The η-η' system: mixing angle, gluonic content and contribution to $(g-2)_\mu$

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Talk based on work
in collaboration with
R. Escribano

SFB 1044
THE LOW-ENERGY FRONTIER
OF THE STANDARD MODEL

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Outline

- Notation for the mixing angle(s) and the gluonic content
- The anomalous magnetic moment of the muon
    - The Hadronic contribution:
        - PS-transition form factor
- Conclusions

- *Notation for the mixing angle:*

mixing of mass eigenstates

octet-singlet basis

quark-flavour basis

1 mixing angle

$$|\eta\rangle = \cos\theta_P|\eta_8\rangle - \sin\theta_P|\eta_0\rangle$$
$$|\eta'\rangle = \sin\theta_P|\eta_8\rangle + \cos\theta_P|\eta_0\rangle$$

$$|\eta\rangle = \cos\phi_P|\eta_q\rangle - \sin\phi_P|\eta_s\rangle$$
$$|\eta'\rangle = \sin\phi_P|\eta_q\rangle + \cos\phi_P|\eta_s\rangle$$

with
$$|\eta_8\rangle = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$
$$|\eta_0\rangle = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

and
$$|\eta_q\rangle = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$
$$|\eta_s\rangle = s\bar{s}$$

$$\theta_P = \phi_P - \arctan\sqrt{2} \simeq \phi_P - 54.7^\circ$$

Assumptions:

- no energy dependence
- $\Gamma_{\eta,\eta'} \ll m_{\eta,\eta'}$
- no mixing with other pseudoscalars (π^0, η_c, glueballs)

- *Notation for the mixing angles of the decay constants*

mixing of decay constants

octet-singlet basis

$$\langle 0|A^a_\mu|P(p)\rangle = i f^a_P p_\mu$$

with $A^a_\mu = \bar{q}\gamma_\mu\gamma_5 \frac{\lambda^a}{\sqrt{2}} q$

$f^a_P \; (a = 8, 0; P = \eta, \eta')$

2 mixing angles

$$\begin{pmatrix} f^8_\eta & f^0_\eta \\ f^8_{\eta'} & f^0_{\eta'} \end{pmatrix} = \begin{pmatrix} f_8 \cos\theta_8 & -f_0 \sin\theta_0 \\ f_8 \sin\theta_8 & f_0 \cos\theta_0 \end{pmatrix}$$

2 decay constants

quark-flavour basis

$$\langle 0|A^i_\mu|P(p)\rangle = i f^i_P p_\mu$$

with

$$A^q_\mu = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d)$$

and $A^s_\mu = \bar{s}\gamma_\mu\gamma_5 s$

$f^i_P \; (i = q, s; P = \eta, \eta')$

$$\begin{pmatrix} f^q_\eta & f^s_\eta \\ f^q_{\eta'} & f^s_{\eta'} \end{pmatrix} = \begin{pmatrix} f_q \cos\phi_q & -f_s \sin\phi_s \\ f_q \sin\phi_q & f_s \cos\phi_s \end{pmatrix}$$

- *Study of the η-η' system in the two mixing angle scheme*

η,η'→γγ decays

R. Escribano, J.-M. Frère, '05

octet-singlet basis

$$\Gamma_{\eta\to\gamma\gamma} = \frac{\alpha^2}{96\pi^3} M_\eta^3 \left(\frac{\cos\theta_0/f_8 - 2\sqrt{2}\sin\theta_8/f_0}{\cos\theta_0\cos\theta_8 + \sin\theta_0\sin\theta_8} \right)^2$$

$$\Gamma_{\eta'\to\gamma\gamma} = \frac{\alpha^2}{96\pi^3} M_{\eta'}^3 \left(\frac{\sin\theta_0/f_8 + 2\sqrt{2}\cos\theta_8/f_0}{\cos\theta_0\cos\theta_8 + \sin\theta_0\sin\theta_8} \right)^2$$

quark-flavour basis

$$\phi_q = \phi_s = \phi$$

$$\Gamma_{\eta\to\gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_\eta^3 \left(\frac{C_q\cos[\phi]}{f_q} - \frac{C_s\sin[\phi]}{f_s} \right)^2$$

$$\Gamma_{\eta'\to\gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_{\eta'}^3 \left(\frac{C_q\sin[\phi]}{f_q} + \frac{C_s\cos[\phi]}{f_s} \right)^2$$

• *Study of the η-η' system in the two mixing angle scheme*

octet-singlet basis

R. Escribano, J.-M. Frère, '05

$$f_8 = (1.51 \pm 0.05) f_\pi \ , \qquad \theta_8 = (-23.8 \pm 1.4)^\circ \ ,$$
$$f_0 = (1.29 \pm 0.04) f_\pi \ , \qquad \theta_0 = (-2.4 \pm 1.9)^\circ \ ,$$

quark-flavour basis

$$f_q = (1.09 \pm 0.03) f_\pi \ , \qquad \phi_q = (39.9 \pm 1.3)^\circ \ ,$$
$$f_s = (1.66 \pm 0.06) f_\pi \ , \qquad \phi_s = (41.4 \pm 1.4)^\circ \ ,$$

- in the octet-singlet basis a two mixing angle scheme is needed to describe experimental data in a better way;
- in the quark-flavour basis a one mixing angle description of data is enough at the current experimental accuracy.

At the present accuracy, our results satisfy the approximate relations

$$f_8 = \sqrt{1/3 f_q^2 + 2/3 f_s^2} \ , \qquad \theta_8 = \phi - \arctan(\sqrt{2} f_s / f_q) \ ,$$
$$f_0 = \sqrt{2/3 f_q^2 + 1/3 f_s^2} \ , \qquad \theta_0 = \phi - \arctan(\sqrt{2} f_q / f_s) \ .$$

• *Notation for the gluonic content: phenomenological parametrization*

We work in a basis consisting of the states

$$|\eta_q\rangle \equiv \frac{1}{\sqrt{2}}|u\bar{u}+d\bar{d}\rangle \qquad |\eta_s\rangle = |s\bar{s}\rangle \qquad |G\rangle \equiv |\mathrm{gluonium}\rangle$$

The physical states η and η' are assumed to be the linear combinations

$$\begin{aligned} |\eta\rangle &= X_\eta|\eta_q\rangle + Y_\eta|\eta_s\rangle + Z_\eta|G\rangle \ , \\ |\eta'\rangle &= X_{\eta'}|\eta_q\rangle + Y_{\eta'}|\eta_s\rangle + Z_{\eta'}|G\rangle \ , \end{aligned}$$

with $X^2_{\eta(\eta')} + Y^2_{\eta(\eta')} + Z^2_{\eta(\eta')} = 1$ and thus $X^2_{\eta(\eta')} + Y^2_{\eta(\eta')} \leq 1$

A significant gluonic admixture in a state is possible only if

$$Z^2_{\eta(\eta')} = 1 - X^2_{\eta(\eta')} - Y^2_{\eta(\eta')} > 0$$

Assumptions:

- no mixing with π^0 (isospin symmetry)
- no mixing with η_c states
- no mixing with radial excitations

- *Notation for the gluonic content*

In absence of gluonium (standard picture)

$$Z_{\eta(\eta')} \equiv 0$$

$$\begin{aligned} |\eta\rangle &= \cos\phi_P |\eta_q\rangle - \sin\phi_P |\eta_s\rangle \\ |\eta'\rangle &= \sin\phi_P |\eta_q\rangle + \cos\phi_P |\eta_s\rangle \end{aligned}$$

with $X_\eta = Y_{\eta'} \equiv \cos\phi_P$ and $X^2_{\eta(\eta')} + Y^2_{\eta(\eta')} = 1$

$$X_{\eta'} = -Y_\eta \equiv \sin\phi_P$$

where ϕ_P is the η-η' mixing angle in the quark-flavour basis related to its octet-singlet analog through

$$\theta_P = \phi_P - \arctan\sqrt{2} \simeq \phi_P - 54.7^\circ$$

Similarly, for the vector states ω and ϕ the mixing is given by

$$\begin{aligned} |\omega\rangle &= \cos\phi_V |\omega_q\rangle - \sin\phi_V |\phi_s\rangle \\ |\phi\rangle &= \sin\phi_V |\omega_q\rangle + \cos\phi_V |\phi_s\rangle \end{aligned}$$

where ω_q and ϕ_s are the analog non-strange and strange states of η_q and η_s, respectively.

- *Euler angles*

In presence of gluonium,

glueball-like state
$\eta(1405)$?

$$\begin{aligned}|\eta\rangle &= X_\eta|\eta_q\rangle + Y_\eta|\eta_s\rangle + Z_\eta|G\rangle \\ |\eta'\rangle &= X_{\eta'}|\eta_q\rangle + Y_{\eta'}|\eta_s\rangle + Z_{\eta'}|G\rangle \\ |\iota\rangle &= X_\iota|\eta_q\rangle + Y_\iota|\eta_s\rangle + Z_\iota|G\rangle\end{aligned}$$

Normalization:

$$\begin{aligned}X_\eta^2 + Y_\eta^2 + Z_\eta^2 &= 1 \\ X_{\eta'}^2 + Y_{\eta'}^2 + Z_{\eta'}^2 &= 1 \\ X_\iota^2 + Y_\iota^2 + Z_\iota^2 &= 1\end{aligned}$$

Orthogonality:

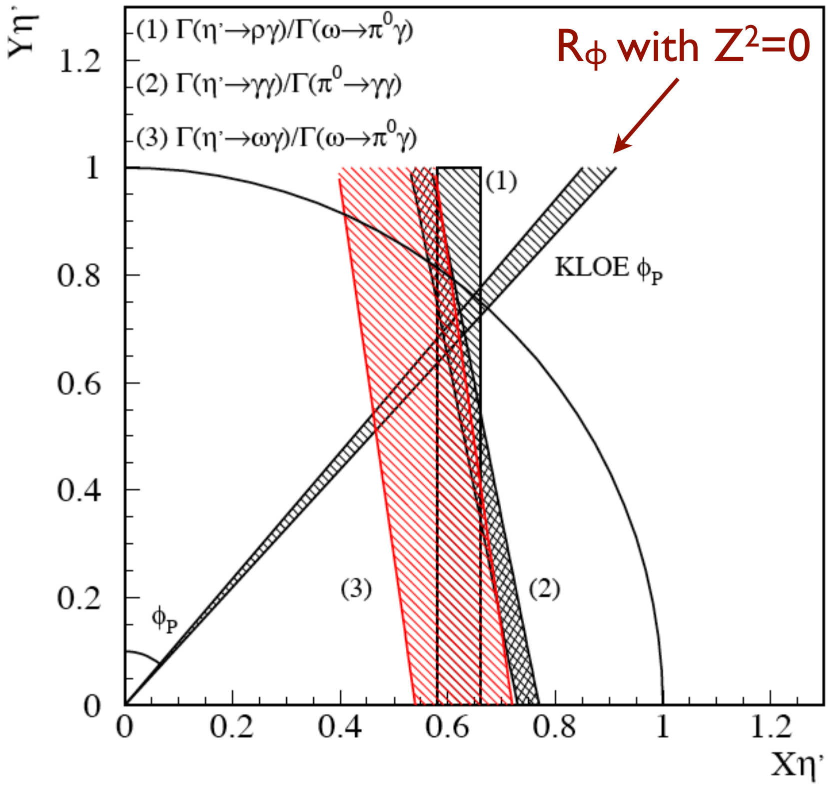
$$\begin{aligned}X_\eta X_{\eta'} + Y_\eta Y_{\eta'} + Z_\eta Z_{\eta'} &= 0 \\ X_\eta X_\iota + Y_\eta Y_\iota + Z_\eta Z_\iota &= 0 \\ X_{\eta'} X_\iota + Y_{\eta'} Y_\iota + Z_{\eta'} Z_\iota &= 0\end{aligned}$$

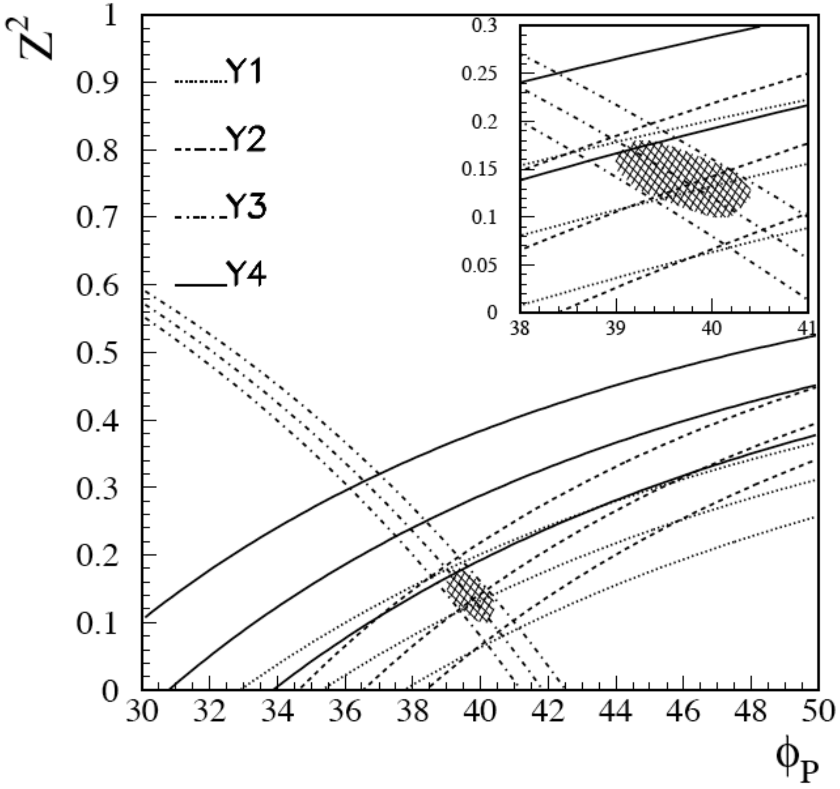
3 independent parameters: ϕ_P, $\phi_{\eta G}$ and $\phi_{\eta' G}$

$$\begin{pmatrix} \eta \\ \eta' \\ \iota \end{pmatrix} = \begin{pmatrix} c\phi_{\eta\eta'}c\phi_{\eta G} & -s\phi_{\eta\eta'}c\phi_{\eta G} & -s\phi_{\eta G} \\ s\phi_{\eta\eta'}c\phi_{\eta' G} - c\phi_{\eta\eta'}s\phi_{\eta' G}s\phi_{\eta G} & c\phi_{\eta\eta'}c\phi_{\eta' G} + s\phi_{\eta\eta'}s\phi_{\eta' G}s\phi_{\eta G} & -s\phi_{\eta' G}c\phi_{\eta G} \\ s\phi_{\eta\eta'}s\phi_{\eta' G} + c\phi_{\eta\eta'}c\phi_{\eta' G}s\phi_{\eta G} & c\phi_{\eta\eta'}s\phi_{\eta' G} - s\phi_{\eta\eta'}c\phi_{\eta' G}s\phi_{\eta G} & c\phi_{\eta' G}c\phi_{\eta G} \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \\ G \end{pmatrix}$$

- *Motivation*

KLOE Collaboration, ’07





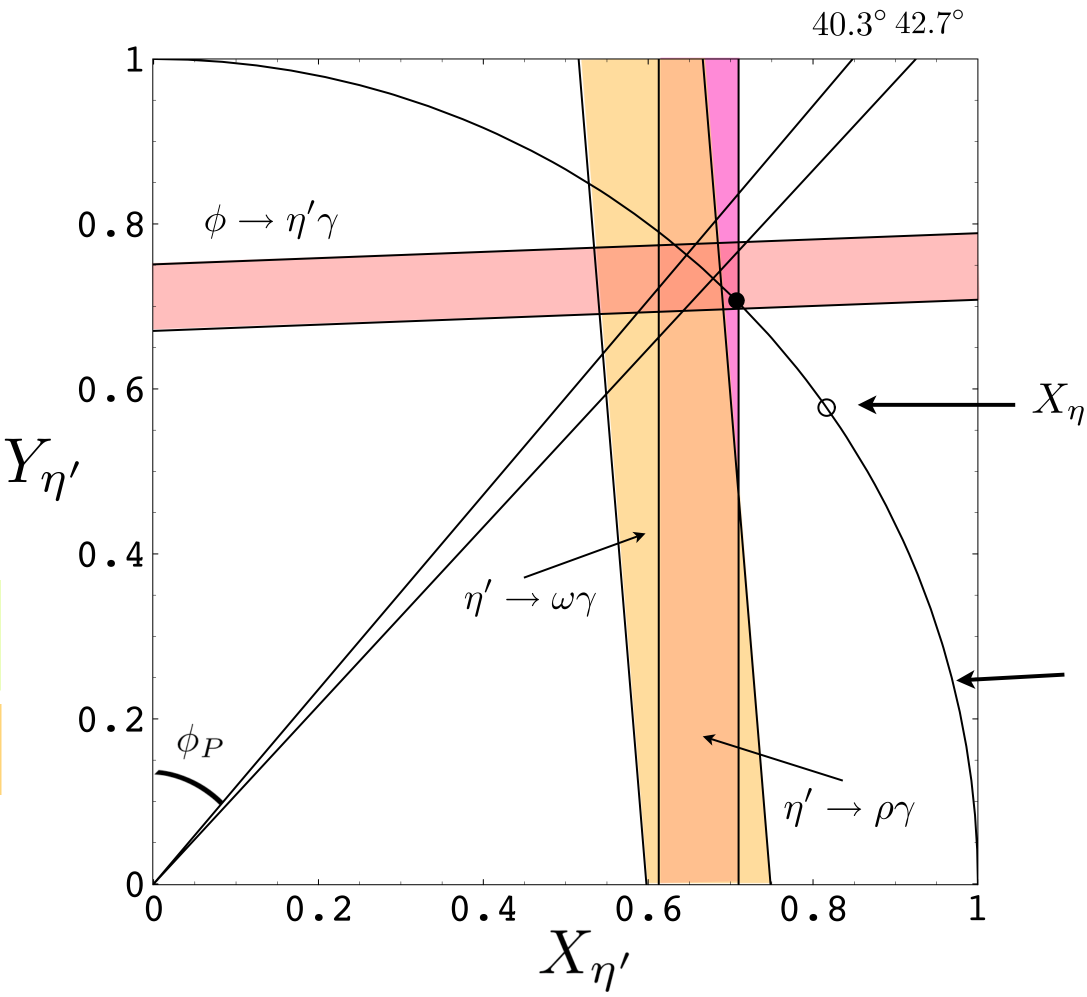
$$\phi_P = (39.7 \pm 0.7)^\circ$$

$$Z^2_{\eta'} = 0.14 \pm 0.04$$

Y1=$\eta'\to\gamma\gamma/\pi^0\to\gamma\gamma$
Y2=$\eta'\to\rho\gamma/\omega\to\pi^0\gamma$
Y3=$\phi\to\eta'\gamma/\phi\to\eta\gamma$
Y4=$\eta'\to\omega\gamma/\omega\to\pi^0\gamma$

• *Results*

R. E.scribano and J. Nadal, '07



$$X_{\eta'} = \sqrt{2} Y_{\eta'} = \frac{1}{\sqrt{3}}$$

η=η0

$$X_\eta^2 + Y_\eta^2 \leq 1$$

ϕP=(41.4±1.3)°

$$Z_{\eta'}^2 = 0.04 \pm 0.09$$

✔ importance of constraining even more ϕ→η'γ

➡ More refined data for this channel will contribute decisively to clarify this issue

The anomalous magnetic moment of the muon

Anomalous magnetic moment a_μ (anomaly):

$$g_\mu = 2\left(1 + a_\mu = \frac{\alpha}{2\pi} + \cdots\right) \qquad a_\mu^{th} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{had}$$

Contribution	Result in 10^{-10} units	
QED(leptons)	11658471.885 ± 0.004	Kinoshita *et al* 2012
HVP(leading order)	692.3 ± 4.2	Davier *et al* 2011
HVP(higher order)	-9.84 ± 0.07	Hagiwara *et al* 2009
HLBL	11.6 ± 4.0	Jegerlehner and Nyffeler 2009
EW	15.4 ± 0.2	Czarnecki *et al* 2003
Total	11659181.3 ± 5.8	

$$\boxed{a_\mu^{exp} - a_\mu^{SM} = 27.6(8.0) \times 10^{-10} \Rightarrow 3.4\sigma}$$

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New g-2 experiment at Fermilab with error

$$1.6 \times 10^{-10}$$

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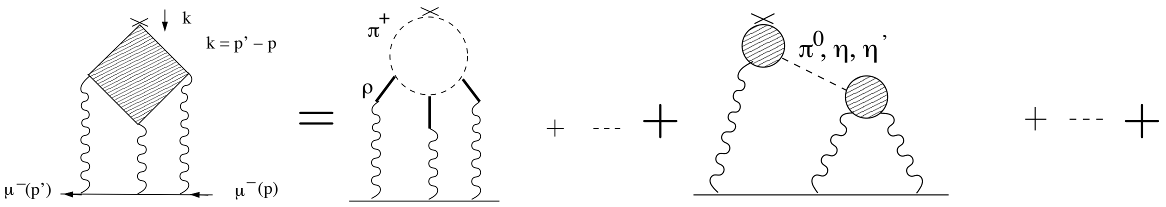
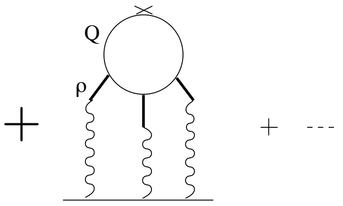
Classification proposal by Eduardo de Rafael '94

Chiral Perturbation Theory counting (p^2)+large-Nc counting



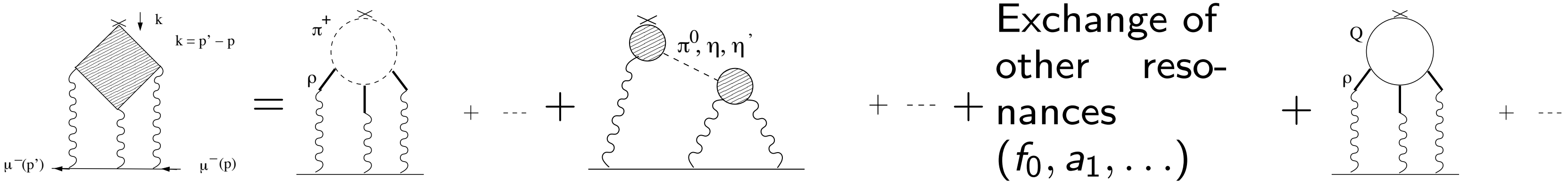
Exchange of other resonances ($f_0, a_1, \dots$)



Chiral counting:	p^4	p^6	p^8	p^8
N_C-counting:	1	N_C	N_C	N_C

Pesudoscalars: numerically dominant contribution (according to most models)

[from A. Nyffeler 2012]



Exchange of other resonances ($f_0, a_1, \ldots$)

Chiral counting:		p^4	p^6	p^8	p^8
N_C-counting:		1	N_C	N_C	N_C

Contribution to $a_\mu \times 10^{11}$:

BPP:	+83 (32)	-19 (13)	+85 (13)	-4 (3) $[f_0, a_1]$	+21 (3)
HKS:	+90 (15)	-5 (8)	+83 (6)	+1.7 (1.7) $[a_1]$	+10 (11)
KN:	+80 (40)		+83 (12)		
MV:	+136 (25)	0 (10)	+114 (10)	+22 (5) $[a_1]$	0
2007:	+110 (40)				
PdRV:	+105 (26)	-19 (19)	+114 (13)	+8 (12) $[f_0, a_1]$	+2.3 [c-quark]
N,JN:	+116 (40)	-19 (13)	+99 (16)	+15 (7) $[f_0, a_1]$	+21 (3)
GFW:	+217 (91)		+81 (12)		+136 (59)
GdR:	+150 (3)		+68 (3)		+82 (6)
		ud.: -45	ud.: $+\infty$		ud.: +60

ud. = undressed, i.e. point vertices without form factors

BPP = Bijnens, Pallante, Prades '96, '02; HKS = Hayakawa, Kinoshita, Sanda '96, '98, '02; KN = Knecht, Nyffeler '02; MV = Melnikov, Vainshtein '04; 2007 = Bijnens, Prades; Miller, de Rafael, Roberts; PdRV = Prades, de Rafael, Vainshtein '09; N,JN = Nyffeler '09; Jegerlehner, Nyffeler '09; GFW = Goecke, Fischer, Williams '11 (total includes estimate of "other contributions" = 0 (20)); GdR = Greynat, de Rafael '12 (given error only reflects variation $M_Q = 240 \pm 10$ MeV, estimated 20%-30% systematic error)

Recall (in units of 10^{-11}): $\delta a_\mu(\text{had. VP}) \approx 45$; $\delta a_\mu(\text{exp [BNL]}) = 63$; $\delta a_\mu(\text{future exp}) = 15$

Dissection of the HLBL contribution

In the large N_c and chiral limits:

$$a^{\mathrm{HLbL}}(\pi^0) = \left(\frac{\alpha}{\pi}\right)^3 N_c \, \frac{m_\mu^2 N_c}{48\pi^2 F_\pi^2}\left[\ln^2\frac{m_\rho}{m_\pi} + \mathcal{O}\left(\ln\frac{m_\rho}{m_\pi}\right) + \mathcal{O}(1)\right]$$

for η and η':

$$\frac{1}{\tilde{f}_\eta^{\mathrm{eff}}} \equiv \frac{1}{C_\pi}\left[C_q\frac{\cos\phi}{f_q} - C_s\frac{\sin\phi}{f_s}\right]$$

$$\frac{1}{\tilde{f}_{\eta'}^{\mathrm{eff}}} \equiv \frac{1}{C_\pi}\left[C_q\frac{\sin\phi}{f_q} + C_s\frac{\cos\phi}{f_s}\right]$$

Ballpark contributions from PS:

$$\pi^0 \;\rightarrow\; \sim 7\cdot 10^{-10}$$

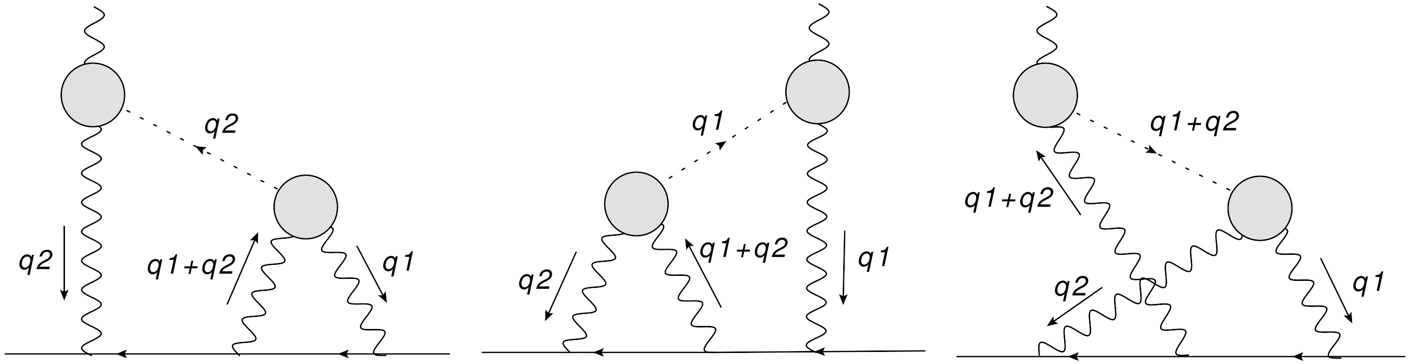
$$\eta \;\rightarrow\; \sim 1.5\cdot 10^{-10}$$

$$\eta' \;\rightarrow\; \sim 1.5\cdot 10^{-10}$$

New g-2 experiment at Fermilab with error

$$\sim 1.6\cdot 10^{-10}$$

Dissection of the HLBL contribution



$$a_\mu^{LbL;P} = -e^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (q_1+q_2)^2 [(p+q_1)^2 - m^2][(p-q_2)^2 - m^2]}$$

$$\times \left(\frac{F_{P^*\gamma^*\gamma^*}(q_2^2, q_1^2, (q_1+q_2)^2) F_{P^*\gamma^*\gamma^*}(q_2^2, q_2^2, 0)}{q_2^2 - M_P^2} T_1(q_1, q_2; p) \right.$$

Use data from
the Transition Form Factor

$$\left. + \frac{F_{P^*\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2) F_{P^*\gamma^*\gamma^*}((q_1+q_2)^2, (q_1+q_2)^2, 0)}{(q_1+q_2)^2 - M_P^2} T_2(q_1, q_2; p) \right)$$

Dissection of the HLBL contribution

Use data from
the Transition Form Factor
for numerical integrals

$$F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)$$

Dissection of the HLBL contribution

~~Use data from
the Transition Form Factor
for numerical integral~~

$$\cancel{F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)}$$

$$\Downarrow$$

$$F_{P\gamma^*\gamma^*}(m_P^2, q_1^2, q_2^2)$$

double-tag method

Dissection of the HLBL contribution

~~Use data from~~
~~the Transition Form Factor~~
~~for numerical integral~~

$$\cancel{F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)}$$

$$\Downarrow$$

$$\cancel{F_{P\gamma^*\gamma^*}(m_P^2, q_1^2, q_2^2)}$$

$$\Downarrow$$

Use data from
the Transition Form Factor
to constrain your
hadronic model

$$\boxed{F_{P\gamma^*\gamma}(m_P^2, q_1^2, 0)}$$

single-tag method

Dissection of the HLBL contribution

~~Use data from
the Transition Form Factor
for numerical integral~~

$$F_{P^*\gamma^*\gamma^*}\cancel{(q_3^2, q_1^2, q_2^2)}$$

$$\Downarrow$$

$$F_{P\gamma^*\gamma^*}\cancel{(m_P^2, q_1^2, q_2^2)}$$

$$\Downarrow$$

Use data from
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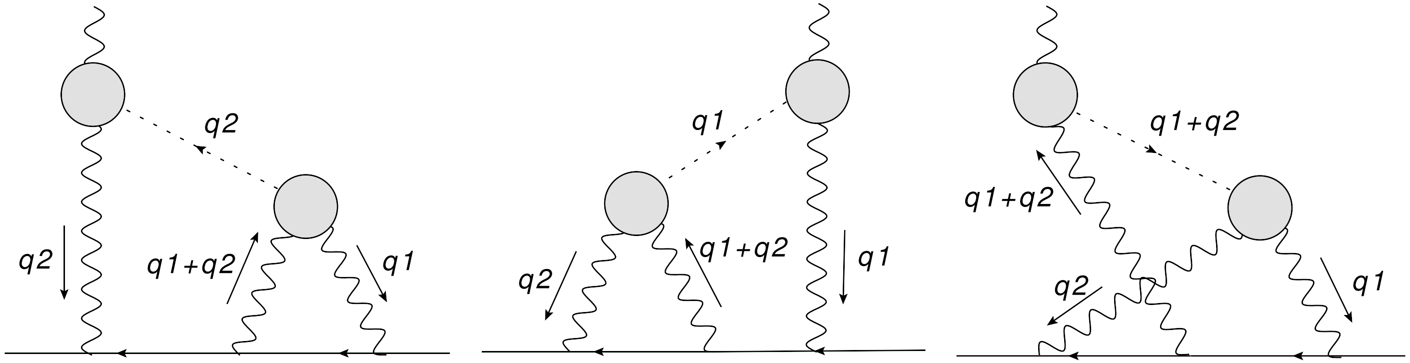
$$\boxed{F_{P\gamma^*\gamma}(m_P^2, q_1^2, 0)}$$

How??

Nice synergy between experiment and theory

Dissection of the HLBL contribution



its calculation requires info. on the pseudoscalar form factors

$$F_{\eta^{(\prime)}\gamma*\gamma}(Q^2,0) = a_0^\eta\left(1 + a_\eta\frac{Q^2}{m_\eta^2} + b_\eta\frac{Q^4}{m_\eta^4} + \ldots\right)$$

$\Gamma_{\eta\to\gamma\gamma}$ slope curvature

Constrain Hadronic Models

P.M. '12

P.M., Vanderhaeghen'12

Our proposal
use Padé Approximants

R. Escribano, P.M., P. Sanchez-Puertas, soon

$$F_{\eta^{(\prime)}\gamma*\gamma}(Q^2,0) = a_0^\eta\left(1 + a_\eta\frac{Q^2}{m_\eta^2} + b_\eta\frac{Q^4}{m_\eta^4} + \dots\right)$$

$\Gamma_{\eta\to\gamma\gamma}$ slope curvature

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$$F_{\eta^{(\prime)}\gamma*\gamma}(Q^2,0) = \underset{\underset{\Gamma_{\eta\to\gamma\gamma}}{\nearrow}}{a_0^\eta}\left(1 + \underset{\underset{\text{slope}}{\uparrow}}{a_\eta}\frac{Q^2}{m_\eta^2} + \underset{\underset{\text{curvature}}{\uparrow}}{b_\eta}\frac{Q^4}{m_\eta^4} + \ldots\right)$$

We have published space-like data for $Q^2F_{\eta^{(\prime)}\gamma*\gamma}(Q^2,0)$

$$Q^2F_{\eta^{(\prime)}\gamma*\gamma}(Q^2,0) = a_0Q^2 + a_1Q^4 + a_2Q^6 + \ldots$$

$$P_M^N(Q^2) = \frac{T_N(Q^2)}{R_M(Q^2)} = a_0Q^2 + a_1Q^4 + a_2 + Q^6 + \cdots + \mathcal{O}((Q^2)^{N+M+1})$$

Our proposal
use Padé Approximants

R. Escribano, P.M., P. Sanchez-Puertas, soon

$$F_{\eta^{(\prime)}\gamma*\gamma}(Q^2,0) = \underset{\underset{\Gamma_{\eta\to\gamma\gamma}}{\nearrow}}{a_0^\eta}\left(1 + \underset{\underset{\text{slope}}{\uparrow}}{a_\eta}\frac{Q^2}{m_\eta^2} + \underset{\underset{\text{curvature}}{\uparrow}}{b_\eta}\frac{Q^4}{m_\eta^4} + \ldots\right)$$

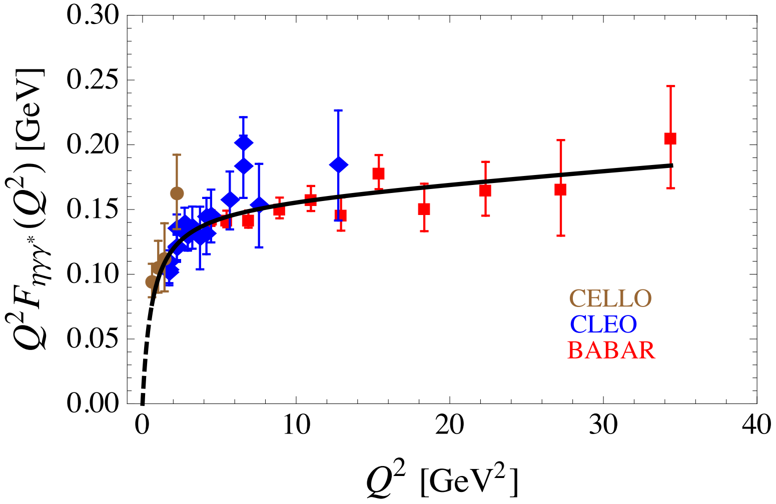
We have published space-like data for $Q^2 F_{\eta^{(\prime)}\gamma*\gamma}(Q^2,0)$

$$Q^2 F_{\eta^{(\prime)}\gamma*\gamma}(Q^2,0) = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \ldots$$

$$P_1^1(Q^2) = \frac{a_0 Q^2}{1 - a_1 Q^2} \longrightarrow \begin{array}{l} P_1^N(Q^2) = P_1^1(Q^2), P_1^2(Q^2), P_1^3(Q^2), \ldots \\ P_N^N(Q^2) = P_1^1(Q^2), P_2^2(Q^2), P_3^3(Q^2), \ldots \end{array}$$

η-TFF

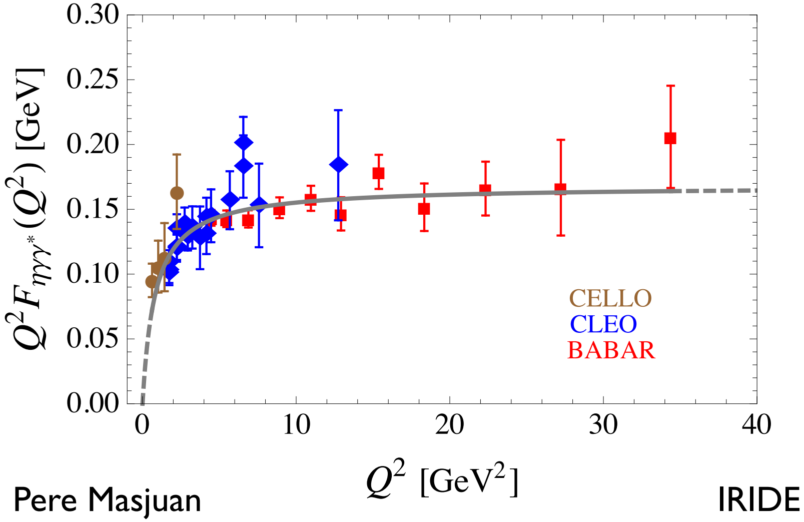
Fit to Space-like data: CELLO'91, CLEO'98, BABAR'11



$P_1^N(Q^2)$ up to N=2

$$\Gamma^{pred}_{\eta\to\gamma\gamma} = (0.41 \pm 0.18)keV$$

$$\Gamma^{PDG}_{\eta\to\gamma\gamma} = (0.51 \pm 0.03)keV$$

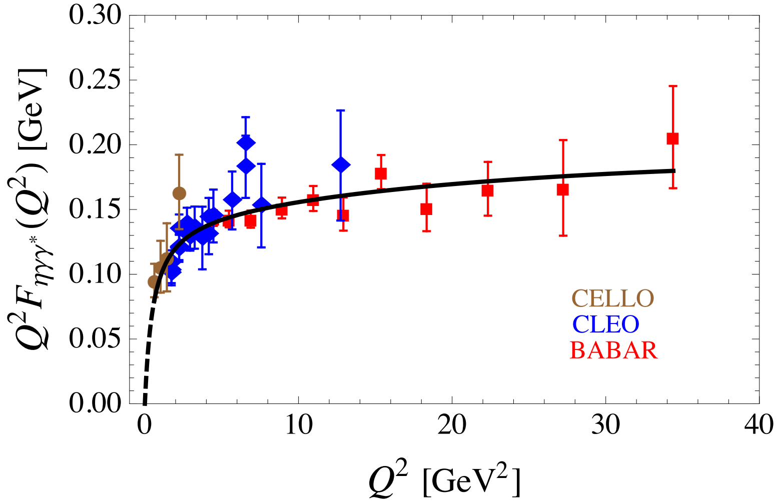


$P_N^N(Q^2)$ up to N=1

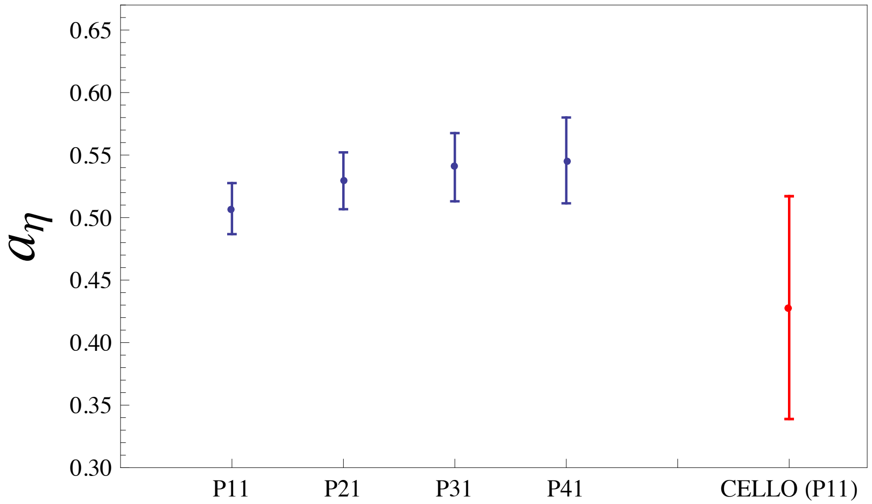
$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma*\gamma}(Q^2,0) = 0.17(6)GeV$$

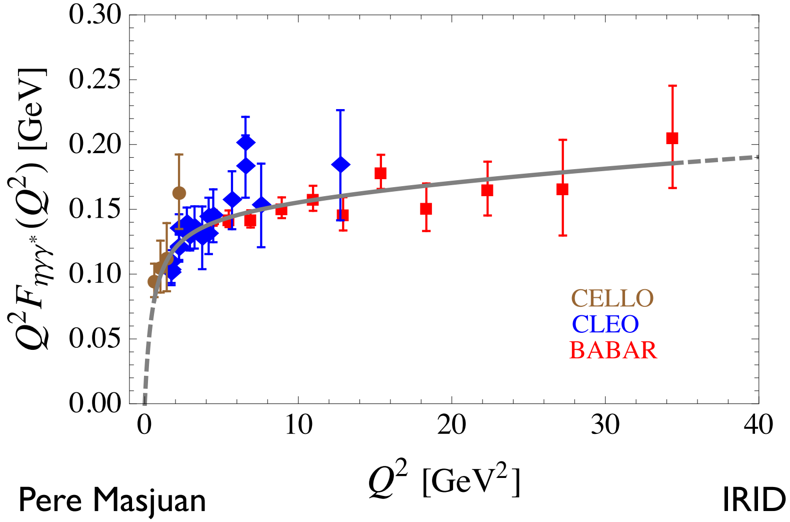
η-TFF

Fit to Space-like data: CELLO'91, CLEO'98, BABAR'11+ $\Gamma_{\eta\to\gamma\gamma}$



$P_1^N(Q^2)$ up to N=4



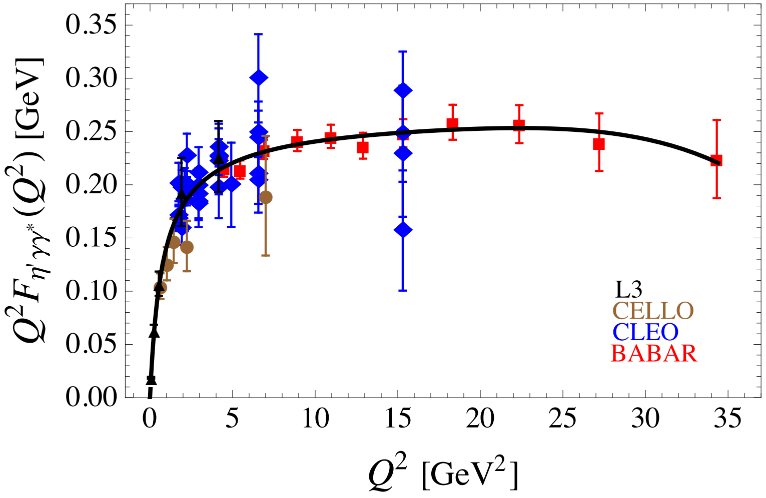


$P_N^N(Q^2)$ up to N=2

$$\lim_{Q^2\to\infty} Q^2F_{\eta\gamma*\gamma}(Q^2,0) = 0.164(2)GeV$$

η'-TFF

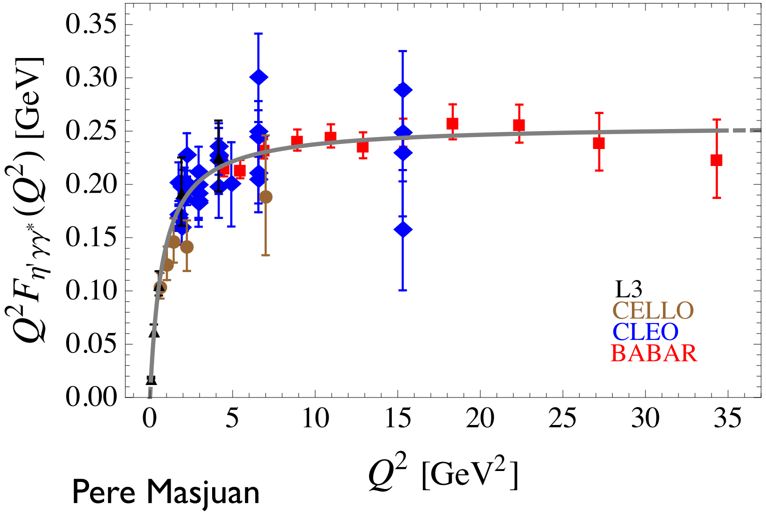
Fit to Space-like data: CELLO'91, CLEO'98, L3'98, BABAR'11



$P_1^N(Q^2)$ up to N=5

$$\Gamma^{pred}_{\eta' \to \gamma\gamma} = (4.21 \pm 0.43) keV$$

$$\Gamma^{PDG}_{\eta' \to \gamma\gamma} = (4.34 \pm 0.14) keV$$

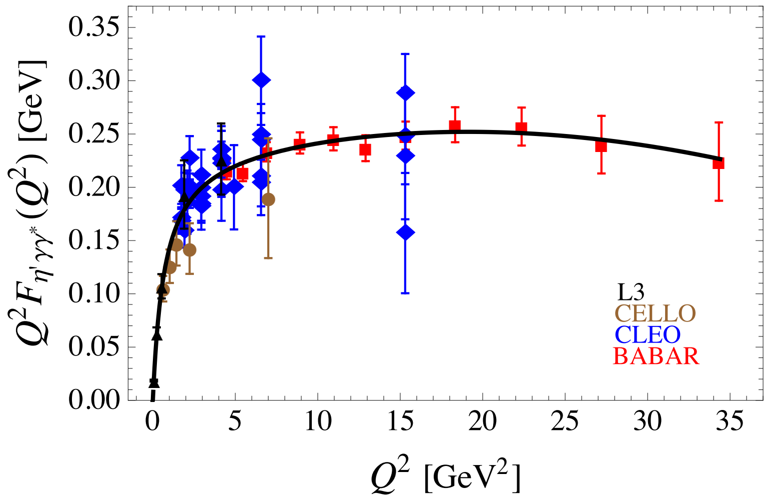


$P_N^N(Q^2)$ up to N=1

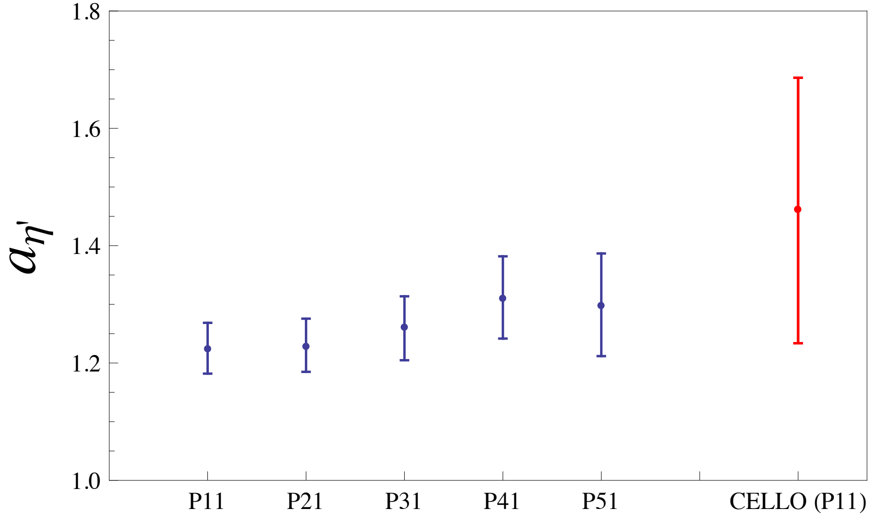
$$\lim_{Q^2 \to \infty} Q^2 F_{\eta' \gamma * \gamma}(Q^2, 0) = 0.256(4) GeV$$

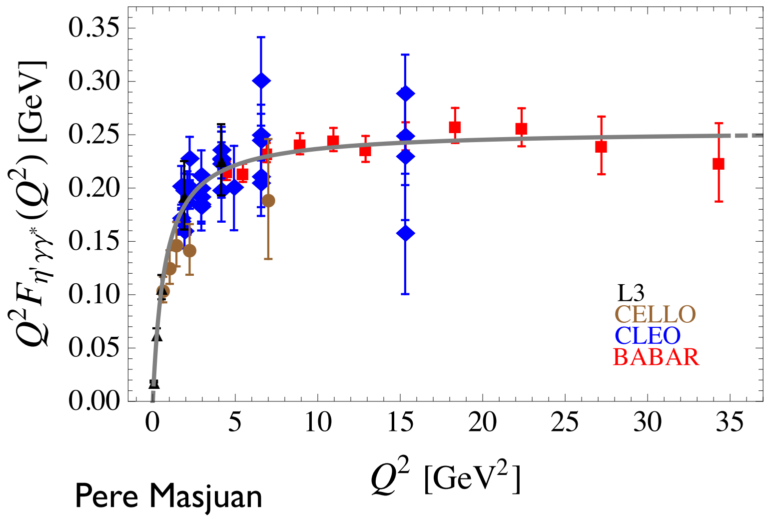
η'-TFF

Fit to Space-like data: CELLO'91, CLEO'98, L3'98, BABAR'11+ $\Gamma_{\eta'\to\gamma\gamma}$



$P_1^N(Q^2)$ up to N=5





$P_N^N(Q^2)$ up to N=1

$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma*\gamma}(Q^2,0) = 0.254(4)GeV$$

η-η’ mixing

η-η’ mixing in the flavor basis

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = \begin{pmatrix} f_q \cos[\phi] & -f_s \sin[\phi] \\ f_q \sin[\phi] & f_s \cos[\phi] \end{pmatrix}$$

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From the TFFs we can determine f_q, f_s, ϕ

$$\Gamma_{\eta\to\gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_\eta^3 \left(\frac{C_q \cos[\phi]}{f_q} - \frac{C_s \sin[\phi]}{f_s} \right)^2 \qquad \lim_{Q^2\to\infty} Q^2 F_{\eta\gamma\gamma^*}(Q^2) = f_\eta^q \frac{10}{3} + f_\eta^s \frac{2\sqrt{2}}{3} \,,$$

$$\Gamma_{\eta'\to\gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_{\eta'}^3 \left(\frac{C_q \sin[\phi]}{f_q} + \frac{C_s \cos[\phi]}{f_s} \right)^2 \qquad \lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma\gamma^*}(Q^2) = f_{\eta'}^q \frac{10}{3} + f_{\eta'}^s \frac{2\sqrt{2}}{3} \,.$$

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$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma\gamma^*}(Q^2) = f_{\eta'}^q \frac{10}{3} + f_{\eta'}^s \frac{2\sqrt{2}}{3} .$$

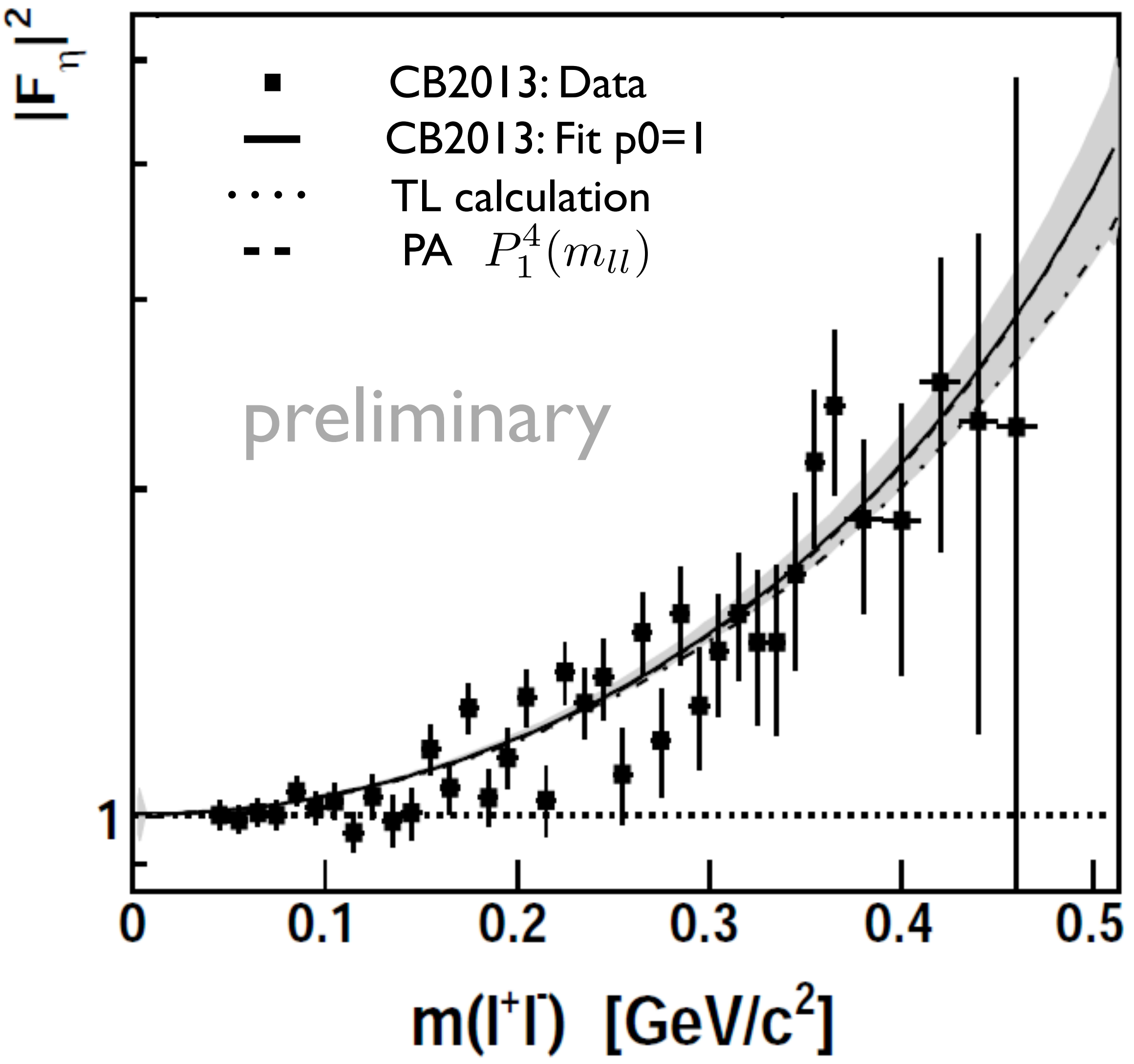
$$f_q = 1.065(13) f_\pi , \quad f_s = 1.53(22) f_\pi , \quad \phi = 40.2(1.5)^\circ$$

Update of Frere-Escribano '05 with PDG12 using 9 inputs

$$f_q = 1.07(1) f_\pi , \quad f_s = 1.63(2) f_\pi , \quad \phi = 40.4(0.3)^\circ$$

Time-like TFF: prediction

$\eta \to e^+e^-\gamma$



Courtesy of
M. Unverzagt

Conclusions

- A precise measurement of $\Gamma_{\eta^{(\prime)}\to\gamma\gamma}$ is crucial for:
  - A precise extraction of eta-eta' mixing
  - A precise constraint for the gluonic content on eta(')
  - Together with theory (nice synergy):
    - Accurate extraction of slope and curvature of FF
- The same applies to $\Gamma_{\pi^0\to\gamma\gamma}$ (see S. Ivashyn)
- And to

$$\pi(1300)\to\gamma\gamma,\quad \eta(1295)\to\gamma\gamma$$

$$\eta(1405)\to\gamma\gamma,\quad \eta(1475)\to\gamma\gamma$$

Conclusions

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Thank you!