Hydrodynamics from black hole

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WHY DO WE NEED STRING THEORY TO UNDERSTAND PROPERTIES OF HYDRODYNAMICS SYSTEM?

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- A remarkable experiment in 2005 in RHIC - Relativistic Heavy Ion Collider at Brookhaven National Lab, New York.
- Gold ions are collided nearly at the speed of light. Energy: 200 GeV
- When the ions collide the high energy of the collision melts the protons and neutrons into their component quarks and gluons, creating thousands of new particles.
- RHIC scientists observed a dramatic asymmetry in the expansion of these particles.
- Expectation : to find a gas, no interaction between the quarks and more uniform expansion.

WHY DO WE NEED STRING THEORY TO UNDERSTAND PROPERTIES OF HYDRODYNAMICS SYSTEM?

- Instead an asymmetric momentum distribution was observed among the particles --- **Elliptic flow** which suggests that particles are **strongly interacting**.

- Elliptic flow was the first important discovery in RHIC which tells that the QGP matter produced in the collider are near perfect fluid.

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  Such a low ratio of shear viscosity to entropy density ratio of a strongly coupled system is difficult to explain by conventional methods.

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Q. What is the AdS/CFT Correspondence?

THE ADS/CFT CORRESPONDENCE

- The famous AdS/CFT correspondence was proposed by Juan M Maldacena in 1998.

  The correspondence follows from the string theory.

  We discuss the correspondence with

  the help of a cartoon.

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- Note that complete information about a *d+1* dimensional theory is encoded in a completely different *d* dimensional theory.

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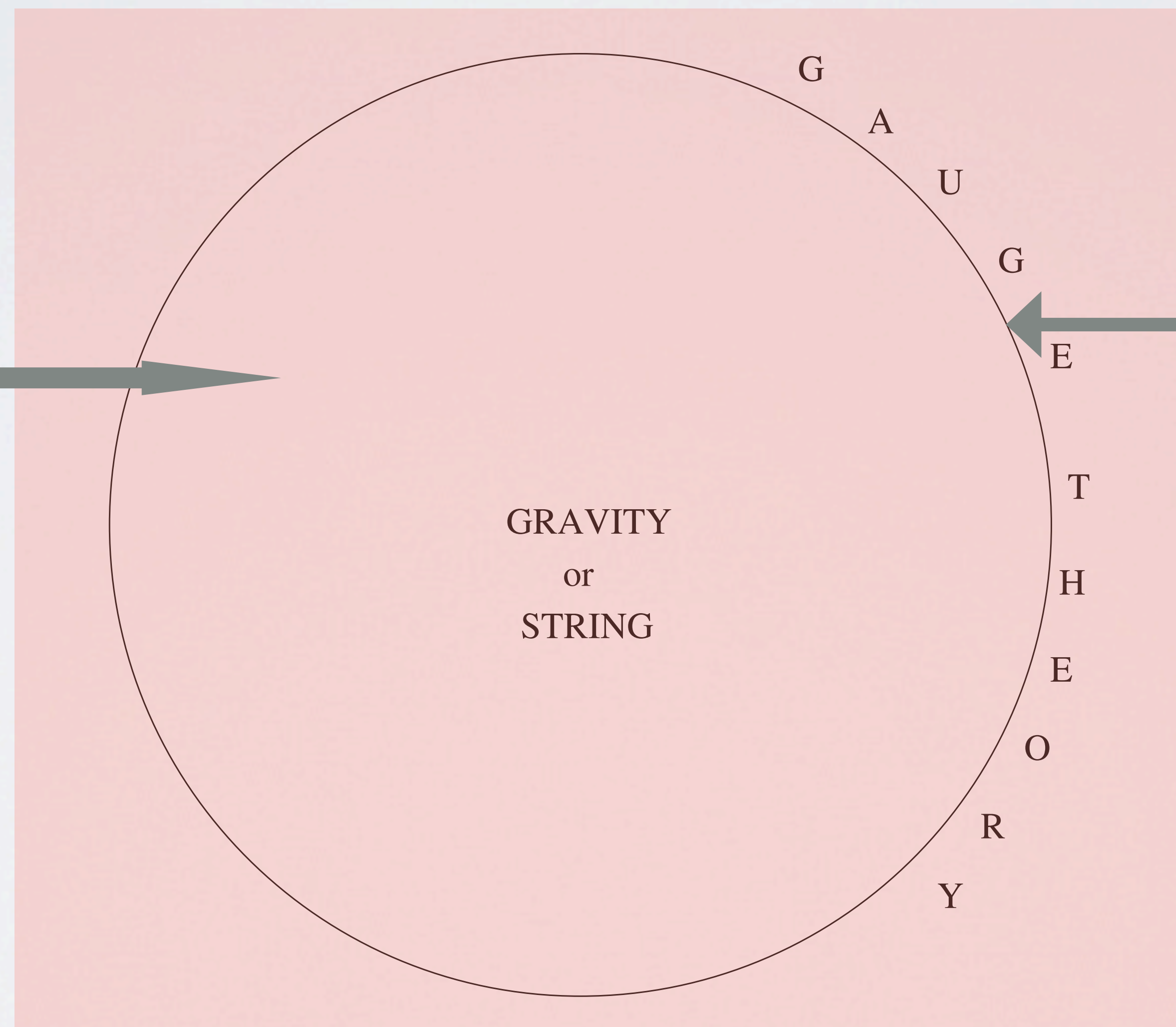
Holography

THE ADS/CFT CORRESPONDENCE

d+1 dimensional
AdS space
(Bulk)



Boundary
theory in d

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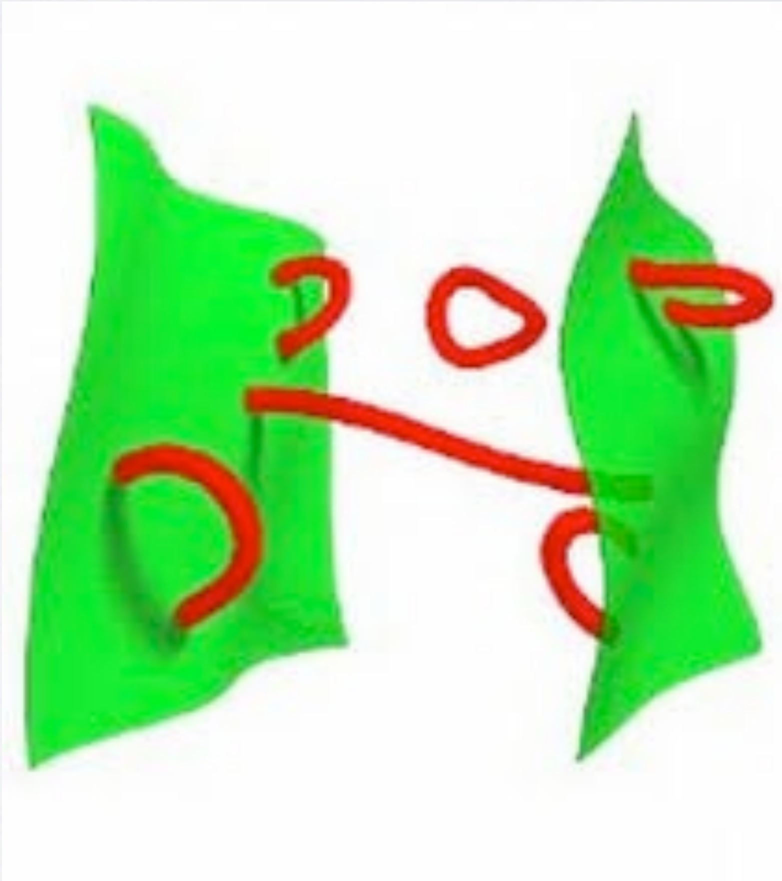
- The correspondence follows from two different descriptions of an object called D - branes.
- Dp brane is a p dimensional extended object.
- Consider type IIB string theory in this D3 brane background.
- The system has two different descriptions.



Description A

- Low energy effective action

$$S = S_{CFT} + S_{gravity} + S_{int}$$

- In low energy limit interaction becomes zero. We have two decoupled systems : **a)** A CFT living on brane and **b)** a (super) gravity excitations living in the bulk.

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Description B

- D3 branes are massive and charged. Source for many supergravity fields.

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CFT = String theory in AdS space

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  Dual means?

- We know every interacting theory has a coupling constant.
- The AdS/CFT says that when the field theory is in strongly coupled side then its equivalent string theory is in weakly coupled regime and vice-versa. That is why it is also called strong-weak duality.

- Therefore, if we want to calculate any thermodynamic or hydrodynamic properties of strong coupled field theory we first map the system to its dual string theory description which is in the weakly coupled regime. We do the calculation there and then using the AdS/CFT dictionary we map the result back to field theory side.

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- Example:

$\mathcal{N}$ = 4 SYM $\equiv$ type IIB string theory in AdS space.

Entropy of strongly coupled $\mathcal{N}$=4 SYM theory is given by,

$$\mathcal{S}(\lambda \to \infty) = \frac{\pi^2}{8} N^2 V_3 T^3$$

IMPORTANT TOOLS IN THE GAME

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- **Field-operator correspondence**

$$\langle \mathcal{O} \rangle = \frac{\partial}{\partial J} \mathcal{Z}_{FT}(J)|_{J=0}$$

$$\mathcal{Z}_{FT}(J) = \int \mathcal{D}\Phi \exp\left[-S(\Phi) + \int J\mathcal{O}\right]$$

Witten conjectured that the there exists an one-to-one correspondence between boundary operators and bulk fields.
The boundary value of a bulk field acts as a source for the corresponding boundary operator and

$$\mathcal{Z}_{ST}(h) = \mathcal{Z}_{FT}(J), \; h = J$$

IMPORTANT TOOLS IN THE GAME

- For example : we want to compute correlation function of energy momentum tensor in a strongly coupled field theory.

  a) we identify the bulk field corresponding to the EM tensor in boundary is the graviton.

  b) calculate bulk partition function, which is easy to calculate.

  c) take derivative of bulk partition function with respect to the boundary value of graviton.

  d) that gives the expectation value of energy momentum tensor in a strongly coupled theory.

FLUID/GRAVITY CORRESPONDENCE

FLUID/GRAVITY CORRESPONDENCE

- The power of AdS/CFT is not confined to characterizing only the thermodynamic properties of boundary field theories.
- It can also explain different hydrodynamic properties of boundary theory.
- Hydrodynamics : long wave length deviation (low frequency fluctuation) from thermal equilibrium.
- Our goal : to study the properties of finite temperature gauge theory plasma living on boundary with hydrodynamic fluctuations.

HYDRODYNAMICS

- Hydrodynamics is an effective theory, describing the dynamics of some field theory at large distances and time-scales.
- Hydrodynamic description does not follow from the action principle rather it is normally formulated in the language of equations of motion : because it is a dissipative system.
- Hydrodynamic equations are just the laws of conservation of energy and momentum.

$$\nabla_\mu T^{\mu\nu} = 0$$

- Energy momentum tensor is given by

$$T_{\mu\nu} = (e+p)u_\mu u_\nu + p g_{\mu\nu} - 2\sigma_{\mu\nu}$$

Ideal fluid part

Viscous part

$$\sigma_{\mu\nu} = P^\alpha_\mu P^\beta_\nu \left[\eta\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{d-1}\eta_{\mu\nu}\partial\cdot u\right) + \zeta\partial\cdot u\right]$$

Shear viscosity

$$P^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$$

Bulk viscosity

HOW TO CALCULATE SHEAR VISCOSITY COEFFICIENTS ?

- **Kubo formula**

Under the small metric fluctuation $g_{xy} = \eta_{xy} + h_{xy}$ the energy momentum tensor is given by

$$T_{xy} = p h_{xy} + \eta \frac{\partial h_{xy}}{\partial t}$$

In linear response theory the energy momentum tensor can be written as

$$T_{xy} \sim \mathcal{G}_{xy,xy}(\omega) h_{xy}$$

where,

$$\mathcal{G}_{xy,xy} = p - i\omega\eta + \mathcal{O}(\omega^2)$$

Therefore,

$$\eta = -\lim_{\omega \to 0} \frac{\Im \mathcal{G}_{xy,xy}(\omega)}{\omega}$$

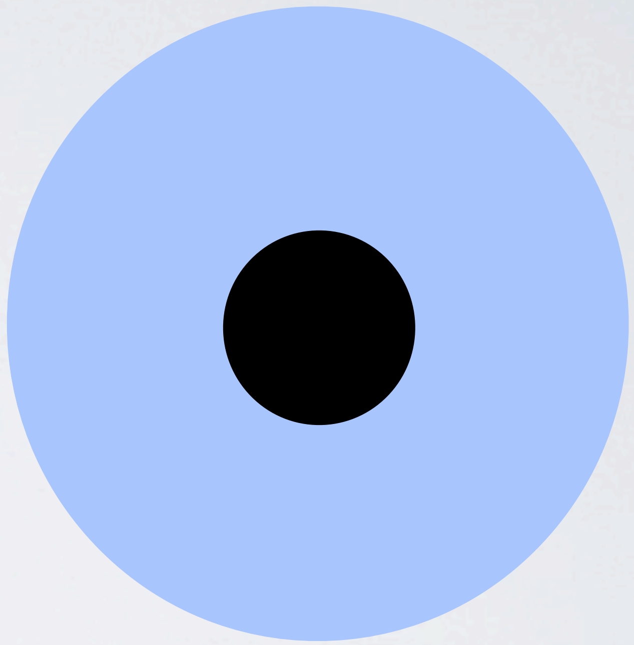
- GOAL : ***To compute***

$$\mathcal{G}_{xy,xy}(\omega)$$

holographically

HOLOGRAPHIC SET UP

- Holographic geometry is given by a finite temperature black hole solution in 5 dimension.
- Black holes are thermodynamic objects.
- Black hole has temperature which is same as the temperature of boundary gauge theory.
- Black hole has entropy.



- Entropy of a black hole is equal to the area of event horizon.

HOLOGRAPHIC CALCULATION OF SHEAR VISCOSITY COEFFICIENT

- To calculate shear viscosity coefficient we need to compute the Green's function.

- Green's function can be written as

$$\mathcal{G}_{xy,xy} = \frac{\partial^2 \mathcal{Z}_{BH}}{\partial h^{(0)}_{xy} \partial h^{(0)}_{xy}}$$

What is black hole partition function?

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What is black hole partition function?

- Black hole partition function is given by

$$\mathcal{Z}_{BH} = \exp[-I_{on}]$$

$$I \sim \int d^5x (R + 12)\sqrt{g}$$

HOLOGRAPHIC CALCULATION OF SHEAR VISCOSITY COEFFICIENT

- Black hole solution

$$ds^2 = -\left(r^2 - \frac{r_0^4}{r^2}\right) dt^2 + \frac{1}{\left(r^2 - \frac{r_0^4}{r^2}\right)} dr^2 + r^2(dx^2 + dy^2 + dz^2)$$

This defines the metric. Boundary is at r = infinity.

- To compute we need to consider small fluctuation about this solution.

$$g_{xy} = g_{xy}^{(0)} + h_{xy}$$

- Then we calculate action I for this fluctuation.

$$\mathcal{Z}_{BH} = \exp[-I(h_{xy}^{(0)})]$$

RESULTS OF HOLOGRAPHIC COMPUTATION

- The final result turns out to be

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B} \sim 6.08 \times 10^{-13} K s$$

CHARGED FLUID

- A charged fluid also has conserved current J

$$J_\mu = n u_\mu - D P^\nu_\mu \mathcal{D}_\nu n + \xi_1 \epsilon^{\nu\lambda\sigma}_\mu u_\nu \partial_\lambda u_\sigma + \xi_2 \frac{1}{2} \epsilon_{\mu\lambda\alpha\beta} u^\lambda F^{\alpha\beta}$$

D - diffusion coefficient

ξ_1 and ξ_2 are two new transport coefficients.

- Effect of these transport coefficients has been observed in QGP produced in RHIC.

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- **Holographic set up**

Charged black hole in AdS space

$$S \sim \int R + 12 - F^2 + \kappa \epsilon^{ABCDE} A_A F_{BC} F_{DE}$$

A_μ is the dual field for boundary current J_μ

- Our result $\xi_1 = \frac{3\kappa q^2}{16\pi G m}$

[JHEP 1101 (2011) 094]

SECOND ORDER HYDRODYNAMICS

- Second derivative correction to hydrodynamics

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$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + Pg^{\mu\nu} - \sigma^{\mu\nu} + \Theta^{\mu\nu}.$$

$$\Theta^{\mu\nu} = \eta\tau_\Pi \left[{}^{<}D\sigma^{\mu\nu>} + \frac{1}{d-1}\sigma^{\mu\nu}(\nabla\cdot u) \right]$$
$$+\kappa\left[R^{<\mu\nu>} - (d-2)u_\alpha R^{\alpha<\mu\nu>\beta}u_\beta\right]$$
$$+\lambda_1 \sigma^{<\mu}{}_\lambda \sigma^{\nu>\lambda} + \lambda_2 \sigma^{<\mu}{}_\lambda \Omega^{\nu>\lambda} + \lambda_3 \Omega^{<\mu}{}_\lambda \Omega^{\nu>\lambda}$$

$$\Omega^{\mu\nu} = \frac{1}{2}P^{\mu\alpha}P^{\nu\beta}(\partial_\alpha u_\beta - \partial_\beta u_\alpha), \quad D = u\cdot\partial$$

SECOND ORDER HYDRODYNAMICS

- In linear response theory

$$G_R^{xy,xy}(\omega, k) = P - i\eta\omega + \eta\tau_{\Pi}\omega^2 - \frac{\kappa}{2}[(d-3)\omega^2 + k^2]$$

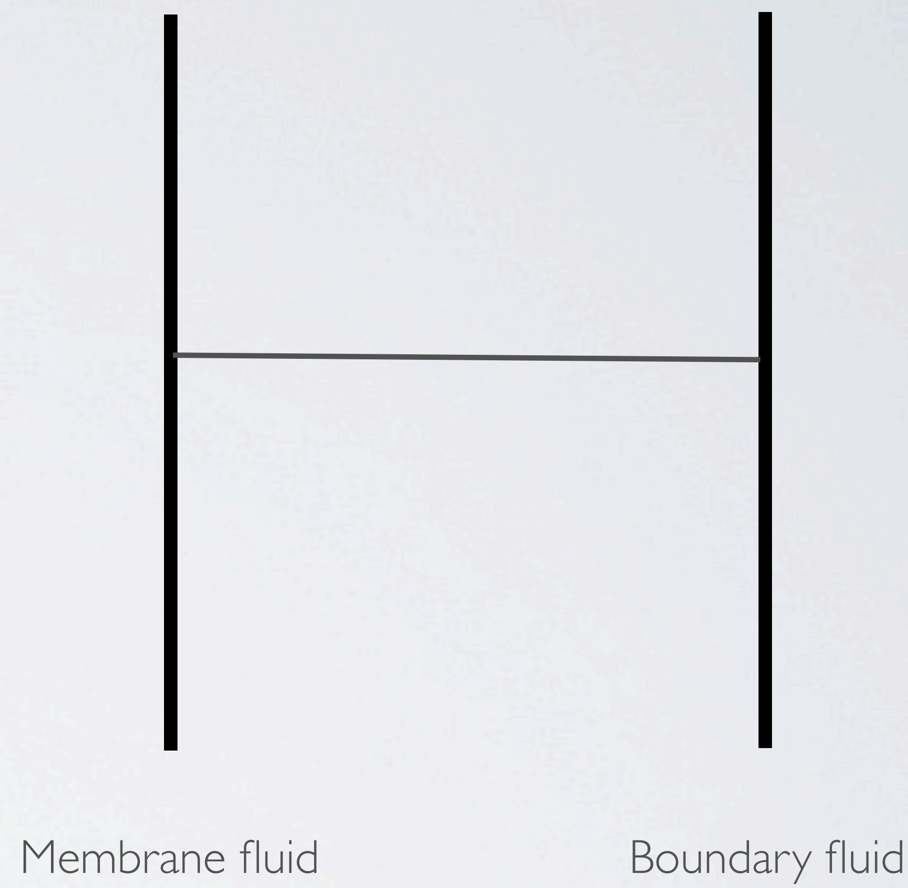
- Calculating Green's function holographically,

$$\kappa = \frac{\eta}{\pi T}, \quad \tau_\pi = \frac{2 - \ln 2}{2\pi T}$$

Other three transport coefficients can not be calculated in linear response theory. One has to write down the stress tensor for boundary fluid using AdS/CFT (Minwalla and group).

BOUNDARY FLUID AND MEMBRANE FLUID

- A black hole can be viewed as a fictitious fluid membrane.
- The membrane has hydrodynamic characteristic.
- The membrane is located at the horizon of the black hole.



BOUNDARY FLUID AND MEMBRANE FLUID

- In Phys.Rev. D79 (2009) 025023 Liu and Iqbal showed that the low energy response of boundary fluid is determined by the membrane fluid.
- Shear viscosity coefficient of boundary fluid is same as shear viscosity coefficient of membrane fluid.
- In JHEP 0903 (2009) 116 N. Banerjee, SD: we have generalized this idea in generic higher derivative gravity. Our result was, the shear viscosity coefficient of boundary fluid is given by the effective coupling constant of graviton.
- In JHEP 1008 (2010) 041 N. Banerjee, SD we have studied finite energy excitations of boundary fluid.
- We should that there exists a response function $\chi(k_\mu, r)$ which satisfies a first order diff. equation

$$\partial_r \chi(k_\mu, r) = f(k_\mu, r; \chi)$$

- We solved this equation imposing boundary conditions at the horizon and extracted value of higher order transport coefficients of boundary fluid.
- UV/IR connection !!

OTHER DIRECTIONS

- We see that the AdS/CFT is a very successful tool to understand the properties of strongly coupled systems.

- Holographic phase transition and phase diagram, Holographic QCD, Holographic super conductor, Holographic fermi liquid ....

THANK YOU