Hydrodynamics from Equilibrium Partition Function

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Plan of the Talk

- Introduction
  - Hydrodynamic Systems
  - Charged Hydrodynamic System
- Equilibrium Partition Function
  - Generic Construction
  - Counting
- Charged Perfect Fluid Dynamics from Equilibrium PF
- Anomalous Charged Fluid Dynamics to First Non-trivial Order
  - Hydrodynamics from Equilibrium PF
  - Equivalence of Equilibrium PF and 2nd law of Thermodynamics
- Results for Uncharged Fluid Dynamics
- Future Directions

Fluid Dynamics

- Fluid dynamics concerns the motion of fluids , i.e. liquids and gases. The motion can be non-relativistic or relativistic.
- Ideal fluids can have motion with out any energy dissipation.
- Fluids loose energy due to internal friction (viscosity) and heat exchange between different parts (thermal conductivity).
- The fluid motion is described by some equations of motion : Continuity equation, Euler/Navier-Stokes equations.

Relativistic Hydrodynamic system

- *Long wavelength* effective description of strongly coupled field theory.
- It is formulated in the language of constituent equations.
- The simplest case: no global conserved currents

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \Rightarrow d + 1 \text{ } \textit{equations}$$

- Need to reduce the number of independent elements of $T^{\mu\nu}$.

Relativistic Hydrodynamic system

- We consider the fluid is in local thermal equilibrium and fluctuations are of small energy.
- At any given time the system is described by the following local quantities
  - Temperature: $T(x)$ $\hookrightarrow 1$
  - Velocity vector: $u^\mu(x)$ $\hookrightarrow d+1$
- Velocity vectors are normalized : $u^\mu u_\mu = -1$

■ Total $d+1$ variables.

- In hydrodynamics we express $T^{\mu\nu}$ through $T(x)$ and $u^\mu(x)$ through the so-called constitutive equations.

Derivative expansion

- As fluid dynamics is a long wavelength effective theory the constitutive relations are usually specified in a derivative expansion.
- At any given order, thermodynamics plus symmetries determine the form of this expansion up to a finite number of undetermined coefficients.
- These coefficients may then be obtained either from measurements or from microscopic computations.

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Charged Fluid

Constitutive Relations

- 0^{th} **order energy-momentum tensor and charged current**

$$
\begin{aligned}
T^{\mu\nu} &= (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} \\
J^\mu &= q u^\mu
\end{aligned}
$$

  - $\epsilon \rightarrow$ energy density, $P \rightarrow$ pressure, $q \rightarrow$ charge density

- 1^{st} **order energy-momentum tensor and charged current**

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\begin{aligned}
T^{\mu\nu} &= (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \Pi^{\mu\nu} \\
J^\mu &= q u^\mu + J^\mu_{diss}
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Charged Fluid

Definitions:

$$\Pi^{\mu\nu} = \eta\,\sigma^{\mu\nu} + \zeta\,\Theta P^{\mu\nu}$$
$$J^{\mu}_{diss} = \sigma\,E^{\mu} + \mathcal{D}\,\partial^{\mu}\nu + \tau\,\partial^{\mu}T + D_{\omega}\,\omega^{\mu} + D_{B}\,B^{\mu}$$

where,

$$\sigma^{\mu\nu} = P^{\mu\alpha}P^{\nu\beta}\left[\frac{\nabla_{\alpha}u_{\beta} + \nabla_{\beta}u_{\alpha}}{2} - \frac{\Theta}{3}g_{\alpha\beta}\right], \quad \Theta = \nabla.u$$

$$\omega^{\mu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_{\nu}\nabla_{\alpha}u_{\beta}, \quad B^{\mu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_{\nu}F_{\alpha\beta}, \quad \nu = \frac{\mu}{T}$$

Here, $P^{\mu\nu} = g^{\mu\nu} + u^{\mu}u^{\nu} \rightarrow$ is the projection operator.

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How to restrict Fluid dynamics ?

- Fluid dynamics is an effective field theory, where we study fluctuations of fluid variables around local thermal equilibrium.

- **Does fluid satisfy any restriction?**

- **Yes, local 2nd law of thermodymanics !**

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How to restrict Fluiddynamics ?

- Let us define an "entropy current" for the fluid as, $J_s^\mu = su^\mu$, where s is the local entropy density of the perfect-fluid.
- The local entropy of the fluid should increase, i.e. $\partial_\mu J_s^\mu \geq 0$, the equality holds at equilibrium .
- For dissipitive fluids, the entropy current also gets derivative corrections and one has to apply the divergence law for the corrected entropy current.
- It puts strong restriction on the form of the constituitive relations.

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Charged Fluid

Charged Fluid Hydrodynamics from Gravity 0809.2596 ; 0809.2488

- We constructed the dual gravity solution by solving Einstein's equation and obtained the boundary fluid stress tensor and charge current. In this process we obtained the new transport coefficient D_ω. For us, the background gauge fields were constant at boundary and thus D_B term did not appear.

- From the gravity analysis we found the transport coefficient to be proportional to the CS coupling C of the corresponding five dimensional gravity theory, as

$$D_\omega = C\nu^2 T^2 \left(1 - \frac{2q}{3(\epsilon + P)}\nu T\right)$$

Charged Fluid

Son-Surowka Analysis 0906.5044

- Son and Surowka studied the system directly in field theory and they found the same transport coefficients. They studied it in presence of the boundary gauge field and found the other transport coefficients also.
- They started with a generic local entropy current and put the constraint that the divergence of the entropy current is positive definite. They found,

$$D_B = C\nu T\left(1 - \frac{q}{2(\epsilon + P)}\nu T\right), \quad D_\omega = C\nu^2 T^2\left(1 - \frac{2q}{3(\epsilon + P)}\nu T\right)$$

- D_ω matches with our expressions, the constants turns out to be zero for the charged black brane system we studied.

- A similar analysis had been done recently for uncharged fluid at second order. It gives non trivial constraints on second order transport coefficients .
- The computation becomes very messy and complicated.

We look at fluid dynamics from a complete new perspective, by assuming that the system reaches an equilibrium.

Results

- We recover all known results of fluid dynamics from this simple physical assumption. To add, computations are much simpler.
- For anomalous charged fluids, we proved that the exsistence of a equilibrium partition function is equivalent to the validity of 2nd law of thermodynamics .

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Equilibrium PF

What are we doing ??

- We consider fluid in a 'special' background given as,

$$
\begin{aligned}
ds^2 &= -e^{2\sigma(\vec{x})}\left(dt + a_i(\vec{x})dx^i\right)^2 + g_{ij}(\vec{x})dx^i dx^j \\
A^\mu &= (A^0(\vec{x}), A^i(\vec{x}))
\end{aligned}
$$

- The metric has a time like killing vector and the gauge field is also time independent.
- Assumptions
  •Fluid in such a background has a time independent solution (equilibrium).
  •Thermodynamics of this system is generated from a partition function (equilibrium partition function).

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Equilibrium Partition Function

- We begin with a most generic partition function expressed in terms of the background fields and their derivative expansion,

$$logZ = W = \int d^3x\sqrt{g_3}S(\sigma, A_0, a_i, A_i, g^{ij})$$

- While writing the partition function we need to maintain the symmetries of the corresponding fluid system.
- We compute equilibrium stress tensor and charge current from the equilibrium partition function as,

$$T^{z}_{\mu\nu} = -\frac{2}{\sqrt{g_4}}\frac{\delta S}{\delta g_4^{\mu\nu}}, \quad J^{\mu}_{z} = \frac{1}{\sqrt{g_4}}\frac{\delta S}{\delta A^4_{\mu}}$$

Equilibrium PF

Equilibrium Solution

- We add the generic possible correction terms to fluid variables in terms of derivative expansion of the background variables with arbitrary coefficients.

$$u^\mu = u_0^\mu + u_1^\mu, \quad T = T_0 + T_1, \quad \mu = \mu_0 + \mu_1$$

- Similarly, we write the most generic constitutive relations allowed by the symmetry of the theory. These will also contain many arbitrary coefficients coming with all possible derivative expansion terms.

$$T^{\mu\nu} = A u^\mu u^\nu + B g^{\mu\nu} + \Pi^{\mu\nu}, \qquad J^\mu = D u^\mu + J^\mu_{diss}.$$

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$$T^{\mu\nu} = Au^\mu u^\nu + Bg^{\mu\nu} + \Pi^{\mu\nu}, \quad J^\mu = Du^\mu + J^\mu_{diss}.$$

Equilibrium PF

- We equate, $T^z_{\mu\nu} = T_{\mu\nu}|_{equilibrium}$ and $J^\mu_z = J^\mu|_{equilibrium}$.

Claims

- We can solve for all fluid variables and determine the transport coefficients in terms of the equilibrium partition function.
- Partition function contains few number of coefficients, thus, we can determine many coefficients in terms of fewer coefficients.

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Equilibrium PF

Counting of Coefficients and Equations

Lets assume at any i-th order in derivative expansion of the background fields, we have,

• T_i tensors, V_i vectors, S_i scalars.

- Number of parameters ::
  - In any arbitrary frame, stress tensor can have $(T_i + V_i + S_i)$ coefficients and charge current can have $(V_i + S_i)$ coefficients. In landau frame, $(V_i + S_i)$ coefficients get eliminated.
  - Number of coefficients appearing in velocity , temperature and chemical potential correction $(V_i + 2S_i)$.
  - Total number of unknown coefficients are $(T_i + 2V_i + 3S_i)$.
- Number of equations ::
  - From the variation of the partition function, we get $(T_{<ij>}, T_{0i}, J^i, T, T_{00}, J^0)$
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Perfect Charged Fluid Dynamics from Equilibrium Partition Function

- The partition function:

$$logZ = W = \int d^3x\sqrt{g_3}S(\sigma, A_0)$$

- Equilibrium solution:

$$u^\mu = e^{-\sigma}(1,0,0,0), \quad T = e^{-\sigma}, \quad \mu = A_0 e^{-\sigma}.$$

- Constitutive relations :

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + Pg^{\mu\nu}, \qquad J^\mu = qu^\mu,$$

where, we have redefined $S(\sigma, A_0) = \frac{P(\mu,T)}{T}$

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Equations of Motion

- Stress tensor conservation equation :

$$\nabla_\mu T^\mu_\nu = F_{\nu\mu}\tilde{J}^\mu$$

- Current conservation equation :

$$\nabla_\mu \tilde{J}^\mu = CE.B$$

Here, $\tilde{J}^\mu$ is the gauge covariant current, which differs from the usual current by a gauge non-invariant term. C is the measure of the U(1) gauge anomaly.

1st Order Charged Fluid

1st derivative data

- Scalars : $\epsilon^{ijk} A_i F_{jk}, \quad \epsilon^{ijk} a_i f_{jk}, \quad \epsilon^{ijk} A_i f_{jk}$ Parity Odd
- Vectors : $\partial_i \sigma, \quad F_{i0}$ Parity Even

  $\epsilon^{ijk} F_{jk}, \quad \epsilon^{ijk} f_{jk}$ Parity Odd
- Tensor : No symmetric tensor

Symmetry Requirements

- Physical observable are gauge invariant. Since the 1st derivative scalars are gauge non invariant, they can not correct scalar fluid variables, namely, temperature T and chemical potential μ.
- The charge current should be gauge invariant. Thus, it is the covariant current that we have to use for all computations.
- Although the scalars are not gauge invariant, but their integrals are. Thus we can construct a local equilibrium partition function with them.
- We know that the $U(1)$ symmetry is anomalous, thus, while writing the action, we have to allow gauge non invariant terms corresponding to the $U(1)$ gauge field, so that we can produce the right anomaly.

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1st Order Charged Fluid

Equilibrium Partition Function

- The most generic equilibrium partition function can be,

$$W = logZ = \int d^3x \epsilon^{ijk} \left[C_0(A_0) A_i F_{jk} + C_1 a_i f_{jk} + C_2(A_0) A_i f_{jk} \right]$$

- All fields appearing in the action are three dimensional fields.
- Since the diffeomorphism symmetry is unbroken, thus the coefficient C_1 has to be a constant.
- The other two coefficients C_0, C_2 are allowed to be functions of A_0, so that it can produce anomaly. They can not be functions of σ.

Stress tensor and charge current

- Various components of equilibrium stress tensor and charged current obtained from the partition function are:

$$
\begin{aligned}
T_{00} &= 0, \quad T^{ij} = 0, \\
J_0 &= -\frac{e^{\sigma}}{\sqrt{g_3}}\epsilon^{ijk}\left[C_0'(A_0)A_iF_{jk} + C_2'(A_0)A_if_{jk}\right] \\
J^i &= \frac{e^{-\sigma}}{\sqrt{g_3}}\epsilon^{ijk}\left[C_0(A_0)F_{jk} + C_2(A_0)f_{jk} + \nabla_j(C_0(A_0)A_k)\right], \\
T_0^i &= e^{\sigma}\epsilon^{ijk}\left[(C_2(A_0) - 2C_0(A_0)A_0)F_{jk} + (2C_1 - C_2(A_0)A_0)f_{jk}\right]
\end{aligned}
$$

- The usual charged current is not gauge invariant. Thus, we have to construct the covariant current $\tilde{J}^\mu$ which appears in the equations of motion. By requiring that the covariant current constructed is gauge invariant, we get,

$$C_0(A_0) = \frac{C}{3}A_0 + \tilde{C}_0, \quad C_2(A_0) = \frac{C}{6}A_0^2 + \tilde{C}_2, \tag{0.1}$$

  where, $\tilde{C}_0, \tilde{C}_2$ are two arbitrary constants.
- The covariant current looks as,

$$\tilde{J}_0 = 0, \quad \tilde{J}^i = T\epsilon^{ijk}\left[2(\frac{C}{6}A_0 + \tilde{C}_0)F_{jk} + (\frac{C}{6}A_0^2 + \tilde{C}_2)f_{jk}\right] \tag{0.2}$$

Constitutive Relations and Fluid variables

- Temperature and chemical potential does not get corrected, thus,

$$T = e^{-\sigma}, \qquad \mu = A_0 e^{-\sigma}. \tag{0.3}$$

- Most generic velocity correction,

$$u^\mu = u_0^\mu + \sum b_m X_m^\mu, \quad m = 1, 4 \tag{0.4}$$

- Most generic charged current.

$$J^\mu = q u_0^\mu + \sum (a_m + q b_m) X_m^\mu \tag{0.5}$$

- Thus we have total 8 coefficients appearing in velocity corrections and constitutive relations.
- We also have exactly 8 equations coming from $\tilde{J}^i$, T_0^i.

1st Order Charged Fluid

Results

$$
\begin{aligned}
b_1 &= \frac{T^3}{\epsilon+P}\left(\frac{2}{3}\nu^3 C + 4\nu^2\tilde{C}_0 - 4\nu^2\tilde{C}_2 + 4C_1\right),\\
b_2 &= \frac{T^2}{\epsilon+P}\left(\frac{1}{2}\nu^2 C + 2\nu\tilde{C}_0 - \tilde{C}_2\right),\\
a_1 &= C\nu^2 T^2\left(1 - \frac{2q}{3(\epsilon+P)}\nu T\right)\\
&+ T^2\left[(4\nu\tilde{C}_0 - 2\tilde{C}_2) - \frac{qT}{\epsilon+P}(4\nu^2\tilde{C}_0 - 4\nu\tilde{C}_2 + 4C_1)\right],\\
a_2 &= C\nu T\left(1 - \frac{q}{2(\epsilon+P)}\nu T\right) + T\left(2\tilde{C}_0 - \frac{qT}{\epsilon+P}(2\nu\tilde{C}_0 - \tilde{C}_2)\right),\\
b_3 &= b_4 = a_3 = a_4 = 0
\end{aligned}
$$

Matches exactly with Son-Surowka results up to the constant $\tilde{C}_0$.

1st Order Charged Fluid

- CPT Analysis : The action has to be invariant under CPT transformation. This restricts the action further and sets $\tilde{C}_0$ and C_1 to zero.

- Thus, the new transport coefficient is actually zero in 4 dimensions. This feature is true for any arbitrary $2n-$dimensional theory for any n. We have proved this in details.

1st Order Charged Fluid

How to accommodate the new constant $\tilde{C}_0$

- What can be the source of this new constant $\tilde{C}_0$?
- Another physical quantity that we can compute is the total entropy for the system.
- To do this, we begin with the most generic entropy current and compute the entropy. This gives us a new possible term in the entropy current compared to Son-Surowka analysis.

Entropy Current

- A local entropy current is not a physical observable. Instead the rate of entropy production and the total entropy are physical.
- This allows us to add non-gauge invariant terms to the entropy current, keeping its divergence and total entropy gauge invariant.
- There are only two terms possible satisfying this criteria,

$$\epsilon^{\mu\nu\alpha\beta}\nabla_\nu(h_1(\sigma, A_0)A_\alpha u_\beta) \quad \text{and} \quad h_2\epsilon^{\mu\nu\alpha\beta}A_\mu\nabla_\alpha A_\beta,$$

  where, h_1 can be a function of (σ, A_0), but h_2 is constant.
- Entropy current looks as,

$$J_s^\mu = su^\mu - \nu J_{diss}^\mu + X\omega^\mu + X_B B^\mu + T_1 + T_2$$

1st Order Charged Fluid

Final Results

- Comparing the total entropy we get,

$$h_2 = \tilde{C}_0, \quad X_B = T\left(\frac{C\nu^2}{2} - \tilde{C}_2\right), \quad X = T^2\left(\frac{C\nu^3}{3} - 2\tilde{C}_2\nu + 2C_1\right)$$

- The new parameter $\tilde{C}_0$ incorporates this extra term in the entropy current.
- The function h_1 remains unfixed as the term, being a total derivative, does not appear in entropy.
- Redoing Son-Surowka analysis with this modified entropy current exactly reproduces this result.

- In 1106.0277 , the author has studid this system in a nice and innovative way. He started with following constitutive relations ,

$$
\begin{aligned}
T^{\mu\nu} &\equiv \varepsilon u^{\mu} u^{\nu} + pP^{\mu\nu} + q^{\mu}_{anom} u^{\nu} + u^{\mu} q^{\nu}_{anom} + T^{\mu\nu}_{diss} \\
J^{\mu} &\equiv q u^{\mu} + J^{\mu}_{anom} + J^{\mu}_{diss} \\
J^{\mu}_{S} &\equiv s u^{\mu} + J^{\mu}_{S,anom} + J^{\mu}_{S,diss}
\end{aligned}
$$

- here, $\{q^{\mu}_{anom}, J^{\mu}_{anom}, J^{\mu}_{S,anom}\}$ are the anomalous heat/charge/entropy currents .

Equilibrium PF and 2nd law of Thermodynamics

- The divergence of entropy current will be

$$\nabla_\mu J^\mu_S = UsualTerms + f(q^\mu_{anom}, J^\mu_{anom}, J^\mu_{S,anom})$$

- Demanding the validity of usual 2nd law with in the dissipative currents, they found an adiabaticity relation among these anomalous currents .

$$f(q^\mu_{anom}, J^\mu_{anom}, J^\mu_{S,anom}) = 0$$

Equilibrium PF and 2nd law of Thermodynamics

- Thus, they concluded that if we can find a set of $\{q^{\mu}_{anom}, J^{\mu}_{anom}, J^{\mu}_{S,anom}\}$ which satisfies the adiabaticity condition, the usual 2nd law of thermodynamics would be valid for the system .

- More over, they obtained that all these anomalous currents can be obtained from a Gibbs current $\bar{\mathcal{G}}_{anom}$ via thermodynamics as,

$$
\begin{aligned}
J_{anom} &= -\frac{\partial \bar{\mathcal{G}}_{anom}}{\partial \mu} \\
J_{S,anom} &= -\frac{\partial \bar{\mathcal{G}}_{anom}}{\partial T} \\
q_{anom} &= \bar{\mathcal{G}}_{anom} + T J_{S,anom} + \mu J_{anom}
\end{aligned}
$$

Equilibrium PF and 2nd law of Thermodynamics

- We have obtained the relation of this Gibbs current $\bar{\mathcal{G}}_{anom}$ with the equilibrium partition function .

$$W_{anom} = \ln \ Z^{anom} = -\int_{space} \frac{1}{T}\bar{\mathcal{G}}_{anom}$$

- Thus, we prove that the exsistence of the equilibrium partition function ensures the exsistence of the $\bar{\mathcal{G}}_{anom}$ which satisfies the right adiabaticity relation .

- This ensures the validity of 2nd law of thermodynamics.

2nd order Uncharged Fluid

Uncharged Fluid Dynamics

- The system is simpler as there is fewer fluid variable, but computationally more involved, as we are looking at second order fluid dynamics.
- There are fifteen possible two derivative corrections for the stress tensor. Out of them, seven terms are zero on the leading equilibrium solution.
- We can put constraints on remaining eight transport coefficients.
- There are only three nontrivial scalars possible $R, f^2, (\nabla\sigma)^2$, which implies that only three of those eight coefficients are free and all other five are determined in terms of them.

2nd order Uncharged Fluid

- Positivity of entropy production predicts five relations (1201.4654). It is an involved computation, in particular it requires computation of divergence of entropy current up to four-th order in derivative expansion.

- We recovered all these relations in a simple way just by requiring the equilibrium partition function.

Summary

- Requirement of entropy production gives us some inequalities and some equalities among the transport coefficients. The equalities are interesting, as they contain physical information of the theory.
- By our equilibrium construction, we can nicely capture all the equality relations.
- The inequalities should come from unitarity.
- We have obtained an elegant and physical conditions to constrain fluid dynamics. By counting the number of independent non trivial scalars at any derivative order, we can estimate the number of transport coefficients !!
- We can also find the entire thermodynamics from the partition function.

Summary

- We have obtained all the transport coefficients for fluids anomalous under multiple $U(1)$ charges in abritrary even dimensions, using the equilibrium partition function technique, up to first non-trivial order.
- For this particular system, we have been able to prove that the exsistence of the equilibrium partition function ensures that the 2nd law of thermodynamics holds.

Future Directions

Conceptual

- We find exactly same constraints on fluid dynamics as can be obtained by the demand of entropy production. Why?? What is the connection (physical as well as algebraical) between these two different approaches ??
- What can we say about dual gravity system?? Does existence of equilibrium implies area increase theorem ??
- How can we move away from equilibrium ?? Can we put dynamics into the problem?? Can we write Fluid action ??

Future Directions

Applications

- Constraints on 2nd order Charged Fluid Dynamics.
- Understanding fluids anomalous under non-abelian gauge symmetry from the partition function.
- Understanding fluids dynamics anomalous under mixed and gravitational anomaly.
- Many more ......

THANK YOU

Future Directions

Applications

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THANK YOU