

Open quantum systems

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Consiglio di Sezione INFN - IS GE41

- 1 General infos on IS GE41
- 2 Open quantum systems
- 3 Collaborations and publications

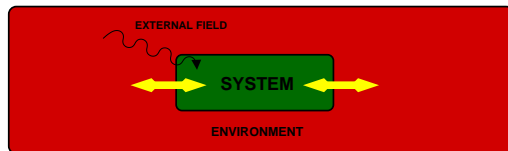
- **Titolo:**
Fundamental Problems in Quantum Physics
- **Sezioni INFN coinvolte:**
Lecce-Cosenza, Milano, Pavia, Trieste, Genova
- **Membri unità di Milano:**
Barchielli Alberto (PO, Polimi)
Gregoratti Matteo (RU, Polimi)
Smirne Andrea
(Assegnista Unimi PRIN 2008
→ Assegnista Units FP7 FET-OPEN *NANOQUESTFIT*)
Toigo Alessandro (RTD, Polimi)
Vacchini Bassano (RU Unimi)
- **Progetti collegati:**
COST Action MP1006:
Fundamental Problems in Quantum Physics
Futuro in ricerca 2010:
Semigrupperi Quantistici Markoviani e la loro stima empirica

Outline

- 1 General infos on IS GE41
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Open quantum systems

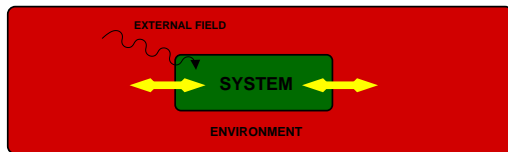
- Quantum system interacting with environment
Quantum optics, condensed matter theory, physical chemistry, quantum measurement, decoherence. . .



- Physical motivations
 - Perfect isolation of a quantum system is not possible
 - Measurement theory
 - Quantum-to-classical transition and decoherence
 - Quantum information
- Mathematical aspects
 - Irreversible dynamical maps
 - Positive operator-valued measures and instruments
 - Insights from classical probability theory

Open quantum systems

- Quantum system interacting with environment



- Bipartite setting** $H \in \mathcal{B}(\mathcal{H}_S \otimes \mathcal{H}_E)$ $\rho_{SE} \in \mathcal{T}(\mathcal{H}_S \otimes \mathcal{H}_E)$
System observables only determined by $\rho_S(t) = \text{Tr}_E \rho_{SE}(t)$

$$\frac{d}{dt} \rho_S(t) = -\frac{i}{\hbar} \text{Tr}_E [H, \rho(t)] \quad \rho_S(0) = \text{Tr}_E \rho_{SE}(0)$$

- Role of correlations**
 - Initial correlations $\rho_{SE}(0) \neq \rho_S(0) \otimes \rho_E(0)$
do not allow closed physical reduced dynamics
 - Time evolved correlations $\rho_{SE}(t) \neq \text{Tr}_E \rho_{SE}(t) \otimes \text{Tr}_S \rho_{SE}(t)$
allow for memory effects and non-Markovianity

Quantum dynamical map

- Reduced dynamics

$$\begin{array}{ccc} \rho(0) = \rho_S(0) \otimes \rho_E & \xrightarrow{\text{unitary evolution}} & \rho(t) = e^{-\frac{i}{\hbar}Ht}(\rho_S(0) \otimes \rho_E)e^{+\frac{i}{\hbar}Ht} \\ \text{Tr}_E \downarrow & & \downarrow \text{Tr}_E \\ \rho_S(0) & \xrightarrow{\text{dynamical map}} & \rho_S(t) = \Phi(t)\rho_S(0) \end{array}$$

- Quantum dynamical map

$$\rho_S(0) \mapsto \rho_S(t) = \Phi(t)\rho_S(0) = \text{Tr}_E(e^{-\frac{i}{\hbar}Ht}(\rho_S(0) \otimes \rho_E)e^{+\frac{i}{\hbar}Ht})$$

Completely positive trace preserving map

Dynamical semigroup evolution

- Markov condition
Separation of system environment time scales
- Semigroup property

$$V(t)V(s) = V(t+s) \quad t, s \geq 0 \quad V(t) = \exp(\mathcal{L}t)$$

- Quantum dynamical semigroups
Quantum Markov process

$$\frac{d}{dt}\rho_s(t) = \mathcal{L}\rho_s(t)$$

$$\mathcal{L}\rho_s(t) = -\frac{i}{\hbar}[H_s + H_{\text{eff}}, \rho_s(t)] + \sum_{\alpha} \gamma_{\alpha} \left[A_{\alpha} \rho_s(t) A_{\alpha}^{\dagger} - \frac{1}{2} \{ A_{\alpha}^{\dagger} A_{\alpha}, \rho_s(t) \} \right]$$

Master equation in Lindblad form

General non-Markovian evolution

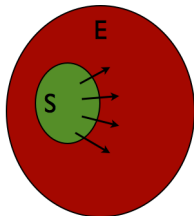
- **Memory kernel** determined by
 - $\mathcal{E} \rightarrow$ completely positive trace preserving map
 - $\mathcal{F}(t) \rightarrow$ collection of time dependent completely positive trace preserving maps
 - $f(t) \rightarrow$ waiting time distribution of renewal process
- **Master equation**

$$\begin{aligned} \frac{d}{dt}\rho(t) = & \int_0^t d\tau \frac{d}{d(t-\tau)} [f(t-\tau)\mathcal{F}(t-\tau)]\mathcal{E}\rho(\tau) \\ & + f(0)\mathcal{E}\rho(t) + \frac{d}{dt} [g(t)\mathcal{F}(t)]\rho(0) \end{aligned}$$

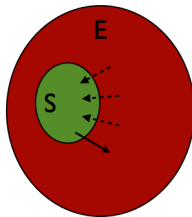
Approaches to obtain general evolution equations

Notion of non-Markovianity

- Non-Markovianity from directionality of information flow

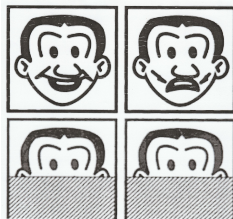


Info flow



Info backflow

- **Markovian dynamics:**
system irreversibly loses info towards the environment
- **Non-Markovian dynamics:**
system partially recovers info from the environment
- **Distinguishability** of different time evolved states as quantifier of info flow



Distinguishability

- Distinguishability can be measured through the trace distance between system states ρ_1 and ρ_2

$$D(\rho_1, \rho_2) = \frac{1}{2} \|\rho_1 - \rho_2\|_1 = \frac{1}{2} \text{Tr} |\rho_1 - \rho_2|$$

- Quantum dynamical semigroups monotonic contractions

$$\begin{aligned} \Phi(s)\Phi(t) &= \Phi(t+s) \quad \forall t, s \geq 0 \\ D(\rho_1(t+s), \rho_2(t+s)) &\leq D(\rho_1(t), \rho_2(t)) \end{aligned}$$

- Non-Markovian quantum dynamics if

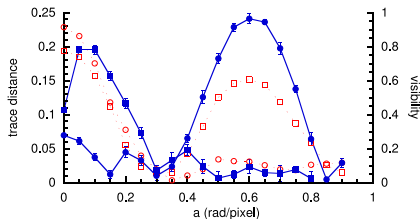
$$\frac{d}{dt} D(\rho_1(t), \rho_2(t)) > 0$$

for some pair of initial states $\rho_{1,2}(0)$ and some time $t > 0$

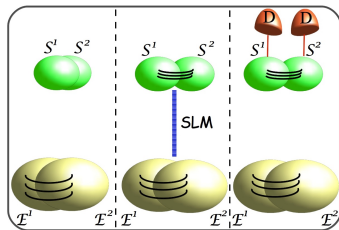
Distinguishability via trace distance

Experimental results

- Bipartite system in quantum optical setting
 - system \rightarrow polarization degrees of freedom
 - environment \rightarrow frequency degrees of freedom



Detection of initial correlations



System as environmental probe

Experimental accessibility of trace distance information

Outline

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Collaborazioni:

- S. Attal: Université Claude Bernard Lyon 1, Francia
- H.-P. Breuer: University of Freiburg, Germania
- G. Chiribella: Tsinghua University, Cina
- T. Heinosaari, E.-M. Laine, J. Piilo: University of Turku, Finlandia
- K. Hornberger: University of Duisburg-Essen, Germany
- L. Mazzola, M. Paternostro: Queen's University, Ireland
- C. Pellegrini: Université Paul Sabatier, Francia
- F. Petruccione: University of KwaZulu-Natal, Sudafrica
- C. Carmeli, V. Umanità: Università di Genova
- S. Cialdi, M. Paris, R. Martinazzo: Unimi

Selezione pubblicazioni recenti (2008-2013): [Open quantum systems]

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Phys. Rev. A vol. 87, 052129 (2013)
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New J. Phys. vol. 14, 113034 (2012)
- A. Smirne, D. Brivio, S. Cialdi, B. Vacchini, M. G. A. Paris
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- B. Vacchini, A. Smirne, E.-M. Laine, J. Piilo, H.-P. Breuer
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- R. Martinazzo, B. Vacchini, K. H. Hughes and I. Burghardt
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- B. Vacchini and K. Hornberger
Phys. Rep. vol. 478, pp. 71-120 (2009)
- H.-P. Breuer and B. Vacchini
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- B. Vacchini
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