How do we observe quarks and gluons orbital angular momentum distributions in the proton?

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Outline

1. The Spin Crisis and a better look into Energy Momentum Tensor of QCD and Sum Rules

2. Partonic interpretation

3. Measurements/Observables

4. Conclusions

Collaborators:

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Main references:

arXiv:1310.5157

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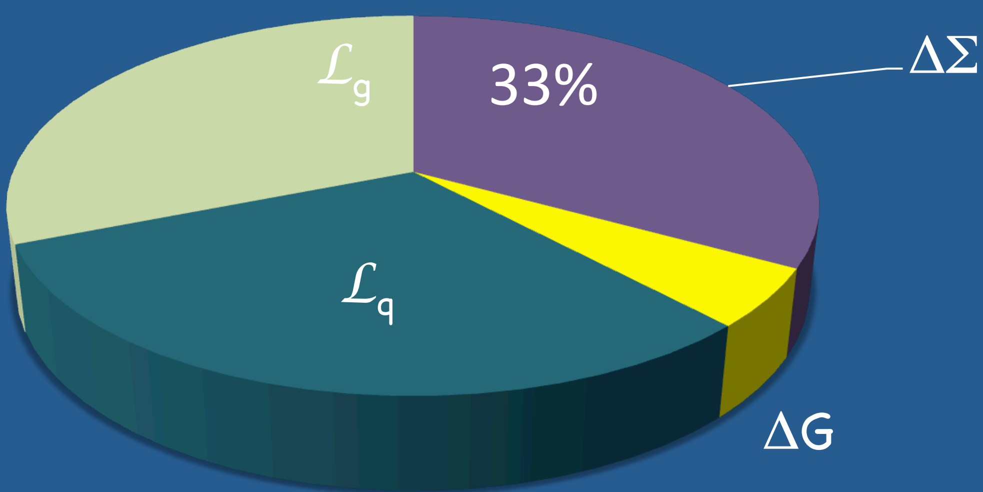
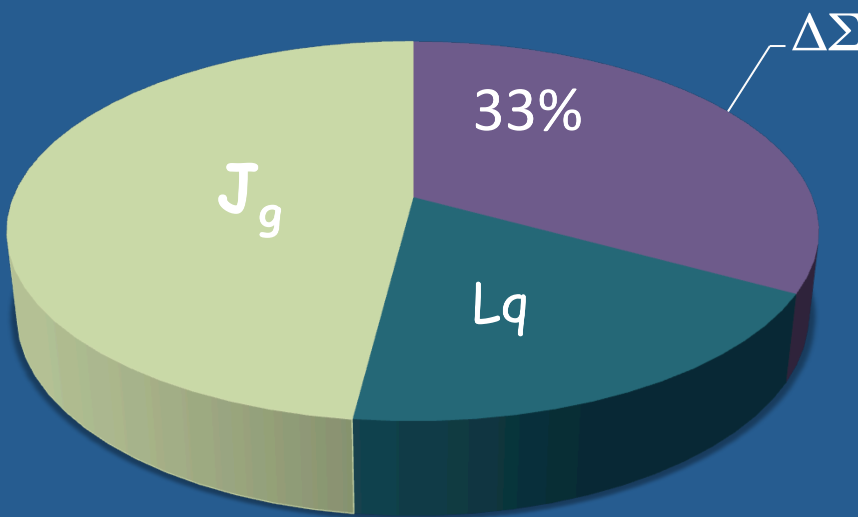
The spin crisis in a "cartoon"

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$





Jaffe Manohar

Ji

How does OAM enter the picture?

?

Define Angular Momentum through the QCD Energy Momentum Tensor

$$T^{\mu\nu} = \frac{1}{4} i q \bar{\psi}\left(\gamma^{\mu}\overleftrightarrow{D}^{\nu} + \gamma^{\nu}\overleftrightarrow{D}^{\mu}\right)\psi + Tr\left\{F^{\mu\alpha}F^{\nu}_{\alpha} - \frac{1}{2}g^{\mu\nu}F^{2}\right\} \rightarrow M^{\mu\nu\lambda} = x^{\nu}T^{\mu\lambda} - x^{\lambda}T^{\mu\nu}$$

AM density

Momentum density

Energy density

Momentum density

T^{00}	T^{01}	T^{02}	T^{03}
T^{10}	T^{11}	T^{12}	T^{13}
T^{20}	T^{21}	T^{22}	T^{23}
T^{30}	T^{31}	T^{32}	T^{33}

Shear stress

Pressure

Sum Rule in two parts

By first defining the angular momentum components

$$M^{\mu\nu\lambda} = x^{\nu} T^{\mu\lambda} - x^{\lambda} T^{\mu\nu} \quad \Rightarrow \quad J_q^i = \varepsilon^{ijk} \int dz^- d^2 z \; M^{+jk}$$

and parametrizing thr EMT in terms of form factors A, B, C

$$\langle p' | T^{\mu\nu}(0) | p \rangle = A(\gamma^{\mu} \bar{P}^{\nu} + \gamma^{\nu} \bar{P}^{\mu}) + B\left(\frac{i\sigma^{\mu\alpha}\Delta_{\alpha}}{2M} \bar{P}^{\nu} + \frac{i\sigma^{\nu\alpha}\Delta_{\alpha}}{2M} \bar{P}^{\mu} \right) + C\, \frac{\Delta^{\mu}\Delta^{\nu} - \Delta^2 g^{\mu\nu}}{M}$$

$$\Downarrow$$

Find connection of the EMT form factors with AM components

$$J_q = \frac{1}{2}\left(A_q + B_q\right) \Rightarrow \sum J_q + J_g = \frac{1}{2}$$

Jaffe Manohar (1990)
Ji (1997)

The second part of the sum rule is about finding a Partonic Interpretation and Observables

Ji: define new type of distribution functions for new process, DVCS

$$J_{q,g} = \frac{1}{2}\left(A_{q,g} + B_{q,g}\right) = \frac{1}{2}\int_{-1}^{1} dx\, x\left(H_{q,g}(x,\xi,t;Q^2) + E_{q,g}(x,\xi,t;Q^2)\right)$$

$J_{q,g}$, $\Delta\Sigma$ (3-way pie)

JM: in LC gauge rewrite AM using Dirac eqn. to isolate spin terms

$$M^{+12} = \frac{1}{2} q_+^\dagger \gamma^5 q_+ + \frac{1}{2} i q_+^\dagger (\vec{x}\times\partial)^3 q_+ + Tr(\varepsilon^{+-ij} F^{+j} A^j) + 2i\, Tr F^{+j} (\vec{x}\times\partial) A^j$$

$\Delta\Sigma$? ΔG ?

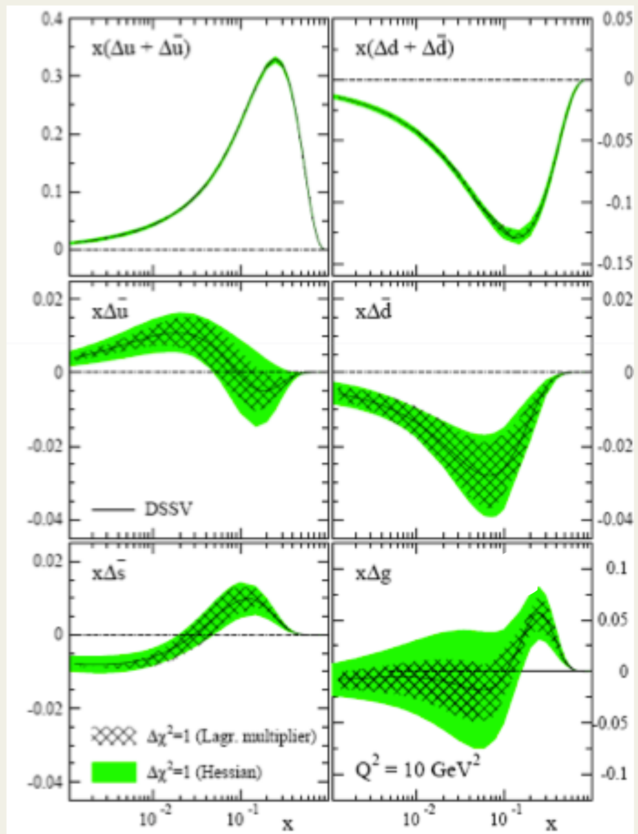
Partonic picture:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

- quark and gluon spin components are identified with the n=1 moments of spin dependent structure functions from DIS, ΔΣ and ΔG.

$$M^{+12} = \frac{1}{2} q_+^\dagger \gamma^5 q_+ + \frac{1}{2} i q_+^\dagger (\vec{x}\times\partial)^3 q_+ + Tr(\varepsilon^{+-ij} F^{+j} A^j) + 2iTrF^{+j}(\vec{x}\times\partial)A^j$$

ΔΣ ? ΔG ?



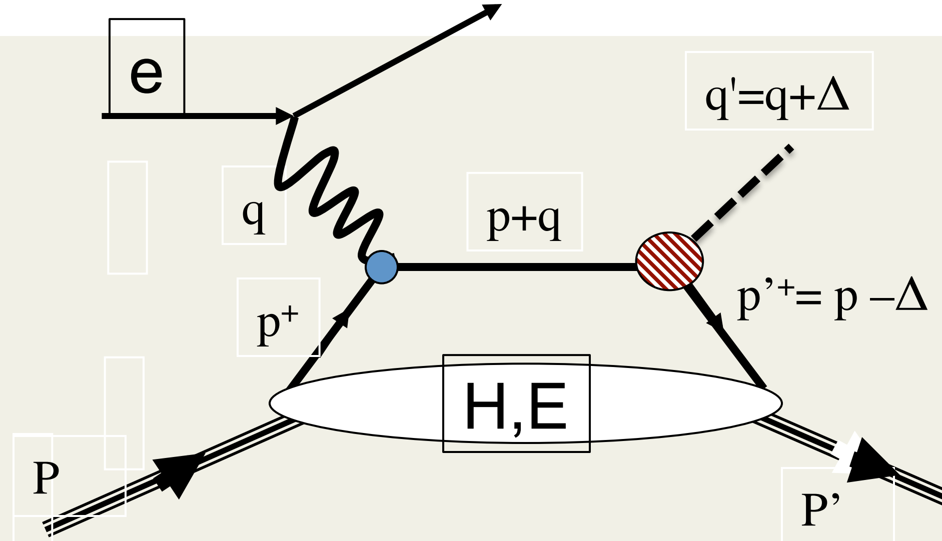
Observables!

1997 New Relation (X.Ji)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \overline{U}(P')[A_{q,g}(\Delta^2)\gamma^{(\mu}\overline{P}^{\nu)} + B_{q,g}(\Delta^2)\overline{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M + C_{q,g}(\Delta^2)(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)/M + \overline{C}_{q,g}(\Delta^2)g^{\mu\nu}M]U(P),$$

New processes (DVCS …) were thought of, whose structure functions – the GPDs - admit n=2 moments that were identified with the (spin+OAM) quark and gluon components of the SR



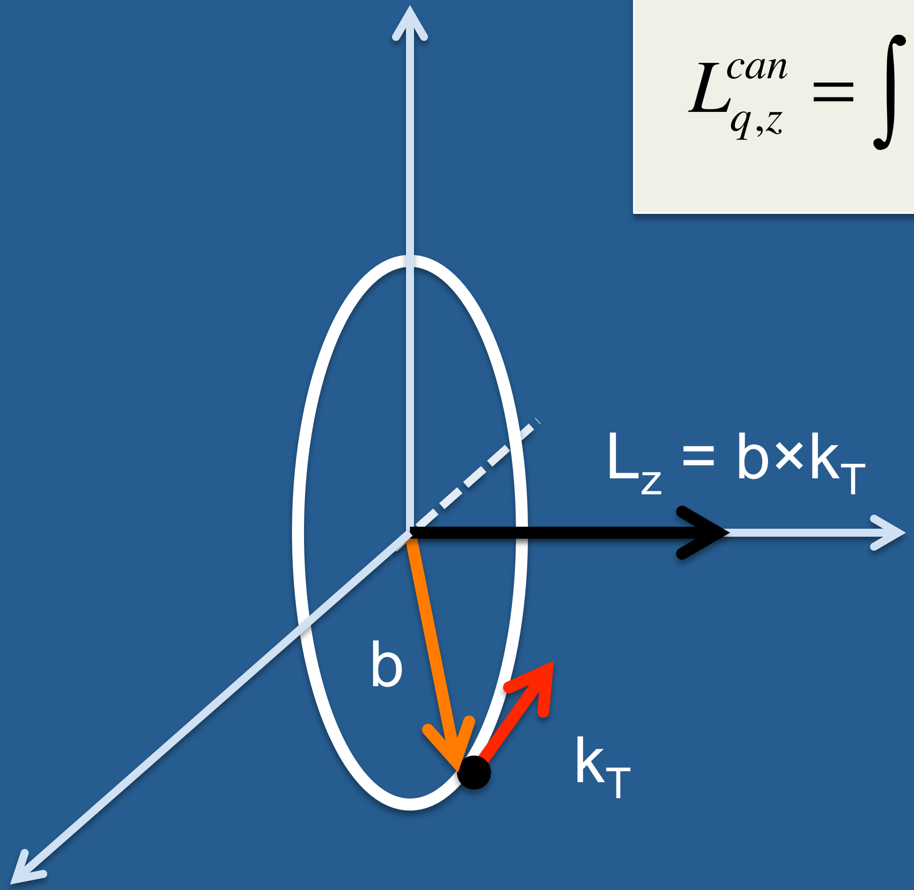
$$\frac{1}{2}\int dx\, x\left[H_{q,g}(x,0,0)+E_{q,g}(x,0,0)\right] = A_{q,g} + B_{q,g} = J_q + J_g$$

$\Delta\Sigma + L_q$

We now know where OAM could enter the picture …
….but what is OAM in QCD

?

OAM: The "naive" picture



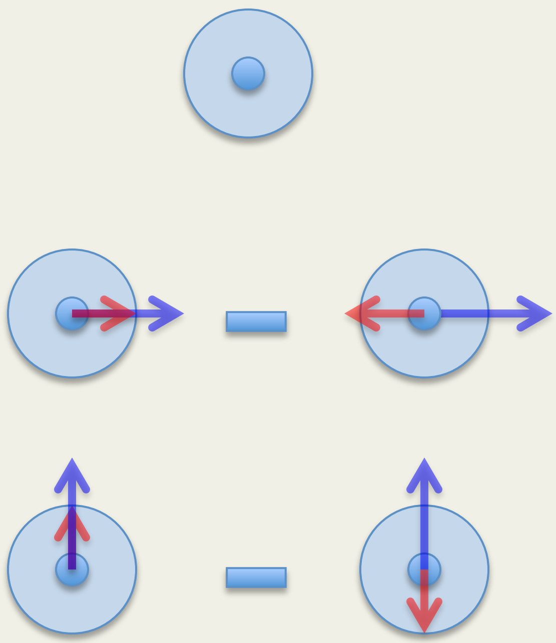
$$L_{q,z}^{can} = \int d^2b\, d^2k_T \left(\vec{b} \times \vec{k}_T\right)_z W_{LC}(x, \vec{b}, \vec{k}_T)$$

Leading twist Wigner distribution with gauge link in LC direction

(a tad more sophisticated)

But any measurement of quark and gluon angular momentum involves the spin of the proton (S_L or S_T)

Through different spin orientations we access different properties, so we need to understand how spin enters

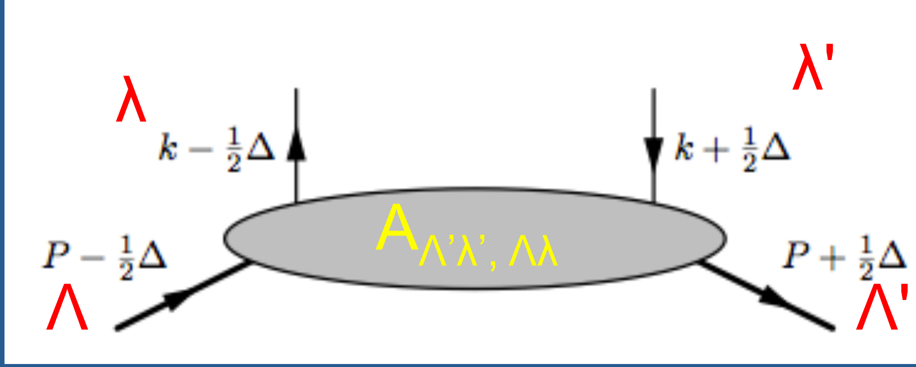


$\mathrm{H} \leftrightarrow f_1$ — charge

$\tilde{\mathrm{H}} \leftrightarrow g_{1L}$ — axial charge

$\mathrm{H}_T \leftrightarrow h_1$ — tensor charge

Technically: GTMD Correlator



integrate to GPDs

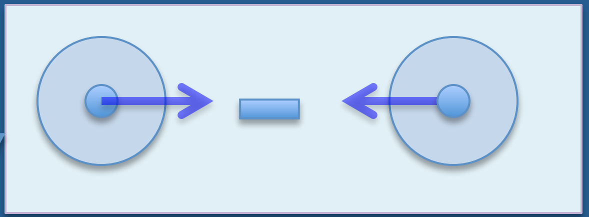
$$
\begin{aligned}
W^{\gamma^+}_{\Lambda\Lambda'} &= \frac{1}{2\overline{P}^+}\Big[\overline{U}(P',\Lambda')\gamma^+ U(P,\Lambda)F_{11} + \overline{U}(P',\Lambda')\frac{i\sigma^{+i}\Delta_T^i}{2M}U(P,\Lambda)(2F_{13}-F_{11}) \\
&+ \overline{U}(P',\Lambda')\frac{i\sigma^{+i}k_T^i}{2M}U(P,\Lambda)(2F_{12}) + \overline{U}(P',\Lambda')\frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}U(P,\Lambda)\frac{\overline{P}^+}{M}F_{14}\Big] \\
&= \delta_{\Lambda,\Lambda'}F_{11} + \delta_{\Lambda,-\Lambda'}\frac{-\Lambda\Delta_1 - i\Delta_2}{2M}(2F_{13}-F_{11}) + \delta_{\Lambda,-\Lambda'}\frac{-\Lambda k_1 - ik_2}{2M}(2F_{12}) + \delta_{\Lambda,\Lambda'}\Lambda F_{14},
\end{aligned}
$$

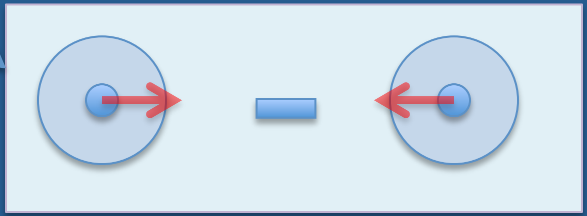
"New" term

Meissner, Metz, Schlegel JHEP 2009

Connect Dirac basis (F_{11}, F_{12}, F_{13}, F_{14}) with helicity basis ($A_{\Lambda'\lambda', \Lambda\lambda}$)

$$F_{14} = A_{++,++} + A_{+-,+-} - A_{-+,-+} - A_{--,--}$$
$$G_{11} = A_{++,++} - A_{+-,+-} + A_{-+,-+} - A_{--,--}$$

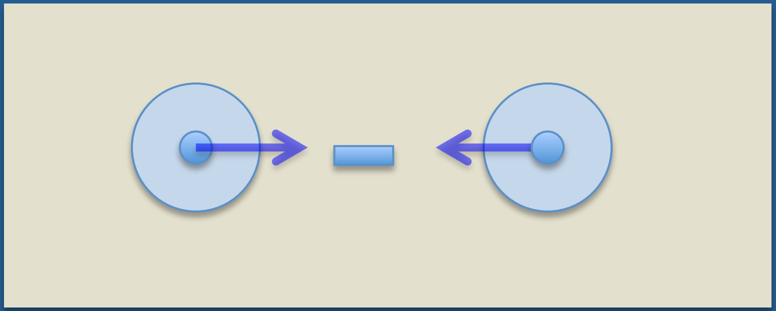




In terms of proton spin

$$\sigma_{ij} k_T^i \Delta_T^j \Rightarrow \vec{S}_L \cdot \left(\vec{k}_T \times \vec{\Delta}_T \right)$$

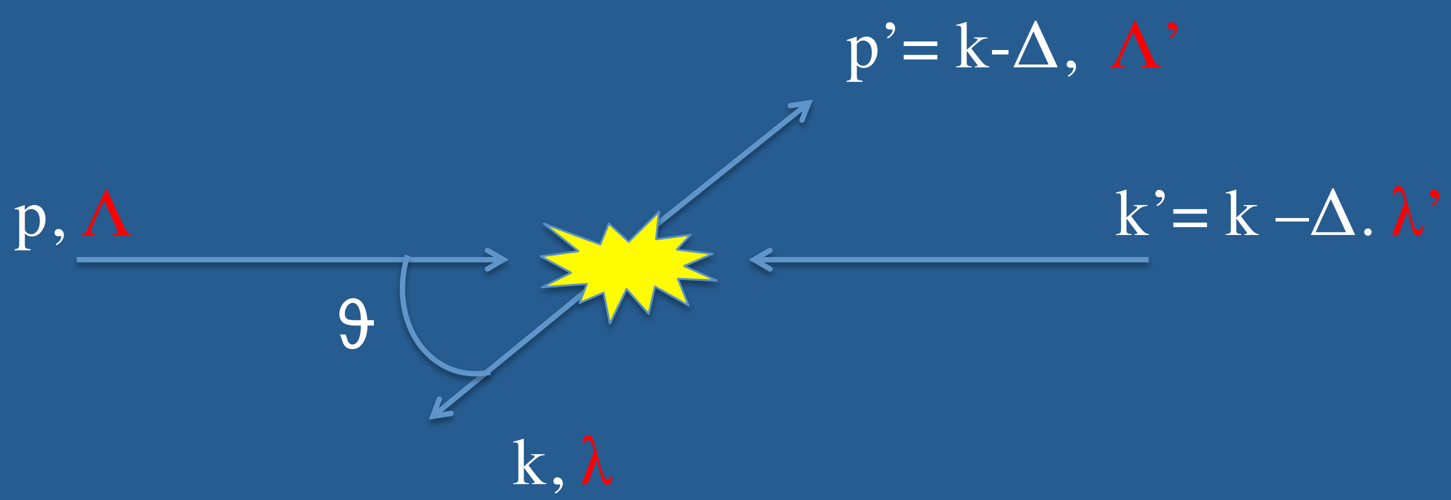
Parity Odd: it is the proton spin dotted into OAM



- It is not OAM (naïve identification)
- It decouples from direct measurements of TMD/GPD observables
- Can it be measured at all? (need a matching Parity odd hard process…work in progress…)

Consider twist 3 contributions

1. They allow us to move out of a two body scattering picture which underlies tw 2 (and sets the Parity relations)



2. OAM can be defined through a sum rule similar to Ji's at tw 2

$$\int dx\, x G_2 = -\frac{1}{2}\int dx\, x(H+E) + \frac{1}{2}\int dx\, \tilde{H}$$

$-L_q$ $-J_q$ S_q

Polyakov et al. (2000)
Hatta (2011)

Twist 3 decomposition of hadronic tensor

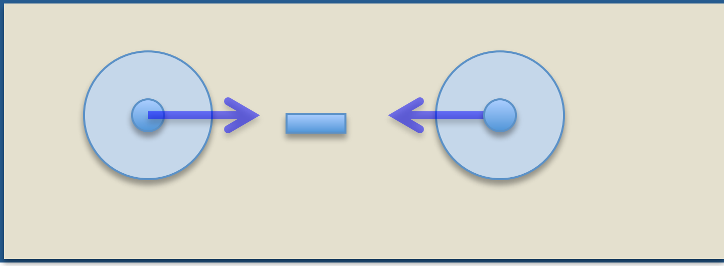
$$F_{\perp}^{\mu} = G_1 \frac{\Delta_{\perp}^{\mu}}{2M} + G_2 \gamma_{\perp}^{\mu} + G_3 \Delta_{\perp}^{\mu} \gamma^{+} + G_4 i \varepsilon_{\mu\nu} \Delta_{\perp}^{\mu} \gamma^{+} \gamma^{5}$$

Polyakov et al. [13]	$2G_1$	G_2	G_3	G_4
Meissner et al. [3]	$2\widetilde{H}_{2T}$	$\widetilde{E}_{2T}$	E_{2T}	H_{2T}
Belitsky et al. [16]	E_{+}^{3}	$\widetilde{H}_{-}^{3}$	$H_{+}^{3} + E_{+}^{3}$	$\frac{1}{\xi}\widetilde{E}_{-}^{3}$

TABLE I: Comparison of notations for different twist 3 GPDs.

arXiv:1310.5157

In arXiv:1310.5157 we asked what is the spin configuration corresponding to quark OAM?



Analysis done using twist 3 GTMDs

$$-\frac{4}{P^+}\left[\frac{\bar{\mathbf{k}}_T\cdot\mathbf{\Delta}_T}{\Delta_T}F_{27}+\Delta_T F_{28}-\left(\frac{\bar{\mathbf{k}}_T\cdot\mathbf{\Delta}_T}{\Delta_T}G_{27}+\Delta_T G_{28}\right)\right] = A^{tw3}_{++,++}+A^{tw3}_{+-,+-}-A^{tw3}_{-+,-+}-A^{tw3}_{--,--}$$

G_2 $\quad$ $\tilde{G}_2$

$G_2 \rightarrow$ $\sigma_{ij}\Delta^j \Rightarrow \vec{S}_L\times\vec{\Delta}$

This is no longer Parity odd but it has a transverse component, OAM is associated with a transverse spin component in the proton

Notice that this is valid for both canonical (Jaffe Manohar) and orbital AM

$$L_q(x) = L_q^{WW}(x) + \overline{L}_q(x)$$
$$\mathcal{L}_q(x) = L_q^{WW}(x) + \overline{\mathcal{L}}_q(x),$$

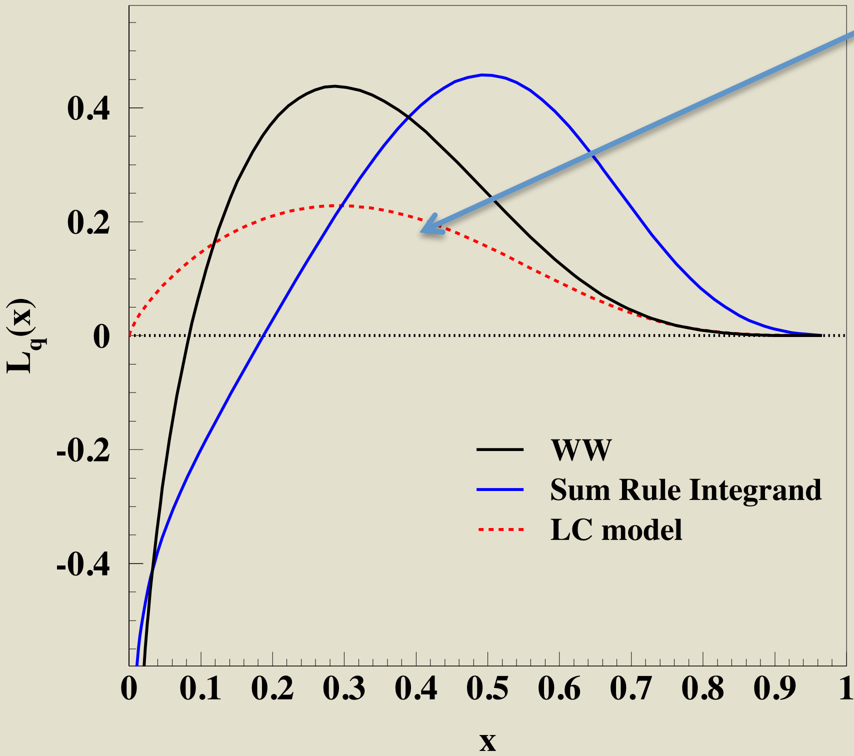
Hatta, 2011

Now that we understand all this, can we measure OAM?

First of all notice that in Wandzura Wilczek approximation

$$L_q(x,0,0) = x\int_x^1 \frac{dy}{y}(H_q(y,0,0)+E_q(y,0,0)) - x\int_x^1 \frac{dy}{y^2}\widetilde{H}_q(y,0,0),$$

$\neq F_{14}$!

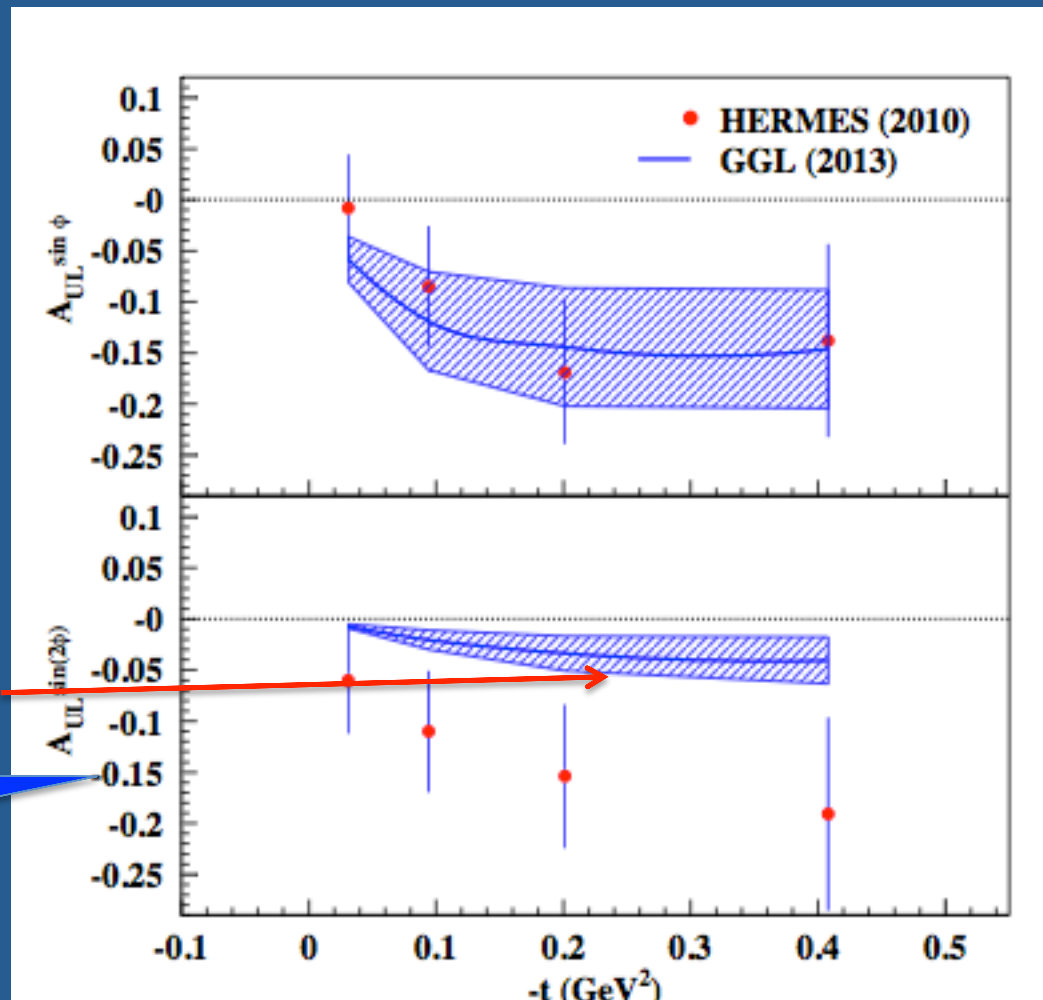


DVCS on a longitudinally polarized target

$$A_{UL,L} = \frac{N_{s_z=+} - N_{s_z=-}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{2\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi}}{F_{UU,T} + \epsilon F_{UU,L}} + \frac{\epsilon \sin 2\phi F_{UL}^{\sin 2\phi}}{F_{UU,T} + \epsilon F_{UU,L}}$$

sin 2φ term is tw 3!

WW, small ξ



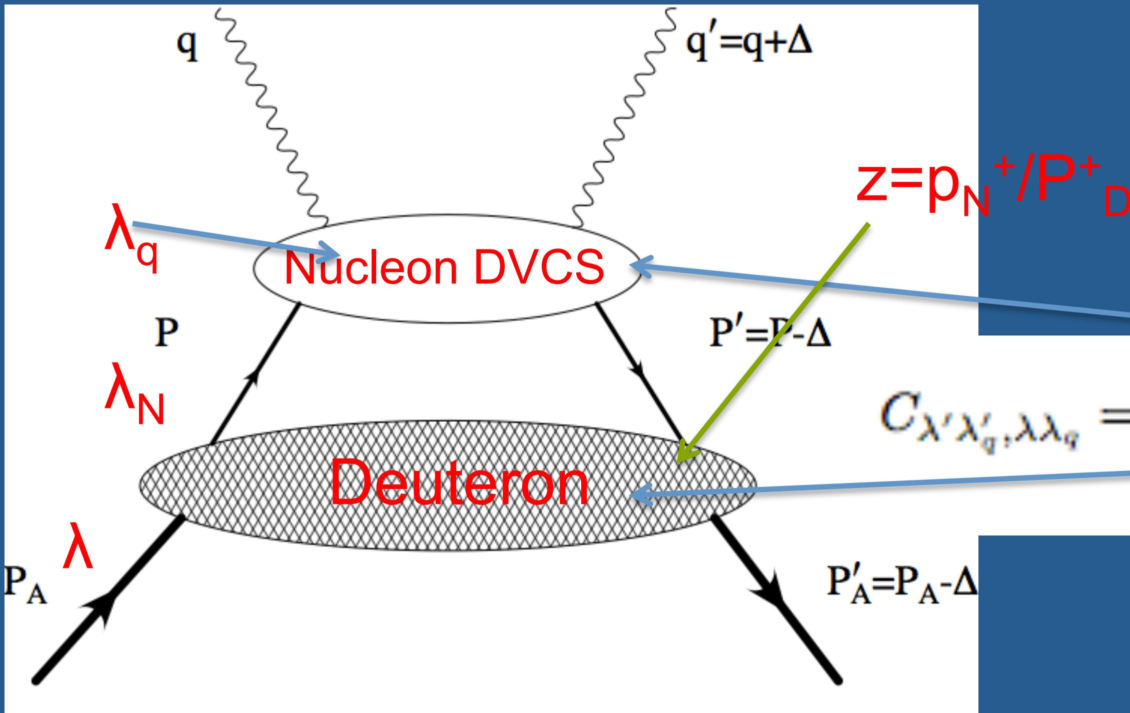
Jlab data in progress!
Avakian, Pisano

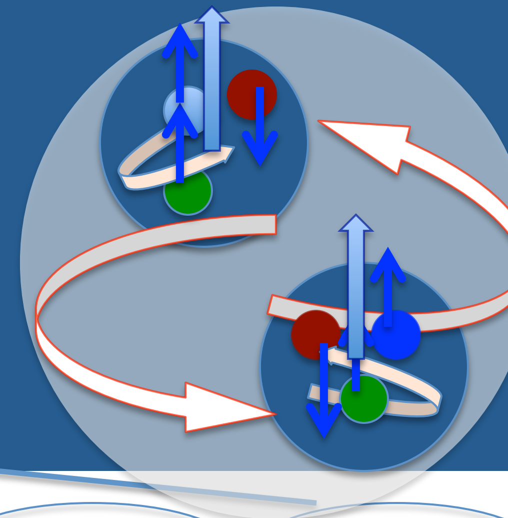
To explore…

- ✓ G_2 can be measured directly, and it does have a partonic interpretation, if one views it as a twist 3 quantity.
- ✓ How does the tw 3 picture connect to M. Burkardt's *gauge-link-based* interpretation (torque experienced by quarks)
- ✓ We need to devise experiments that are sensitive to this contributions (some data already exist, HERMES + S. Pisano)
- ✓ Question of L vs. T interpretation
- ✓ We focused mainly on quarks, question of gluon component

One way to shed light on these questions is to look at deuteron

Sum Rules in Deuteron (Kunal Kathuria)





$$C_{\lambda'\lambda'_q,\lambda\lambda_q} = \sum_{\lambda_N,\lambda'_N} B_{\lambda'\lambda'_N,\lambda\lambda_N} \otimes A_{\lambda'_N\lambda'_q,\lambda_N\lambda_q}$$

Spin 1 systems, due to

1) The presence of additional L components (S+D-waves)
2) Isoscalarity

provide a crucial test the working of the angular momentum sum rules

Deuteron Angular Momentum Sum Rule

Phys.Rev. D86 (2012) 036008

Longitudinal

$$J_q = \frac{1}{2}\int dx\, x\left[H_q(x,0,0) + E_q(x,0,0)\right],$$

Nucleon

→

$$J_q = \frac{1}{2}\int dx\, x\, H_2^q(x,0,0),$$

Deuteron

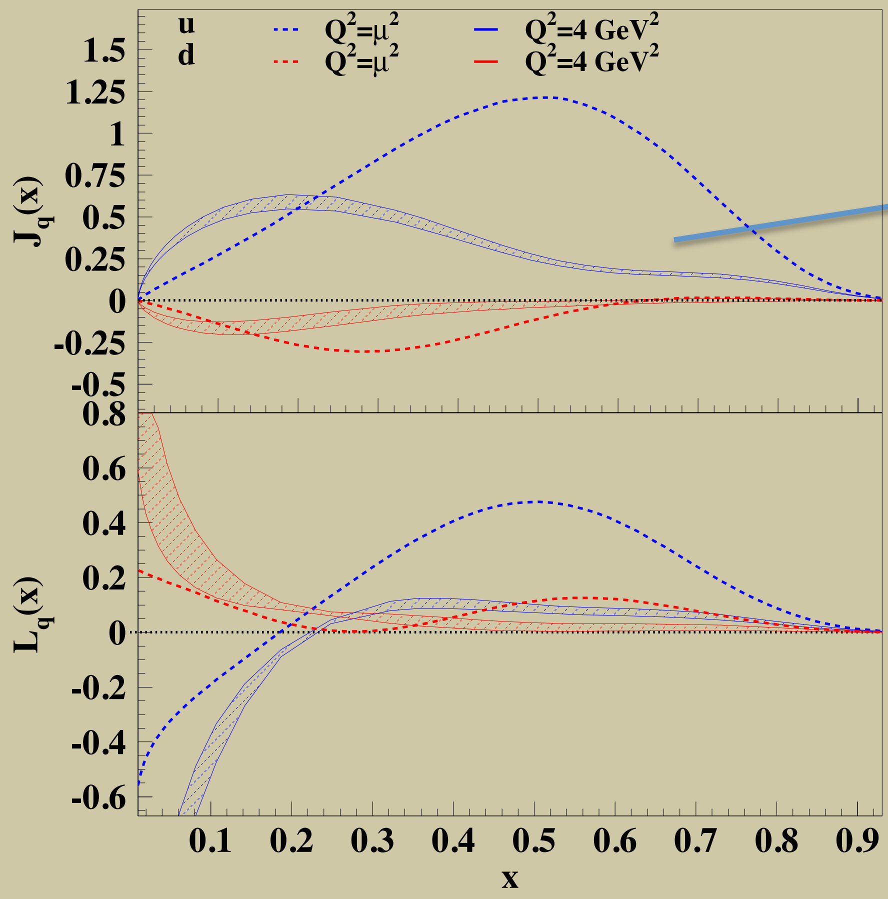
Nucleon: $F_1+F_2= G_M$

Deuteron: G_M

Transverse Deuteron

Differently from the nucleon, in the transverse case, we are not finding the same relation (other GPDs describing charge and tensor component enter…). More details later…

How does Ji sum rule differ from JM in the deuteron?



Effect of evolution

Using GPDs from Goldstein, Gonzalez Hernandez, SL, PRD84

Summary

OAM has (misleadingly) been associated with the distribution of an unpolarized quark in a longitudinally polarized proton. This quantity is Parity odd and it cannot be observed directly

One could match F_{14} with a hard part that is also Parity Odd, and generate an observable, however that does would change the partonic content of the soft part, it still would not carry the meaning of OAM

However, at twist three a new quantity, G_2, representing OAM was found (Polyakov sum rule)

We showed the helicity and spin structure of this quantity, that it can replace the previously suggested one, and that it does not violate Parity

We identified a possible direct measurement of OAM, DVCS on a longitudinally polarized proton

Back up

AM sum rule is obtained by connecting the quark matrix elements of n=2 term

$$\left\langle p' \left| \bar{\psi}(0)\gamma^{\mu} iD^{\nu}\psi(0) \right| p \right\rangle = \bar{U}(p',\Lambda')\gamma^{\mu}U(p,\Lambda)\bar{P}^{\nu}A_{20}(t) -$$

$$\bar{U}(p',\Lambda')\frac{i\sigma^{\alpha\mu}\Delta_{\mu}}{2M}U(p,\Lambda)\bar{P}^{\nu}B_{20}(t) + \frac{\Delta_{\mu}\Delta_{\nu}}{M}\bar{U}(p',\Lambda')U(p,\Lambda)C_{20}(t)$$

with the matrix elements of the energy momentum tensor

$$\left\langle p' \left| T_q^{\mu\nu} \right| p \right\rangle = \bar{U}(p',\Lambda')\left[\gamma^{(\mu}\bar{P}^{\nu)}A(t) - \bar{P}^{(\nu}i\sigma^{\mu)\alpha}\frac{\Delta_{\alpha}}{2M}B(t) + \frac{\Delta^{\mu}\Delta^{\nu} - g^{\mu\nu}\Delta^{2}}{M}C(t)\right]U(p,\Lambda)$$

$$\frac{1}{2}\int dx\, x\Big(H(x,0,0) + E(x,0,0)\Big) = J_z^{q,g}$$