Holographic Q-Lattices and Metal-Insulator Transitions

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Holographic tools provide a powerful framework for investigating strongly coupled systems using weakly coupled theories of gravity

Make contact with real systems?

Greatly enriched our understanding of holography and of black holes

Examples
- Superconducting phases - s,p and d
- Spatially modulated phases
- Non fermi liquids
- New ground states - Lifshitz, Schrodinger, hyperscaling violating,....

Metal Insulator Transitions - can we realise them and can we find new ground states? How do we realise insulators?

[Hartnoll, Donos]

Metal - Insulator transition

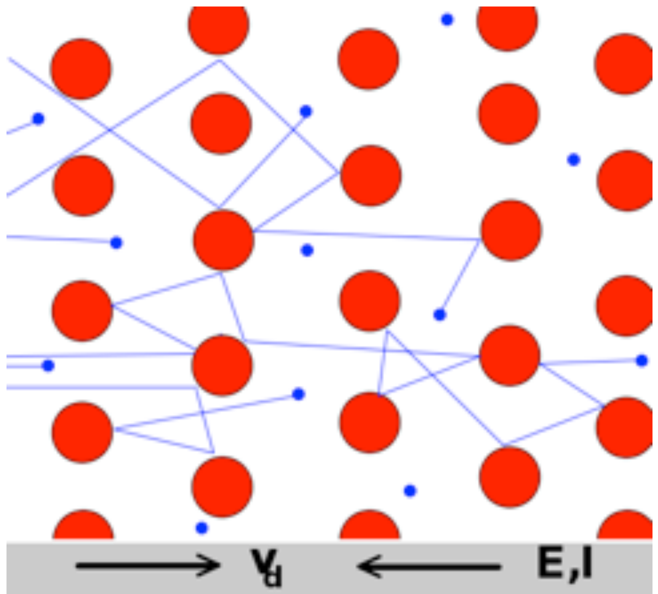
Dramatic reorganisation of degrees of freedom

Examples:

- Band insulators: Fermi energy moves to gap due Fermi surface instabilities
- Mott transitions: coulomb interactions
- Anderson Localisation: localisation caused by impurities

Drude Model of transport in a metal

Quasi-particle interactions ignored



$$m\frac{d}{dt}v = qE - \frac{m}{\tau}v \qquad \Rightarrow v = \frac{q\tau E}{m}$$

$$J = nqv$$

$$J = \sigma_{DC}E \qquad \sigma_{DC} = \frac{nq^2\tau}{m}$$

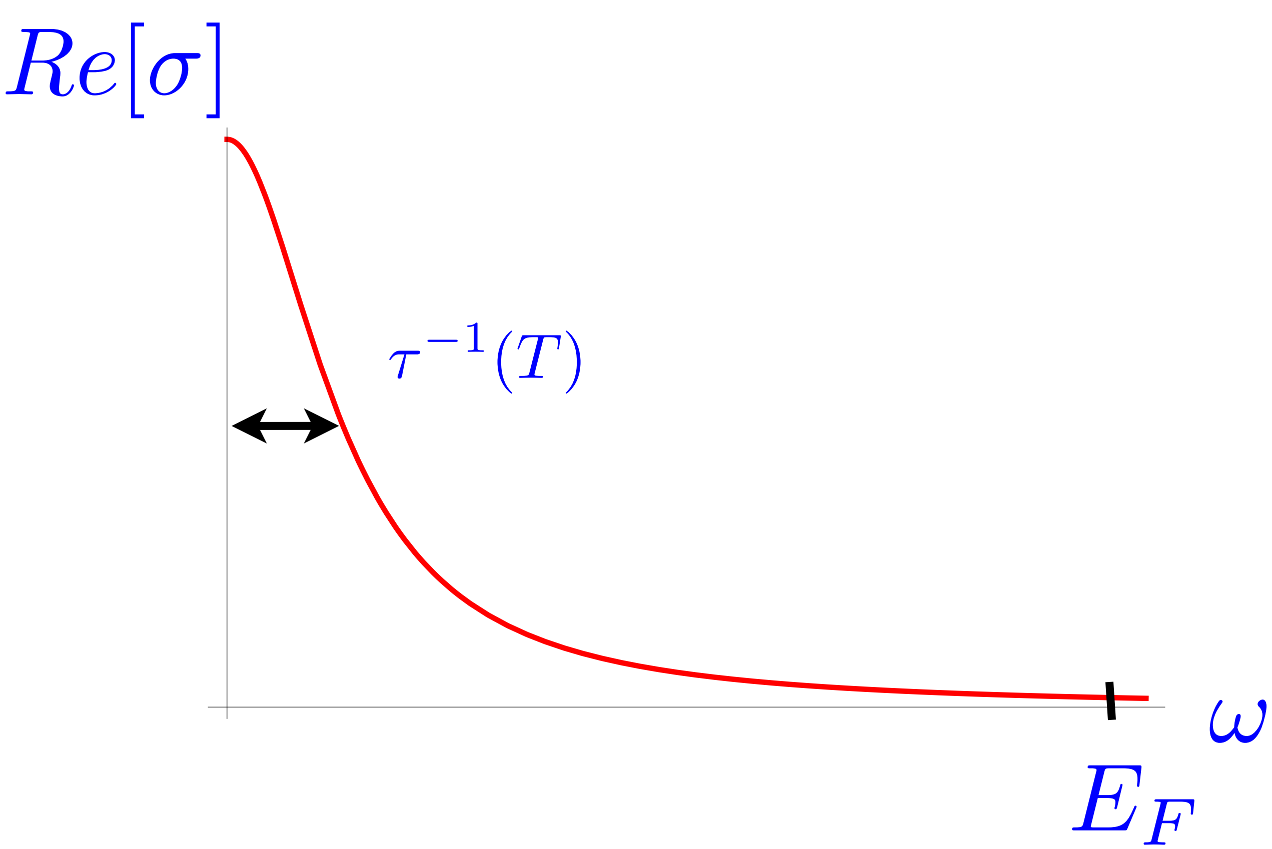
Drude Model

$$E = E(\omega)e^{-i\omega t}$$

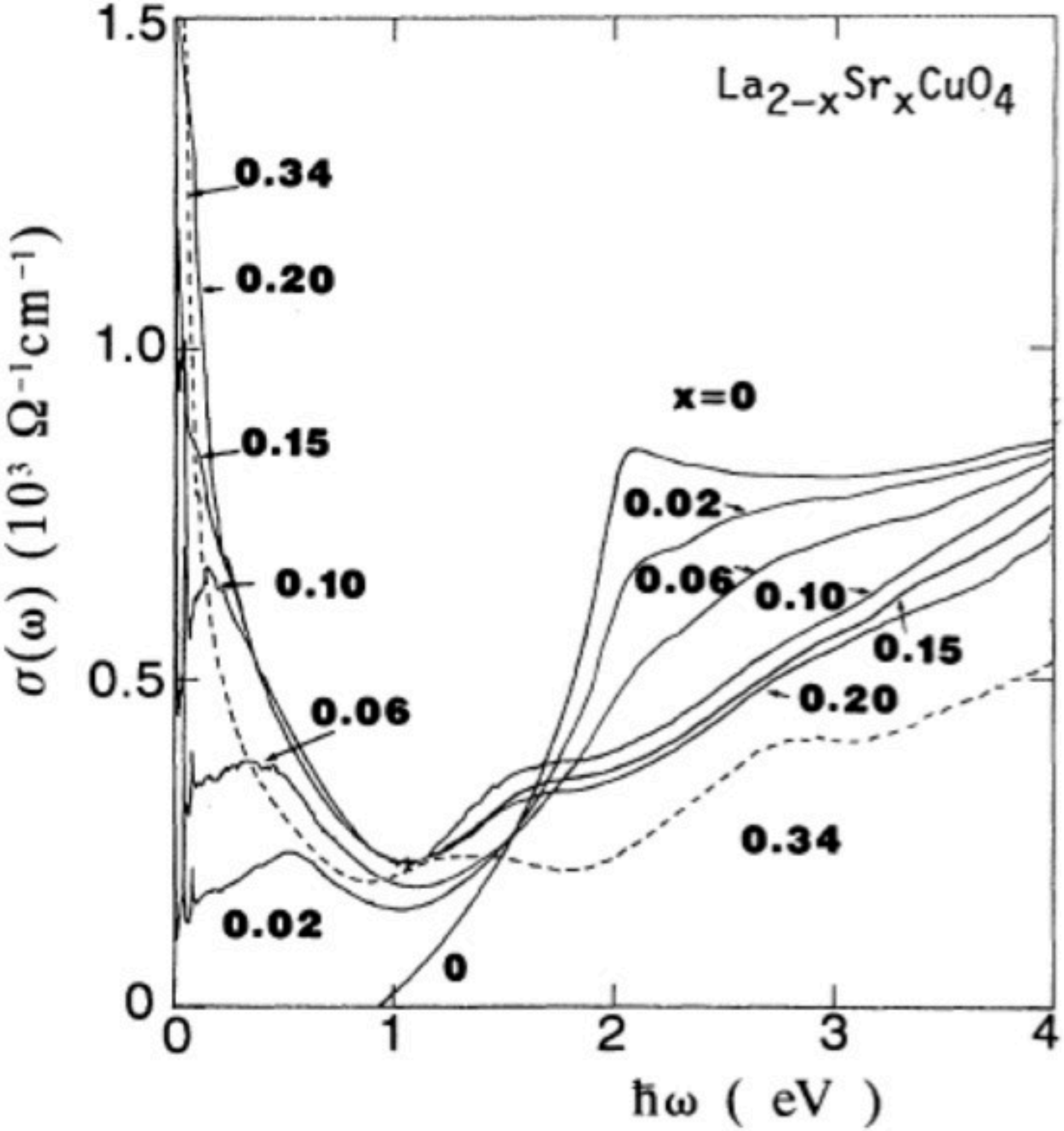
$$J = J(\omega)e^{-i\omega t}$$

$$J(\omega) = \sigma(\omega)E(\omega)$$

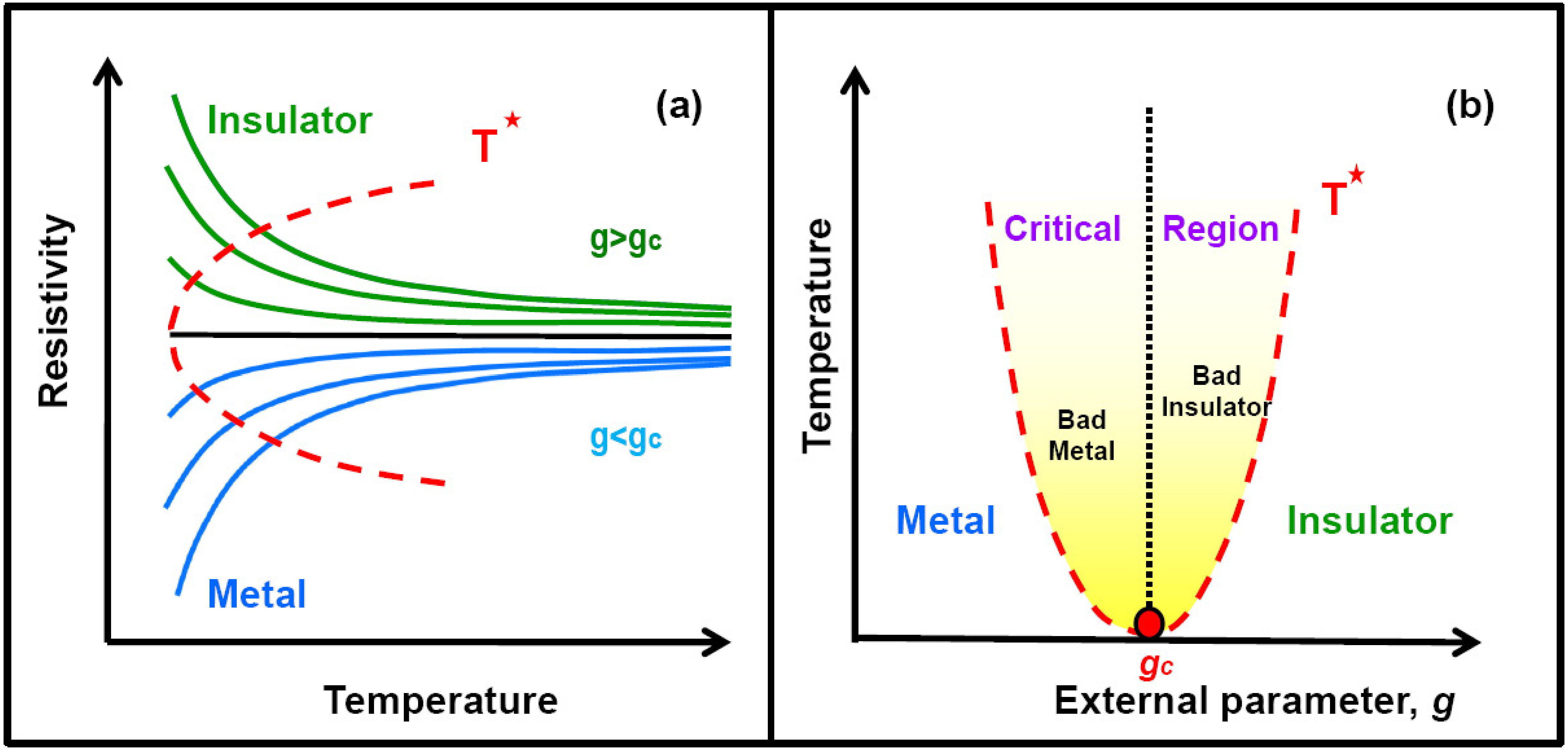
$$\sigma(\omega) = \frac{\sigma_{DC}}{1 - i\omega\tau}$$



"Good" metal



Interaction driven and strongly coupled


(a)
Insulator
T*
g>gc
g<gc
Metal
Resistivity
Temperature
(b)
T*
Critical
Region
Bad Metal
Bad Insulator
Metal
Insulator
Temperature
gc
External parameter, g

Electrically charged AdS-RN black hole

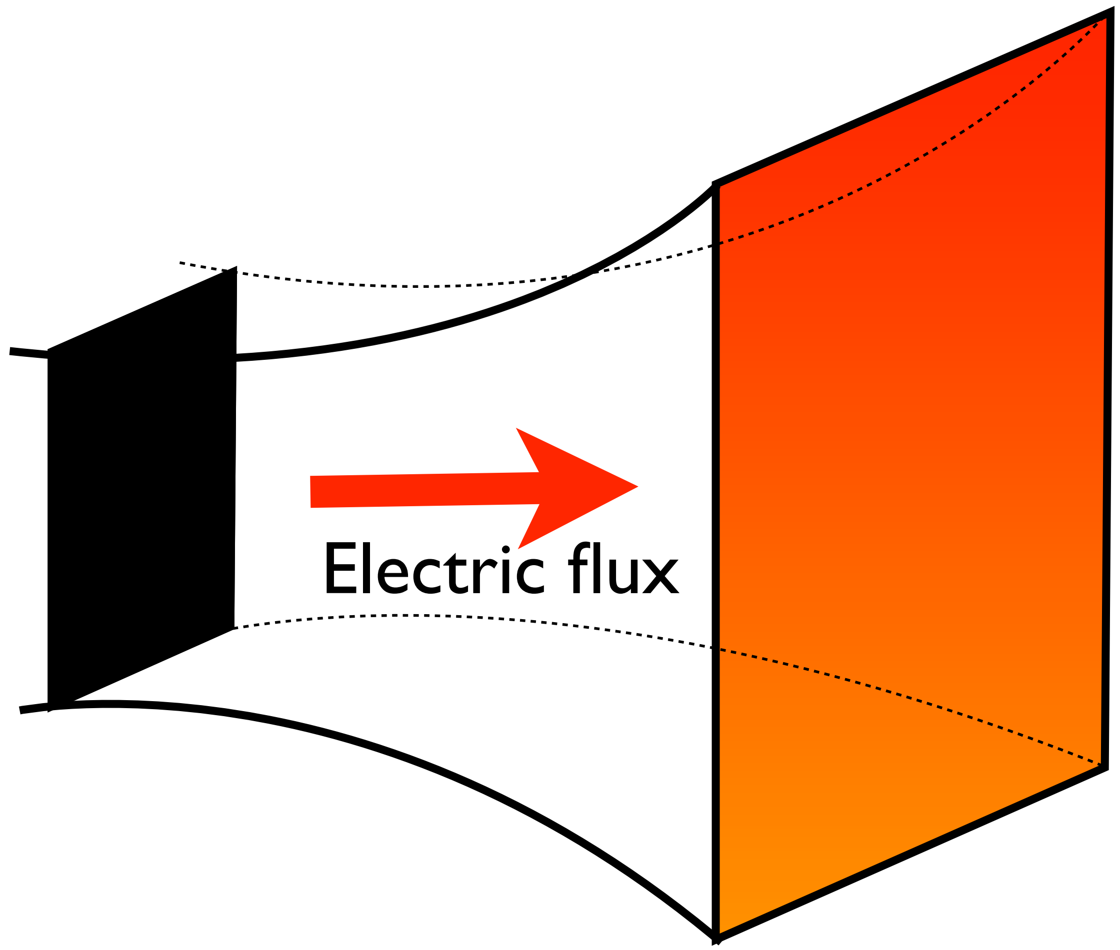
Holographic matter at finite charge density and translationally invariant

$$ds^2 = -U dt^2 + \frac{dr^2}{U} + r^2(dx^2 + dy^2)$$

$$A_t = \mu(1 - \frac{r_+}{r})$$

UV: $r \to \infty$ $\quad AdS_5$

IR: $r \to r_+$ black hole horizon topology $\mathbb{R}^2$ and temp T



At T=0 AdS-RN black hole interpolates between

$$AdS_4 \longrightarrow AdS_2 \times \mathbb{R}^2$$

UV IR

In the far IR a locally quantum critical fixed point emerges

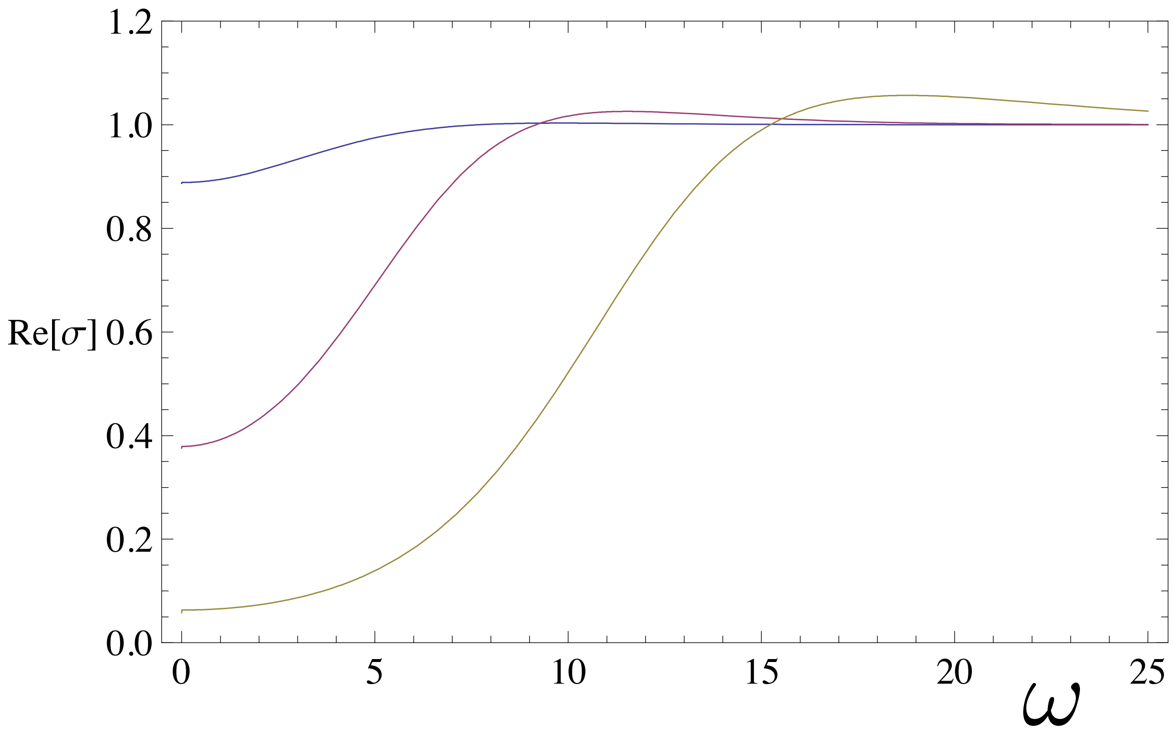
Conductivity calculation

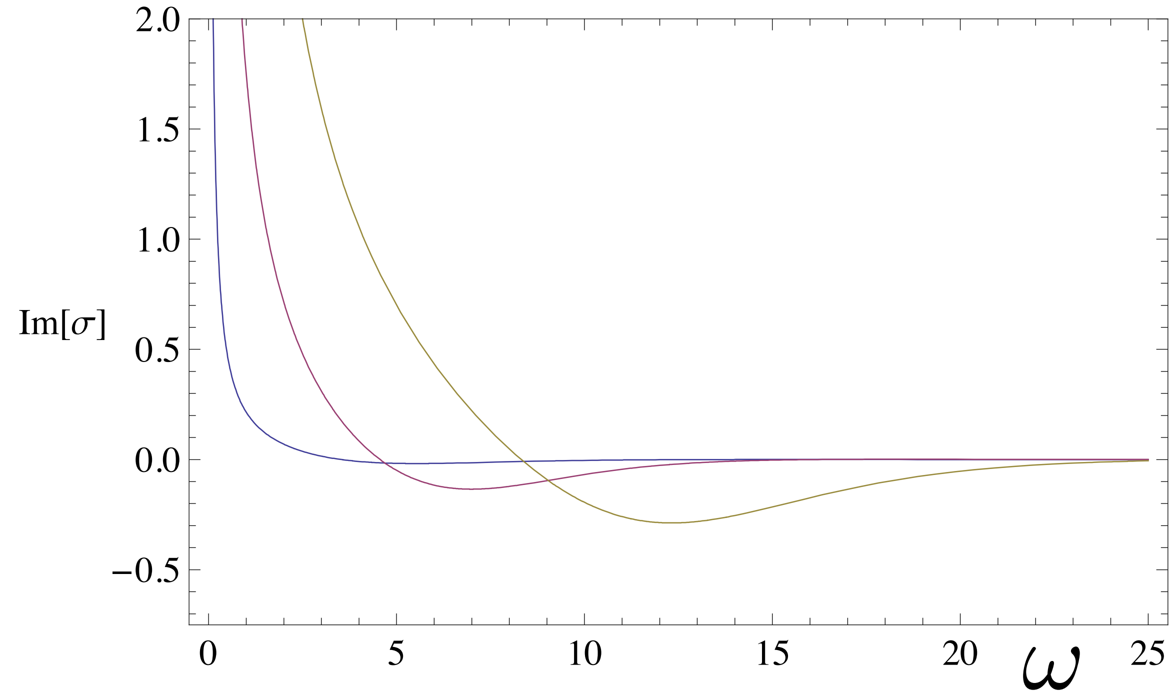
$$\delta A_x = e^{-i\omega t} a_x(r)$$

$$\delta g_{tx} = e^{-i\omega t} h_{tx}(r)$$

$$\sigma(\omega) = -i\frac{G_{J_x J_x}(\omega)}{\omega}$$

[Hartnoll]





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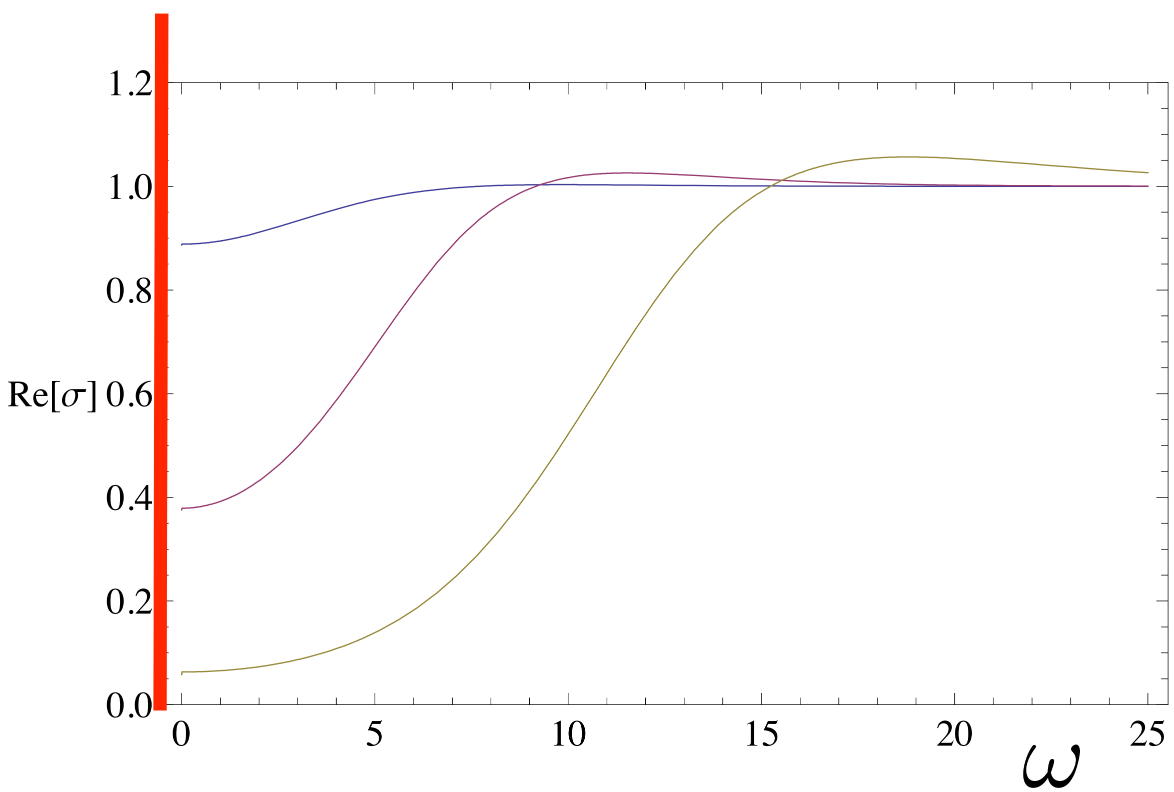
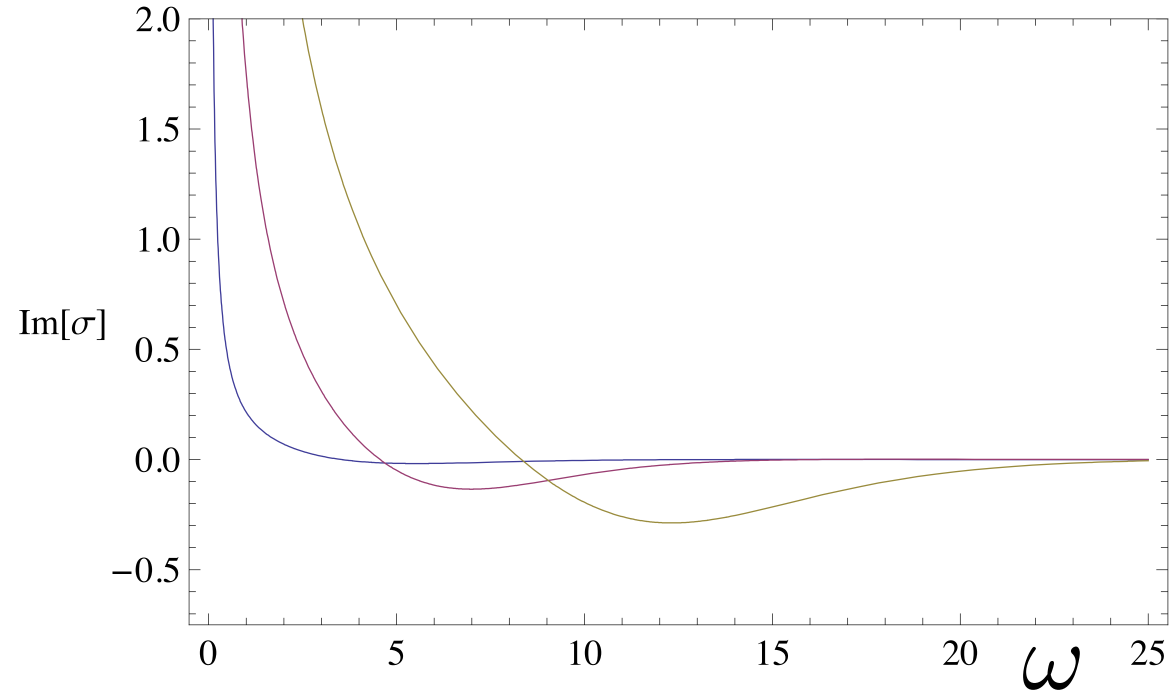
[Hartnoll]





Kramers-Kronig implies $\sigma(\omega) \sim \delta(\omega) + \frac{i}{\omega}$

Delta function arises because translation invariance implies there is no momentum dissipation

Any translation invariant theory will have a delta function

To realise more realistic metals and/or insulators need to break translations

Holographic lattices

Probe brane constructions [Hartnoll,Polchinski,Silverstein,Tong]...
and massive gravity [Vegh]....
have also been used to study momentum dissipation

Holographic Lattices and metals

Break translation invariance explicitly using a deformation of the CFT

A few examples of periodic monochomratic lattices have been studied [Horowitz, Santos, Tong]

In Einstein-Maxwell theory construct black holes with

$$\mu(x) = \mu_0 + \lambda \cos(kx) \qquad A_t(r,x) \sim \mu(x) + \mathcal{O}(\frac{1}{r})$$

Need to solve PDEs

Alternatively add a real scalar field to Einstein-Maxwell and consider

$$\phi(r,x) \sim \frac{\lambda \cos(kx)}{r^{3-\Delta}} + \dots$$

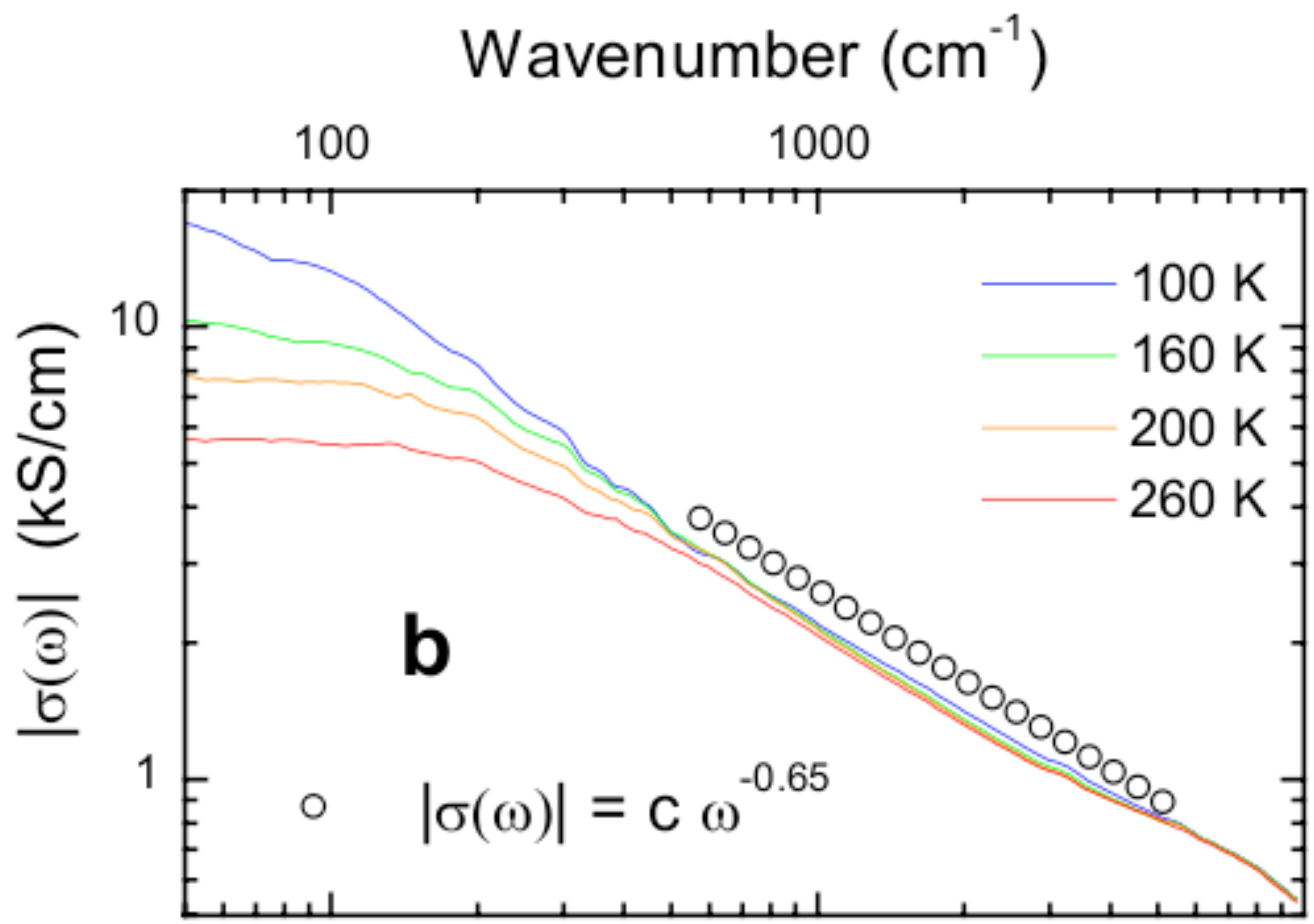
Key results [Horowitz, Santos, Tong]

- Holographic metals with Drude peaks
- Claim that there is an intermediate scaling

$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

and moreover that is universal

Reminiscent of the cuprates



At T=0 these holographic lattices approach $AdS_2 \times \mathbb{R}^2$ in the IR with a deformation by an irrelevant operator of the locally quantum critical theory $\mathcal{O}(k_{IR})$ with dimension $\Delta(k_{IR}, \mu_0)$

Hartnoll, Hoffman:

Using field theory and holographic arguments predict

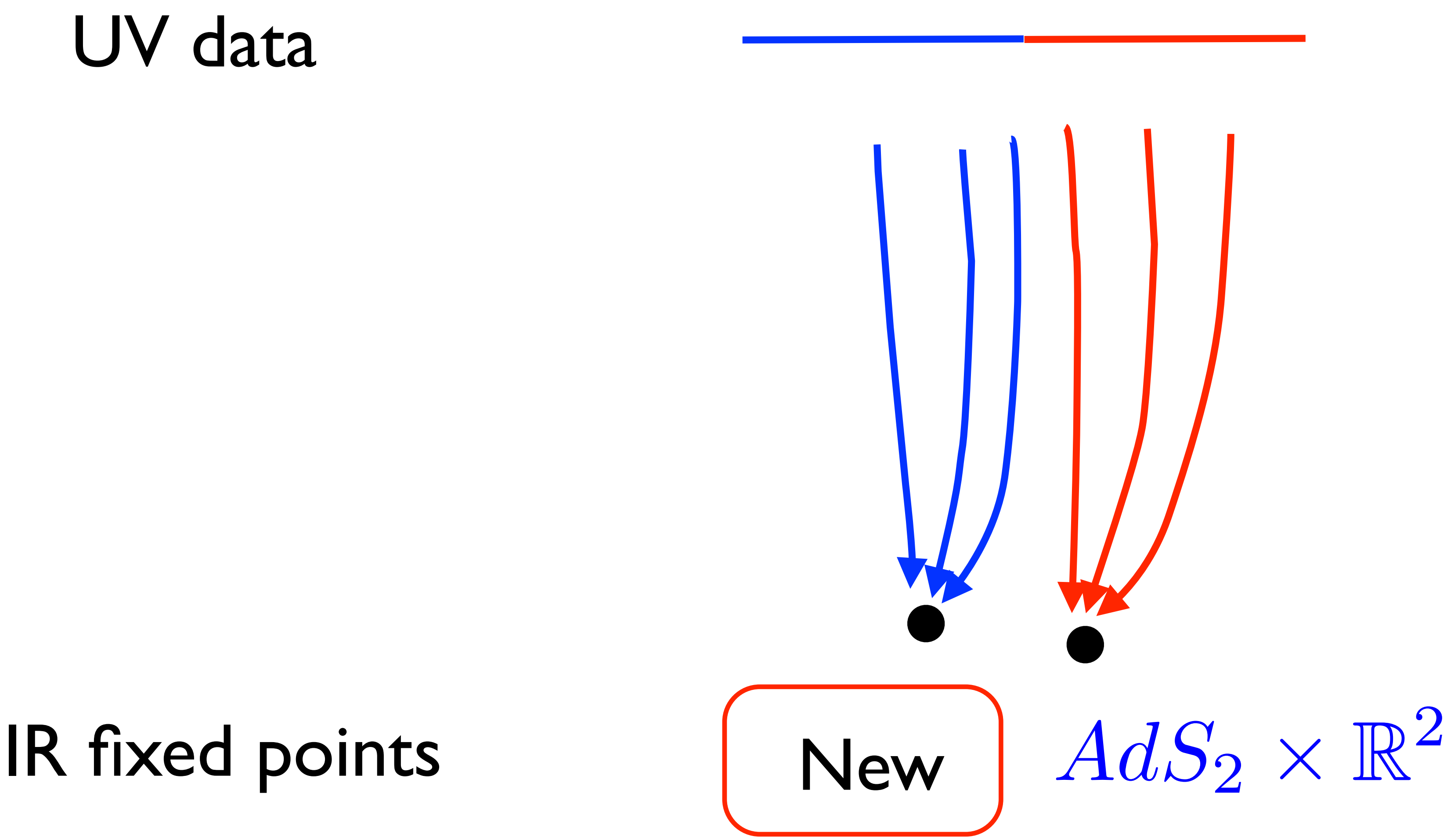
$$\rho_{DC} = \sigma_{DC}^{-1} \sim T^{2\Delta(k_{IR})-2} \qquad T << \mu$$

Can be more than one irrelevant operator in the IR and then it is the least irrelevant

Subtleties:

- Generically $k_{IR} \neq k_{UV}$
- Used matching argument in $T << \omega << \mu$

Holographic Metal-Insulator transition



Strategy: vary UV data aiming to destabilise $AdS_2 \times \mathbb{R}^2$

First examples in D=5 using helical lattices [Donos,Hartnoll]

We will find new M-I transitions as well as M-M transitions

Holographic Q-lattices - Part I

Exploit a global symmetry to break translation invariance

Consider a model with gauged U(1) and global U(1) in bulk:

$$\mathcal{L} = R + 6 - \frac{1}{4}F^2 - |\partial\phi|^2 - m^2|\phi|^2$$

$$R_{\mu\nu} = g_{\mu\nu}(-3 + \frac{m^2}{2}|\phi|^2) + \partial_{(\mu}\phi\partial_{\nu)}\phi^* + \tfrac{1}{2}\left(F^2_{\mu\nu} - \tfrac{1}{4}g_{\mu\nu}F^2\right) ,$$
$$\nabla_\mu F^{\mu\nu} = 0, \qquad (\nabla^2 - m^2)\phi = 0\,,$$

$$ds^2 = -U dt^2 + U^{-1}dr^2 + e^{2V_1}dx_1^2 + e^{2V_2}dx_2^2$$
$$A = a dt$$
$$\phi = e^{ikx_1}\varphi$$

Homogeneous and anisotropic holographic lattice

Reminiscent of Coleman's construction of Q-balls

Equivalent to two real holographic lattices with phase shift

$$\phi_1 = \cos(kx)\varphi(r)$$

$$\phi_2 = \sin(kx)\varphi(r)$$

Many generalisations are possible by allowing for more general global symmetries e.g. [Andrade,Withers]

[No global symmetries expected in string theory?]

Q-lattice black holes:

$$ds^2 = -U dt^2 + U^{-1} dr^2 + e^{2V_1} dx_1^2 + e^{2V_2} dx_2^2$$
$$A = a dt$$
$$\phi = e^{ikx_1}\varphi$$

UV expansion:

$$U = r^2 + \ldots, \quad V_1 = \log r + \ldots \quad V_2 = \log r + \ldots$$
$$a = \mu + \frac{q}{r} \ldots,$$
$$\varphi = \frac{\lambda}{r^{3-\Delta}} + \cdots + \frac{\varphi_c}{r^\Delta} + \ldots$$

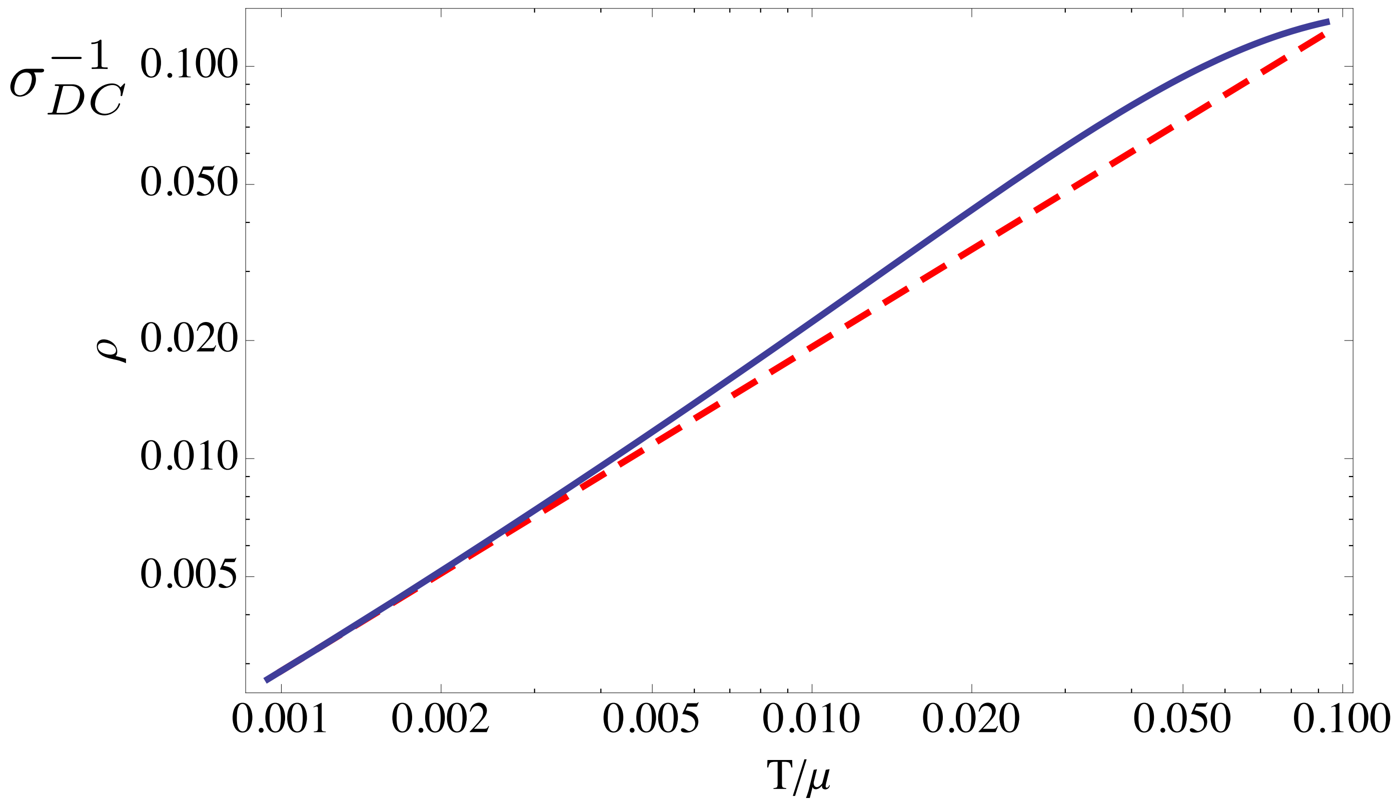
$$m^2 = -\frac{3}{2} \quad \leftrightarrow \quad \Delta = \frac{3+\sqrt{3}}{2}$$

UV data: $T/\mu \quad \lambda/\mu^{3-\Delta} \quad k/\mu$

IR expansion: regular black hole horizon

Metallic phase

$\lambda/\mu^{3-\Delta} = 1/2$ $\quad k/\mu = 1/\sqrt{2}$



At T=0 the black holes approach $AdS_2 \times \mathbb{R}^2$

Metallic phase

The irrelevant operator driving the T=0 flow from the IR has

$$\Delta(k) = \frac{1}{2} + \frac{1}{2\sqrt{3}}\sqrt{3 + 2m^2 + 2k_{IR}^2}$$

There is a renormalisation of length scales from IR to UV

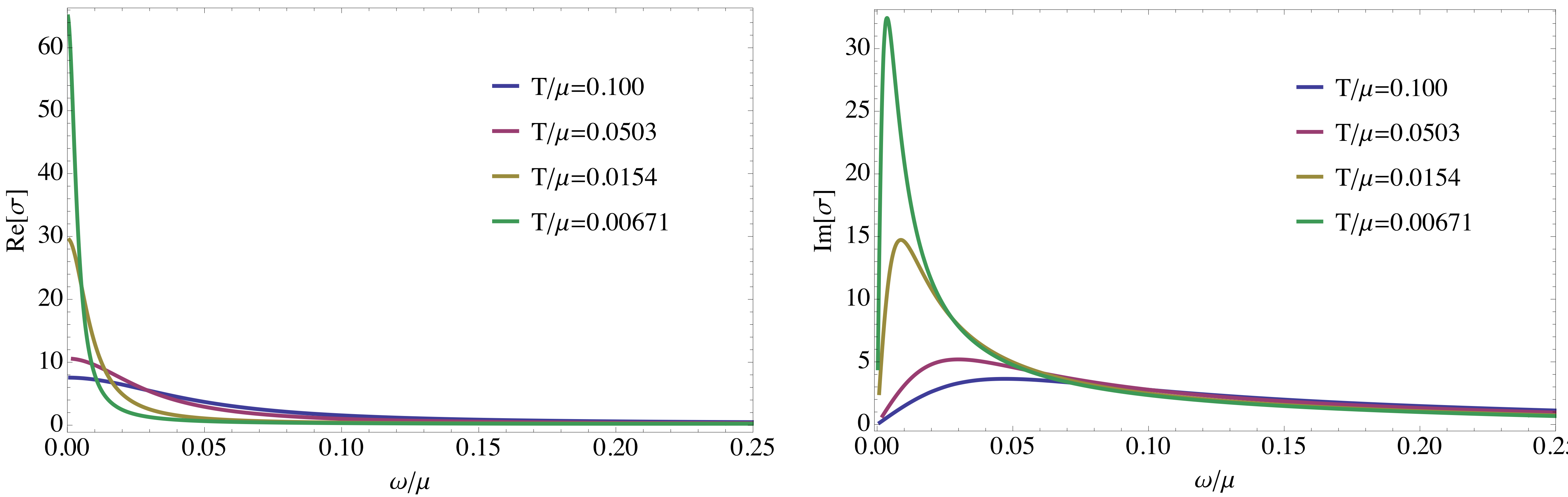
$$k_{IR} = e^{-v_{10}} k$$

[Hartnoll, Hoffman]

$$\rho \sim T^{2\Delta(k_{IR})-2}$$

Metallic phase

Drude peaks

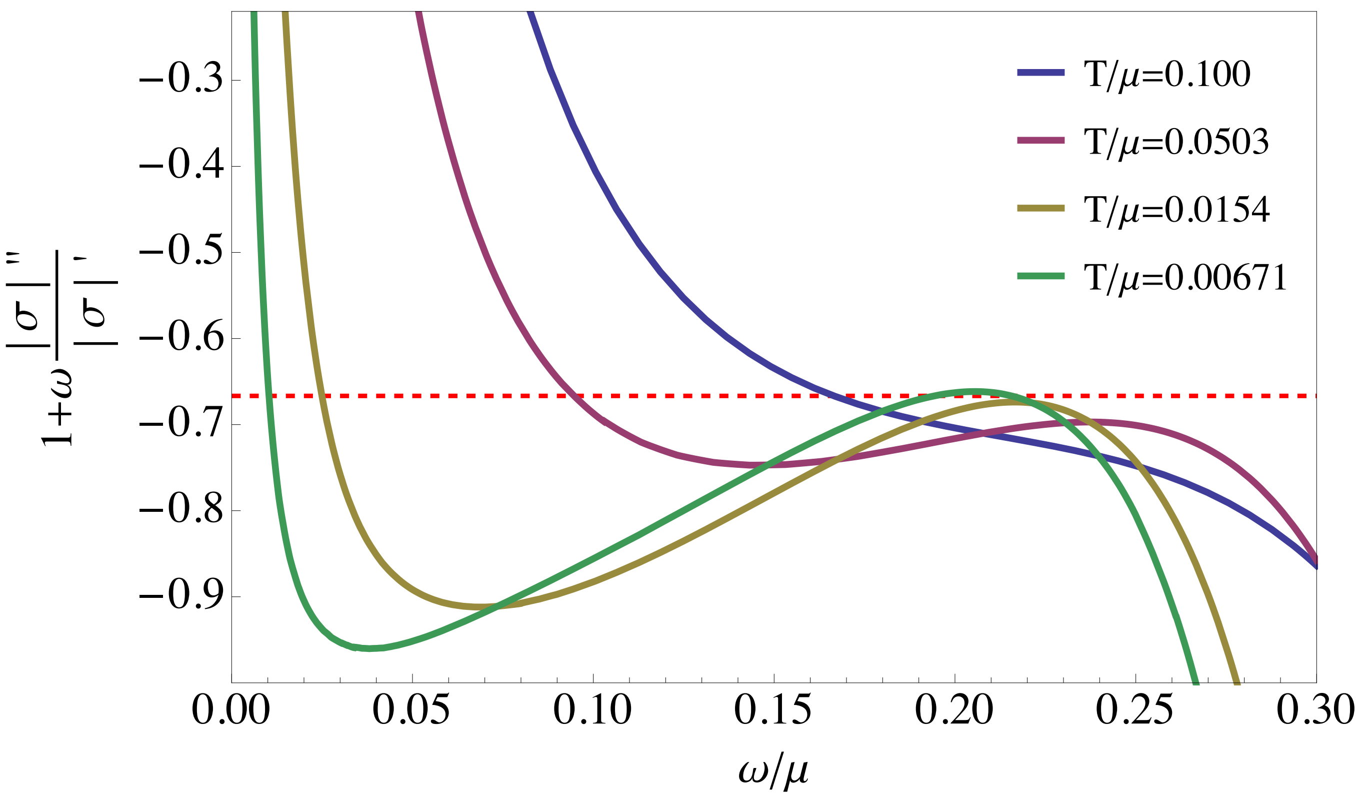


Sum rule

$$\int_0^\infty Re[\sigma(\omega)]d\omega$$

fixed by UV data

Metallic phase



Intermediate scaling is NOT universal

Note also that we could consider holographic lattices of the form

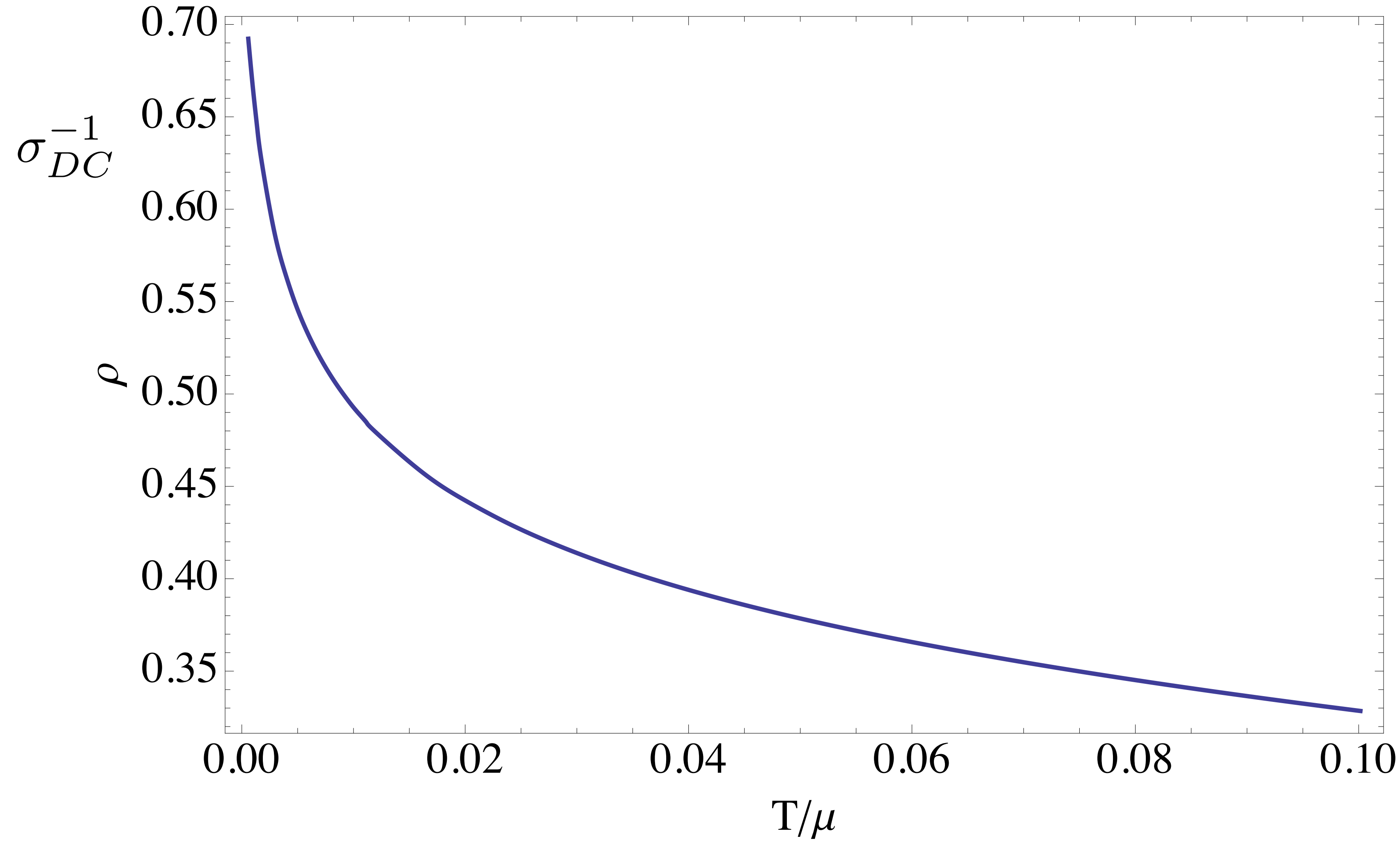
$$\phi \sim \lambda\,(\cos\alpha\cos kx_1 + i\sin\alpha\sin kx_1)\,\frac{1}{r^{3-\Delta}} + \ldots \qquad\qquad 0 \leq \alpha \leq \pi/4$$

When $\alpha \neq \pi/4$ more fields involved and need to solve PDES. However higher harmonics exponentialy suppressed

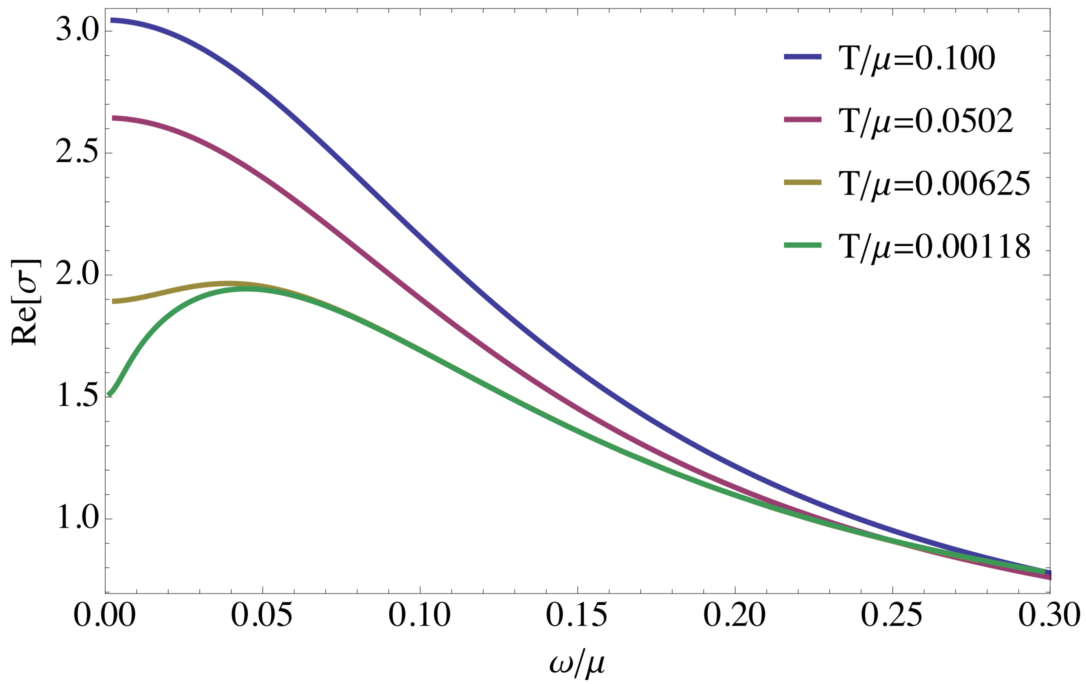
Tricky using log-log plots and fitting using PDES?

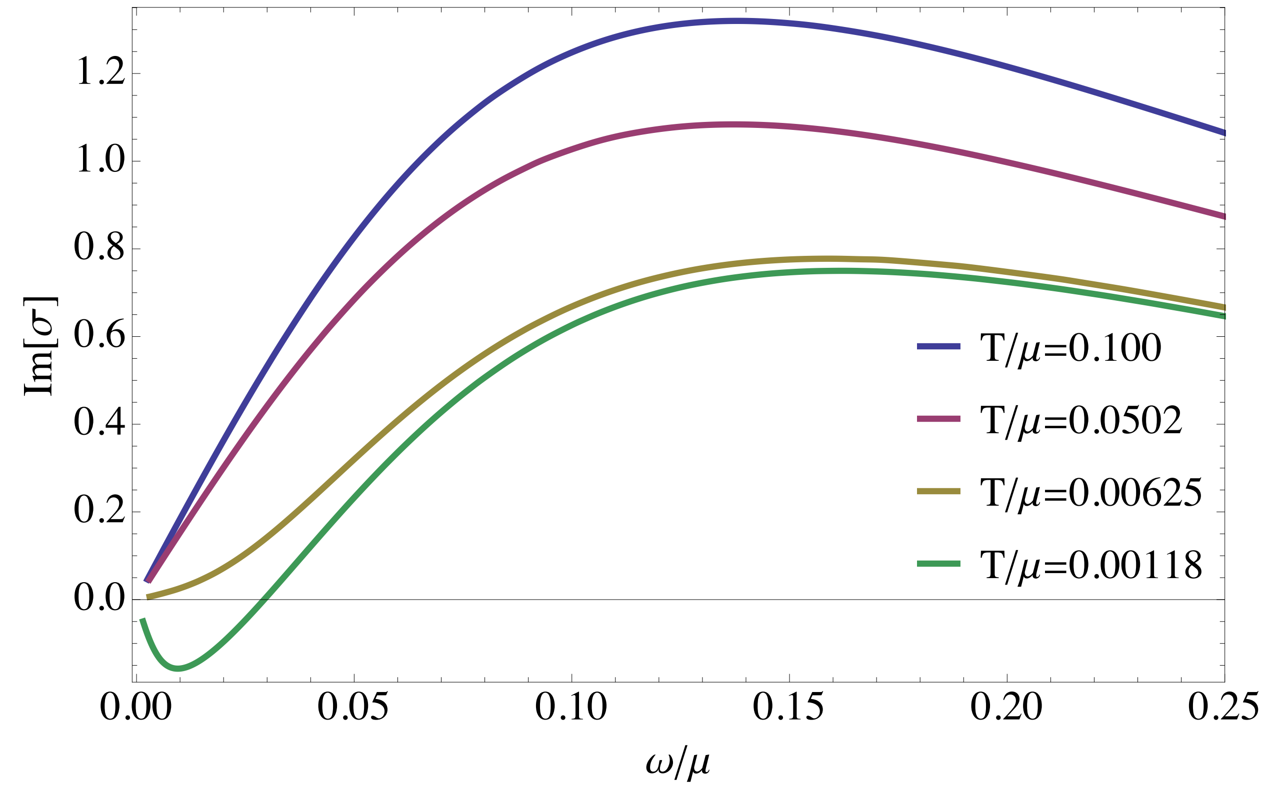
Insulating phase

$$\lambda/\mu^{3-\Delta} = 2 \qquad k/\mu = 1/2^{3/2}$$



Insulating phase





Notice the appearance of a mid-frequency hump. Spectral weight is being transferred, consistent with sum rule

What are the T=0 insulating ground states??

Obscure in this model. Seem to have s=0, but apparently not simple scaling solutions

Holographic Q-lattices - Part 2

$$\mathcal{L} = R - \frac{1}{2}\left[(\partial\phi)^2 + \Phi(\phi)(\partial\chi)^2\right] + V(\phi) - \frac{Z(\phi)}{4}F^2$$

Choose Φ, V, Z so that we have an AdS_4 vacuum and that AdS-RN is a solution

Want to construct black holes that approach novel ground states in the far IR at T=0 in addition to $AdS_2 \times \mathbb{R}^2$

Focus on the ground states which are solutions with $\phi \to -\ln r$ as $r \to 0$

and

$$\mathcal{L} \to R - \frac{3}{2}\left[(\partial\phi)^2 + e^{2\phi}(\partial\chi)^2\right] + e^{\phi} - \frac{e^{\gamma\phi}}{4}F^2$$

IR “fixed point” solutions

$$ds^2 \sim -r^u dt^2 + r^{-u} dr^2 + r^{v_1} dx_1^2 + r^{v_2} dx_2^2$$

$$e^{\phi} \sim r^{-\phi_0} \qquad A \sim r^a dt \qquad \chi = k x_1$$

with exponents fixed by k, γ

Comments:

- Solutions are a kind of generalisation of hyperscaling violating solutions
- Can arise as T=0 limits of black holes with s=0
- Similar ground states also found by [Gouteraux]

- Calculate AC conductivity

Obtained using a matching argument with ground state correlators at T=0. Valid when $T << \omega << \mu$

$$\sigma_{AC} \sim \omega^{c(\gamma)}$$

- Calculate DC conductivity

Analytic result for all T! (see later)

For $T << \mu$ the scaling is obtained from the IR fixed point solutions

$$\sigma_{DC} \sim T^{b(\gamma)}$$

In these models we have $b = c$
(as we have for the $AdS_2 \times \mathbb{R}^2$ metals)

$$\sigma_{DC} \sim T^{b(\gamma)} \qquad \sigma_{AC} \sim \omega^{c(\gamma)}$$

$-1 < \gamma < 3$ $\quad b = c > 0$ Have new type of insulating ground states

$3 < \gamma$ $\quad b = c < 0$ Have new type of incoherent metallic ground states not associated with Drude physics

$\gamma = 3$ $\quad b = c = 0$ Novel metallic ground states with finite conductivity at T=0

For fixed γ can find transitions between AdS2 metals and the new insulating and metallic ground states by varying strength of the lattice

Holographic Q-lattices - Part 3

Generalise to models that have two axion like fields

$$\mathcal{L} = R - \frac{1}{2}\left[(\partial\phi)^2 + \Phi_1(\phi)(\partial\chi_1)^2 + +\Phi_2(\phi)(\partial\chi_2)^2\right] + V(\phi) - \frac{Z(\phi)}{4}F^2$$

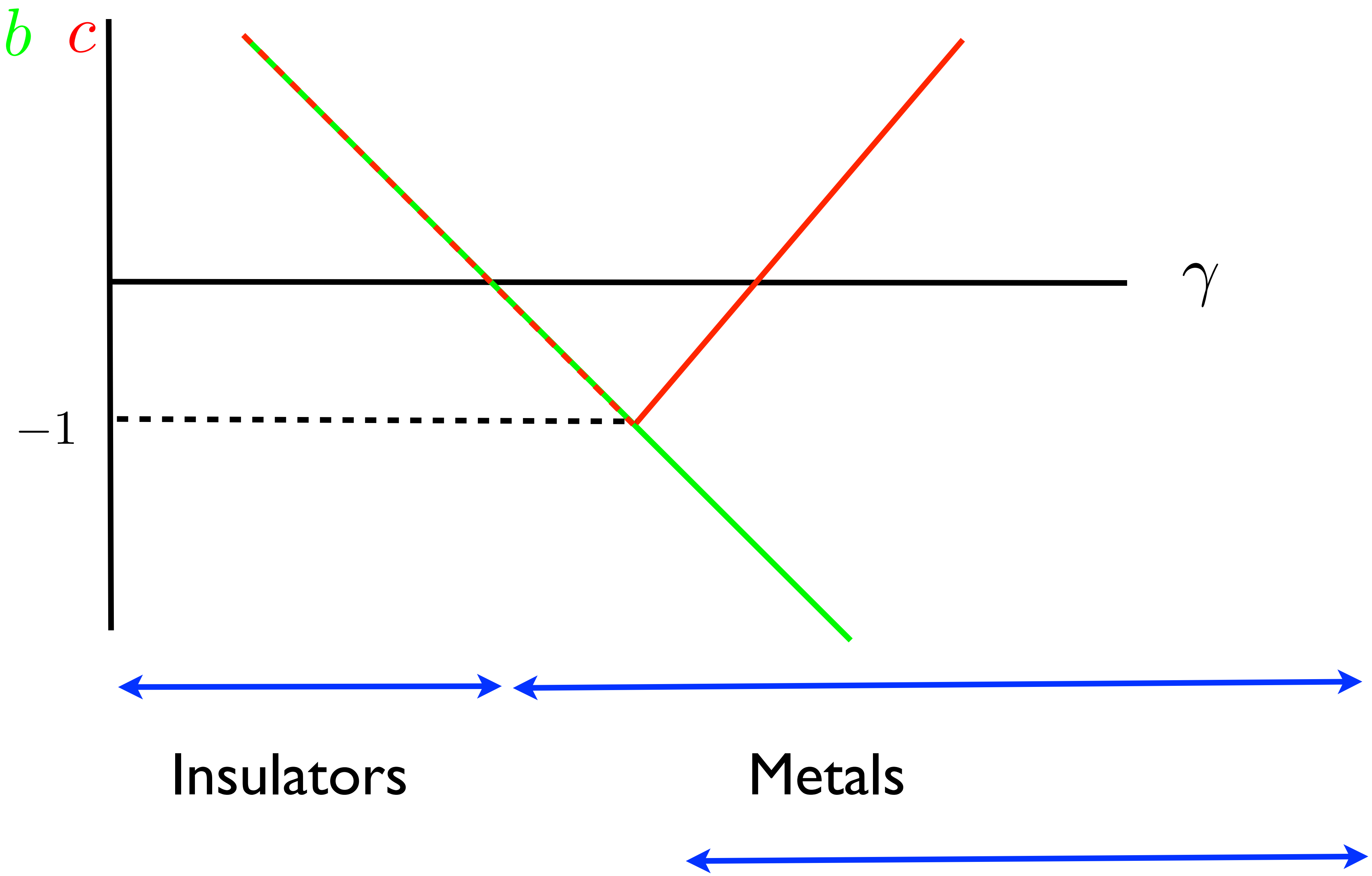
Homogeneous and isotropic Q-lattice

$$ds^2 = -U dt^2 + U^{-1} dr^2 + e^{2V_1}\left[dx_1^2 + dx_2^2\right]$$

$$A = a dt$$

$$\chi_1 = k x_1, \qquad \chi_2 = k x_2$$

$\sigma_{DC} \sim T^b$ $\sigma_{AC} \sim \omega^c$



Parametric separation of thermal and momentum relaxation scales. Reappearance of delta functions not related to the charge density and Drude physics

Analytic result for DC in terms of horizon data

Related work [Iqbal,Liu][Blake,Tong,Vegh][Withers]

Key steps:

Switch on constant electric field from start

$$A_x = -Et + \delta a_x(r)$$

Use gauge equation of motion to solve for the current J

Use timelike Killing vector to solve for the momentum and demand regularity at the black hole horizon to relate J and E

$$\mathcal{L} = R - \frac{1}{2}\left[(\partial\phi)^2 + \Phi(\phi)(\partial\chi)^2\right] + V(\phi) - \frac{Z(\phi)}{4}F^2$$

$$\sigma_{DC} = \left[e^{-V_1+V_2}Z(\phi) + \frac{q^2 e^{-V_1-V_2}}{k^2\Phi_1(\phi)}\right]_{r=r_+}$$

First term gives finite result of Iqbal, Liu for AdS-Schwarzschild

Can use this to obtain analytic results for low T scaling of IR fixed points

For the case that the IR approaches $AdS_2 \times \mathbb{R}^2$ or other fixed points where $c = b$ we recover the prediction of Hartnoll and Hofman. Also works when $c \neq b$

Summary

- Metals, insulators and transitions between them are interesting

- Holographic Q-lattices are a powerful and tractable tool to study them

- Novel new insulating and metallic phases

- Analytic result for DC conductivity in terms of horizon data