Pairing effects on spinodal decomposition of asymmetric nuclear matter

International Workshop on Multi facets of Eos and Clustering - IWM-EC 2014

Dipartimento di Fisica e Astronomia and Laboratori Nazionali del Sud

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Authors: Burrello S.\(^1\), Colonna M.\(^1\), Matera F.\(^2\)

\(^1\) INFN - LNS, Catania

\(^2\) INFN - Dipartimento di Fisica e Astronomia - Firenze
Introduction: interacting many-body systems

- Mean-field approximation:
  effective interactions

- Equilibrium limit: EoS
  energy functional $E(\rho,T)$

- Nuclear matter at low density:
  liquid-gas phase transition

- Spinodal (mechanical) instability
  and multifragmentation

- Interparticle correlations:
  pairing effects

From nuclei...
Isotope spectra
...to neutron stars
Glitch phenomena, cooling processes, ...
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Asymmetric nuclear matter (ANM)

- Effective interaction $\Rightarrow$ simplified Skyrme-like interaction
- Energy density functional $(T = 0)$

$$\rho \frac{E}{A} = \frac{3}{5} \sum_{q=p,n} \rho_q \epsilon_q^F + \rho \left[ \frac{A}{2} \left( \frac{\rho}{\rho_0} \right) + \frac{B}{\sigma + 1} \left( \frac{\rho}{\rho_0} \right)^\sigma + \frac{1}{2} C_{\text{sym}}(\rho) I^2 \right]$$

- Chemical potentials
  $$\mu_q = \frac{\partial (\rho E/A)}{\partial \rho_q} = \epsilon_q^F + U_{\text{ANM}}^q$$

- Curvature matrix $C$
  $$C = \begin{pmatrix} \frac{\partial \mu_p/\partial \rho_p}{\partial \rho_p/\partial \rho_n} & \frac{\partial \mu_p/\partial \rho_n}{\partial \rho_p/\partial \rho_n} \\ \frac{\partial \mu_n/\partial \rho_p}{\partial \rho_n/\partial \rho_n} & \frac{\partial \mu_n/\partial \rho_n}{\partial \rho_n/\partial \rho_n} \end{pmatrix} = \begin{pmatrix} a & \frac{c}{2} \\ \frac{c}{2} & b \end{pmatrix}$$

- Symmetry term $C_{\text{sym}}(\rho)$: stiff - soft
  $\Rightarrow$ nuclear structure
  $\Rightarrow$ inner crust of neutron stars
  $\Rightarrow$ heavy ion reactions

- Negative eigenvalues
  $\Rightarrow$ unstable oscillations (clusters)
Asymmetric nuclear matter (ANM)

- **Effective interaction** \( \Rightarrow \) simplified *Skyrme-like* interaction
- **Energy density functional** \((T = 0)\)

\[
\frac{E}{A} = \frac{3}{5} \sum_{q=p,n} \rho_q \epsilon_q^F + \rho \left[ \frac{A}{2} \left( \frac{\rho}{\rho_0} \right) + \frac{B}{\sigma + 1} \left( \frac{\rho}{\rho_0} \right)^{\sigma} + \frac{1}{2} C_{\text{sym}}(\rho) I^2 \right]
\]

- **Chemical potentials**

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\mu_q = \frac{\partial (E/A)}{\partial \rho_q} = \epsilon_q^F + U_q^{\text{ANM}}
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- **Curvature matrix** \( \mathbf{C} \)

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\mathbf{C} = \begin{pmatrix}
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\end{pmatrix} = \begin{pmatrix}
a & c \\
c & 2b
\end{pmatrix}
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\frac{E}{A} = \frac{3}{5} \sum_{q=p,n} \rho_q \epsilon_F^q + \rho \left[ A \left( \frac{\rho}{\rho_0} \right) + \frac{B}{\sigma + 1} \left( \frac{\rho}{\rho_0} \right)^\sigma + \frac{1}{2} C_{sym}(\rho) I^2 \right]
\]

\(I = \frac{\rho_n - \rho_p}{\rho}\)

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\[ \mathcal{I} = \frac{\rho_n - \rho_p}{\rho} \]

- Chemical potentials

$$\mu_q = \frac{\partial (\rho E/A)}{\partial \rho_q} = \epsilon_q^F + U^q_{\text{ANM}}$$

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\end{array} \right) = \left( \begin{array}{cc}
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Isospin distillation mechanism

- Isoscalar-like $\Rightarrow$ unstable mode
- Rotation of $\theta$ in the plane $(\rho_p, \rho_n)$
  \[
  \tan \theta = \frac{\delta \rho_n}{\delta \rho_p} \quad \Leftrightarrow \quad \tan 2\theta = \frac{c}{a - b}
  \]
- Asymmetry $\delta I$ of the density fluctuation
  \[
  \delta I = \rho_n - \rho_p = \frac{\delta \rho_n}{\delta \rho_p} - 1
  \]
- Isospin distillation mechanism
  \[
  \delta I \ll I
  \]
Isospin distillation mechanism

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- Asymmetry $\delta I$ of the density fluctuation

$$\delta I = \frac{\delta \rho_n - \delta \rho_p}{\delta \rho_n + \delta \rho_p} = \frac{\delta \rho_n/\delta \rho_p - 1}{\delta \rho_n/\delta \rho_p + 1}$$

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- Isospin distillation mechanism

$\delta I < I$
Isospin distillation mechanism

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- Neutron distillation
  Liquid phase is more symmetric than gas phase!
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Neutron distillation
Liquid phase is more symmetric than gas phase!
Pairing effect on ANM instability: why and how?

- Nucleons form paired states (Cooper pairs)
  ⇒ Pairing treatment: BCS theory (analogous to electrons in metals in the superconducting phase)
- Pairing correlations active at low density ⇒ impact on spinodal instability
- ANM ⇒ only nn or pp pairing
- Pairing interaction: nucleons of the same type vs.
  Symmetry potential: nucleons of different type
  ⇒ Different isotopic feature of ANM
- Extension of mean-field approach: Hartree-Fock-Bogoliubov (HFB) theory
  ⇒ unified formalism for pairing and mean-field effective interaction
Pairing effect on ANM instability: why and how?

- Nucleons form paired states (**Cooper pairs**)
  - Pairing treatment: **BCS theory** (analogous to electrons in metals in the superconducting phase)

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  - unified formalism for pairing and mean-field effective interaction
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  Symmetry potential: nucleons of different type
  ⇒ Different isotopic feature of ANM

- Extension of mean-field approach: Hartree-Fock-Bogoliubov (HFB) theory
  ⇒ unified formalism for pairing and mean-field effective interaction
Pairing effect on ANM instability: why and how?

- Nucleons form paired states (Cooper pairs)
  \[ \Rightarrow \text{Pairing treatment: BCS theory (analogous to electrons in metals in the superconducting phase)} \]
- Pairing correlations active at low density \( \Rightarrow \) impact on spinodal instability
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Burrello S., Colonna M., Matera F. Pairing effects on spinodal decomposition of ANM
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**BCS theory at zero temperature: gap equation**

Ground state: **Fermi sea**

**HF theory**

\[
|HF\rangle = \prod_{k<k_F} \hat{a}^\dagger_k |0\rangle \\
E_n = \sum_{k<k_F} (2\xi_k \nu_k^2|n - \Gamma_k \nu_k^2|n)
\]

**BCS theory**

\[
|BCS\rangle = \prod_{k>0} (u_k|s + v_k|s \hat{a}^\dagger_k \hat{a}^\dagger_k)|0\rangle \\
E_s = \sum_{k>0} (2\xi_k \nu_k^2|s - \Gamma_k \nu_k^2|s - \Delta_k \nu_k u_k|s)
\]

\[
\nu_k^2 \quad \text{Occupation number}
\]

\[
\xi_k = \epsilon_k - \mu_k^* \quad \mu_k^* = \mu - \Gamma_k \quad \text{Effective chemical potential}
\]

\[
\Gamma_k = \sum_{k'} \tilde{V}_{kk'kk'} \nu_{k'}^2|n,s \quad \text{Mean-field potential}
\]

\[
\Delta_k = -\sum_{k'>0} \tilde{V}_{kk'kk'} \nu_{k'} u_{k'}|s \quad \text{Energy gap} \Rightarrow \text{Gap equation}
\]

\[
N = 2 \sum_{k>0} \nu_k^2|s \quad \text{Particle number conservation} \Rightarrow \text{Density equation}
\]

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\]

**Ground state:** only Cooper pairs

**BCS theory**

\[
|BCS\rangle = \prod_{k>0} (u_k |s\rangle + v_k |s\rangle \hat{a}_k^\dagger \hat{a}_{k'}^\dagger) |0\rangle
\]

\[
E_s = \sum_{k>0} (2\xi_k v_k^2 |s\rangle - \Gamma_k v_k^2 |s\rangle - \Delta_k v_k u_k |s\rangle)
\]

- \(v_k^2\) Occupation number
- \(\xi_k = \epsilon_k - \mu_k^*\) Effective chemical potential
- \(\mu_k^* = \mu - \Gamma_k\)
- \(\Gamma_k = \sum_{k'} \tilde{V}_{kk'kk'} v_{k'}^2 |n,s\rangle\) Mean-field potential
- \(\Delta_k = - \sum_{k'>0} \tilde{V}_{kk'kk'} v_{k'} u_{k'} |s\rangle\) Energy gap \(\Rightarrow\) Gap equation
- \(N = 2 \sum_{k>0} v_k^2 |s\rangle\) Particle number conservation \(\Rightarrow\) Density equation
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---

**BCS theory at zero temperature: gap equation**

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|HF\rangle = \prod_{k<k_F} \hat{a}_{k}^{\dagger} |0\rangle
\]

\[
E_n = \sum_{k<k_F} (2\xi_k v_k^2 |n - \Gamma_k v_k^2 |n)
\]

Ground state: **only Cooper pairs**

\[
|BCS\rangle = \prod_{k>0} (u_k|s + v_k|s \hat{a}_{k}^{\dagger} \hat{a}_{k}^{\dagger}) |0\rangle
\]

\[
E_s = \sum_{k>0} (2\xi_k v_k^2 |s - \Gamma_k v_k^2 |s - \Delta_k v_k u_k |s)
\]

**Occupation number**

\[ v_k^2 \]

**Effective chemical potential**

\[ \xi_k = \epsilon_k - \mu_k^* \quad \mu_k^* = \mu - \Gamma_k \]

**Mean-field potential**

\[ \Gamma_k = \sum_{k'} \tilde{V}_{kk'k} v_{k'}^2 |n,s \]

**Energy gap** \( \Rightarrow \) **Gap equation**

\[ \Delta_k = - \sum_{k'>0} \tilde{V}_{kk'k} v_{k'} u_{k'} |s \]

**Particle number conservation** \( \Rightarrow \) **Density equation**

\[ N = 2 \sum_{k>0} v_k^2 |s \]

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**BCS theory**

\[ |BCS\rangle = \prod_{k>0} (u_k |s\rangle + v_k |s\rangle \hat{a}_k^\dagger \hat{a}_{k'}^\dagger) |0\rangle \]

\[ E_s = \sum_{k>0} (2\xi_k v_k^2 |s\rangle - \Gamma_k v_k^2 |s\rangle - \Delta_k v_k u_k |s\rangle) \]

- **Occupation number**
  \[ v_k^2 \]

- **Effective chemical potential**
  \[ \xi_k = \epsilon_k - \mu_k^* \]
  \[ \mu_k^* = \mu - \Gamma_k \]

- **Mean-field potential**
  \[ \Gamma_k = \sum_{k'} \tilde{V}_{kk'kk'} v_{k'}^2 |n,s\rangle \]

- **Energy gap**
  \[ \Delta_k = - \sum_{k'<0} \tilde{V}_{kk'kk'} v_{k'} u_{k'} |s\rangle \]

- **Particle number conservation**
  \[ N = 2 \sum_{k>0} v_k^2 |s\rangle \]

**Burrello S., Colonna M., Matera F.**

**Pairing effects on spinodal decomposition of ANM**
Gap and density equations: effective interaction

- **Effective pairing interaction**
  
  \[ V_\pi(r_i, r_j) = \frac{1}{2} (1 - P_\sigma) \nu_q^q(\rho_q) \delta(r_{ij}) \quad q = p, n \]

- **Zero range** \( \Rightarrow \) energy cutoff \( \epsilon_\Lambda = 16 \text{ MeV} \)

  \[ \rho_q = \frac{(2m)^{3/2}}{4\pi^2 h^3} \int_{0}^{\mu_q^* + \epsilon_\Lambda} d\epsilon \sqrt{\epsilon} \left[ 1 - \frac{\xi}{E_\Delta} \tanh \left( \frac{\beta E_\Delta}{2} \right) \right] \]

- **Density equation**

- **Gap equation**

  \[ 1 = -\nu_\pi(\rho_q) \frac{(2m)^{3/2}}{8\pi^2 h^3} \int_{0}^{\mu_q^* + \epsilon_\Lambda} d\epsilon \sqrt{\epsilon} \tanh \left( \frac{\beta E_\Delta}{2} \right) \]

  \[ \beta = \frac{1}{T}, \quad E_\Delta = \sqrt{\xi^2 + \Delta^2}, \quad \xi = \epsilon - \mu_q^*, \quad \mu_q^* = \mu_q - U_q \]

- **T = 0** \( \Rightarrow \) strength \( \nu_q^q(\rho_q) \equiv \nu_\pi(\rho_q) \)

  \[ \nu_\pi(\rho_q) = V_\pi^\Lambda \left[ 1 - \eta \left( \frac{\rho_q}{\rho_0} \right)^\alpha \right] \text{ also for pp} \]

- **T \neq 0** \( \Rightarrow \) \( \Delta(T) \)

  \[ T = T_C \text{ superfluid} \leftrightarrow \text{normal phase} \]

1\text{S}_0 \text{ pairing gap of neutron matter (Brueckner calculations with Argonne \( v_{14} \) potential)}
Gap and density equations: effective interaction

- **Effective pairing interaction**

\[ V_\pi(r_i, r_j) = \frac{1}{2} (1 - P_\sigma) \nu_\pi^q(\rho_q) \delta(r_{ij}) \quad q = p, n \]

*zero range* \(\Rightarrow\) *energy cutoff* \(\epsilon_\Lambda = 16\) MeV

- **Density equation**

\[ \rho_q = \frac{(2m)^{3/2}}{4\pi^2 \hbar^3} \int_0^{\mu^*_q + \epsilon_\Lambda} d\epsilon \sqrt{\epsilon} \left[ 1 - \frac{\xi}{E_\Delta} \tanh \left( \frac{\beta E_\Delta}{2} \right) \right] \]

- **Gap equation**

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\[ \beta = \frac{1}{T}, \quad E_\Delta = \sqrt{\xi^2 + \Delta^2}, \quad \xi = \epsilon - \mu^*_q, \quad \mu^*_q = \mu_q - U_q \]

\[ T = 0 \Rightarrow \text{strength} \quad \nu_\pi^q(\rho_q) \equiv \nu_\pi(\rho_q) \]

\[ \nu_\pi(\rho_q) = V_\Lambda^q \left[ 1 - \eta \left( \frac{\rho_q}{\rho_0} \right)^\alpha \right] \]

*also for pp*

\[ T \neq 0 \Rightarrow \Delta(T) \]

\[ T = T_C \quad \text{superfluid} \leftrightarrow \text{normal phase} \]

---

**$^1S_0$ pairing gap of neutron matter**


(Brucekner calculations with Argonne $v_{14}$ potential)
Gap and density equations: effective interaction

- Effective pairing interaction

\[
V_\pi(r_i, r_j) = \frac{1}{2} (1 - P_\sigma) \nu_\pi^q(\rho_q) \delta(r_{ij}) \quad q = p, n
\]

zero range \(\Rightarrow\) energy cutoff \(\epsilon_\Lambda = 16\) MeV

- Density equation

\[
\rho_q = \frac{(2m)^{3/2}}{4\pi^2 \hbar^3} \int_0^{\mu_q^* + \epsilon_\Lambda} d\epsilon \sqrt{\epsilon} \left[ 1 - \frac{\xi}{E_\Delta} \tanh \left( \frac{\beta E_\Delta}{2} \right) \right]
\]

- Gap equation

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1 = -\nu_\pi(\rho_q) \frac{(2m)^{3/2}}{8\pi^2 \hbar^3} \int_0^{\mu_q^* + \epsilon_\Lambda} d\epsilon \sqrt{\epsilon} \frac{\xi}{E_\Delta} \tanh \left( \frac{\beta E_\Delta}{2} \right)
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\[1S_0\] pairing gap of neutron matter


(Bruceckner calculations with Argonne \(v_{14}\) potential)
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\[ V_{\pi}(r_i, r_j) = \frac{1}{2}(1 - P_{\sigma})v_{\pi}^q(\rho_q)\delta(r_{ij}) \quad q = p, n \]

zero range \(\Rightarrow\) energy cutoff \(\epsilon_\Lambda = 16\) MeV

- Density equation

\[ \rho_q = \frac{(2m)^{3/2}}{4\pi^2\hbar^3} \int_{0}^{\mu_q^* + \epsilon_\Lambda} d\epsilon \sqrt{\epsilon} \left[ 1 - \frac{\xi}{E_\Delta} \tanh \left( \frac{\beta E_\Delta}{2} \right) \right] \]

- Gap equation

\[ 1 = -v_{\pi}(\rho_q)\frac{(2m)^{3/2}}{8\pi^2\hbar^3} \int_{0}^{\mu_q^* + \epsilon_\Lambda} d\epsilon \sqrt{\epsilon} \frac{E_\Delta}{E_\Delta} \tanh \left( \frac{\beta E_\Delta}{2} \right) \]

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\(1S_0\) pairing gap of neutron matter

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\[ \beta = \frac{1}{T}, \quad E_\Delta = \sqrt{\xi^2 + \Delta^2}, \quad \xi = \epsilon - \mu_q^*, \quad \mu_q^* = \mu_q - U_q \]

\[ T = 0 \Rightarrow \text{strength} \quad v_\pi^q(\rho_q) \equiv v_\pi (\rho_q) \]

\[ v_\pi (\rho_q) = V_\pi^\Lambda \left[ 1 - \eta \left( \frac{\rho_q}{\rho_0} \right)^\alpha \right] \quad \text{also for pp} \]

\[ T \neq 0 \Rightarrow \Delta(T) \]

\[ \Delta(T) = \Delta(0) \left( \frac{T}{T_C} \right)^\nu \quad \text{with} \quad \nu = 0.31 \quad \text{for neutrons} \quad \text{and} \quad \nu = 0.5 \quad \text{for protons} \]

\[ T = T_C \quad \text{superfluid} \leftrightarrow \text{normal phase} \]

\[ ^1S_0 \quad \text{pairing gap of neutron matter} \]

\[ Burrello S., Colonna M., Matera F. \]

Pairing effects on spinodal decomposition of ANM
**Liquid-gas phase transition and spinodal instability**

**HFB theory: pairing effective interaction**

**Effect on isospin distillation**

---

**Gap and density equations: effective interaction**

- **Effective pairing interaction**

\[
V_\pi(r_i, r_j) = \frac{1}{2}(1 - P_\sigma)\nu_\pi^q(\rho_q)\delta(r_{ij}) \quad q = p, n
\]

zero range \(\Rightarrow\) energy cutoff \(\epsilon_\Lambda = 16\) MeV

- **Density equation**

\[
\rho_q = \frac{(2m)^{3/2}}{4\pi^2\hbar^3} \int_{0}^{\mu_q^* + \epsilon_\Lambda} d\epsilon \sqrt{\epsilon} \left[ 1 - \frac{\xi}{E_\Delta} \tanh \left( \frac{\beta E_\Delta}{2} \right) \right]
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also for pp

**$T \neq 0 \Rightarrow \Delta(T)$**

*Burrello S., Colonna M., Matera F.*

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$^1S_0$ pairing gap of neutron matter (Brueckner calculations with Argonne $v_{14}$ potential)
Gap and density equations: effective interaction

- Effective pairing interaction
  \[ V_\pi(r_i, r_j) = \frac{1}{2} (1 - P_\sigma) v_\pi^q(\rho_q) \delta(r_{ij}) \quad q = p, n \]

  zero range ⇒ energy cutoff \( \epsilon_\Lambda = 16 \text{ MeV} \)

- Density equation
  \[ \rho_q = \frac{(2m)^{3/2}}{4\pi^2\hbar^3} \int_0^{\mu_q^* + \epsilon_\Lambda} d\epsilon \sqrt{\epsilon} \left[ 1 - \frac{\xi}{E_\Delta} \tanh\left( \frac{\beta E_\Delta}{2} \right) \right] \]

- Gap equation
  \[ 1 = -v_\pi(\rho_q) \frac{(2m)^{3/2}}{8\pi^2\hbar^3} \int_0^{\mu_q^* + \epsilon_\Lambda} d\epsilon \sqrt{\epsilon} \tanh\left( \frac{\beta E_\Delta}{2} \right) \]

  \[ \beta = \frac{1}{T}, \quad E_\Delta = \sqrt{\xi^2 + \Delta^2}, \quad \xi = \epsilon - \mu_q^*, \quad \mu_q^* = \mu_q - U_q \]

- \( T = 0 \) ⇒ strength \( v_\pi^q(\rho_q) \equiv v_\pi(\rho_q) \)
  \[ v_\pi(\rho_q) = V_\pi^\Lambda \left[ 1 - \eta \left( \frac{\rho_q}{\rho_0} \right)^{\alpha} \right] \] also for pp

- \( T \neq 0 \) ⇒ \( \Delta(T) \)

- \( T = T_C \) superfluid ⇔ normal phase

\[ T = 0 \Rightarrow \text{strength } v_\pi^q(\rho_q) \equiv v_\pi(\rho_q) \]

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Burrello S., Colonna M., Matera F. Pairing effects on spinodal decomposition of ANM
Effect on chemical potential derivatives \((T = 0)\)

- Energy density functional in paired ANM \((\tilde{\rho} \equiv 2\Delta/v_\pi)\):

\[
\rho \frac{E}{A} = \sum_q \left[ 2 \int \frac{dp}{\hbar^3} f^q(p) \frac{p^2}{2m} + v_\pi(\rho_q) \frac{|\tilde{\rho}_q|^2}{4} \right] + \rho \left[ \frac{A}{2} \left( \frac{\rho}{\rho_0} \right) + \frac{B}{\sigma + 1} \left( \frac{\rho}{\rho_0} \right)^\sigma + \frac{C_{\text{sym}}}{2} \right] \]

- Effect on the asymmetry of unstable oscillation

\(\Rightarrow\) (isospin distillation)

- Increase at most of 20\% of the asymmetry \(\mathcal{I}\)
- Competition between pairing and \(C_{\text{sym}}\)
- Confirmed the leading role of \(C_{\text{sym}}\) on distillation

\[
\mu^q_\pi = \frac{\partial (\rho E/A)}{\partial \rho_q} \bigg|_{\tilde{\rho}_q} = \mu^*_q + U^q_\pi + U^q_{\text{ANM}}
\]

\[
\frac{\partial \mu^q_\pi}{\partial \rho_q} = \frac{\partial \mu^*_q}{\partial \rho_q} + \frac{\partial U^q_\pi}{\partial \rho_q} = 0
\]

\[
\frac{\partial \mu^q_q}{\partial \rho_q} = \frac{\partial \epsilon_F^q}{\partial \rho_q} = 0
\]

\[
\tan 2\theta_\pi = \frac{c}{a_\pi - b_\pi}
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\]

- Effect on the asymmetry of unstable oscillation

  ⇒ (isospin distillation)
  - Increase at most of 20% of the asymmetry $I$
  - Competition between pairing and $C_{\text{sym}}$
  - Confirmed the leading role of $C_{\text{sym}}$ on distillation

\[
\partial \mu_q^\pi / \partial \rho_q = \partial \mu_q^\pi / \partial \rho_q + \partial U^q_\pi / \partial \rho_q = \delta_q
\]

\[
\frac{\partial \mu_q}{\partial \rho_q} = \frac{\partial \epsilon_F^q}{\partial \rho_q} = \frac{2 \epsilon_F^q}{3 \rho_q}
\]

\[
\tan 2\theta_\pi = \frac{c}{a_\pi - b_\pi}
\]

Burrello S., Colonna M., Matera F. | Pairing effects on spinodal decomposition of ANM
Effect on chemical potential derivatives ($T = 0$)

- Energy density functional in paired ANM ($\tilde{\rho} \equiv 2\Delta/\nu_\pi$):

$$\rho \frac{E}{A} = \sum_q \left[ 2 \int \frac{d\mathbf{p}}{h^3} f^q(p) \frac{p^2}{2m} + \nu_\pi(\rho_q) \frac{\tilde{\rho}_q}{4} \right] + \rho \left[ \frac{A}{2} \left( \frac{\rho}{\rho_0} \right) + \frac{B}{\sigma + 1} \left( \frac{\rho}{\rho_0} \right)^\sigma + \frac{C_{\text{sym}}}{2} \mathcal{I}^2 \right]$$

- Effect on the asymmetry of unstable oscillation

  ⇒ (isospin distillation)

  - Increase at most of 20% of the asymmetry $\mathcal{I}$
  - Competition between pairing and $C_{\text{sym}}$

  - Confirmed the leading role of $C_{\text{sym}}$ on distillation

$$\mu^q_\pi = \left. \frac{\partial (\rho E/A)}{\partial \rho_q} \right|_{\tilde{\rho}_q} = \mu^*_q + U^q_\pi + U^q_{\text{ANM}}$$

$$a_\pi, b_\pi = \frac{\partial \mu^q_\pi}{\partial \rho_q} = \frac{\partial \mu^*_q}{\partial \rho_q} + \frac{\partial U^q_\pi}{\partial \rho_q} + \frac{\partial U^q_{\text{ANM}}}{\partial \rho_q} \equiv \delta_q$$

$$a, b = \frac{\partial \mu_q}{\partial \rho_q} = \frac{\partial \epsilon_F^q}{\partial \rho_q} + \frac{\partial U^q_{\text{ANM}}}{\partial \rho_q} = \frac{2}{3} \frac{\epsilon_F^q}{\rho_q}$$

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Effect on chemical potential derivatives ($T = 0$)

- Energy density functional in paired ANM ($\tilde{\rho} \equiv 2\Delta/\nu_\pi$):
  
  $$
  \rho \frac{E}{A} = \sum_q \left[ 2 \int \frac{dp}{h^3} f^q(p) \frac{p^2}{2m} + v_\pi(\rho_q) \frac{|\tilde{\rho}_q|^2}{4} \right] + \rho \left[ \frac{A}{2} \left( \frac{\rho}{\rho_0} \right) + \frac{B}{\sigma + 1} \left( \frac{\rho}{\rho_0} \right)^\sigma + \frac{C_{\text{sym}}}{2} \right]
  $$

- Effect on the asymmetry of unstable oscillation
  
  $$
  \mu^q_\pi = \frac{\partial (\rho E/A)}{\partial \rho_q} \bigg|_{\tilde{\rho}} = \mu^*_q + U^q_\pi + U^q_{ANM}
  $$

  $$
  \frac{\partial \mu^q_\pi}{\partial \rho_q} = \frac{\partial \mu^*_q}{\partial \rho_q} + \frac{\partial U^q_\pi}{\partial \rho_q} \equiv \delta_q
  $$

  $$
  \frac{\partial \mu_q}{\partial \rho_q} = \frac{\partial \epsilon^q_F}{\partial \rho_q} = \frac{2}{3} \frac{\epsilon^q_F}{\rho_q}
  $$

- Effect on the asymmetry of unstable oscillation
  
  $$
  \tan 2\theta_\pi = \frac{c}{a_\pi - b_\pi}
  $$

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Effect on chemical potential derivatives \((T = 0)\)

- **Energy density functional in paired ANM** \((\tilde{\rho} \equiv 2\Delta/\nu_\pi)\):
  \[
  \rho \frac{E}{A} = \sum_q \left[ 2 \int \frac{dp}{\hbar^3} f^q(p) \frac{p^2}{2m} + v_\pi(\rho_q) \frac{\left| \tilde{\rho}_q \right|^2}{4} \right] + \rho \left[ \frac{A}{2} \left( \frac{\rho}{\rho_0} \right) + \frac{B}{\sigma + 1} \left( \frac{\rho}{\rho_0} \right)^\sigma + \frac{C_{\text{sym}}}{2} \right]^2
  \]

- **Effect on the asymmetry of unstable oscillation**
  \[\Rightarrow \text{(isospin distillation)}\]

- Increase at most of 20% of the asymmetry \(I\)
Effect on chemical potential derivatives ($T = 0$)

- Energy density functional in paired ANM ($\tilde{\rho} \equiv 2\Delta/v_\pi$):
  \[
  \rho \frac{E}{A} = \sum_q \left[ 2 \int \frac{dp}{h^3} f^q(p) \frac{p^2}{2m} + v_\pi(\rho_q) \left| \tilde{\rho}_q \right|^2 \right] + \rho \left[ \frac{A}{2} \left( \frac{\rho}{\rho_0} \right) + \frac{B}{\sigma + 1} \left( \frac{\rho}{\rho_0} \right)^\sigma + \frac{C_{\text{sym}}}{2} I^2 \right].
  \]

- Effect on the asymmetry of unstable oscillation
  \[\Rightarrow \text{(isospin distillation)}\]
  - increase at most of 20% of the asymmetry $I$
  - competition between pairing and $C_{\text{sym}}$
  - confirmed the leading role of $C_{\text{sym}}$ on distillation

\[\mu^q_\pi = \frac{\partial (\rho E/A)}{\partial \rho_q} \bigg|_{\tilde{\rho}_q} = \mu^*_q + U^q_\pi + U^q_{\text{ANM}}\]

\[\frac{\partial \mu^q_\pi}{\partial \rho_q} = \frac{\partial \mu^*_q}{\partial \rho_q} + \frac{\partial U^q_\pi}{\partial \rho_q} \equiv \delta_q\]

\[\frac{\partial \mu_q}{\partial \rho_q} = \frac{\partial \epsilon^q_F}{\partial \rho_q} = \frac{2 \epsilon^q_F}{3 \rho_q}\]

\[\tan 2\theta_\pi = \frac{c}{a_\pi - b_\pi}\]
Effect on chemical potential derivatives ($T = 0$)

- Energy density functional in paired ANM ($\tilde{\rho} \equiv 2\Delta/\nu_\pi$):
  \[
  \frac{\rho E}{A} = \sum_q \left[ 2 \int \frac{d^3p}{h^3} f^q(p) \frac{p^2}{2m} + \nu_\pi(\rho_q) \frac{|\tilde{\rho}_q|^2}{4} \right] + \rho \left[ \frac{A}{2} \left( \frac{\rho}{\rho_0} \right) + \frac{B}{\sigma + 1} \left( \frac{\rho}{\rho_0} \right)^\sigma + \frac{C_{\text{sym}}}{2} \right]
  \]

- Effect on the asymmetry of unstable oscillation
  ⇒ (isospin distillation)
  - increase at most of 20% of the asymmetry $I$
  - competition between pairing and $C_{\text{sym}}$
  - confirmed the leading role of $C_{\text{sym}}$ on distillation

\[
\delta \mu_{\pi} = \frac{\partial (\rho E/A)}{\partial \rho_q} \bigg|_{\tilde{\rho}_q} = \mu_{\pi}^* + U^q_{\pi} + U^q_{\text{ANM}}
\]

\[
\frac{\partial \mu_{\pi}^*}{\partial \rho_q} = \frac{\partial \mu_{\pi}^*}{\partial \rho_q} + \frac{\partial U^q_{\pi}}{\partial \rho_q} \equiv \delta_q
\]

\[
\frac{\partial \mu_q}{\partial \rho_q} = \frac{\partial \epsilon_F^q}{\partial \rho_q} = \frac{2 \epsilon_F^q}{3 \rho_q}
\]

\[
\tan 2\theta_{\pi} = \frac{c}{a_{\pi} - b_{\pi}}
\]
Effect on chemical potential derivatives ($T = 0$)

- Energy density functional in paired ANM ($\tilde{\rho} \equiv 2\Delta/\nu_\pi$):

$$\frac{\rho E}{A} = \sum_q \left[ 2 \int \frac{dp}{h^3} f^q(p) \frac{p^2}{2m} + \nu_\pi (\rho_q) \frac{|\tilde{\rho}_q|^2}{4} \right] + \rho \left[ \frac{A}{2} \left( \frac{\rho}{\rho_0} \right) + \frac{B}{\sigma + 1} \left( \frac{\rho}{\rho_0} \right)^\sigma + \frac{C_{\text{sym}}}{2} I^2 \right]$$

- Effect on the asymmetry of unstable oscillation
  \[ \Rightarrow \text{(isospin distillation)} \]
  - Increases at most of 20% of the asymmetry $I$
  - Competition between pairing and $C_{\text{sym}}$

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Effect on chemical potential derivatives ($T = 0$)

- Energy density functional in paired ANM ($\tilde{\rho} \equiv 2\Delta/v_\pi$):
  \[
  \rho \frac{E}{A} = \sum_q \left[ 2 \int \frac{dp}{h^3} f^q(p) \frac{p^2}{2m} + v_\pi(\rho_q) \frac{|\tilde{\rho}_q|^2}{4} \right] + \rho \left[ \frac{A}{2} (\frac{\rho}{\rho_0})^{\sigma} + \frac{B}{\sigma + 1} (\frac{\rho}{\rho_0})^{\sigma} + \frac{C_{\text{sym}}}{2} I^2 \right]
  \]

- Effect on the asymmetry of unstable oscillation
  \[
  \Rightarrow \text{(isospin distillation)}
  \]
  - increase at most of 20% of the asymmetry $I$
  - competition between pairing and $C_{\text{sym}}$
  - confirmed the leading role of $C_{\text{sym}}$ on distillation

\[
\mu^q_\pi = \frac{\partial (\rho E/A)}{\partial \rho_q} \bigg|_{\tilde{\rho}_q} = \mu_q^* + U^q_\pi + U^q_{\text{ANM}}
\]

\[
\frac{\partial \mu^q_\pi}{\partial \rho_q} = \frac{\partial \mu^*_q}{\partial \rho_q} + \frac{\partial U^q_\pi}{\partial \rho_q} \equiv \delta_q
\]

\[
\frac{\partial \mu_q}{\partial \rho_q} = \frac{\partial \epsilon^q_F}{\partial \rho_q} = \frac{2}{3} \frac{\epsilon^q_F}{\rho_q}
\]

\[
\tan 2\theta_\pi = \frac{c}{a_\pi - b_\pi}
\]
Effect on chemical potential derivatives \((T \neq 0)\)

- Normal phase: 
  \[
  \frac{\partial \epsilon_q}{\partial \rho_q} \approx \frac{2}{3} \frac{\epsilon_q}{\rho_q} \left[ 1 + \frac{\pi^2}{12} \left( \frac{T}{\epsilon_q} \right)^2 \right]
  \]

- Normal ↔ superfluid: discontinuity on the 2nd derivatives of the energy
  \Rightarrow 2nd order phase transition

- Compressibility \Rightarrow jump depends on the density (disappears at \(\rho_q = \rho_M\))

- Larger effect than \(T = 0\)
- Double discontinuity
  \Rightarrow n and p critical \(T\)
- Jump \(\sim \langle \delta I \rangle\)
  \Rightarrow sizable effects on the isotopic properties of clusters
Effect on chemical potential derivatives \((T \neq 0)\)

- Normal phase: \(\frac{\partial \epsilon_F}{\partial \rho_q} \simeq \frac{2}{3} \frac{\epsilon_F}{\rho_q} \left[ 1 + \frac{\pi^2}{12} \left( \frac{T}{\epsilon_F} \right)^2 \right] \frac{\text{superfluid}}{\text{phase}} \) \(\delta_q\) decreases with \(T\)

- Normal \(\leftrightarrow\) superfluid: discontinuity on the 2nd derivatives of the energy
  \(\Rightarrow\) 2nd order phase transition

- Compressibility \(\Rightarrow\) jump depends on the density (disappears at \(\rho_q = \rho_M\))

- Larger effect than \(T = 0\)

- Double discontinuity

  \(\Rightarrow\) \(n\) and \(p\) critical \(T\)

- Jump \(\sim \langle \delta I \rangle / T\)

  \(\Rightarrow\) sizable effects on the isotopic properties of clusters

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Effect on chemical potential derivatives \( (T \neq 0) \)

- Normal phase: \[
\frac{\partial \epsilon_F^q}{\partial \rho_q} \approx \frac{2}{3} \frac{\epsilon_F^q}{\rho_q} \left[ 1 + \frac{\pi^2}{12} \left( \frac{T}{\epsilon_F^q} \right)^2 \right] \rightarrow \delta_q \text{ decreases with } T!
\]

- Normal ↔ superfluid: discontinuity on the 2nd derivatives of the energy
  \( \Rightarrow \) 2nd order phase transition

- Compressibility \( \Rightarrow \) jump depends on the density (disappears at \( \rho_q = \rho_M \))

- Larger effect than \( T = 0 \)
- Double discontinuity
  \( \Rightarrow \) n and p critical \( T \)
- Jump \( \sim \langle \delta I \rangle \)
  \( \Rightarrow \) sizable effects on the isotopic properties of clusters
Effect on chemical potential derivatives ($T \neq 0$)

- Normal phase: $\frac{\partial \epsilon^q_F}{\partial \rho^q_F} \simeq \frac{2}{3} \frac{\epsilon^F_q}{\rho^q_F} \left[ 1 + \frac{\pi^2}{12} \left( \frac{T}{\epsilon^F_q} \right)^2 \right] \overset{\text{superfluid phase}}{\rightarrow} \delta^q_q \text{ decreases with } T!$

- Normal $\leftrightarrow$ superfluid: discontinuity on the 2nd derivatives of the energy
  $\Rightarrow$ 2nd order phase transition

- Compressibility $\Rightarrow$ jump depends on the density (disappears at $\rho^q_q = \rho^q_M$)

Larger effect than $T = 0$

Double discontinuity $\Rightarrow$ sizable effects on the isotopic properties of clusters

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Pairing effects on spinodal decomposition of ANM
Effect on chemical potential derivatives \((T \neq 0)\)

- Normal phase: \(\frac{\partial \epsilon^q}{\partial \rho_q} \approx \frac{2}{3} \frac{\epsilon^q}{\rho_q} \left[ 1 + \frac{\pi^2}{12} \left( \frac{T}{\epsilon^q} \right)^2 \right] \frac{\text{superfluid}}{\text{phase}} \delta_q \) decreases with \(T\)!

- Normal ↔ superfluid: discontinuity on the 2nd derivatives of the energy
  \(\Rightarrow\) 2nd order phase transition

- Compressibility \(\Rightarrow\) jump depends on the density (disappears at \(\rho_q = \rho_M\))

Larger effect than \(T = 0\)

Burrello S., Colonna M., Matera F.  
Pairing effects on spinodal decomposition of ANM
Effect on chemical potential derivatives \((T \neq 0)\)

- Normal phase: \(\frac{\partial e_q}{\partial \rho_q} \simeq \frac{2}{3} \frac{e_q^F}{\rho_q} \left[ 1 + \frac{\pi^2}{12} \left( \frac{T}{e_q^F} \right)^2 \right] \) 
  \(\text{superfluid} \rightarrow \text{phase}\) \(\delta_q\) decreases with \(T\)!

- Normal ↔ superfluid: discontinuity on the 2nd derivatives of the energy 
  \(\Rightarrow\) 2nd order phase transition

- Compressibility \(\Rightarrow\) jump depends on the density (disappears at \(\rho_q = \rho_M\))

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- Larger effect than \(T = 0\)
- Double discontinuity
  \(\Rightarrow\) \(n\) and \(p\) critical \(T\)
Effect on chemical potential derivatives \((T \neq 0)\)

- Normal phase: 
  \[ \frac{\partial \epsilon_F}{\partial \rho_q} \approx \frac{2}{3} \frac{\epsilon_F}{\rho_q} \left[ 1 + \frac{\pi^2}{12} \left( \frac{T}{\epsilon_F} \right)^2 \right] \]
  \(\text{superfluid} \overset{\text{phase}}{\rightarrow} \delta_q \text{ decreases with } T!\)

- Normal ↔ superfluid: discontinuity on the 2nd derivatives of the energy
  \(\Rightarrow 2\text{nd order phase transition}\)

- **Compressibility** \(\Rightarrow\) jump depends on the density (disappears at \(\rho_q = \rho_M\))

- **Heat capacity** \(\Rightarrow\) implications on the astrophysical context

- Larger effect than \(T = 0\)

- Double discontinuity

\[\rho_q = \rho_M\]

\[\rho_q = \frac{\rho_M}{2}\]

\[\rho_q = 2 \rho_M\]

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Pairing effects on spinodal decomposition of ANM
Effect on chemical potential derivatives ($T \neq 0$)

- Normal phase: \( \frac{\partial \epsilon_q}{\partial \rho_q} \approx \frac{2}{3} \frac{\epsilon_q}{\rho_q} \left[ 1 + \frac{\pi^2}{12} \left( \frac{T}{\epsilon_q} \right)^2 \right] \) \( \frac{\text{superfluid}}{\text{phase}} \rightarrow \delta_q \) decreases with $T$!

- Normal $\leftrightarrow$ superfluid: discontinuity on the 2nd derivatives of the energy
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- Compressibility $\Rightarrow$ jump depends on the density (disappears at $\rho_q = \rho_M$)

- Heat capacity $\Rightarrow$ implications on the astrophysical context

- Larger effect than $T = 0$

- Double discontinuity
  \( \Rightarrow \) $n$ and $p$ critical $T$

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Pairing effects on spinodal decomposition of ANM
Effect on chemical potential derivatives \((T \neq 0)\)

- Normal phase: 
  \[
  \frac{\partial \epsilon_q}{\partial \rho_q} \approx \frac{2}{3} \frac{\epsilon_q F}{\rho_q} \left[ 1 + \frac{\pi^2}{12} \left( \frac{T}{\epsilon_q F} \right)^2 \right] \]
  superfluid phase \( \delta_q \) decreases with \( T \! \)

- Normal \(\leftrightarrow\) superfluid: discontinuity on the 2nd derivatives of the energy
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- Larger effect than \( T = 0 \)
- Double discontinuity
  \( \Rightarrow \) \( n \) and \( p \) critical \( T \)
- Jump \( \sim \left\langle \frac{\delta I}{I} \right\rangle \)
  \( \Rightarrow \) sizable effects on the isotopic properties of clusters

\[\rho = 0.08 \text{ fm}^{-3}\]
Effect on chemical potential derivatives ($T \neq 0$)

- Normal phase: \( \frac{\partial \epsilon_F}{\partial \rho} \simeq \frac{2}{3} \frac{\epsilon_F}{\rho_F} \left[ 1 + \frac{\pi^2}{12} \left( \frac{T}{\epsilon_F} \right)^2 \right] \)\( \frac{\text{superfluid}}{\text{phase}} \) \( \delta_q \) decreases with \( T \)!

- Normal ↔ superfluid: discontinuity on the 2nd derivatives of the energy
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- Compressibility \( \Rightarrow \) jump depends on the density (disappears at \( \rho_q = \rho_M \))

- Larger effect than \( T = 0 \)
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  \( \Rightarrow \) \( n \) and \( p \) critical \( T \)
- Jump \( \sim \langle \frac{\delta I}{T} \rangle \)
  \( \Rightarrow \) sizable effects on the isotopic properties of clusters

\[ \rho = 0.08 \text{ fm}^{-3} \]

- \( I = 0.1 \)
- \( I = 0.2 \)
- \( I = 0.5 \)
Effect on chemical potential derivatives \((T \neq 0)\)

- Normal phase: \[ \frac{\partial \epsilon_F^q}{\partial \rho_q} \approx \frac{2}{3} \frac{\epsilon_F^q}{\rho_q} \left[ 1 + \frac{\pi^2}{12} \left( \frac{T}{\epsilon_F^q} \right)^2 \right] \]
  \[ \xrightarrow{\text{superfluid phase}} \delta_q \text{ decreases with } T! \]

- Normal \(\leftrightarrow\) superfluid: discontinuity on the 2nd derivatives of the energy
  \[ \Rightarrow 2\text{nd order phase transition} \]

- Compressibility \(\Rightarrow\) jump depends on the density (disappears at \(\rho_q = \rho_M\))

- Larger effect than \(T = 0\)
- Double discontinuity
  \[ \Rightarrow n \text{ and } p \text{ critical } T \]
- Jump \(\sim \langle \frac{\delta I}{I} \rangle\)
  \[ \Rightarrow \text{sizable effects on the isotopic properties of clusters} \]

\[
\begin{array}{ccc}
\text{Temperature } T \text{ (MeV)} & 0 & 0.5 & 1 & 1.5 & 2 & 2.5 & 3 \\
\text{I = 0.1} & \text{I} & \text{I} & \text{I} & \text{I} & \text{I} & \text{I} & \text{I} \\
\text{I = 0.2} & \text{I} & \text{I} & \text{I} & \text{I} & \text{I} & \text{I} & \text{I} \\
\text{I = 0.5} & \text{I} & \text{I} & \text{I} & \text{I} & \text{I} & \text{I} & \text{I} \\
\end{array}
\]

\[ \rho = 0.08 \text{ fm}^{-3} \]
Conclusions

- Clusterization studies with pairing confirm the leading role of the symmetry energy in determining isotopic properties of density fluctuations.
- At the transition temperature (from superfluid to normal phase), significant effects may appear for the asymmetry of the density oscillations.
- Implications in the astrophysical context.
Conclusions

- Clusterization studies with *pairing* confirm the leading role of the *symmetry energy* in determining isotopic properties of density fluctuations.
- At the *transition temperature* (from superfluid to normal phase) significant effects may appear for the asymmetry of the density oscillations.

⇒ Implications in the astrophysical context.
Conclusions

- Clusterization studies with **pairing** confirm the leading role of the **symmetry energy** in determining isotopic properties of density fluctuations.
- At the **transition temperature** (from superfluid to normal phase) significant effects may appear for the asymmetry of the density oscillations.
- $\Rightarrow$ Implications in the **astrophysical** context.
Conclusions

- Clusterization studies with pairing confirm the leading role of the symmetry energy in determining isotopic properties of density fluctuations.
- At the transition temperature (from superfluid to normal phase) significant effects may appear for the asymmetry of the density oscillations.

⇒ Implications in the astrophysical context.

Thank You!