Hadronic light-by-light scattering in the muon $g-2$: current status, open problems and impact of form factor measurements

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Outline

- Muon $g-2$: current status
- Hadronic light-by-light scattering in the muon $g-2$
- Had. LbyL scattering: anno 2010
- Recent developments: Had. LbyL scattering anno 2013
- Off-shell versus on-shell pion form factors
- Relevant momentum regions in pion-exchange
- Impact of form factor measurements (example KLOE-2)
- Quark-loop, pion-loop: the recent developments
- Conclusions and Outlook

Muon $g-2$: current status

- Experimental value (world average dominated by BNL experiment '06; shifted $+9.2 \times 10^{-11}$ due to new $\lambda = \mu_\mu/\mu_p$ from CODATA '08):

  $a_\mu^{\text{exp}} = (116\ 592\ 089 \pm 63) \times 10^{-11}$

- Theory: total SM contribution (based on various recent papers):

  $$a_\mu^{\text{SM}} = (116\ 591\ 795 \pm \underbrace{47}_{\text{VP}} \pm \underbrace{40}_{\text{LbyL}} \pm \underbrace{1.8}_{\text{QED + EW}} \quad [\pm 62]) \times 10^{-11}$$

  Hadronic contributions are largest source of error: vacuum polarization (VP) and light-by-light (LbyL) scattering.

  $a_\mu^{\text{had. LbyL}} = (116 \pm 40) \times 10^{-11}$ (Nyffeler '09; Jegerlehner, Nyffeler '09)
  Sometimes used: $a_\mu^{\text{had. LbyL}} = (105 \pm 26) \times 10^{-11}$ (Prades, de Rafael, Vainshtein '09)

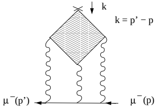
- $\Rightarrow a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (294 \pm 88) \times 10^{-11}$ $\quad [3.3\ \sigma]$

- Other evaluations: $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} \sim (250 - 400) \times 10^{-11}$ $\quad [2.9 - 4.9\ \sigma]$
  (Jegerlehner, Nyffeler '09; Davier et al. '10; Jegerlehner, Szafron '11; Hagiwara et al. '11; Aoyama et al. '12; Benayoun et al. '13)

- Discrepancy a sign of New Physics ?

- Note: Hadronic contributions need to be better controlled in order to fully profit from future muon $g-2$ experiments at Fermilab or JPARC with $\delta a_\mu = 16 \times 10^{-11}$

Hadronic light-by-light scattering in the muon $g-2$

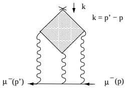


$\mathcal{O}(\alpha^3)$ hadronic contribution to muon $g-2$: four-point function $\langle VVVV \rangle$ projected onto a_μ (soft external photon $k \to 0$).

Had. LbyL: not directly related to experimental data, in contrast to had. VP which can be obtained from $\sigma(e^+e^- \to \text{hadrons})$ $\Rightarrow$ need hadronic model (or lattice QCD)

Current approach: use some hadronic model at low energies with exchanges and loops of resonances and some form of (dressed) "quark-loop" at high energies.
Problem: $\langle VVVV \rangle$ depends on several invariant momenta $\Rightarrow$ distinction between low and high energies is not as easy as for two-point function $\langle VV \rangle$ (had. VP).

Classification of de Rafael '94: Chiral counting p^2 (ChPT) and large-N_C counting as guideline (all higher orders in p^2 and N_C contribute):



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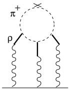

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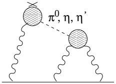
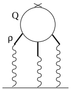


+ ··· + Exchanges of other resonances ($f_0, a_1, \ldots$) +



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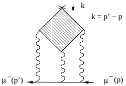
Chiral counting:	p^4	p^6	p^8	p^8
N_C-counting:	1	N_C	N_C	N_C
	pion-loop (dressed)	pseudoscalar exchanges		quark-loop (dressed)

Constrain models using experimental data (form factors of hadrons with photons) and theory (ChPT at low energies; short-distance constraints from pQCD / OPE at high momenta).

Open problem: on-shell versus off-shell form factors, see pages 8 – 10.

Relevant scales in had. LbyL ($\langle VVVV \rangle$ with off-shell photons): $0-2$ GeV, i.e. larger than m_μ !
See page 11.

Had. LbyL scattering: anno 2010



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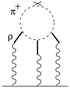

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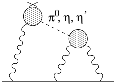

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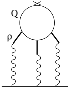

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Contribution to $a_\mu \times 10^{11}$:

BPP: +83 (32)	-19 (13)	+85 (13)	-4 (3) [f_0, a_1]	+21 (3)
HKS: +90 (15)	-5 (8)	+83 (6)	+1.7 (1.7) [a_1]	+10 (11)
KN: +80 (40)		+83 (12)		
MV: +136 (25)	0 (10)	+114 (10)	+22 (5) [a_1]	0
2007: +110 (40)				
PdRV: +105 (26)	-19 (19)	+114 (13)	+8 (12) [f_0, a_1]	+2.3 [c-quark]
N,JN: +116 (40)	-19 (13)	+99 (16)	+15 (7) [f_0, a_1]	+21 (3)
	ud.: -45	ud.: $+\infty$		ud.: +60

ud. = undressed, i.e. point vertices without form factors

BPP = Bijnens, Pallante, Prades '96, '02: Extended Nambu-Jona-Lasinio (ENJL) model; but for some contributions also other models used (in particular for pseudoscalars, pion-loop)

HKS = Hayakawa, Kinoshita, (Sanda) '96, ('98), '02: Hidden Local Symmetry (HLS) model (often = VMD)

KN = Knecht, Nyffeler '02: large-N_C QCD for pion-pole (lowest meson dominance LMD, LMD+V)

MV = Melnikov, Vainshtein '04: large-N_C QCD, short-distance constraint from $\langle VVVV \rangle$ on pion-pole and axial-vector contribution, mixing of two axial-vector nonets

2007 = Bijnens, Prades; Miller, de Rafael, Roberts; PdRV = Prades, de Rafael, Vainshtein '09 (compilation)

N = Nyffeler '09: large-N_C for pion-exchange with off-shell LMD+V form factor, new short-distance constraint at external vertex; JN = Jegerlehner, Nyffeler '09 (compilation)

- 2001: sign change in dominant pseudoscalar contribution: $a_\mu^{\text{had. LbyL}} \sim 85 \times 10^{-11}$ with discussion about estimate of error (adding errors of individual contributions linearly or in quadrature).
- 2004: MV $\Rightarrow$ enhanced pion-pole and axial-vector contributions. Estimate shifted upwards.
- 2010: (almost) consensus reached on central value $a_\mu^{\text{had. LbyL}} \sim 110 \times 10^{-11}$, still discussion about error estimate. Conservative in N, JN: $\pm 40 \times 10^{-11}$, more progressive in PdRV: $\pm 26 \times 10^{-11}$.

Recent developments

- Other recent partial evaluations (mostly pseudoscalars):

$$a_\mu^{\mathrm{LbyL};\pi^0} \sim (50-69) \times 10^{-11}$$
$$a_\mu^{\mathrm{LbyL;PS}} \sim (59-107) \times 10^{-11}$$

  Most evaluations agree at level of 15%, but some estimates are quite low or high (details: see page 21)

- Open problem: Dressed quark-loop (details: see pages 14 + 15)
  Dyson-Schwinger equation (DSE) approach (Fischer, Goecke, Williams '11, '13):

$$a_\mu^{\mathrm{LbyL;quark-loop}} = 107 \times 10^{-11}$$

  Large contribution, no damping seen, in contrast to BPP, HKS.

- Open problem: Dressed pion-loop (details: see pages 16 + 17)
  Potentially important effect from pion polarizability and a_1 resonance (Engel, Patel, Ramsey-Musolf '12; Engel '13; Engel, Ramsey-Musolf '13):

$$a_\mu^{\mathrm{LbyL};\pi-\mathrm{loop}} = -(11-71) \times 10^{-11}$$

  Large negative contribution, no damping seen, in contrast to BPP, HKS.

- Combining the extreme estimates:

$$a_\mu^{\mathrm{had.\ LbyL}} = (64-202) \times 10^{-11}$$

or:

$$a_\mu^{\mathrm{had.\ LbyL}} = (133 \pm 69) \times 10^{-11}$$

Had. LbyL scattering: anno (late) 2013

- We do not understand had. LbyL scattering at all !?
  Unless we ignore those new estimates for the quark-loop and pion-loop.

- Option 1: Wait for final result from Lattice QCD . . .

  One idea: put QCD + QED on the lattice !
  Blum et al. '05, '08, '09; Chowdhury '09; Blum, Hayakawa, Izubuchi '12 + poster at Lattice 2013 (private communication):

$$F_2(0.18\ \mathrm{GeV}^2) = (127 \pm 29) \times 10^{-11} \qquad \text{(result } 4.4\sigma \text{ from zero)}$$

$$F_2(0.11\ \mathrm{GeV}^2) = (-15 \pm 39) \times 10^{-11} \qquad \text{(result consistent with zero)}$$

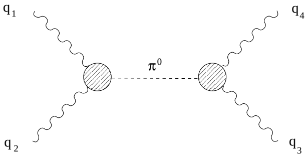
$$a_\mu^{\mathrm{had.\ LbyL;models}} = F_2(0) = (116 \pm 40) \times 10^{-11} \qquad \text{(Jegerlehner, Nyffeler '09)}$$

  For $m_\mu = 190$ MeV, $m_\pi = 329$ MeV. Still large statistical errors, systematic errors not yet under control, still quenched QED, potentially large "disconnected" contributions missing !

- Option 2: Maybe we (non-Lattice theorists and experimentalists) can still do some work in the coming years, as far as had. LbyL scattering in muon $g-2$ is concerned !

A reminder: pion-pole in $\langle VVVV\rangle$ versus pion-exchange in $a_\mu^{\mathrm{LbyL};\pi^0}$

- To uniquely identify contribution of exchanged neutral pion π^0 in Green's function $\langle VVVV\rangle$, we need to pick out pion-pole:

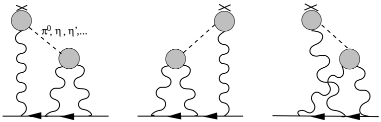


+ crossed diagrams

$$\lim_{(q_1+q_2)^2 \to m_\pi^2} ((q_1+q_2)^2 - m_\pi^2)\langle VVVV\rangle$$

Residue of pole: on-shell vertex function $\langle 0|VV|\pi\rangle \to$ on-shell form factor $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$

- But in contribution to muon $g-2$, we evaluate Feynman diagrams, integrating over photon momenta with exchanged off-shell pions.
For all the pseudoscalars:



Shaded blobs represent off-shell form factor $\mathcal{F}_{\mathrm{PS}^*\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2)$ where $\mathrm{PS} = \pi^0, \eta, \eta', \pi^{0\prime}, \ldots$

Off-shell form factors are either inserted "by hand" starting from constant, pointlike Wess-Zumino-Witten (WZW) form factor or using e.g. some resonance Lagrangian.

- Similar statements apply for exchanges (or loops) of other resonances.

Off-shell pion form factor from $\langle VVP \rangle$

- Following Bijnens, Pallante, Prades '96; Hayakawa, Kinoshita, (Sanda) '96, ('98), we can define off-shell form factor for π^0:

$$\int d^4x\, d^4y\, e^{i(q_1\cdot x+q_2\cdot y)} \langle 0|T\{j_\mu(x)j_\nu(y)P^3(0)\}|0\rangle$$

$$= \quad \varepsilon_{\mu\nu\alpha\beta}\, q_1^\alpha q_2^\beta \frac{i\langle\overline{\psi}\psi\rangle}{F_\pi} \frac{i}{(q_1+q_2)^2 - m_\pi^2} \mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2) + \ldots$$

Up to small mixing effects of P^3 with η and η' and neglecting exchanges of heavier states like $\pi^{0\prime}, \pi^{0\prime\prime}, \ldots$

$$j_\mu(x) = (\overline{\psi}\hat{Q}\gamma_\mu\psi)(x), \quad \psi \equiv \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \quad \hat{Q} = \mathrm{diag}(2,-1,-1)/3$$

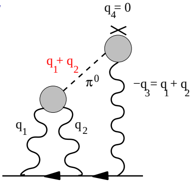
(light quark part of electromagnetic current)

$$P^3 = \overline{\psi} i\gamma_5 \tfrac{\lambda^3}{2}\psi = \left(\overline{u}i\gamma_5 u - \overline{d}i\gamma_5 d\right)/2, \quad \langle\overline{\psi}\psi\rangle = \text{single flavor quark condensate}$$

Bose symmetry: $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2) = \mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_2^2, q_1^2)$

- Note: for off-shell pions, instead of $P^3(x)$, we could use any other suitable interpolating field, like $(\partial^\mu A_\mu^3)(x)$ or even an elementary pion field $\pi^3(x)$! Off-shell form factor is therefore model dependent and not a physical quantity !

Pion-exchange versus pion-pole contribution to $a_\mu^{\mathrm{LbyL};\pi^0}$



- Off-shell form factors have been used to evaluate the pion-exchange contribution in Bijnens, Pallante, Prades '96 and Hayakawa, Kinoshita, (Sanda) '96, ('98). "Rediscovered" by Jegerlehner in '07, '08. Consider diagram:

$$\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2) \;\times\; \mathcal{F}_{\pi^{0*}\gamma^*\gamma}((q_1+q_2)^2, (q_1+q_2)^2, 0)$$

- On the other hand, Knecht, Nyffeler '02 used on-shell form factors:

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2, q_1^2, q_2^2) \;\times\; \mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, (q_1+q_2)^2, 0)$$

- But form factor at external vertex $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, (q_1+q_2)^2, 0)$ for $(q_1+q_2)^2 \neq m_\pi^2$ violates momentum conservation, since momentum of external soft photon vanishes !
  Often the following misleading notation was used:
  $\mathcal{F}_{\pi^0\gamma^*\gamma^*}((q_1+q_2)^2, 0) \equiv \mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2, (q_1+q_2)^2, 0)$
  At external vertex identification with transition form factor was made (wrongly !).
- Melnikov, Vainshtein '04 had observed this inconsistency and proposed to use

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2, q_1^2, q_2^2) \;\times\; \mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, m_\pi^2, 0)$$

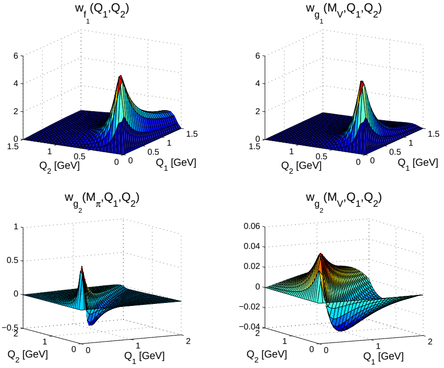
  i.e. a constant form factor at the external vertex given by the WZW term.
- However, this prescription will only yield the so-called pion-pole contribution and not the full pion-exchange contribution !
- The pion-exchange contribution with off-shell pions is model dependent. Only the sum of all contributions in a given model is relevant.

Relevant momentum regions in $a_\mu^{\mathrm{LbyL};\pi^0}$

- In Knecht, Nyffeler '02, a 2-dimensional integral representation was derived for a certain class (VMD-like) of form factors (schematically):

$$a_\mu^{\mathrm{LbyL};\pi^0} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \sum_i w_i(Q_1, Q_2)\, f_i(Q_1, Q_2)$$

  with universal weight functions w_i. Dependence on form factors resides in the f_i.
- Expressions with on-shell form factors are in general not valid as they stand. One needs to set form factor at external vertex to a constant to obtain pion-pole contribution (Melnikov, Vainshtein '04). Expressions valid for WZW and off-shell VMD form factors.
- Plot of weight functions w_i from Knecht, Nyffeler '02:

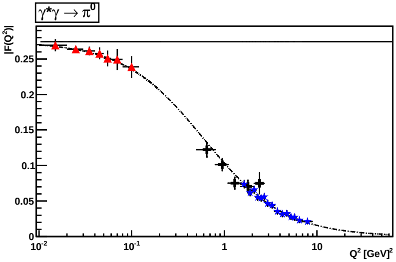


- Relevant momentum regions around 0.25 – 1.25 GeV. As long as form factors in different models lead to damping, expect comparable results for $a_\mu^{\mathrm{LbyL};\pi^0}$, at level of 20%.
- Jegerlehner, Nyffeler '09 derived 3-dimensional integral representation for general (off-shell) form factors (hyperspherical approach). Integration over $Q_1^2, Q_2^2, \cos\theta$, where $Q_1 \cdot Q_2 = |Q_1||Q_2|\cos\theta$.
- Idea recently taken up by Dorokhov et al. '12 (for scalars) and Bijnens, Zahiri Abyaneh '12, '13 (for all contributions, work in progress).

Impact of form factor measurements: example KLOE-2

On the possibility to measure the $\pi^0 \to \gamma\gamma$ decay width and the $\gamma^*\gamma \to \pi^0$ transition form factor with the KLOE-2 experiment

Babusci et al. '12



Simulation of KLOE-2 measurement of $F(Q^2)$ (red triangles). MC program EKHARA 2.0 (Czyż, Ivashyn '11) and detailed detector simulation.
Solid line: $F(0)$ given by chiral anomaly (WZW).
Dashed line: form factor according to on-shell LMD+V model (Knecht, Nyffeler '01).
CELLO (black crosses) and CLEO (blue stars) data at higher Q^2.

Within 1 year of data taking, collecting 5 fb^{-1}, KLOE-2 will be able to measure:

- $\Gamma_{\pi^0 \to \gamma\gamma}$ to 1% statistical precision.
- $\gamma^*\gamma \to \pi^0$ transition form factor $F(Q^2)$ in the region of very low, space-like momenta $0.01 \text{ GeV}^2 \leq Q^2 \leq 0.1 \text{ GeV}^2$ with a statistical precision of less than 6% in each bin.
  KLOE-2 can (almost) directly measure slope of form factor at origin (note: logarithmic scale in Q^2 in plot !).

Impact of form factor measurements: example KLOE-2 (continued)

- Error in $a_\mu^{\mathrm{LbyL};\pi^0}$ related to the model parameters determined by $\Gamma_{\pi^0\to\gamma\gamma}$ (normalization of form factor; not taken into account in most papers) and $F(Q^2)$ will be reduced as follows (details: see pages 22 – 27):

  - $\delta a_\mu^{\mathrm{LbyL};\pi^0} \approx 4 \times 10^{-11}$ (with current data for $F(Q^2) + \Gamma^{\mathrm{PDG}}_{\pi^0\to\gamma\gamma}$)
  - $\delta a_\mu^{\mathrm{LbyL};\pi^0} \approx 2 \times 10^{-11}$ ($+\ \Gamma^{\mathrm{PrimEx}}_{\pi^0\to\gamma\gamma}$)
  - $\delta a_\mu^{\mathrm{LbyL};\pi^0} \approx (0.7 - 1.1) \times 10^{-11}$ (+ KLOE-2 data)

- Note that this error does not account for other potential uncertainties in $a_\mu^{\mathrm{LbyL};\pi^0}$, e.g. related to the off-shellness of the pion or the choice of model.

- Simple models with few parameters, like VMD (two parameters: F_π, M_V), which are completely determined by the data on $\Gamma_{\pi^0\to\gamma\gamma}$ and $F(Q^2)$, can lead to very small errors in $a_\mu^{\mathrm{LbyL};\pi^0}$. For illustration:

  $a_{\mu;\mathrm{VMD}}^{\mathrm{LbyL};\pi^0} = (57.3 \pm 1.1) \times 10^{-11}$

  $a_{\mu;\mathrm{LMD+V}}^{\mathrm{LbyL};\pi^0} = (72 \pm 12) \times 10^{-11}$ (off-shell LMD+V form factor, including all errors)

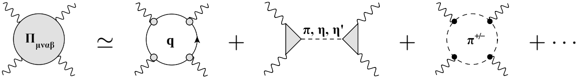
  But this might be misleading ! Results differ by about 20% ! VMD form factor has wrong high-energy behavior $\Rightarrow$ too strong damping.

Open problem: Dressed quark-loop

Dyson-Schwinger equation (DSE) approach [Fischer, Goecke, Williams '11, '13]

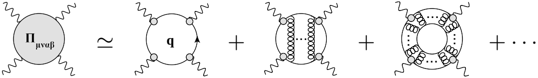
Claim: no double-counting between quark-loop and pseudoscalar exchanges (or exchanges of other resonances)

Had. LbyL in Effective Field Theory (hadronic) picture:

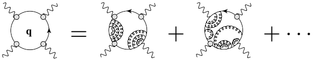


Quarks here may have different interpretation than below !

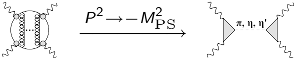
Had. LbyL using functional methods (all propagators and vertices fully dressed):



Expansion of quark-loop in terms of planar diagrams (rainbow-ladder approx.):



Pole representation of ladder-exchange contribution:



Truncate DSE using well tested model for dressed quark-gluon vertex (Maris, Tandy '99).

Large contribution from quark-loop (even after recent correction), in contrast to all other approaches, where coupling of (constituent) quarks to photons is dressed by form factors ($\rho - \gamma$-mixing, VMD).

Open problem: Dressed quark-loop (continued)

- Dyson-Schwinger equation approach [Fischer, Goecke, Williams '11, '13]

  $a_\mu^{\mathrm{LbyL};\pi^0} = 57.5(6.9) \times 10^{-11}$ (off-shell), $\quad a_\mu^{\mathrm{LbyL;PS}} = 81(2) \times 10^{-11}$
  $a_\mu^{\mathrm{LbyL;quark-loop}} = 107(2) \times 10^{-11}$, $\quad a_\mu^{\mathrm{had.\ LbyL}} = 188(4) \times 10^{-11}$

  Error for PS, quark-loop and total only from numerics. Quark-loop: still some parts are missing. Systematic error ? Not yet all contributions calculated.
  Note: numerical error in quark-loop in earlier paper (GFW PRD83 '11):
  $a_\mu^{\mathrm{LbyL;quark-loop}} = 136(59) \times 10^{-11}$, $\quad a_\mu^{\mathrm{had.\ LbyL}} = 217(91) \times 10^{-11}$

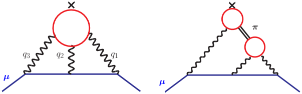
- Constituent quark loop [Boughezal, Melnikov '11]

  $a_\mu^{\mathrm{had.\ LbyL}} = (118 - 148) \times 10^{-11}$

  Consider ratio of had. VP and had. LbyL with pQCD corrections. Paper was reaction to earlier results using DSE yielding large values for the quark-loop and the total.

- Constituent Chiral Quark Model [Greynat, de Rafael '12]

  $a_\mu^{\mathrm{LbyL;CQloop}} = 82(6) \times 10^{-11}$
  $a_\mu^{\mathrm{LbyL};\pi^0} = 68(3) \times 10^{-11}$ (off-shell)
  $a_\mu^{\mathrm{had.\ LbyL}} = 150(3) \times 10^{-11}$



  Error only reflects variation of constituent quark mass $M_Q = 240 \pm 10$ MeV, fixed to reproduce had. VP in $g-2$. Determinations from other quantities give larger value for $M_Q \sim 300$ MeV and thus smaller value for quark-loop. 20%-30% systematic error estimated. Not yet all contributions calculated.

- Padé approximants [Masjuan, Vanderhaeghen '12]

  $a_\mu^{\mathrm{had.\ LbyL}} = (76(4) - 125(7)) \times 10^{-11}$
  Quark-loop with running mass $M(Q) \sim (180 - 220)$ MeV, where the average momentum $\langle Q \rangle \sim (300 - 400)$ MeV is fixed from relevant momenta in 2-dim. integral representation for pion-pole in Knecht, Nyffeler '02.

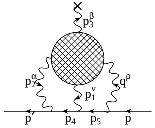
Open problem: Dressed pion-loop

1. ENJL/VMD versus HLS

Model	$a_\mu^{\pi-\text{loop}} \times 10^{11}$
scalar QED (no FF)	-45
HLS	-4.5
ENJL	-19
full VMD	-15

Strong damping if form factors are introduced, very model dependent: compare ENJL (BPP '96) versus HLS (HKS '96). See also discussion in Melnikov, Vainshtein '04.

Origin: different behavior of integrands in contribution to $g-2$ (Zahiri Abyaneh '12; Bijnens, Zahiri Abyaneh '12; Talk by Bijnens at MesonNet 2013, Prague)

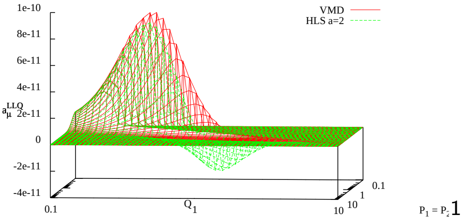


π loop

One can do 5 of the 8 integrations in the 2-loop integral for $g-2$ analytically, using the hyperspherical approach / Gegenbauer polynomials (Jegerlehner, Nyffeler '09, taken up in Bijnens, Zahiri Abyaneh '12):

$$a_\mu^{\mathrm{X}} = \int dl_{P_1} dl_{P_2}\, a_\mu^{\mathrm{XLL}} = \int dl_{P_1} dl_{P_2} dl_Q\, a_\mu^{\mathrm{XLLQ}}, \quad \text{with} \quad l_P = \ln(P/\mathrm{GeV})$$

Contribution of type X at given scale P_1, P_2, Q is directly proportional to volume under surface when a_μ^{XLL} and a_μ^{XLLQ} are plotted versus the energies on a logarithmic scale.



Momentum distribution of the full VMD and HLS pion-loop contribution for $P_1 = P_2$.

HLS: Integrand changes from positive to negative at high momenta. Leads to cancellation and therefore smaller absolute value. Usual HLS model ($a=2$) known to not fullfill certain QCD short-distance constraints.

Open problem: Dressed pion-loop (continued)

2. Role of pion polarizability and a_1 resonance

- Engel, Patel, Ramsey-Musolf '12: ChPT analysis of LbyL up to order p^6 in limit $p_1, p_2, q \ll m_\pi$. Identified potentially large contributions from pion polarizability ($L_9 + L_{10}$ in ChPT) which are not fully reproduced in ENJL / HLS models used by BPP '96 and HKS '96.

- Pure ChPT approach is not predictive. Loops not finite, would need new a_μ counterterm (Knecht et al. '02).

- Engel, Ph.D. Thesis '13; Engel, Ramsey-Musolf '13: tried to include a_1 resonance explicitly in EFT. Problem: contribution to $g-2$ in general not finite (loops with resonances).

  $\Rightarrow$ Form factor approach with a_1 that reproduces pion polarizability at low energies, has correct QCD scaling at high energies and generates a finite result in a_μ. Depending on how models with ρ and a_1 are combined, potentially large results (absolute value):

$$a_\mu^{\pi-\text{loop}} \sim -(11 - 71) \times 10^{-11}$$

  Variation of 60×10^{-11} ! Uncertainty underestimated in earlier calculations ?

- Issue taken up in Zahiri Abyaneh '12; Bijnens, Zahiri Abyaneh '12; Bijnens, Relefors (to be published); Talk by Bijnens at MesonNet 2013, Prague. Tried various ways to include a_1, but again no finite result for $g-2$ achieved. With a cutoff of 1 GeV:

$$a_\mu^{\pi-\text{loop}} = (-20 \pm 5) \times 10^{-11} \qquad \text{(preliminary)}$$

Conclusions and Outlook

- Had. LbyL in muon $g-2$: not directly related to data $\Rightarrow$ need hadronic model (or lattice QCD). Goal: to match precision of new $g-2$ experiments $\delta a_\mu = 16 \times 10^{-11}$.

- Note: only Bijnens, Pallante, Prades '96, '02 and Hayakawa, Kinoshita, Sanda '96, '98, '02 are "full" calculations so far ! But the models used have their deficiencies.
  Need one consistent (as much as possible) hadronic model !

- Need more information from experiment for various form factors of photons with hadrons at small and intermediate momenta $|Q| \leq 2$ GeV, decays like $\pi^0 \to \gamma\gamma$ to fix normalization of form factors and from cross-section measurements like $\gamma\gamma \to \pi\pi$ to gain information on the relevant $\gamma\pi\pi$ and $\gamma\gamma\pi\pi$ form factors (with off-shell pions !). Also needed as input for dispersion relations (see talks by Moussallam; Hoferichter; poster by Schneider at PHIPSI 2013). In this way one can hopefully test the models.

- The inclusion of radiative corrections and the development of Monte Carlo generators will be crucial to properly interpret such experimental measurements and to connect them with theoretical models and to achieve the needed precision.

Conclusions and Outlook (continued)

- Need more theoretical constraints on form factors and $\langle VVVV \rangle$ at low energies from ChPT and short-distance constraints from OPE and pQCD.
  Also useful to constrain models: sum rules for the (on-shell) hadronic light-by-light scattering (Pascalutsa, Pauk, Vanderhaeghen '12)
- Pseudoscalars: under control at level of 15%. Issue: off-shell form factors (pion-exchange) versus on-shell form factors (pion-pole; Melnikov, Vainshtein '04).
- Quark-loop: more work needed. Problem for theory only !
  First let Fischer et al. complete DSE calculation of quark-loop and the rest of the contributions ?
- Pion-loop: more work needed. Theory and experiment have to work together.
  Need more information on pion-polarizability, e.g. from radiative pion decay $\pi^+ \to e^+ \nu_e \gamma$, radiative pion photoproduction $\gamma p \to \gamma' \pi^+ n$, the hadronic Primakov process $\pi A \to \pi' \gamma A$ (with some heavy nucleus A) or $\gamma A \to \pi^+ \pi^- A$.
  Conflicting values from previous experiments, some new measurements are ongoing or planned. Also the properties of the a_1 resonance should be better determined, e.g. its decay modes $a_1 \to \rho\pi$ and $a_1 \to \pi\gamma$. Also important for axial-vector exchange contribution.
- Error estimates: small error does not necessarily imply that the estimate is "better", maybe the model used is too simple ! Overall error: combine errors from different contributions, where different models are used, linearly or in quadrature ?

Backup

Other recent partial evaluations (mostly pseudoscalars)

- Nonlocal chiral quark model (off-shell) [Dorokhov et al.; Talk by Radzhabov at PHIPSI 2013]

  2008: $a_\mu^{\mathrm{LbyL};\pi^0} = 65(2) \times 10^{-11}$

  2011: $a_\mu^{\mathrm{LbyL};\pi^0} = 50.1(3.7) \times 10^{-11}, \quad a_\mu^{\mathrm{LbyL;PS}} = 58.5(8.7) \times 10^{-11}$

  2012: $a_\mu^{\mathrm{LbyL};\pi^0+\sigma} = 54.0(3.3) \times 10^{-11}, \quad a_\mu^{\mathrm{LbyL};a_0+f_0} \sim 0.1 \times 10^{-11}$

  $a_\mu^{\mathrm{LbyL;PS+S}} = 62.5(8.3) \times 10^{-11}$

  Strong damping for off-shell form factors. Positive and small contribution from scalar $\sigma(600)$, differs from other estimates (BPP '96, '02; Blokland, Czarnecki, Melnikov '02).
- Holographic (AdS/QCD) model 1 (off-shell ?) [Hong, Kim '09]

  $a_\mu^{\mathrm{LbyL};\pi^0} = 69 \times 10^{-11}, \quad a_\mu^{\mathrm{LbyL;PS}} = 107 \times 10^{-11}$
- Holographic (AdS/QCD) model 2 (off-shell) [Cappiello, Cata, D'Ambrosio '10]

  $a_\mu^{\mathrm{LbyL};\pi^0} = 65.4(2.5) \times 10^{-11}$

  Used AdS/QCD to fix parameters in ansatz by D'Ambrosio et al. '98.
- Resonance saturation in odd-intrinsic parity sector (off-shell) [Kampf, Novotny '11]

  $a_\mu^{\mathrm{LbyL};\pi^0} = 65.8(1.2) \times 10^{-11}$
- Padé approximants (on-shell, but not constant FF at external vertex)

  $a_\mu^{\mathrm{LbyL};\pi^0} = 54(5) \times 10^{-11}$ [Masjuan '12 (using on-shell LMD+V FF)]

  $a_\mu^{\mathrm{LbyL};\pi^0} = 64.9(5.6) \times 10^{-11}, \quad a_\mu^{\mathrm{LbyL;PS}} = 89(7) \times 10^{-11}$

  [Escribano, Masjuan, Sanchez-Puertas '13]
  Fix parameters in Padé approximants from data on transition form factors.

The VMD form factor

Vector Meson Dominance:

$$\mathcal{F}^{\mathrm{VMD}}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2) = \frac{N_C}{12\pi^2 F_\pi} \frac{M_V^2}{q_1^2 - M_V^2} \frac{M_V^2}{q_2^2 - M_V^2}$$

on-shell = off-shell form factor !

Only two model parameters even for off-shell form factor: F_π and M_V

Transition form factor:

$$F^{\mathrm{VMD}}(Q^2) = \frac{N_C}{12\pi^2 F_\pi} \frac{M_V^2}{Q^2 + M_V^2}$$

The LMD+V form factor (off-shell)

Knecht, Nyffeler, EPJC '01; Nyffeler '09

- Ansatz for $\langle VVP \rangle$ and thus $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}$ in large-N_C QCD in chiral limit with 1 multiplet of lightest pseudoscalars (Goldstone bosons) and 2 multiplets of vector resonances, ρ, ρ' (lowest meson dominance (LMD) + V)
- $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}$ fulfills all leading (and some subleading) QCD short-distance constraint from Operator Product Expansion (OPE)
- Reproduces Brodsky-Lepage (BL): $\lim_{Q^2 \to \infty} \mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(m_\pi^2, -Q^2, 0) \sim 1/Q^2$

  (OPE and BL cannot be fulfilled simultaneously with only one vector resonance)
- Normalized to decay width $\Gamma_{\pi^0 \to \gamma\gamma}$

Off-shell LMD+V form factor:

$$\mathcal{F}^{\mathrm{LMD+V}}_{\pi^{0*}\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2) = -\frac{F_\pi}{3} \frac{q_1^2\, q_2^2\, (q_1^2 + q_2^2 + q_3^2) + P_H^V(q_1^2, q_2^2, q_3^2)}{(q_1^2 - M_{V_1}^2)\,(q_1^2 - M_{V_2}^2)\,(q_2^2 - M_{V_1}^2)\,(q_2^2 - M_{V_2}^2)}$$

$$P_H^V(q_1^2, q_2^2, q_3^2) = h_1\,(q_1^2 + q_2^2)^2 + h_2\, q_1^2\, q_2^2 + h_3\,(q_1^2 + q_2^2)\, q_3^2 + h_4\, q_3^4 + h_5\,(q_1^2 + q_2^2) + h_6\, q_3^2 + h_7$$

$$q_3^2 = (q_1 + q_2)^2$$

$F_\pi = 92.4$ MeV, $M_{V_1} = M_\rho = 775.49$ MeV, $M_{V_2} = M_{\rho'} = 1.465$ GeV

Free parameters: h_i

The LMD+V form factor (on-shell)

On-shell LMD+V form factor:

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}^{\mathrm{LMD+V}}(q_1^2, q_2^2)$$
$$= \quad -\frac{F_\pi}{3} \frac{q_1^2\, q_2^2\,(q_1^2 + q_2^2) + h_1\,(q_1^2 + q_2^2)^2 + \bar{h}_2\, q_1^2\, q_2^2 + \bar{h}_5\,(q_1^2 + q_2^2) + \bar{h}_7}{(q_1^2 - M_{V_1}^2)\,(q_1^2 - M_{V_2}^2)\,(q_2^2 - M_{V_1}^2)\,(q_2^2 - M_{V_2}^2)}$$

$$\bar{h}_2 \quad = \quad h_2 + m_\pi^2$$
$$\bar{h}_5 \quad = \quad h_5 + h_3\, m_\pi^2$$
$$\bar{h}_7 \quad = \quad h_7 + h_6\, m_\pi^2 + h_4\, m_\pi^4$$

Transition form factor:

$$F^{\mathrm{LMD+V}}(Q^2) = -\frac{F_\pi}{3}\, \frac{1}{M_{V_1}^2\, M_{V_2}^2}\, \frac{h_1\, Q^4 - \bar{h}_5\, Q^2 + \bar{h}_7}{(Q^2 + M_{V_1}^2)(Q^2 + M_{V_2}^2)}$$

- $h_1 = 0$ in order to reproduce Brodsky-Lepage behavior.
- Can treat h_1 as free parameter to fit the BABAR data, but the form factor does then not vanish for $Q^2 \to \infty$, if $h_1 \neq 0$.
  As pointed out by Dorokhov '10, this violates the Terazawa-West inequality $|F(Q^2)| \leq 1/Q$ which follows from unitarity ('72, '73).

Form factor $F(Q^2)$: data sets and normalization

Data sets used for fits:

A0 : CELLO, CLEO, PDG
A1 : CELLO, CLEO, PrimEx
A2 : CELLO, CLEO, PrimEx, KLOE-2

B0 : CELLO, CLEO, BABAR, PDG
B1 : CELLO, CLEO, BABAR, PrimEx
B2 : CELLO, CLEO, BABAR, PrimEx, KLOE-2

Normalization for $F(0)$:

- $\Gamma^{\mathrm{PDG}}_{\pi^0 \to \gamma\gamma} = 7.74 \pm 0.48$ eV (6.2% precision) for current PDG value
- $\Gamma^{\mathrm{PrimEx}}_{\pi^0 \to \gamma\gamma} = 7.82 \pm 0.22$ eV (2.8% precision) from PrimEx experiment
- $\Gamma^{\mathrm{KLOE-2}}_{\pi^0 \to \gamma\gamma} = 7.73 \pm 0.08$ eV (1% precision) for the KLOE-2 simulation

As noted in Nyffeler, PoS '09, the uncertainty in the normalization of the form factor was not taken into account in most evaluations of $a_\mu^{\mathrm{LbyL};\pi^0}$ (with the exception later of Dorokhov et al. '11).

In most papers, simply $F_\pi = 92.4$ MeV is used without any error attached to it. Value is close to $F_\pi = (92.20 \pm 0.14)$ MeV obtained from $\pi^+ \to \mu^+ \nu_\mu(\gamma)$.

Fitting the models

Model	Data	$\chi^2/d.o.f.$	Parameters		
VMD	A0	6.6/19	$M_V = 0.778(18)$ GeV	$F_\pi = 0.0924(28)$ GeV	
VMD	A1	6.6/19	$M_V = 0.776(13)$ GeV	$F_\pi = 0.0919(13)$ GeV	
VMD	A2	7.5/27	$M_V = 0.778(11)$ GeV	$F_\pi = 0.0923(4)$ GeV	
VMD	B0	77/36	$M_V = 0.829(16)$ GeV	$F_\pi = 0.0958(29)$ GeV	
VMD	B1	78/36	$M_V = 0.813(8)$ GeV	$F_\pi = 0.0925(13)$ GeV	
VMD	B2	79/44	$M_V = 0.813(5)$ GeV	$F_\pi = 0.0925(4)$ GeV	
LMD+V, $h_1 = 0$	A0	6.5/19	$\bar{h}_5 = 6.99(32)$ GeV^4	$\bar{h}_7 = -14.81(45)$ GeV^6	
LMD+V, $h_1 = 0$	A1	6.6/19	$\bar{h}_5 = 6.96(29)$ GeV^4	$\bar{h}_7 = -14.90(21)$ GeV^6	
LMD+V, $h_1 = 0$	A2	7.5/27	$\bar{h}_5 = 6.99(28)$ GeV^4	$\bar{h}_7 = -14.83(7)$ GeV^6	
LMD+V, $h_1 = 0$	B0	65/36	$\bar{h}_5 = 7.94(13)$ GeV^4	$\bar{h}_7 = -13.95(42)$ GeV^6	
LMD+V, $h_1 = 0$	B1	69/36	$\bar{h}_5 = 7.81(11)$ GeV^4	$\bar{h}_7 = -14.70(20)$ GeV^6	
LMD+V, $h_1 = 0$	B2	70/44	$\bar{h}_5 = 7.79(10)$ GeV^4	$\bar{h}_7 = -14.81(7)$ GeV^6	
LMD+V, $h_1 \neq 0$	A0	6.5/18	$\bar{h}_5 = 6.90(71)$ GeV^4	$\bar{h}_7 = -14.83(46)$ GeV^6	$h_1 = -0.03(18)$ GeV^2
LMD+V, $h_1 \neq 0$	A1	6.5/18	$\bar{h}_5 = 6.85(67)$ GeV^4	$\bar{h}_7 = -14.91(21)$ GeV^6	$h_1 = -0.03(17)$ GeV^2
LMD+V, $h_1 \neq 0$	A2	7.5/26	$\bar{h}_5 = 6.90(64)$ GeV^4	$\bar{h}_7 = -14.84(7)$ GeV^6	$h_1 = -0.02(17)$ GeV^2
LMD+V, $h_1 \neq 0$	B0	18/35	$\bar{h}_5 = 6.46(24)$ GeV^4	$\bar{h}_7 = -14.86(44)$ GeV^6	$h_1 = -0.17(2)$ GeV^2
LMD+V, $h_1 \neq 0$	B1	18/35	$\bar{h}_5 = 6.44(22)$ GeV^4	$\bar{h}_7 = -14.92(21)$ GeV^6	$h_1 = -0.17(2)$ GeV^2
LMD+V, $h_1 \neq 0$	B2	19/43	$\bar{h}_5 = 6.47(21)$ GeV^4	$\bar{h}_7 = -14.84(7)$ GeV^6	$h_1 = -0.17(2)$ GeV^2

Main improvement in normalization parameter, F_π for VMD and $\bar{h}_7$ for LMD+V. But more data also better determine the other parameters M_V or $\bar{h}_5$.

Results for $a_\mu^{\mathrm{LbyL};\pi^0}$

Model	Data	$a_\mu^{\mathrm{LbyL};\pi^0} \times 10^{11}$
VMD	A0	$(57.2 \pm 4.0)_{JN}$
VMD	A1	$(57.7 \pm 2.1)_{JN}$
VMD	A2	$(57.3 \pm 1.1)_{JN}$
LMD+V, $h_1 = 0$	A0	$(72.3 \pm 3.5)_{JN}^{*}$
		$(79.8 \pm 4.2)_{MV}$
LMD+V, $h_1 = 0$	A1	$(73.0 \pm 1.7)_{JN}^{*}$
		$(80.5 \pm 2.0)_{MV}$
LMD+V, $h_1 = 0$	A2	$(72.5 \pm 0.8)_{JN}^{*}$
		$(80.0 \pm 0.8)_{MV}$
LMD+V, $h_1 \neq 0$	A0	$(72.4 \pm 3.8)_{JN}^{*}$
LMD+V, $h_1 \neq 0$	A1	$(72.9 \pm 2.1)_{JN}^{*}$
LMD+V, $h_1 \neq 0$	A2	$(72.4 \pm 1.5)_{JN}^{*}$
LMD+V, $h_1 \neq 0$	B0	$(71.9 \pm 3.4)_{JN}^{*}$
LMD+V, $h_1 \neq 0$	B1	$(72.4 \pm 1.6)_{JN}^{*}$
LMD+V, $h_1 \neq 0$	B2	$(71.8 \pm 0.7)_{JN}^{*}$

* error does not include uncertainty due to off-shellness of pion

Error in $a_\mu^{\mathrm{LbyL};\pi^0}$ related to model parameters determined by $\Gamma_{\pi^0\to\gamma\gamma}$ (normalization; not taken into account before) and $F(Q^2)$ is reduced as follows:

- Sets A0, B0: $\delta a_\mu^{\mathrm{LbyL};\pi^0} \approx 4 \times 10^{-11}$
- Sets A1, B1: $\delta a_\mu^{\mathrm{LbyL};\pi^0} \approx 2 \times 10^{-11}$ ($+\ \Gamma^{\mathrm{PrimEx}}_{\pi^0\to\gamma\gamma}$)
- Sets A2, B2: $\delta a_\mu^{\mathrm{LbyL};\pi^0} \approx (0.7 - 1.1) \times 10^{-11}$ (+ KLOE-2 data)

Relevant momentum regions in $a_\mu^{\mathrm{LbyL;PS}}$

Result for pseudoscalar exchange contribution $a_\mu^{\mathrm{LbyL;PS}} \times 10^{11}$ for off-shell LMD+V and VMD form factors obtained with momentum cutoff Λ in 3-dimensional integral representation of JN '09 (integration over square). In brackets, relative contribution of the total obtained with $\Lambda = 20$ GeV.

Λ [GeV]	π^0 LMD+V ($h_3 = 0$)	π^0 LMD+V ($h_4 = 0$)	π^0 VMD	η VMD	η' VMD
0.25	14.8 (20.6%)	14.8 (20.3%)	14.4 (25.2%)	1.76 (12.1%)	0.99 (7.9%)
0.5	38.6 (53.8%)	38.8 (53.2%)	36.6 (64.2%)	6.90 (47.5%)	4.52 (36.1%)
0.75	51.9 (72.2%)	52.2 (71.7%)	47.7 (83.8%)	10.7 (73.4%)	7.83 (62.5%)
1.0	58.7 (81.7%)	59.2 (81.4%)	52.6 (92.3%)	12.6 (86.6%)	9.90 (79.1%)
1.5	64.9 (90.2%)	65.6 (90.1%)	55.8 (97.8%)	14.0 (96.1%)	11.7 (93.2%)
2.0	67.5 (93.9%)	68.3 (93.8%)	56.5 (99.2%)	14.3 (98.6%)	12.2 (97.4%)
5.0	71.0 (98.8%)	71.9 (98.8%)	56.9 (99.9%)	14.5 (99.9%)	12.5 (99.9%)
20.0	71.9 (100%)	72.8 (100%)	57.0 (100%)	14.5 (100%)	12.5 (100%)

π^0:

- Although weight functions plotted earlier are not applicable to off-shell LMD+V form factor, region below $\Lambda = 1$ GeV gives the bulk of the result: 82% for LMD+V, 92% for VMD.
- No damping from off-shell LMD+V form factor at external vertex since $\chi \neq 0$ (new short-distance constraint). Note: VMD falls off too fast, compared to OPE.

η, η':

- Mass of intermediate pseudoscalar is higher than pion mass $\rightarrow$ expect a stronger suppression from propagator.
- Peak of relevant weight functions shifted to higher values of Q_i. For η', vector meson mass is also higher $M_V = 859$ MeV. Saturation effect and the suppression from the VMD form factor only fully set in around $\Lambda = 1.5$ GeV: 96% of total for η, 93% for η'.