Gravity and fundamental physics: Probing the Equivalence Principle at Classical and Quantum Level

Salvatore Capozziello[a] and Gaetano Lambiase[b]

[a] Università di Napoli "Federico II"
and INFN, Sez. di Napoli, Italy

INFN Istituto Nazionale di Fisica Nucleare

[b] Università di Salerno
and INFN, Sez. di Napoli, Italy

Summary

- Foundation: gravity and space-time
- Shortcomings in General Relativity
- Alternatives, way out and extensions
- Metric or connections?
- The role of Equivalence Principle
- Testing EP at classical and quantum level
- STE-QUEST: a possibility
- Conclusions

Foundation: gravity and space-time

Einstein worked hard at finding a theory of gravity based on the following requirements:

- principle of equivalence → Gravity and Inertia are indistinguishable; there exist observers in free fall (inertial motion)
- principle of relativity → Special Relativity holds pointwise; the structure of the spacetime is pointwise Minkowskian
- principle of general covariance → "democracy" in Physics
- principle of causality → all physical phenomena propagate respecting the light cones
- Riemann's teachings about the link between matter and curvature →

Foundation: gravity and space-time

The distribution of matter influences Gravity through 10 second order equations, nowadays called Einstein equations:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

A linear concomitant of the Riemann tensor, nowadays called the Einstein tensor, equals the stress-energy tensor that reflects the properties of matter.

They have a structure that suitably reduces to Newtonian equations in the "weak field limit."

So, is g the gravitational field?

Einstein knows that it is not, since g is a tensor, while the principle of equivalence holds true! Free fall is described by the geodesics of (*M*, *g*):

$$\ddot{x}^\lambda + \left\{ {\lambda \atop \mu\nu} \right\}_g \dot{x}^\mu \, \dot{x}^\nu = 0$$

This is the right object to represent the gravitational field: g is just the potential of the gravitational field... but being $\left\{ {\lambda \atop \mu\nu} \right\}_g$ constructed since g, the metric remains the fundamental variable: g gives rise to the gravitational field, to causality, to the principle of equivalence, to rods & clocks.

Foundation: gravity and space-time

Working on the theory of "parallelism" in manifolds, Tullio Levi-Civita understands that it is not a metric property of space, but rather a property of "affine" type, having to do with "congruences of privileged lines."

Generalizing the case of Christoffel symbols $\left\{ {\lambda \atop \mu\nu} \right\}_g$

Levi-Civita introduces the notion of linear connection as the more general object $\Gamma^{\lambda}{}_{\mu\nu}$

such that the equation of geodesics

$$\ddot{x}^{\lambda} + \Gamma^{\lambda}{}_{\mu\nu}\, \dot{x}^{\mu}\, \dot{x}^{\nu} = 0$$

is generally covariant.

A connection in a 4D space has 64 components. Only 40 if it is symmetric.

Any linear connection defines a (different) covariant derivative.

Foundation: gravity and space-time

If Γ is the Levi-Civita connection of g, then the covariant derivative of g vanishes:

$$\nabla_\nu \left(g^{\alpha\beta} \sqrt{|g|} \right) = 0$$

If has no torsion (i.e. it is symmetric) this is a characteristic property of $\Gamma_{L\text{-}C}(g)$:

$$\nabla_\nu \left(g^{\alpha\beta} \sqrt{|g|} \right) = 0 \quad \Rightarrow \quad \Gamma^\alpha_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\}_g$$

Its 40 components are function of 10 fields.

Foundation: gravity and space-time

Einstein was not so happy with the fact that the gravitational field is not the fundamental object, but just a by-product of the metric. Using a method invented few years before by Attilio Palatini, he realizes that one can obtain Einstein equations by working on a theory that depends on *two* variables, varied independently:

a metric g and a linear connection Γ assumed to be symmetric.

$$R \equiv R(g, \Gamma) = g^{\mu\nu} R_{\mu\nu}(\Gamma) \qquad \mathcal{L}_{PE} = g^{\mu\nu} R_{\mu\nu}(\Gamma, \partial\Gamma)$$

There are 10 + 40 independent variables and the equations are:

$$R_{(\mu\nu)} - \frac{1}{2} R\, g_{\mu\nu} = \kappa\, T_{\mu\nu}$$

$$\nabla^{\Gamma}_{\alpha} \left(\sqrt{g} g^{\mu\nu}\right) = 0$$

Foundation: gravity and space-time

Field equations for the 40 components of Γ ensure that Γ is the Levi-Civita connection of g (Levi-Civita theorem):

$$R_{(\mu\nu)} - \frac{1}{2} R\, g_{\mu\nu} = \kappa\, T_{\mu\nu}$$

$$\nabla^{\Gamma}_{\alpha} \left(\sqrt{g} g^{\mu\nu}\right) = 0$$

$$\Longrightarrow \quad \Gamma^{\alpha}_{\mu\nu} = \left\{ {}^{\alpha}_{\mu\nu} \right\}_g$$

Field equations for the remaining 10 variables transform directly into Einstein equations.

In Palatini formalism, the metric g determines rods & clocks, while the connection Γ the free fall.

Foundation: gravity and space-time

Hermann Weyl makes a celebrated attempt to unify Gravity with Electromagnetism. He understands that Electromagnetism is a gauge field.

He introduces a scalar factor ϕ (a "gauge") that point by point calibrates the interaction.

The metric g (Gravitation) and the scalar factor ϕ (the "phase") determine in fact a linear connection in spacetime.

Weyl's idea fails. The Lagrangian is not appropriate and field equations describe a "massive photon" (it is in fact a Proca-Yukawa interaction in modern language).

Weyl's idea generates however a keypoint: connections may have an interesting dynamics. Fields may be gauge fields - i.e. fields with group properties coming from further principles and "internal symmetries".

Shortcomings in General Relativity

Is still *g* the fundamental object of Gravity?

Einstein tries to consider directly the connection as the fundamental object of Gravity, but he never completes the process of "dethronizing" *g*.

Shortcomings in General Relativity

But after all, what are the problems with GR?

GR is *simple*, beautiful.. but seems to be not self-consistent at all scales:

- cosmological constant Λ
- Inflation
- Dark Matter + Dark Energy
- Quantum Gravity problem
- Consistency of EP at classical and quantum level

Today observations say that there is too few matter in the Universe! Thence the need, in order to save GR, for dark energy and dark matter:

$$G_{\mu\nu} = T_{\mu\nu} + T_{\mu\nu}^{dark}$$

In summary: We must suitably discriminate among different theories of gravity!!!

$$G_{\mu\nu} = (8\pi G)\, T_{\mu\nu}$$

$G_{\mu\nu} \Rightarrow \tilde{G}_{\mu\nu}$

$T_{\mu\nu} \Rightarrow \tilde{T}_{\mu\nu}$

Extended theories of Gravity

WHY?

- QFT on curved spacetimes
- String/M-theory corrections
- Brane-world models

$$R,\ R^{\mu\nu}R_{\mu\nu},\ R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta},\ R\Box^{l}R,$$

Curvature invariants or scalar fields have to be taken into account

Dark Energy and Λ

- Cosmological constant (Λ)
- Time varing Λ
- Scalar field theories
- Phantom fields
- Phenomenological Theories
- Exotic matter

Is there any way out to these shortcomings?

Alternatives, way out and extensions

A particular family of NLTG is that of f(R) theories in metric formalism, in which the Hilbert Lagrangian is replaced by any non-linear density depending on R. GR is retrieved in (and only in) the particular case f(R)=R.

In these theories there is a second order part that resembles Einstein tensor (and reduces to it if and only if f (R) = R) and a fourth order "curvature part" (that reduces to zero if and only if f (R) = R):

$$f'(R(g))\, R_{\mu\nu}(g) - \frac{1}{2} f(R(g)) g_{\mu\nu} - \nabla_\mu \nabla_\nu f'(R(g)) + g_{\mu\nu} \Box f'(R(g)) = \kappa T_{\mu\nu}$$

Higher order Gravity (4^{th})!

Pushing the 4th order part to the r.h.s. lets interpret it as an "extra gravitational stress" $T^{\mathrm{curv}}_{\mu\nu}$, much in the spirit of Riemann.
These theories can be recast in scalar-tensor form so the paradigm is that higher order terms can be dealt under the standard of scalar fields (see Weyl appraach!).

Alternatives, way out and extensions

From f(R) theories GR is retrieved in (and only in) the particular case f(R)=R.

f '(R) = 1

f ''(R) = 0

Second Order Field Equations

degenerate theory

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 4\,\pi\, T_{\mu\nu}$$

Gravitational contribution

Matter contribution

The role of Equivalence Principle

We need to investigate the EP:

- discriminating among theories of gravity
- its validity at classical and quantum level
- investigating geodesic and causal structure

The role of Equivalence Principle

Einstein Equivalence Principle states:

- Weak Equivalence Principle is valid;
- the outcome of any local non-gravitational test experiment is independent of velocity of free-falling apparatus;
- the outcome of any local non-gravitational test experiment is independent of where and when in the Universe it is performed.

The role of Equivalence Principle

One of the predictions of this principle is the gravitational red-shift, experimentally verified by Pound and Rebka in 1960

Gravitational interactions are excluded from WEP and Einstein EP

In order to classify alternative theories of gravity, the Gravitational WEP and the Strong Equivalence Principle (SEP) has to be introduced

The role of Equivalence Principle

The SEP extends the Einstein EP by including all the laws of physics in its terms:

- WEP is valid for self-gravitating bodies as well as for test bodies (Gravitational Weak Equivalence Principle);
- the outcome of any local test experiment is independent of the velocity of the free-falling apparatus;
- the outcome of any local test experiment is independent of where and when in the Universe it is performed.

The SEP contains the Einstein Equivalence Principle, when gravitational forces are neglected

The role of Equivalence Principle

Two different classes of experiments can be considered:

- the first ones testing the foundations of gravitation theory (among them the EP)
- the second one testing the metric theories of gravity where space-time is endowed with a metric tensor and where the Einstein EP is valid.

For several fundamental reasons extra fields might be necessary to describe the gravitation, e.g. scalar fields or higher-order corrections in curvature invariants.

Several theories are characterized by the fact that a scalar field (or more than one scalar field) is coupled or not to gravity and ordinary matter

There are several reasons to introduce a scalar field:

* Scalar fields are unavoidable for theories aimed to unify gravity with the other fundamental forces: e.g. Superstring, Supergravity (SUGRA), M-theories.

* Scalar fields appear both in particle physics and cosmology:
  - the Higgs boson in the Standard Model
  - the dilaton entering the supermultiplet of higher dimensional gravity
  - the super-partner of spin ½ in SUGRA.

- The introduction of a scalar field gives rise typically to a possible “violation” of the Einstein Equivalence Principle (EEP).

In order to distinguish competing theories, a possibility is related to the so-called “fifth force” approach.
For example, the case of f(R)-gravity:

$$\mathcal{A} = \int d^4x\sqrt{-g}\Big[f(R) + \mathcal{X}\mathcal{L}_m\Big], \qquad \mathcal{X} = \frac{16\pi G}{c^4}$$

The variation with respect to the metric tensor gives

$$f'R_{\mu\nu} - \frac{1}{2}f g_{\mu\nu} - f'_{;\mu\nu} + g_{\mu\nu}\Box f' = \frac{\mathcal{X}}{2}T_{\mu\nu}$$

$$3\Box f' + f'R - 2f = \frac{\mathcal{X}}{2}T$$

Trace equation

$$f(R) = \sum_n \frac{f^n(R_0)}{n!}(R-R_0)^n \simeq f_0 + f_1 R + f_2 R^2 + f_3 R^3 + \ldots .$$

In the Newtonian limit,
let us consider the perturbation of the metric
with respect to the Minkowskin background

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

The metric entries can be developed as

$$\begin{cases} g_{tt}(t,r) \simeq 1 + g^{(2)}_{tt}(t,r) + g^{(4)}_{tt}(t,r) \\ g_{rr}(t,r) \simeq -1 + g^{(2)}_{rr}(t,r) \\ g_{\theta\theta}(t,r) = -r^2 \\ g_{\phi\phi}(t,r) = -r^2 \sin^2\theta \end{cases} ,$$

As general solution:

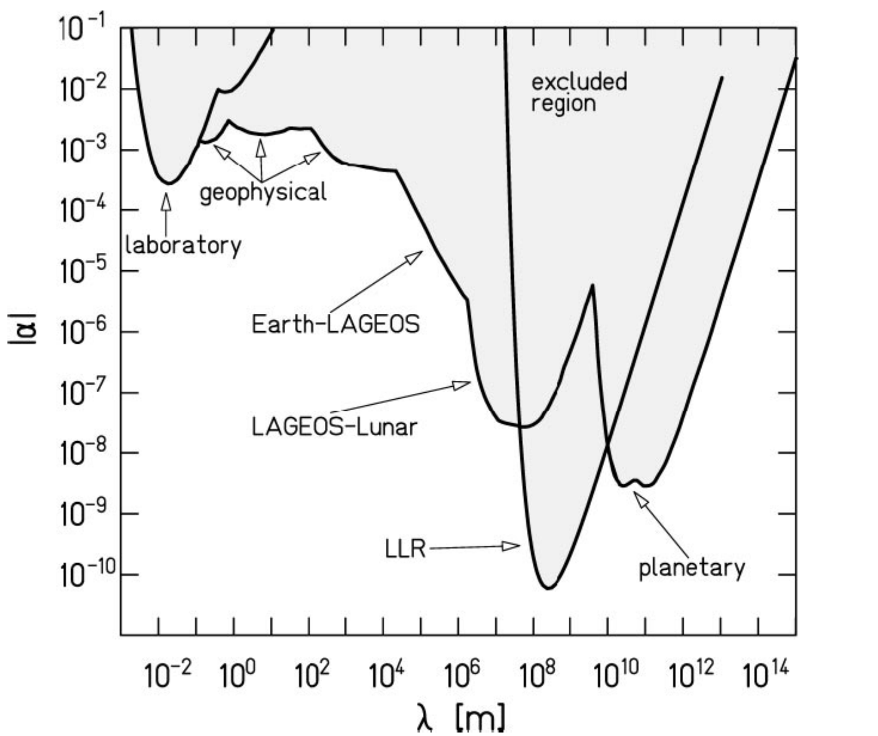
$$\Phi_{grav} = -\left(\frac{GM}{f_1 r} + \frac{\delta_1(t)e^{-r\sqrt{-\xi}}}{6\xi r}\right)$$

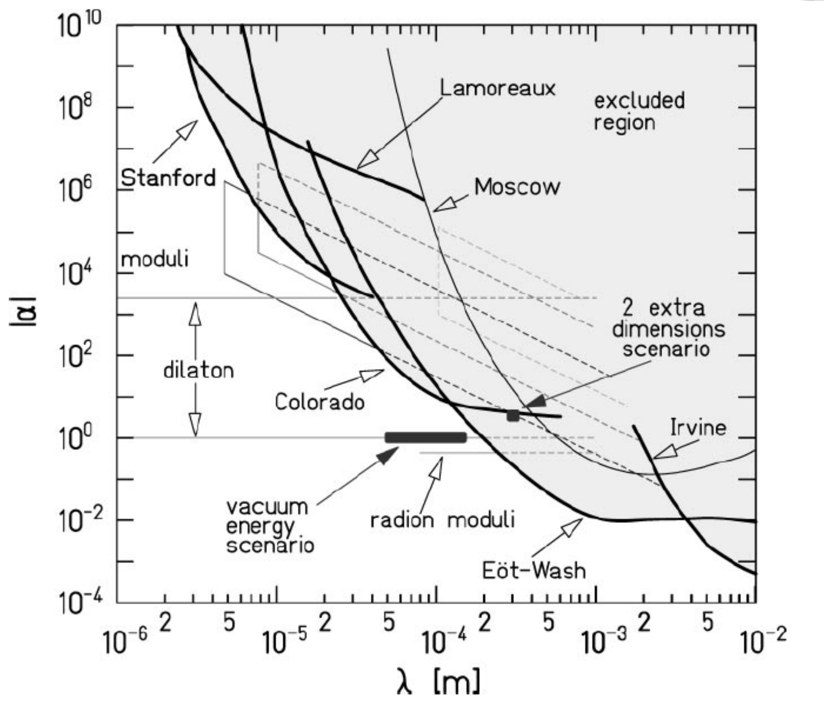
Fifth force $V(r) = -G\frac{m_1 m_2}{r}[1 + \alpha e^{-r/\lambda}]$

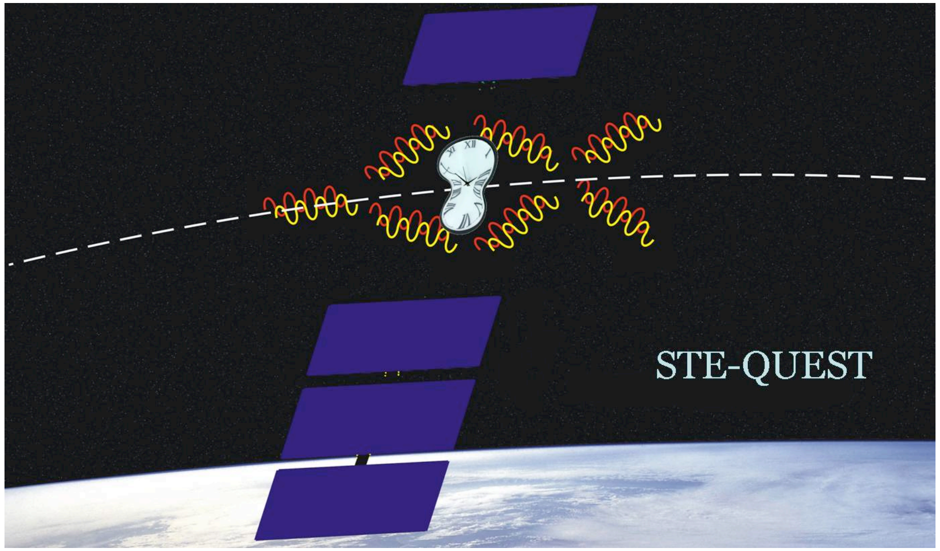
α is a dimensionless strength parameter

λ is a length scale or range

Experimental bounds







The STE-QUEST Experiment could allow:

- to set and refine the bounds on space parameters
- to discriminate among competing theories
- fifth force
- to test EP and SEP at quantum level

Mission Summary

- Primary Goal: To test Einstein's Equivalence Principle to high precision and search for new fundamental constituents and interactions in the Universe

- Observables: Clock redshift measurements;
  Differential acceleration measurements of freely falling atoms

- Spacecraft and Instruments: Single spacecraft carrying:
  A microwave clock based on laser cooled rubidium atoms;
  A differential atom interferometer operating on the two rubidium isotopes;
  Time and frequency transfer links in the microwave and optical domain for space-to-ground comparisons of clocks.

- Orbit: Highly elliptical orbit around the Earth

- Lifetime: 5 years

- Type: M-class mission.

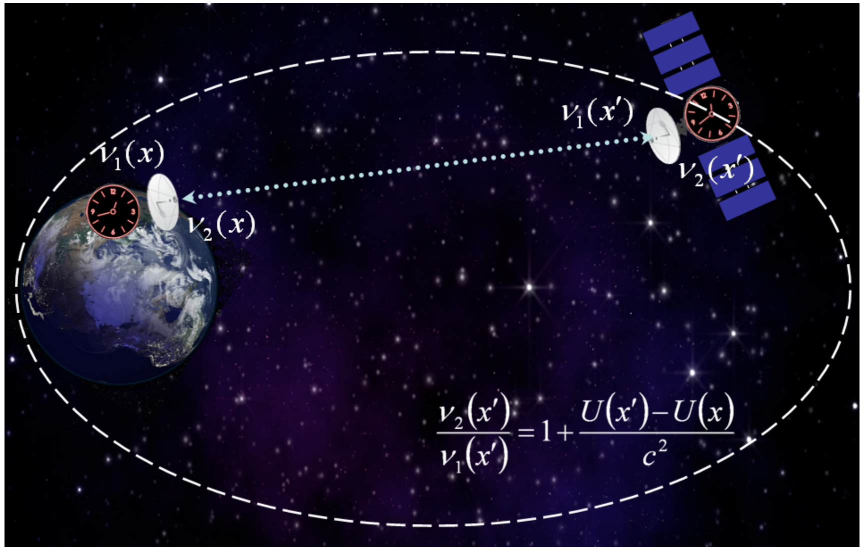
Primary Science Objectives

Scientific objective	Target accuracy
Gravitational Redshift Test	
Earth gravitational redshift	Measurement of Earth’s gravitational redshift to a fractional frequency uncertainty lower than 2×10^{-7}, with an ultimate goal of 4×10^{-8}
Sun gravitational redshift	Measurement of the Sun's gravitational redshift to a fractional frequency uncertainty better than 2×10^{-6}, with an ultimate goal of 6×10^{-7}
Weak Equivalence Principle Tests	
Universality of propagation of matter waves	To test the universality of the free propagation of matter waves to an uncertainty in the Eötvös parameter better than 1×10^{-15}

Additional Science with STE-QUEST

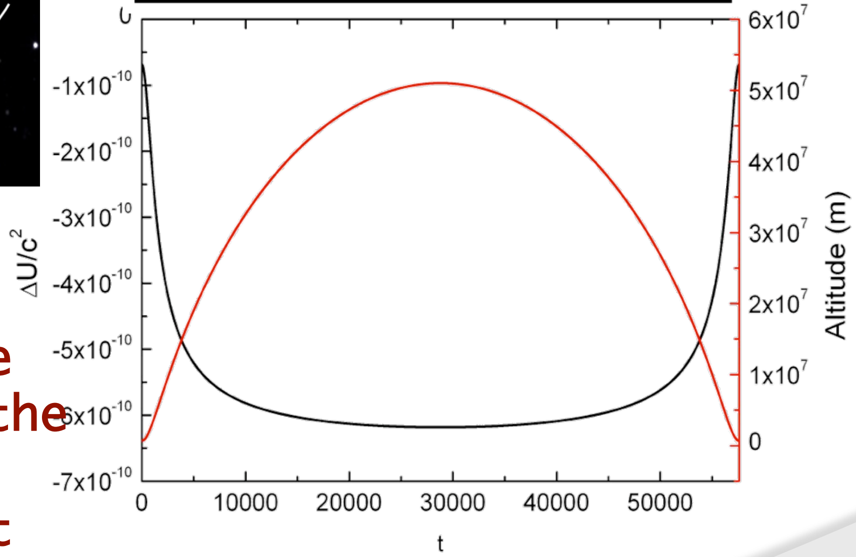
Scientific objective	Target accuracy
Time and frequency metrology	STE-QUEST will connect atomic clocks on ground in a worldwide network, bringing important contributions to the generation of atomic time scales and to the synchronization of clocks
Relativistic geodesy:	The comparison of clocks on Earth will give access, via the red-shift formula, to differential geopotential measurements on the Earth's surface. A resolution at the level of 1 cm on the differential geoid height can be achieved by STE-QUEST
Cold-atom and matter wave physics in conditions of weightlessness	STE-QUEST will study the evolution of ultra-cold atomic samples in an environment free from perturbations and over long free-propagation times
Optical and microwave ranging	The optical and microwave links will allow the cross-comparison of different ranging techniques and the measurement of differential atmospheric propagation delays (optical vs microwave)

Earth Gravitational Redshift Measurements



Typical orbit parameters

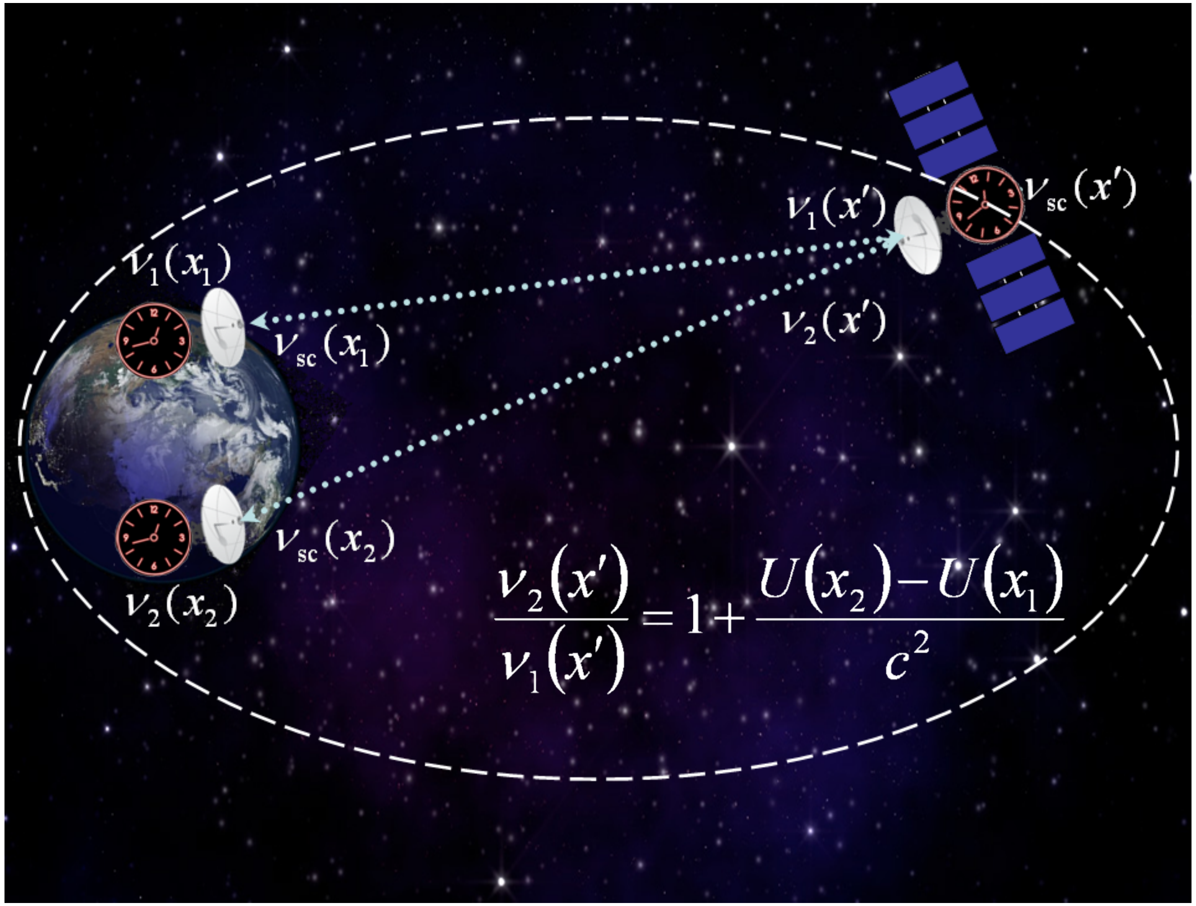
Period	16 h
SMA	32203.7 km
Eccentricity	0.7802
Inclination	63.43 deg
RAAN	336 deg
Argument of perigee	342 deg
True anomaly	0 deg



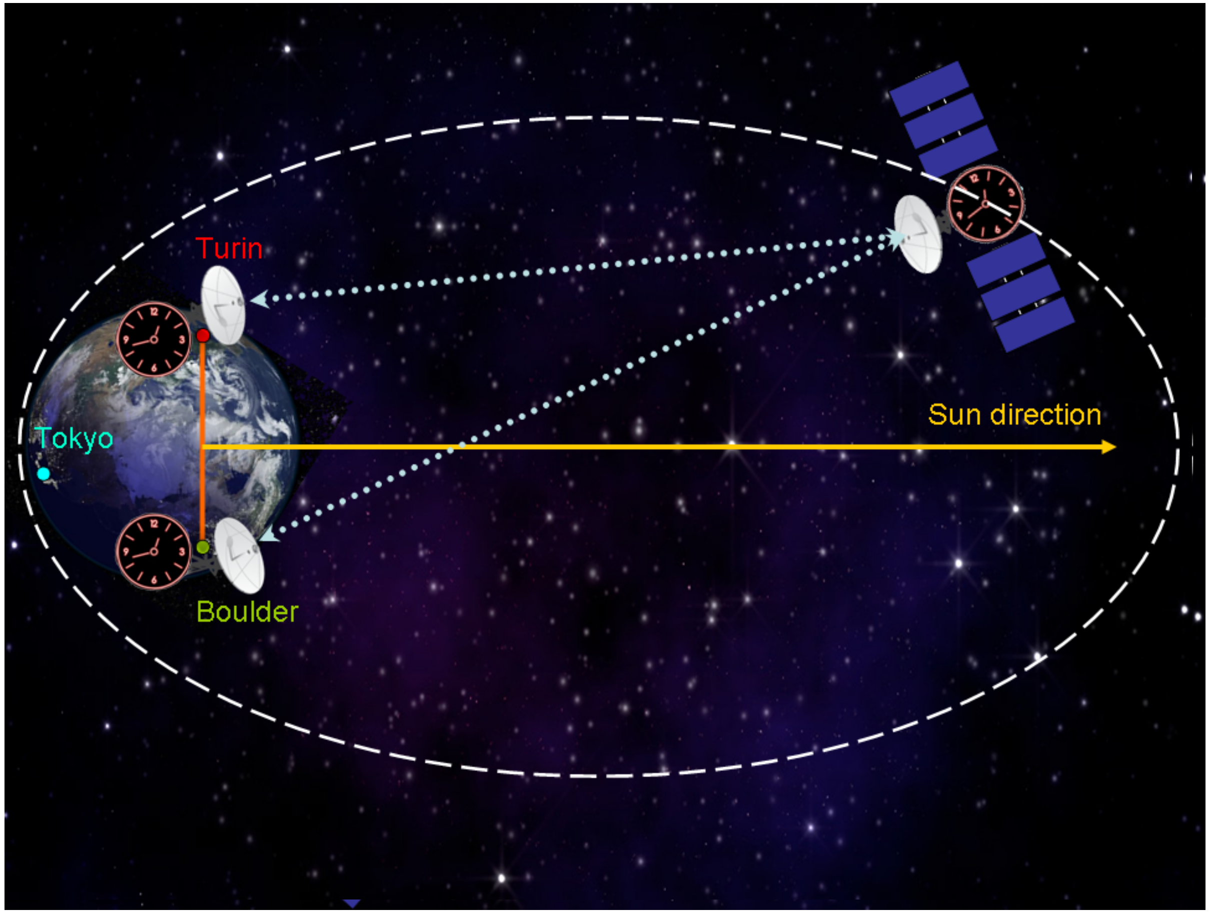
Gravitational redshift measurement:

- Absolute comparison of the space clocks to clock on the ground
- Modulation of the redshift effect between perigee and apogee

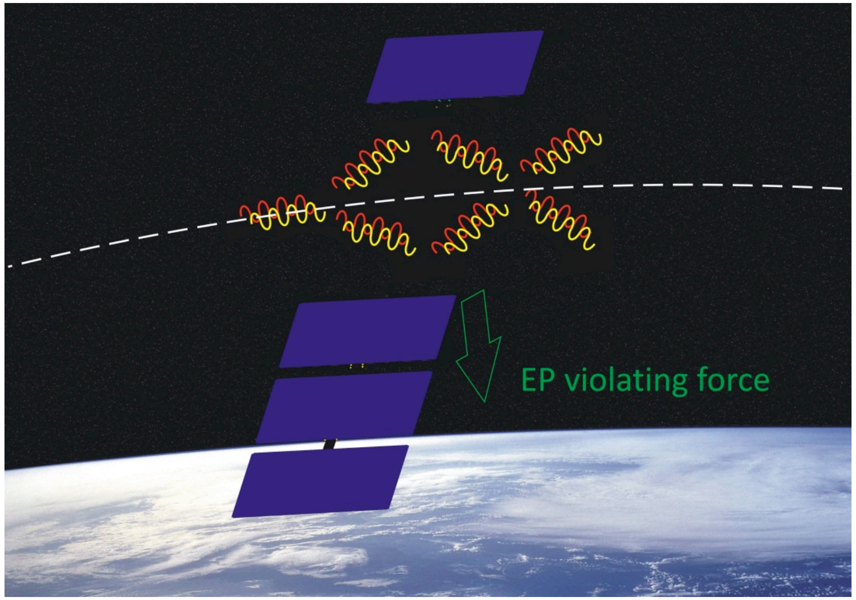
Comparison of Clocks on the Ground



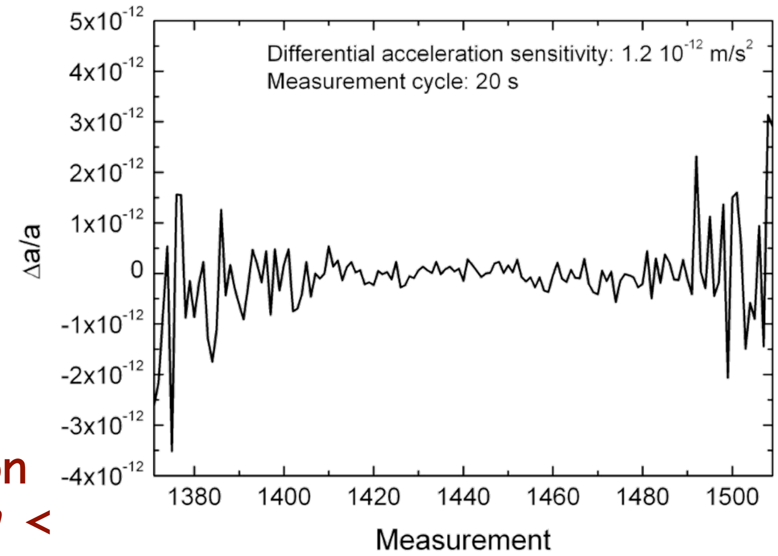
Sun Gravitational Redshift Measurements



Universality of Free Motion for Matter Waves



$$\eta = \frac{\Delta a}{a} = 2 \cdot \frac{(m_g/m_i)_A - (m_g/m_i)_B}{(m_g/m_i)_A + (m_g/m_i)_B}$$



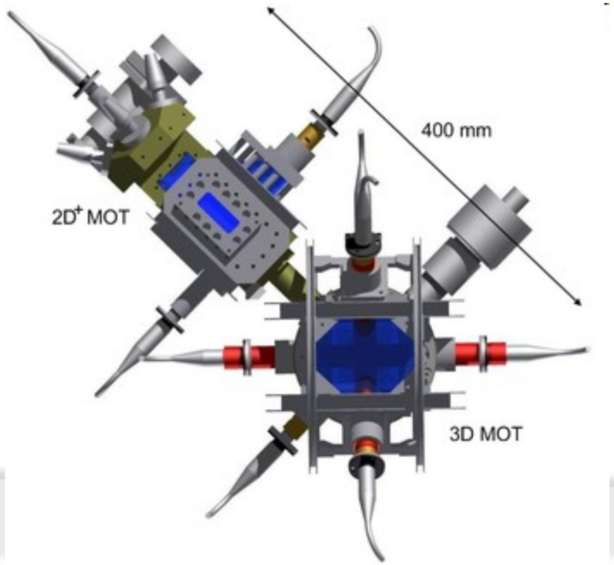
WEP measurement:

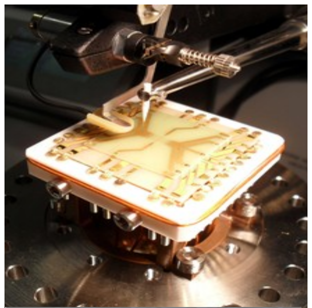
- Spacecraft with fixed orientation around perigee (g > 4.5 m/s2, h < 3000 km)
- Differential acceleration measurement (^{85}Rb – ^{87}Rb) by atom interferometry

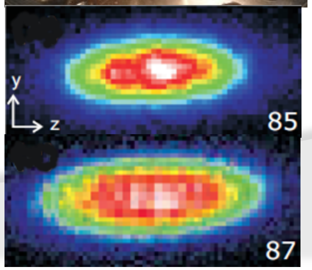
^{85}Rb-^{87}Rb Differential Atom Interferometer

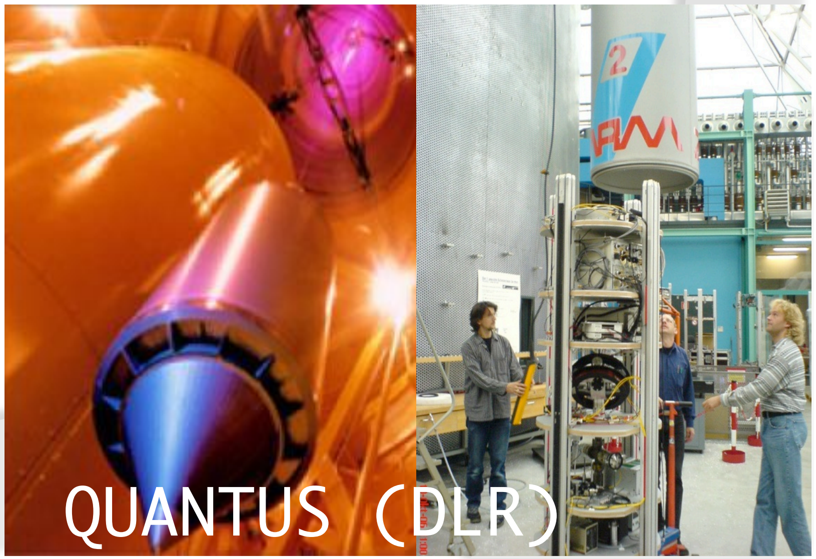
Testing the Weak Equivalence Principle:

- Differential acceleration measurement on two sample of ultra-cold ^{85}Rb and ^{87}Rb atoms;
- Long interrogation times (T~1-10 s): 2 to 3 orders of magnitude improvement with respect to ground based instruments;
- 10^7 rejection ratio of common mode acceleration noise (drag and mechanical vibration)
- Absolute sensor with precisely known scale factor;
- Atom interferometry measurements performed in a small size vacuum system: simplified control of external perturbations (magnetic, thermal, etc).









Conclusions

- Several shortcoming in standard General Relativity
- We have to test the fundamental gravitational fields. Is it g or Γ ?
- Coincidence of geodesic and causal structures strictly depends on the validity of the Equivalence Principle (Levi-Civita)
- Discrimination of gravitational theories at quantum level
- Tools: atomic clocks, free fall, space-craft (STE-QUEST)

WORK IN PROGRESS!!!