Gravitational interactions of the Higgs boson

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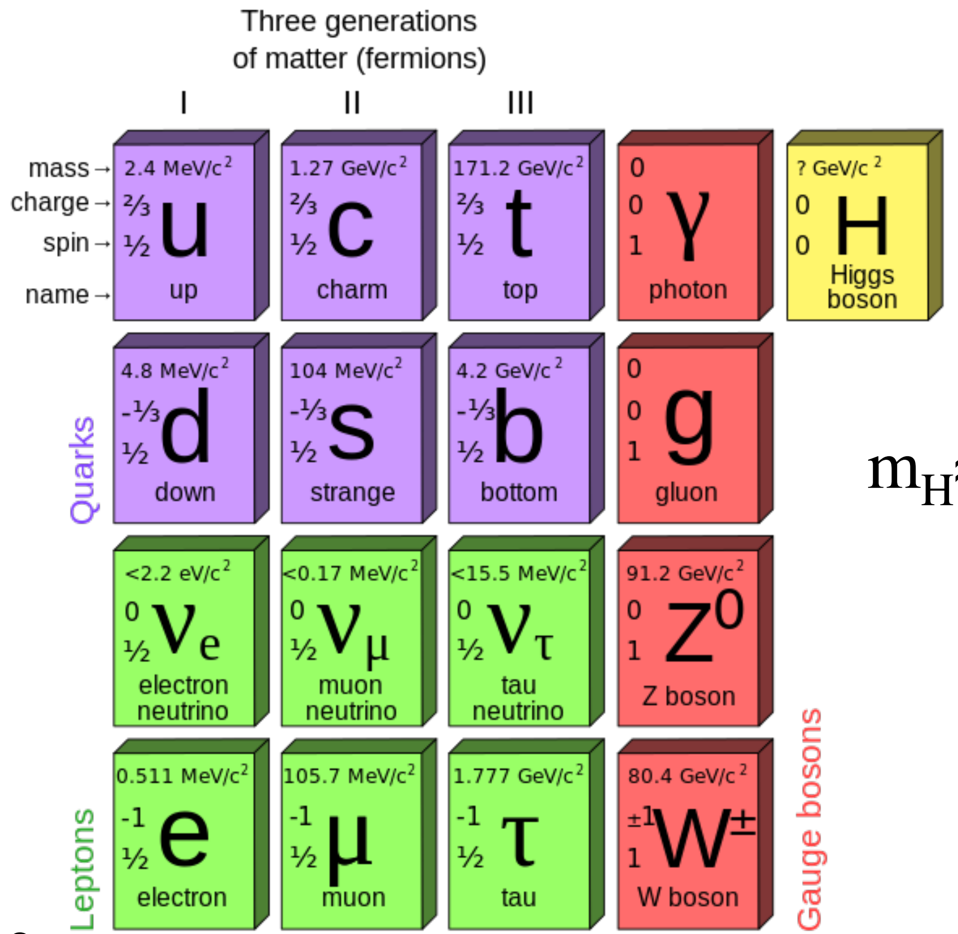
Outline

- Introduction
- Higgs boson's nonminimal coupling to R
- How does the nonminimal coupling of the Higgs impact physics?
  - Running of the Planck mass
  - Higgs inflation
  - Unitarity
- Frame dependence of gravitational theories

We live at an exciting time!

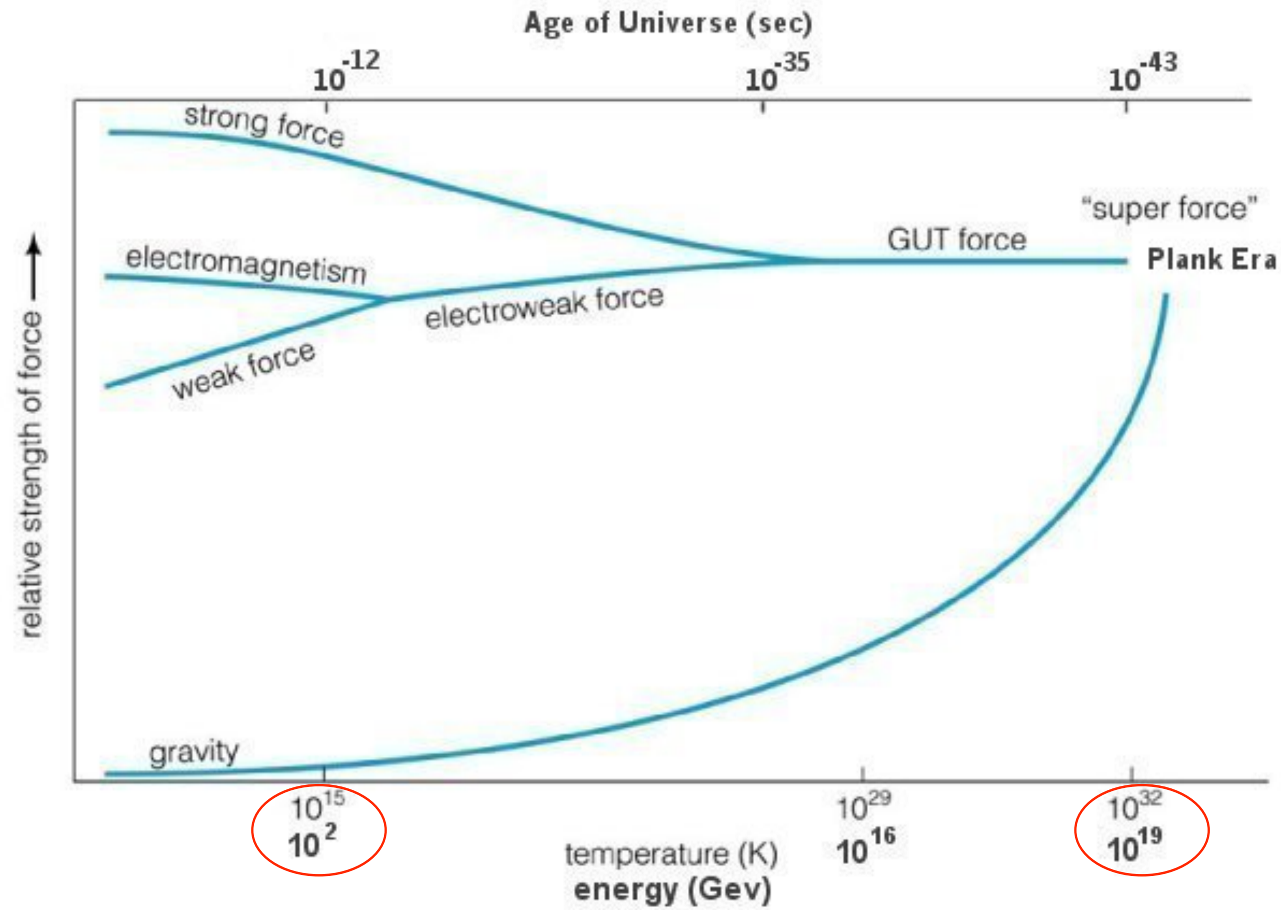


$m_H \approx 125$ GeV

However no sign of new physics

Why is the Higgs boson so light?

Or why is gravity so much weaker than the forces?



Is there something special about the Higgs boson from a gravitational point of view?

Is the Higgs boson the source of problems?

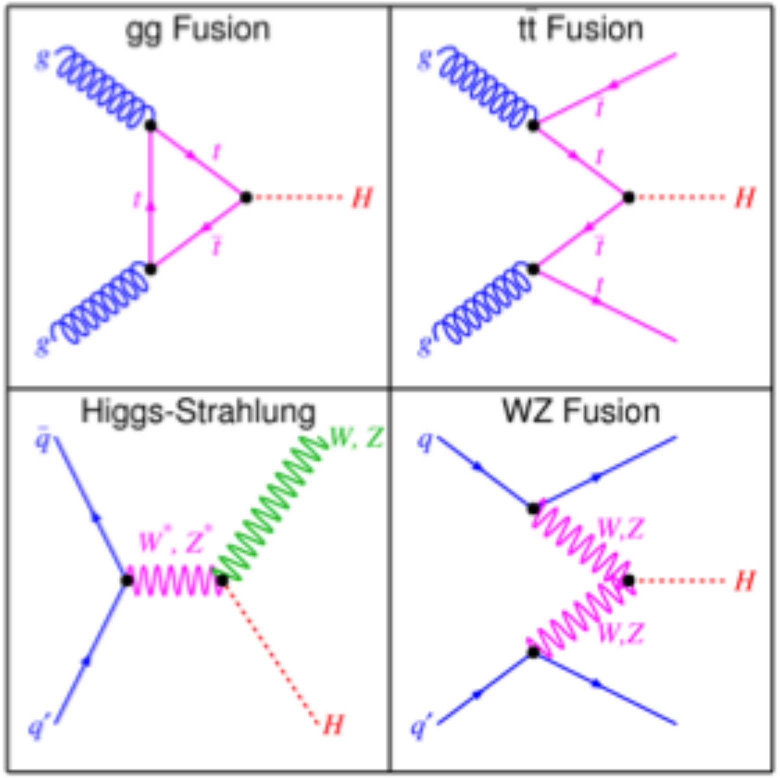
- Dominant point of view for the last 40 years: the Higgs boson’s mass is not stable in the SM and should be protected by a symmetry (or there is no fundamental scalar)
- My point of view: it is not a question that we can address within our current theories of physics.
- We are only dealing with renormalizable theories: the Higgs mass is not calculable.
- Wilson called the hierarchy problem a blunder!
- Note that the lack of new physics at the LHC could be the second nail in the coffin for naturalness after the cosmological constant.

Or rather is the Higgs boson the solution to other fine tuning problems?

- It is difficult to imagine why our universe is so flat and homogenous.
- Why are there no monopols?
- What is at the origin of the cosmological perturbations?
- This is really an initial condition problem.
- Either we had very special initial conditions or something created them: inflation.
- Could the Higgs boson be the inflaton?
- Obviously inflation with scalar fields also has fine-tuning/ stability issues.

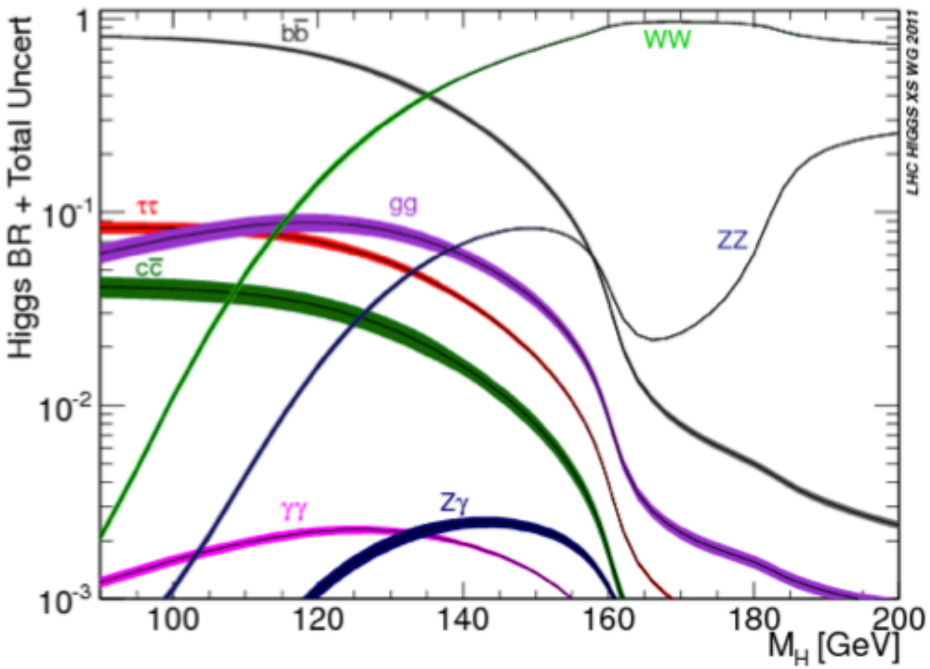
It is the Higgs boson

The Standard Model predicts precisely how the Higgs boson should be produced

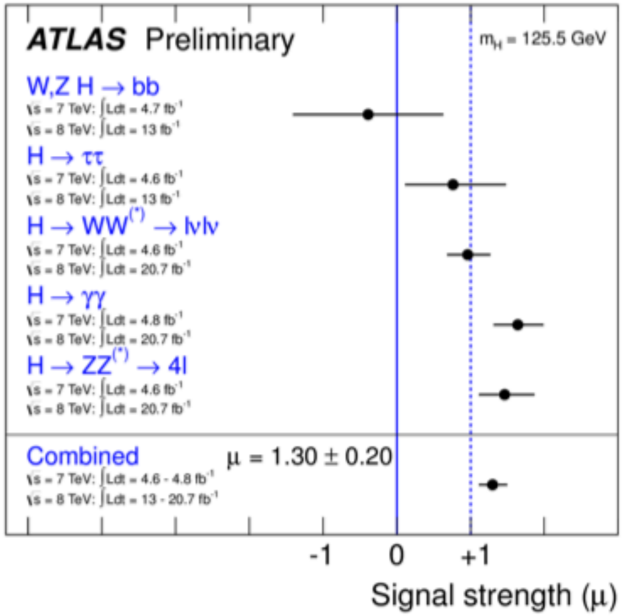


It is the Higgs boson

The Standard Model predicts precisely how the Higgs boson should decay



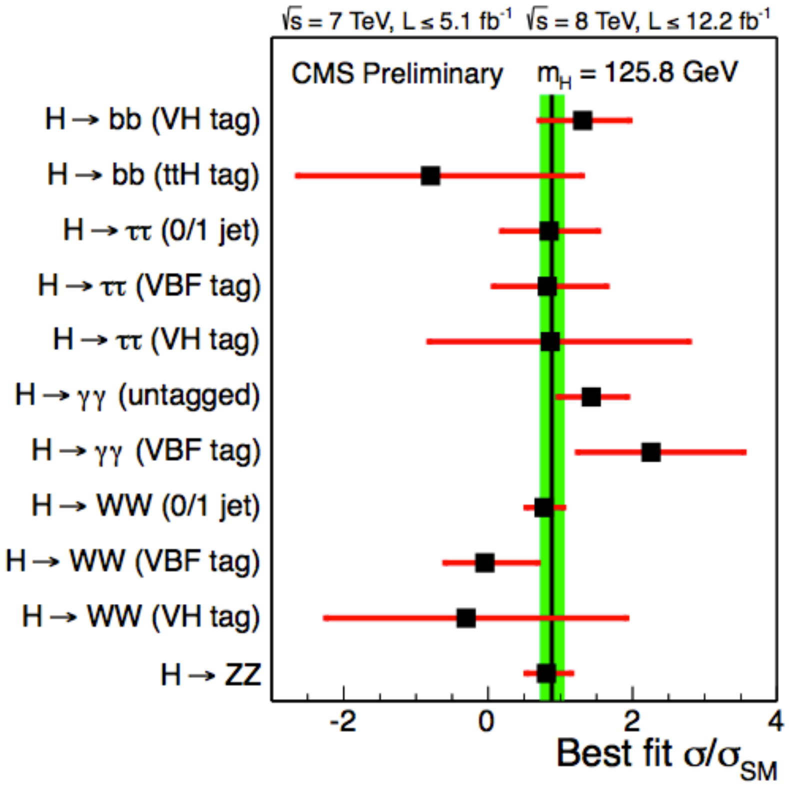
It is the Higgs boson



From arxiv:1305.3315

FIG. 19. Measurements of the signal strength parameter μ for $m_H = 125.5\,\mathrm{GeV}$ for the individual channels and for their combination [8].

It is the Higgs boson



From arxiv:1305.4775

Something special about the Higgs boson

- It can be coupled in a nonminimal way to gravity.

$$S \supset \int d^4x \sqrt{-g}\, \xi H^\dagger H \mathcal{R},$$

- This is a dimension 4 operator: it is a fundamental constant of nature.
- Is there any bound on its value?

The decoupling effect

- Let's consider the SM with a nonminimal coupling to R

$$S = -\int d^4x\,\sqrt{-g}\left[\left(\frac{1}{2}M^2 + \xi H^\dagger H\right)R - (D^\mu H)^\dagger(D_\mu H) + \mathcal{L}_{SM} + \mathcal{O}(M_P^{-2})\right]$$

- We can always go from the Jordan frame to the Einstein frame

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$$

$$\tilde{g}^{\mu\nu} = \Omega^{-2} g^{\mu\nu}\,, \quad \sqrt{-\tilde{g}} = \Omega^d \sqrt{-g}\,.$$

$$R = \Omega^2\left[\tilde{R} - 2(n-1)\tilde{\Box}\omega - (n-1)(n-2)\tilde{g}^{\mu\nu}\partial_\mu\omega\partial_\nu\omega\right]$$

$$\omega \equiv \ln\Omega\,, \quad \tilde{\Box}\omega = \frac{1}{\sqrt{-\tilde{g}}}\partial_\mu(\sqrt{-\tilde{g}}\,\tilde{g}^{\mu\nu}\partial_\nu\omega)$$

$$\Omega^2 = (M^2 + 2\xi H^\dagger H)/M_P^2$$

The decoupling effect

- In the Einstein frame, the action reads

$$S = -\int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2} M_P^2 \tilde{R} - \frac{3\xi^2}{M_P^2 \Omega^4} \partial^\mu (H^\dagger H) \partial_\mu (H^\dagger H) - \frac{1}{\Omega^2} (D^\mu H)^\dagger (D_\mu H) + \frac{\mathcal{L}_{SM}}{\Omega^4} \right]$$

- One notices that the Higgs boson kinetic term is not canonically normalized. We need to diagonalize this term.

- Let me now use the unitary gauge

$$H = \frac{1}{\sqrt{2}} (0, h + v)^\top$$

- The Planck mass is defined by

$$(M^2 + \xi v^2) = M_P^2$$

The decoupling effect

- To diagonalize the Higgs boson kinetic term:

$$\frac{d\chi}{dh} = \sqrt{\frac{1}{\Omega^2} + \frac{6\xi^2 v^2}{M_P^2 \Omega^4}}$$

- To leading order in Ω^{-1} $\qquad \Omega^2 = (M^2 + 2\xi H^\dagger H)/M_P^2$

$$h = \frac{1}{\sqrt{1+\beta}}\,\chi \qquad \beta = 6\xi^2 v^2/M_P^2$$

The decoupling effect

- The couplings of the Higgs boson to particles of the SM are rescaled! E.g.

$$y h \bar{\psi} \psi \to \frac{y}{\sqrt{1+\beta}} \chi \bar{\psi} \psi$$

- For a large nonminimal coupling, the Higgs boson decouples from the Standard Model:

$$\xi^2 \gg M_P^2 / v^2 \simeq 10^{32}$$

The decoupling effect

- The decoupling can also be seen in the Jordan frame:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$\mathcal{L}^{(2)} = -\frac{M^2 + \xi v^2}{8}\left(h^{\mu\nu}\Box h_{\mu\nu} + 2\partial_\nu h^{\mu\nu}\partial^\rho h_{\mu\rho} - 2\partial_\nu h^{\mu\nu}\partial_\mu h^\rho_\rho - h^\mu_\mu \Box h^\nu_\nu\right) + \frac{1}{2}(\partial_\mu h)^2 + \xi v(\Box h^\mu_\mu - \partial_\mu\partial_\nu h^{\mu\nu})h.$$

$$h = \frac{1}{\sqrt{1+\beta}}\chi,$$

$$h_{\mu\nu} = \frac{1}{M_P}\tilde{h}_{\mu\nu} - \frac{2\xi v}{M_P^2\sqrt{1+\beta}}\bar{g}_{\mu\nu}\chi.$$

same renormalization factor!

Bound on the nonminimal coupling from the LHC

- The LHC experiments produce fits to the data assuming that all Higgs boson couplings are modified by a single parameter (arXiv:1209.0040 [hep-ph]):

$$\kappa = 1/\sqrt{1+\beta}$$

- In the narrow width approximation, one finds:

$$\begin{aligned}\sigma(ii \to h \to ff) &= \sigma(ii \to h) \cdot \mathrm{BR}(h \to ff) \\ &= \kappa^2\, \sigma_{\mathrm{SM}}(ii \to h) \cdot \mathrm{BR}_{\mathrm{SM}}(h \to ff).\end{aligned}$$

Bound on the nonminimal coupling from the LHC

- Current LHC data allows to bound

$$\mu = \sigma/\sigma_{\rm SM} = \quad 1.4 \pm 0.3 \qquad \text{ATLAS}$$

$$0.87 \pm 0.23 \qquad \text{CMS}$$

- Combining these two bounds one gets:

$$\mu = 1.07 \pm 0.18$$

- which excludes

$$|\xi| > 2.6 \times 10^{15} \text{ at the } 95\% \text{ C.L.}$$

Atkins & xc, PRL 110 (2013) 051301

Bound on the nonminimal coupling from the LHC

- At a 14 TeV LHC with an integrated luminosity of 300 fb^{-1}, could lead to an improved bound on the nonminimal coupling:

$$|\xi| < 1.6 \times 10^{15}$$

- while an ILC with a center of mass energy of 500 GeV and an integrated luminosity of 500 fb^{-1}, could give

$$|\xi| < 4 \times 10^{14}$$

- It seems tough to push the bound below this limit within the foreseeable future.

How does the nonminimal coupling of the Higgs impact physics?

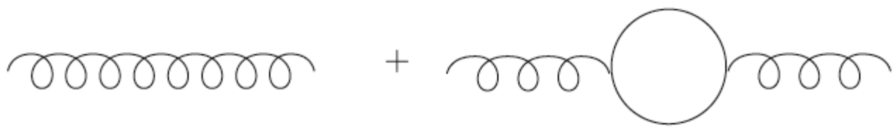
- Running of the Planck mass
- Higgs inflation
- Unitarity
- Much more but work in progress

Running of Newton's constant

- Consider GR with a massive scalar field

$$S = \int d^4x\sqrt{-g}\left[\frac{\bar{M}_P^2}{2}R - \xi\mathcal{H}^\dagger\mathcal{H}R + \mathcal{L}_{SM}\right]$$

- Let me consider the renormalization of the Planck mass:



$$\bar{M}(\mu)^2 = \bar{M}(0)^2 - \frac{1}{16\pi^2}\left(\frac{1}{6}N_l + 2\xi N_\xi\right)\mu^2$$

- Can be derived using the heat kernel method (regulator preserves symmetries!)

Running of Newton's constant

- Consider GR with a massive scalar field

$$S = \int d^4x\sqrt{-g}\left[\frac{\bar{M}_P^2}{2}R - \xi\mathcal{H}^\dagger\mathcal{H}R + \mathcal{L}_{SM}\right]$$

- Let me consider the renormalization of the Planck mass:

$$\bar{M}(\mu)^2 = \bar{M}(0)^2 - \frac{1}{16\pi^2}\left(\frac{1}{6}N_l + 2\xi N_\xi\right)\mu^2$$

- Gravity becomes strong if:

$$M(\mu_*) \sim \mu_*$$

- To give you an idea $\xi=10^{15}$ implies $\mu_\star \approx 10^{11}$GeV

Like any other coupling constant: Newton’s constant runs!

Theoretical physics can lead to anything…
even business ideas!

NEWTON®
running

Higgs as the inflaton?

- Nice idea: try to unify two scalar fields

$$S = \int d^4x \sqrt{-g} \left[\frac{\bar{M}_P^2}{2} R - \xi \mathcal{H}^\dagger \mathcal{H} R + \mathcal{L}_{SM} \right]$$

- Successful inflation requires $\xi \sim 10^4$

- This is suspiciously large!

- What can we learn from unitarity considerations?

Quick review of inflation

- Assuming a Friedmann, Robertson Walker (FRW) metric for the universe

$$ds^2 = dt^2 - a(t)^2 d\vec{x}^2$$

- where a(t) is the scale factor, using Einstein's equations, one gets

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3p)$$

- where ρ and p are the density and pressure appearing in the stress energy tensor of the vacuum of the universe.

- Inflation can be described as the condition $\ddot{a} > 0$

occurs for $p < -\rho/3$

Quick review of inflation

- One can fix the potential of a scalar field such that

$$p < -\rho/3$$

- with

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

- So one finds:

$$\dot{\phi}^2 < V(\phi) \iff \ddot{a} > 0$$

- The potential needs to be flat enough.

Quick review of inflation

- With a at enough potential, this criteria will be met and inflation will occur as the scalar field slowly rolls down the slope.

- The potential also requires a minimum where inflation can eventually end.

- During the period of inflation the universe is supercooled.

- Following inflation, the inflaton oscillates around its final minimum transferring its potential energy into the standard model particles that fill the universe including electromagnetic radiation which starts the radiation dominated phase of the universe.

- This period after inflation ends and before the inflaton comes to rest is known as reheating.

Quick review of Higgs inflation

- Since we know of one scalar field in nature it is natural to try to describe inflation with it.

- The SM Higgs potential

$$V(H) = \lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2$$

  is not flat enough!
- But a nonminimal coupling will change the shape of the potential

$$S = -\int d^4x \sqrt{-g} \left(\frac{1}{2} M^2 + \xi H^\dagger H \right) R$$

Quick review of Higgs inflation

- In Einstein frame the action becomes

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - U(\chi) \right\}$$

- with

$$U(\chi) = \frac{1}{\Omega(\chi)^4} \frac{\lambda}{4} \left(h(\chi)^2 - v^2 \right)^2$$

- For small Higgs values $h \simeq \chi$ and $\Omega^2 \simeq 1$

the potential is the same as for the initial Higgs one, however for large field values $h \gg M_P/\sqrt{\xi}$

$$h \simeq \frac{M_P}{\sqrt{\xi}} \exp\left(\frac{\chi}{\sqrt{6} M_P} \right)$$

i.e. the potential is exponentially flat

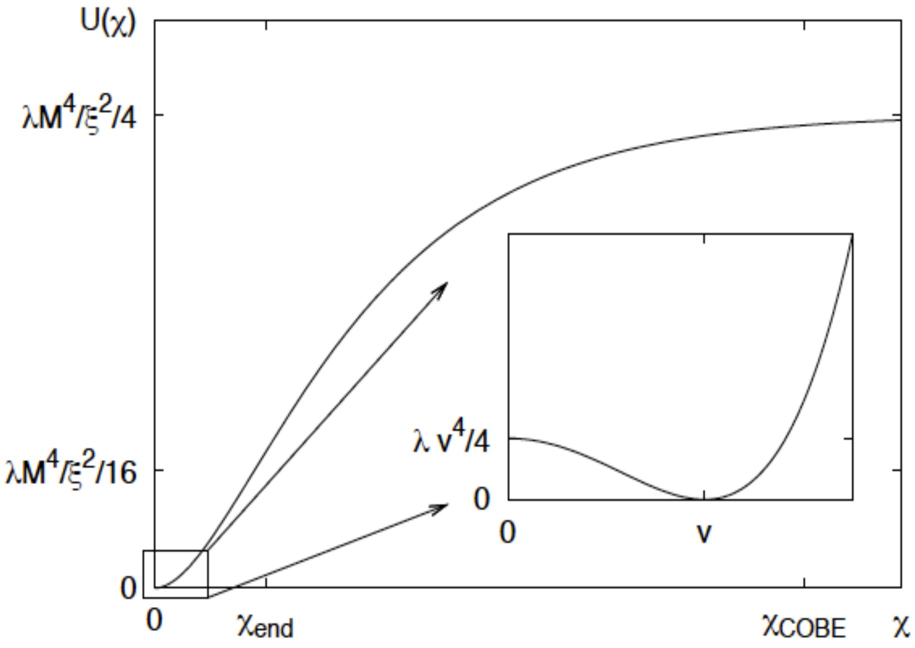


Fig. 1. Effective potential in the Einstein frame.

From 0710.3755 (Bezrukov&Shaposhnikov)

Standard analysis, slow role parameters:

$$\epsilon = \frac{M_P^2}{2}\left(\frac{dU/d\chi}{U}\right)^2 \simeq \frac{4M_P^4}{3\xi^2 h^4}\,,$$

$$\eta = M_P^2 \frac{d^2U/d\chi^2}{U} \simeq -\frac{4M_P^2}{3\xi h^2}\,,$$

$$\zeta^2 = M_P^4 \frac{(d^3U/d\chi^3)dU/d\chi}{U^2} \simeq \frac{16M_P^4}{9\xi^2 h^4}\,.$$

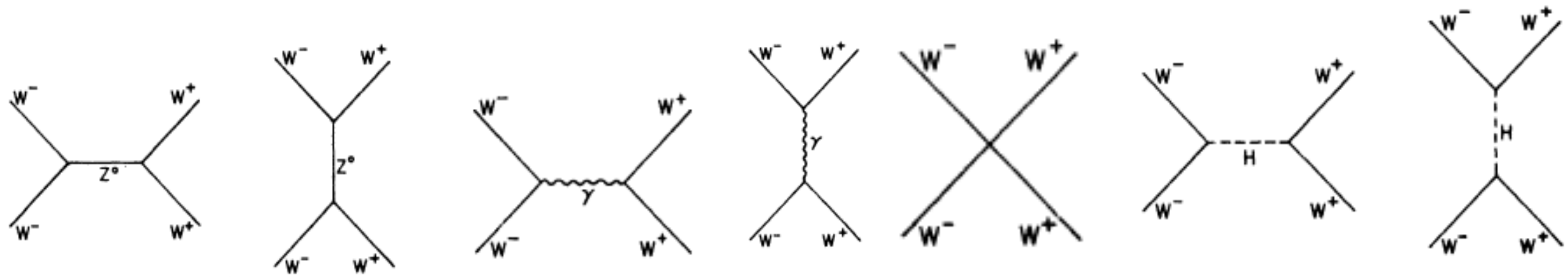
Number of e-foldings: $$N = \int_{h_{\rm end}}^{h_0} \frac{1}{M_P^2}\frac{U}{dU/dh}\left(\frac{d\chi}{dh}\right)^2 dh \simeq \frac{6}{8}\frac{h_0^2 - h_{\rm end}^2}{M_P^2/\xi}$$

$$\xi \simeq \sqrt{\frac{\lambda}{3}}\frac{N_{\rm COBE}}{0.027^2} \simeq 49000\sqrt{\lambda} = 49000\frac{m_H}{\sqrt{2}v}$$

$$\xi \sim 10^4$$

Unitarity in quantum field theory

- Follows from the conservation of probability in quantum mechanics.
- Implies that amplitudes do not grow too fast with energy.
- One of the few theoretical tools in quantum field theory to get information about the parameters of the model.
- Well known example is the bound on the Higgs boson's mass in the Standard Model (m<790 GeV).



$$\mathcal{A} = 16\pi \sum_J (2J+1) a_J d^J_{\mu,\mu'} \qquad |\mathrm{Re}\ a_J| \leq 1/2$$

M. Atkins & xc, 2010

Let us consider gravitational scattering of the particles included in that model (s-channel, we impose different in and out states)

(calculation also done by Han & Willenbrock 2004, but without RG considerations)

$\rightarrow$	$s'\bar{s}'$	$\psi'_+\bar{\psi}'_-$	$\psi'_-\bar{\psi}'_+$	$V'_+V'_-$	$V'_-V'_+$
$s\bar{s}$	$-2\pi G_N s(1/3d^2_{0,0} - 1/3(1+12\xi)^2 d^0_{0,0})$	$-2\pi G_N s\sqrt{1/3}\ d^2_{0,1}$	$-2\pi G_N s\sqrt{1/3}\ d^2_{0,-1}$	$-4\pi G_N s\sqrt{1/3}\ d^2_{0,2}$	$-4\pi G_N s\sqrt{1/3}\ d^2_{0,-2}$
$\psi_+\bar{\psi}_-$	$-2\pi G_N s\sqrt{1/3}\ d^2_{1,0}$	$-2\pi G_N s d^2_{1,1}$	$-2\pi G_N s d^2_{1,-1}$	$-4\pi G_N s\ d^2_{1,2}$	$-4\pi G_N s\ d^2_{1,-2}$
$\psi_-\bar{\psi}_+$	$-2\pi G_N s\sqrt{1/3}\ d^2_{-1,0}$	$-2\pi G_N s d^2_{-1,1}$	$-2\pi G_N s d^2_{-1,-1}$	$-4\pi G_N s 2\ d^2_{-1,2}$	$-4\pi G_N s 2\ d^2_{-1,-2}$
V_+V_-	$-4\pi G_N s\sqrt{1/3}\ d^2_{2,0}$	$-4\pi G_N s\ d^2_{2,1}$	$-4\pi G_N s\ d^2_{2,-1}$	$-8\pi G_N s\ d^2_{2,2}$	$-8\pi G_N s\ d^2_{2,-2}$
V_-V_+	$-4\pi G_N s\sqrt{1/3}\ d^2_{-2,0}$	$-4\pi G_N s\ d^2_{-2,1}$	$-4\pi G_N s\ d^2_{-2,-1}$	$-8\pi G_N s\ d^2_{-2,2}$	$-8\pi G_N s\ d^2_{-2,-2}$

$$\mathcal{A} = 16\pi \sum_J (2J+1) a_J d^J_{\mu,\mu'} \qquad |\mathrm{Re}\ a_J| \leq 1/2$$

Higgs as the inflaton?

We obtained a bound on the non minimal gravitational coupling of scalar fields

$$-\int d^4x\sqrt{-\det(g)}\ \xi R\phi^2$$

In the minimal model, we have the SM + gravity and no new physics. The cutoff should be the red. Planck mass!

From the J=0 partial wave, we get:

$$-\frac{4\sqrt{6\pi N_S}+N_S}{12N_S}\leq\xi\leq\frac{4\sqrt{6\pi N_S}-N_S}{12N_S}$$

In today's background (small Higgs vev, flat spacetime)

$$E_\star = 3.5\times 10^{14}\ \text{GeV}$$

$$-0.81\leq\xi\leq 0.64$$

$$\mu_\star\sim 1\times 10^{17}\ \text{GeV}$$

- However one needs to be careful, the bound depends on the background.
- In inflationary background, one finds

$$\bar{M}_P/\sqrt{\xi}$$

- This does not affect our conclusion though: the tightest bound on ξ is the one obtained in flat space-time and for a small Higgs vev

- Minimal model does not work, one needs new physics between the inflationary scale and the red. Planck mass.

- Way out: asymptotically safe gravity. If the Planck mass and ξ get weaker in the UV, their running can compensate the growth of the amplitude with energy.

What happens for N=1?

Singlet scalar field $S + S \to S + S$

$$A_\xi = \frac{-2}{\bar{M}_P^2}\left(m^4(s^{-1}+t^{-1}+u^{-1}) + \frac{(2m^2-s)(2m^2-t)}{2u} + \frac{(2m^2-s)(2m^2-u)}{2t} + \right.$$
$$\left. + \frac{(2m^2-s)(2m^2-u)}{2t} + 2m^2\xi(6\xi-5)\right)$$

In the high energy limit $s \sim |t| \sim |u| > s_{\rm min}$

$$A_\xi \sim \frac{-2}{\bar{M}_P^2}\left(\frac{3}{2}s + 2m^2\xi(6\xi-5)\right)$$

Terms proportional to ξ (and not *m*) do not grow with energy: no bound.

Higgs+singlet inflation

- Singlets have been advocated as a possible solution

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = \frac{1}{2}\left(\bar{M}^2 + \xi\bar{\sigma}^2 + 2\zeta\mathcal{H}^\dagger\mathcal{H}\right)R - \frac{1}{2}(\partial_\mu\bar{\sigma})^2 - |D_\mu\mathcal{H}|^2$$
$$-\frac{1}{4}\kappa\left(\bar{\sigma}^2 - \bar{\Lambda}^2 - 2\alpha\mathcal{H}^\dagger\mathcal{H}\right)^2 - \lambda\left(\mathcal{H}^\dagger\mathcal{H} - \frac{v^2}{2}\right)^2.$$

Giudice & Lee

- but one finds

$$\Lambda \sim \bar{M}_P/(\sqrt{\lambda}\xi)$$

by looking at 2 to 4 scattering. A self coupling must be finely tuned!

New Higgs inflation (Germani & Kehagias)

$$S = \int d^4x\sqrt{-g}\left[\frac{\bar{M}_P^2}{2}R - \frac{1}{2}(g^{\mu\nu} - w^2G^{\mu\nu})\partial_\mu\Phi\partial_\nu\Phi - \frac{\lambda}{4}\Phi^4\right]$$

This action generates operators of the type:

$$I \simeq \frac{1}{2H^2\bar{M}_P}\partial^2 h^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

Looking at scalars scattering again, we find that unitarity is violated at:

$\Lambda \simeq 2\times 10^{-3}\bar{M}_P$ in inflationary background

$\Lambda = 3.4\times 10^{-5}\bar{M}_P$ in today's background

Again, new physics is needed below the red. Planck mass

- SM Higgs + Gravity alone cannot provide a full description of particle physics and inflation up to the Planck mass unless gravity is asymptotically safe.

- New Higgs inflation does not work.

- Singlets could work, but you need to fine-tune the self-coupling.

- Is there a connection to dark matter?

More nonminmal couplings!

- We can describe any theory of quantum gravity below the Planck scale using effective field theory techniques:

$$S = \int d^4x \sqrt{-g} \left[\left(\frac{1}{2} M^2 + \xi H^\dagger H \right) \mathcal{R} - \Lambda_C^4 + c_1 \mathcal{R}^2 + c_2 \mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} + \mathcal{L}_{SM} + \mathcal{O}(M_\star^{-2}) \right]$$

- Electroweak symmetry breaking:

$$(M^2 + \xi v^2) = M_P^2 \qquad M_P = 2.4335 \times 10^{18} \text{ GeV}$$

- Several energy scale:

  - $\Lambda_C \sim 10^{-12}$ GeV cosmological constant
  - M_P or equivalently Newton's constant $G = 1/(8\pi\, M_P^2)$
  - $M_\star$ energy scale up to which one trusts the effective theory

- Dimensionless coupling constants ξ, c_1, c_2 etc

What values to expect for the coefficients?

- It all depends whether they are truly new fundamental constants or whether the operators are induced by quantum gravitational effects.
  - If fundamental constants, they are arbitrary
  - If induced by quantum gravity we can estimate their magnitude.
- Usually induced dimension four operators are expected to be small

$$\exp\left(-\lambda/\Lambda_{NP}\right)$$

- However, $\xi H^\dagger H \mathcal{R}$ translates into $\xi H^\dagger H h \Box h / M_P^2$ in terms of the graviton h. $\mathcal{R}^2$-type operators lead to $h \Box h h \Box h / M_P^4$

- We thus expect the coefficients of these operators to be $O(1)$.

- Naturalness arguments would imply $M_\star \sim \Lambda_C$. However, there is not sign of new physics at this energy scale.

What do experiments tell us?

- In 1977, Stelle has shown that one obtains a modification of Newton's potential at short distances from R^2 terms

$$\Phi(r) = -\frac{Gm}{r}\left(1 + \frac{1}{3}e^{-m_0 r} - \frac{4}{3}e^{-m_2 r}\right)$$

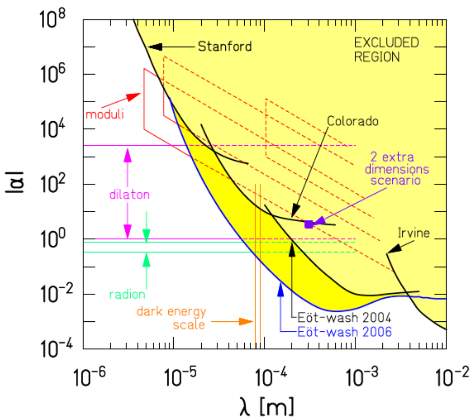
$$m_0^{-1} = \sqrt{32\pi G\left(3c_1 - c_2\right)}$$

$$m_2^{-1} = \sqrt{16\pi G c_2}$$

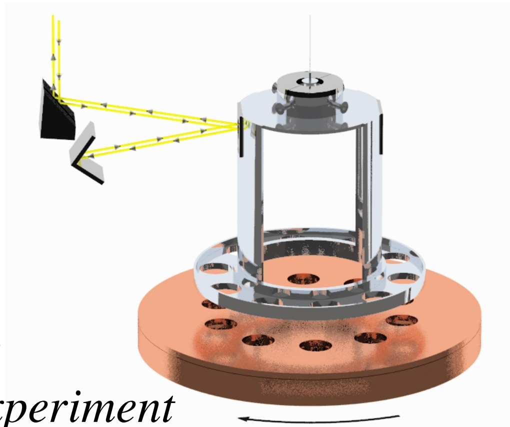
$$V(r) = -G_N \frac{m_1 m_2}{r}\left[1 + \alpha \exp\left(-r/\lambda\right)\right]$$

c_1 and c_2 $<10^{61}$

xc, Hsu and Reeb (2008)



NB: Bound has improved by 10 order of magnitude since Stelle's paper!



Schematic drawing of the Eöt-Wash Short-range Experiment

Can better bounds be obtained in astrophysics?

- Bounds on Earth are obtained in weak curvature, binary pulsar systems are probing high curvature regime.

- Approximation: Ricci scalar in the binary system of pulsars by G M/(r^3c^2) where M is the mass of the pulsar and r is the distance to the center of the pulsar.

- But: if the distance is larger than the radius of the pulsar, then the Ricci scalar vanishes. This is a rather crude estimate.

Can better bounds be obtained in astrophysics?

- Let me be optimistic and assume one can probe gravity at the surface of the pulsar. I take r=13.1km and M=2 solar masses.

- I now request that the R^2 term should become comparable to the leading order Einstein-Hilbert term ($1/2\ M_P^2\ R$)

- One could reach bounds of the order of 10^{78} only on c_1 or c_2

- Such limits are obviously much weaker that those obtained on Earth.

Frame dependence in GR?

- Short and obvious answer: no you are just doing field redefinitions. BUT: semantic is important!
- The real question is what do you mean by the equivalence of two frames.
- Starting from the Jordan frame

$$S_J = \int d^4x\sqrt{-g}\left(\left(\frac{1}{16\pi G} - \frac{1}{2}\xi\phi^2\right)R + \frac{1}{2}g_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi - V(\phi)\right)$$

Frame dependence in GR?

- Starting from the Jordan frame action

$$S_J = \int d^4x\sqrt{-g}\left(\left(\frac{1}{16\pi G} - \frac{1}{2}\xi\phi^2\right)R + \frac{1}{2}g_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi - V(\phi)\right)$$

- using

$$\begin{aligned}
\tilde{g}_{\mu\nu} &= \Omega^2 g_{\mu\nu} \\
\sqrt{-\tilde{g}} &= \Omega^4\sqrt{-g} \\
d\tilde{\phi} &= \frac{(1-8\pi G\xi(1-6\xi)\phi^2)^{1/2}}{1-8\pi G\xi\phi^2}d\phi \\
\tilde{V}(\tilde{\phi}) &= \Omega^{-4}V(\phi) \\
\tilde{R} &= \Omega^{-2}(R - \frac{12\Box\sqrt{\Omega}}{\sqrt{\Omega}} - 3\frac{g^{ab}\nabla_a\Omega\nabla_b\Omega}{\Omega^2})
\end{aligned}$$

$$\Omega^2 = \exp[\sigma(x)] = 1 - 8\pi G\xi\phi^2$$

Frame dependence in GR?

- Starting from the Jordan frame action

$$S_J = \int d^4x\sqrt{-g}\left(\left(\frac{1}{16\pi G} - \frac{1}{2}\xi\phi^2\right)R + \frac{1}{2}g_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi - V(\phi)\right)$$

- one obtains

$$S_{E+boundary} =$$

$$\int d^4x\sqrt{-\tilde{g}}\left(\frac{\tilde{R}}{16\pi G} - \frac{1}{2}\tilde{\phi}\tilde{\Box}\tilde{\phi} - \tilde{V}(\tilde{\phi}) + \frac{1}{2}\partial_\mu(\tilde{g}^{\mu\nu}\tilde{\phi}\partial_\nu\tilde{\phi}) - \frac{3\Omega^2\tilde{\Box}\ln\Omega}{8\pi G\Omega^2}\right)$$

Frame dependence in GR?

- We thus find

$$\mathcal{L}_J = \mathcal{L}_E + \partial_\mu(\mathbf{boundary\,terms})$$

- with the boundary term given by

$$\begin{aligned}
&\int d^4x\,\sqrt{-\tilde{g}}\left(\frac{1}{2}\partial_\mu(\tilde{g}^{\mu\nu}\tilde{\phi}\partial_\nu\tilde{\phi}) - \frac{3\Omega^2\tilde{\Box}\ln\Omega}{8\pi G\Omega^2}\right) \\
&= \int d^4x\sqrt{-\tilde{g}}\,\partial_\mu[\frac{1}{2}\tilde{g}^{\mu\nu}\tilde{\phi}\partial_\nu\tilde{\phi} - \frac{3\tilde{g}^{\mu\nu}\partial_\nu\ln\Omega}{8\pi G}] \\
&= \int d\sigma\frac{1}{2}[\tilde{g}^{\mu\nu}\tilde{\phi}\partial_\nu\tilde{\phi} - \frac{3\tilde{g}^{\mu\nu}\partial_\nu\ln\Omega}{4\pi G}]\,|_\partial \\
&\equiv (\mathbf{surface\,terms}),
\end{aligned}$$

Frame dependence in GR?

- At the quantum level, it is easier to work backwards

$$Z_E \;=\; \tilde{N} \int d\mu[\tilde{\phi}]\, \exp\left(\frac{i}{\hbar}\left(\int d^4x \mathcal{L}_E + \int d^4x \sqrt{-\tilde{g}}\, \tilde{J}_\phi \tilde{\phi}\right)\right)$$

- Doing the same field transformation, we obtain

$$Z_E = \tilde{N} \int \det C_{N'N} d\mu[\phi]$$
$$\exp \frac{i}{\hbar}\left(\int d^4x(\mathcal{L}_J - \partial_\mu(\textbf{boundary terms})) + \sqrt{-g}\, J_\phi \phi\right)$$

with a Jacobian given by $\quad d\mu[\tilde{\phi}_{N'}] = \det C_{N'N} d\mu[\phi_N]$

Frame dependence in GR?

- One can easily show that the Jacobian is related to the expectation value of the energy momentum tensor

$$i\hbar \ln(\det C) = \frac{1}{2}\int dx^4 \langle T^\mu{}_\mu \rangle$$

with

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} \qquad \begin{aligned} \langle T^\mu{}_\mu \rangle &= \frac{2}{\sqrt{-g(x)}} g^{\mu\nu} \frac{\delta W}{\delta g^{\mu\nu}} \\ &= -\frac{\Omega}{\sqrt{-g(x)}}\frac{\delta W}{\delta \Omega} \end{aligned}$$

$$W = -i \ln Z[0]$$

Frame dependence in GR?

- Partitions functions are the same up to 2 terms

$$Z_E = \tilde{N} \int d\mu[\phi] \exp \frac{i}{\hbar} \left(\int d^4x \left(\mathcal{L}_J - \frac{1}{2} \langle T^\mu_{\ \mu} \rangle - \partial_\mu (\textbf{boundary terms}) \right) + \sqrt{-g}\, J_\phi \phi \right)$$

- To be compared with

$$Z_J = N \int d\mu[\phi] \exp \left(\frac{i}{\hbar} \left(\int d^4x \mathcal{L}_J + \int d^4x \sqrt{-g}\, J_\phi \phi \right) \right)$$

Physics is not affected as long as the transformations are done properly!

Frame dependence in GR?

- In the Higgs inflation case

$$
\begin{aligned}
\langle T^{\mu}{}_{\mu}\rangle &= \left(\frac{1}{64\pi^2} g^{\mu}{}_{\mu}\right) \times \\
&\left(m^2\left[m^2+\left(\xi-\frac{1}{6}\right)R\right]\left[\Psi\left(\frac{3}{2}+\nu\right)+\Psi\left(\frac{3}{2}-\nu\right)-\ln\left(12m^2R^{-1}\right)\right]\right. \\
&\left.-m^2\left(\xi-\frac{1}{6}\right)R-\frac{1}{18}m^2R-\frac{1}{2}\left(\xi-\frac{1}{6}\right)^2R^2+\frac{1}{2160}R^2\right)
\end{aligned}
$$

with

$$\Psi(z)=\frac{\Gamma'(z)}{\Gamma(z)},\ \nu=\sqrt{\frac{9}{4}-m^2\frac{12}{R}-12\xi}\quad \text{and } m=\sqrt{\lambda}v/2$$

This clearly does not affect the inflationary calculation.

Conclusions

- The SM Higgs has been found: there is at least one more new fundamental constant in nature: the nonminimal coupling of the Higgs boson to gravity.
- This new parameter can have a dramatic impact on physics:
  - It can make Newton's constant run
  - It can lead to Higgs inflation within the SM
  - It creates issues with unitarity (unless there is a self-healing mechanism at work)
  - Much more to come
- Physics does not depend on the frame.

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Thanks for your attention!