Comparing Initial-Sate Radiation at Leading Order to Initial-Sate Radiation at Next-to-Leading-Order

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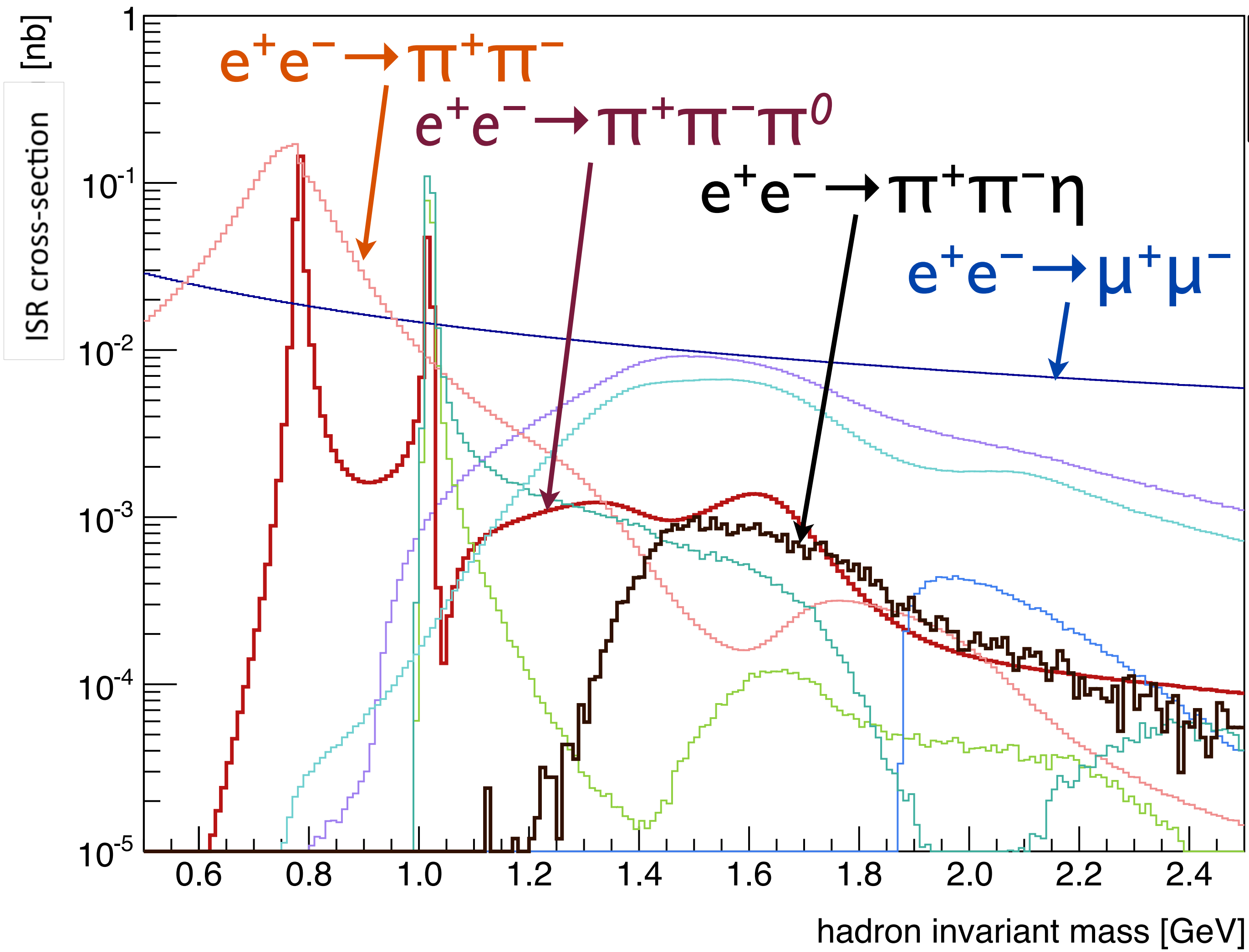
Physics Motivation: SM LO HVP contribution to the a_μ is calculated from low-energy e^+e^- production cross sections

1. **Obtain exclusive *σ(e⁺e⁻ → hadrons)***
2. Divide *σ(e⁺e⁻ → hadrons)* by *σ(e⁺e⁻ → μ⁺μ⁻)*
3. Multiply by known integral kernel function
4. Integrate over final-state invariant mass



Produced with PHOKHARA

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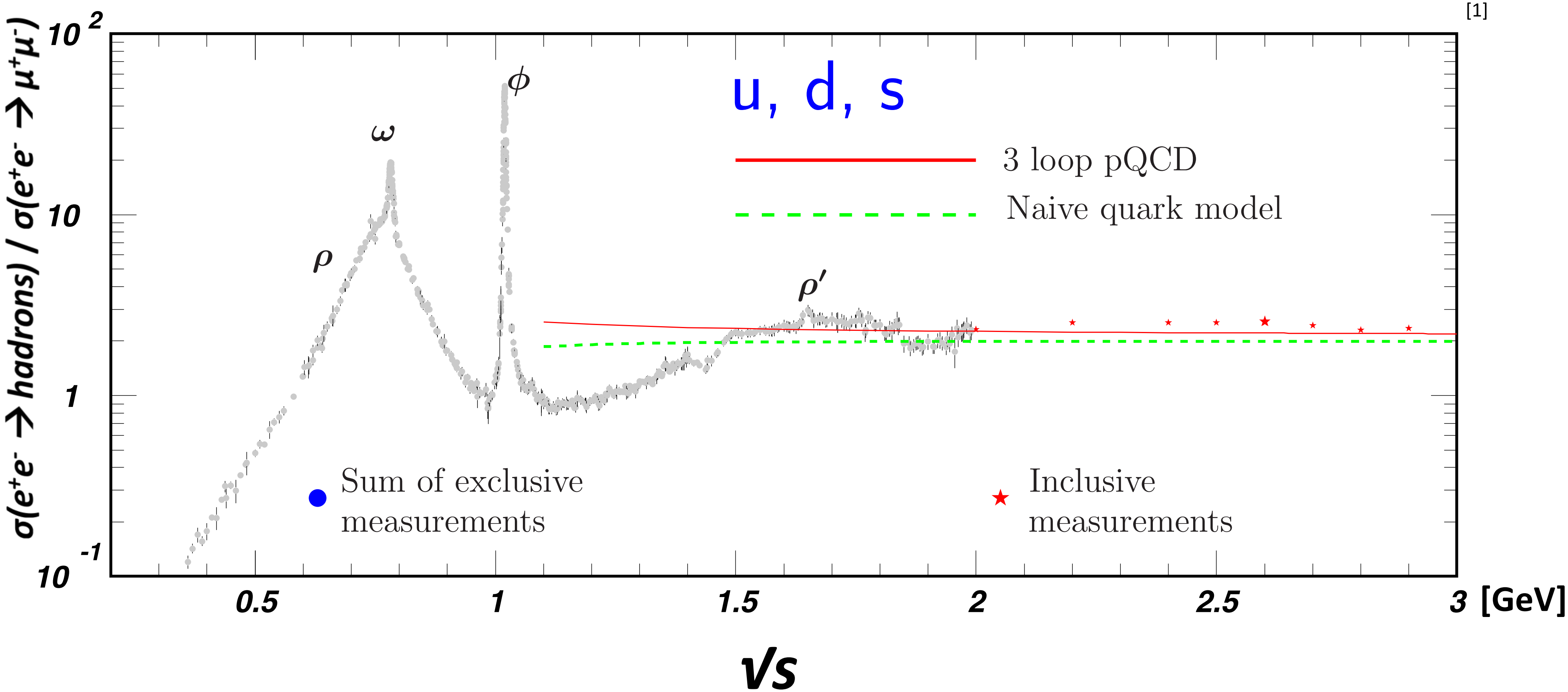
[1]



J. Beringer et al. (Particle Data Group), Phys. Rev. D86, 010001 (2012).

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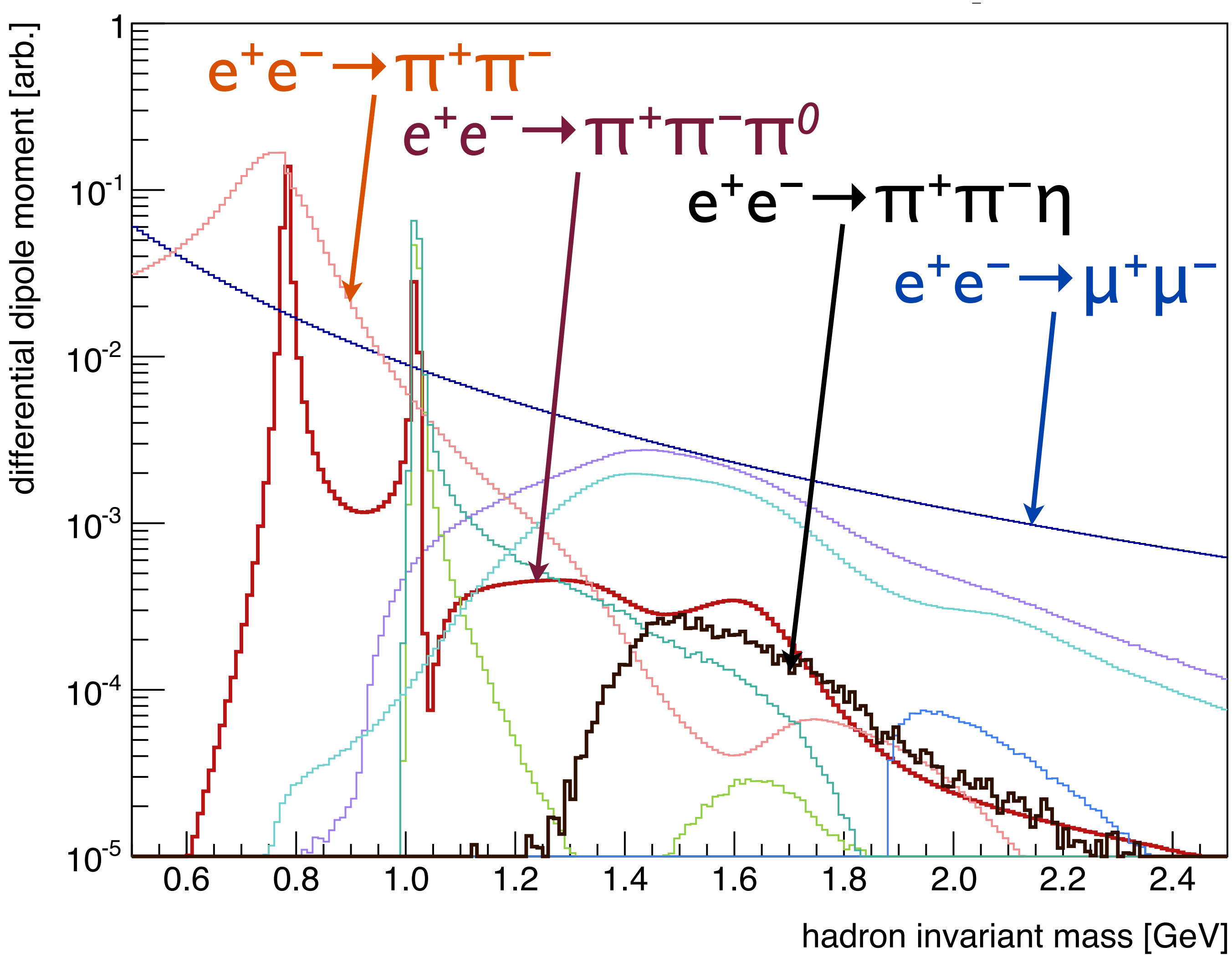

Produced with PHOKHARA

Physics Motivation: SM LO HVP contribution to the a_μ is calculated from low-energy e^+e^- production cross sections

1. Measure exclusive *σ(e⁺e⁻ → hadrons)*
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3. Multiply by known integral kernel function
4. **Integrate over final-state invariant mass**

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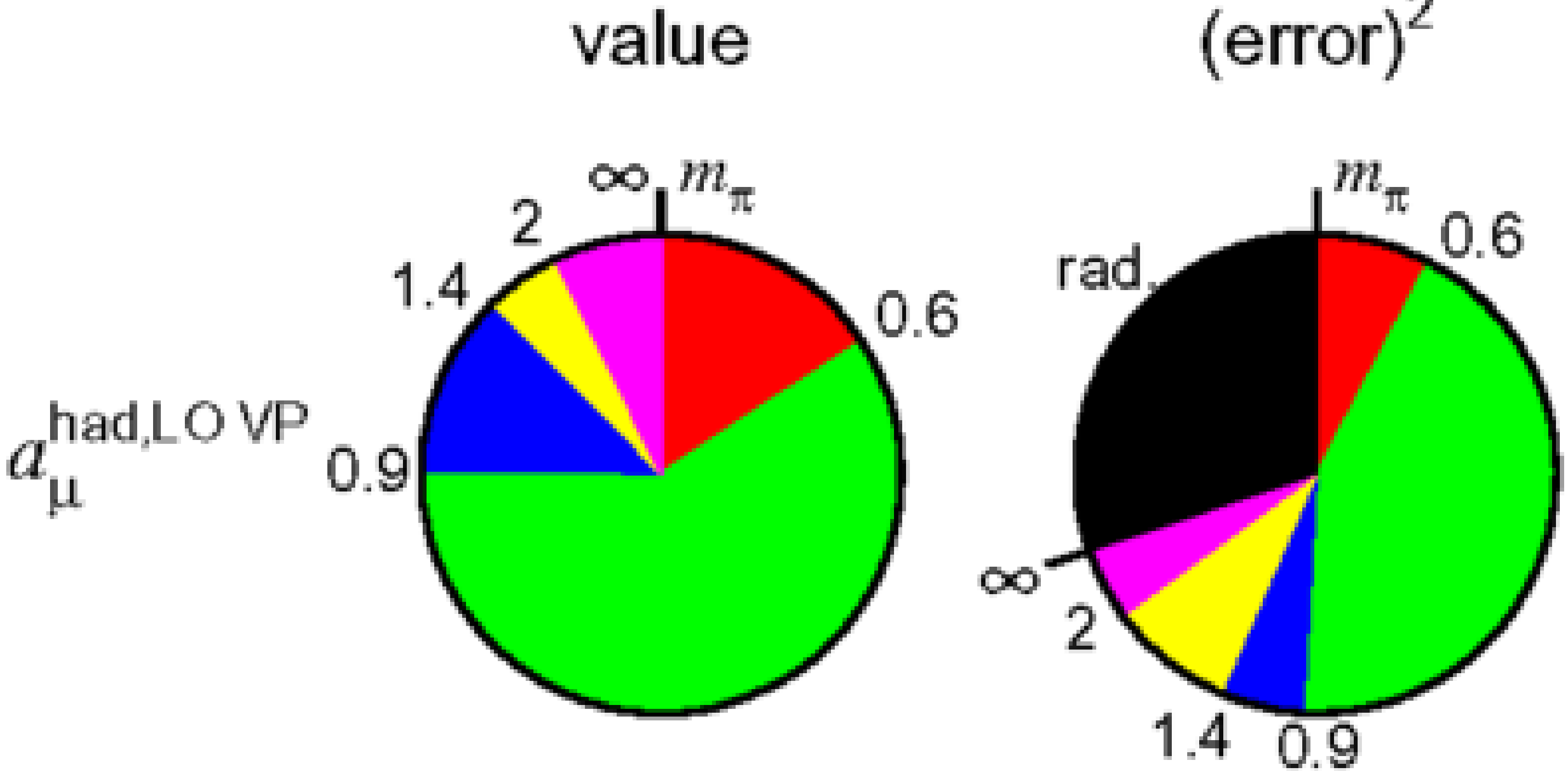


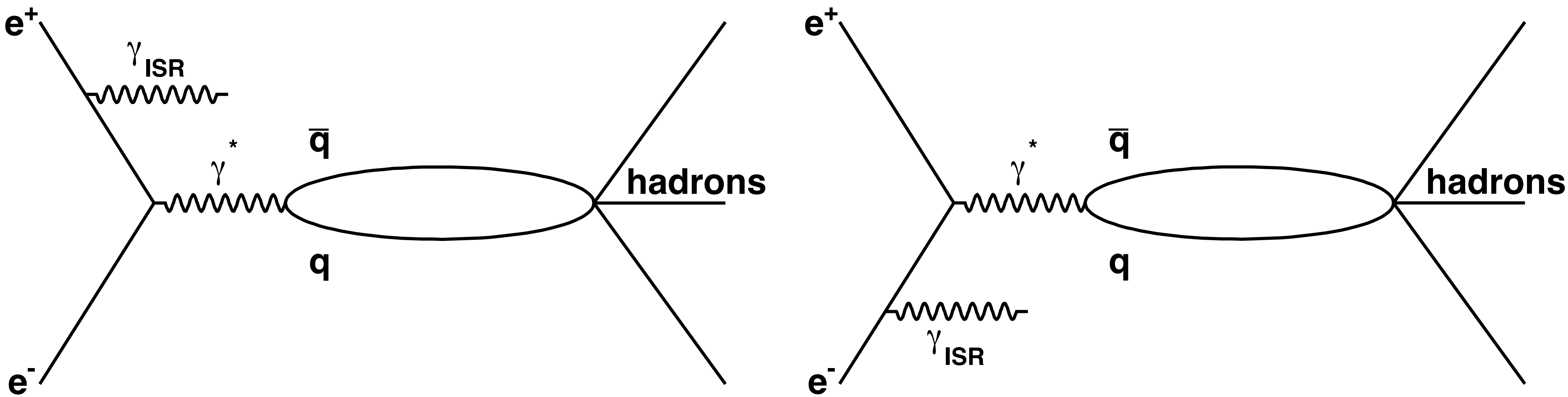
[1] T. Teubner, K. Hagiwara, R. Liao, A. D. Martin and D. Nomura, Nucl. Phys. Proc. Suppl. **225-227**, 282 (2012).

RMC WG Talk Motivation: Get feedback on our NLO procedure for calculating effective luminosity

- ✔ a) Signal Yield: Apply signal selection & background suppression cuts; Carry out kinematic fitting; Subtract backgrounds; Apply mass unfolding
- ✔ b) Detector Efficiency: Start with PHOKHARA in NLO mode; Apply θ_{ISR} cut (Belle detector fiducial volume); PHOKHARA output is used as Belle GEANT MC input; Process Belle GEANT MC outputs the same as data.
- ✔ c) Correct For FSR: PHOKHARA output is used as EvtGen (decay hadrons) input; EvtGen output is used as PHOTOS (produce FSR) input; PHOTOS output and PHOKHARA output (not run through PHOTOS) are combined to get FSR correction
- ✔ d) Remove Vacuum Polarization: Vacuum polarization calculate by other; Apply vacuum polarization bin-by-bin to the cross section
- ? e) Apply Radiator Function: Compensate for $\theta(\gamma_{ISR})$ cut; Account for likelihood of emitting γ_{ISR}'s (likelihood of event ending up in a particular final-state mass bin)

Radiator function at LO is not a problem.
Radiator function at NLO?

Radiator function accounts for ISR effects

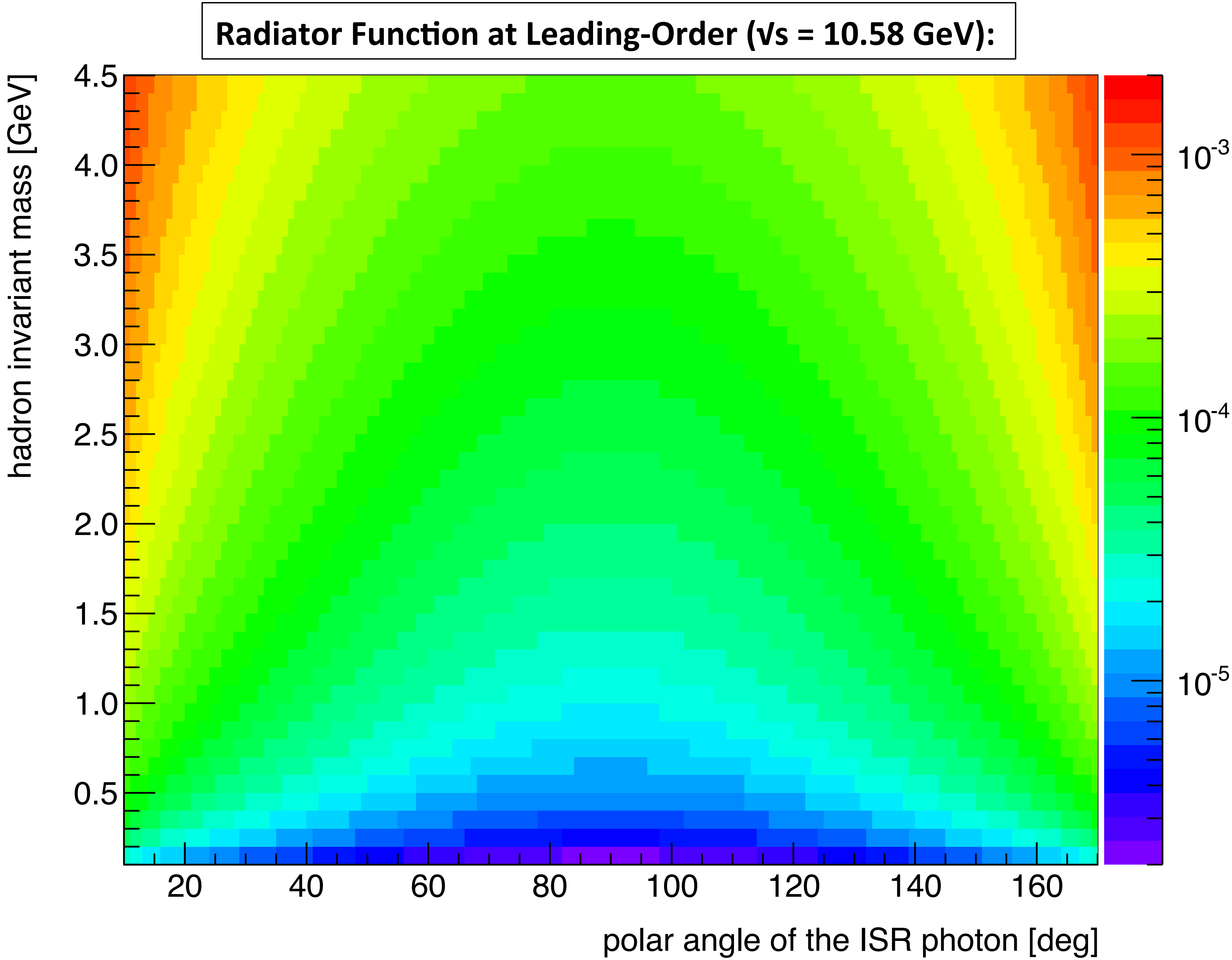


Radiator Function at Leading-Order:

$$\frac{\pi x}{\alpha}\frac{dW(x,s,\theta)}{\sin\theta d\theta} = \frac{2\left(1-x+\frac{x^2}{2}\right)\sin^2\theta - \frac{x^2}{2}\sin^4\theta}{\left(\sin^2\theta + \frac{4m_e^2}{s}\cos^2\theta\right)^2} - \frac{4m_e^2}{s}\frac{(1-2x)\sin^2\theta - x^2\cos^4\theta}{\left(\sin^2\theta + \frac{4m_e^2}{s}\cos^2\theta\right)^2}$$

W(x,s,θ) **= radiator function**
θ = ISR photon angle
m_e **= electron mass**
√s = e^+e^- **c.m.s. energy**
x = 1 - (m^2/s)
m = final-state hadron system invariant mass

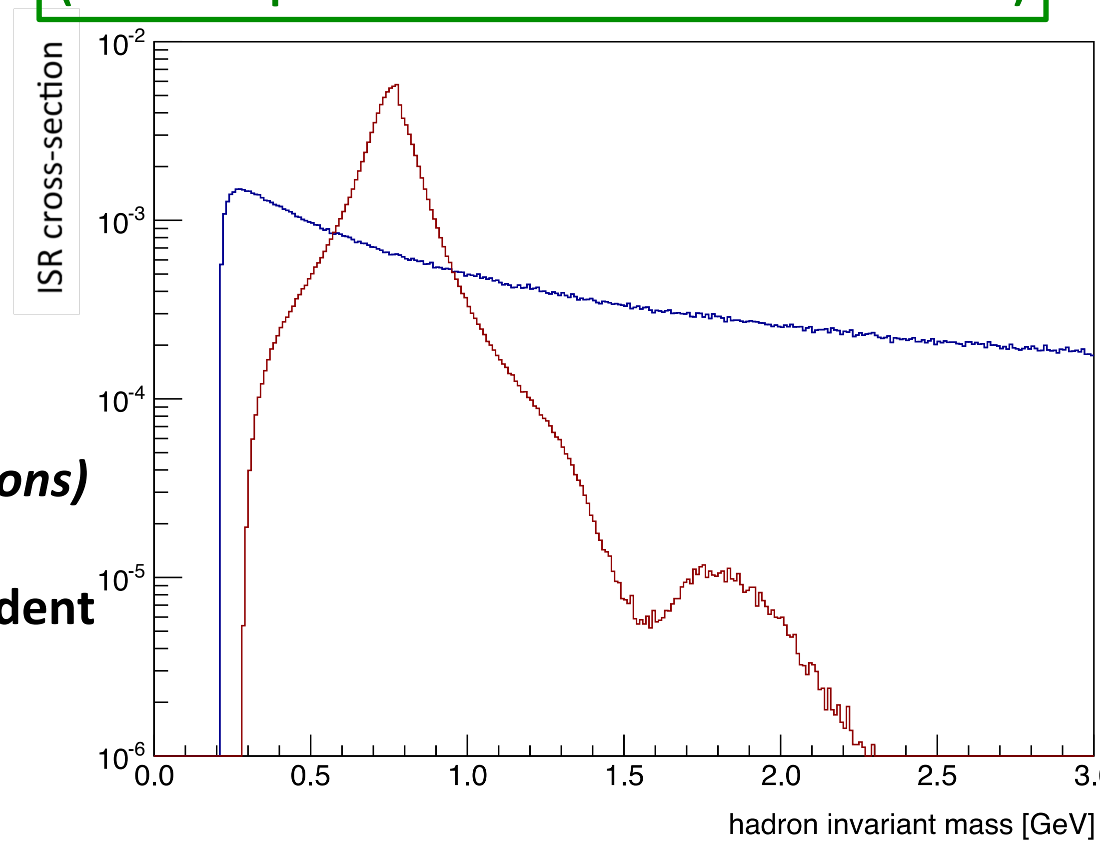
Radiator function accounts for ISR effects



We are interested in understanding and using the Next-to-Leading-Order radiator function

- ***NO*** **"simple" textbook formula**
- **Use realistic NLO event generator (PHOKARA)**
- **Write NLO radiator function as a correction factor with respect to the LO radiator function**

Process used for this study:
$e^+e^- \rightarrow \pi^+\pi^-\gamma_{ISR}(\gamma_{ISR})$
(most complete PHOKHARA hadronic final-state)



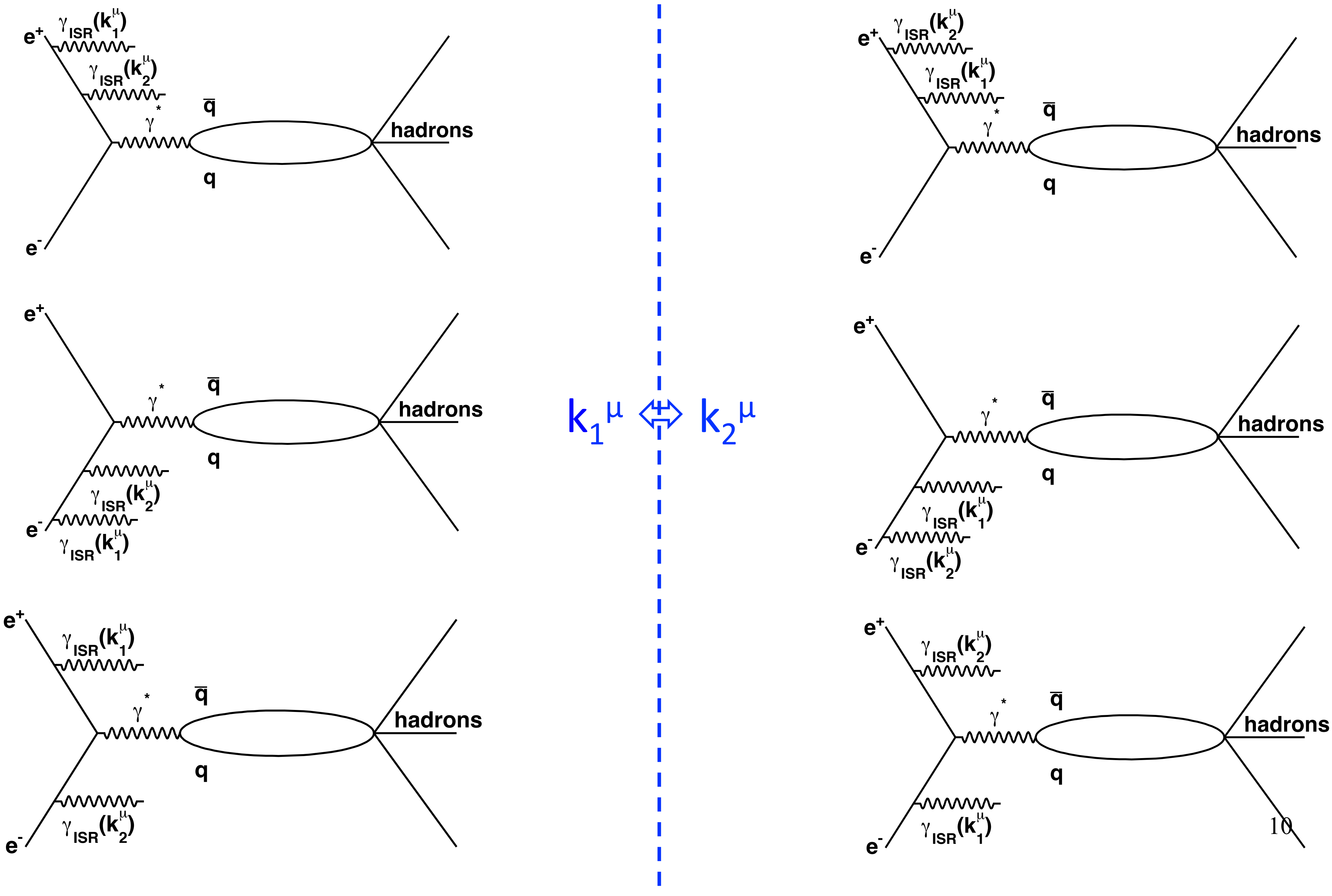
Using PHOKHARA 7.0:

For an exclusive final state:

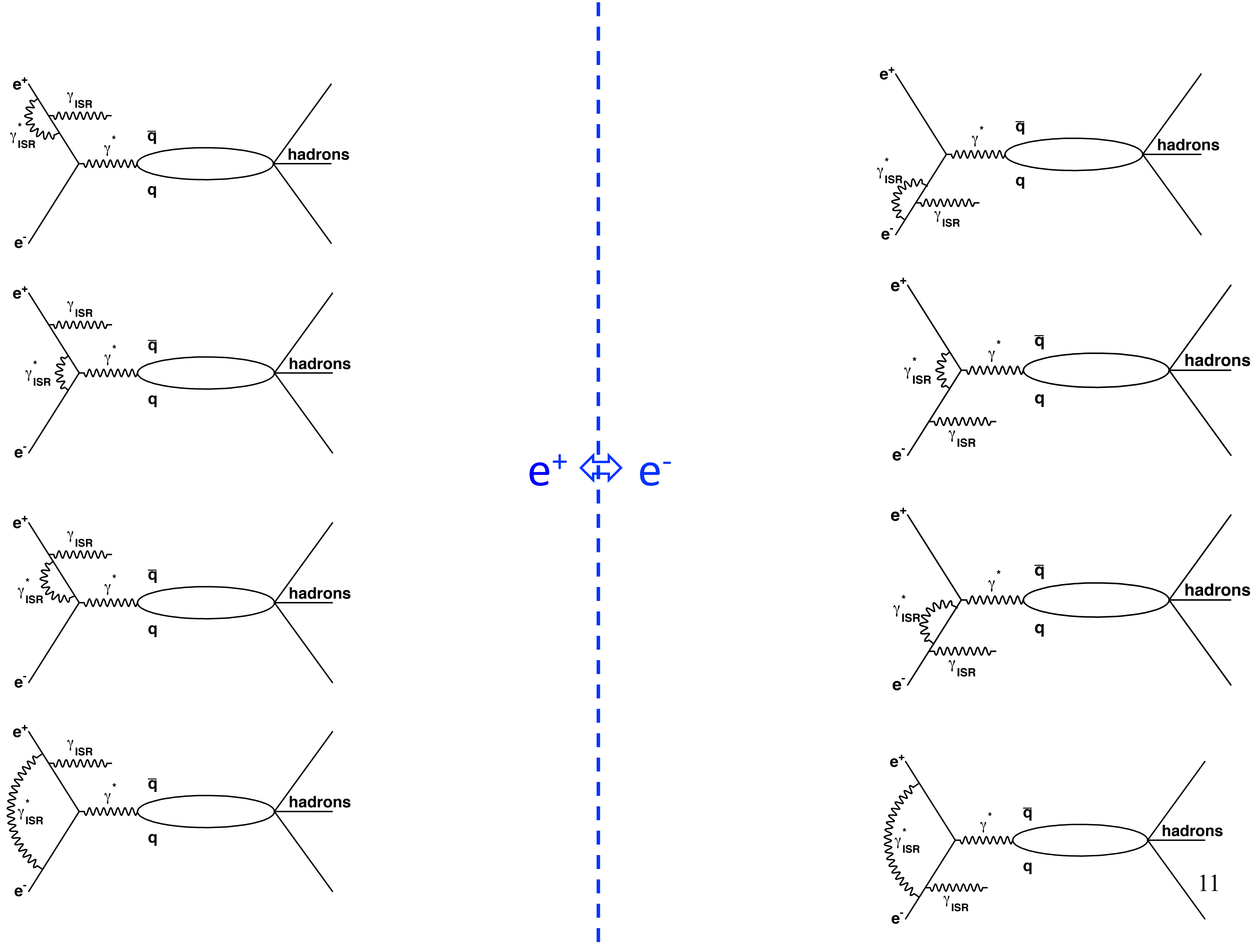
$\boldsymbol{\sigma_{ISR}^{NLO}(e^+e^- \rightarrow hadrons) / \sigma_{ISR}^{LO}(e^+e^- \rightarrow hadrons)}$

a) **Ratio is hadronic current independent**
b) **Ratio is absolute normalization independent**

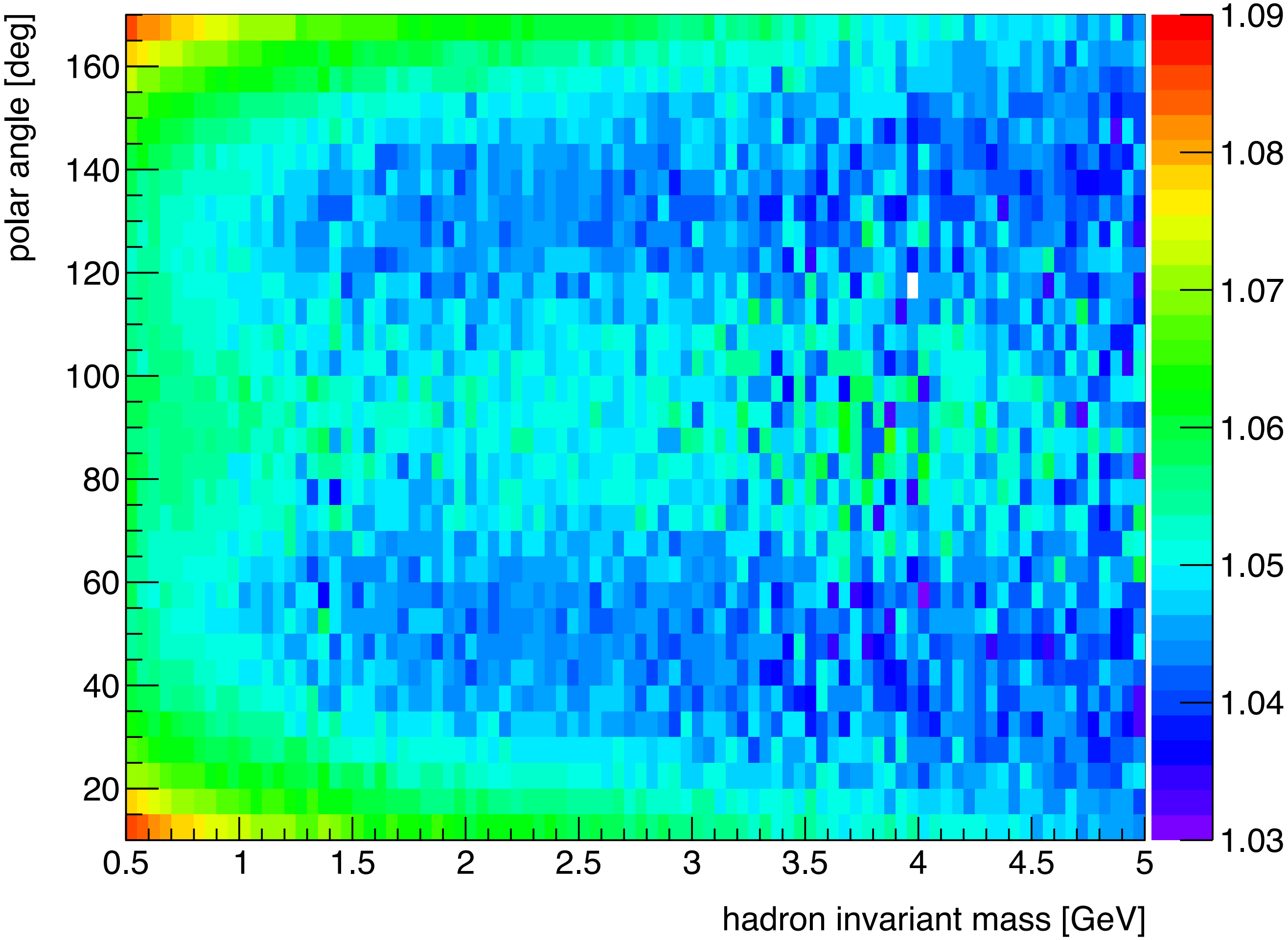
Next-to-Leading-Order radiator function includes the emission of 2 real γ_{ISR}



Next-to-Leading-Order radiator function includes 1-loop corrections



We first looked at the ratio of the LO & NLO differential radiator functions



$$\frac{dW^{NLO}(s,x,\theta)}{dW^{LO}(s,x,\theta)}$$

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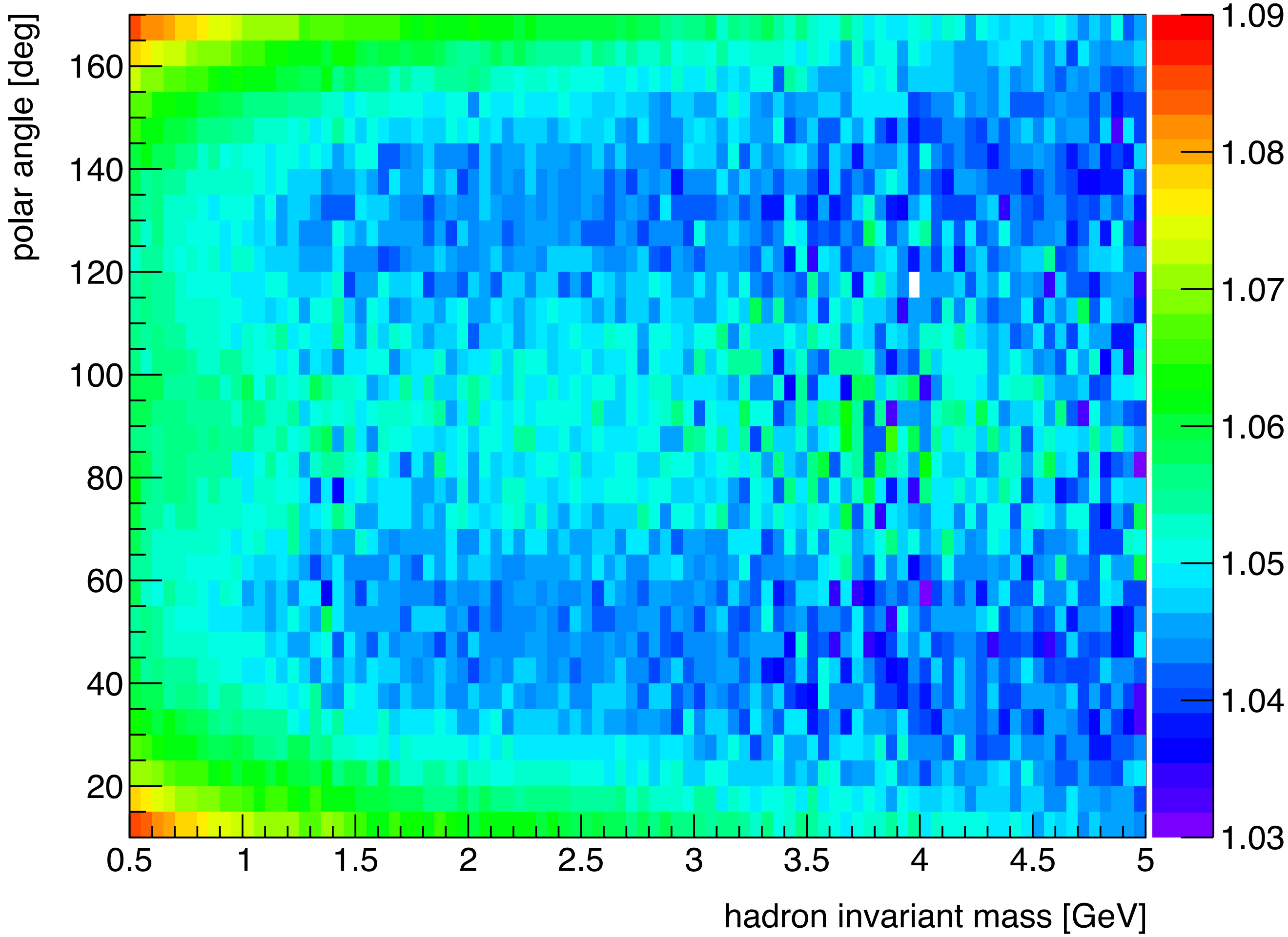
$$\frac{dW^{NLO}(s,x,\theta)}{dW^{LO}(s,x,\theta)} = \frac{\dfrac{d^2\sigma_{ISR}^{NLO}(s,m,\theta)}{dmd\theta}}{\dfrac{d^2\sigma_{ISR}^{LO}(s,m,\theta)}{dmd\theta}} = \frac{\dfrac{2m}{s}\varepsilon(s,m,\theta)\,dW^{NLO}(s,x,\theta)\,\sigma_{Born}(m,\theta)\sin\theta}{\dfrac{2m}{s}\varepsilon(s,m,\theta)\,dW^{LO}(s,x,\theta)\,\sigma_{Born}(m,\theta)\sin\theta}$$

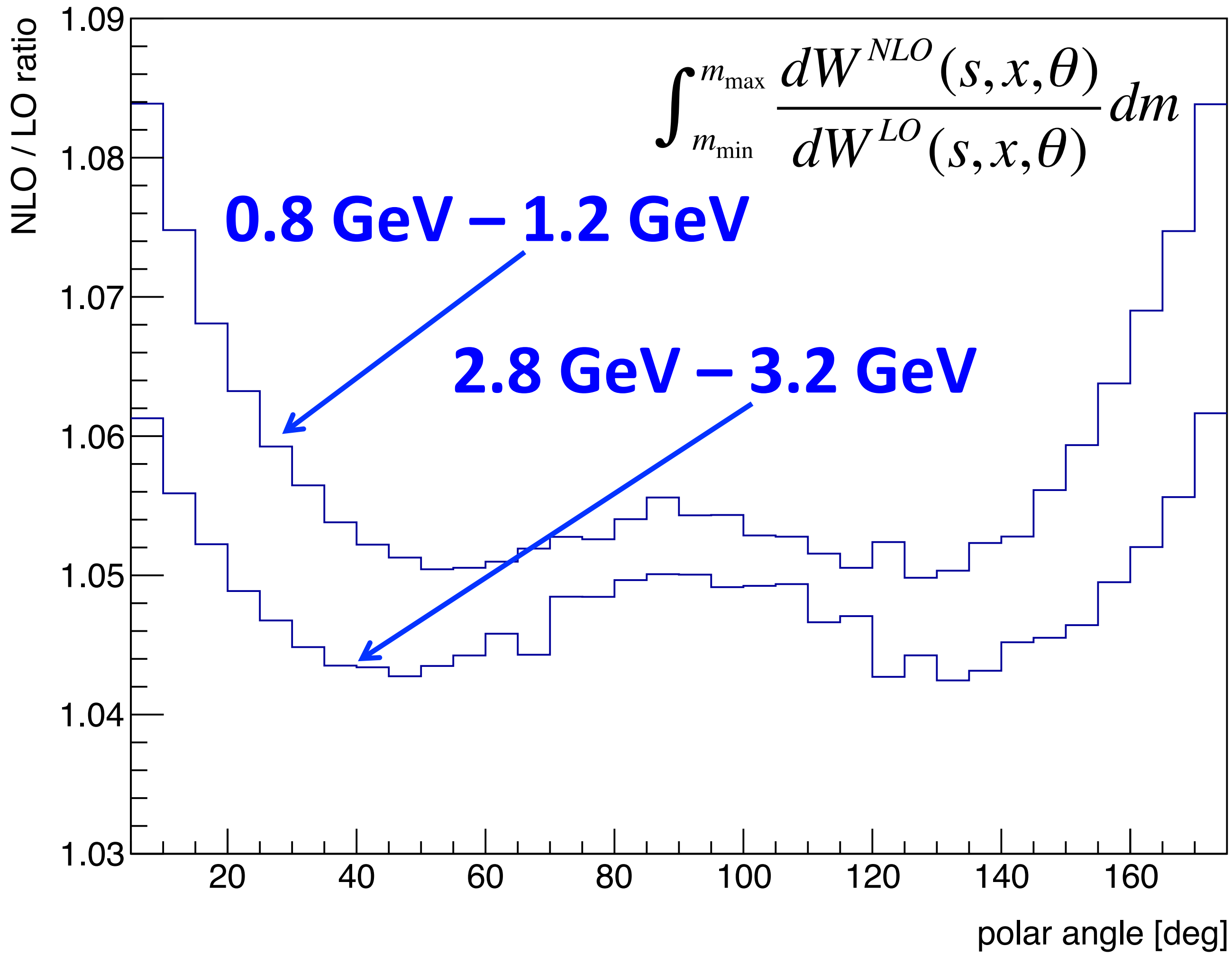


$$\frac{dW^{NLO}(s,x,\theta)}{dW^{LO}(s,x,\theta)}$$

a) Ratio is hadronic current independent

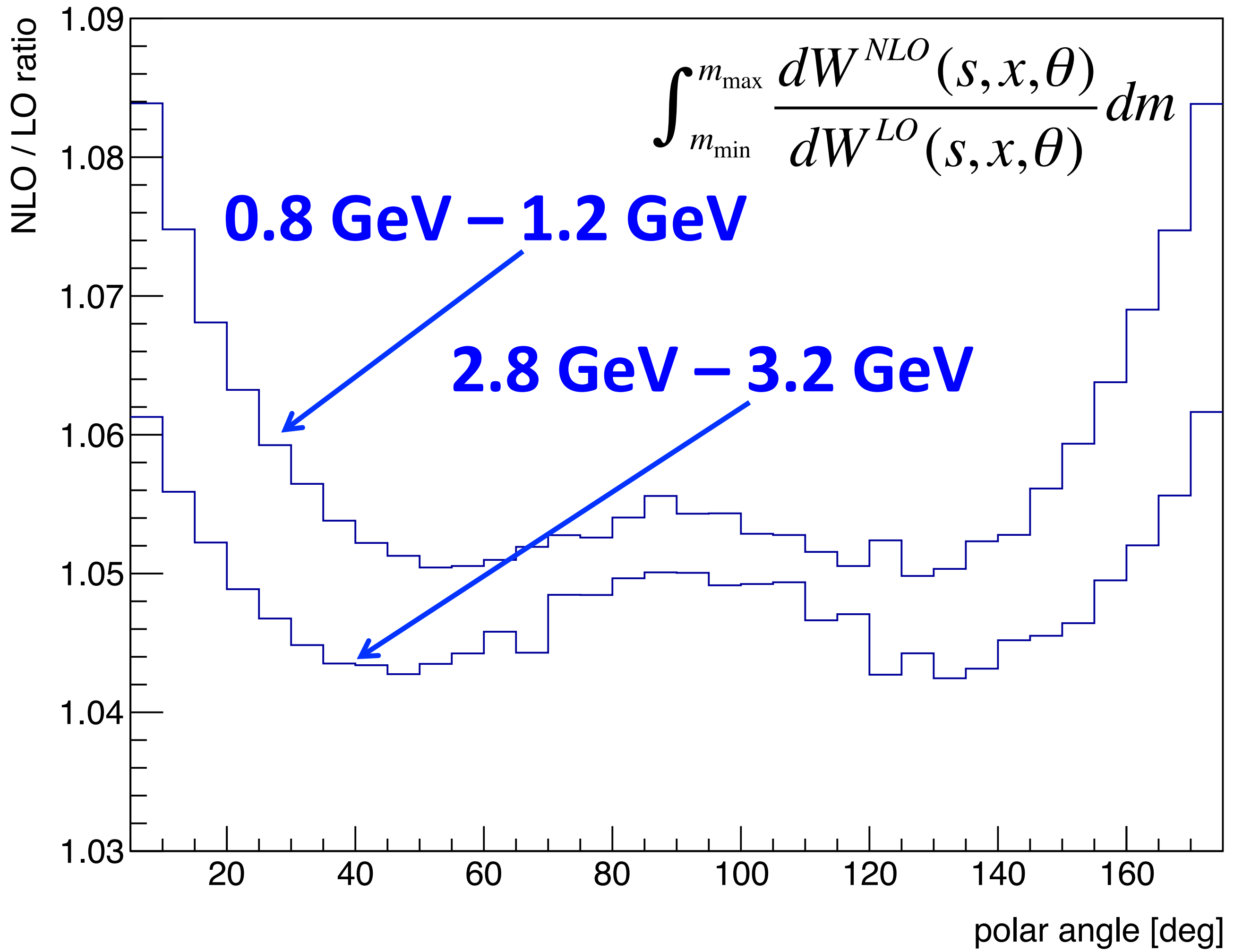
b) Ratio is absolute normalization independent

We also looked at the ratio of the LO & NLO radiator functions over the ISR polar angle

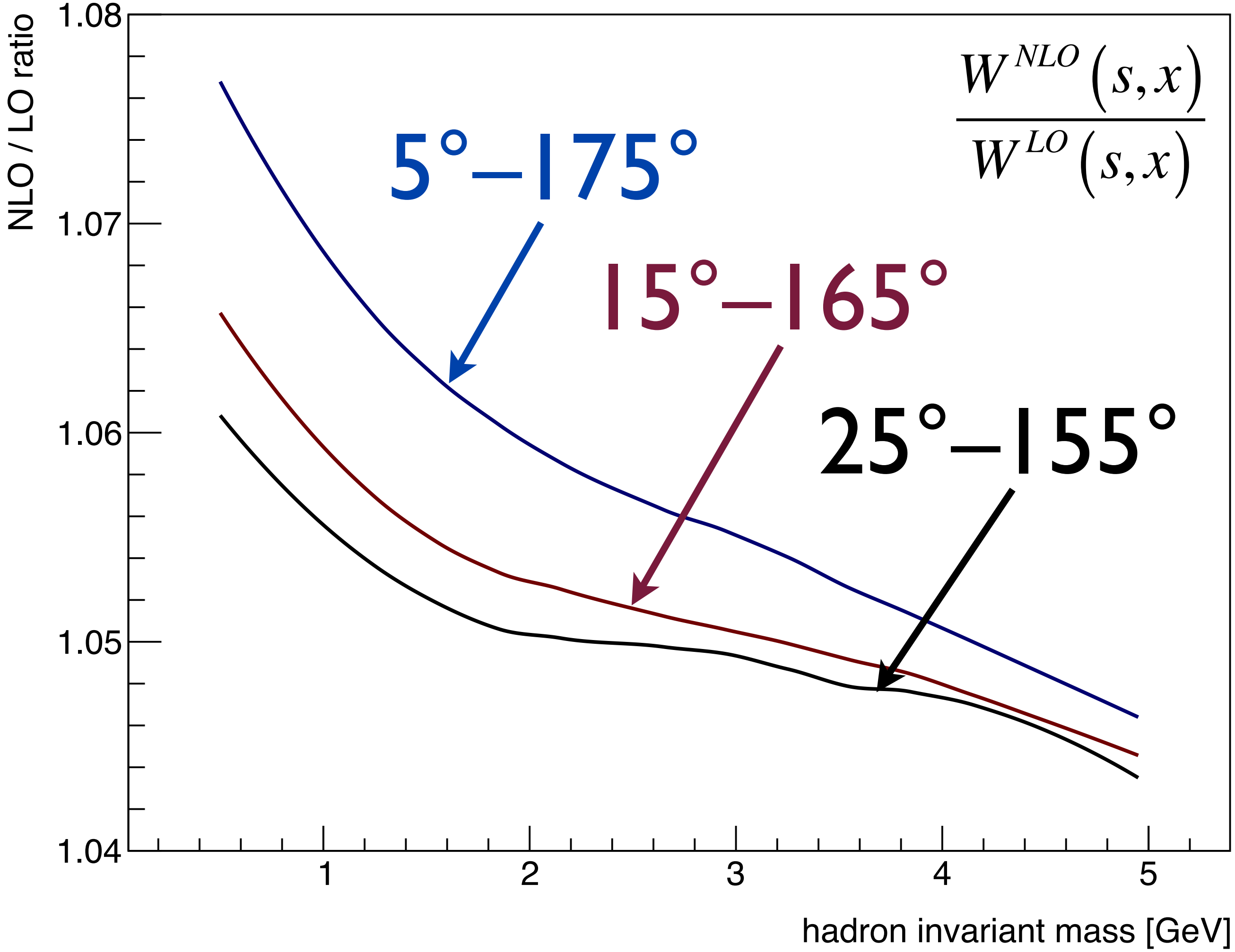


We also looked at the ratio of the LO & NLO radiator functions over the ISR polar angle

$$\int_{m_{\min}}^{m_{\max}} \frac{dW^{NLO}(s,x,\theta)}{dW^{LO}(s,x,\theta)}\,dm = \int_{m_{\min}}^{m_{\max}} \frac{\dfrac{d^2\sigma_{ISR}^{NLO}(s,m,\theta)}{dm\,d\theta}}{\dfrac{d^2\sigma_{ISR}^{LO}(s,m,\theta)}{dm\,d\theta}}\,dm$$

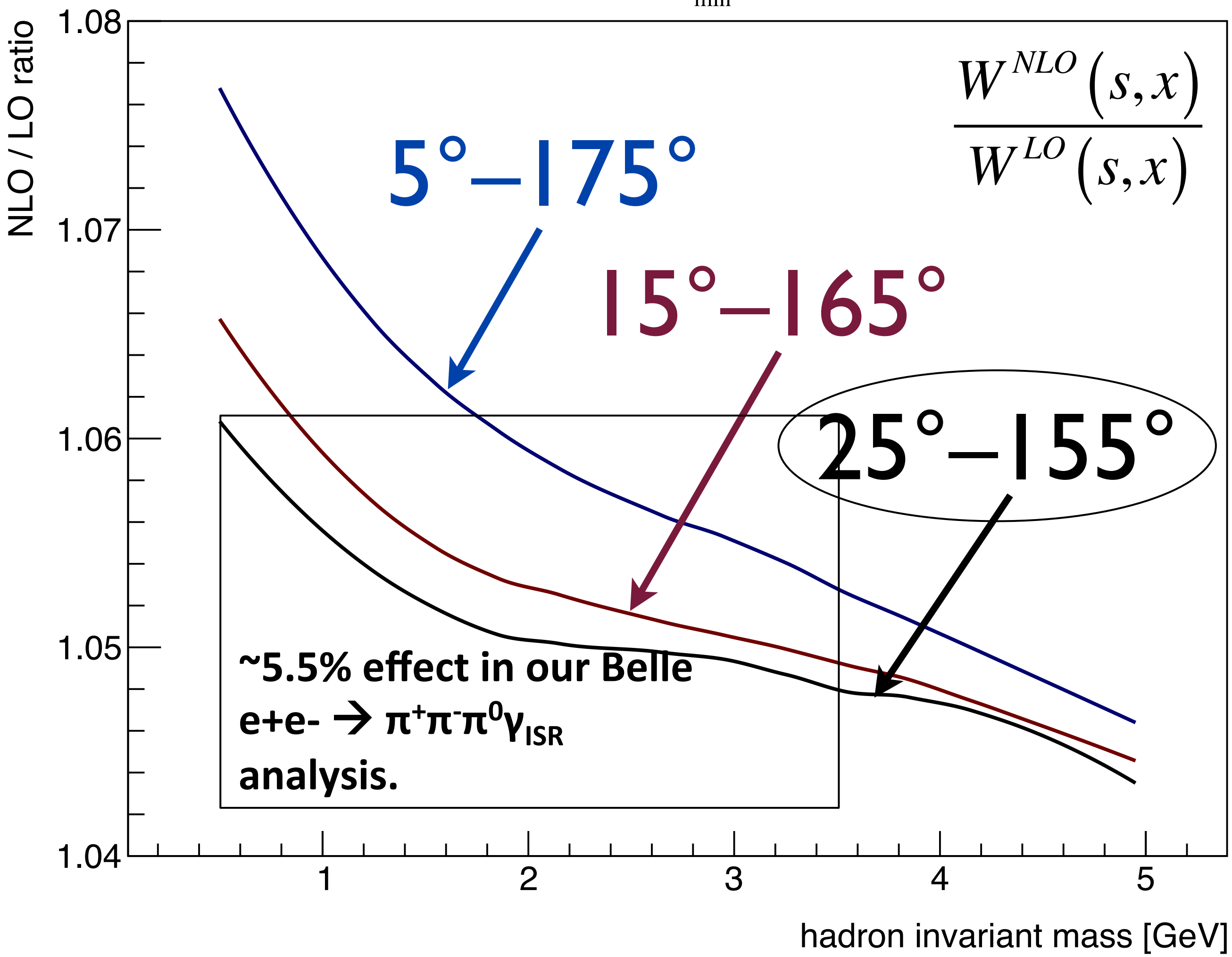


We finally look at the ratio of the LO & NLO radiator functions over the hadronic final-state invariant mass



We finally look at the ratio of the LO & NLO radiator functions over the hadronic final-state invariant mass

$$\frac{W^{NLO}(s,x)}{W^{LO}(s,x)} = \frac{\int_{\theta_{\min}}^{\theta_{\max}} dW^{NLO}(s,x,\theta)}{\int_{\theta_{\min}}^{\theta_{\max}} dW^{LO}(s,x,\theta)} = \frac{\int_{\theta_{\min}}^{\theta_{\max}} \frac{\frac{d^2\sigma_{ISR}^{NLO}(s,m,\theta)}{dmd\theta}}{\frac{d^2\sigma_{ISR}^{LO}(s,m,\theta)}{dmd\theta}} dW^{LO}(s,x,\theta)}{\int_{\theta_{\min}}^{\theta_{\max}} dW^{LO}(s,x,\theta)}$$



Cut used in our Belle e+e- → $\pi^+\pi^-\pi^0\gamma_{ISR}$ analysis.

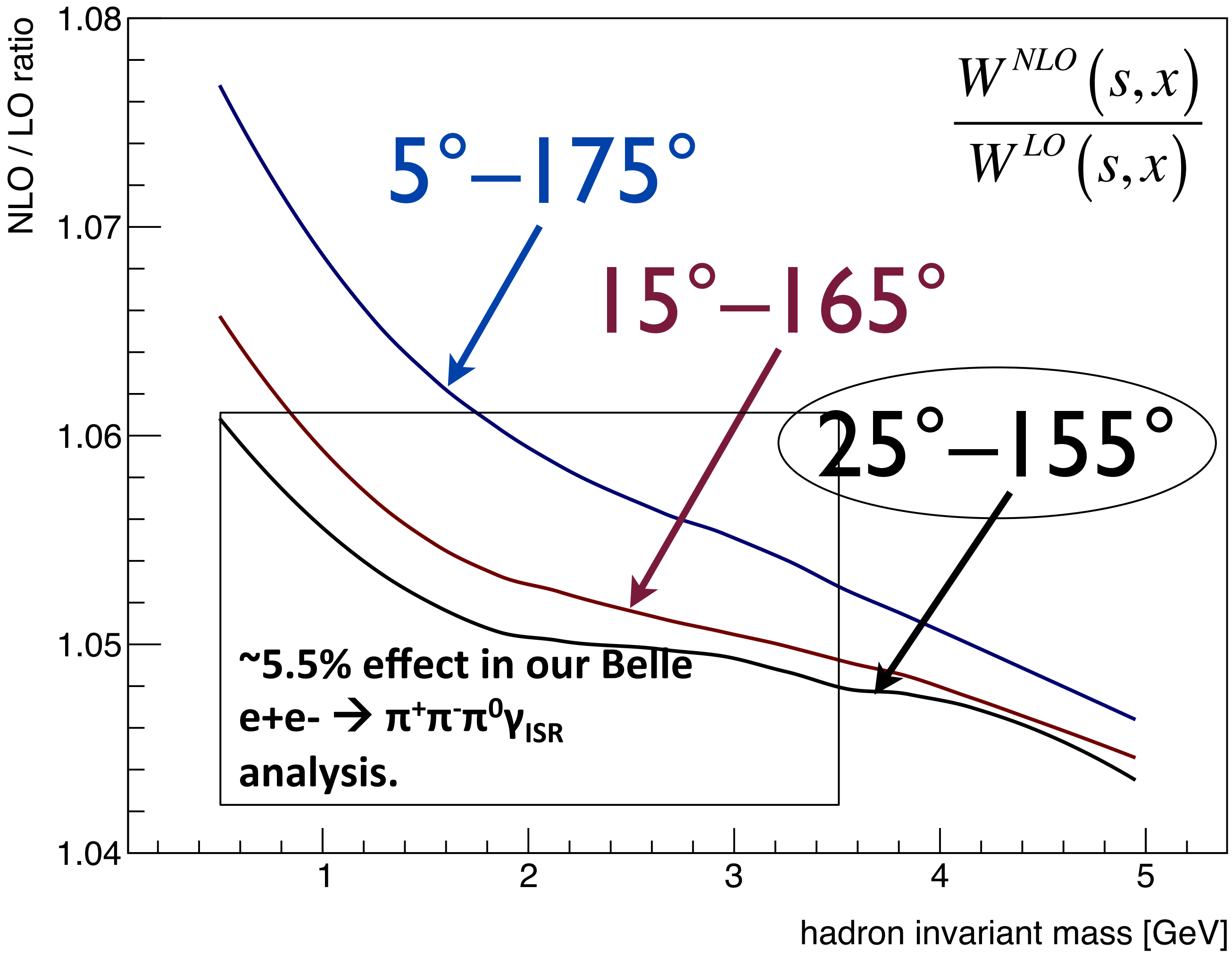
Questions we have concerning the assignment of a 0.5% error for NLO PHOKHARA:

1) **What about the γ_{ISR} undergoing pair-production?** Omitting the effect lowers the cross section: we are including e+e- pair production from the fusion of 2 γ_{ISR}.
2) **What about the beam cross angle?** A non-zero beam crossing angle adds terms to the leptonic matrix.
3) **What about assuming the hadronic tensor being proportional to $Q^2 g^{\mu\upsilon}$-$Q^{\mu}Q^{\upsilon}$ for 2-particle final states?** For instance, this assumption does not apply to $e^+e^- \rightarrow \gamma_{ISR}\gamma^* \rightarrow \gamma_{ISR}\gamma\pi^{0*} \rightarrow \gamma_{ISR}\gamma\pi^0\pi^+\pi^-$
4) **Is 0.5% error conservative for a *B*-factory?** We use loose cuts on final-state invariant mass, and m_e^2/s is smaller at a *B*-factory than a φ-factory.

Conclusion

We have a PHOKHARA based procedure for calculating the NLO radiator function.

- **Is this procedure correct?**
- **Is it reasonable to expect a NLO ~5.5% effect?**
- **Should we being using 0.5% error for using PHOKHARA?**



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Bonus Questions:

1. **What is the state-of-art fitting technique?**
    1. Several kinds exist: VMD, HLS, and broken HLS.
    2. All use BW for resonances. Several BW formulas exist: widths are s-weighted in various ways. Recipes are not clear on what BW form they use.
2. **Need to bootstrap the fitter with something (all fitters). 3pi channel doesn't give us all needed fit parameters.**
    1. What external data should we use?
    2. If we add more channels later, we'd become independent of any external data at some point. How exactly should we start?
3. **All fitting recipes are very clear up to phi, but become a little bit blurry around 1.5 GeV.**
    1. Is there an exact recipe for this region within the bHLS framework?
4. **Need more details in how to apply VP. We have very narrow resonances: detector resolution matters a lot. Is data unfolding sufficient?**
    1. Should we fold the BW resonances with an additional Gaussian to account for detector resolution BEFORE we apply VPL?
    2. What VP data should we use: Jegerlehner, Novosibirsk, some other one.
5. **What about a NLO kernel function?**
    1. Couple of different kernel functions exist that can be combined with different types of cross-sections (visible, dressed, bare).
    2. We use the the bare cross section method. Is there a more straightforward way? What kernel function is best? Why?

Bonus Questions:

6. How to handle FSR?

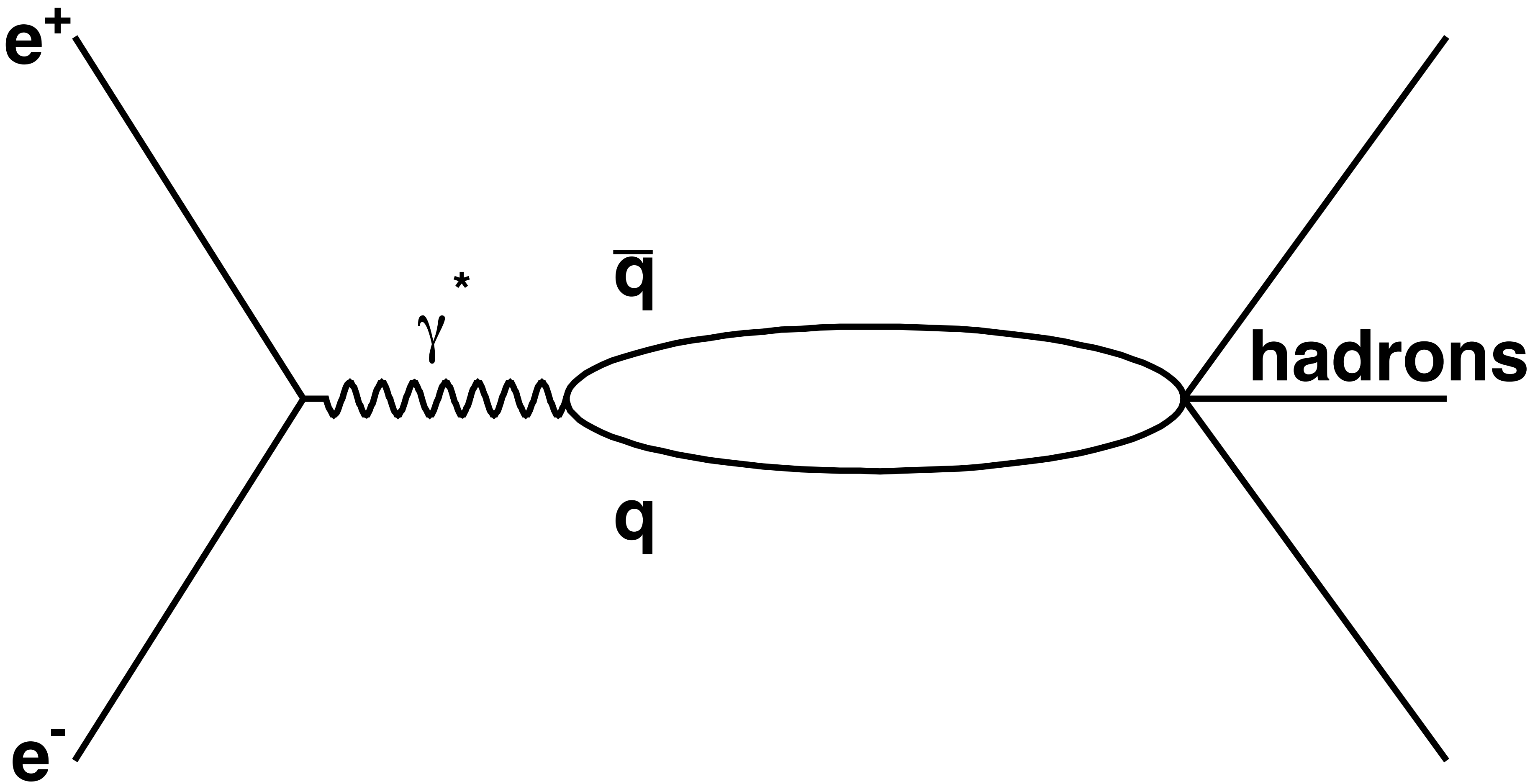
1. PHOTOS only does kinematics. Can we break virtual vertex corrections (we need to undo) from the total FSR cross-section? How do we break those two (real photon vs. virtual one) apart?
2. Is an upper limit on these FSR cross-sections known? Something for our systematics table. These are small, but all corrections go in the same direction: lowering our dressed cross-section.

7. All published $a_\mu^{LO\ HVP}$ from e^+e^- data avoid J/ψ: J/ψ added separately from theory.

1. Why?
2. If we don't need the J/ψ cross section for $a_\mu^{LO\ HVP}$, what could we use it for.

Backup Slides

The Born cross section *excludes* initial-state radiation, final-state radiation, and vacuum polarization



Final-state radiation processes are an important background for initial-state radiation processes