

# X-ray spectroscopy of heavy kaonic atoms

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SMI, Vienna

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# Our recent focus on kaonic atoms

- Precise measurement of kaonic hydrogen 1s state (done by SIDDHARTA)
- Shifts and widths of kaonic  $^3,^4\text{He}$  2p states (done by E570, SIDDHARTA, to be done by J-PARC E17)
- First observation of kaonic deuterium 1s state (to be done by SIDDHARTA-2)

**Targets with  $Z=1$  and 2 have been used recently.**

**Targets with  $Z>2$  are FULLY understood? No interest?**

Heavy kaonic atoms

-- Again, kaonic atom puzzle!!--

# Heavy kaonic atoms

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## Puzzle of kaonic atoms with $Z > 2$

### THE KAONIC ATOMS 'PUZZLE': WHAT NEXT?

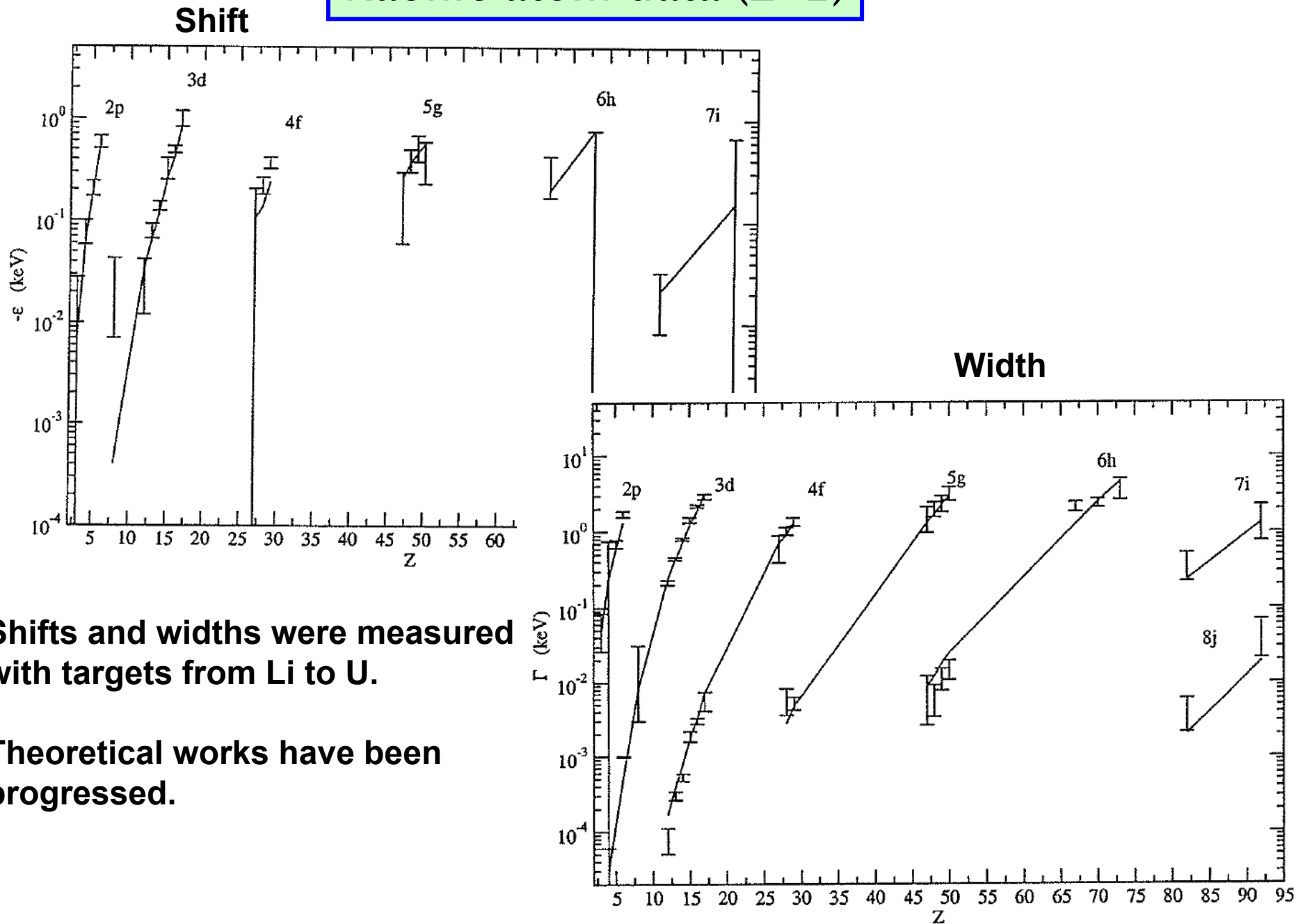
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## Messages to Experimentalists

See also, Friedman, Gal: arXiv:1108.2156v1, updated version of Proc. INPC2010

# Kaonic atom data ( $Z > 2$ )



Shifts and widths were measured with targets from Li to U.

Theoretical works have been progressed.

# Kaonic hydrogen puzzle

## Deser Formula

$$\varepsilon + i\frac{1}{2}\Gamma = 2\alpha^3 \mu^2 a_{K^-p}$$

$$a(K^-p) = \frac{1}{2}(a_0 + a_1)$$

Scattering data (by Martin) gave

$$a_0 = -1.70 + i0.68 \text{ fm}$$

$$a_1 = +0.37 + i0.60 \text{ fm}$$

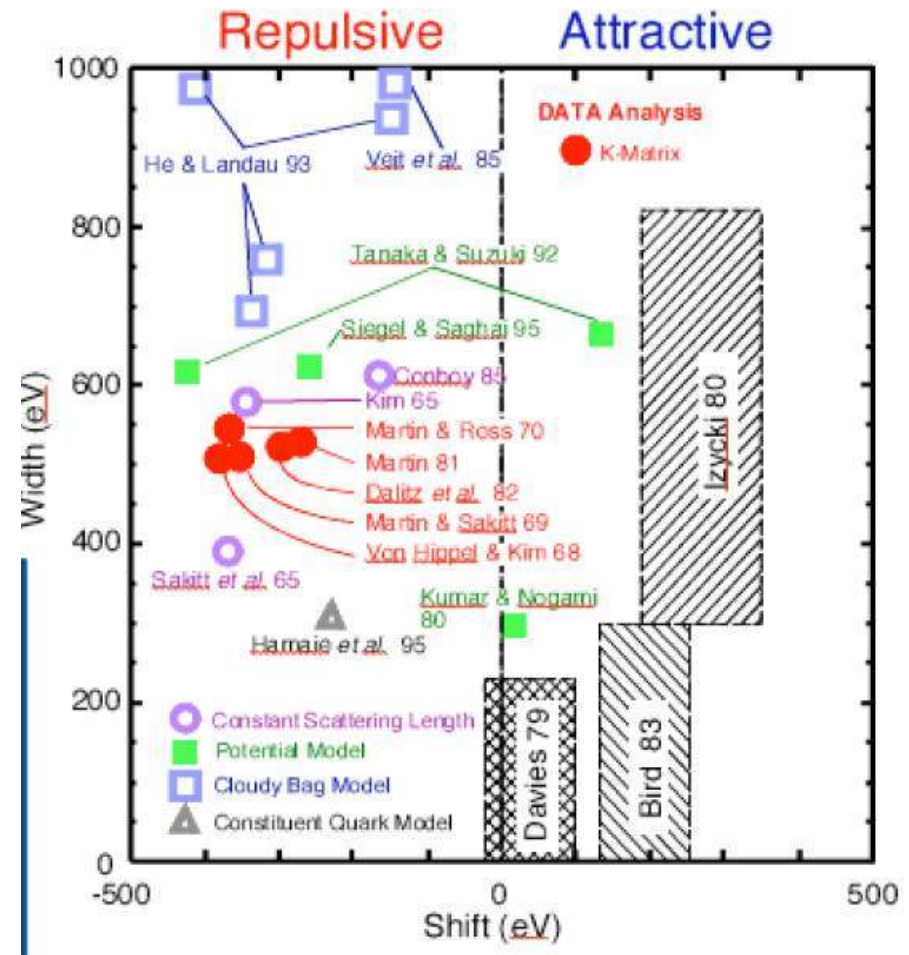


$$a(K^-p) = -0.66 + i0.64 \text{ fm}$$

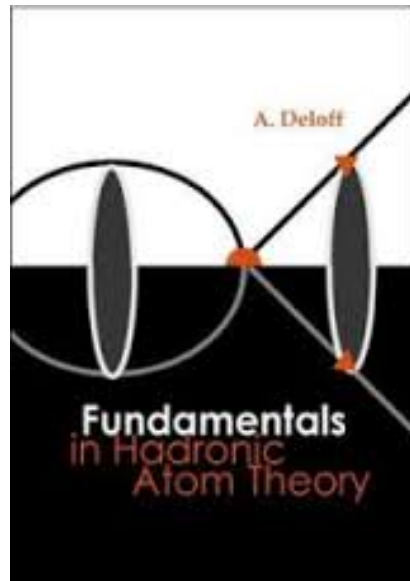
**Negative shift!**

**Experiments: positive shift!**

**KpX, DEAR, SIDDHARTA measured negative shifts, solving the puzzle...**



## Kaonic atom data ( $Z > 2$ )



Text book of exotic atoms  
by A. Deloff (2003)

**Early calculations revealed two facts:**

- A simple optical potential proportional to the nuclear density reproduces quite well the data
- The best fit depth of the optical potential violates the low-density limit.

**A simple potential:**

$$2\mu V(r) = -4\pi A \left(1 + \frac{\mu}{M}\right) b_0 \rho(r)$$

**Solving Klein-Gordon eq. (Schroedinger eq.),  
Energy shifts and widths can be determined.**

**Compare with experimental data**

$$\varepsilon, \Gamma \Leftrightarrow b_0$$

**A simple potential:**  $2\mu V(r) = -4\pi A(1 + \frac{\mu}{M})b_0\rho(r)$

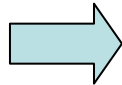
$$b_0 = \frac{1}{2}[a(K^- p) + a(K^- n)]$$

$$a(K^- p) = \frac{1}{2}(a_0 + a_1), \quad a(K^- n) = a_1$$

**Scattering data (by Martin) gave**

$$a_0 = -1.70 + i0.68 \text{ fm}$$

$$a_1 = +0.37 + i0.60 \text{ fm}$$



$$b_0 = -0.15 + i0.62 \text{ fm}$$

**Negative b0**

**A fit to kaonic atom data shows a big departure from the above free-space value**

$$b_0 = (0.35 \pm 0.03) + i(0.82 \pm 0.03) \text{ fm}$$

Earlier work by Batty

**Positive b0**

$$b_0 = (0.62 \pm 0.05) + i(0.92 \pm 0.05) \text{ fm}$$

Recent work using refined potential

**Interesting sign inversion!! See e.g. Sec.4 of Phys. Rep. 287 (97) 385**



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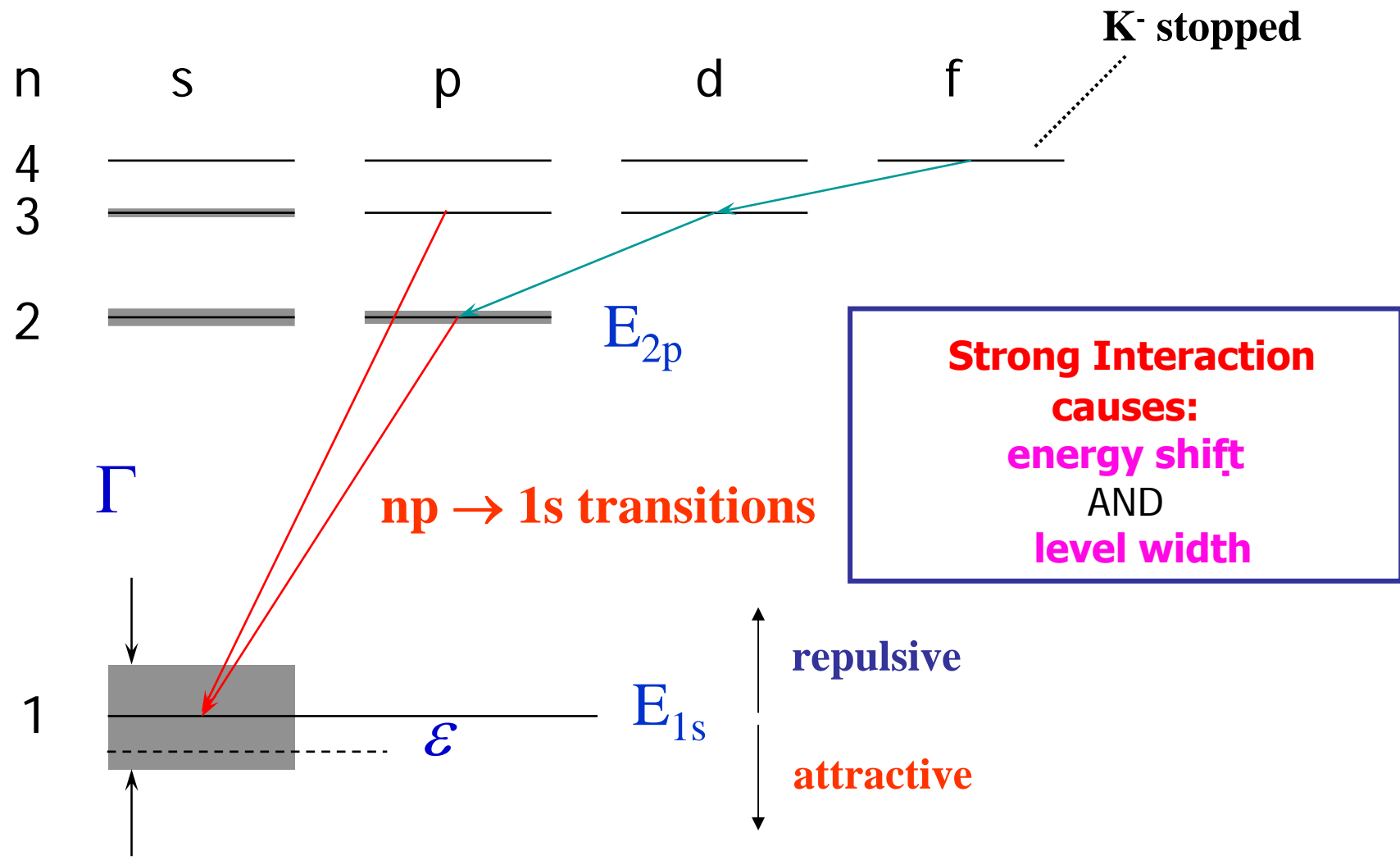
See also, Friedman, Gal: arXiv:1108.2156v1, updated version of Proc. INPC2010

## Different scenarios for different exotic atoms

| particle   | real potl.                                 | imaginary potl. | comments                          |
|------------|--|-----------------|-----------------------------------|
| $\pi^-$    | repulsive in bulk<br>attractive on surface | moderate        | excellent data<br>well understood |
| $K^-$      | attractive<br>deep or shallow?             | moderate        | good data<br>open problems        |
| $\bar{p}$  | model dependent                            | very absorptive | excellent data<br>understood      |
| $\Sigma^-$ | repulsive in bulk<br>attractive outside    | moderate        | limited data<br>poorly understood |

**Data of kaonic atoms = “good” data sets,  
but questions for potential depths—deep or shallow?**

# Kaonic atoms

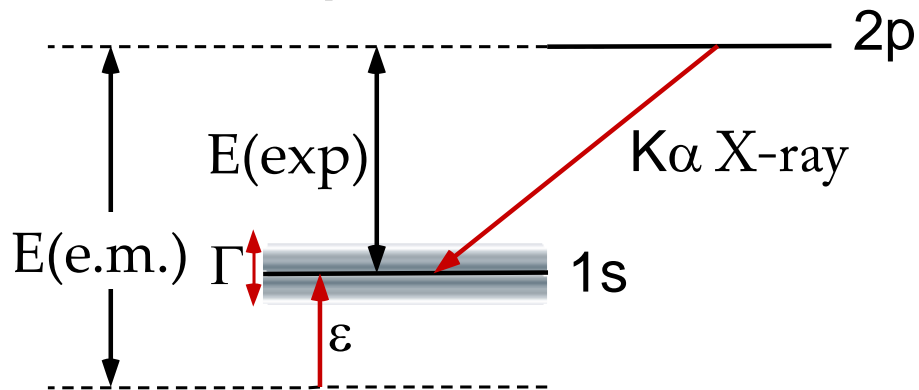


$$\epsilon = E_{np \rightarrow 1s}(\text{exp}) - E_{np \rightarrow 1s}(\text{e.m.})$$

# Energy shifts and widths

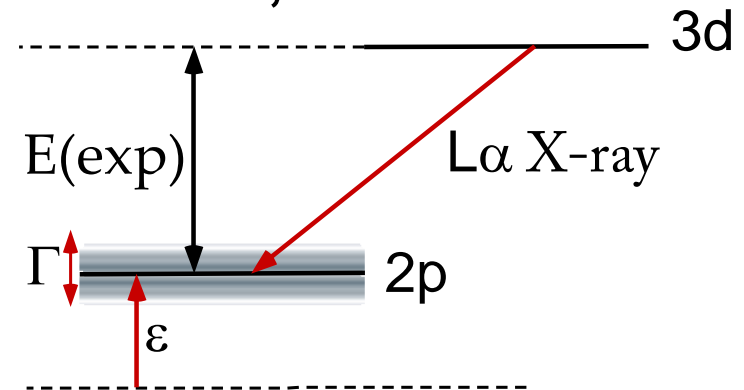
Z=1

K-p, K-d



Z=2

K-<sup>3</sup>He, K-<sup>4</sup>He

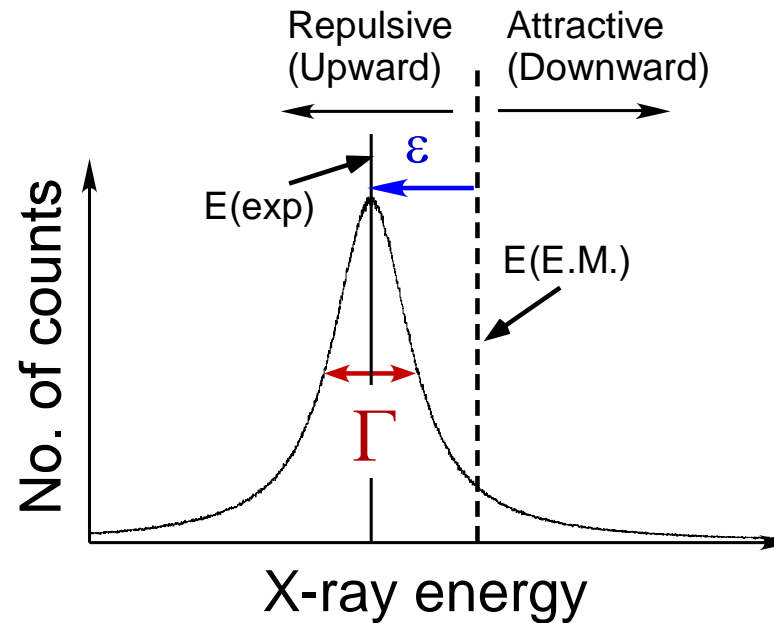


$$E(e.m.) \approx -\frac{1}{2} \mu c^2 (Z\alpha)^2 \cdot \left[ \frac{1}{n_i^2} - \frac{1}{n_f^2} \right]$$

$$\varepsilon = E(\text{exp}) - E(e.m.)$$

$$\varepsilon < 0 \text{ (repulsive)}$$

$$\varepsilon > 0 \text{ (attractive)}$$



## Shifts and widths of “Last orbits” (measured targets)

1s:  $Z=1$  (H)

2p:  $Z=2$  (He)~6 (C)

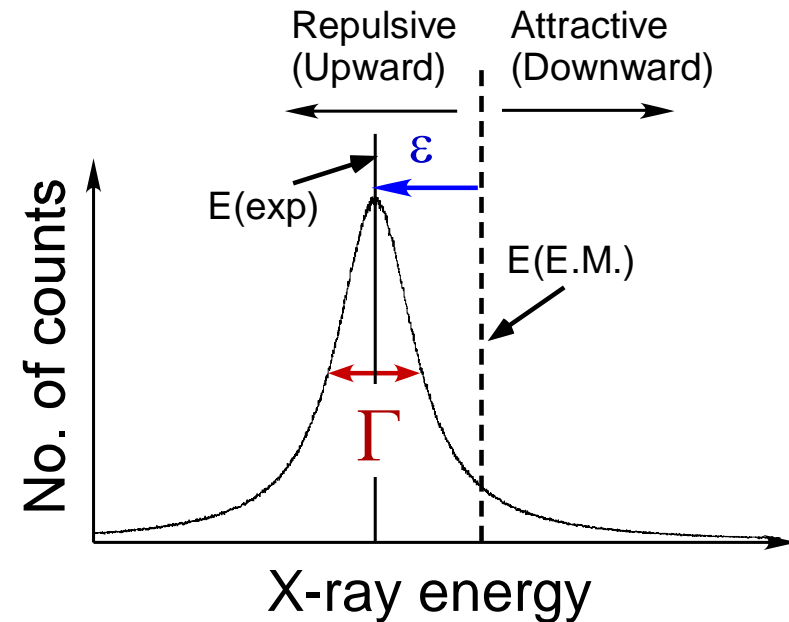
3d:  $Z=8$  (O)~17 (Cl)

4f:  $Z=27$  (Co)~29 (Cu)

5g:  $Z=47$  (Ag)~50 (Sn)

6h:  $Z=67$  (Ho)~73 (Ta)

7i:  $Z=82$  (Pb)~92 (U)



•“Last orbits” do not mean physically really last.

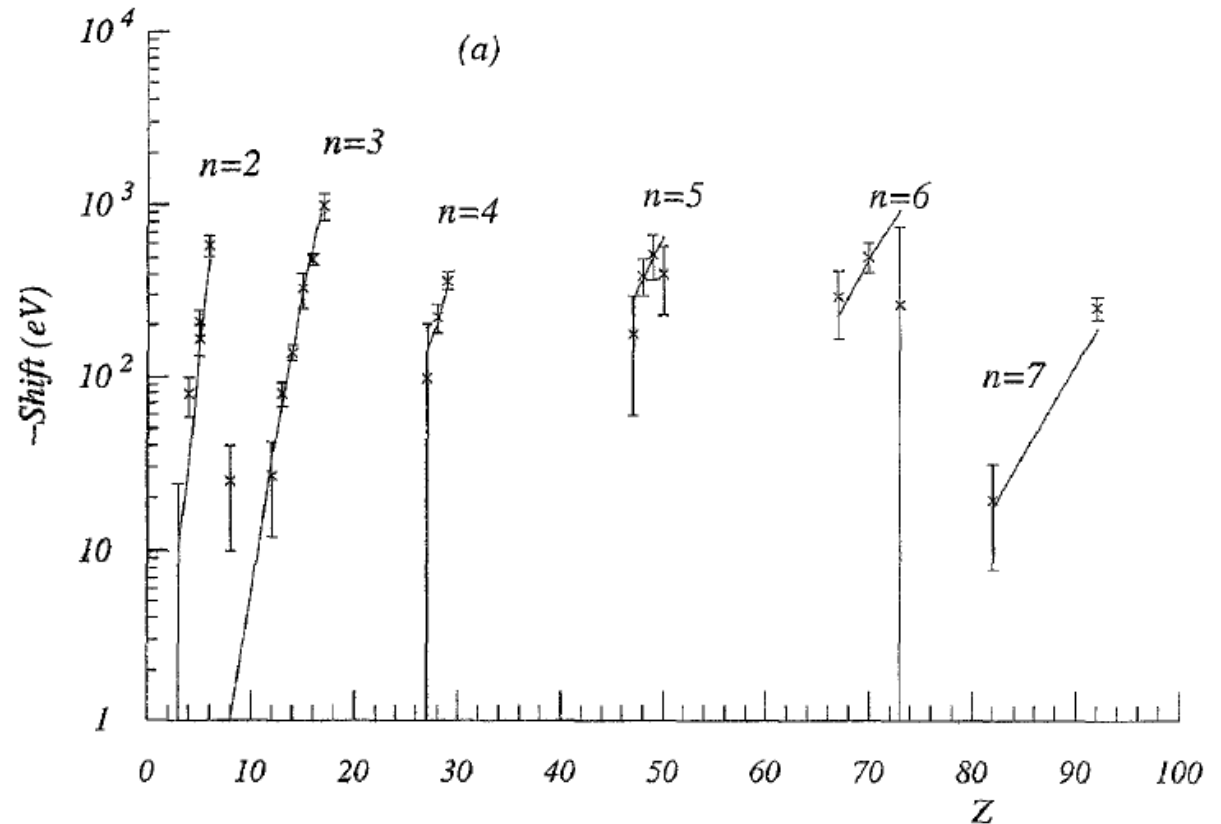
K-O 2p → very large width and very small yield

K-C 3d → very small shift and very small width

No data for Nitrogen → 2p: Large width, 3d: small shift

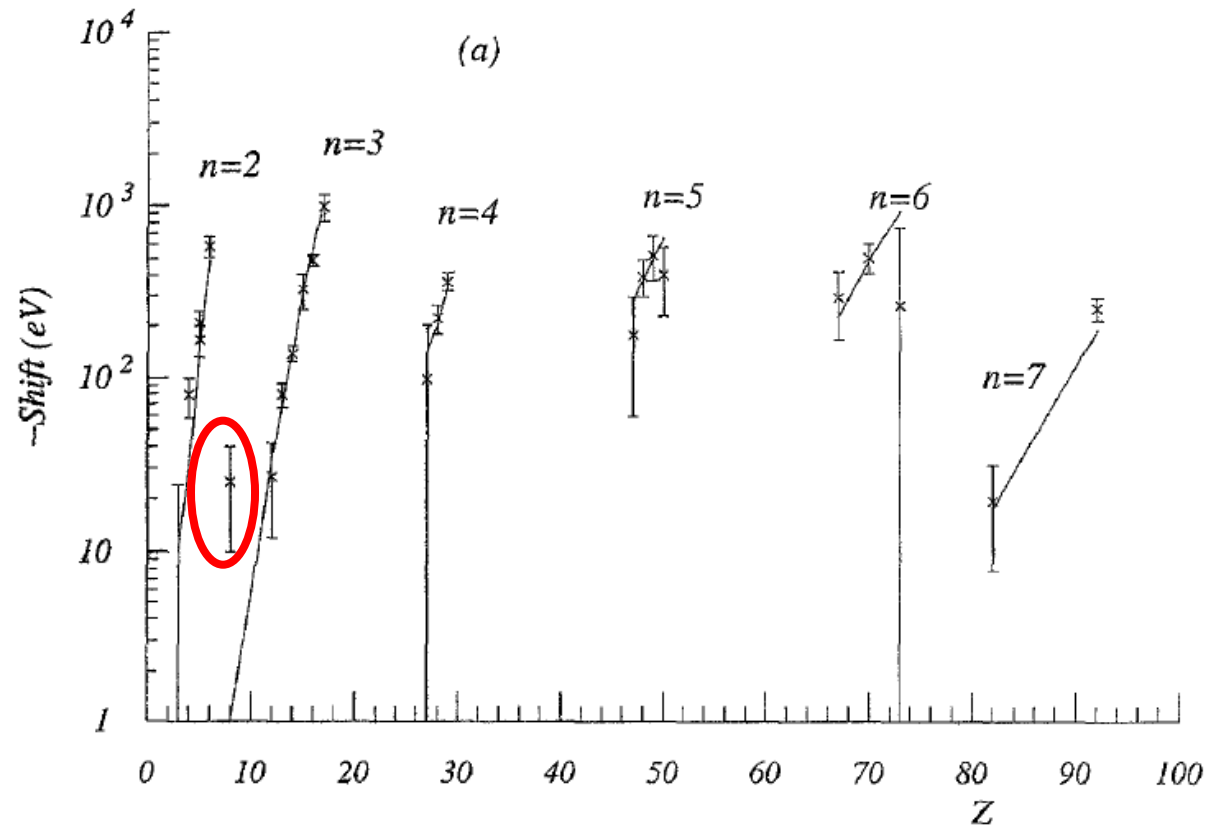
## Shifts and widths of “Last orbits” (measured targets)

- 1s:  $Z = 1$  (H)
- 2p:  $Z = 2$  (He)~6 (C)
- 3d:  $Z = 8$  (O)~17 (Cl)
- 4f:  $Z = 27$  (Co)~29 (Cu)
- 5g:  $Z = 47$  (Ag)~50 (Sn)
- 6h:  $Z = 67$  (Ho)~73 (Ta)
- 7i:  $Z = 82$  (Pb)~92 (U)



**One target → One shift and One width**

**It is interesting : One target → more than one shifts and widths**



K-O 4-3:

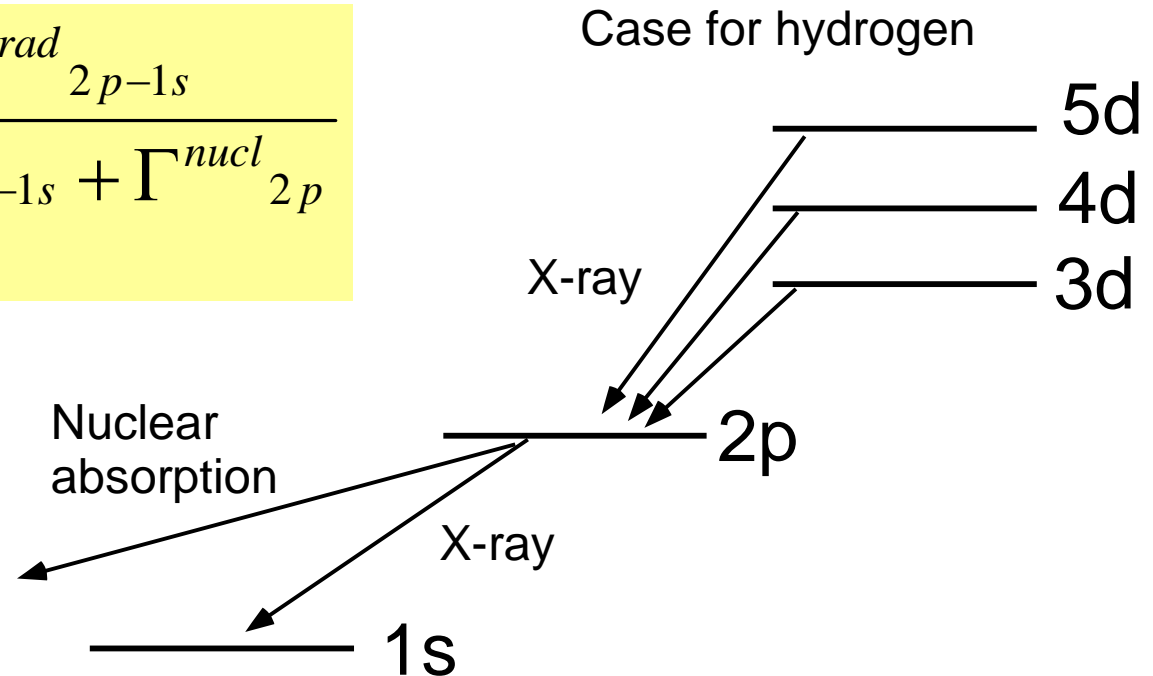
Exp:  $-25 \pm 18$  eV [NPA329 (79) 407]

Theory: 0.1~1 eV [NPA 673 (2000) 335]

## Alternative way to determine “upper” level width

Missing X-rays → related to “upper” level width

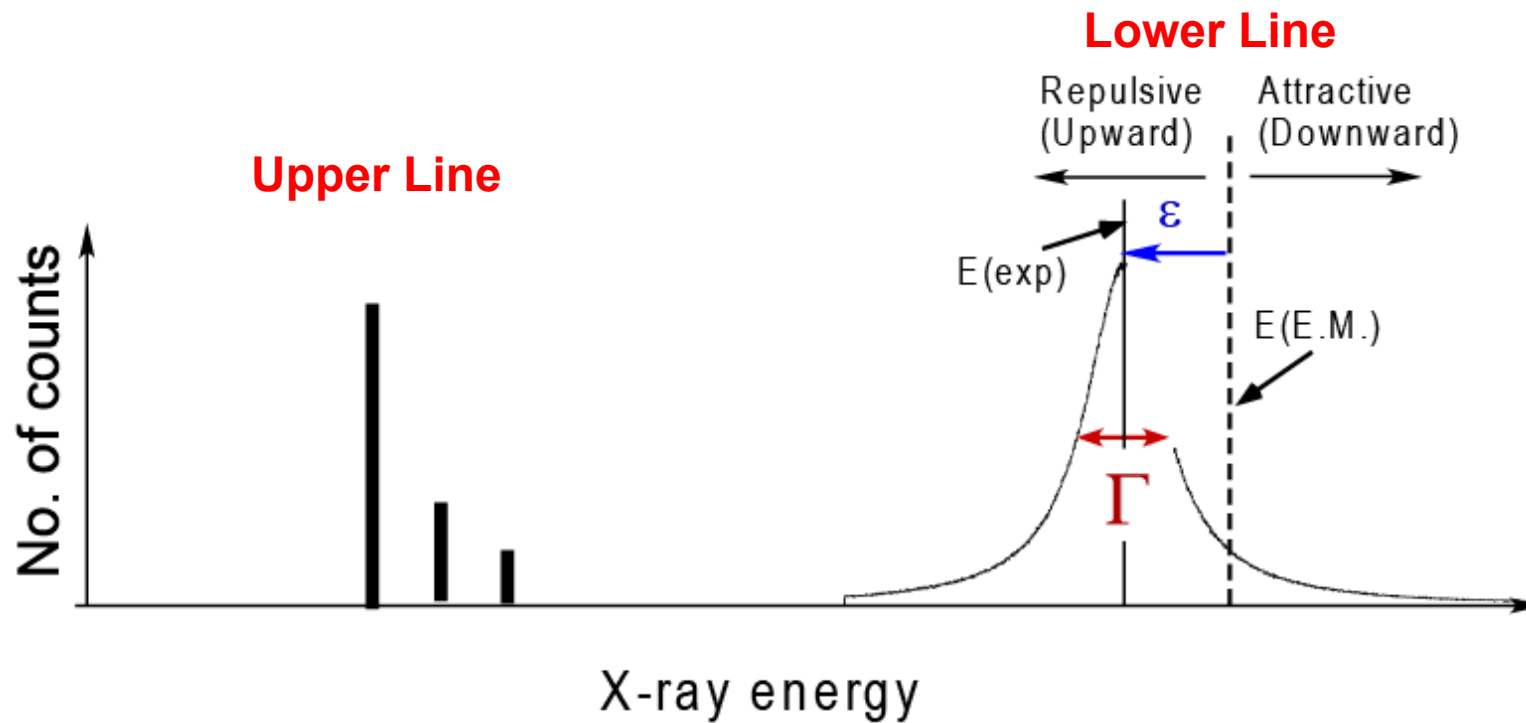
$$Y = \frac{Y_{K\alpha}}{\sum_{i=\alpha,\beta,\dots} Y_{Li}} = \frac{\Gamma^{rad}_{2p-1s}}{\Gamma^{rad}_{2p-1s} + \Gamma^{nucl}_{2p}}$$



Measurements of  $K\alpha$  yield and L-total yields give 2p width  
Similar for other transitions

More precisely, need rates of Auger transitions, electron refilling, kaon decay, etc.  
(from Cascade calculations)



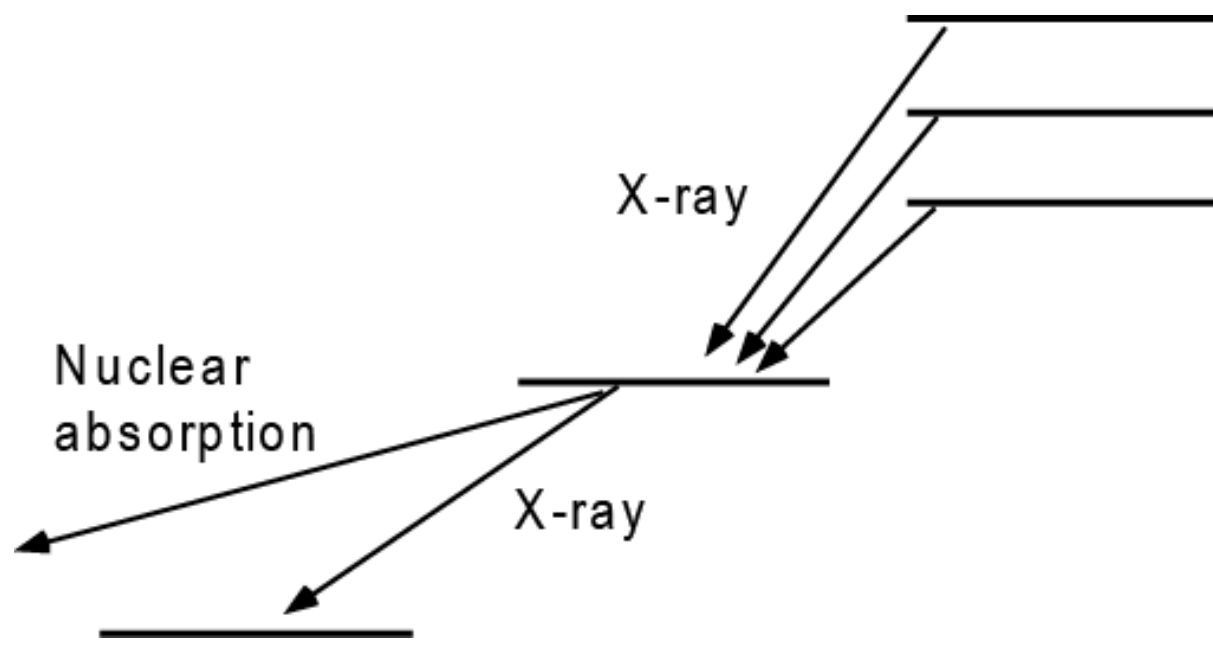


**Measurements:**

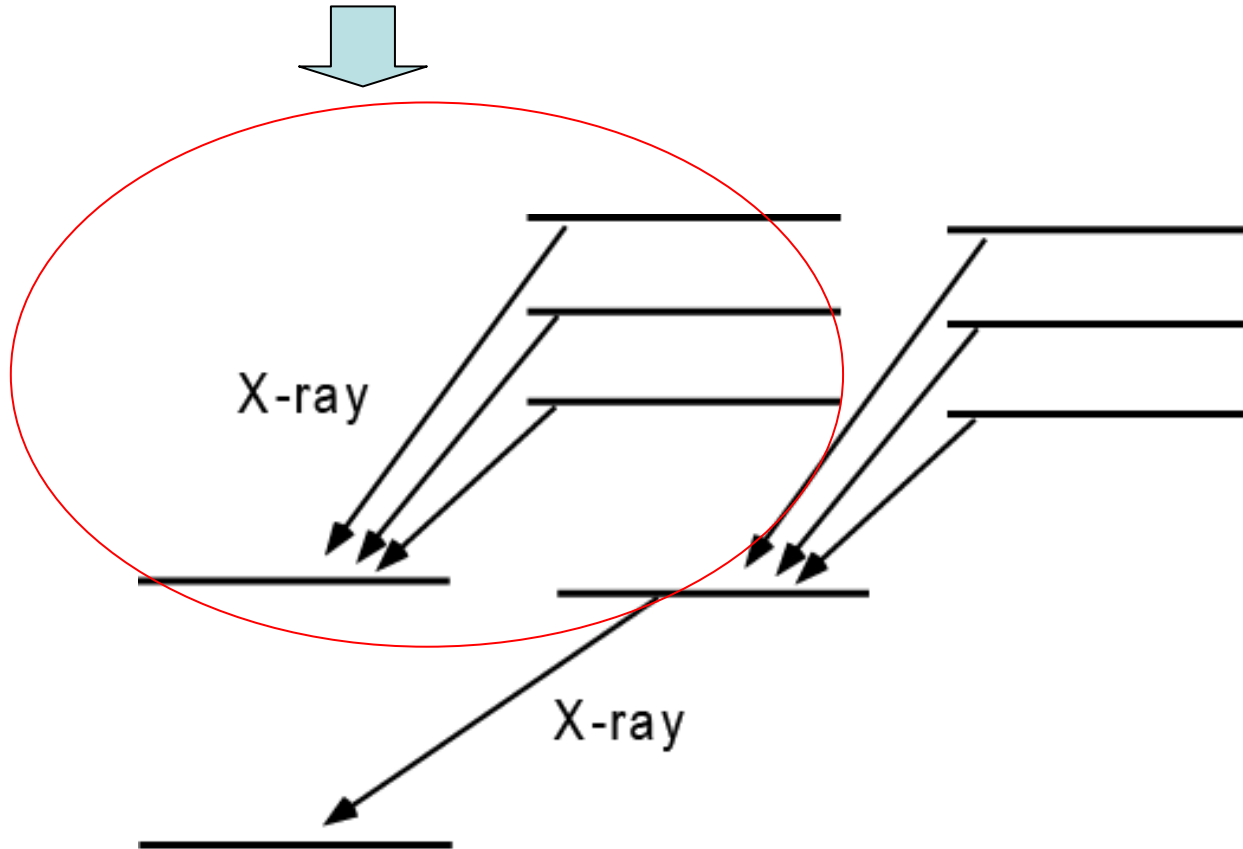
1. shift and width for Lower
2. Relative yield of Lower and Upper

•Energy dependence of X-ray detection efficiency and X-ray attenuation in target

Thin targets OR Low momentum spread and low energy kaons preferred



## X-rays from parallel transitions

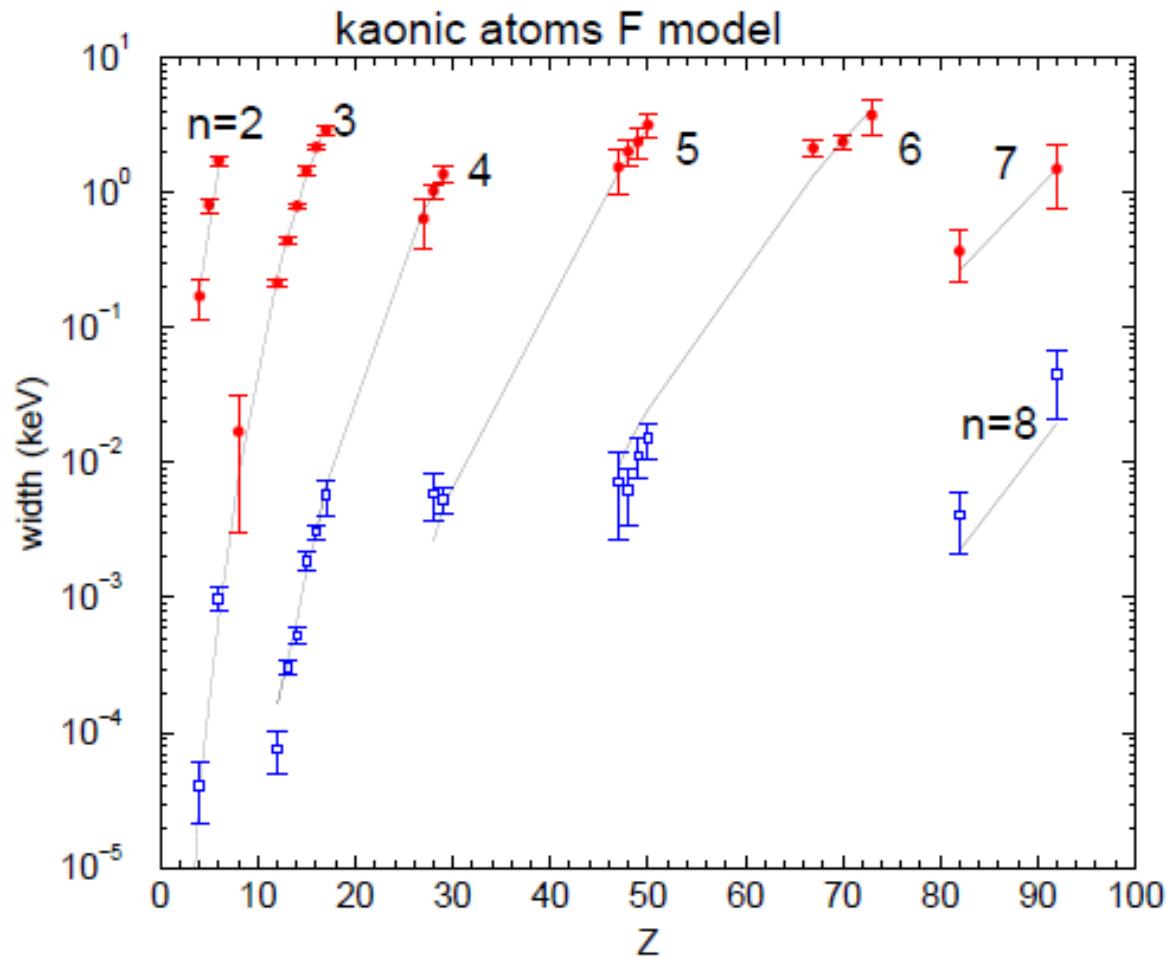


Need subtraction of X-rays from parallel transitions.

However, the energy differences are smaller than detector resolution.

**Estimation by cascade calculations is needed.**

Same for kaon mass measurements (unless using high resolution detector)



**Red points =  
direct measurements**

**Blue points =  
upper level widths  
(determined  
from yield balance)**

**One target has data of  
2 widths  
→ Important for  
constrains**

Quality fits over five orders of magnitude.

Results of global fits apply to average behaviour.

# Deep or Shallow?

Table 1. Global fits to kaonic atom data (65 points).

| model         | $\chi^2$ | $-\text{Re}V(0)$ (MeV) | $-\text{Im}V(0)$ (MeV) |
|---------------|----------|------------------------|------------------------|
| $t\rho$       | 130      | 81( $\pm 10\%$ )       | 122( $\pm 5\%$ )       |
| $t(\rho)\rho$ | 84       | 180( $\pm 3.5\%$ )     | 82( $\pm 8\%$ )        |
| chiral        | 266      | 33                     | 45                     |

Phenomenological optical potential:  $t\rho$  (fixed  $t$  effective amplitude)  
 $t(\rho)\rho$  (empirical density-dependent (DD))

Chiral-based  $t(\rho)$ : potentials constructed from more fundamental approaches

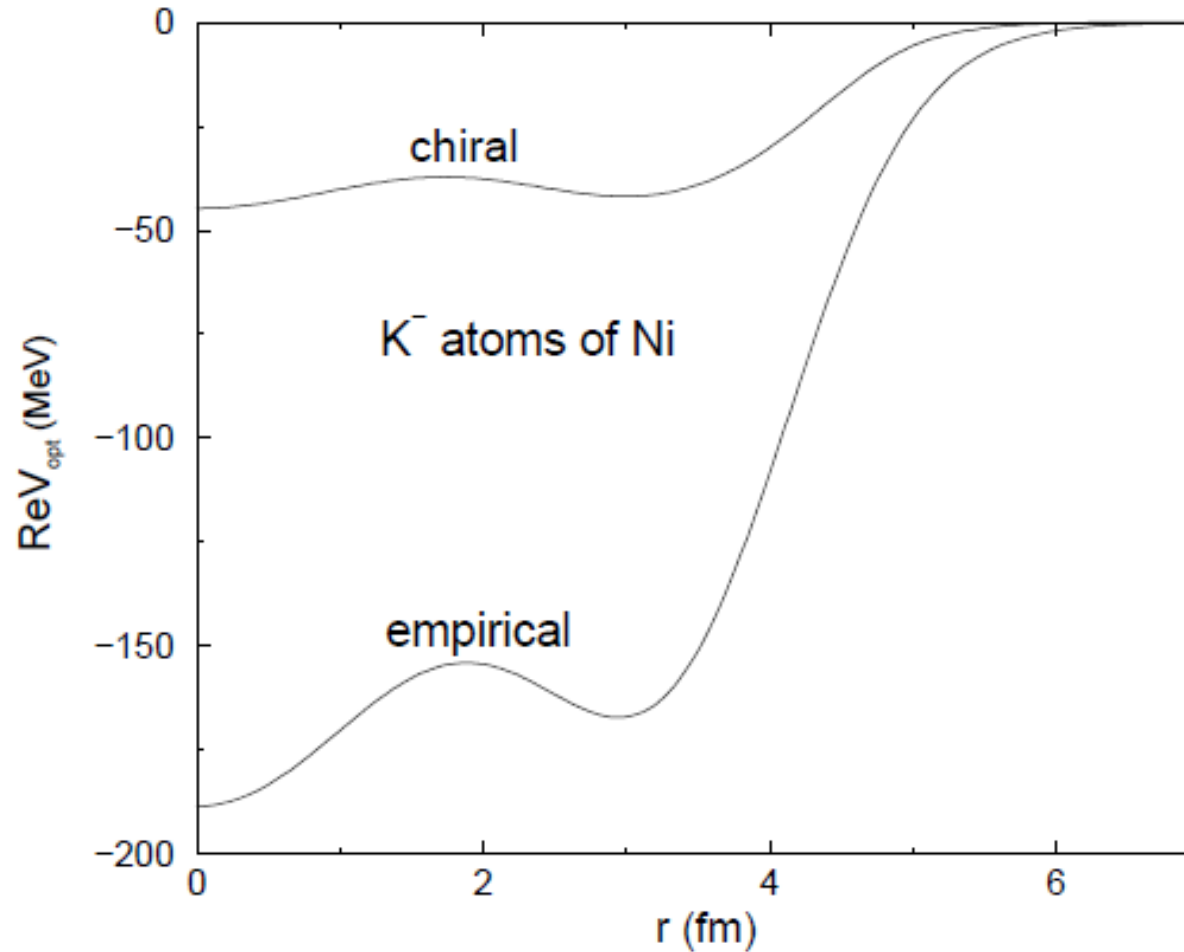
**Phenomenological approaches gave better chi2-fit!**

Selection of 65 points

(=24 shifts, 24 widths, 17 upper widths; some data excluded)

See 2.2.2 Physics Reports 287 (1997) 385

## Real K-Ni potential from chiral and empirical DD potentials



**Chi2-analysis prefers deep potential by empirical DD**  
**→ Related to kaonic nuclei and kaon condensation in neutron stars,**

## Existence of a $K^-$ strong-interaction $l=3$ quasibound state

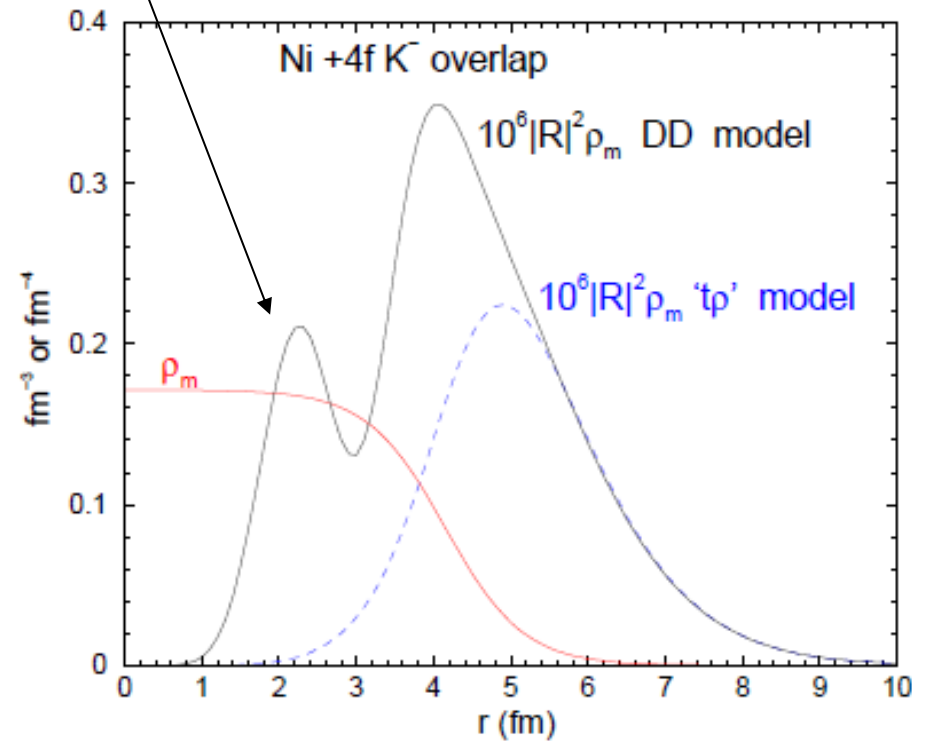
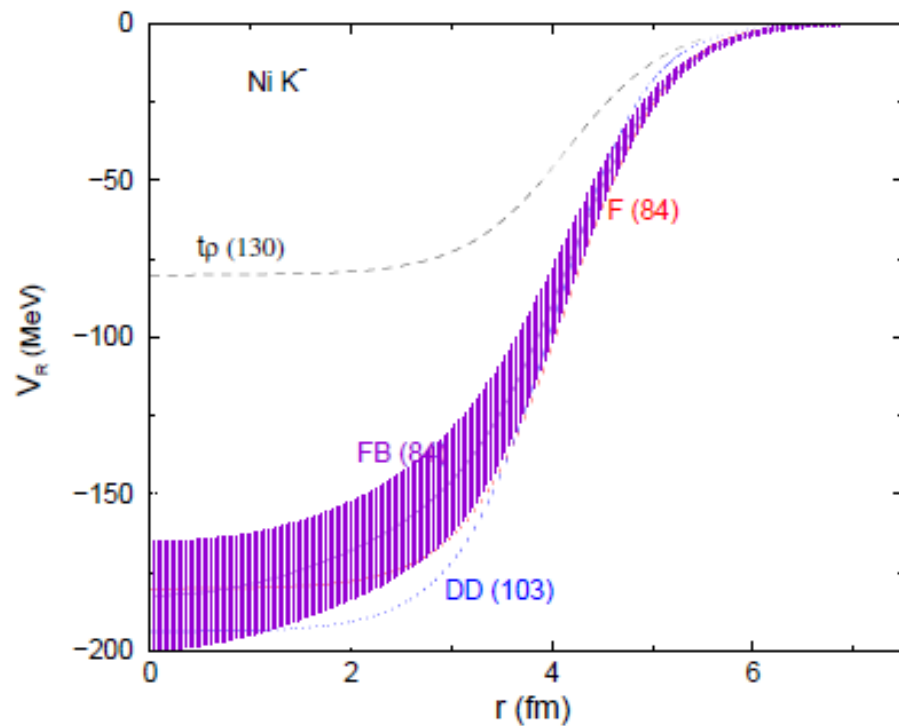
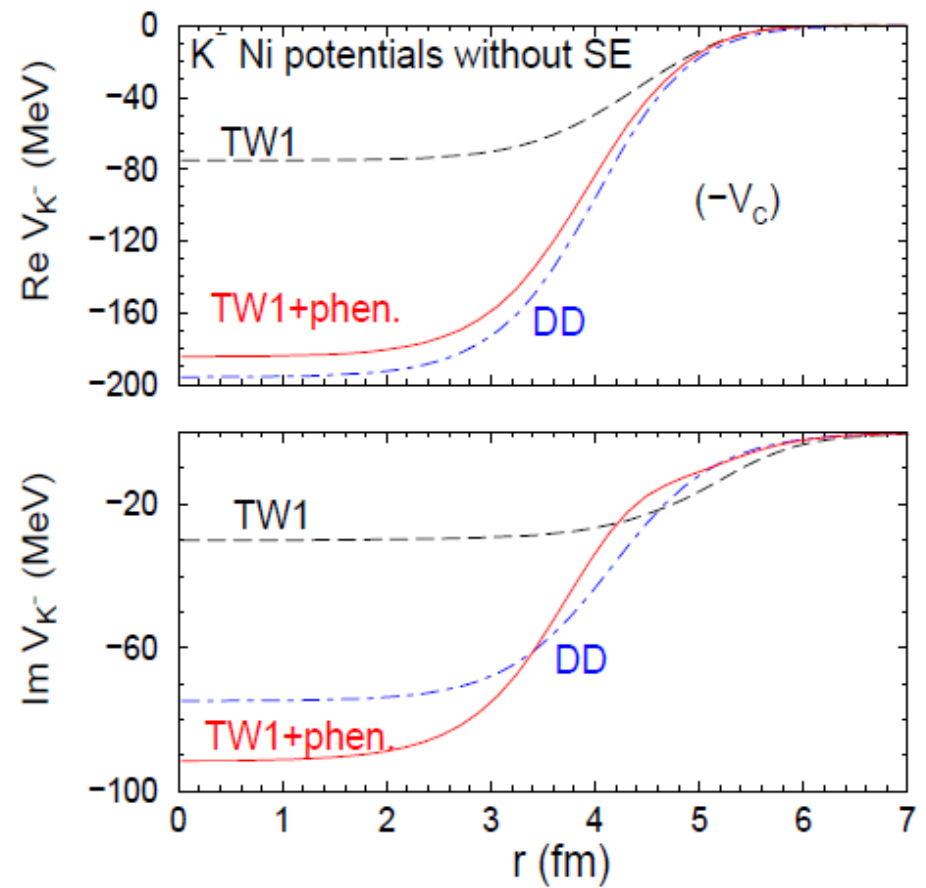
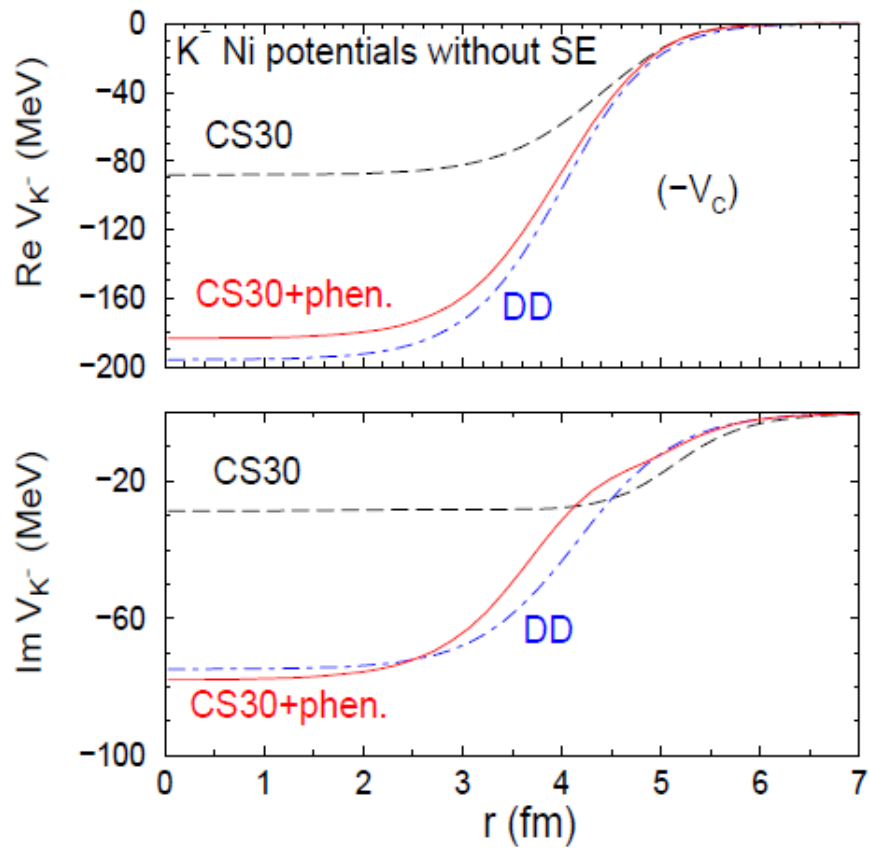


Figure 6. Left:  $V_R^{\bar{K}}$  in Ni from global fits to 65  $K^-$  atom data points, with  $\chi^2$  values in parentheses [2]. The shaded area approximates the uncertainty in the Fourier-Bessel fit. Right: Overlap of  $K^-$  atomic  $4f$  radial wavefunctions  $R$  squared with Ni matter density  $\rho_m$ .



Phys.Rev.C 84, 045206 (2011)  
 NPA 881 (2012) 150; 899 (2013) 60



# Question of data quality/selection

- Phenomenological potentials gave better  $\chi^2$  than chiral potentials
- Is it general tendency, even if we exclude some data?
- Remove 3 nuclear species, which introduced larger  $\chi^2$  values (i.e. “bad” data)

Table 2. Comparing full and 'less' data.

| N  | $\chi^2$ | Re <i>b</i> (fm) | Im <i>b</i> (fm) | $\chi^2$ | Re <i>B</i> (fm) | Im <i>B</i> (fm) |
|----|----------|------------------|------------------|----------|------------------|------------------|
| 65 | 130      | 0.62±0.05        | 0.93±0.04        | 84       | 1.44±0.03        | 0.59±0.03        |
| 56 | 78       | 0.57±0.05        | 0.97±0.04        | 66       | 1.44±0.04        | 0.60±0.04        |

Shallow [*t* $\rho$  (fixed *t*)]

Deep [*t*( $\rho$ ) $\rho$  (DD)]

Removing C, Mg, Si data, chi2 are better,  
and **the results are still same**

Table 4. Comparing full and reduced data.

| N  | $\chi^2$ | Re <i>b</i> (fm) | Im <i>b</i> (fm) | $\chi^2$ | Re <i>B</i> (fm) | Im <i>B</i> (fm) |
|----|----------|------------------|------------------|----------|------------------|------------------|
| 65 | 130      | 0.62±0.05        | 0.93±0.04        | 84       | 1.44±0.03        | 0.59±0.03        |
| 12 | 37       | 0.80±0.15        | 0.95±0.12        | 22       | 1.47±0.05        | 0.56±0.06        |

Only data of C, Si, Ni, Sn, Pb, **the results does not change so much**

Shallow best-fit kaonic atoms potentials

| targets           | $N$ | $\chi^2$ | Re(fm)          | Im(fm)          |
|-------------------|-----|----------|-----------------|-----------------|
| all               | 65  | 130      | $0.59 \pm 0.05$ | $0.94 \pm 0.05$ |
| C, Si, Ni, Sn, Pb | 15  | 44       | $0.78 \pm 0.13$ | $0.92 \pm 0.11$ |
| C, Si, Ni, Pb     | 12  | 37       | $0.80 \pm 0.15$ | $0.95 \pm 0.12$ |
| C, Si, Ni, Sn,    | 12  | 43       | $0.78 \pm 0.15$ | $0.90 \pm 0.14$ |
| Si, Ni, Sn,       | 9   | 31       | $0.68 \pm 0.16$ | $0.91 \pm 0.14$ |

Deep best-fit kaonic atoms potentials

| targets           | $N$ | $\chi^2$ | Re(fm)          | Im(fm)          |
|-------------------|-----|----------|-----------------|-----------------|
| all               | 65  | 84       | $1.44 \pm 0.03$ | $0.59 \pm 0.03$ |
| C, Si, Ni, Sn, Pb | 15  | 26       | $1.47 \pm 0.05$ | $0.55 \pm 0.06$ |
| C, Si, Ni, Pb     | 12  | 22       | $1.47 \pm 0.05$ | $0.56 \pm 0.06$ |
| C, Si, Ni, Sn,    | 12  | 24       | $1.47 \pm 0.05$ | $0.55 \pm 0.06$ |
| Si, Ni, Sn,       | 9   | 13       | $1.47 \pm 0.05$ | $0.52 \pm 0.05$ |

**Even more reduced data set, the results do not change so much**  
**Always deep potentials give better chi2.**

# What's next?

- Even if reduced data set were used, the fit with **the deep real potential is again significantly better** than the fit with the shallow potential. Thus, it appears to be an inherent property of the kaonic atom data.
- An obvious line of action to resolve the puzzle of the depth of the kaonic atom real potential is **to repeat some of the 30-40 years old measurements** with the presently available techniques.

# What's next?

- Unrealistic to repeat experiments on all the 23 targets. Select a small number of targets that are representative of the full set. 5 targets cover the whole range of the original data. The widths are not too large and the relative yields of the upper levels are of the order of 10% or more.

| target                                 | C                 | Si                | Ni                | Sn              | Pb                |
|--|-------------------|-------------------|-------------------|-----------------|-------------------|
| ref                                    | 12                | 13                | 13,14             | 13              | 15                |
| (n,l)                                  | 2p                | 3d                | 4f                | 5g              | 7i                |
| $-\epsilon$ (keV)                      | $0.50 \pm 0.08$   | $0.130 \pm 0.015$ | $0.223 \pm 0.042$ | $0.41 \pm 0.18$ | $0.020 \pm 0.012$ |
| $\Gamma$ (keV)                         | $1.73 \pm 0.15$   | $0.800 \pm 0.033$ | $1.03 \pm 0.12$   | $3.18 \pm 0.64$ | $0.37 \pm 0.15$   |
| relative yield                         | $0.070 \pm 0.013$ | $0.49 \pm 0.03$   | $0.30 \pm 0.08$   | $0.39 \pm 0.07$ | $0.70 \pm 0.08$   |
| $\Gamma_u$ (eV)                        | $0.99 \pm 66$     | $0.53 \pm 0.06$   | $5.9 \pm 2.3$     | $15.1 \pm 4.4$  | $4.1 \pm 2.0$     |
| EM $n+1 \rightarrow n$<br>energy (keV) | 63.3              | 123.7             | 231.6             | 403.9           | 426.2             |

New results give still deep potential?? Indeed so??

#### 4. Summary

possible existence of antikaon-nucleon clusters. Global fits of optical potentials to the world's data on kaonic atoms lead to good agreement with data for a shallow type of real potential and to very good agreement for a deep type of real potential.

**Although deep potentials favors in terms of  $\chi^2$ ,  
but available data cannot exclude shallow potentials.**

**New precise measurements:**

- 1. check old data**
- 2. Improvement of data can provide further  $\chi^2$ -discrepancy  
between shallow and deep potentials.**
- 3. Can we confirm that the exclusion of shallow potential??**

The depth of the antikaon-nucleus real potential has received renewed attention in the last decade because of its relevance to recent theoretical speculations<sup>8</sup> and experimental indications<sup>9,10</sup> for the existence of bound clusters of nucleons and an antikaon with binding energies of the order of 100 MeV. If confirmed they will exclude the shallow potentials, assuming that the depth does not vary strongly with the binding energy.

Last but not least, if the empirical deep potentials are indeed confirmed, then there will remain the question of why they differ so much from potentials constructed using more fundamental approaches.

by E. Friedman

**Fitting to such a data set with improved accuracy could resolve the issue of deep vs. shallow potentials and determine how deep is 'deep'**

by A. Gal

**Under this classification E. Friedman has performed a survey of nuclear targets that could be used in new kaonic atom experiments.**

**The aim is to be able to determine strong interaction widths of TWO levels in the same atom, referred to as 'lower' and 'upper' level. The interest in this topic arises from our recent work (Nucl. Phys. A 899 (2013) 60-75) where we showed that such measurement could help to **disentangle one-nucleon absorptions from multi-nucleon processes.****

**The accuracy of upper level widths in old measurements leaves much to be desired. The present work has identified 6-8 suitable targets. A paper is being written up for publication.**

LEANNIS report by A. Gal

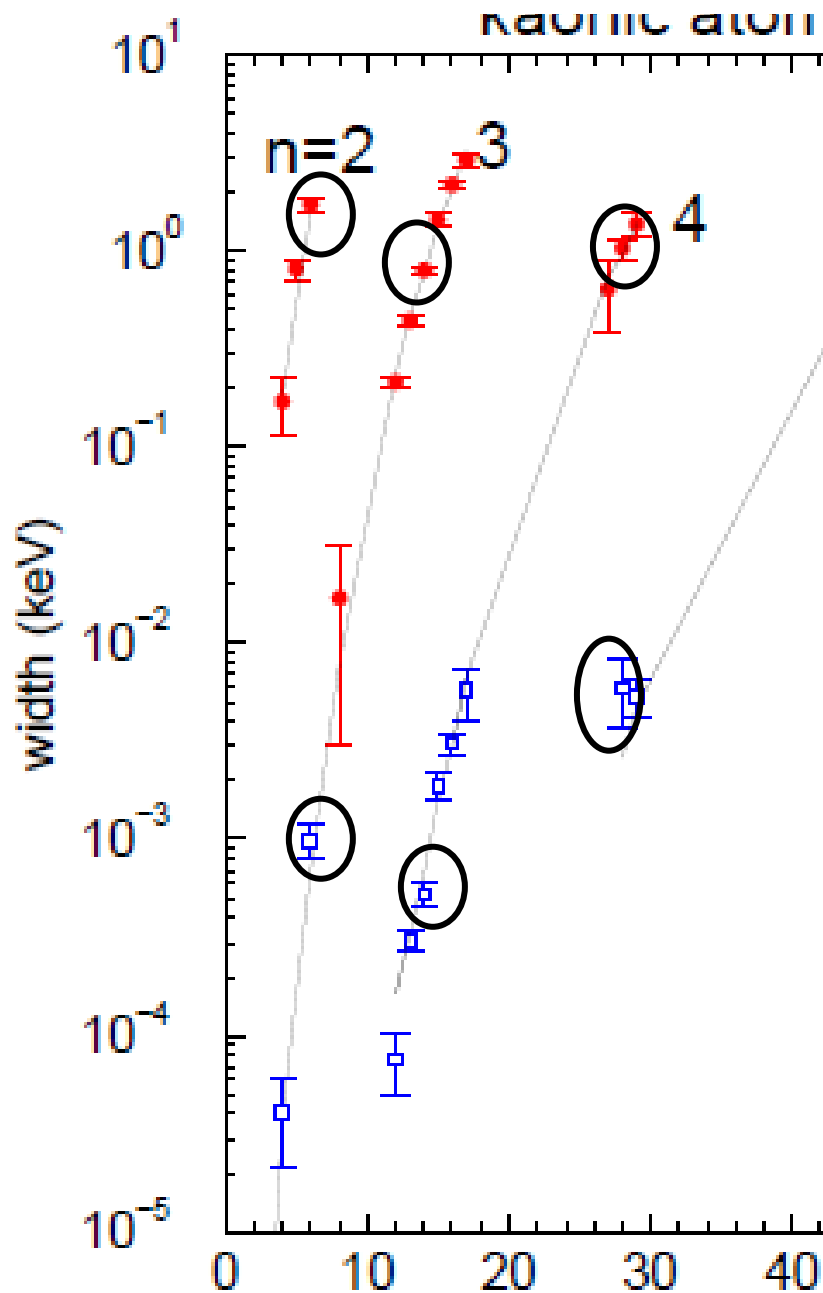


## Minimum data sets (by Friedman)

| target                                 | C                 | Si                | Ni                | Sn              | Pb                |
|--|-------------------|-------------------|-------------------|-----------------|-------------------|
| ref                                    | (a)               | (b)               | (b),(c)           | (b)             | (d)               |
| (n,l)                                  | 2p                | 3d                | 4f                | 5g              | 7i                |
| $-\epsilon$ (keV)                      | $0.50 \pm 0.08$   | $0.130 \pm 0.015$ | $0.223 \pm 0.042$ | $0.41 \pm 0.18$ | $0.020 \pm 0.012$ |
| $\Gamma$ (keV)                         | $1.73 \pm 0.15$   | $0.800 \pm 0.033$ | $1.03 \pm 0.12$   | $3.18 \pm 0.64$ | $0.37 \pm 0.15$   |
| yield                                  | $0.070 \pm 0.013$ | $0.49 \pm 0.03$   | $0.30 \pm 0.08$   | $0.39 \pm 0.07$ | $0.70 \pm 0.08$   |
| $\Gamma_u$ (eV)                        | $0.99 \pm 66$     | $0.53 \pm 0.06$   | $5.9 \pm 2.3$     | $15.1 \pm 4.4$  | $4.1 \pm 2.0$     |
| EM $n+1 \rightarrow n$<br>energy (keV) | 63.3              | 123.7             | 231.6             | 403.9           | 426.2             |

Meson2010, Friedman

**Let's look existing data carefully**



**Suggestion:**  
**C,Si,Ni,Sn,Pb**

**Old method:**  
 Error reduction using “averaging”

Average of Be,B,C → n=2  
 Average of Be-Cl → n=3

...

**New method:**  
 Error reduction using “Precision”

Precise C → n=2  
 Precise C,Si → n=3  
 Precise Si,Ni → n=4

...

## Minimum data sets (by Friedman)

| target                                 | C                 | Si                | Ni                | Sn              | Pb                |
|--|-------------------|-------------------|-------------------|-----------------|-------------------|
| ref                                    | (a)               | (b)               | (b),(c)           | (b)             | (d)               |
| (n,l)                                  | 2p                | 3d                | 4f                | 5g              | 7i                |
| $-\epsilon$ (keV)                      | $0.50 \pm 0.08$   | $0.130 \pm 0.015$ | $0.223 \pm 0.042$ | $0.41 \pm 0.18$ | $0.020 \pm 0.012$ |
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| yield                                  | $0.070 \pm 0.013$ | $0.49 \pm 0.03$   | $0.30 \pm 0.08$   | $0.39 \pm 0.07$ | $0.70 \pm 0.08$   |
| $\Gamma_u$ (eV)                        | $0.99 \pm 66$     | $0.53 \pm 0.06$   | $5.9 \pm 2.3$     | $15.1 \pm 4.4$  | $4.1 \pm 2.0$     |
| EM $n+1 \rightarrow n$<br>energy (keV) | 63.3              | 123.7             | 231.6             | 403.9           | 426.2             |

Meson2010, Friedman

### Yield per stopped kaon

|                            | C(3-2)       | Si(4-3)     | Ni(5-4)      | Sn(6-5)      | Pb(8-7)     |
|----------------------------|--------------|-------------|--------------|--------------|-------------|
| <b>X-ray per stopped K</b> | <b>0.028</b> | <b>0.13</b> | <b>0.045</b> | <b>0.076</b> | <b>0.27</b> |

**Friedman, NPA579(1994)518**

**Wiegand, PRA9 (1974) 2282**

| target                                 | C                 | Si                | Ni                | Sn              | Pb                |
|--|-------------------|-------------------|-------------------|-----------------|-------------------|
| ref                                    | (a)               | (b)               | (b),(c)           | (b)             | (d)               |
| (n,l)                                  | 2p                | 3d                | 4f                | 5g              | 7i                |
| $-\epsilon$ (keV)                      | $0.50 \pm 0.08$   | $0.130 \pm 0.015$ | $0.223 \pm 0.042$ | $0.41 \pm 0.18$ | $0.020 \pm 0.012$ |
| $\Gamma$ (keV)                         | $1.73 \pm 0.15$   | $0.800 \pm 0.033$ | $1.03 \pm 0.12$   | $3.18 \pm 0.64$ | $0.37 \pm 0.15$   |
| yield                                  | $0.070 \pm 0.013$ | $0.49 \pm 0.03$   | $0.30 \pm 0.08$   | $0.39 \pm 0.07$ | $0.70 \pm 0.08$   |
| $\Gamma_w$ (eV)                        | $0.99 \pm 66$     | $0.53 \pm 0.06$   | $5.9 \pm 2.3$     | $15.1 \pm 4.4$  | $4.1 \pm 2.0$     |
| EM $n+1 \rightarrow n$<br>energy (keV) | 63.3              | 123.7             | 231.6             | 403.9           | 426.2             |

A very simple optical model calculation

| K-Pb                 | Exp             |                  | Calc.           |                 |
|----------------------|-----------------|------------------|-----------------|-----------------|
|                      | Shift           | Width            | Shift           | width           |
| 9-8                  | --              | $4.1 \pm 2.0$ eV | 0.08 eV         | 2 eV            |
| 8-7                  | $-20 \pm 12$ eV | $370 \pm 150$ eV | -16 eV          | 220 eV          |
| <b>7-6 (650 keV)</b> | --              | --               | <b>-2.8 keV</b> | <b>10.5 keV</b> |

Detector resolution ~1.5 keV

Calculated absolute Yield at 7-6 ~0.1%

## Minimum data sets (by Friedman)

| target                                 | C                 | Si                | Ni                | Sn              | Pb                |
|--|-------------------|-------------------|-------------------|-----------------|-------------------|
| ref                                    | (a)               | (b)               | (b),(c)           | (b)             | (d)               |
| (n,l)                                  | 2p                | 3d                | 4f                | 5g              | 7i                |
| $-\epsilon$ (keV)                      | $0.50 \pm 0.08$   | $0.130 \pm 0.015$ | $0.223 \pm 0.042$ | $0.41 \pm 0.18$ | $0.020 \pm 0.012$ |
| $\Gamma$ (keV)                         | $1.73 \pm 0.15$   | $0.800 \pm 0.033$ | $1.03 \pm 0.12$   | $3.18 \pm 0.64$ | $0.37 \pm 0.15$   |
| yield                                  | $0.070 \pm 0.013$ | $0.49 \pm 0.03$   | $0.30 \pm 0.08$   | $0.39 \pm 0.07$ | $0.70 \pm 0.08$   |
| $\Gamma_u$ (eV)                        | $0.99 \pm 66$     | $0.53 \pm 0.06$   | $5.9 \pm 2.3$     | $15.1 \pm 4.4$  | $4.1 \pm 2.0$     |
| EM $n+1 \rightarrow n$<br>energy (keV) | 63.3              | 123.7             | 231.6             | 403.9           | 426.2             |

Meson2010, Friedman

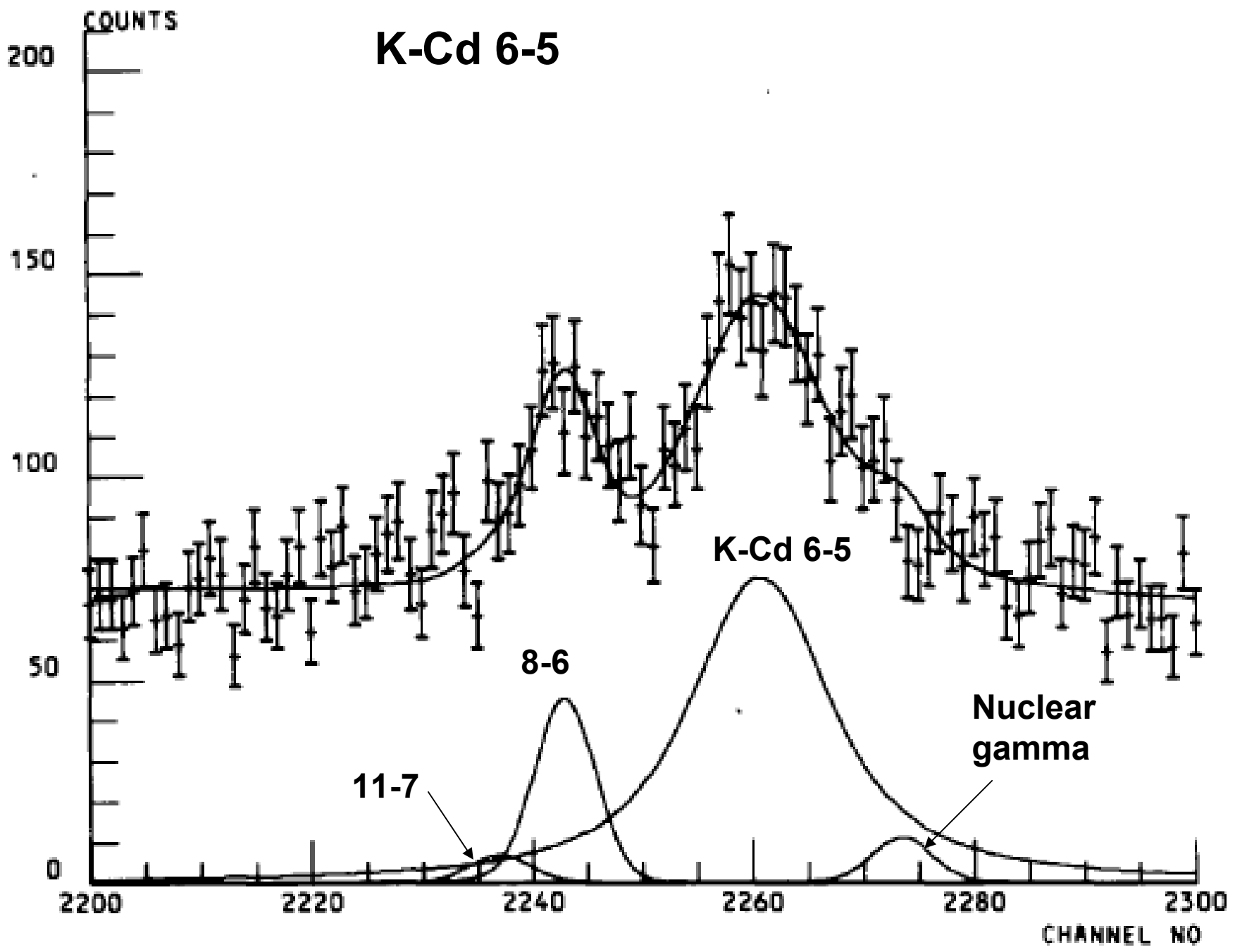
**Sn 6-5: be care for overlap of 6-5 and 8-6**

**Energy difference of E(6-5) and E(8-6) is smaller**

In addition E(11-7)..

Not easy to extract 6-5 contamination.

Same for  $\Xi$ -atoms (J-PARC E03)



| target            | C                 | Si                | Ni                | Sn              | Pb                |
|-------------------|-------------------|-------------------|-------------------|-----------------|-------------------|
| ref               | 12                | 13                | 13,14             | 13              | 15                |
| (n,l)             | 2p                | 3d                | 4f                | 5g              | 7i                |
| $-\epsilon$ (keV) | $0.50 \pm 0.08$   | $0.130 \pm 0.015$ | $0.223 \pm 0.042$ | $0.41 \pm 0.18$ | $0.020 \pm 0.012$ |
| $\Gamma$ (keV)    | $1.73 \pm 0.15$   | $0.800 \pm 0.033$ | $1.03 \pm 0.12$   | $3.18 \pm 0.64$ | $0.37 \pm 0.15$   |
| relative yield    | $0.070 \pm 0.013$ | $0.49 \pm 0.03$   | $0.30 \pm 0.08$   | $0.39 \pm 0.07$ | $0.70 \pm 0.08$   |
| $\Gamma_u$ (eV)   | $0.99 \pm 66$     | $0.53 \pm 0.06$   | $5.9 \pm 2.3$     | $15.1 \pm 4.4$  | $4.1 \pm 2.0$     |

|    | Shift [eV]    | Width [eV]     | Upper Width [eV]         | Ref                    |
|----|---------------|----------------|--------------------------|------------------------|
| Si | $-240 \pm 50$ | $810 \pm 120$  | -- (due to thick target) | NPA 231 (74) 477       |
| Si | $-130 \pm 15$ | $800 \pm 33$   | $0.54^{+0.07}_{-0.06}$   | NPA 329 (79) 407       |
| Ni | $-180 \pm 70$ | $590 \pm 210$  | $6.0 \pm 2.3$            | NPA 231 (74) 477       |
| Ni | $-246 \pm 52$ | $1230 \pm 140$ | --                       | NPA 329 (79) 407       |
| Pb | $-20 \pm 12$  | --             | --                       | NPA254(75)381          |
| Pb | --            | $370 \pm 140$  | $4.1 \pm 2.$ (Y=79±8%)   | J.P. Miller, Ph.D (75) |
| Pb | +72+(+27-57)  | $284 \pm 14$   | Y=82±4%                  | PRC 40 (89) 2154       |

**\*Ni widths are small compared to calculations [NPA 231 (74) 477]**

**•Pb data: several problems...**

|           | Shift [eV]       | Width [eV]       | Upper Width [eV] | Ref                     |
|-----------|------------------|------------------|------------------|-------------------------|
| <b>Ni</b> | <b>-180 ± 70</b> | <b>590 ± 210</b> | <b>6.0 ± 2.3</b> | <b>NPA 231 (74) 477</b> |
| Ni        | -246 ± 52        | 1230 ± 140       | --               | NPA 329 (79) 407        |

### NPA 231 (74) 477

| Element                            | Transition<br>$n_i, l_i \rightarrow n_f, l_f$ | Width of level<br>$\Gamma_{n_f}$ (eV) |             | Shift of level<br>$\Delta E_{n_f}$ (eV) |             | $\bar{A}^{fit}$ (fm)<br>(does not include yield) |
|------------------------------------|---|---------------------------------------|-------------|---|-------------|--|
|                                    |   | meas.                                 | calc<br>(3) | meas.                                   | calc<br>(3) |  |
| $_{13}\text{Al}$                   | 4, 3 $\rightarrow$ 3, 2                       | 490 ± 160                             | 545         | -130 ± 50                               | -63         | $0.3^{+0.6}_{-0.5} + i(1.1 \pm 0.5)$             |
| $_{14}\text{Si}$                   | 4, 3 $\rightarrow$ 3, 2                       | 810 ± 120                             | 895         | -240 ± 50                               | -120        | $0.25^{+0.30}_{-0.25} + i(1.15^{+0.25}_{-0.20})$ |
| <b><math>_{28}\text{Ni}</math></b> | <b>5, 4 <math>\rightarrow</math> 4, 3</b>     | <b>590 ± 210</b>                      | <b>1031</b> | <b>-180 ± 70</b>                        | <b>-97</b>  | $-0.1^{+0.3}_{-0.2} + i(0.6^{+0.3}_{-0.2})$      |
| $_{29}\text{Cu}$                   | 5, 4 $\rightarrow$ 4, 3                       | 1650 ± 720                            | 1504        | -240 ± 220                              | -181        | $0.7^{+1.0}_{-0.7} + i(0.9^{+0.6}_{-0.8})$       |

Ni width disagreement is serious! Also, it related to upper width.  
[Wycheck told me that the upper width of Ni is strange.]



|           | Shift [eV]                      | Width [eV]                       | Upper Width [eV] | Ref                     |
|-----------|---------------------------------|----------------------------------|------------------|-------------------------|
| Ni        | $-180 \pm 70$                   | $590 \pm 210$                    | $6.0 \pm 2.3$    | NPA 231 (74) 477        |
| <b>Ni</b> | <b><math>-246 \pm 52</math></b> | <b><math>1230 \pm 140</math></b> | --               | <b>NPA 329 (79) 407</b> |

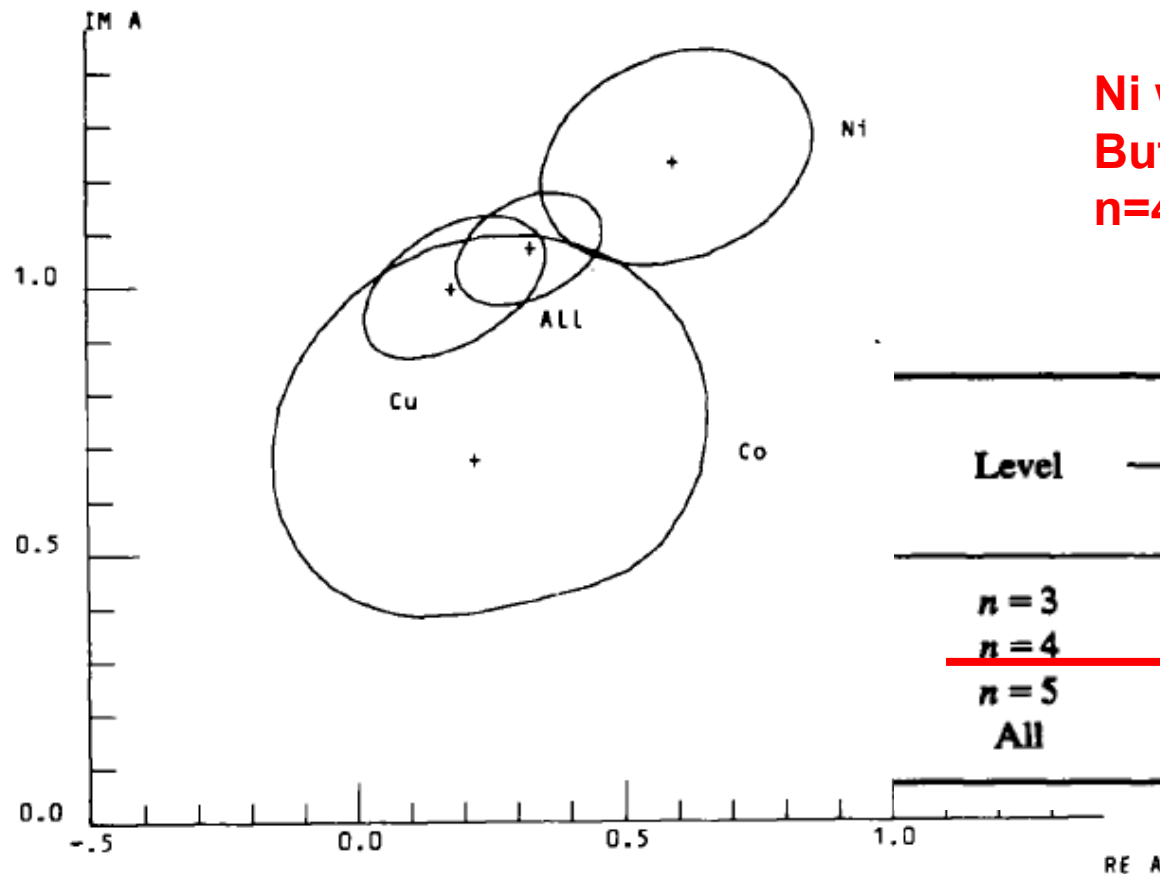
**NPA 329 (79) 407**

### **Calibration problem**

**The data for Co and Ni were obtained prior to the introduction of the stabiliser and simultaneous collection of calibration data. In both cases there appears to be a small energy shift between the data and the calibration runs. This was observed by looking at unshifted X-ray lines and identified  $\gamma$ -lines in the data. These led to an additional error of  $\pm 30$  eV being introduced.**

|           | Shift [eV]                      | Width [eV]                       | Upper Width [eV] | Ref                     |
|-----------|---------------------------------|----------------------------------|------------------|-------------------------|
| Ni        | $-180 \pm 70$                   | $590 \pm 210$                    | $6.0 \pm 2.3$    | NPA 231 (74) 477        |
| <b>Ni</b> | <b><math>-246 \pm 52</math></b> | <b><math>1230 \pm 140</math></b> | --               | <b>NPA 329 (79) 407</b> |

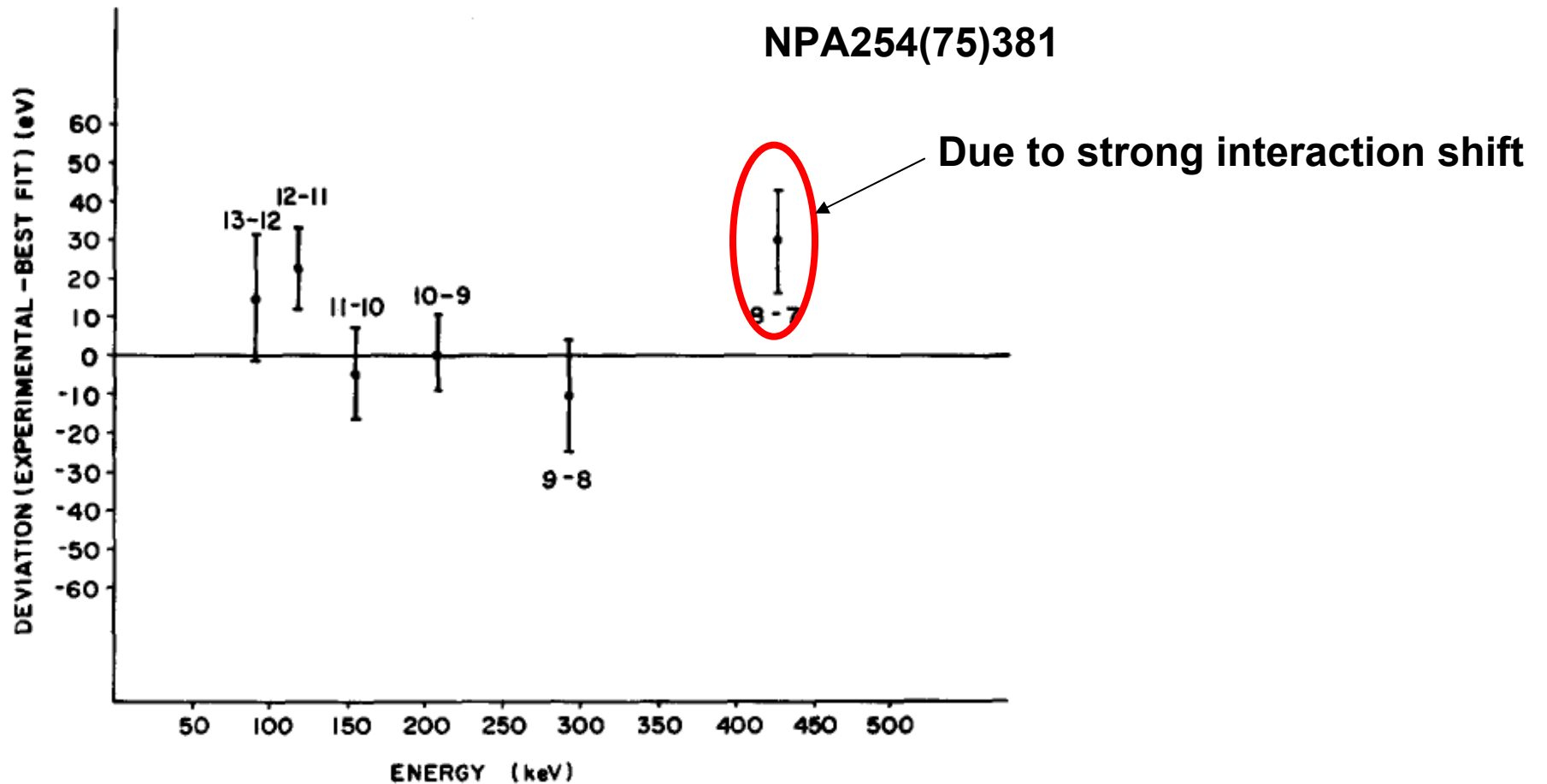
### NPA 329 (79) 407



**Ni width was too large??  
But it seems unclear whether  
n=4 level problem.**

| Level                     | $\bar{d}$                         |                                   | $\chi^2/N$ |
|---------------------------|-----------------------------------|-----------------------------------|------------|
|                           | $a_R(\text{fm})$                  | $a_I(\text{fm})$                  |            |
| $n = 3$                   | $0.33 \pm 0.04$                   | $0.78 \pm 0.03$                   | 0.6        |
| <b><math>n = 4</math></b> | <b><math>0.33 \pm 0.14</math></b> | <b><math>1.07 \pm 0.11</math></b> | <b>0.8</b> |
| $n = 5$                   | $0.57 \pm 0.21$                   | $0.84 \pm 0.11$                   | 0.4        |
| All                       | $0.34 \pm 0.03$                   | $0.81 \pm 0.03$                   | 1.0        |

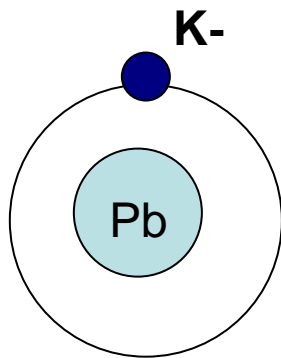
|           | Shift [eV]      | Width [eV] | Upper Width [eV]     | Ref                    |
|-----------|-----------------|------------|----------------------|------------------------|
| <b>Pb</b> | <b>-20 ± 12</b> | --         | --                   | <b>NPA254(75)381</b>   |
| Pb        | --              | 370 ± 140  | 4.1 ± 2. (Y=79 ± 8%) | J.P. Miller, Ph.D (75) |
| Pb        | +72+(+27-57)    | 284 ± 14   | Y=82 ± 4%            | PRC 40 (89) 2154       |



|           | Shift [eV]      | Width [eV] | Upper Width [eV]     | Ref                    |
|-----------|-----------------|------------|----------------------|------------------------|
| <b>Pb</b> | <b>-20 ± 12</b> | --         | --                   | <b>NPA254(75)381</b>   |
| Pb        | --              | 370 ± 140  | 4.1 ± 2. (Y=79 ± 8%) | J.P. Miller, Ph.D (75) |
| Pb        | +72+(+27-57)    | 284 ± 14   | Y=82 ± 4%            | PRC 40 (89) 2154       |

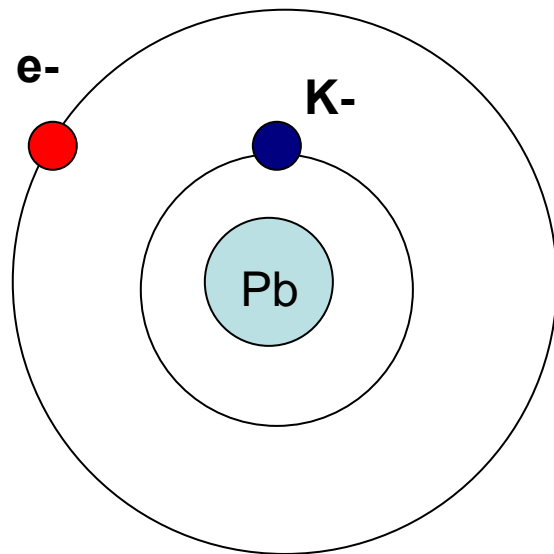
|   | Energy [keV]    | Shift [eV] | Kaon mass [MeV]                       |
|---|-----------------|------------|---------------------------------------|
| Measurement   | 426.181 ± 0.012 |            |                                       |
| Calculated  | 426.201         | -20 ± 12   | 493.715 (used)                        |
| [Optical model]   |                 | [-23]      |                                       |
| Measurement   |                 |            | 493.657 ± 0.020<br>(determined)       |
| Cal,+V(α <sup>2</sup> (Zα <sup>2</sup> ))<br>(but not sure) | 426.173 ± 0.012 | (+8 ± 17)  | 493.696 ± 0.025<br>(determined)       |
| Calculated<br><b>[PRC 40(89) 2154]</b>                      | 426.149         | +32 ± 12   | 493.6364 (used)                       |
| Calculated<br><b>[PRA 71 (2005)<br/>032501 ]</b>            | 426.180         | (+1 ± 12)  | 493.677 (used)<br>[current PDG value] |

|           | Shift [eV]      | Width [eV] | Upper Width [eV]     | Ref                    |
|-----------|-----------------|------------|----------------------|------------------------|
| <b>Pb</b> | <b>-20 ± 12</b> | --         | --                   | <b>NPA254(75)381</b>   |
| Pb        | --              | 370 ± 140  | 4.1 ± 2. (Y=79 ± 8%) | J.P. Miller, Ph.D (75) |
| Pb        | +72+(+27-57)    | 284 ± 14   | Y=82 ± 4%            | PRC 40 (89) 2154       |



Z(Pb)=82, so **total charge=+81** (=+82(Pb)-1(K-))  
very highly ionized.

**Target=solid**  
**electron refilling from surrounding atoms**  
**(+Auger e- emission)**



**Electron screening causes energy shifts,**  
**depending on the number of electrons in kaonic Pb**

|           | Shift [eV]      | Width [eV] | Upper Width [eV]     | Ref                    |
|-----------|-----------------|------------|----------------------|------------------------|
| <b>Pb</b> | <b>-20 ± 12</b> | --         | --                   | <b>NPA254(75)381</b>   |
| Pb        | --              | 370 ± 140  | 4.1 ± 2. (Y=79 ± 8%) | J.P. Miller, Ph.D (75) |
| Pb        | +72+(+27-57)    | 284 ± 14   | Y=82 ± 4%            | PRC 40 (89) 2154       |

TABLE 8  
Calculated theoretical K<sup>-</sup> Pb transition energies

| Transition | Zeroth order<br>(keV) | $\alpha(Z\alpha)$<br>(eV) | $\alpha(Z\alpha)$ |                     | $\alpha^2(Z\alpha)$<br>(eV) | $\alpha(Z\alpha)^3$<br>(eV) | $\alpha(Z\alpha)^{5.7}$<br>(eV) | Finite size high order<br>(eV) | Electron screen<br>(eV) | Nuclear pol.<br>(eV) | $E_{calc}$<br>(keV) |
|------------|-----------------------|---------------------------|-------------------|---------------------|-----------------------------|-----------------------------|---------------------------------|--------------------------------|-------------------------|----------------------|---------------------|
|            |                       |                           | iter.<br>(eV)     | finite size<br>(eV) |                             |                             |                                 |                                |                         |                      |                     |
| 13 → 12    | 90.697                | 284.0                     | 0.5               | 0.4                 | 2.0                         | -7.7                        | -1.1                            | 0.5                            | -51.8                   | 0                    | 90.924              |
| 12 → 11    | 116.575               | 419.5                     | 0.8               | 0.8                 | 2.9                         | -10.6                       | -1.5                            | 0.8                            | -44.9                   | 0                    | 116.943             |
| 11 → 10    | 153.328               | 632.8                     | 1.4               | 1.5                 | 4.3                         | -14.7                       | -2.1                            | 1.2                            | -38.2                   | 1                    | 153.916             |
| 10 → 9     | 207.340               | 980.0                     | 2.3               | 3.1                 | 6.7                         | -20.7                       | -3.0                            | 2.1                            | -31.9                   | 2                    | 208.280             |
| 9 → 8      | 290.081               | 1569.5                    | 3.9               | 7.1                 | 10.9                        | -29.8                       | -4.3                            | 3.6                            | -25.9                   | 5                    | 291.621             |
| 8 → 7      | 423.579               | 2625.5                    | 7.2               | 18.2                | 18.6                        | -44.5                       | -6.4                            | 5.9                            | -20.4                   | 18                   | 426.201             |

NPA254(75)381 **assumed two 1S electrons** (but how evaluated?)  
Electron screening = **20 eV** at 8-7 transition

# Electron screening effects

PRA 71 (2005) 032501

| Nucleus | Transition<br>$n_i \rightarrow n_f$ | Shift [eV] |      |      |      |      |      | $\Delta E_m$ |
|---------|-------------------------------------|------------|------|------|------|------|------|--------------|
|         |                                     | No Elect.  | [H]  | [He] | [Li] | [Be] | [Ar] |              |
| Be      | 4 $\rightarrow$ 3                   | 0.5        | 0.5  | 0.5  | 0.5  |      |      | 1            |
| Al      | 9 $\rightarrow$ 8                   | -1.0       | -0.4 | 0    | 0.1  |      |      | 1            |
| Pb      | 8 $\rightarrow$ 7                   | -9         | 0    | 8    | 10   | 11   | 12   | 12           |
|         | 9 $\rightarrow$ 8                   | -49        | -37  | -26  | -25  | -23  | -22  | 13           |
|         | 10 $\rightarrow$ 9                  | -41        | -26  | -12  | -10  | -8   | -7   | 8            |
|         | 11 $\rightarrow$ 10                 | -50        | -33  | -16  | -13  | -10  | -9   | 11           |
|         | 12 $\rightarrow$ 11                 | -27        | -6   | 14   | 17   | 20   | 22   | 10           |
|         | 13 $\rightarrow$ 12                 | -40        | -10  | 8    | 12   | 15   | 17   | 15           |

**M\_K=493.677 (current PDG value)**; [H]=1 1S electron, [He]=2 1S electrons, etc.

|           | Shift [eV]      | Width [eV] | Upper Width [eV]     | Ref                    |
|-----------|-----------------|------------|----------------------|------------------------|
| <b>Pb</b> | <b>-20 ± 12</b> | --         | --                   | <b>NPA254(75)381</b>   |
| Pb        | --              | 370 ± 140  | 4.1 ± 2. (Y=79 ± 8%) | J.P. Miller, Ph.D (75) |
| Pb        | +72+(+27-57)    | 284 ± 14   | Y=82 ± 4%            | PRC 40 (89) 2154       |

|             | Energy [keV] | Shift [eV] | Kaon mass [MeV]                 |
|-------------|--------------|------------|---------------------------------|
| Measurement |              |            | 493.657 ± 0.020<br>(determined) |

### Calibration done by 75Se and 198Au gamma-rays

Yu.M. Ivanov, representing DENISOV 91, has estimated corrections needed for the older experiments because of **improved 192Ir and 198Au calibration  $\gamma$ -ray energies**. He estimates that **CHENG 75** and **BACKENSTOSS 73 mK ±** values could be raised by **about 15 keV** and 22 keV, respectively

PDG

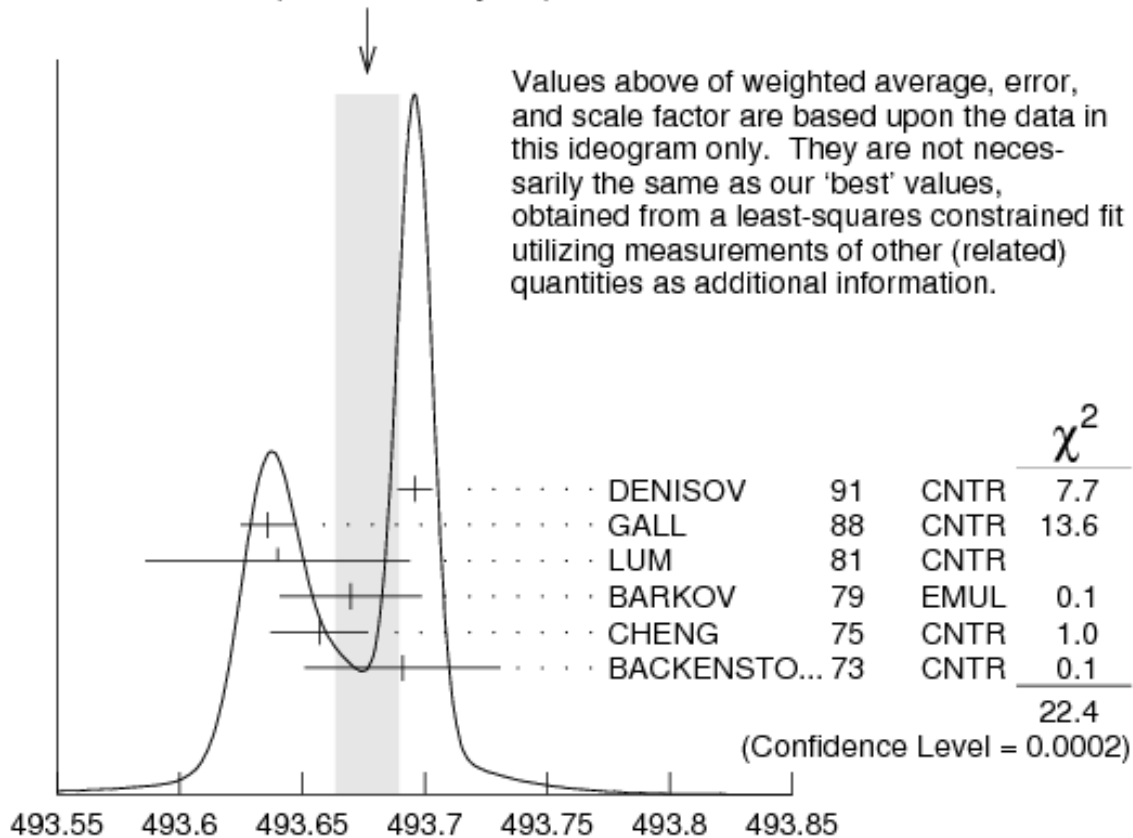
➡ **If so, need recalculation of shift**

In addition, Nuclear gamma-ray contaminations??  
(K-Pb X-rays + gamma-rays in a peak area)



|           | Shift [eV]                          | Width [eV]                     | Upper Width [eV]                 | Ref                     |
|-----------|-------------------------------------|--------------------------------|----------------------------------|-------------------------|
| Pb        | $-20 \pm 12$                        | --                             | --                               | NPA254(75)381           |
| Pb        | --                                  | $370 \pm 140$                  | $4.1 \pm 2.$ (Y= $79 \pm 8\%$ )  | J.P. Miller, Ph.D (75)  |
| <b>Pb</b> | <b><math>+72^{+(+27-57)}</math></b> | <b><math>284 \pm 14</math></b> | <b>Y=<math>82 \pm 4\%</math></b> | <b>PRC 40 (89) 2154</b> |

WEIGHTED AVERAGE  
 $493.677 \pm 0.013$  (Error scaled by 2.4)



**Group of PRC is same as  
Gall PRL60(88)186  
M\_K= $493.636 \pm 0.011$  MeV**

**Shift was evaluated using  
this value.**

|           | Shift [eV]          | Width [eV]                     | Upper Width [eV]                 | Ref                     |
|-----------|---------------------|--------------------------------|----------------------------------|-------------------------|
| Pb        | $-20 \pm 12$        | --                             | --                               | NPA254(75)381           |
| Pb        | --                  | $370 \pm 140$                  | $4.1 \pm 2.$ (Y= $79 \pm 8\%$ )  | J.P. Miller, Ph.D (75)  |
| <b>Pb</b> | <b>+72+(+27-57)</b> | <b><math>284 \pm 14</math></b> | <b>Y=<math>82 \pm 4\%</math></b> | <b>PRC 40 (89) 2154</b> |

### Calibration problem?

#### 511 keV line is not reliable for energy calibration?

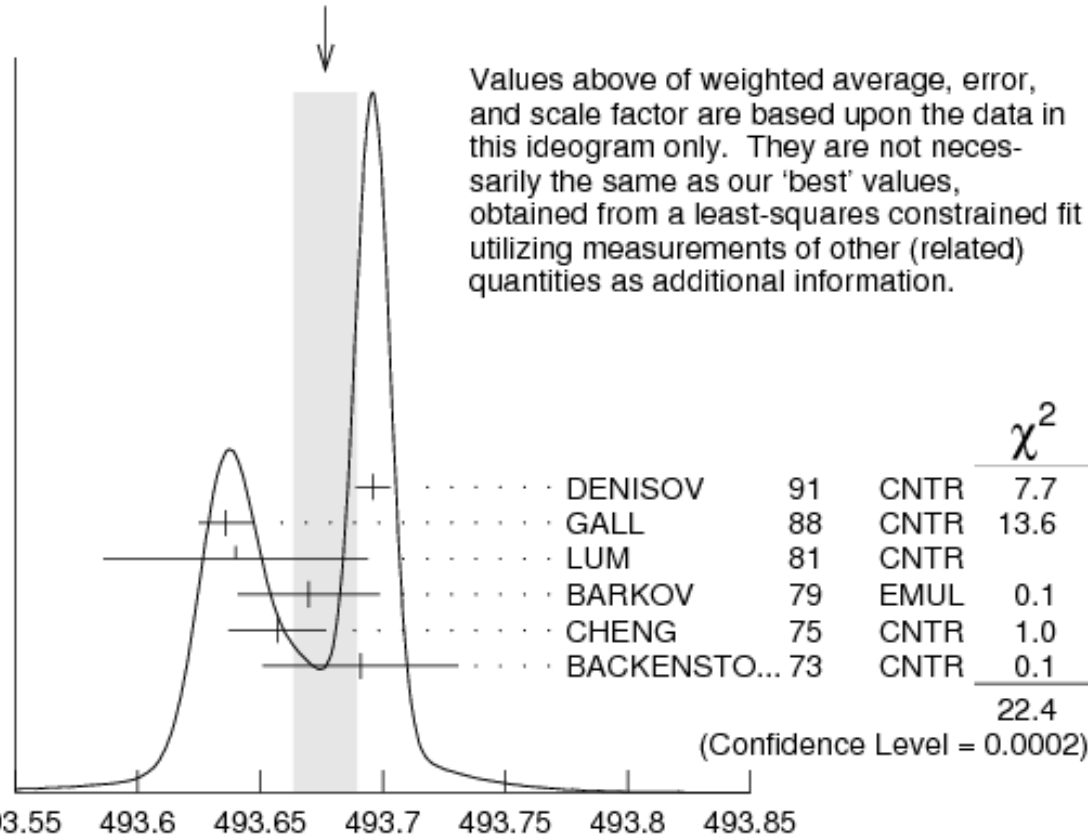
These uncertainties resulted in part because **the e+e- 511 keV annihilation line does not provide a reliable energy calibration.** The energy may be shifted by an amount which depends on the specific material in which the positron annihilates.

**Depending on possible calibration methods, the error was increased.**

**However, positive shift is inconsistent with other lighter nuclei.**

|           | Shift [eV]                          | Width [eV]                     | Upper Width [eV]                 | Ref                     |
|-----------|-------------------------------------|--------------------------------|----------------------------------|-------------------------|
| Pb        | $-20 \pm 12$                        | --                             | --                               | NPA254(75)381           |
| Pb        | --                                  | $370 \pm 140$                  | $4.1 \pm 2.$ (Y= $79 \pm 8\%$ )  | J.P. Miller, Ph.D (75)  |
| <b>Pb</b> | <b><math>+72^{+(+27-57)}</math></b> | <b><math>284 \pm 14</math></b> | <b>Y=<math>82 \pm 4\%</math></b> | <b>PRC 40 (89) 2154</b> |

WEIGHTED AVERAGE  
 $493.677 \pm 0.013$  (Error scaled by 2.4)



**Gall PRL60(88)186**  
**M\_K=493.636 ± 0.011 MeV**

**Denisov JETPL 54(91)558**  
**[with new error estimation\*]**  
**M\_K=493.696 ± 0.007 MeV**

\*Method unpublished  
 [Only thesis]

**Reconsideration of Pb**

**NPA585(95)229c**

|           | Shift [eV]          | Width [eV]                     | Upper Width [eV]                 | Ref                     |
|-----------|---------------------|--------------------------------|----------------------------------|-------------------------|
| Pb        | $-20 \pm 12$        | --                             | --                               | NPA254(75)381           |
| Pb        | --                  | $370 \pm 140$                  | $4.1 \pm 2.$ (Y= $79 \pm 8\%$ )  | J.P. Miller, Ph.D (75)  |
| <b>Pb</b> | <b>+72+(+27-57)</b> | <b><math>284 \pm 14</math></b> | <b>Y=<math>82 \pm 4\%</math></b> | <b>PRC 40 (89) 2154</b> |

### NPA585(95)229c

| Transition | $\epsilon_{\text{meas}}$ | (1)<br>$\epsilon_{\text{calc}}$ | (2)<br>$\epsilon_{\text{corr}}$ | (3)<br>$\epsilon_{\text{corr}}$ |
|------------|--------------------------|---------------------------------|---------------------------------|---------------------------------|
| W (8 → 7)  | $0.079 \pm 0.025$        | -0.003                          | $0.024 \pm 0.025$               | $-0.020 \pm 0.025$              |
| W (7 → 6)  | $-0.353 \pm 0.092$       | -0.967                          | $-0.744 \pm 0.092$              | $-0.811 \pm 0.092$              |
| Pb (8 → 7) | $0.072 \pm 0.057$        | -0.023                          | $0.072 \pm 0.057$               | $0.018 \pm 0.057$               |
| U (8 → 7)  | $-0.40 \pm 0.17$         | -0.189                          | $-0.40 \pm 0.17$                | $-0.47 \pm 0.17$                |
|            | $\chi^2 = 66$            |                                 | 17                              | 12                              |

All values are in keV.

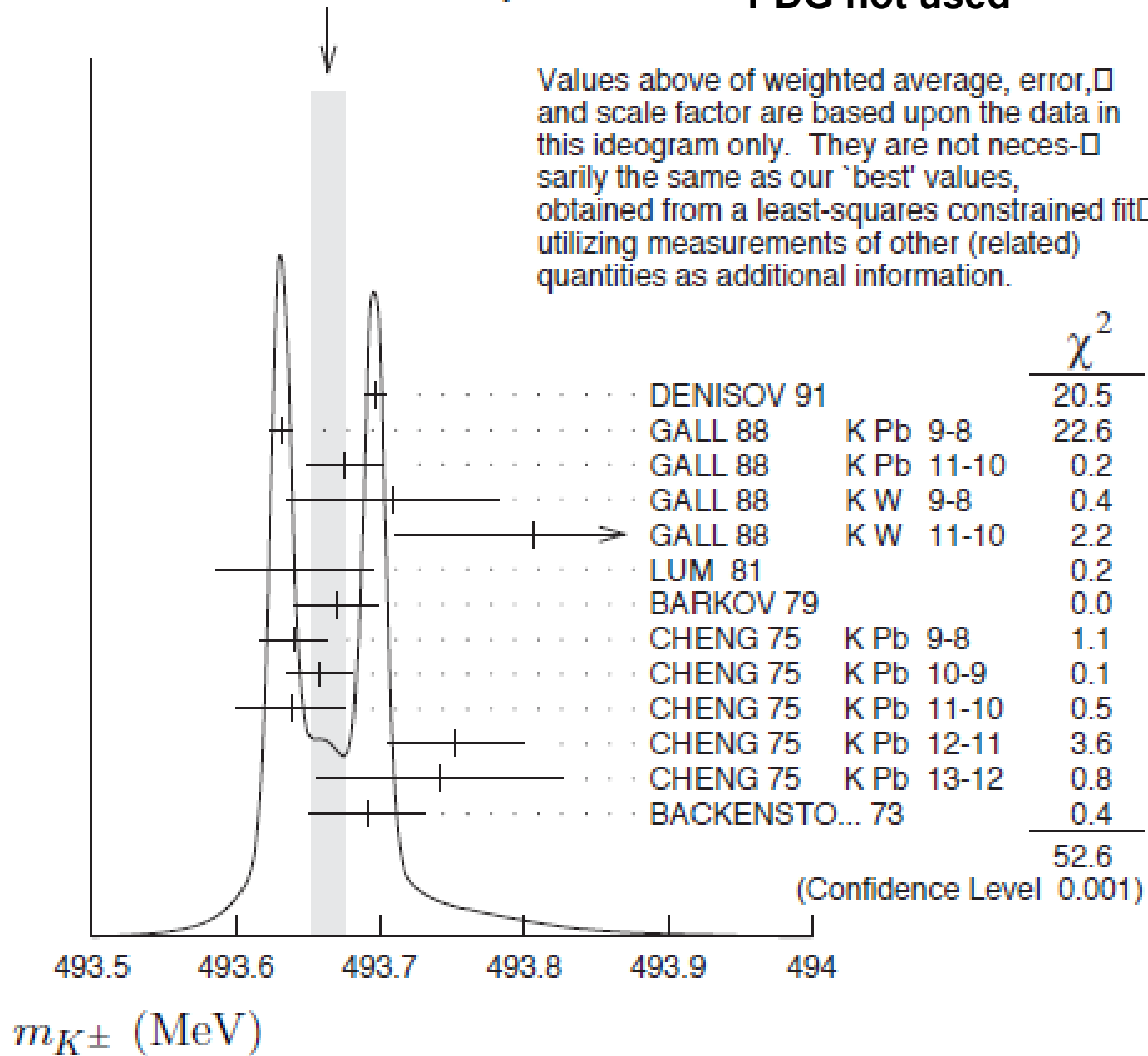
- (1) Optical model prediction
- (2) Including E2 mixing effects
- (3) Using Kmass by Denisov

**Anyway, PRC data are not used for optical model calculations! Even now..**

WEIGHTED AVERAGED  
 $493.664 \pm 0.011$  (Error scaled by 2.5)

PDG not used

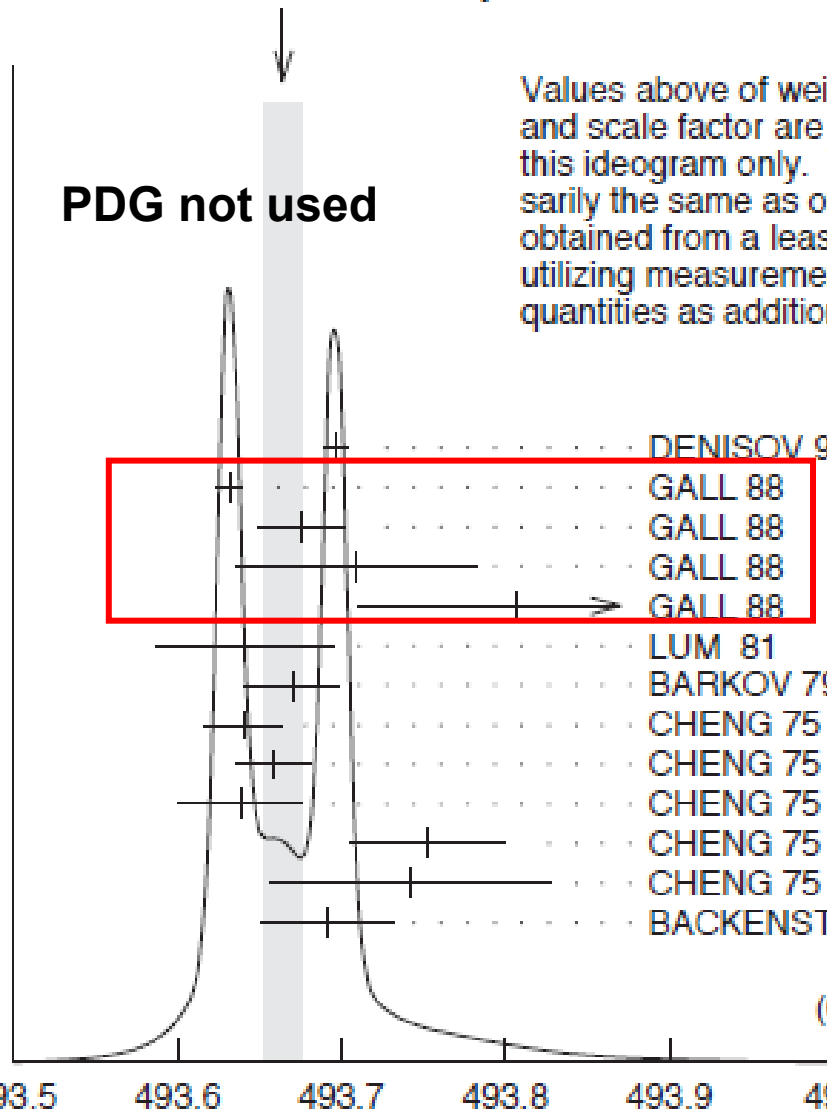
Values above of weighted average, error,  $\chi^2$  and scale factor are based upon the data in this ideogram only. They are not necessarily the same as our 'best' values, obtained from a least-squares constrained fit utilizing measurements of other (related) quantities as additional information.



WEIGHTED AVERAGED  
 $493.664 \pm 0.011$  (Error scaled by 2.5)

PDG not used

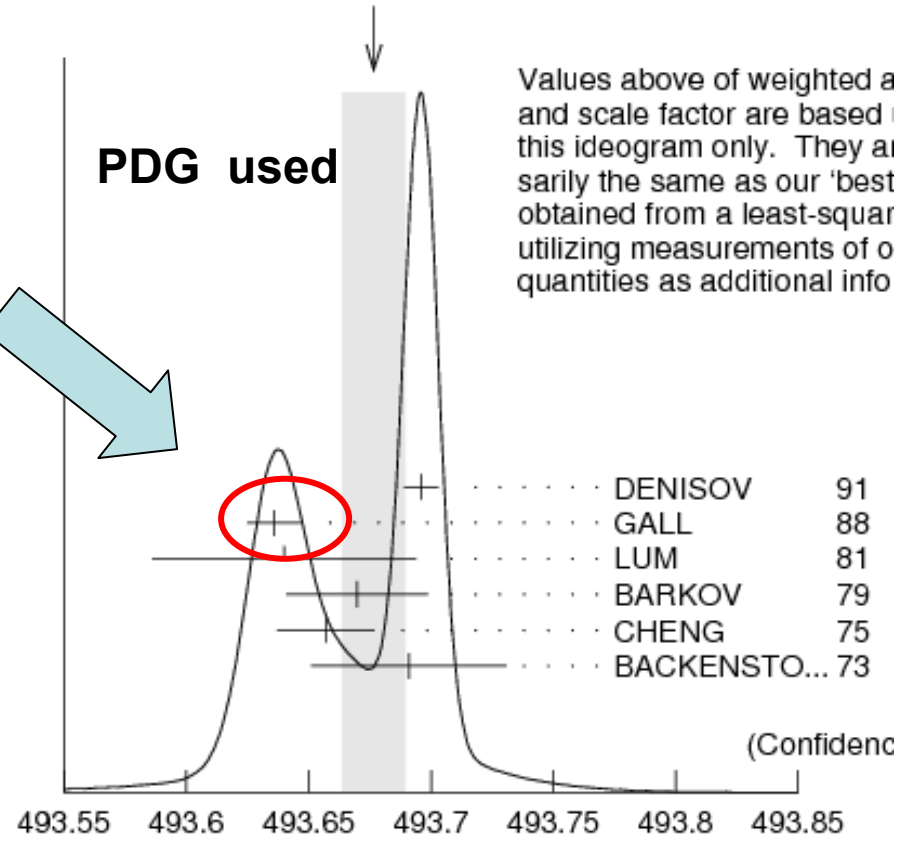
Values above of weighted average, error, and scale factor are based upon the data in this ideogram only. They are not necessarily the same as our 'best' values obtained from a least-squares utilizing measurements of quantities as additional



WEIGHTED AVERAGE  
 $493.677 \pm 0.013$  (Error scaled by 2.4)

PDG used

Values above of weighted average and scale factor are based upon the data in this ideogram only. They are not necessarily the same as our 'best' values obtained from a least-squares utilizing measurements of quantities as additional info



$m_{K^\pm}$  (MeV)

# Best way for K-Pb

- High resolution  
better than 200 eV upto 400 keV energy  
to avoid parallel & nuclear-gamma
- Gaseous Pb target  
to avoid electron screening

However, it is unrealistic!?

How to estimate possible higher order radiative corrections..

## Kaon Mass used for determination of shifts

|           | Kaon Mass [MeV]               | Ref              |
|-----------|-------------------------------|------------------|
| C         | 493.73                        | PLB 38 (72) 181  |
| Si,Ni     | $493.715 \pm 0.037$           | NPA 231 (74) 477 |
| Pb        | 493.715                       | NPA 254 (75) 381 |
| Si,Ni,Sn  | 493.707                       | NPA 329 (79) 407 |
| Pb        | 493.6364                      | PRC 40 (89) 2154 |
| PDG       | $493.677 \pm 0.016$ [our fit] | PRD86(12)821     |
| C (4f-3d) | $493.696 \pm 0.007$           | JETPL 54(91)558  |
| Pb (9-8)  | $493.631 \pm 0.007$           | PRL 60 (88) 186  |

**Need corrections for shift evaluation!**



# To do lists

- We need new measurements
- **How accurately & precisely?**
  - Theoretical requirements: errors of xx eV for KC,, xx eV for KPb
- **What physics we can get?**
  - **Potential can be determined with xxx MeV accuracy**
  - **Exclude deep potential with xxx sigma**
  - **Information for kaonic nuclei**
- **Above important for making goal of experiments**
- Selection of detector (Ge, CdZnTe,...)
  - Need Compton suppressors (NaI, BGO,..)?
  - Detector performance (stability, energy resolution, linearity, etc)
  - Calibration methods (some radioactive sources)
- DAFNE or J-PARC?
- Need Monte Carlo studies and Detector tests
- Request for beam time (How long?)

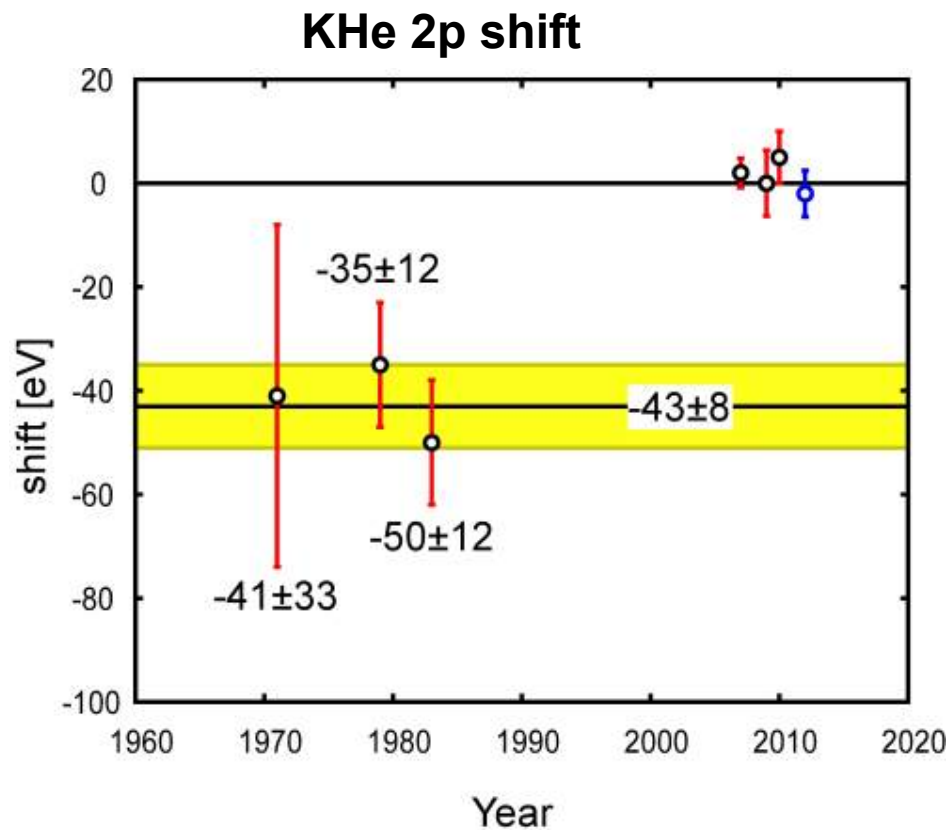
5 shifts and 10 widths determined from C,Si,Ni,Sn,Pb

→ Determine parameters

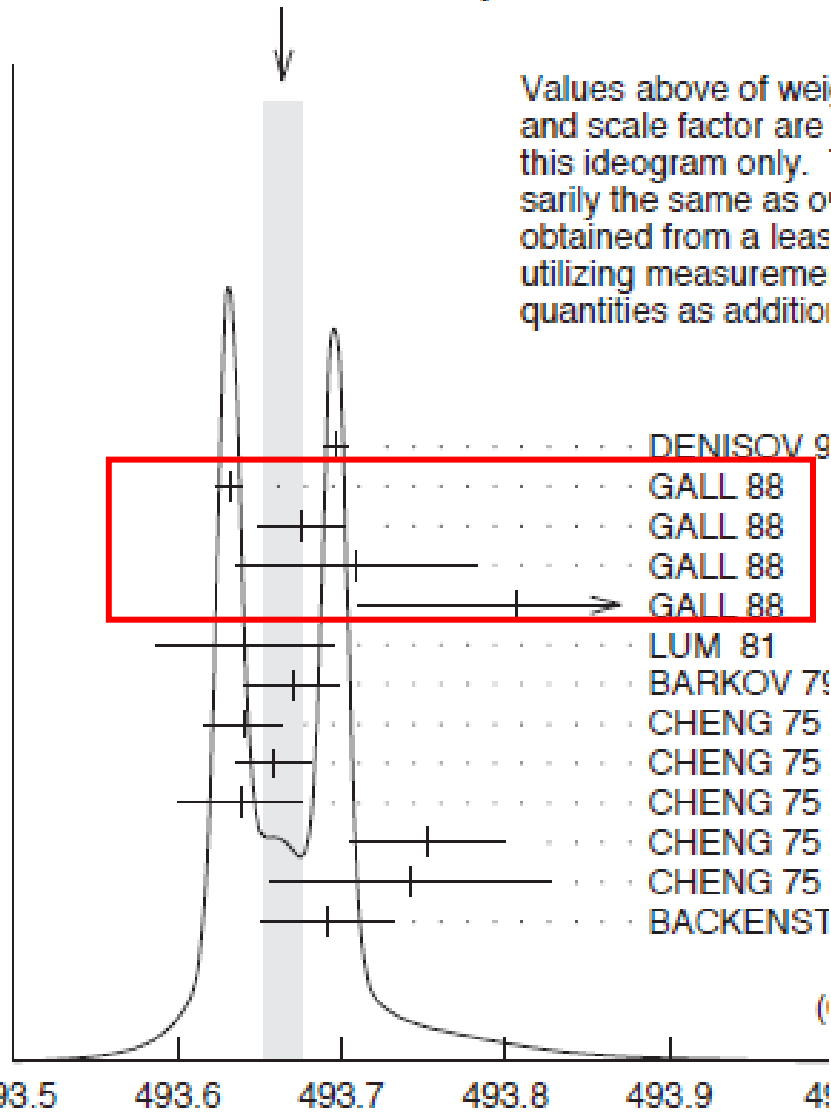
→ Be careful for determination of

**accurate central values & systematic errors**

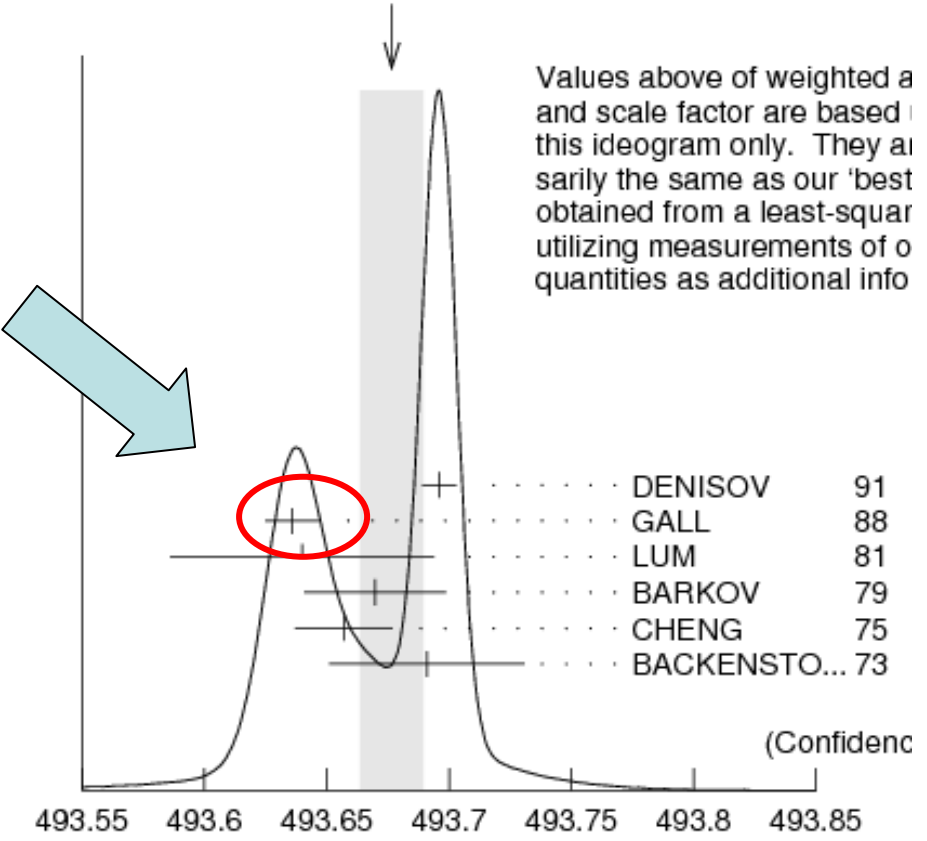
which can be used for “averaging” results of C,Si,Ni,Sn,Pb



WEIGHTED AVERAGED  
 $493.664 \pm 0.011$  (Error scaled by 2.5)



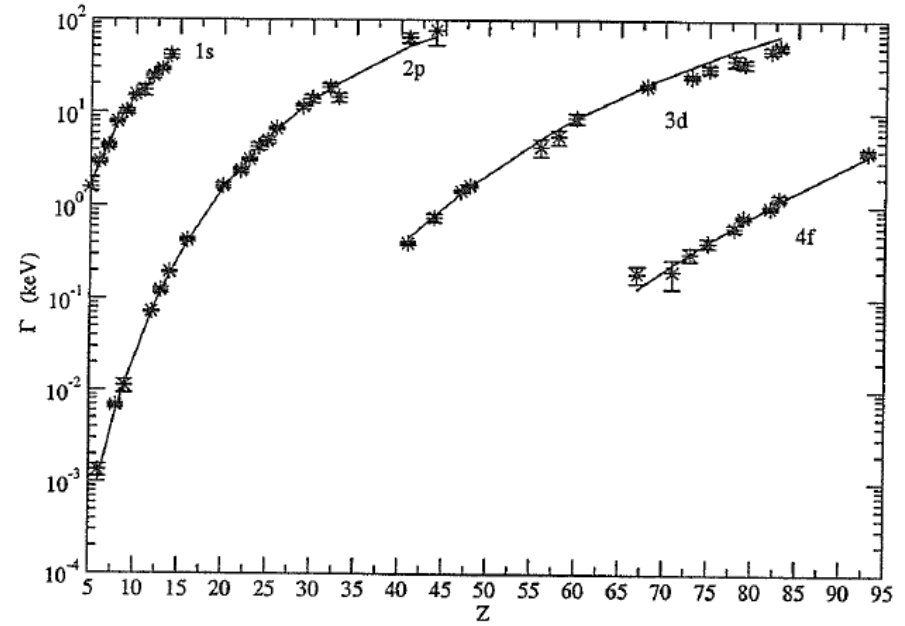
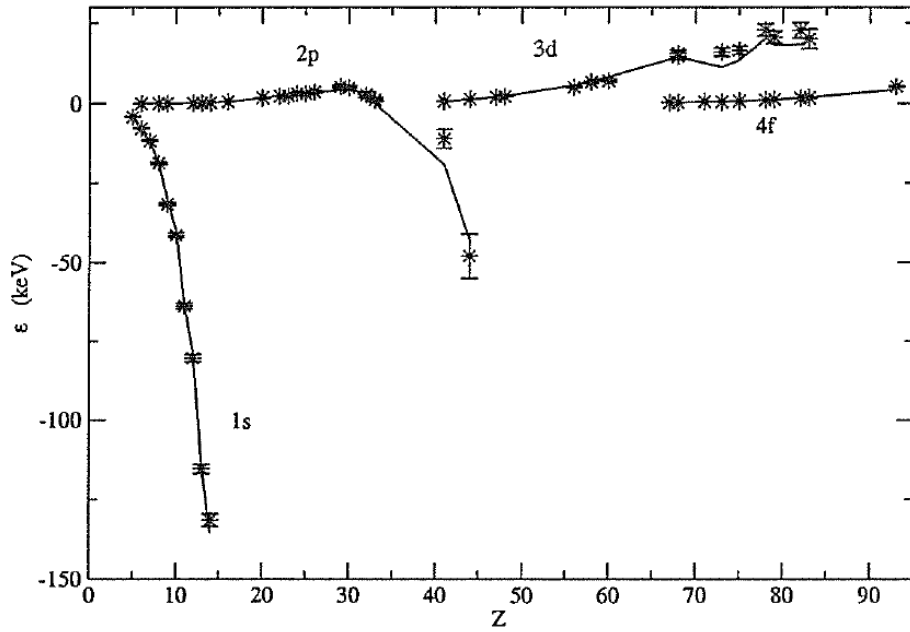
WEIGHTED AVERAGE  
 $493.677 \pm 0.013$  (Error scaled by 2.4)



$m_{K^\pm}$  (MeV)

# Further steps

- Do we need more data set?
- Isotope differences such as  $^{208}\text{Pb}$ ;  $^{112}\text{Sn}$ ,  $^{116}\text{Sn}$ ,  $^{120}\text{Sn}$ ,  $^{124}\text{Sn}$ ;  $^{40}\text{Ca}$ .. $^{48}\text{Ca}$  (suggested by Wycheck); O isotopes (by Gal)
- According to Wycheck, the widths can be related to the subthreshold region of  $\Lambda(1405)$
- Yb E2 mixing problem
- Anyway, the Kaonic atom data are still poor than the pionic atom data.



simulating absorptive effects. This “standard” optical potential takes the form

$$2\mu V(r) = -4\pi \left[ q(r) - \nabla \cdot \frac{\alpha(r)}{1 + (4\pi/3)\xi\alpha(r)} \nabla \right], \quad (16.62)$$

where  $\mu$  is  $\pi$ -nucleus reduced mass and

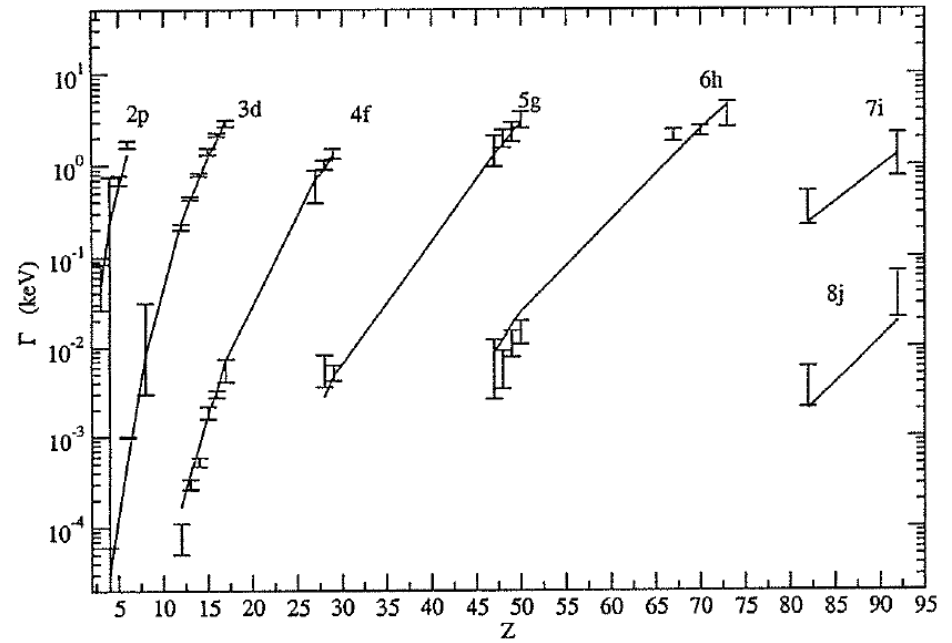
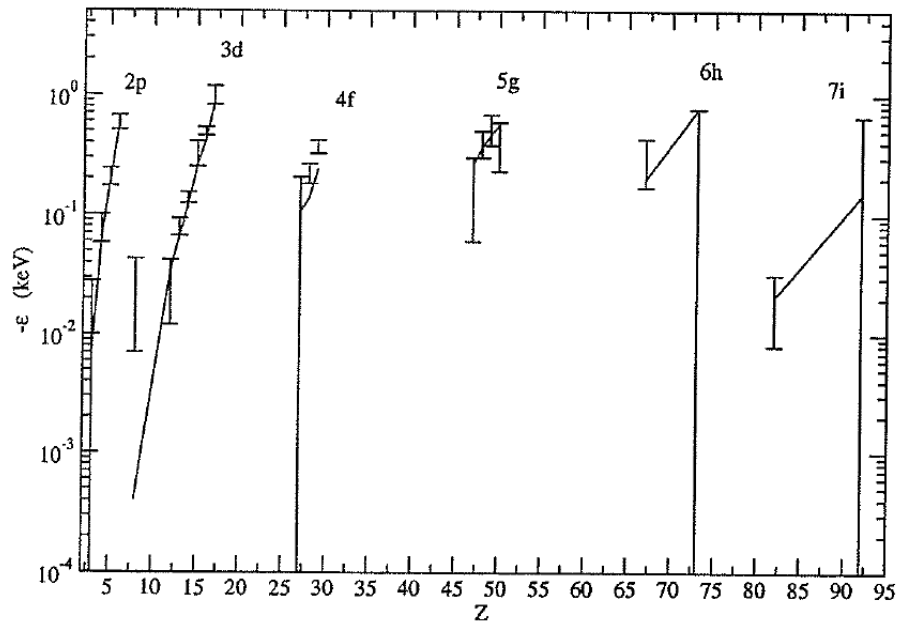
$$q(r) = \left(1 + \frac{m}{M}\right) [b_0 \rho(r) + b_1 \delta\rho(r)] + \left(1 + \frac{m}{2M}\right) B_0 \rho(r)^2; \quad (16.63)$$

## Pionic atom

$$\alpha(r) = \left(1 + \frac{m}{M}\right)^{-1} [c_0 \rho(r) + c_1 \delta\rho(r)] + \left(1 + \frac{m}{2M}\right)^{-1} C_0 \rho(r)^2; \quad (16.64)$$

where  $m$  and  $M$  denote the masses of the pion and the nucleon, respectively. We have introduced the densities

$$\rho(r) = \rho_n(r) + \rho_p(r); \quad \delta\rho(r) = \rho_n(r) - \rho_p(r); \quad (16.65)$$



The simplest class of optical potentials  $V_{\text{opt}}$  is the generic  $t\rho(r)$  potential: (isoscalar)

### Kaonic atom

$$2\mu V_{\text{opt}}(r) = -4\pi\left(1 + \frac{A-1}{A}\frac{\mu}{M}\right)b_0[\rho_n(r) + \rho_p(r)]$$

$\rho_n$  and  $\rho_p$  are neutron and proton densities normalized to  $N$  and  $Z$ , respectively,  $M$  is the mass of the nucleon.

**Let's think about new experiments  
on heavy kantic atoms**