Pairing effects on spinodal decomposition of asymmetric nuclear matter

International Workshop on Multi facets of Eos and Clustering - IWM-EC 2014

Dipartimento di Fisica e Astronomia and Laboratori Nazionali del Sud

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Effective interactions: ANM Characterization of the instability in ANM

Introduction: interacting many-body systems

- Mean-field approximation: effective interactions
- Equilibrium limit: EoS
- Nuclear matter at low density: liquid-gas phase transition
- Spinodal (mechanical) instability
- Interparticle correlations: pairing effects

From nuclei...

Isotope spectra

... to neutron stars

Glitch phenomena, cooling processes,

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Asymmetric nuclear matter (ANM)

- Effective interaction ⇒ simplified Skyrme-like interaction
- Energy density functional (T = 0)

- Symmetry term C_{sym}(ρ): stiff soft
 - \Rightarrow nuclear structure
 - \rightarrow inner crust of neutron stars
 - \rightarrow heavy ion reactions

- Chemical potentials $\mu_q = \frac{\partial(\rho E/A)}{\partial \rho_q} = \epsilon_F^q + U_{ANN}^q$
- Curvature matrix C

$$\mathbf{C} = \begin{pmatrix} \partial \mu_p / \partial \rho_p & \partial \mu_p / \partial \rho_n \\ \partial \mu_n / \partial \rho_p & \partial \mu_n / \partial \rho_n \end{pmatrix} = \begin{pmatrix} \mathbf{a} & \frac{\mathbf{c}}{2} \\ \frac{\mathbf{c}}{2} & \mathbf{b} \end{pmatrix}$$

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 - \Rightarrow inner crust of neutron stars
 - \Rightarrow heavy ion reactions

Burrello S., Colonna M., Matera F.

- Chemical potentials $\mu_q = \frac{\partial(\rho E/A)}{\partial \rho_q} = \epsilon_F^q + U_{ANM}^q$
- Curvature matrix C

$$\mathbf{C} = \begin{pmatrix} \partial \mu_p / \partial \rho_p & \partial \mu_p / \partial \rho_n \\ \partial \mu_n / \partial \rho_p & \partial \mu_n / \partial \rho_n \end{pmatrix} = \begin{pmatrix} \mathbf{a} & \frac{\mathbf{c}}{2} \\ \frac{\mathbf{c}}{2} & \mathbf{b} \end{pmatrix}$$

Negative eigenvalues

 \Rightarrow unstable oscillations (clusters

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Effective interactions: ANM Characterization of the instability in ANM

Asymmetric nuclear matter (ANM)

- Effective interaction ⇒ simplified Skyrme-like interaction
- Energy density functional (T = 0)

$$\rho \frac{E}{A} = \frac{3}{5} \sum_{q=p,n} \rho_q \epsilon_F^q + \rho \left[\frac{\mathcal{A}}{2} \left(\frac{\rho}{\rho_0} \right) + \frac{\mathcal{B}}{\sigma+1} \left(\frac{\rho}{\rho_0} \right)^{\sigma} + \frac{1}{2} \mathcal{C}_{\text{sym}}(\rho) \mathcal{I}^2 \right] \quad \mathcal{I} = \frac{\rho_n - \rho_p}{\rho}$$

- Symmetry term C_{sym}(ρ): stiff soft
 Asy-soft
 Asy-soft
 Asy-soft
 Asy-stiff
 asy-stiff</
 - \Rightarrow nuclear structure
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Burrello S., Colonna M., Matera F.

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 Pairing effects on spinodal decomposition of ANM

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Effective interactions: ANM Characterization of the instability in ANM

Isospin distillation mechanism

- Isoscalar-like \Rightarrow unstable mode
- Rotation of θ in the plane (ρ_p, ρ_n)

 $an heta = rac{\delta
ho_n}{\delta
ho_p} \quad \Leftarrow \quad an 2 heta = rac{c}{a-b}$

• Asymmetry $\delta \mathcal{I}$ of the density fluctuation

 $\delta I = \frac{\delta \rho_0 - \delta \rho_p}{\delta \rho_0 + \delta \rho_0} = \frac{\delta \rho_0 / \delta \rho_p - 1}{\delta \rho_0 + \delta \rho_0}$

Isospin distillation mechanism

 $L > I\delta$

Liquid phase is more symmetric than gas phase!

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Isospin distillation mechanism

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Liquid phase is more symmetric than gas phase!

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Neutron distillation

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HFB theory: pairing effective interaction Effect on isospin distillation

Pairing effect on ANM instability: why and how?

• Nucleons form paired states (Cooper pairs)

 \Rightarrow Pairing treatment: BCS theory (analogous to electrons in metals in the superconducting phase)

- $\bullet\,$ Pairing correlations active at low density \Rightarrow impact on spinodal instability
- ANM ⇒ only nn or pp pairing
- Pairing interaction: nucleons of the same type

VS.

Symmetry potential: nucleons of different type

 \rightarrow Different isotopic feature of ANM

● Extension of mean-field approach: Hartree-Fock-Bogoliubov (HFB) theory ⇒ unified formalism for pairing and mean-field effective interaction

HFB theory: pairing effective interaction Effect on isospin distillation

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HFB theory: pairing effective interaction Effect on isospin distillation

BCS theory at zero temperature: gap equation

Ground state: Fermi sea

HF theory

$$|HF\rangle = \prod_{k < k_F} \hat{a}_k^{\dagger} |0\rangle$$

 $E_n = \sum_{k < k_F} (2\xi_k v_k^2 |_n - \Gamma_k v_k^2 |_n)$

Ground state: only Cooper pairs

$\begin{array}{l} \mathsf{BCS theory} \\ |\mathsf{BCS}\rangle &= \prod_{k>0} \left(u_k |_s + v_k |_s \hat{a}_k^{\dagger} \hat{a}_k^{\dagger} \right) |0\rangle \\ \\ \mathsf{E}_s &= \sum_{k>0} \left(2\xi_k v_k^2 |_s - \Gamma_k v_k^2 |_s - \Delta_k v_k u_k |_s \right) \end{array}$

Occupation number

- $\xi_k = \epsilon_k \mu_k^*$ $\mu_k^* = \mu \Gamma_k$ Effective chemical potential
- $\Gamma_k = \sum_{i'} \bar{V}_{kk'kk'} v_{k'}^2|_{n,s} \qquad I$

 v_{ν}^2

Mean-field potential

 $V_k = -\sum_{k'>0} ar{V}_{kar{k}k'ar{k}'} v_{k'} u_{k'}|_s$ Energy gap \Rightarrow Gap equation

 $V = 2 \sum_{k>0} v_k^2 |_s \quad \text{Particle number conservation} \Rightarrow \underbrace{\text{Density equation}}_{\substack{k>0}} \bullet \underbrace{\langle \Xi \rangle \langle \Xi \rangle \langle \Xi \rangle}_{\substack{k>0}} = \Xi$

HFB theory: pairing effective interaction Effect on isospin distillation

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BCS theory at zero temperature: gap equation

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HFB theory: pairing effective interaction Effect on isospin distillation

Gap and density equations: effective interaction

Effective pairing interaction

$$V_{\pi}(\mathbf{r}_i,\mathbf{r}_j) = \frac{1}{2}(1-P_{\sigma})v_{\pi}^q(\rho_q)\delta(\mathbf{r}_{ij}) \qquad q=\rho, n$$

zero range \Rightarrow energy cutoff $\epsilon_{\Lambda} = 16$ MeV

Density equation

$$\rho_q = \frac{(2m)^{3/2}}{4\pi^2\hbar^3} \int_0^{\mu_q^* + \epsilon_{\Lambda}} d\epsilon \sqrt{\epsilon} \left[1 - \frac{\xi}{E_{\Delta}} \tanh\left(\frac{\beta E_{\Delta}}{2}\right) \right]$$

Gap equation

$$1 = -v_{\pi}(\rho_q) \frac{(2m)^{3/2}}{8\pi^2 \hbar^3} \int_0^{\mu_q^* + \epsilon_{\Lambda}} d\epsilon \frac{\sqrt{\epsilon}}{E_{\Delta}} \tanh\left(\frac{\beta E_{\Delta}}{2}\right)$$

$$\beta = rac{1}{T}, \ E_{\Delta} = \sqrt{\xi^2 + \Delta^2}, \ \xi = \epsilon - \mu_q^*, \ \mu_q^* = \mu_q - U_q$$

• $T = 0 \Rightarrow$ strength $v_{\pi}^{q}(\rho_{q}) \equiv v_{\pi}(\rho_{q})$

 $V_{\pi}(
ho_q) = V_{\pi}^{\Lambda} \left[1 - \eta \left(rac{
ho_q}{
ho_0}
ight)^{lpha}
ight]$ also for pp

•
$$T \neq 0 \Rightarrow \Delta(T)$$

• $T = T_C$ superfluid \leftrightarrow normal phase

 1S_0 pairing gap of neutron matter

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HFB theory: pairing effective interaction Effect on isospin distillation

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$$p_q = \frac{(2m)^{3/2}}{4\pi^2 \hbar^3} \int_0^{\mu_q^* + \epsilon_{\Lambda}} d\epsilon \sqrt{\epsilon} \left[1 - \frac{\xi}{E_{\Delta}} \tanh\left(\frac{\beta E_{\Delta}}{2}\right) \right]$$

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(Brueckner calculations with Argonne v_{14} potential)

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ight]$$
 also for pp

 $T \neq 0 \Rightarrow \Delta(T)$

• $T = T_C$ superfluid \leftrightarrow normal phase



(Brueckner calculations with Argonne v_{14} potential)

HFB theory: pairing effective interaction Effect on isospin distillation

Gap and density equations: effective interaction

Effective pairing interaction

$$V_{\pi}(\mathbf{r}_{i},\mathbf{r}_{j}) = \frac{1}{2}(1-P_{\sigma})v_{\pi}^{q}(\rho_{q})\delta(\mathbf{r}_{ij}) \qquad q=p,r$$

zero range \Rightarrow energy cutoff $\epsilon_{\Lambda} = 16$ MeV • Density equation

$$\rho_q = \frac{(2m)^{3/2}}{4\pi^2 \hbar^3} \int_0^{\mu_q^* + \epsilon_{\Lambda}} d\epsilon \sqrt{\epsilon} \left[1 - \frac{\xi}{E_{\Delta}} \tanh\left(\frac{\beta E_{\Delta}}{2}\right) \right]$$

Gap equation

$$\begin{split} \mathbf{1} &= -\mathbf{v}_{\pi}(\rho_q) \frac{(2m)^{3/2}}{8\pi^2 \hbar^3} \int_{\mathbf{0}}^{\mu_q^* + \epsilon_{\Lambda}} d\epsilon \frac{\sqrt{\epsilon}}{E_{\Delta}} \tanh\left(\frac{\beta E_{\Delta}}{2}\right) \\ \beta &= \frac{1}{T}, \ E_{\Delta} = \sqrt{\xi^2 + \Delta^2}, \ \xi = \epsilon - \mu_q^*, \ \mu_q^* = \mu_q - U_q \end{split}$$

• $T = 0 \Rightarrow \text{strength } v_{\pi}^{q}(\rho_{q}) \equiv v_{\pi}(\rho_{q})$

$$v_{\pi}(
ho_q) = V_{\pi}^{\Lambda} \left[1 - \eta \left(rac{
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HFB theory: pairing effective interaction Effect on isospin distillation

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• $T = T_C$ superfluid \leftrightarrow normal phase 1S_0 pairing gap of neutron matter

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S₀ pairing gap of neutron matter

HFB theory: pairing effective interaction Effect on isospin distillation

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• Energy density functional in paired ANM ($\tilde{\rho} \equiv 2\Delta/\nu_{\pi}$):

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$$\mu_{\pi}^{q} = \frac{\partial \left(\rho E/A\right)}{\partial \rho_{q}}\Big|_{\tilde{\rho}_{q}} = \mu_{q}^{*} + U_{\pi}^{q} + U_{ANM}^{q}$$

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 - \sim increase at most of 20% of the asymmetry ${\cal I}$
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HFB theory: pairing effective interaction Effect on isospin distillation

Effect on chemical potential derivatives (T = 0)

• Energy density functional in paired ANM ($\tilde{\rho} \equiv 2\Delta/v_{\pi}$):

$$\rho \frac{E}{A} = \sum_{q} \left[2 \int \frac{d\mathbf{p}}{h^3} f^q(\mathbf{p}) \frac{\mathbf{p}^2}{2m} + \mathbf{v}_{\pi}(\rho_q) \frac{|\tilde{\rho}_q|^2}{4} \right] + \rho \left[\frac{\mathcal{A}}{2} \left(\frac{\rho}{\rho_0} \right) + \frac{\mathcal{B}}{\sigma+1} \left(\frac{\rho}{\rho_0} \right)^{\sigma} + \frac{\mathcal{C}_{\text{sym}}}{2} \mathcal{I}^2 \right]$$

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Effect on the asymmetry of unstable oscillation
 ⇒ (isospin distillation)

- \sim increase at most of 20% of the asymmetry ${\cal I}$
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HFB theory: pairing effective interaction Effect on isospin distillation

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$$\mathbf{a}, \mathbf{b} = \frac{\partial \mu_{q}}{\partial \rho_{q}} = \frac{\partial \epsilon_{F}^{e}}{\partial \rho_{q}} + \frac{\partial U_{ANM}^{q}}{\partial \rho_{q}} = \frac{2}{3} \frac{\epsilon_{F}^{e}}{\rho_{q}}$$

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HFB theory: pairing effective interaction Effect on isospin distillation

Effect on chemical potential derivatives ($T \neq 0$)

- Normal phase: $\frac{\partial \epsilon_{T}^{p}}{\partial \rho_{q}} \simeq \frac{2}{3} \frac{\epsilon_{T}^{p}}{\rho_{q}} \left[1 + \frac{\pi^{2}}{12} \left(\frac{T}{\epsilon_{T}^{p}} \right)^{2} \right] \xrightarrow{\text{superflux}}_{\text{plane}} \epsilon_{T}$ decreases with T1
- Normal ↔ superfluid: discontinuity on the 2nd derivatives of the energy
 ⇒ 2nd order phase transition
- Compressibility \Rightarrow jump depends on the density (disappears at $\rho_q = \rho_M$)
- Larger effect than T = 0
- Double discontinuity
 - \Rightarrow n and p critical T
- Jump $\sim \left\langle \frac{\delta I}{T} \right\rangle$ \Rightarrow sizable effects on the

isotopic properties of clusters

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 decreases with T!

Normal ↔ superfluid: discontinuity on the 2nd derivatives of the energy
 ⇒ 2nd order phase transition

• Compressibility \Rightarrow jump depends on the density (disappears at $\rho_q = \rho_M$)

- Larger effect than T = 0
- Double discontinuity
 - \Rightarrow n and p critical T
- Jump $\sim \left\langle \frac{\delta I}{T} \right\rangle$ \Rightarrow sizable effects on the

isotopic properties of clusters

HFB theory: pairing effective interaction Effect on isospin distillation

Effect on chemical potential derivatives ($T \neq 0$)

- Normal phase: $\frac{\partial \epsilon_F^e}{\partial \rho_q} \simeq \frac{2}{3} \frac{\epsilon_q^F}{\rho_q} \left[1 + \frac{\pi^2}{12} \left(\frac{T}{\epsilon_q^F} \right)^2 \right] \xrightarrow{superfluid}{phase} \delta_q$ decreases with T!
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Pairing effects on spinodal decomposition of ANM

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Thank You!

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