

Pairing effects on spinodal decomposition of asymmetric nuclear matter

International Workshop on Multi facets
of Eos and Clustering - IWM-EC 2014

Dipartimento di Fisica e Astronomia and Laboratori Nazionali del Sud

Catania, 6 - 9 May 2014



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Introduction: interacting many-body systems

- Mean-field approximation:
effective interactions
- Equilibrium limit: EoS
energy functional $E(\rho, T)$
- Nuclear matter at low density:
liquid-gas phase transition
- Spinodal (mechanical) instability
spinodal decomposition
- Interparticle correlations:
pairing effects

From nuclei...

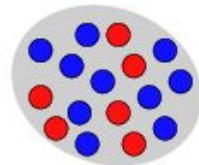
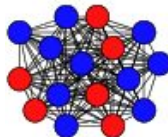
Isotope spectra

...to neutron stars

Glitch phenomena, cooling processes, ...

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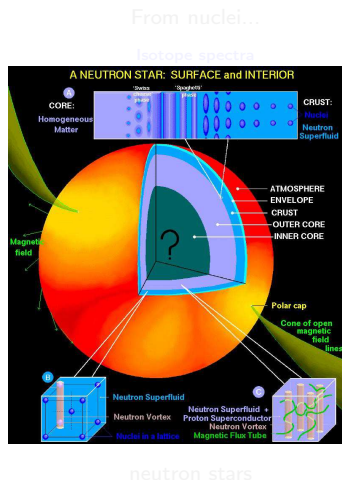
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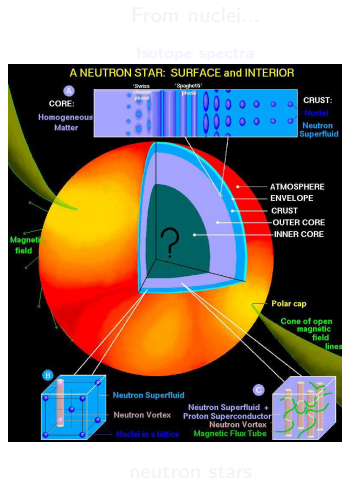
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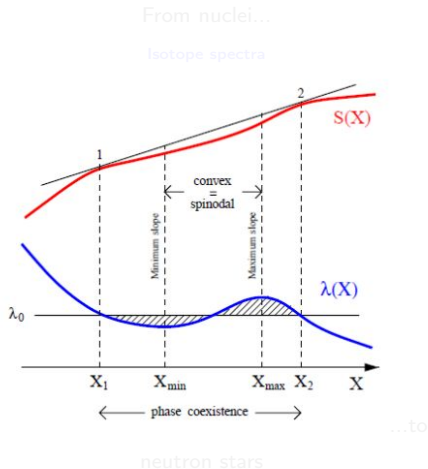
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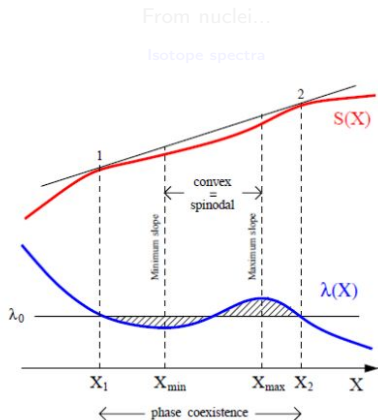
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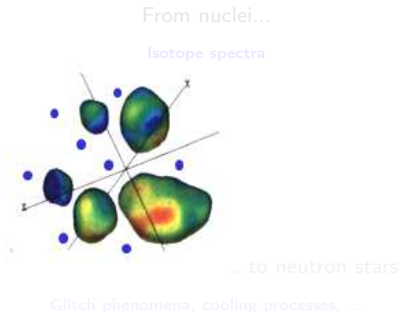
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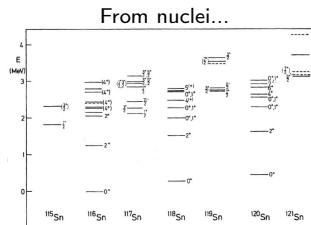
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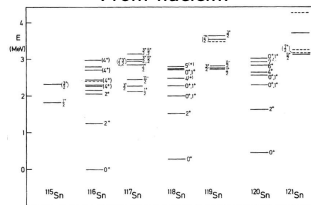
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Asymmetric nuclear matter (ANM)

- Effective interaction \Rightarrow simplified Skyrme-like interaction
- Energy density functional ($T = 0$)

$$\rho \frac{E}{A} = \frac{3}{5} \sum_{q=p,n} \rho_q \epsilon_F^q + \rho \left[\frac{A}{2} \left(\frac{\rho}{\rho_0} \right) + \frac{B}{\sigma + 1} \left(\frac{\rho}{\rho_0} \right)^\sigma + \frac{1}{2} C_{\text{sym}}(\rho) \mathcal{I}^2 \right]$$

- Symmetry term $C_{\text{sym}}(\rho)$: stiff - soft

\Rightarrow spinodal instability

\Rightarrow spinodal instability of neutron stars

\Rightarrow pairing

- Chemical potentials

$$\mu_q = \frac{\partial(\rho E/A)}{\partial \rho_q} = \epsilon_F^q + U_{ANM}^q$$

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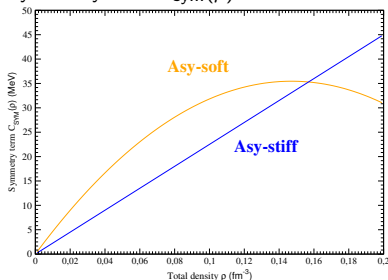
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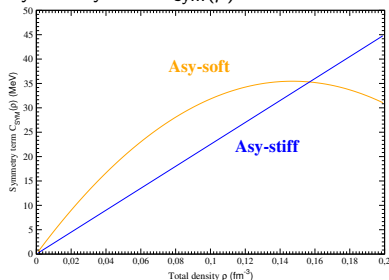
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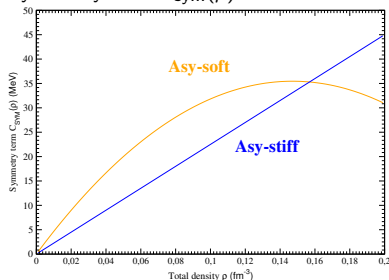
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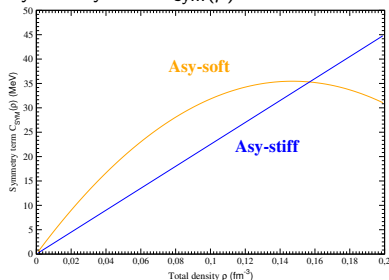
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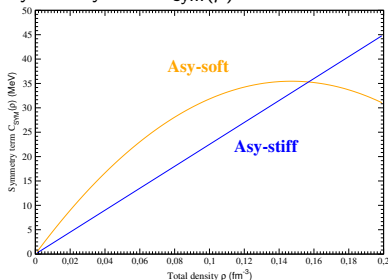
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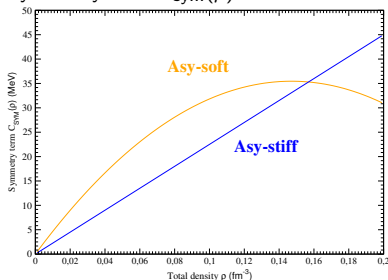
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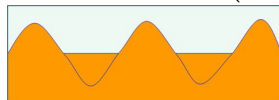
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Isospin distillation mechanism

- Isoscalar-like \Rightarrow unstable mode
- Rotation of θ in the plane (ρ_p, ρ_n)

$$\tan \theta = \frac{\delta \rho_n}{\delta \rho_p} \quad \Leftrightarrow \quad \tan 2\theta = \frac{c}{a-b}$$

- Asymmetry $\delta\mathcal{I}$ of the density fluctuation

$$\frac{\delta \rho_p}{\delta \rho_n} = \frac{\delta \rho_p}{\delta \rho_n} \frac{\delta \rho_n}{\delta \rho_n} = \frac{\delta \rho_p \delta \rho_n}{\delta \rho_n^2}$$

Liquid phase is more compressible than gas phase

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Physical picture of isospin distillation mechanism
was proposed.

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Isospin plane is more anisotropic than
 was assumed.

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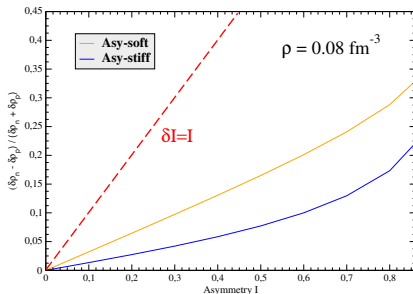
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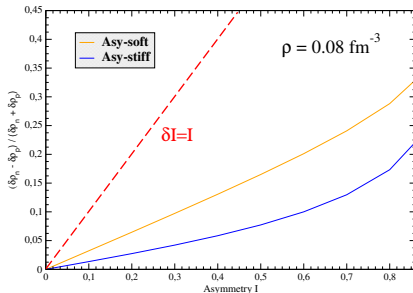
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Neutron distillation

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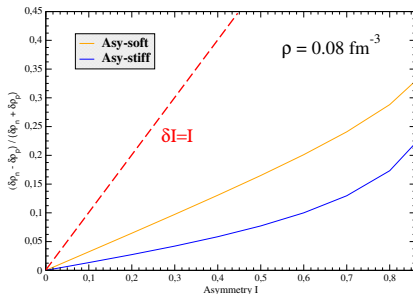
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- Asymmetry $\delta \mathcal{I}$ of the density fluctuation

$$\delta \mathcal{I} = \frac{\delta \rho_n - \delta \rho_p}{\delta \rho_n + \delta \rho_p} = \frac{\delta \rho_n / \delta \rho_p - 1}{\delta \rho_n / \delta \rho_p + 1}$$

- Isospin distillation mechanism

$$\delta \mathcal{I} < \mathcal{I}$$



Neutron distillation

Liquid phase is more symmetric than gas phase!

Pairing effect on ANM instability: why and how?

- Nucleons form **paired** states (**Cooper pairs**)
⇒ Pairing treatment: **BCS theory** (analogous to electrons in metals in the superconducting phase)
- Pairing correlations active at **low density** ⇒ impact on spinodal instability
- ANM ⇒ only **nn** or **pp** pairing
- **Pairing interaction**: nucleons of the same type
vs.
Symmetry potential: nucleons of different type
⇒ different symmetry factors of ANM
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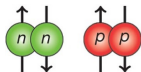
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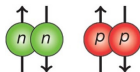
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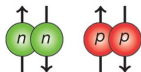
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BCS theory at zero temperature: gap equation

Ground state: **Fermi sea**

Ground state: **only Cooper pairs**

HF theory

$$|HF\rangle = \prod_{k < k_F} \hat{a}_k^\dagger |0\rangle$$

$$E_n = \sum_{k < k_F} (2\xi_k v_k^2 |n\rangle - \Gamma_k v_k^2 |n\rangle)$$

BCS theory

$$|BCS\rangle = \prod_{k > 0} (u_k |s\rangle + v_k |s\rangle \hat{a}_k^\dagger \hat{a}_{\bar{k}}^\dagger) |0\rangle$$

$$E_s = \sum_{k > 0} (2\xi_k v_k^2 |s\rangle - \Gamma_k v_k^2 |s\rangle - \Delta_k v_k u_k |s\rangle)$$

$$v_k^2$$

Occupation number

$$\xi_k = \epsilon_k - \mu_k^* \quad \mu_k^* = \mu - \Gamma_k$$

Effective chemical potential

$$\Gamma_k = \sum_{k'} \bar{V}_{kk'kk'} v_{k'}^2 |n, s\rangle$$

Mean-field potential

$$\Delta_k = - \sum_{k' > 0} \bar{V}_{k\bar{k}k'\bar{k}'} v_{k'} u_{k'} |s\rangle \quad \text{Energy gap} \Rightarrow \text{Gap equation}$$

$$N = 2 \sum_{k > 0} v_k^2 |s\rangle \quad \text{Particle number conservation} \Rightarrow \text{Density equation}$$

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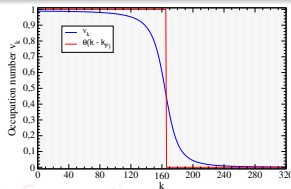
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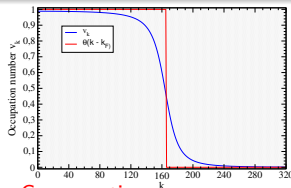
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$$V_{\pi}(\mathbf{r}_i, \mathbf{r}_j) = \frac{1}{2}(1 - P_{\sigma})v_{\pi}^q(\rho_q)\delta(\mathbf{r}_{ij}) \quad q = p, n$$

zero range \Rightarrow energy cutoff $\epsilon_{\Lambda} = 16$ MeV

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$$\rho_q = \frac{(2m)^{3/2}}{4\pi^2\hbar^3} \int_0^{\mu_q^* + \epsilon_{\Lambda}} d\epsilon \sqrt{\epsilon} \left[1 - \frac{\xi}{E_{\Delta}} \tanh\left(\frac{\beta E_{\Delta}}{2}\right) \right]$$

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1S_0 pairing gap of neutron matter
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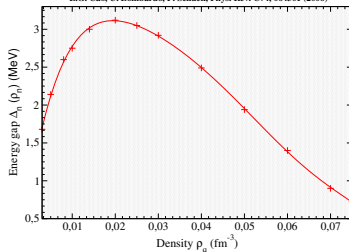
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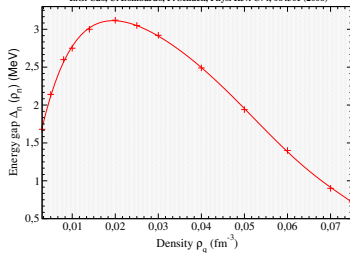
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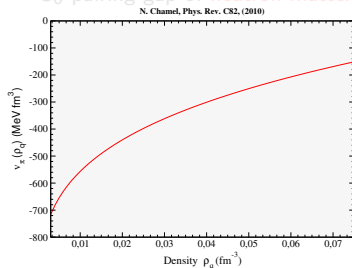
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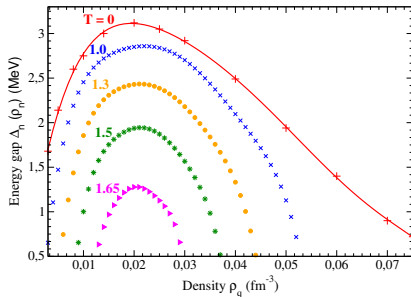
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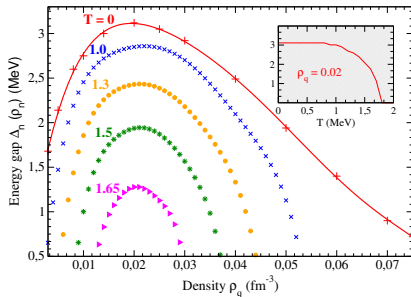
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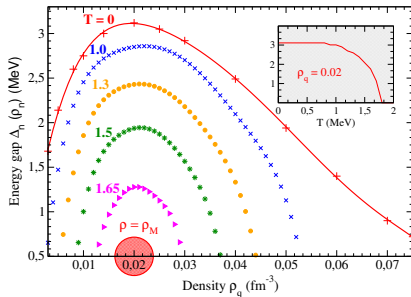
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1S_0 pairing of neutron matter

Effect on chemical potential derivatives ($T = 0$)

- Energy density functional in paired ANM ($\tilde{\rho} \equiv 2\Delta/v_\pi$):

$$\rho \frac{E}{A} = \sum_q \left[2 \int \frac{d\mathbf{p}}{h^3} f^q(\mathbf{p}) \frac{\mathbf{p}^2}{2m} + v_\pi(\rho_q) \frac{|\tilde{\rho}_q|^2}{4} \right] + \rho \left[\frac{A}{2} \left(\frac{\rho}{\rho_0} \right) + \frac{B}{\sigma+1} \left(\frac{\rho}{\rho_0} \right)^\sigma + \frac{C_{\text{sym}}}{2} \mathcal{J}^2 \right]$$

$$\mu_\pi^q = \left. \frac{\partial(\rho E/A)}{\partial \rho_q} \right|_{\tilde{\rho}_q} = \mu_q^* + U_\pi^q + U_{ANM}^q$$

- Effect on the asymmetry of unstable oscillation

\Rightarrow (isospin distillation)

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$$a_\pi, b_\pi = \frac{\partial \mu_\pi^q}{\partial \rho_q} = \frac{\partial \mu_q^*}{\partial \rho_q} + \frac{\partial U_\pi^q}{\partial \rho_q} + \frac{\partial U_{\text{ANM}}^q}{\partial \rho_q} \equiv \delta_\pi^q$$

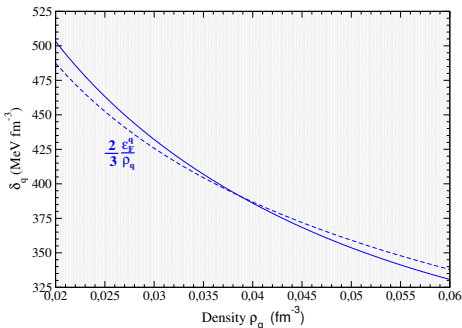
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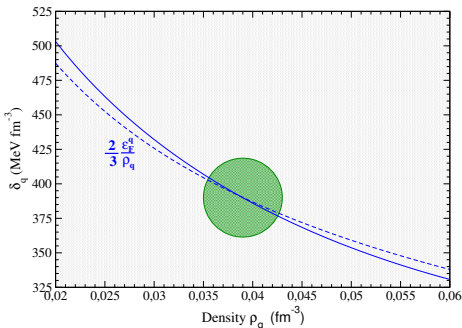
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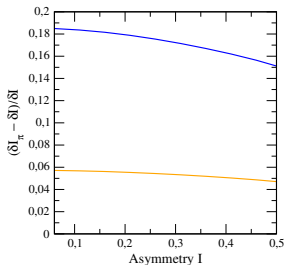
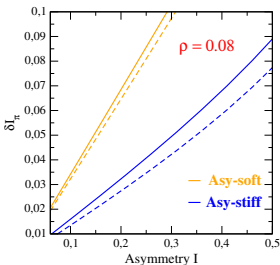
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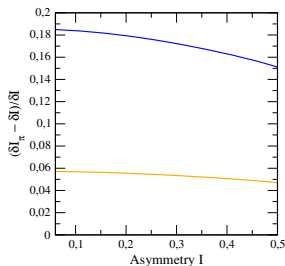
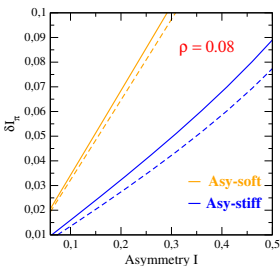
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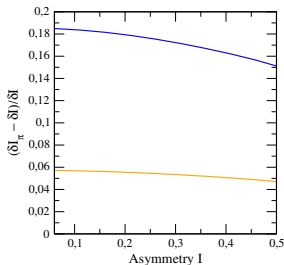
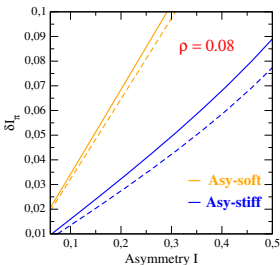
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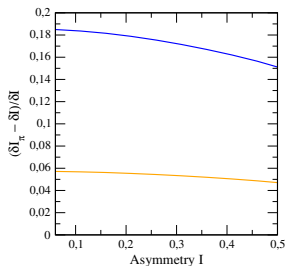
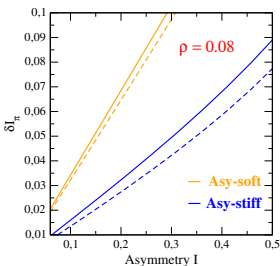
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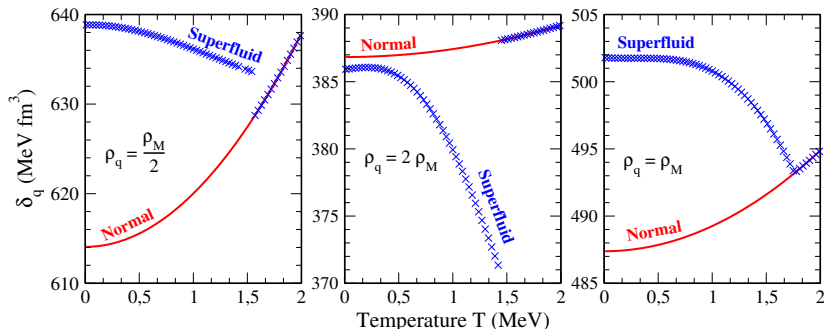
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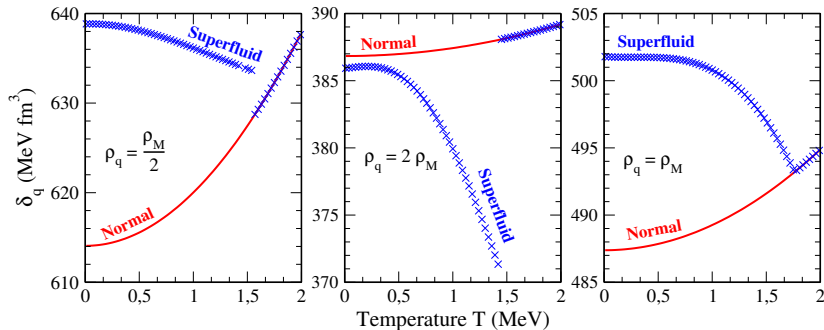
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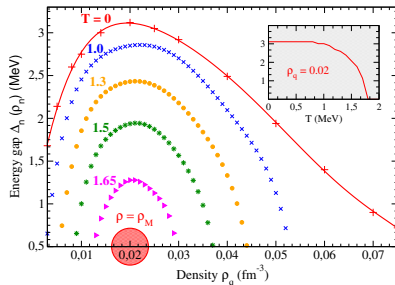
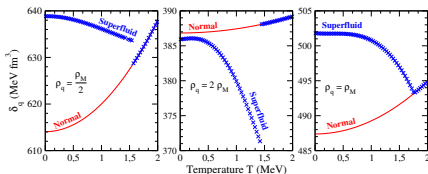
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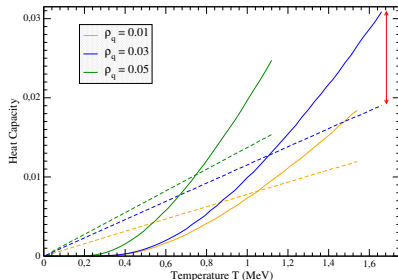
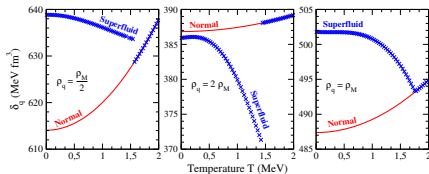
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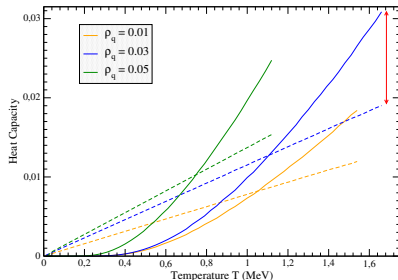
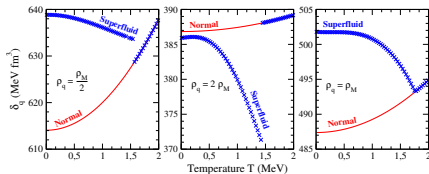
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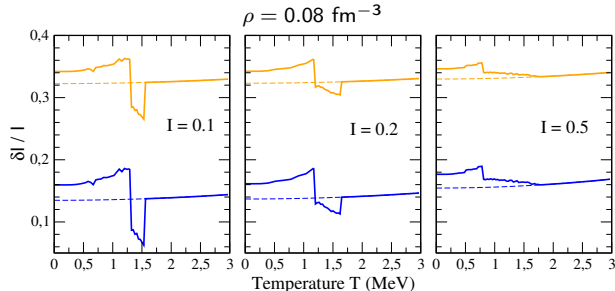
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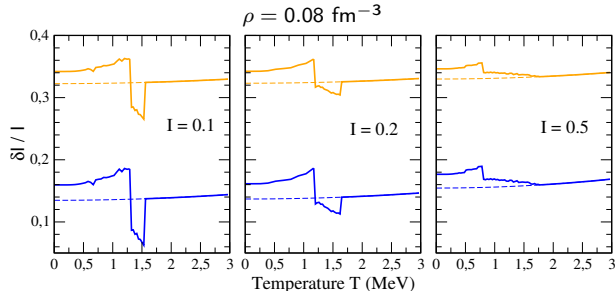
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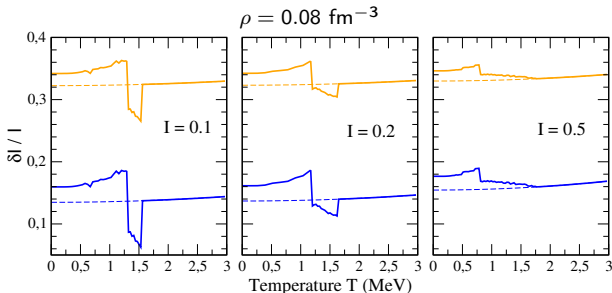
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Thank You!