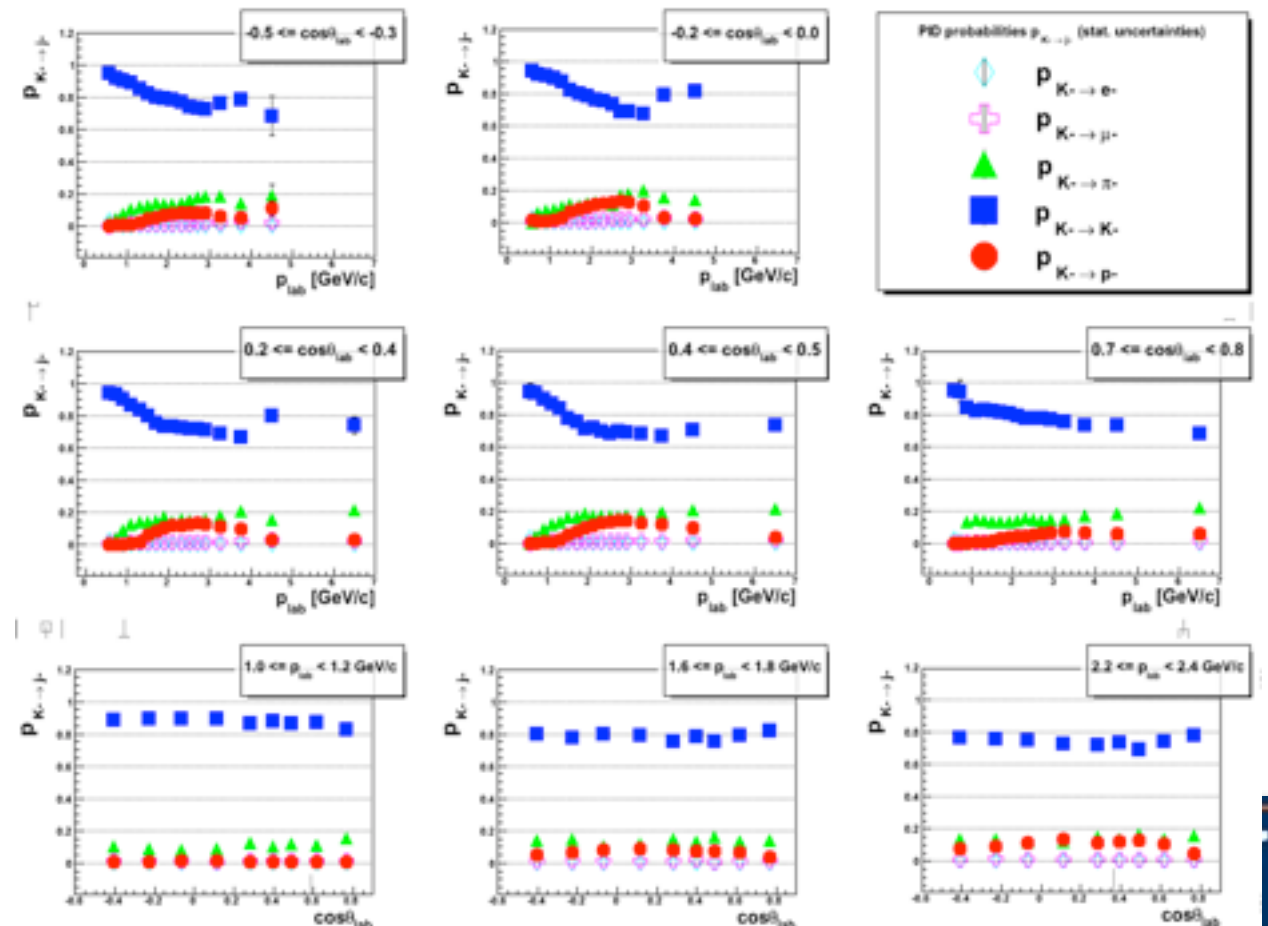


# PID correction

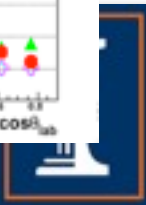
$\pi^-$   
 $< 15\%$



$K^-$



$30\% >$



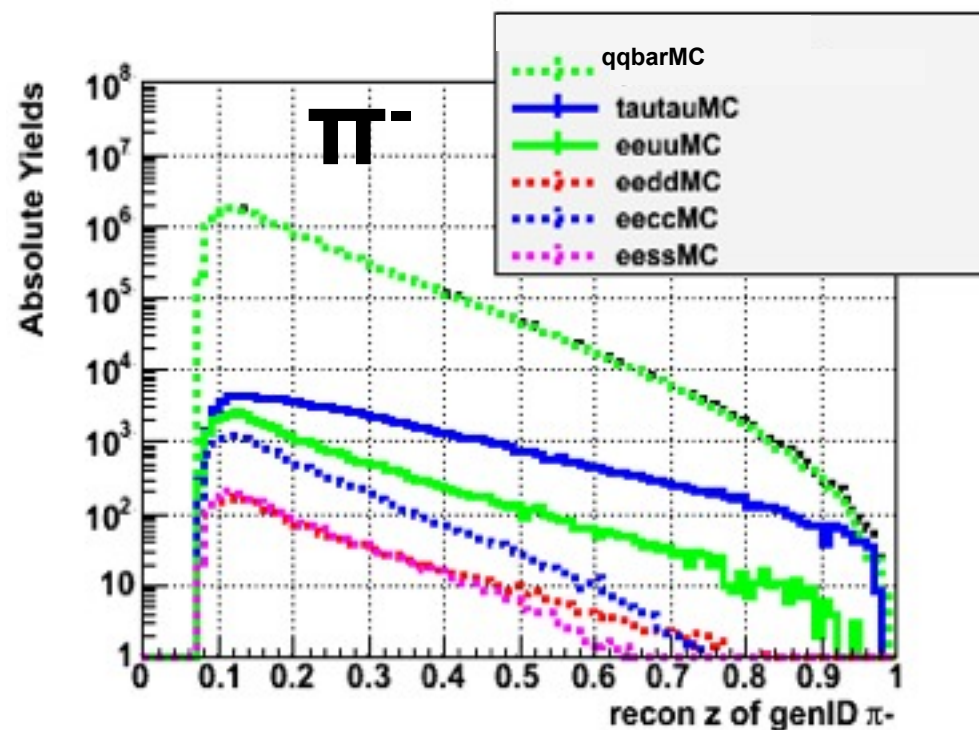


# Impurity correction

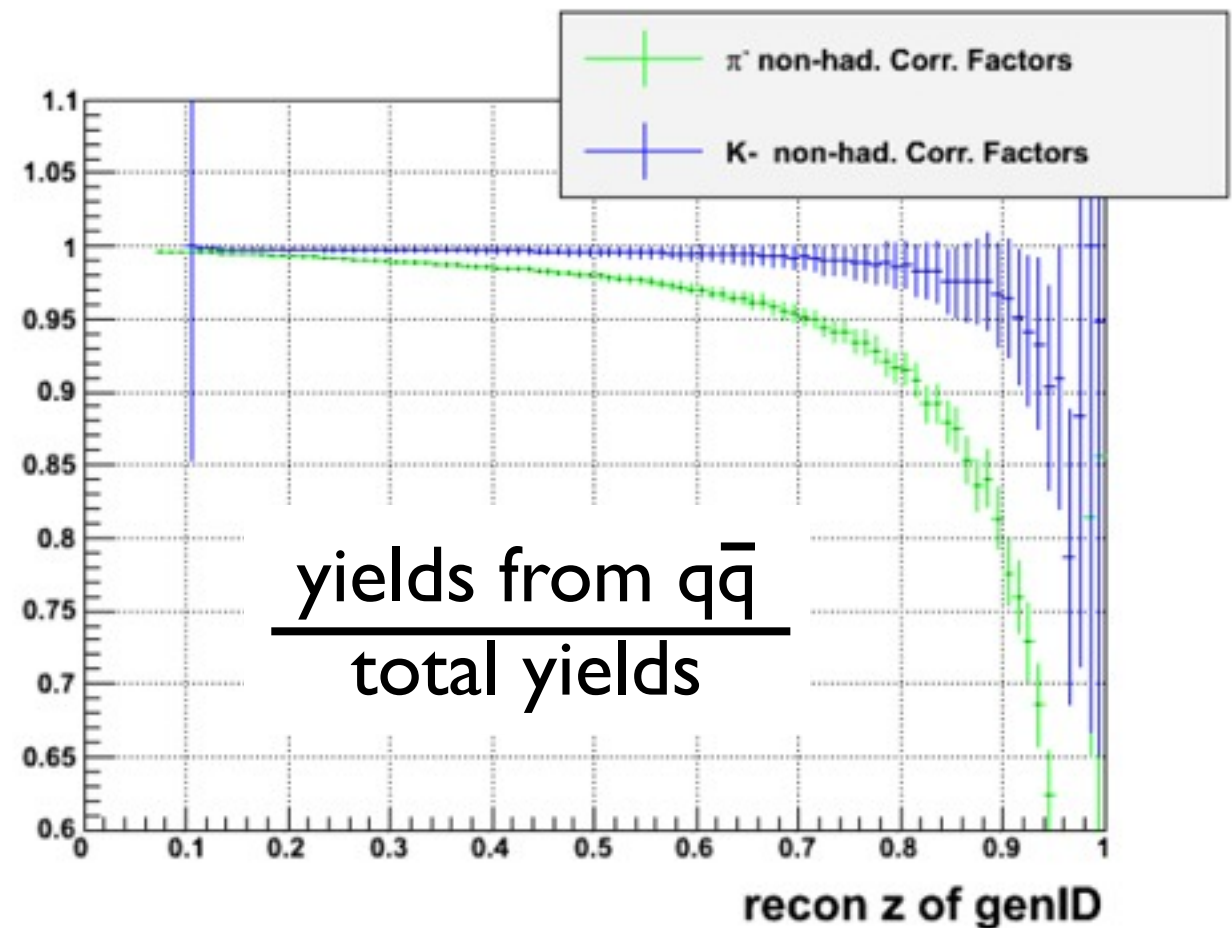
$$i = \pi, K$$

$$\frac{d\sigma_i}{dz} = \frac{1}{L_{tot}} \epsilon_{joint}^i(z) \epsilon_{ISR/FSR}^i(z) S_{zz_m}^{-1} \epsilon_{impu}^i(z_m) P_{ij}^{-1} N^{j,raw}(z_m)$$

Correction for hadrons  
generated by  $\tau, 2\gamma$



non-had. Corr. Factors



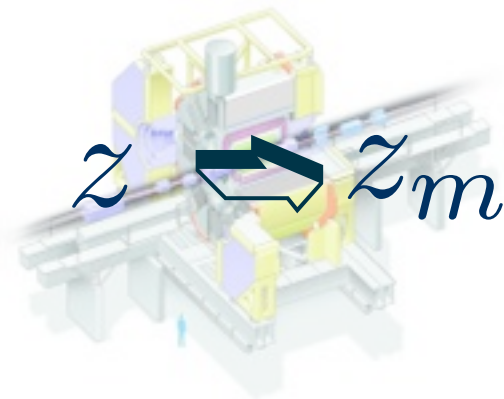
yields from  $q\bar{q}$   
total yields

# Smearing correction

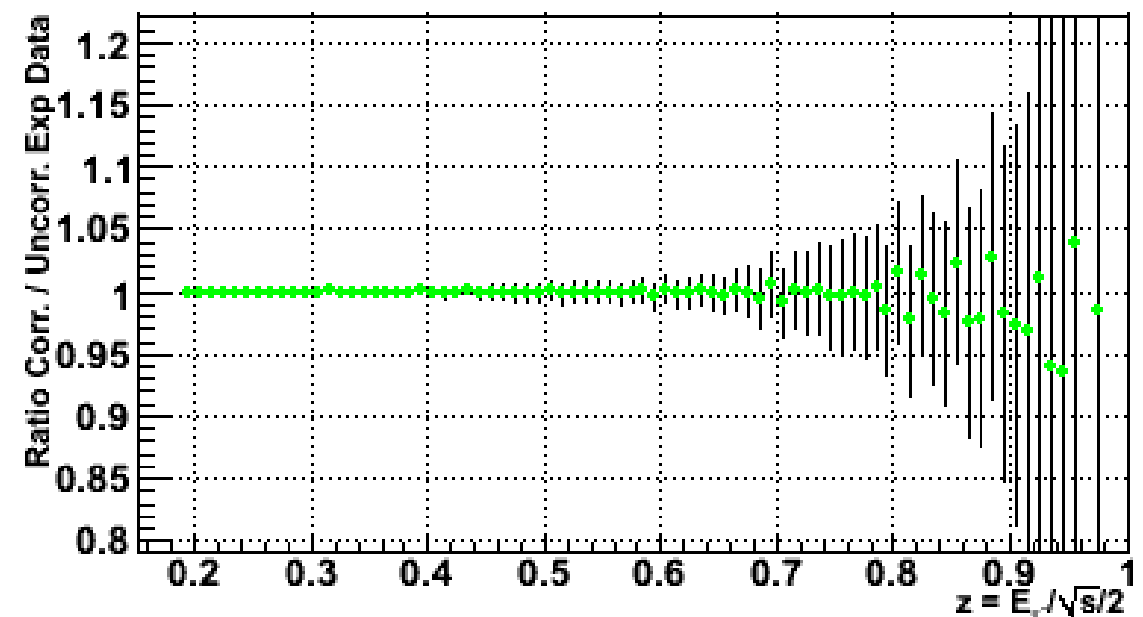
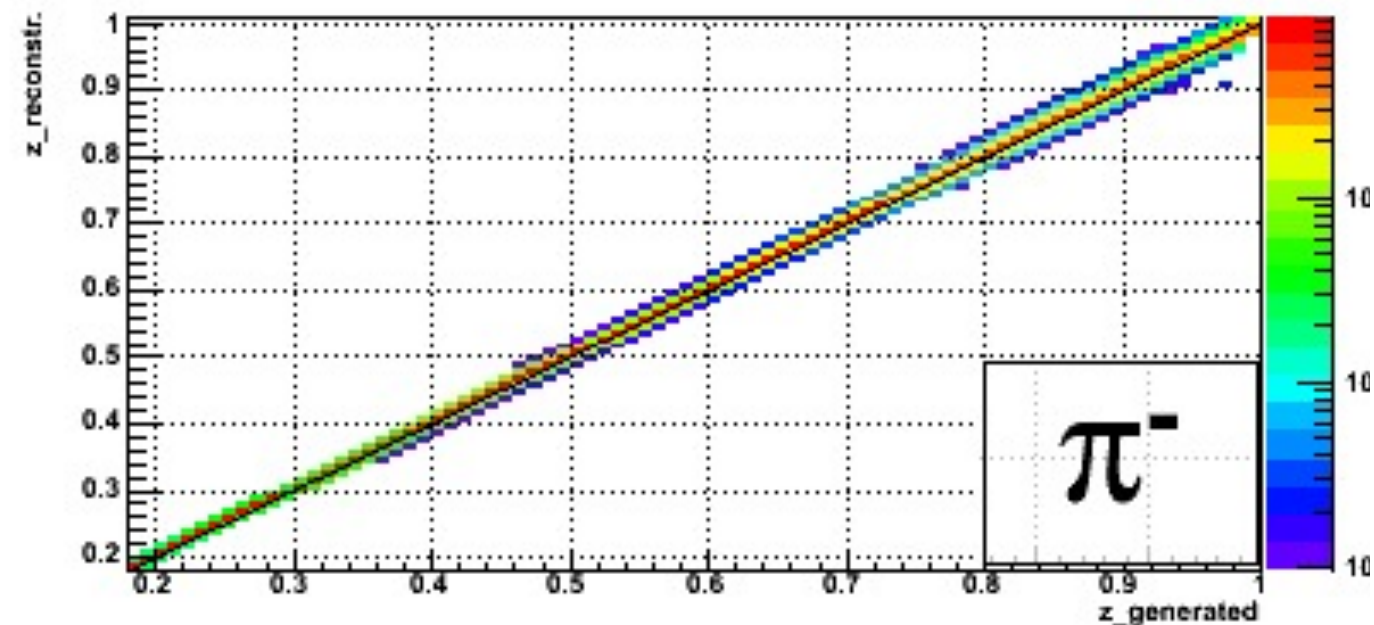
$$i = \pi, K$$

$$\frac{d\sigma_i}{dz} = \frac{1}{L_{tot}} \epsilon_{joint}^i(z) \epsilon_{ISR/FSR}^i(z) S_{zz_m}^{-1} \epsilon_{impu}^i(z_m) P_{ij}^{-1} N^{j,raw}(z_m)$$

Detector smearing correction



before/after

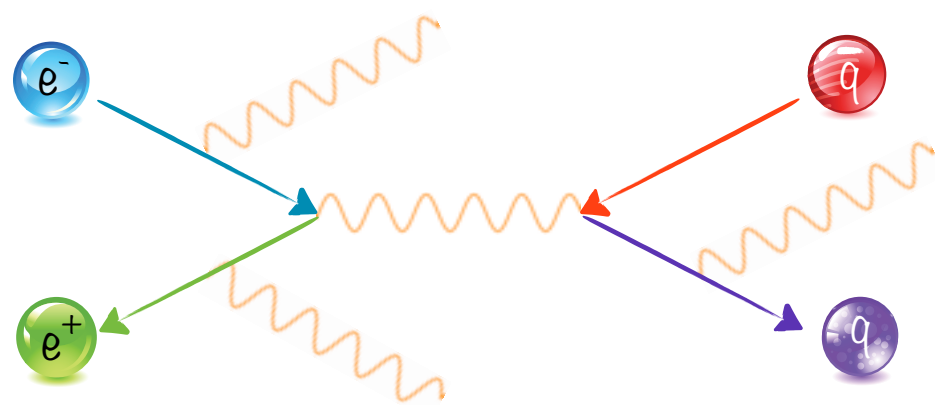


# ISR/FSR correction

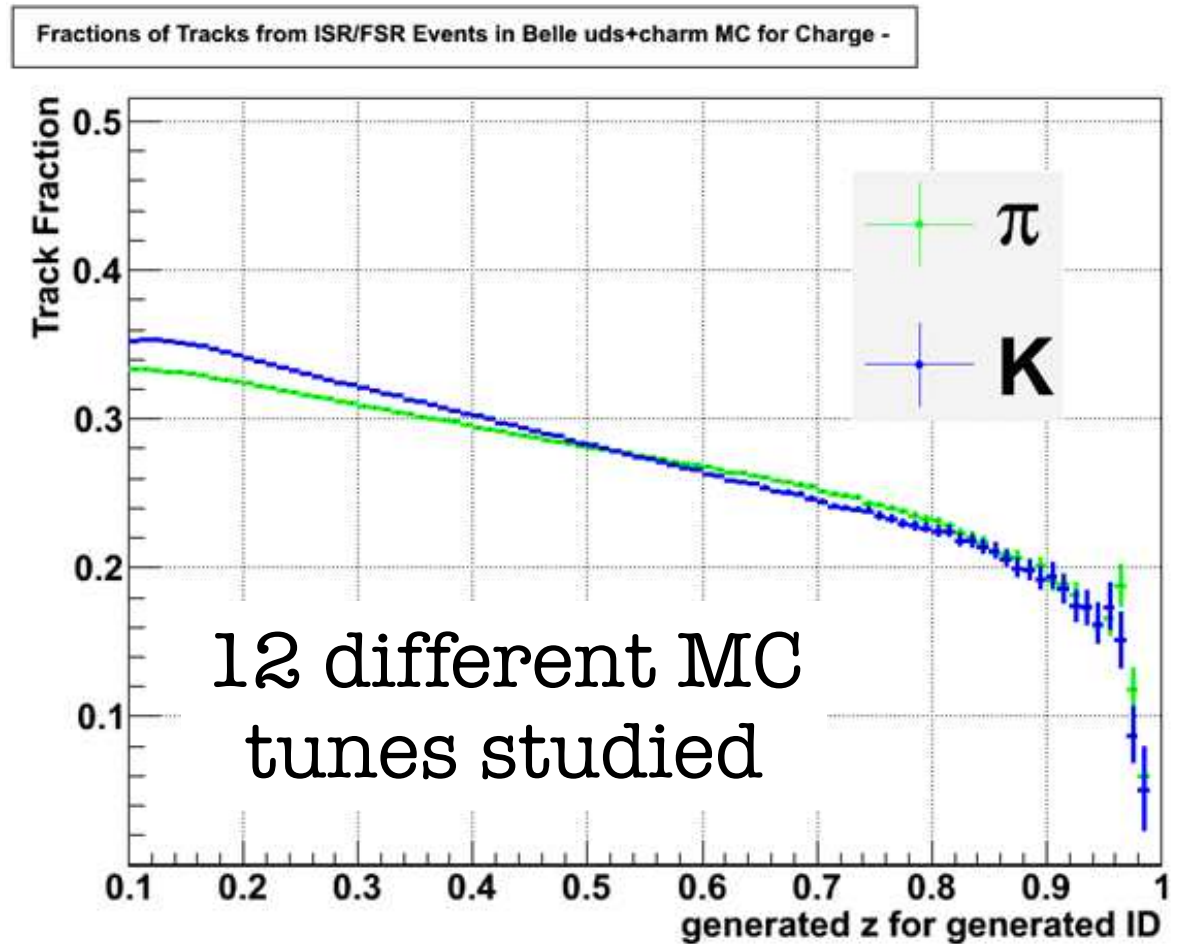
$$i = \pi, K$$

$$\frac{d\sigma_i}{dz} = \frac{1}{L_{tot}} \epsilon_{joint}^i(z) \epsilon_{ISR/FSR}^i(z) S_{zz_m}^{-1} \epsilon_{impu}^i(z_m) P_{ij}^{-1} N^{j,raw}(z_m)$$

Emission of a real photon changes the fragmentation energy scale



> 0.5% change in cms energy



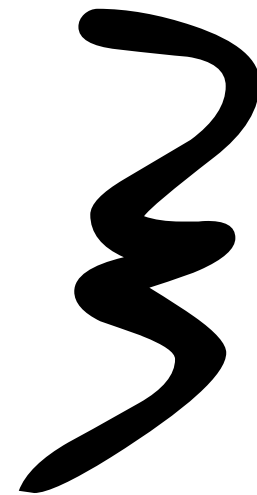
# More corrections

$$i = \pi, K$$

$$\frac{d\sigma_i}{dz} = \frac{1}{L_{tot}} \epsilon_{joint}^i(z) \epsilon_{ISR/FSR}^i(z) S_{zz_m}^{-1} \epsilon_{impu}^i(z_m) P_{ij}^{-1} N^{j,raw}(z_m)$$

Correction for particles lost due to

- decay in flight
- interaction with detectors
- detector/tracking inefficiencies
- geometric/kinematic acceptance

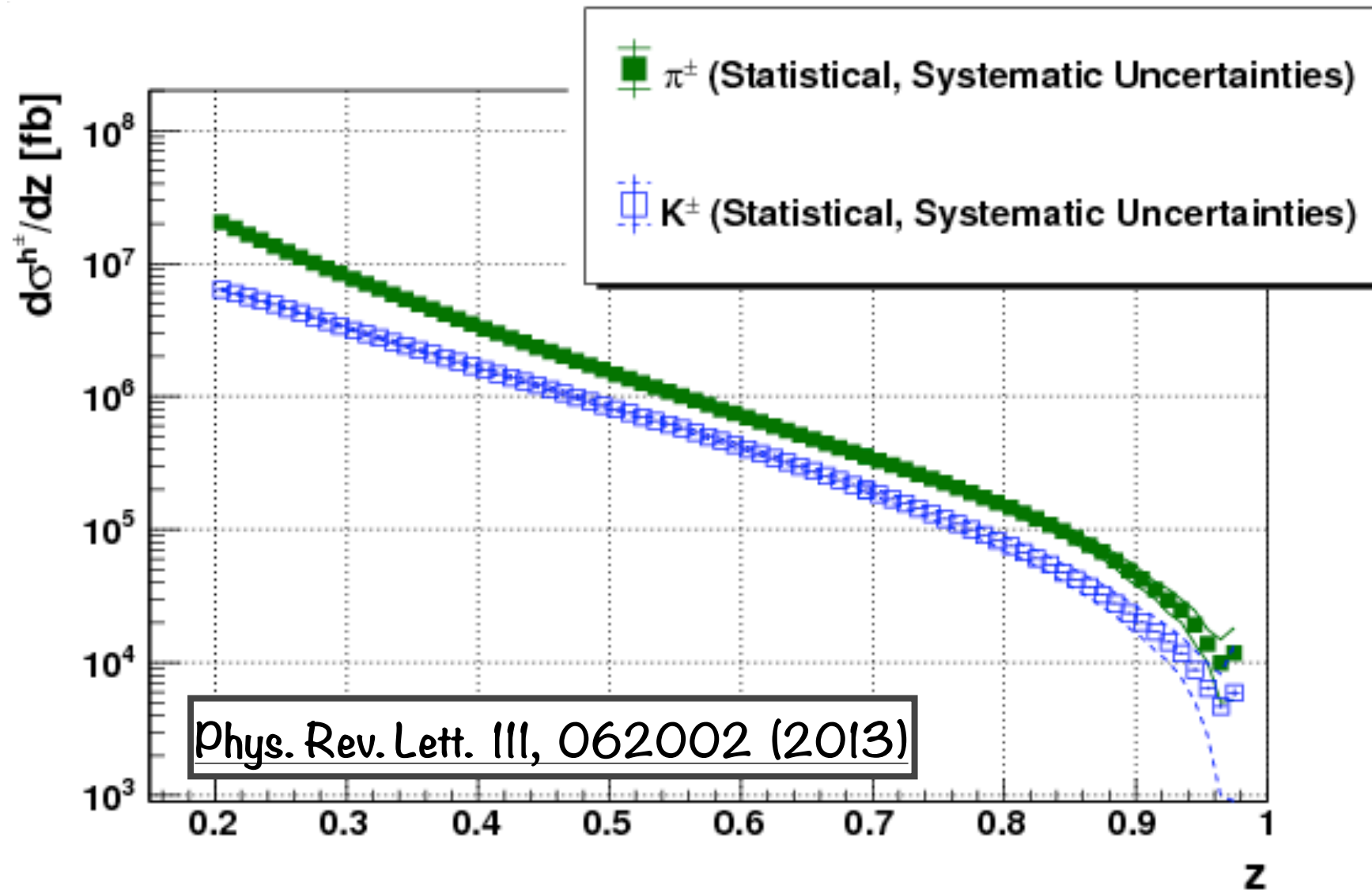
 < 10%



# Cross sections

$i = \pi, K$

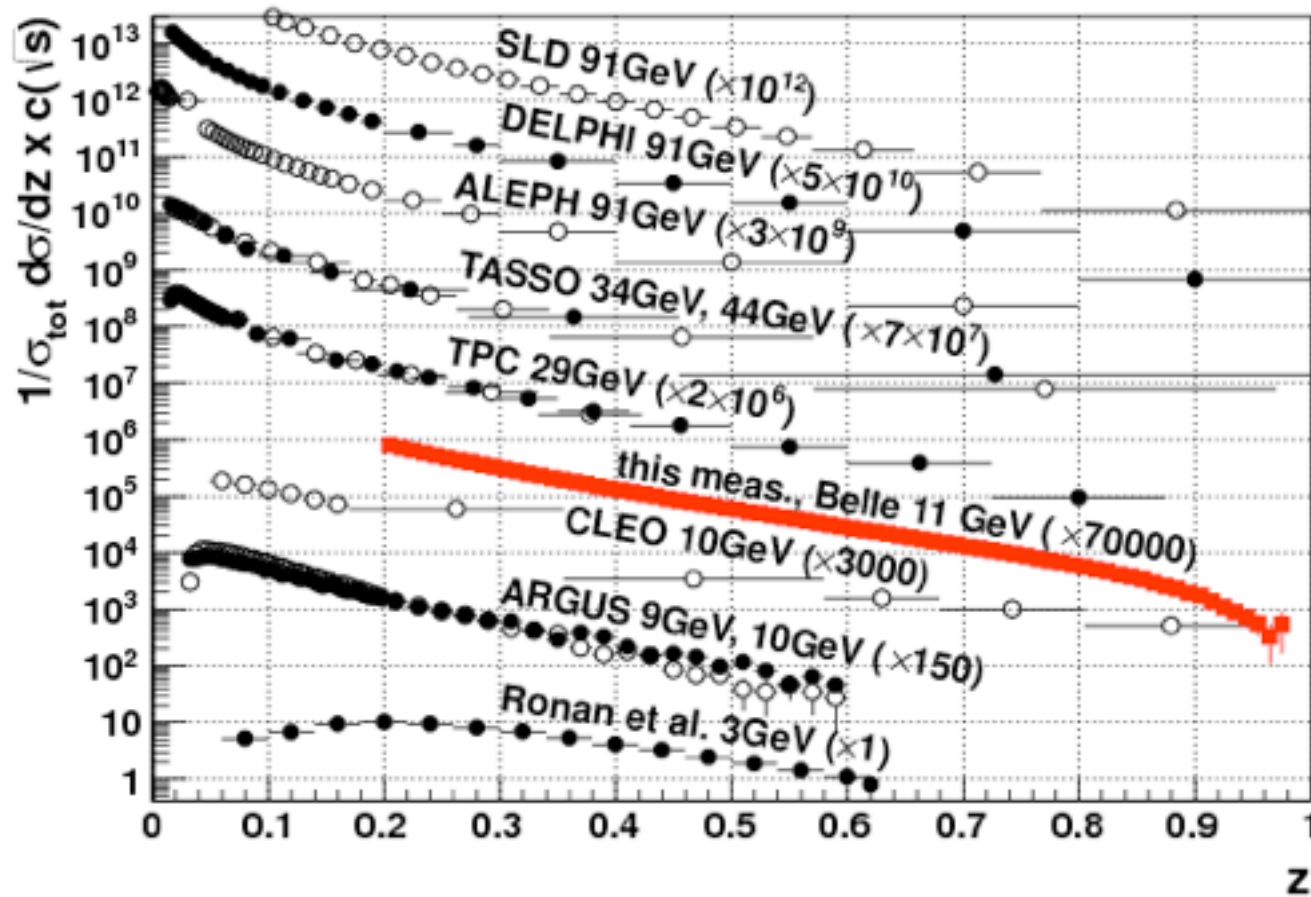
$$\frac{d\sigma_i}{dz} = \frac{1}{L_{tot}} \epsilon_{joint}^i(z) \epsilon_{ISR/FSR}^i(z) S_{zz_m}^{-1} \epsilon_{impu}^i(z_m) P_{ij}^{-1} N^{j,raw}(z_m)$$



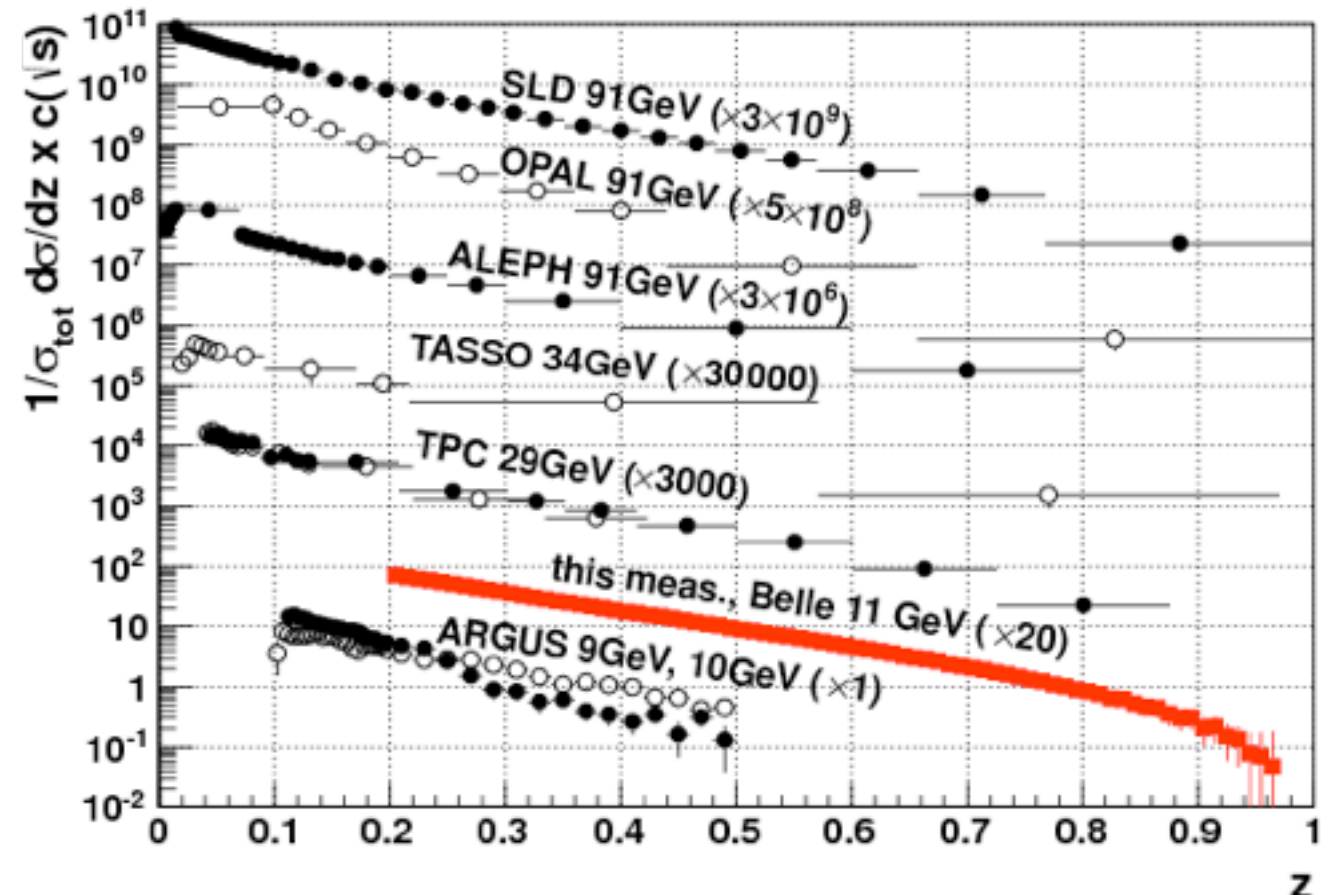


# Updated $e^+e^-$ world data

World Data (Sel.) for  $e^+e^- \rightarrow \pi^+ + X$  Multiplicities



World Data (Sel.) for  $e^+e^- \rightarrow K^+ + X$  Multiplicities




Phys. Rev. Lett. 111, 062002 (2013)





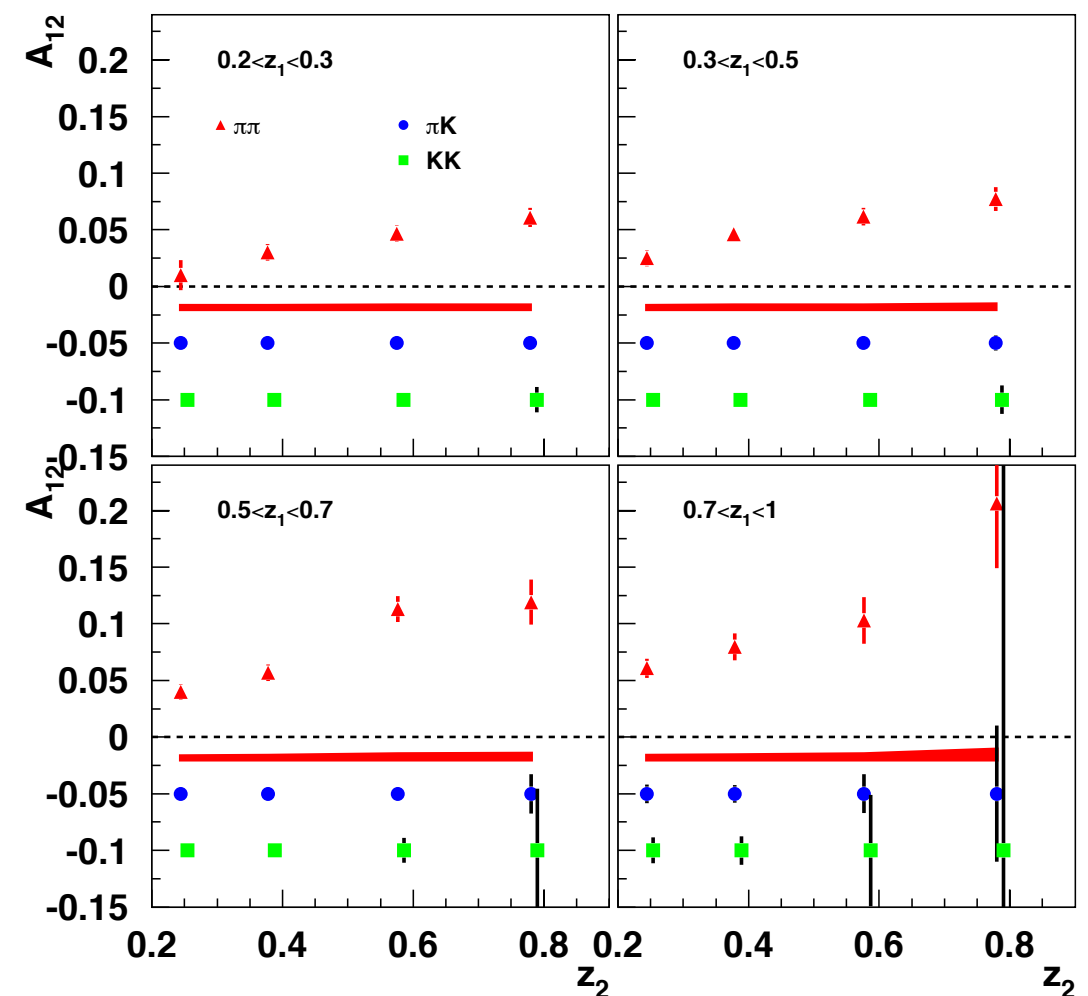
 Precise pion and kaon cross sections published!

Phys. Rev. Lett. 111, 062002 (2013)

 More high precision measurements to come

**$k_T$  spin-averaged FF for single and di-hadron**

**$k_T$  Collins FF for kaons**



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ON FRAGMENTATION  
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BLOOMINGTON, IN,  
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Daniel Boer, Elena Boglione, Francesca Giordano (co-chair),  
Matthias Grosse-Perdekamp, Marco Stratmann, Anselm Vossen (co-chair)

Registration and details:

<http://www.indiana.edu/~ffwrkshp/>

Thank you!

