# Theory motivations for improved precision data on hadronic τ decays

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Tau-Charm Workshop
Elba, Italy, May 29, 2013

# Outline :

1. Introduction and Motivation

2. Inclusive hadronic $\tau$-decays as a probe of strong and electroweak interactions

3. Exclusive hadronic $\tau$-decays :
    - Prediction of strange Brs and $V_{us}$
    - CPV
    - g-2
    - LFV decays

4. Conclusion and outlook

# 1. Introduction and Motivation

# 1.1 Hadronic τ-decays

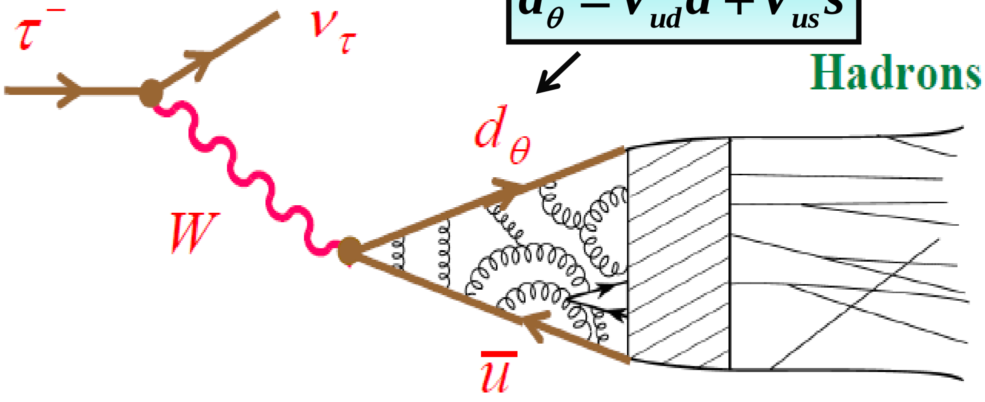
- τ lepton discovered in 1976 by M. Perl et al. (SLAC-LBL group)

PDG'12

  - Mass : $\boxed{m_\tau = 1.77682(16)\ \mathrm{GeV}}$
  - Lifetime : $\boxed{\tau_\tau = 2.096(10) \cdot 10^{-13}\ s}$

- The only lepton heavy enough to decay into hadrons : lots of semileptonic decays !

$$\boxed{d_\theta = V_{ud} d + V_{us} s}$$



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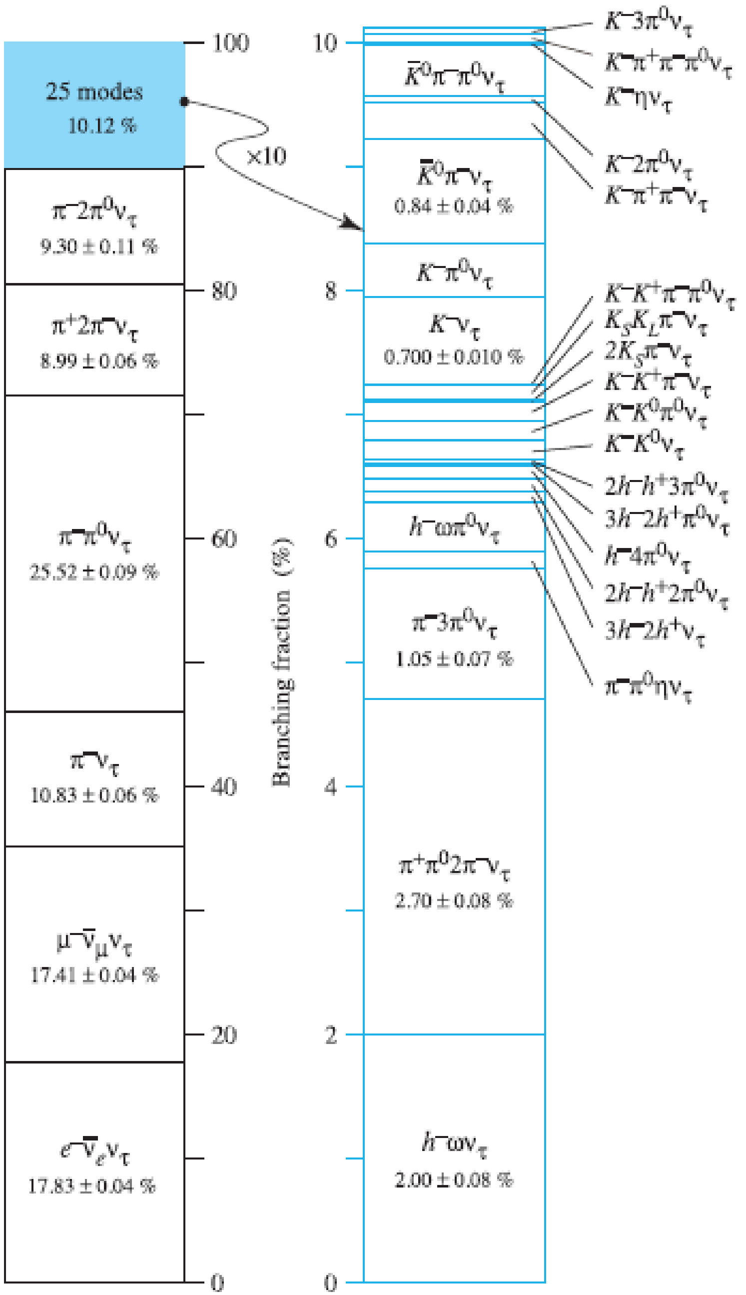

# 1.1 Hadronic τ-decays

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  PDG'12

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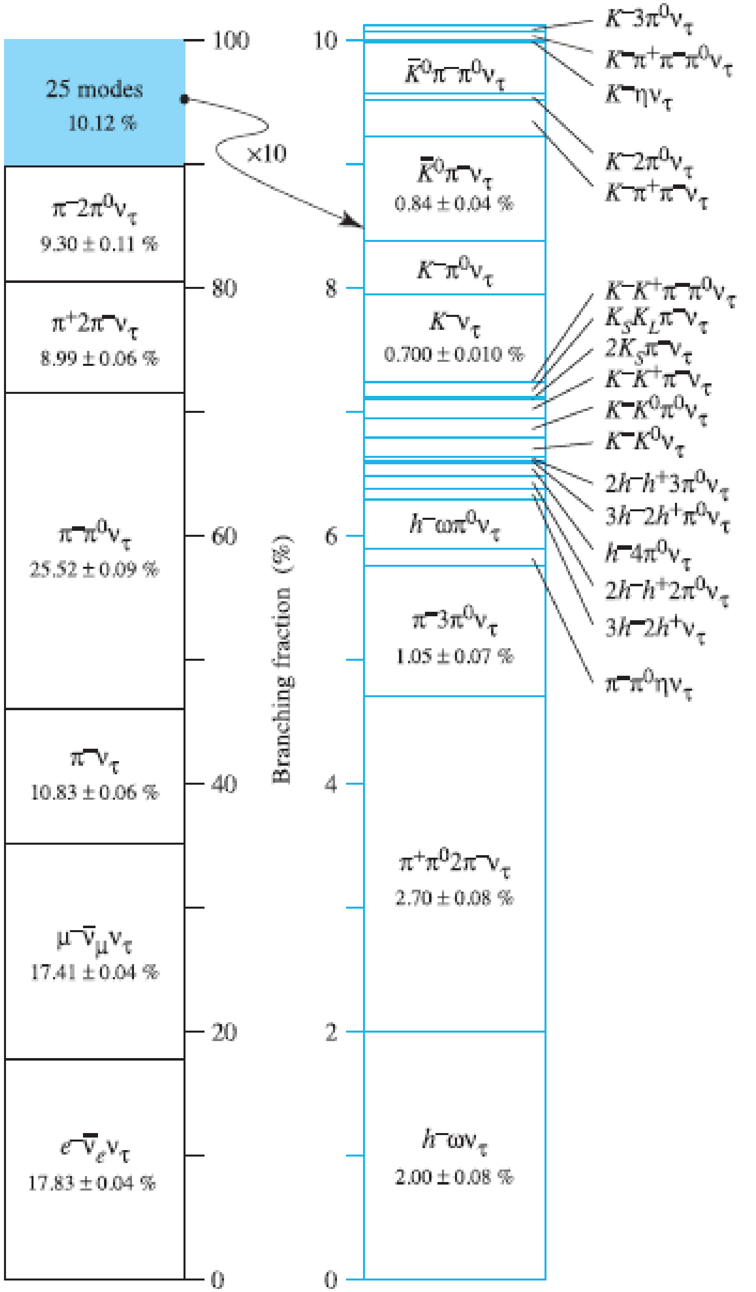
- The only lepton heavy enough to decay into hadrons : lots of semileptonic decays !

  ➡ Very rich phenomenology

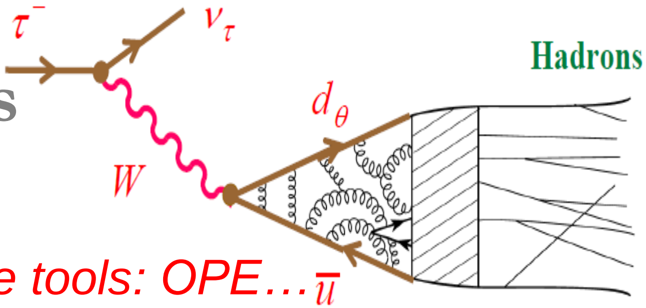
  *Test of QCD and EW interactions*

- For the tests:
  - Precise measurements needed
  - Hadronic uncertainties under control

PDG'12



# 1.2 Test of QCD and EW interactions



- Inclusive τ decays : full hadron spectra, *perturbative tools: OPE…*

$\boxed{\tau \rightarrow (\bar{u}d, \bar{u}s)\nu_\tau}$ ⇨ fundamental SM parameters: $\alpha_S(m_\tau)$, $|V_{us}|$, $m_s$
QCD studies

- Exclusive τ decays : specific hadron spectrum, *non perturbative tools*

$\boxed{\tau \rightarrow (PP, PPP, \ldots)\nu_\tau}$ ⇨ Study of ffs, resonance parameters ($M_R$, $\Gamma_R$)
Hadronization of QCD currents

- τ decays: tool to search for New Physics in inclusive and exclusive decays :

⇨ Unitarity test, CPV, LFV, EDMs, etc.

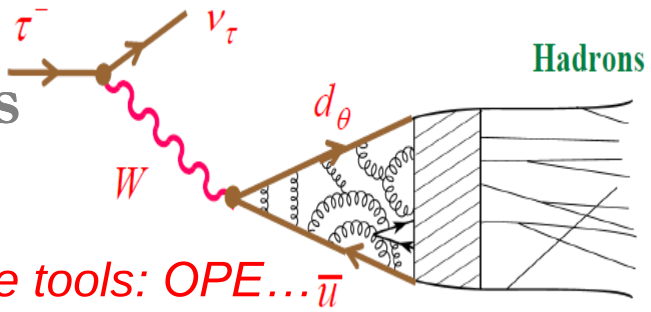
Test of unitarity $\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \overset{?}{=} 1}$

$0^+ \rightarrow 0^+$
β decays

$K_{l3}$ decays
or τ decays

Negligible
(B decays)

# 1.2 Test of QCD and EW interactions



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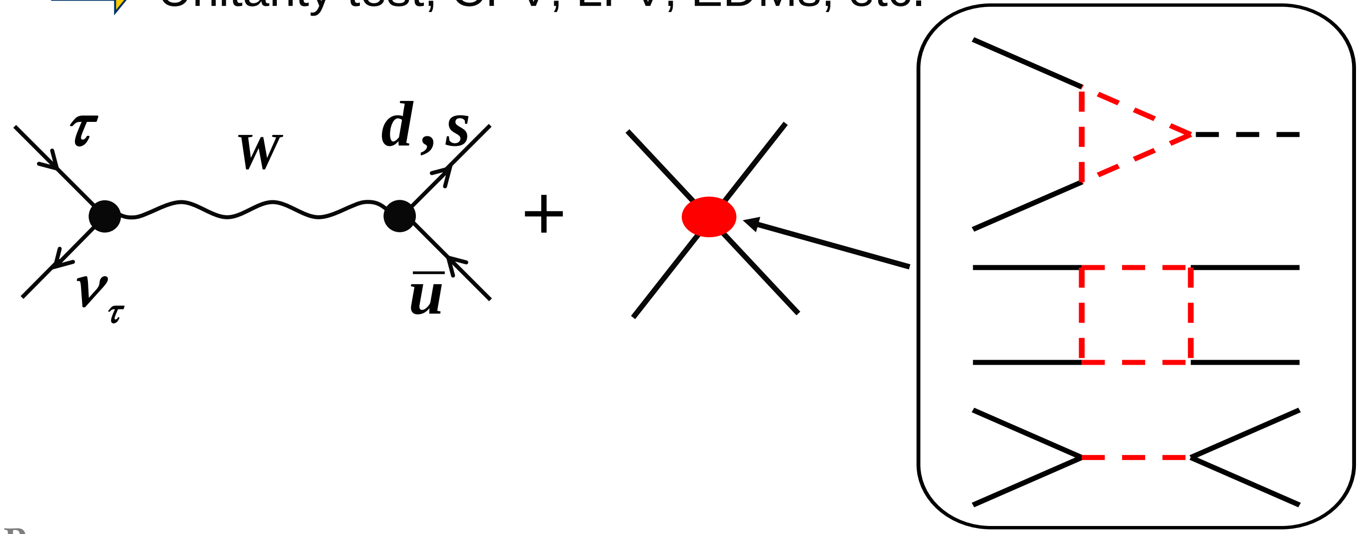


SUSY loops,
Leptoquarks,
Z', Charged Higgs,
Right-Handed
Currents,….

# 2. Inclusive hadronic τ-decays as a probe of strong and electroweak interactions

# 2.1 Introduction

- $$\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \textbf{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)} = R_\tau^{NS} + R_\tau^{S} \approx \left|V_{ud}\right|^2 N_C + \left|V_{us}\right|^2 N_C}$$ naïve QCD prediction

⇨ Experimentally $R_\tau = \dfrac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086$

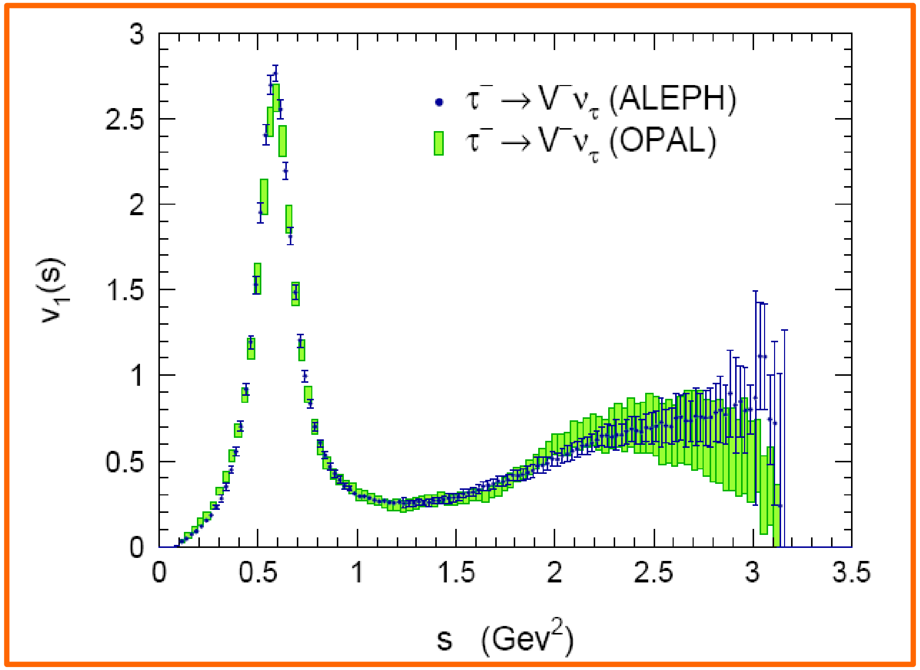
- Difficulty ⇨ QCD corrections : $R_\tau = \left|V_{ud}\right|^2 N_C + \left|V_{us}\right|^2 N_C + O\left(\alpha_S\right)$

- Extraction of the strong coupling constant : $R_\tau^{NS} = f\left(\alpha_S\right)$ ⇨ $\alpha_S$

  measured calculated

- Determination of $V_{us}$ :

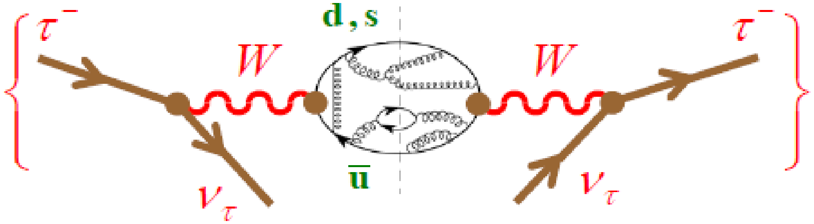
⇨ $$\boxed{\left|V_{us}\right|^2 = \frac{R_\tau^S}{\dfrac{R_\tau^{NS}}{\left|V_{ud}\right|^2} - \delta R_{\tau,th}}}$$



# 2.2 Theoretical Method

*Braaten, Narison, Pich'92*

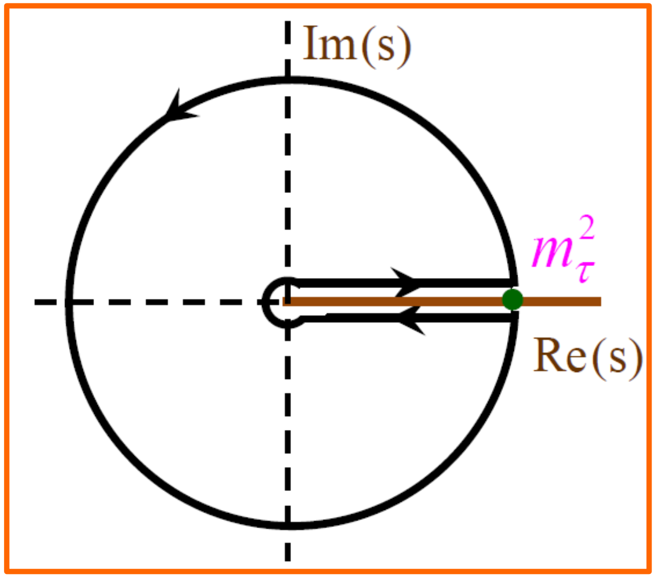
- Calculation of $R_\tau$ : $\Gamma_{\tau\to\nu_\tau+\text{had}} \sim \text{Im}\left\{ \quad \right\}$



$$R_\tau(m_\tau^2) = 12\pi S_{EW}\int_0^{m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\text{Im}\Pi^{(1)}(s+i\varepsilon)+\text{Im}\Pi^{(0)}(s+i\varepsilon)\right]$$

- Analyticity: Π analytic in the entire complex plane except for s real positive ⇒ Cauchy theorem:

$$R_\tau(m_\tau^2) = 6i\pi S_{EW}\oint_{|s|=m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\Pi^{(1)}(s)+\Pi^{(0)}(s)\right]$$



- Sufficient high energy for *OPE*
  Kinematic factor : decreases weight close real axis

$$\Pi^{(J)}(s) = \sum_{D=0,2,4...}\frac{1}{(-s)^{D/2}}\sum_{\dim O=D}\mathcal{C}^{(J)}(s,\mu)\,\langle O_D(\mu)\rangle$$

Wilson coefficients

Operators

$\mu$ separation scale between short and long distances

# 2.2 Theoretical Method

Braaten, Narison, Pich'92

- $$\boxed{R_\tau\left(m_\tau^2\right) = N_C\ S_{EW}\left(1+\delta_P+\delta_{NP}\right)}$$

  $S_{EW} = 1.0201(3)$ Marciano &Sirlin'88, Braaten & Li'90, Erler'04

- Perturbative part : (D=0) $$\boxed{\delta_P = a_\tau + 5.20\ a_\tau^2 + 26\ a_\tau^3 + 127\ a_\tau^4 + \ldots \approx 20\%}$$ $$a_\tau = \frac{\alpha_s(m_\tau)}{\pi}$$

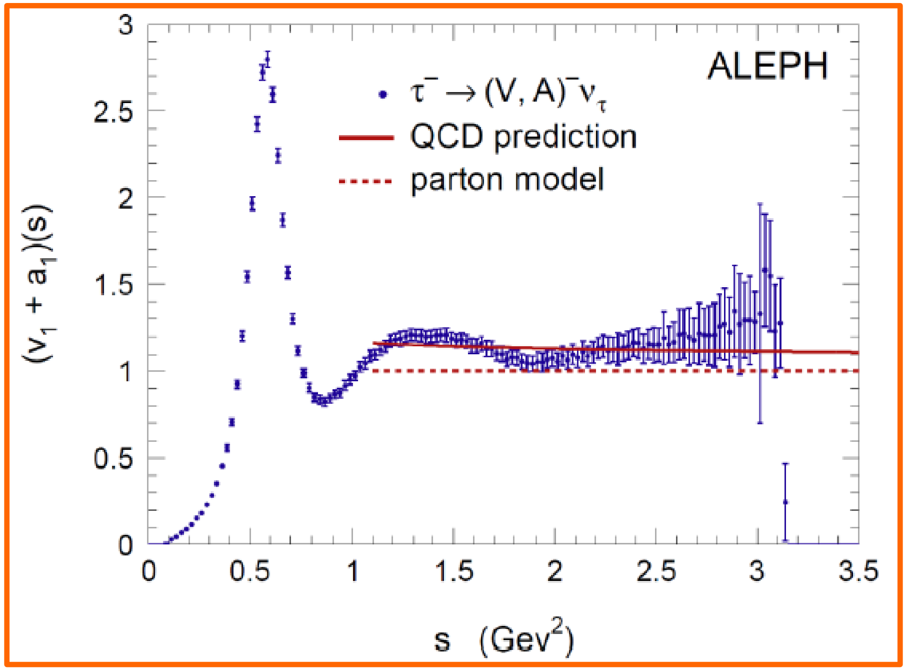
  Baikov, Chetyrkin, Kühn'08

- D=2 : quark mass corrections, neglected for $R_\tau^{NS}\ \left(\propto m_u, m_d\right)$ but not for $R_\tau^S\ \left(\propto m_s\right)$

- Non perturbative part : D ≥ 4
  Not known, fitted from the data
  Use of weighted distributions

  ⇨ $$\boxed{\delta_{NP} = -0.0059 \pm 0.0014}$$

  Davier et al'08



- Small unknown NP part $\left(\delta_{NP} \sim 3\%\ \delta_P\right)$

  ⇨ very precise extraction of $\alpha_S$

# 2.3 Determination of $\alpha_s$

*Pich'Tau12*

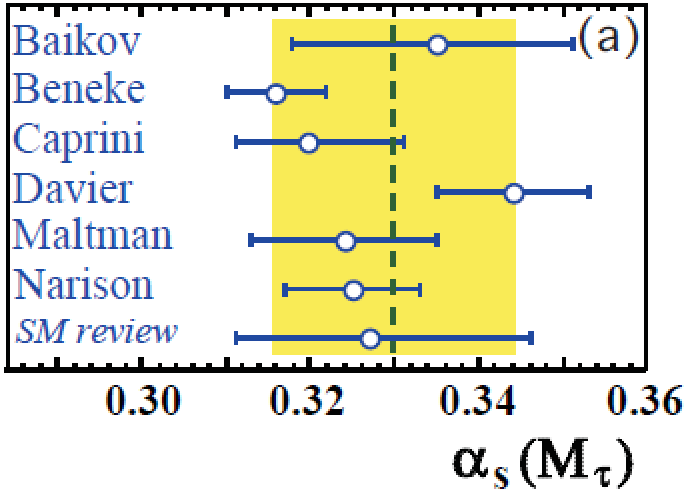
| Reference | Method | $\delta_{NP}$ | $\delta_P$ | $\alpha_s(m_\tau)$ | $\alpha_s(m_Z)$ |
|---|---|---|---|---|---|
| Baikov et al '08 | CIPT, FOPT | | 0.1998 (43) | 0.332 (16) | 0.1202 (19) |
| Davier et al '08 | CIPT | – 0.0059 (14) | 0.2066 (70) | 0.344 (09) | 0.1212 (11) |
| Beneke-Jamin'08 | BSR + FOPT | – 0.007 (3) | 0.2042 (50) | 0.316 (06) | 0.1180 (08) |
| Maltman-Yavin'08 | PWM + CIPT | + 0.012 (18) | – | 0.321 (13) | 0.1187 (16) |
| Menke '09 | CIPT, FOPT | | 0.2042 (50) | 0.342 (11) | 0.1213 (12) |
| Narison '09 | CIPT, FOPT | | – | 0.324 (08) | 0.1192 (10) |
| Caprini-Fischer '09 | BSR + CIPT | | 0.2037 (54) | 0.322 (16) | – |
| Abbas et al '10 | IFOPT | | 0.2037 (54) | 0.338 (10) | |
| Cvetič et al '10 | $\beta_{exp}$ + CIPT | | 0.2040 (40) | 0.341 (08) | 0.1211 (10) |
| Boito et al '12 | CIPT, DV | – 0.002 (12) | – | 0.347 (25) | 0.1216 (27) |
| | FOPT, DV | – 0.004 (12) | | 0.325 (18) | 0.1191 (22) |
| Pich '12 | CIPT | – 0.0059 (14) | 0.2030 (33) | 0.344 (14) | 0.1215 (15) |
| | FOPT | | | 0.321 (15) | 0.1188 (18) |
| **Pich** | **CIPT, FOPT** | | **0.2030 (33)** | **0.334 (14)** | **0.1204 (16)** |

CIPT: Contour-improved perturbation theory
FOPT: Fixed-order perturbation theory
BSR: Borel summation of renormalon series
IFOPT Improved FOPT

$\beta_{exp}$: Expansion in derivatives of $\alpha_s$ ($\beta$ function)
PWM: Pinched-weight moments
CIPTm: Modified CIPT (conformal mapping)
DV: Duality violation (OPAL only)

# 2.3 Determination of $\alpha_S$

PDG'12



- $$\boxed{\alpha_s\left(m_\tau^2\right) = 0.329 \pm 0.013}$$

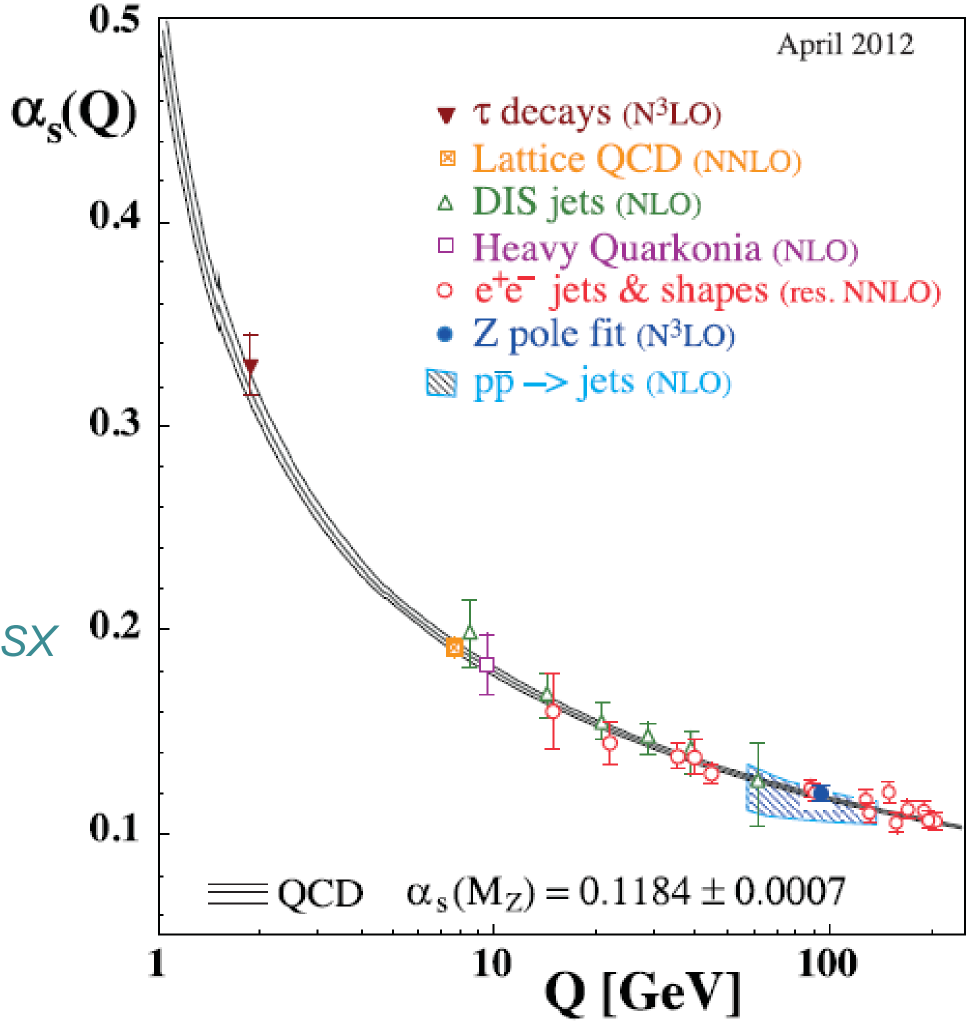
$$\Downarrow$$

Pich'QCHSX

$$\boxed{\alpha_s\left(M_Z^2\right) = 0.1198 \pm 0.0015}$$

to be compared to

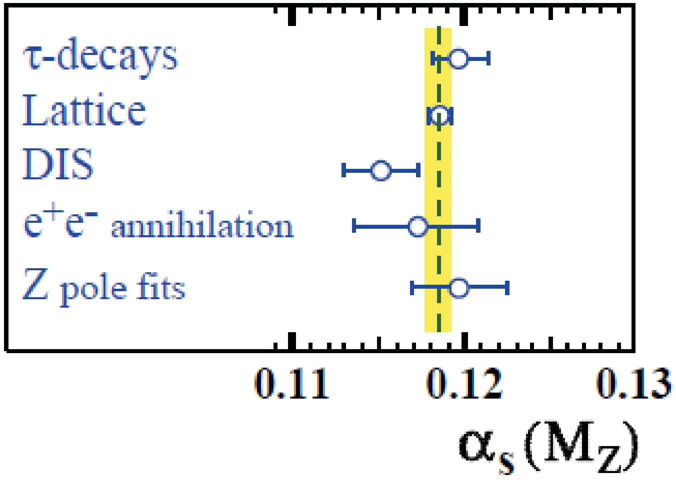
$$\boxed{\alpha_s\left(M_Z^2\right)_{Z\ \text{width}} = 0.1197 \pm 0.0028}$$

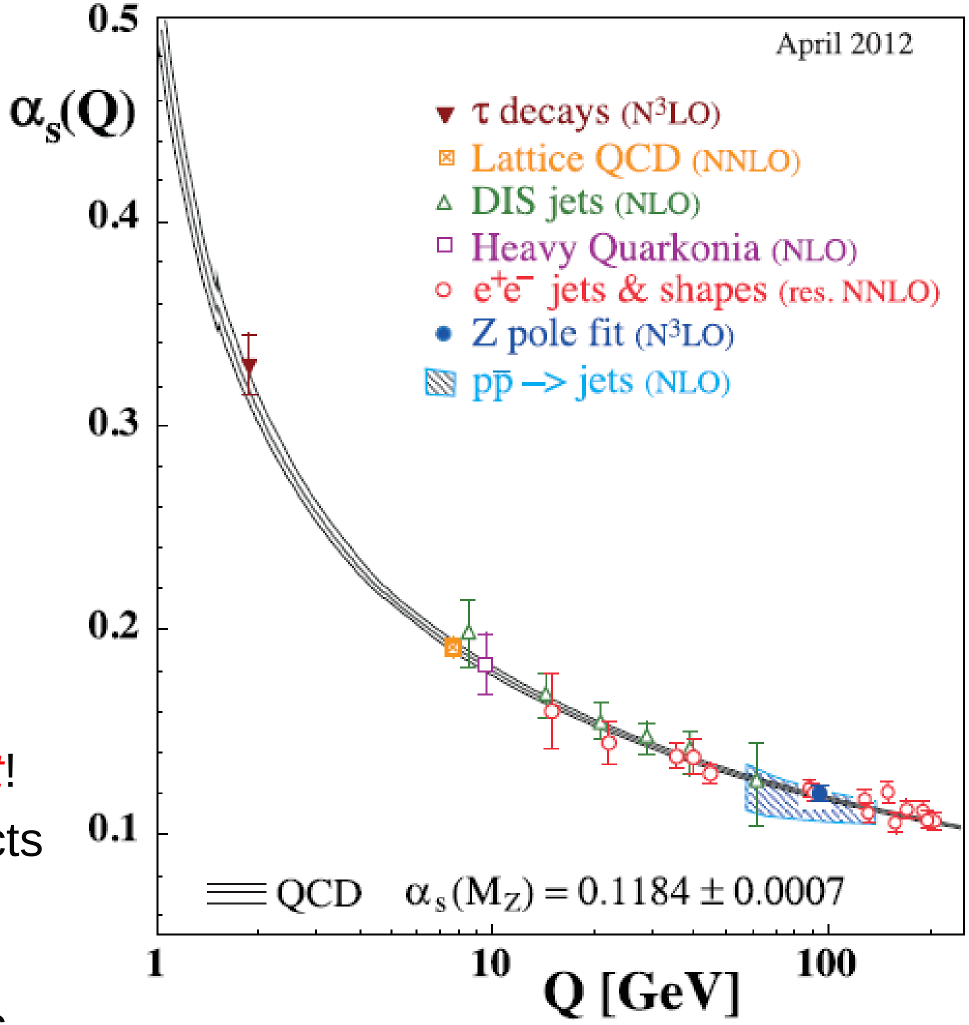


- Impressive test of the running of $\alpha_S$!

# 2.3 Determination of $\alpha_s$

*PDG'12*





- *Extraction of $\alpha_S$* from hadronic τ decays very *competitive*!
- If new data room for *improvement*!
  - Study of duality violation effects
  - Improve precision on non-perturbative determination : higher order condensates, etc
  - New physics?

# 2.4 Extraction of $V_{us}$

- $$\boxed{\delta R_\tau \equiv \frac{R_\tau^{NS}}{|V_{ud}|^2} - \frac{R_\tau^{S}}{|V_{us}|^2}} = N_C S_{EW}\left(\delta_{NP}^{NS} - \delta_{NP}^{S}\right)$$ $SU(3)$ breaking quantity
  ⇨ 0 in the $SU(3)$ limit, small, calculable with *OPE*

$$\delta R_\tau = f\left(m_s\right) \Rightarrow \delta R_{\tau,th} = 0.240(32)$$

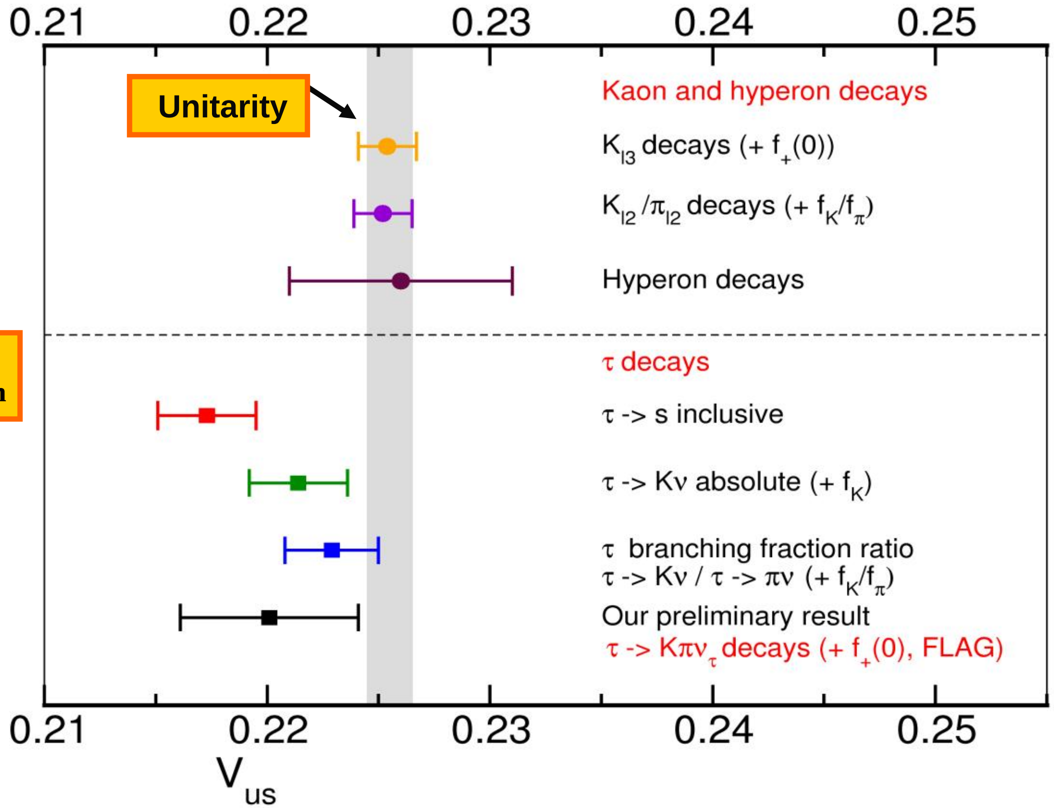
*Gamiz, Jamin, Pich, Prades, Schwab'07, Maltman'11*

- $$\boxed{|V_{us}|^2 = \frac{R_\tau^S}{\frac{R_\tau^{NS}}{|V_{ud}|^2} - \delta R_{\tau,th}}}$$

⇨ $$\boxed{|V_{us}| = 0.2173 \pm 0.0020_{\exp} \pm 0.0010_{th}}$$

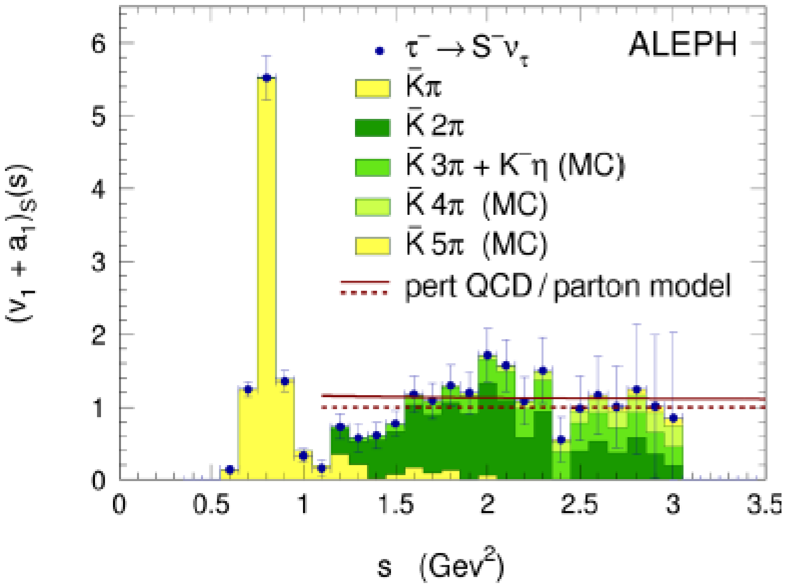
2.6σ away from unitarity!
Dominated by exp. uncertainties contrary to $K_{l3}$

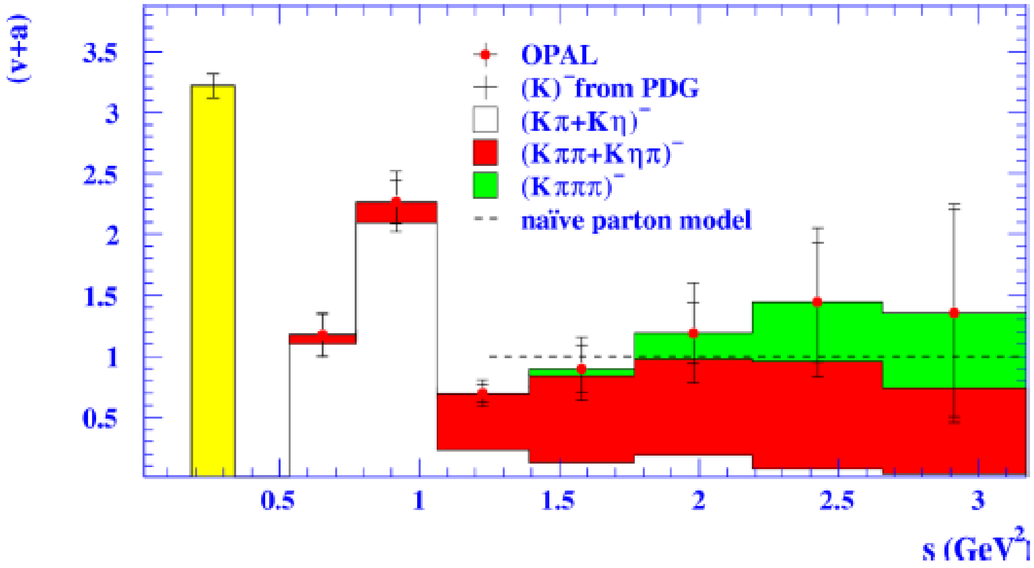
⇨ Potentially the more precise determination of $V_{us}$



# 2.5 Prospects : τ strange Brs

- Experimental measurements of the strange spectral functions not very precise





⇨ New measurements are needed !

- Before B-factories
- With B-factories new measurements :

Smaller τ → K branching ratios ⇨ smaller $R_{\tau,S}$ ⇨ smaller $\boxed{V_{us}}$

$$\boxed{R_\tau^S\Big|_{\text{old}} = 0.1686(47)} \quad \Rightarrow \quad \boxed{R_\tau^S\Big|_{\text{new}} = 0.1612(28)}$$

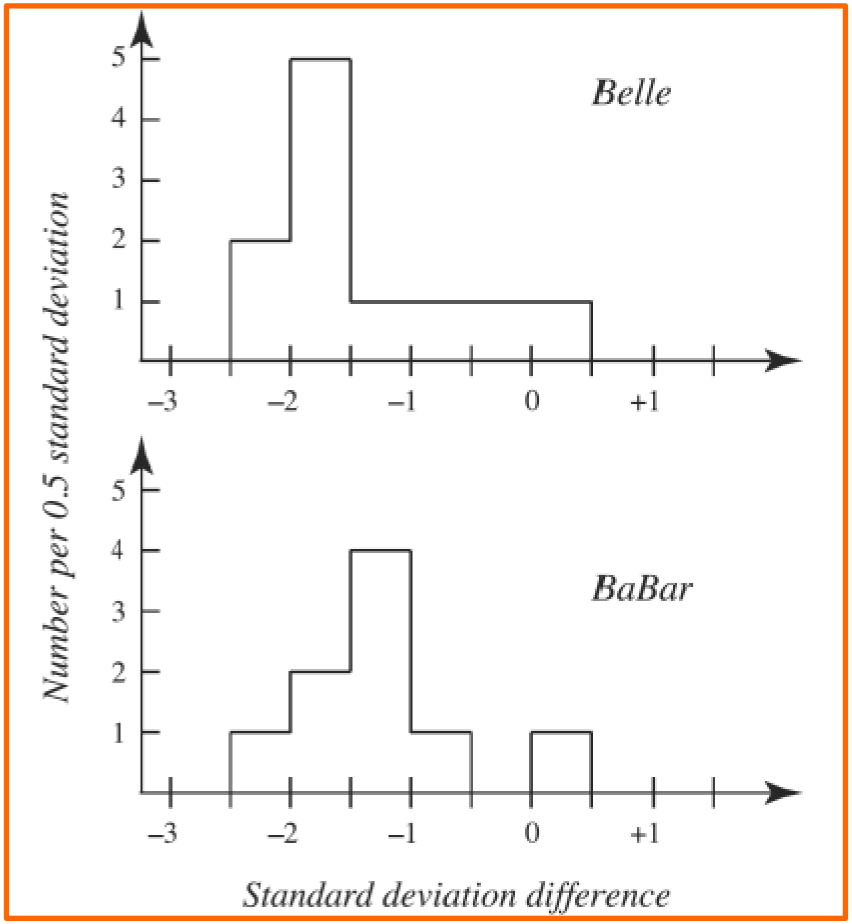
$$\boxed{\left|V_{us}\right|_{\text{old}} = 0.2214 \pm 0.0031_{\text{exp}} \pm 0.0010_{\text{th}}} \quad \Rightarrow \quad \boxed{\left|V_{us}\right|_{\text{new}} = 0.2173 \pm 0.0020_{\text{exp}} \pm 0.0010_{\text{th}}}$$

# 2.5 Prospects : τ strange Brs

- *PDG 2012*: « Eigtheen of the 20 *B*-factory branching fraction measurements are smaller than the non-*B*-factory values. The average normalized difference between the two sets of measurements is -1.30 » (-1.41 for the 11 Belle measurements and -1.24 for the 9 BaBar measurements)

- Measured modes by the 2 B factories:

| Mode | BaBar – Belle<br>Normalized Difference (#$\sigma$) |
|---|---|
| $\pi^-\pi^+\pi^-\nu_\tau$ (ex. $K^0$) | +1.4 |
| $K^-\pi^+\pi^-\nu_\tau$ (ex. $K^0$) | −2.9 |
| $K^-K^+\pi^-\nu_\tau$ | −2.9 |
| $K^-K^+K^-\nu_\tau$ | −5.4 |
| $\eta\ K^-\nu_\tau$ | −1.0 |
| $\phi\ K^-\nu_\tau$ | −1.3 |



# 2.6 New Physics in $R_\tau$

- Models with modifications of the couplings:
  - Right-handed currents

*Bernard, Oertel, E.P., Stern'07*

- $$\Pi^{(J)}(s) = \left|V_{ud}\right|^2 \left(\Pi^{(J)}_{ud,VV}(s) + \Pi^{(J)}_{ud,AA}(s)\right) + \left|V_{us}\right|^2 \left(\Pi^{(J)}_{us,VV}(s) + \Pi^{(J)}_{us,AA}(s)\right)$$

$$\Pi^{(J)}(s) = \left|V^{eff}_{ud}\right|^2 \Pi^{(J)}_{ud,VV}(s) + \left|A^{eff}_{ud}\right|^2 \Pi^{(J)}_{ud,AA}(s) + \left|V^{eff}_{us}\right|^2 \Pi^{(J)}_{us,VV}(s) + \left|A^{eff}_{us}\right|^2 \Pi^{(J)}_{us,AA}(s)$$

$$\frac{R_A}{R_V} = \frac{\left|A^{ud}_{eff}\right|^2 S_{EW}\left(1+\delta^{(0)} + \sum_{D=2,4..} \delta^{(D)}_{ud,A}\right)}{\left|V^{ud}_{eff}\right|^2 S_{EW}\left(1+\delta^{(0)} + \sum_{D=2,4..} \delta^{(D)}_{ud,V}\right)} = (1-4\varepsilon_{ns})\,\frac{\left(1+\delta^{(0)} + \sum_{D=2,4..} \delta^{(D)}_{ud,A}\right)}{\left(1+\delta^{(0)} + \sum_{D=2,4..} \delta^{(D)}_{ud,V}\right)}$$

- Tensor & scalar interactions : ex: leptoquarks, charged Higgs etc

$$R^{NS}_\tau(s_0) = 6\pi i \left|V^{ud}\right|^2 \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \{\left|\kappa_V\right|^2 \left[\left(1+\frac{2s}{m_\tau^2}\right)\Pi^{(1)}_{ud,VV}(s) + \Pi^{(0)}_{ud,VV}(s)\right]$$
$$+\left|\kappa_A\right|^2 \left[\left(1+\frac{2s}{m_\tau^2}\right)\Pi^{(1)}_{ud,AA}(s) + \Pi^{(0)}_{ud,VV}(s)\right] + 2\,\mathrm{Re}\left(\kappa_V \kappa_S^*\right)\frac{\Pi_{ud,VS}(s)}{m_\tau} + 2\,\mathrm{Re}\left(\kappa_A \kappa_P^*\right)\frac{\Pi_{ud,AP}(s)}{m_\tau}$$
$$+12\,\mathrm{Re}\left(\kappa_V \kappa_T^*\right)\frac{\Pi_{ud,VT}(s)}{m_\tau}\}\left[1-2\tilde{v}_L\right]$$

*Cirigliano, Filipuzzi, Gonzalez-Alonso, E.P. in progress*

# 2.6 New Physics in $R_\tau$

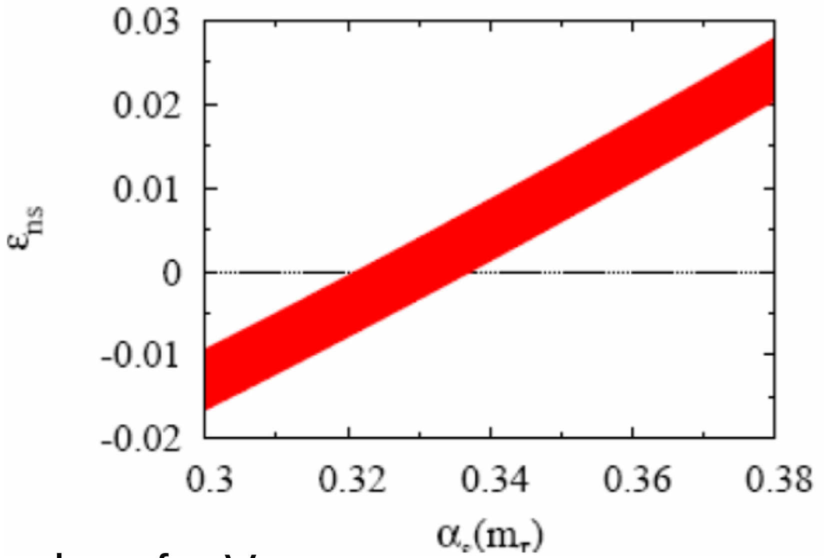
- Disentangle New Physics from QCD effects:
  - Take QCD observables from other sources or more data : Ex: $\alpha_s(m_\tau)$

    *Lattice QCD*, *SCET*, *moments…*

  - Experimental separation V/A very important ⇒ only data from OPAL, need more data

- Possible constraint on NP parameters
  Ex: RHCs

  *Bernard, Oertel, E.P., Stern'07*



⇒ Could explain the difference in the values for $V_{us}$

# 3. Exclusive hadronic τ-decays

# 3.1 Introduction

- For the exclusive hadronic processes $\tau \to H\nu_\tau$ :

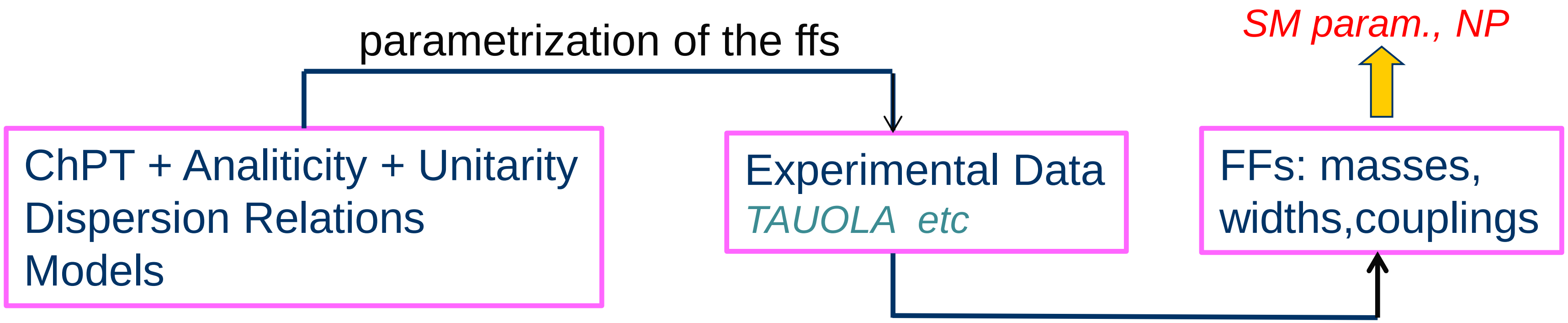
$$M\left(\tau\to H\nu_\tau\right)=\frac{G_F}{\sqrt{2}}V_{CKM}\bar{u}_{\nu_\tau}\gamma^\mu\left(1-\gamma^5\right)u_\tau H_\mu$$

- The hadronic matrix element : $H_\mu=\langle H|\left(V_\mu-A_\mu\right)e^{iL_{QCD}}|0\rangle=\left(Lorentz\ \text{struct.}\right)^i_\mu F_i\left(q^2\right)$

- Experimental measurement : decay rate

$$d\Gamma\left(\tau\to H\nu_\tau\right)=\frac{G_F^2}{4m_\tau}\left|V_{CKM}\right|^2 L_{\mu\nu}H^{\mu\nu}dP_S$$

- Challenge : determination of the form factors to extract SM parameters or NP



parametrization of the ffs

*SM param., NP*

ChPT + Analiticity + Unitarity
Dispersion Relations
Models

Experimental Data
*TAUOLA etc*

FFs: masses, widths,couplings

# 3.1 Introduction

Experimental situation :

- $\tau \to PP\nu_\tau$

$$\begin{cases} \pi^-\pi^0, K^-K^0 \\ K^-\pi^0, \bar{K}^0\pi^- \\ \eta \text{ modes} \end{cases}$$

Branching fractions
Spectrum

Branching fractions

*ALEPH, CLEOIII, OPAL*
*Belle, BaBar*

- $\tau \to PPP\nu_\tau$

$$\begin{cases} \pi\ \pi\ \pi \\ KK\pi \\ K\pi\pi \\ \eta \text{ modes} \\ \text{KKK} \end{cases}$$

Branching fractions
Spectrum

Branching fractions

*ALEPH, CLEOIII, OPAL*
*Belle, BaBar*

- $\tau \to > 3P\nu_\tau$

Theoretical situation

Parametrization using

ChPT + Analiticity + Unitarity
Dispersion relations on the market

➡ Reasonably good control

Parametrization using
ChPT + Analiticity + Unitarity+ Resonances

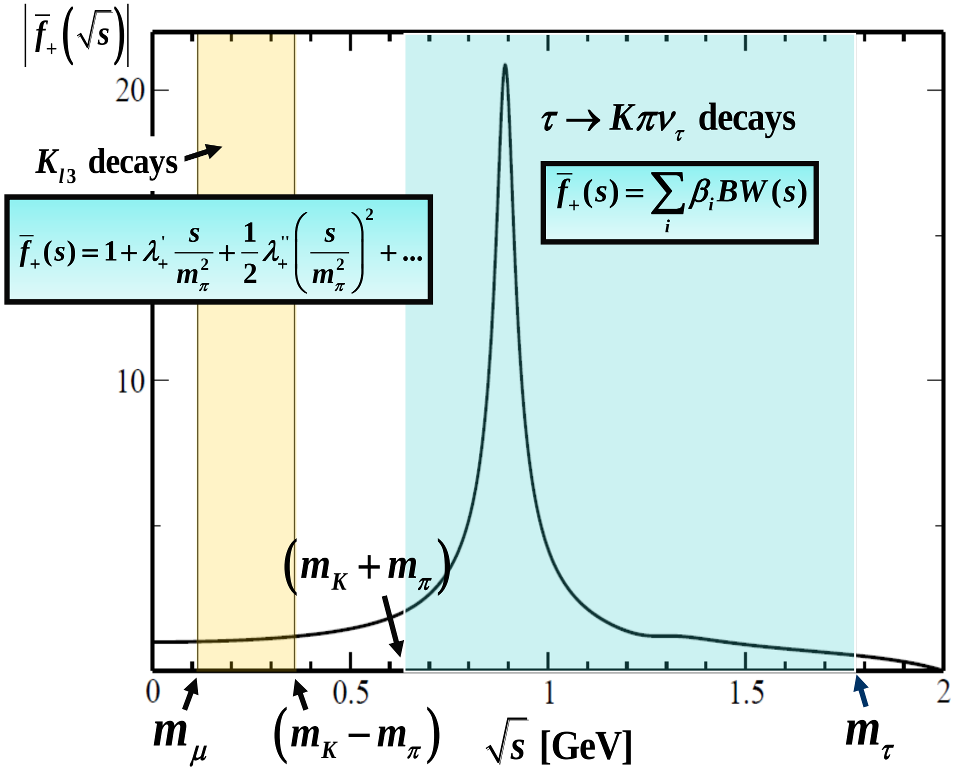
➡ Much more difficult and model dependent

➡ Poor knowledge

# 3.2 Determination of the Kπ form factors

- Use a *dispersive parametrization* to combine experimental information on $K_{l3}$ $\left(K \to \pi l \nu_l\right)$ and $\tau \to K\pi\nu_\tau$ decays

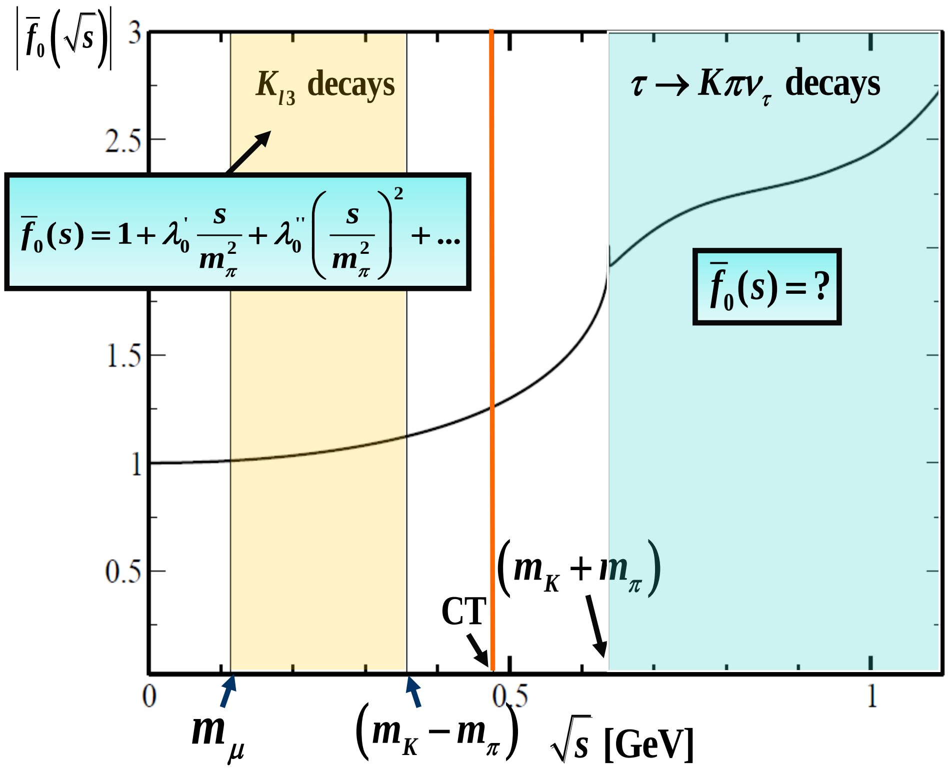
Vector form factor



$$\bar{f}_+(s) = 1 + \lambda_+^{'} \frac{s}{m_\pi^2} + \frac{1}{2}\lambda_+^{''}\left(\frac{s}{m_\pi^2}\right)^2 + \ldots$$

$$\bar{f}_+(s) = \sum_i \beta_i BW(s)$$

Scalar form factor



$$\bar{f}_0(s) = 1 + \lambda_0^{'} \frac{s}{m_\pi^2} + \lambda_0^{''}\left(\frac{s}{m_\pi^2}\right)^2 + \ldots$$

$$\bar{f}_0(s) = ?$$

⇒ Dominance of K*(892) resonance

⇒ No obvious dominance of a resonance

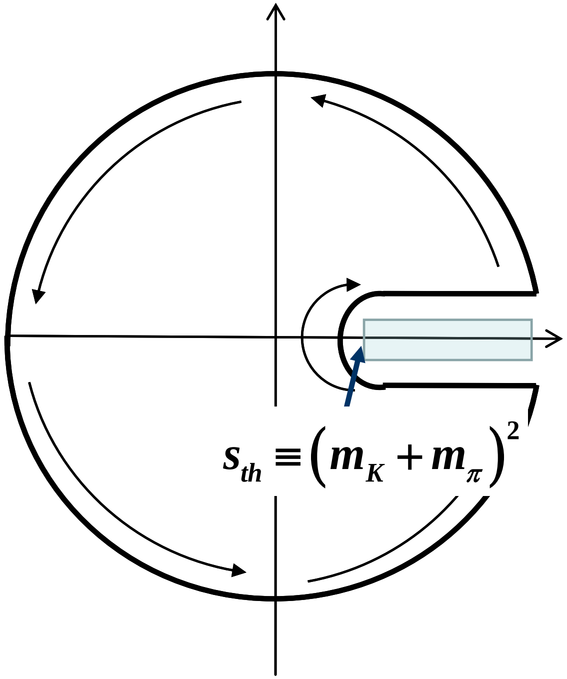
# Dispersive representation

- Parametrization to analyse both $K_{l3}$ and τ

  ⇒ Use dispersion relations

- Omnès representation: ⇒

$$\bar{f}_{+,0}(s) = \exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\phi_{+,0}(s')}{s'-s-i\varepsilon}\right]$$



$\phi_{+,0}(s)$ : phase of the form factor

- $s < s_{in}$ : $\phi_{+,0}(s) = \delta_{K\pi}(s)$

  Kπ scattering phase

- $s \geq s_{in}$ : $\phi_{+,0}(s)$ unknown

  ⇒ $\phi_{+,0}(s) = \phi_{+,0as}(s) = \pi \pm \pi$ $\left(\bar{f}_{+,0}(s) \to 1/s\right)$

*Brodsky & Lepage*

- Subtract dispersion relation to weaken the high energy contribution of the phase. Improve the convergence but sum rules to be satisfied!

# Dispersive representation

*Bernard, Boito, E.P., in progress*

- Dispersion relation with n subtractions in $\bar{s}$:

$$\boxed{\bar{f}_{+,0}(s) = \exp\left[P_{n-1}(s) + \frac{(s-\bar{s})^n}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{(s'-\bar{s})^n}\frac{\phi_{+,0}(s')}{s'-s-i\varepsilon}\right]}$$

➢ $\bar{f}_0(s)$ ⇨ dispersion relation with 3 subtractions: 2 in s=0 and 1 in s=$\Delta_{K\pi}$

*Callan-Treiman*

$$\boxed{\bar{f}_0(s) = \exp\left[\frac{s}{\Delta_{K\pi}}\left(\ln C + (s-\Delta_{K\pi})\left(\frac{\ln C}{\Delta_{K\pi}} - \frac{\lambda_0'}{m_\pi^2}\right) + \frac{\Delta_{K\pi}s(s-\Delta_{K\pi})}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{ds'}{s'^2}\frac{\phi_0(s')}{(s'-\Delta_{K\pi})(s'-s-i\varepsilon)}\right)\right]}$$

➢ $\bar{f}_+(s)$ ⇨ dispersion relation with 3 subtractions in s=0

*Boito, Escribano, Jamin'09,'10*

$$\boxed{\bar{f}_+(s) = \exp\left[\lambda_+'\frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda_+'' - \lambda_+'^2\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{ds'}{s'^3}\frac{\phi_+(s')}{(s'-s-i\varepsilon)}\right]}$$

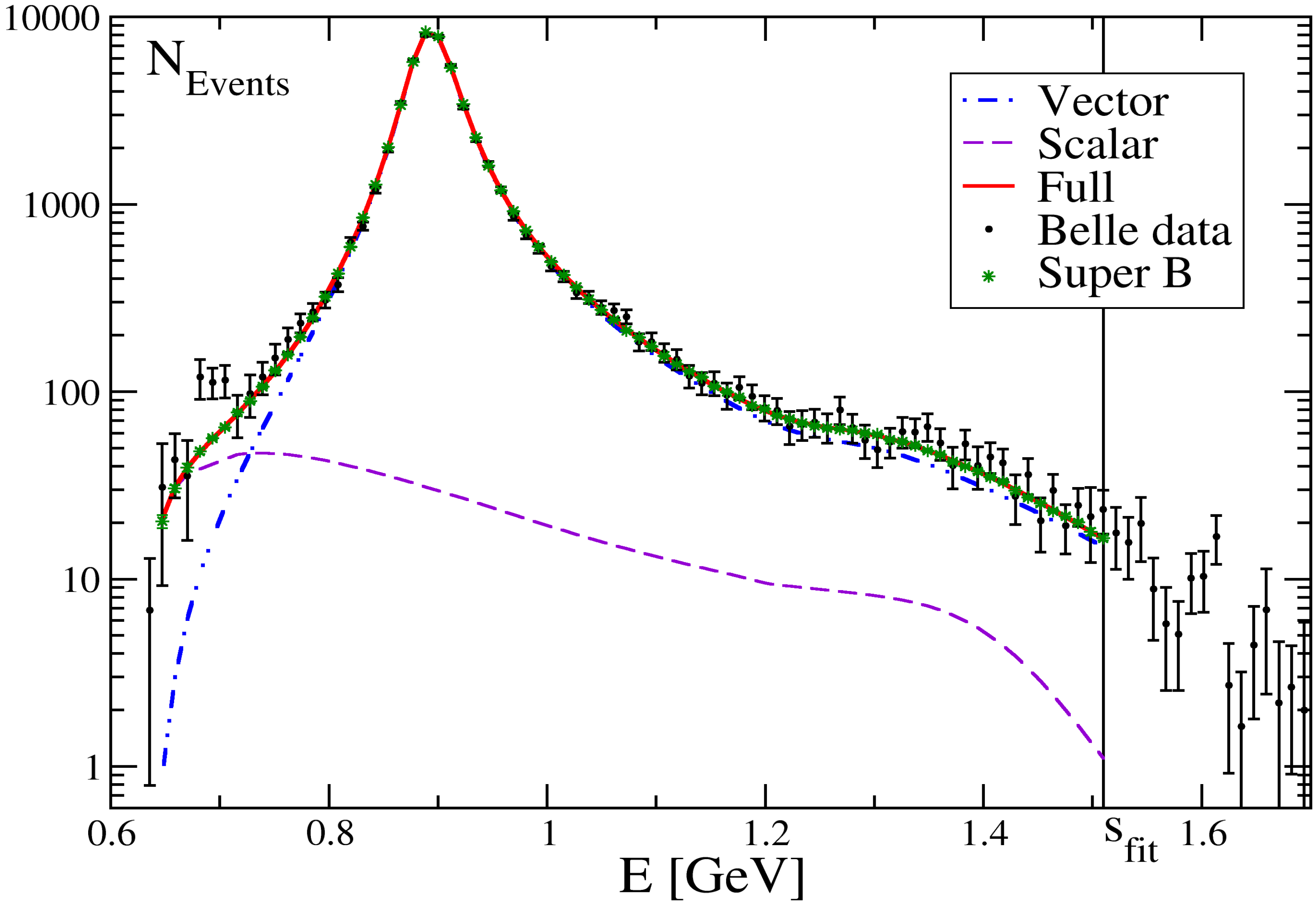
*Jamin, Pich, Portolés'08*

Extracted from a model including
2 resonances K*(892) and K*(1414)

# $K\pi$ form factors from $\tau \to K\pi\nu_\tau$ and $K_{l3}$ decays

*Bernard, Boito, E.P., in progress*

*Antonelli, Cirigliano, Lusiani, E.P. '13*



# $K\pi$ form factors from $\tau \rightarrow K\pi\nu_\tau$ and $K_{l3}$ decays

- Precise extraction of $K\pi$ scattering phase and good determination of K*

$\boxed{m_{K^*} = 892.02 \pm 0.21 \text{ MeV}}$ and $\boxed{\Gamma_{K^*} = 46.300 \pm 0.426 \text{ MeV}}$

PDG : $m_{K^*} = 891.66 \pm 0.26 \text{ MeV}$ and $\Gamma_{K^*} = 50.8 \pm 0.9 \text{ MeV}$

⇨ Tau-Charm: $\boxed{m_{K^*} = 892.02 \pm 0.02 \text{ MeV}}$ and $\boxed{\Gamma_{K^*} = 46.300 \pm 0.044 \text{ MeV}}$

- Callan-Treiman test or lattice QCD test ($F_K/F_\pi$ and $f_+(0)$)

- $V_{us}$ from $\tau \rightarrow K\pi\nu_\tau$ $\boxed{\Gamma_{\tau\rightarrow K\pi\nu_\tau} = N \left| f_+(0) V_{us} \right|^2 I_K^\tau}$ with $\boxed{I_K^\tau = \int ds\, F\left(s, \overline{f}_+(s), \overline{f}_0(s)\right)}$

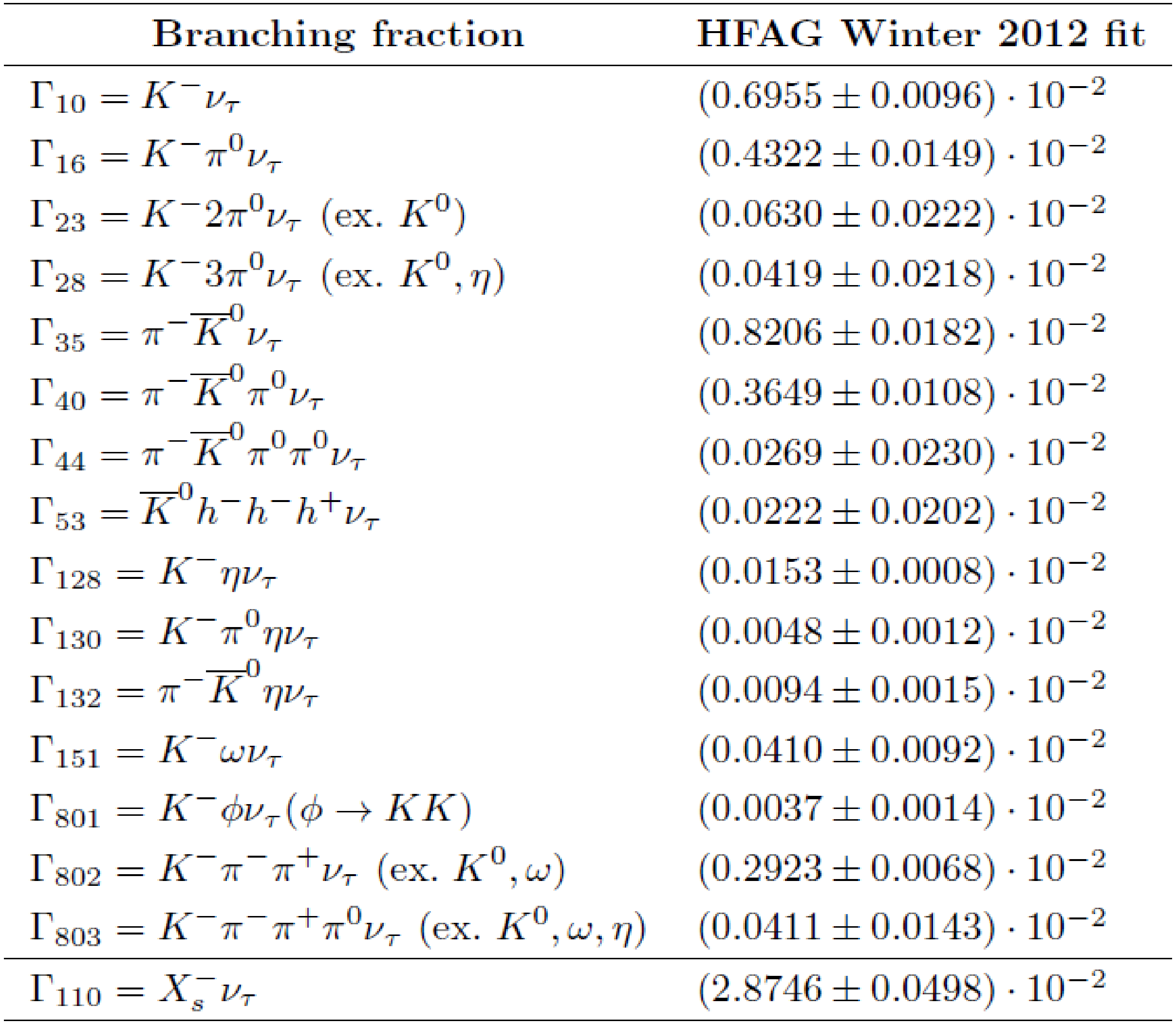
- Prediction of the strange Brs and $V_{us}$

- Use of the form factors for CPV tests, etc.

# 3.3 Application: Prediction of $\tau$ strange Brs and $V_{us}$

*Antonelli, Cirigliano, Lusiani, E.P.'13*

- Modes measured in the strange channel for $\tau \to s$ :



| Branching fraction | HFAG Winter 2012 fit |
|---|---|
| $\Gamma_{10} = K^- \nu_\tau$ | $(0.6955 \pm 0.0096) \cdot 10^{-2}$ |
| $\Gamma_{16} = K^- \pi^0 \nu_\tau$ | $(0.4322 \pm 0.0149) \cdot 10^{-2}$ |
| $\Gamma_{23} = K^- 2\pi^0 \nu_\tau$ (ex. $K^0$) | $(0.0630 \pm 0.0222) \cdot 10^{-2}$ |
| $\Gamma_{28} = K^- 3\pi^0 \nu_\tau$ (ex. $K^0, \eta$) | $(0.0419 \pm 0.0218) \cdot 10^{-2}$ |
| $\Gamma_{35} = \pi^- \overline{K}^0 \nu_\tau$ | $(0.8206 \pm 0.0182) \cdot 10^{-2}$ |
| $\Gamma_{40} = \pi^- \overline{K}^0 \pi^0 \nu_\tau$ | $(0.3649 \pm 0.0108) \cdot 10^{-2}$ |
| $\Gamma_{44} = \pi^- \overline{K}^0 \pi^0 \pi^0 \nu_\tau$ | $(0.0269 \pm 0.0230) \cdot 10^{-2}$ |
| $\Gamma_{53} = \overline{K}^0 h^- h^- h^+ \nu_\tau$ | $(0.0222 \pm 0.0202) \cdot 10^{-2}$ |
| $\Gamma_{128} = K^- \eta \nu_\tau$ | $(0.0153 \pm 0.0008) \cdot 10^{-2}$ |
| $\Gamma_{130} = K^- \pi^0 \eta \nu_\tau$ | $(0.0048 \pm 0.0012) \cdot 10^{-2}$ |
| $\Gamma_{132} = \pi^- \overline{K}^0 \eta \nu_\tau$ | $(0.0094 \pm 0.0015) \cdot 10^{-2}$ |
| $\Gamma_{151} = K^- \omega \nu_\tau$ | $(0.0410 \pm 0.0092) \cdot 10^{-2}$ |
| $\Gamma_{801} = K^- \phi \nu_\tau (\phi \to KK)$ | $(0.0037 \pm 0.0014) \cdot 10^{-2}$ |
| $\Gamma_{802} = K^- \pi^- \pi^+ \nu_\tau$ (ex. $K^0, \omega$) | $(0.2923 \pm 0.0068) \cdot 10^{-2}$ |
| $\Gamma_{803} = K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. $K^0, \omega, \eta$) | $(0.0411 \pm 0.0143) \cdot 10^{-2}$ |
| $\Gamma_{110} = X_s^- \nu_\tau$ | $(2.8746 \pm 0.0498) \cdot 10^{-2}$ |

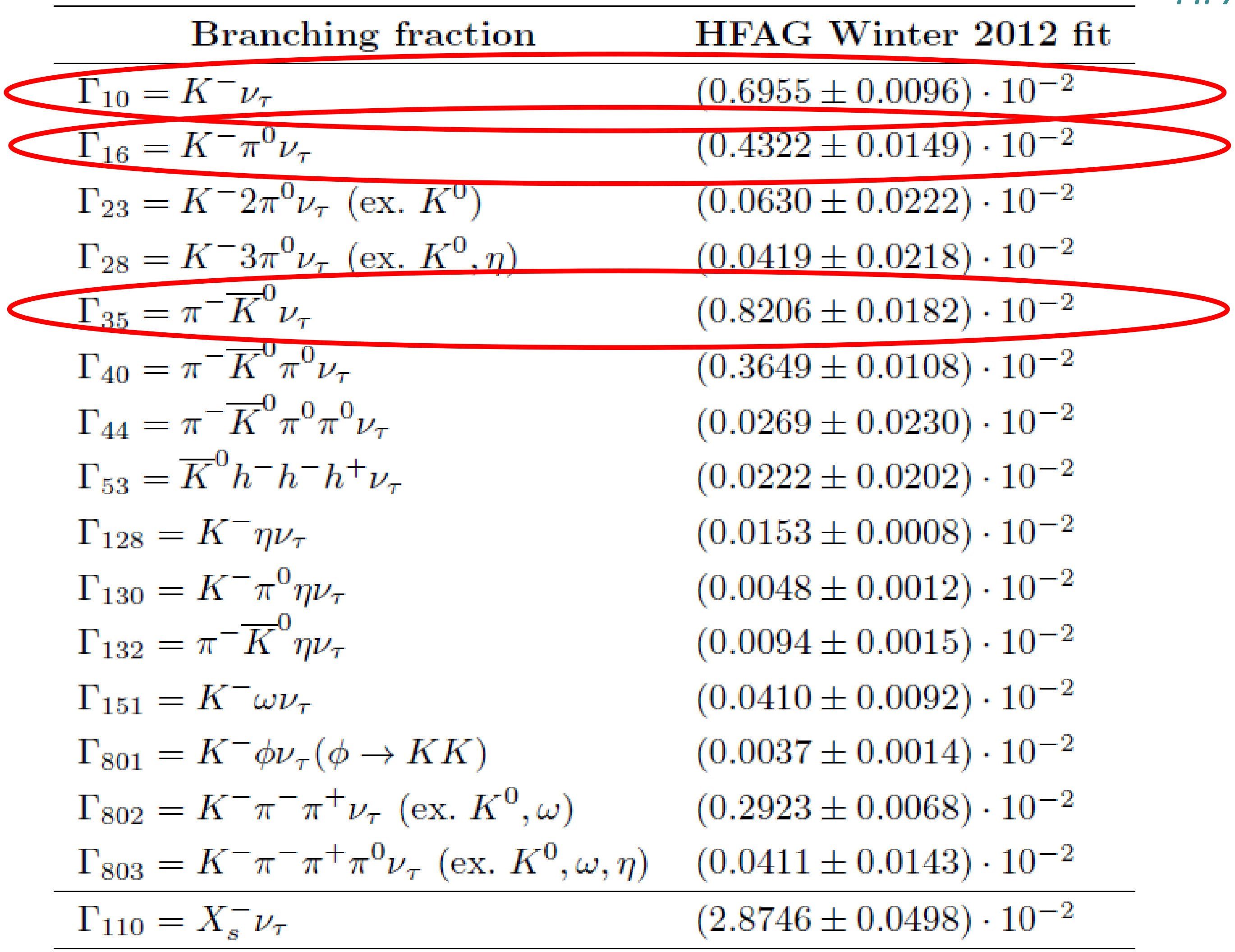
*HFAG'12*

# 3.3 Application: Prediction of τ strange Brs and $V_{us}$

*Antonelli, Cirigliano, Lusiani, E.P.'13*

- Modes measured in the strange channel for $\tau \to s$ :

*HFAG'12*



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| $\Gamma_{803} = K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. $K^0, \omega, \eta$) | $(0.0411 \pm 0.0143) \cdot 10^{-2}$ |
| $\Gamma_{110} = X_s^- \nu_\tau$ | $(2.8746 \pm 0.0498) \cdot 10^{-2}$ |

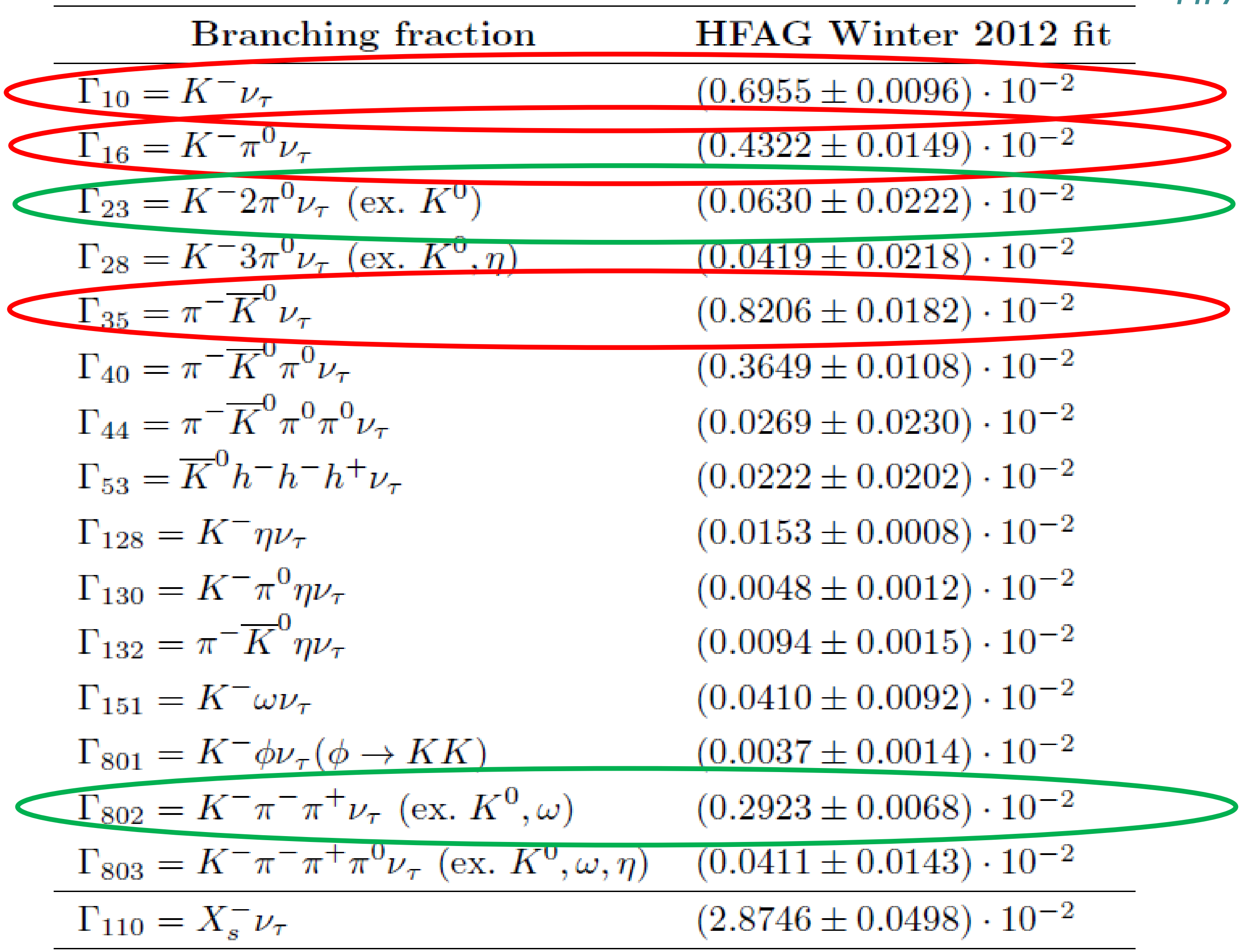
~70% of the decay modes crossed channels from Kaons!

# 3.3 Application: Prediction of τ strange Brs and $V_{us}$

*Antonelli, Cirigliano, Lusiani, E.P.'13*

- Modes measured in the strange channel for $\tau \to s$ :

*HFAG'12*



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| $\Gamma_{110} = X_s^- \nu_\tau$ | $(2.8746 \pm 0.0498) \cdot 10^{-2}$ |

~70% of the decay
modes crossed
channels
from Kaons!

Up to ~90%
Including the
2π modes

# 3.3 Application: Prediction of τ strange Brs and $V_{us}$

*Antonelli, Cirigliano, Lusiani, E.P.'13*

- The Brs of these 3 modes can be predicted using Kaon Brs very precisely measured + form factor information

➢ $\tau \to K\nu_\tau$ :

$$\boxed{\mathrm{BR}(\tau \to K\nu_\tau) = \frac{m_\tau^3}{2 m_K m_\mu^2} \boxed{\frac{S_{\mathrm{EW}}^{\tau}}{S_{\mathrm{EW}}^{K}}} \left(\frac{1 - m_K^2/m_\tau^2}{1 - m_\mu^2/m_K^2}\right)^2 \boxed{\frac{\tau_\tau}{\tau_K}} \boxed{\delta_{\mathrm{EM}}^{\tau/K}} \boxed{\mathrm{BR}(K_{\ell 2})}}$$

➢ Inputs needed:

→ Experimental : BR($K_{l2}$), lifetimes

→ Theoretical : Short distance EW corrections
Long distance EM corrections

⇨ $$\boxed{BR\left(\tau^- \to K^-\nu_\tau\right) = (0.713 \pm 0.003)\%}$$

# 3.3 Application: Prediction of τ strange Brs and $V_{us}$

*Antonelli, Cirigliano, Lusiani, E.P.'13*

- The Brs of these 3 modes can be predicted using Kaon Brs very precisely measured + form factor information

  - $\tau \to K\pi\nu_\tau$ :

$$\boxed{\mathrm{BR}(\tau \to \bar{K}\pi\nu_\tau) = \frac{2m_\tau^5}{m_K^5} \boxed{\frac{S_{\mathrm{EW}}^\tau}{S_{\mathrm{EW}}^K}} \boxed{\frac{I_K^\tau}{I_K^\ell}} \frac{\left(1 + \boxed{\delta_{\mathrm{EM}}^{K\tau}} + \boxed{\tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}}\right)^2}{\left(1 + \boxed{\delta_{\mathrm{EM}}^{K\ell}} + \boxed{\delta_{\mathrm{SU(2)}}^{K\pi}}\right)^2} \boxed{\frac{\tau_\tau}{\tau_K}} \boxed{\mathrm{BR}(K \to \pi e \bar{\nu}_e)}}$$

  - Inputs needed :
    - The $K_{e3}$ branching ratios, lifetimes
    - Phase space integrals ⇨ use the dispersive parametrization for the form factors
    - The electromagnetic and isospin-breaking corrections

⇨ $\boxed{BR\left(\tau \to \overline{K}^0 \pi^- \nu_\tau\right) = (0.8569 \pm 0.0293)\%}$ and $\boxed{BR\left(\tau \to K^- \pi^0 \nu_\tau\right) = (0.4709 \pm 0.0178)\%}$

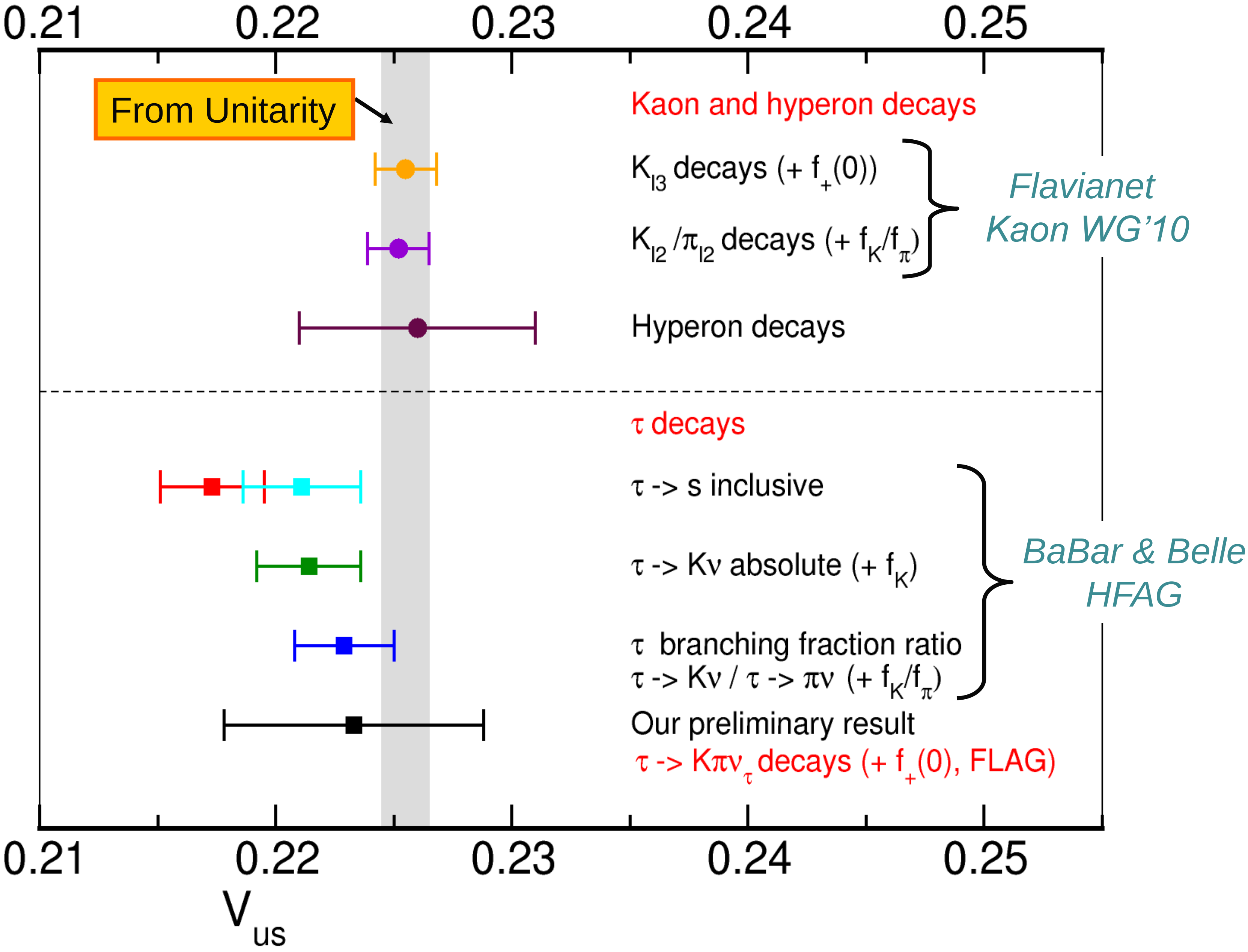
# 3.3 Application: Prediction of τ strange Brs and V$_{us}$

*Antonelli, Cirigliano, Lusiani, E.P.'13*

| Mode | BR | % err | BR($K_{e3}$) | $\tau_K$ | $\tau_\tau$ | $I^\tau_K/I^e_K$ | $\Delta_{\rm EM}$ | $\Delta_{\rm SU(2)}$ |
|---|---|---|---|---|---|---|---|---|
| $\tau^- \to \bar{K}^0\pi^-\nu_\tau$ | $0.8569 \pm 0.0293$ | 3.42 | 0.22 | 0.41 | 0.35 | 3.34 | 0.46 | 0 |
| $\tau^- \to K^-\pi^0\nu_\tau$ | $0.4709 \pm 0.0178$ | 3.79 | 0.06 | 0.12 | 0.34 | 3.60 | 0.47 | 1.00 |

| **Branching fraction** | **HFAG Winter 2012 fit** | **Prediction** |
|---|---|---|
| $\Gamma_{10} = K^-\nu_\tau$ | $(0.6955 \pm 0.0096)\cdot 10^{-2}$ | $(0.713 \pm 0.003)\cdot 10^{-2}$ |
| $\Gamma_{16} = K^-\pi^0\nu_\tau$ | $(0.4322 \pm 0.0149)\cdot 10^{-2}$ | $(0.4709 \pm 0.0178)\cdot 10^{-2}$ |
| $\Gamma_{35} = \pi^-\bar{K}^0\nu_\tau$ | $(0.8206 \pm 0.0182)\cdot 10^{-2}$ | $(0.8569 \pm 0.0293)\cdot 10^{-2}$ |
| $\Gamma_{110} = X_s^-\nu_\tau$ | $(2.8746 \pm 0.0498)\cdot 10^{-2}$ | $(2.9714 \pm 0.0561)\cdot 10^{-2}$ |

$$\boxed{|V_{us}| = 0.2173 \pm 0.0022} \quad \Rightarrow \quad \boxed{|V_{us}| = 0.2211 \pm 0.0025}$$


0.21
0.22
0.23
0.24
0.25
From Unitarity
Kaon and hyperon decays
$K_{l3}$ decays (+ $f_+(0)$)
$K_{l2}/\pi_{l2}$ decays (+ $f_K/f_\pi$)
Flavianet
Kaon WG'10
Hyperon decays
$\tau$ decays
$\tau$ -> s inclusive
$\tau$ -> K$\nu$ absolute (+ $f_K$)
$\tau$ branching fraction ratio
$\tau$ -> K$\nu$ / $\tau$ -> $\pi\nu$ (+ $f_K/f_\pi$)
BaBar & Belle
HFAG
Our preliminary result
$\tau$ -> K$\pi\nu_\tau$ decays (+ $f_+(0)$, FLAG)
$V_{us}$

# 3.3 Application : $\tau \to K\pi\nu_\tau$ CP violating asymmetry

- CPV in hadronic τ decays : Ex: $\tau \to K\pi\nu_\tau$ CP violating asymmetry

$$\boxed{A_Q = \frac{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \overline{\nu}_\tau\right) - \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \overline{\nu}_\tau\right) + \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}}$$

$$\left|K_S^0\right\rangle = p\left|K^0\right\rangle + q\left|\overline{K}^0\right\rangle$$

$$\left|K_L^0\right\rangle = p\left|K^0\right\rangle - q\left|\overline{K}^0\right\rangle$$

$= |p|^2 - |q|^2$ $\boxed{\approx (0.36 \pm 0.01)\,\%}$ in the SM

$$\boxed{\left\langle K_L | K_S \right\rangle = |p|^2 - |q|^2 \simeq 2\,\mathrm{Re}\left(\varepsilon_K\right)}$$

*Bigi & Sanda'05*

*Grossman & Nir'11*

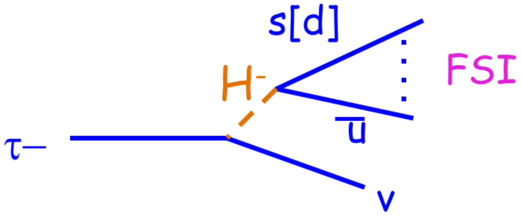
- Experimental measurement : $\boxed{A_{Q\,\mathrm{exp}} = \left(-0.45 \pm 0.24_{\mathrm{stat}} \pm 0.11_{\mathrm{syst}}\right)\%}$

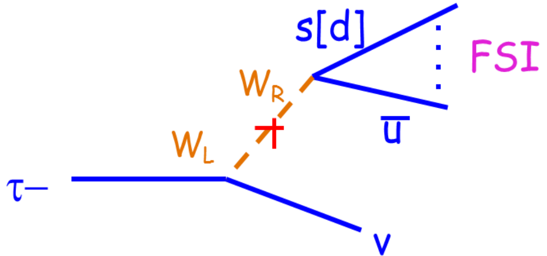
⇨ $\boxed{\sim 3\sigma}$ from the SM!

*BaBar'11*

- New physics explanation : Charged Higgs, $W_L$-$W_R$ mixings, leptoquarks?

*Bigi'Tau12*





⇨ Need to know the Kπ form factors

# 3.3 Application : $\tau \to K\pi\nu_\tau$ CP violating asymmetry

- Belle doesn't see any CP asymmetry in the angular distribution

⇨ finds a null result at 0.2 - 0.3% level

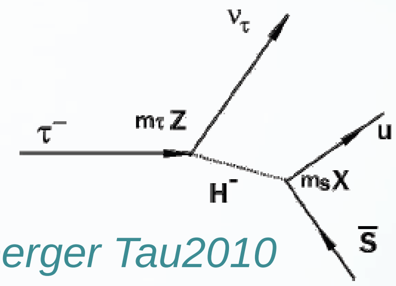
*Belle'11*

- Need new measurements on the angular CP violating asymmetry

⇨ measure other asymmetries to disentangle scalar and vector $K\pi$ ffs

Ex: Forward-Backward asymmetry

$$\boxed{A_{FB} = \frac{d\Gamma(\cos\theta) - d\Gamma(-\cos\theta)}{d\Gamma(\cos\theta) + d\Gamma(-\cos\theta)}}$$



- A variety of CPV observables can be studied : rate, angular asymmetries, triple products, ….

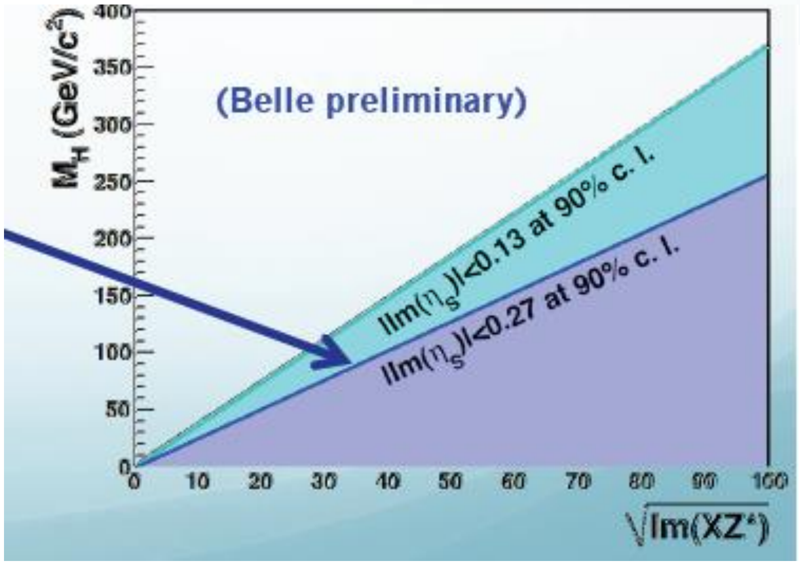
*Bishchofberger Tau2010*

⇨ Consider CPV in $\tau \to K\pi\pi\nu_\tau$ , $\tau \to V\pi\nu_\tau$

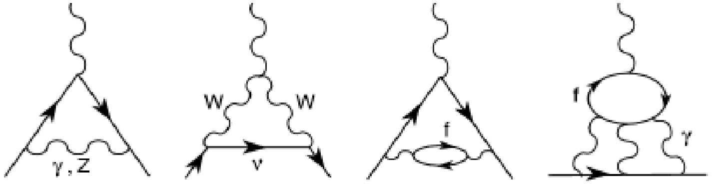
*Datta, Kiers, London, O'Donnell', Szynkman'07*

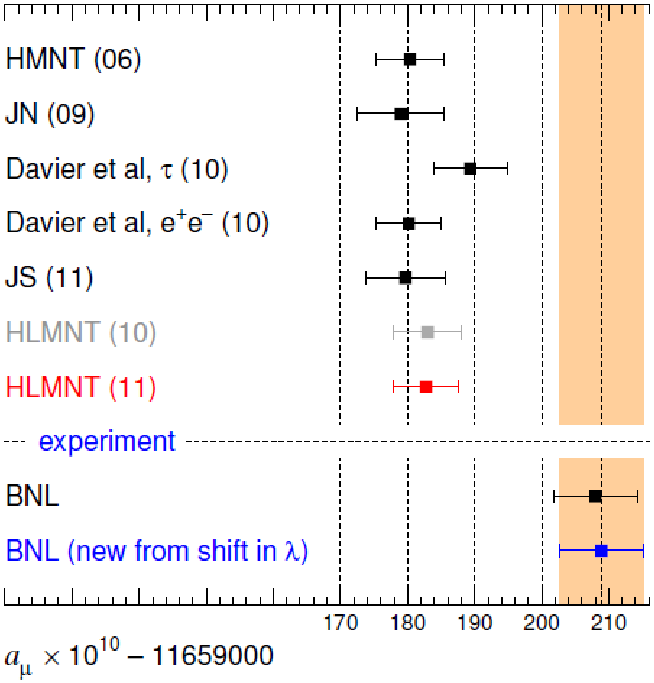
Difficulty : hadronic final state interactions

- Interest : Constrain mass and couplings of new particles ⇨ correlations



# 3.4 Anomalous magnetic moment of the muon





$$a_\mu^{\rm exp} = (11\,659\,208.9 \pm 6.3)\cdot 10^{-10}$$

**BNL-E821**

$$10^{10}\cdot a_\mu^{\rm th} = 11\,658\,471.895 \pm 0.008 \quad \text{QED}$$

**Aoyama-Hayakawa-Kinoshita-Nio**

$$+\ 15.4 \pm 0.2 \quad \text{EW}$$

**Czarnecki et al**, Knecht et al

$$+\ 696.4 \pm 4.6 \quad \text{hvp} \quad (701.5 \pm 4.7)_\tau \ , \ (692.4 \pm 4.1)_{e^+e^-}$$

**Davier et al,** Hagiwara et al, Jegerlehner-Nyffeler

$$-\ 9.8 \pm 0.1 \quad \text{hvp NLO}$$

**Krause**, Hagiwara et al

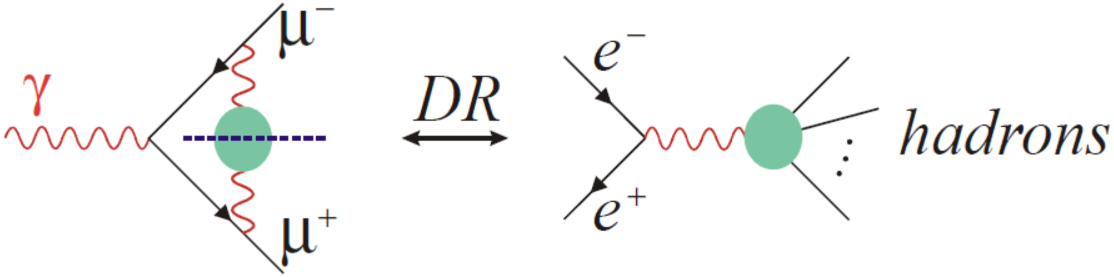
$$+\ 10.5 \pm 2.6 \quad \text{light-by-light}$$

**de Rafael-Prades-Vainshtein,** Melnikov-Vainshtein, Knecht et al, Bijnens et al, Hayakawa et al, Nyffeler

$$= 11\,659\,184.4 \pm 5.3 \qquad (11\,659\,189.5 \pm 5.4)_\tau \ , \ (11\,659\,180.4 \pm 4.9)_{e^+e^-}$$

$$a_\mu^{\rm exp} - a_\mu^{\rm th} = 3.0\,\sigma \qquad 2.3\,\sigma \qquad 3.6\,\sigma$$

# 3.4 Anomalous magnetic moment of the muon



- Leading order hadronic vacuum polarization :

$$\boxed{a_\mu^{had,LO} = \frac{\alpha^2 m_\mu^2}{(3\pi)^2}\int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s^2} R_V(s)}$$

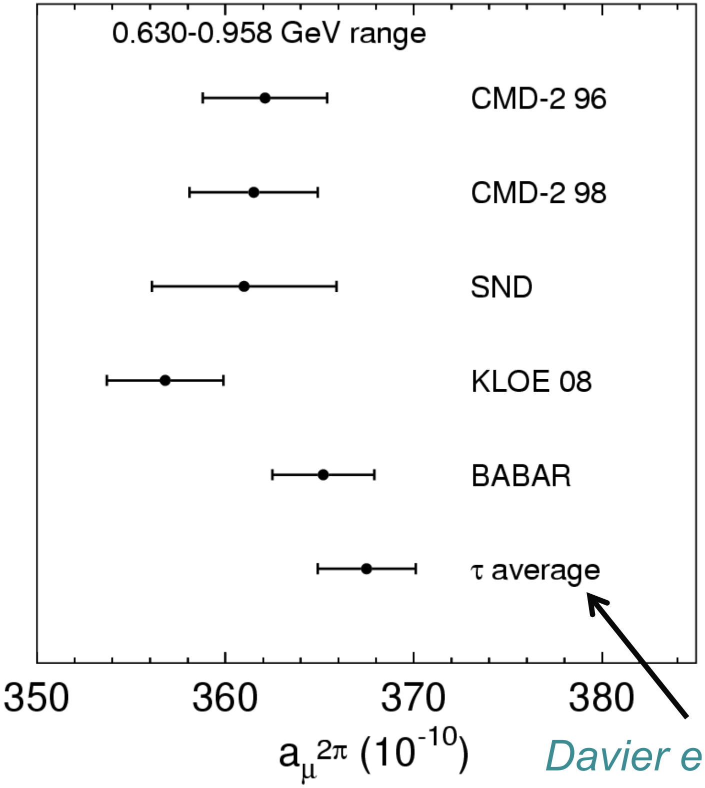
$$\boxed{R_V(s) = \frac{\sigma\left(e^+e^- \to hadrons\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)}}$$

- Low energy contribution dominates :
  ~75% comes from s < (1 GeV)$^2$ ⇨ ππ loops dominate

$$a_\mu^{had,LO} \Rightarrow a_\mu^{2\pi} \quad \text{and} \quad R_V(s) = \frac{1}{4}\left(1 - \frac{4m_\pi^2}{s}\right)^{3/2} \left|F_\pi\right|^2$$

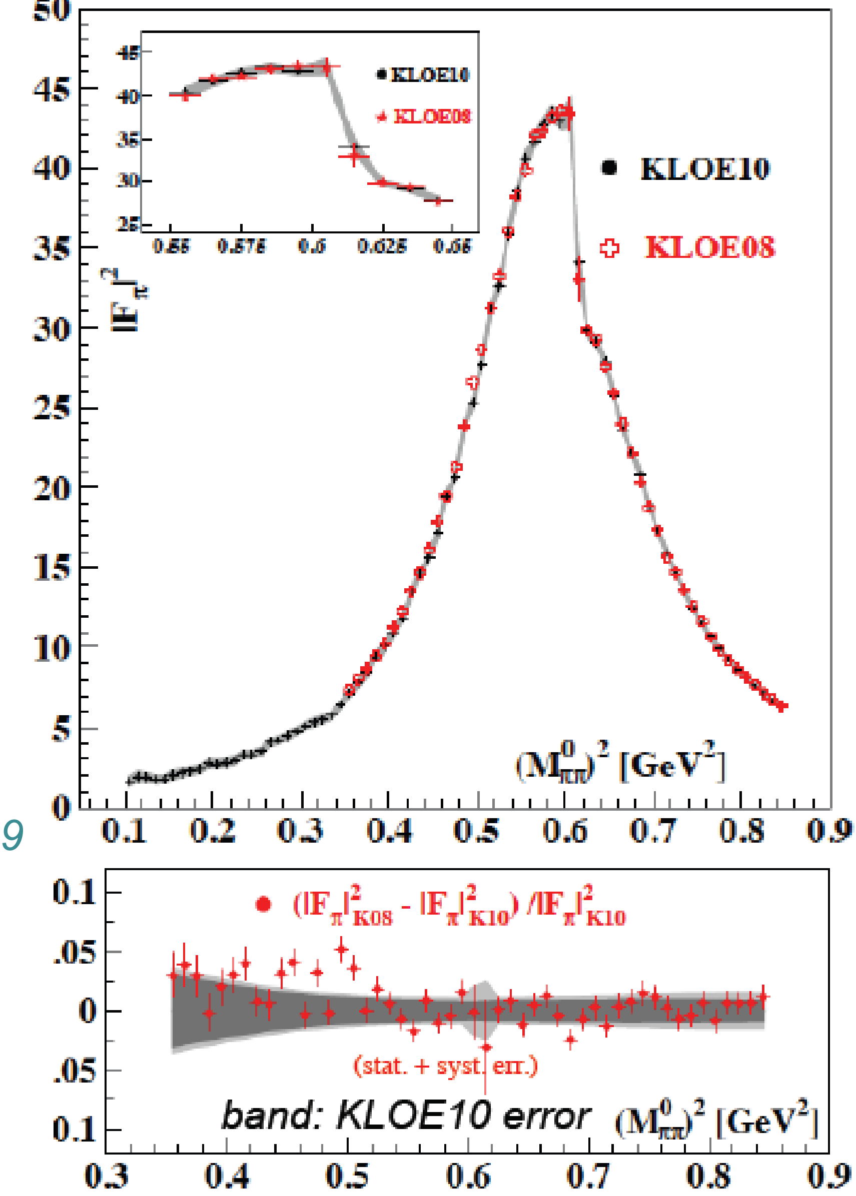
# 3.4 Anomalous magnetic moment of the muon

Malaescu'Photon13



Davier et al.'09

Passeri'Photon13

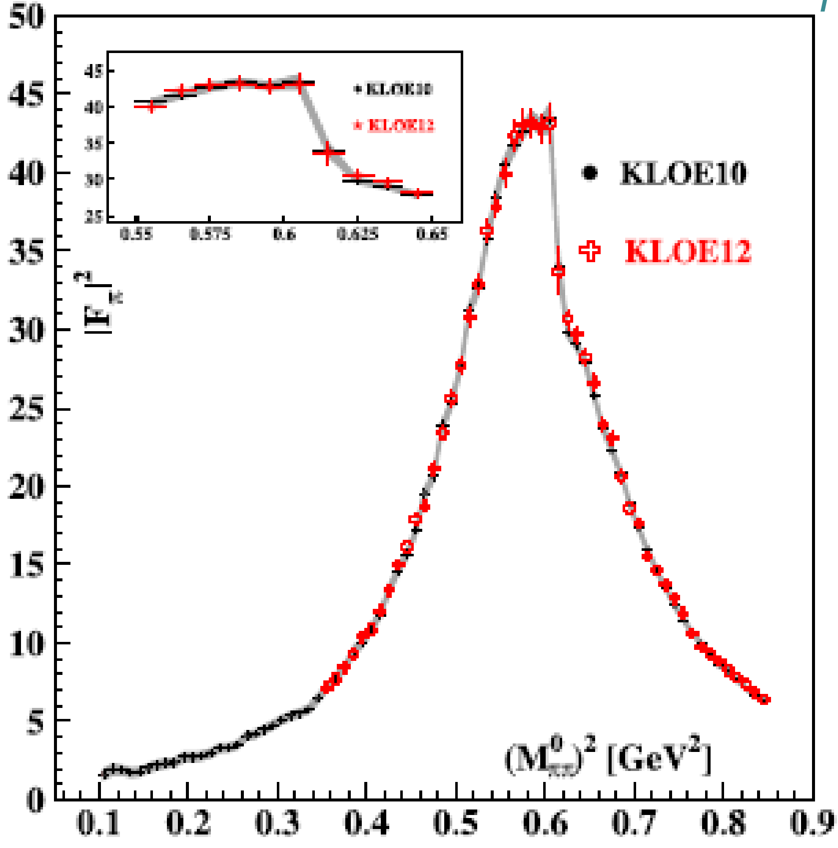


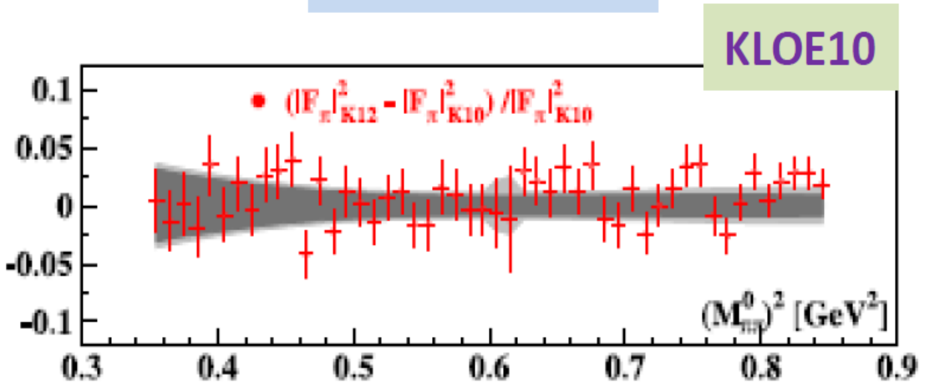
*band: KLOE10 error*

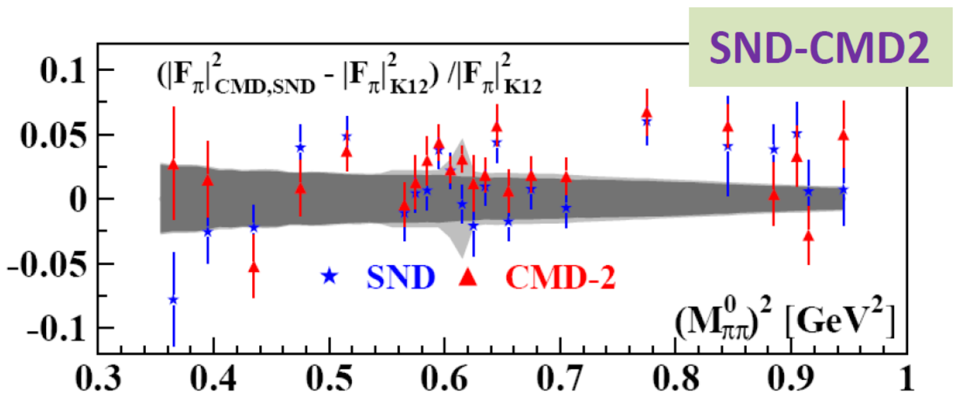
# 3.4 Anomalous magnetic moment of the muon

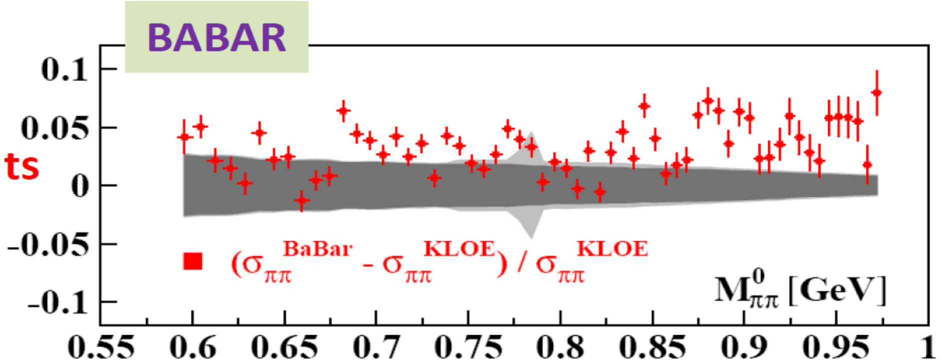
*Passeri'Photon13*

**KLOE12 vs :**









- Parametrization of the ffs
- FSR contributions from $\pi$
- Disagreement betwenn KLOE and BaBar ⇨ Tau-Charm factory?

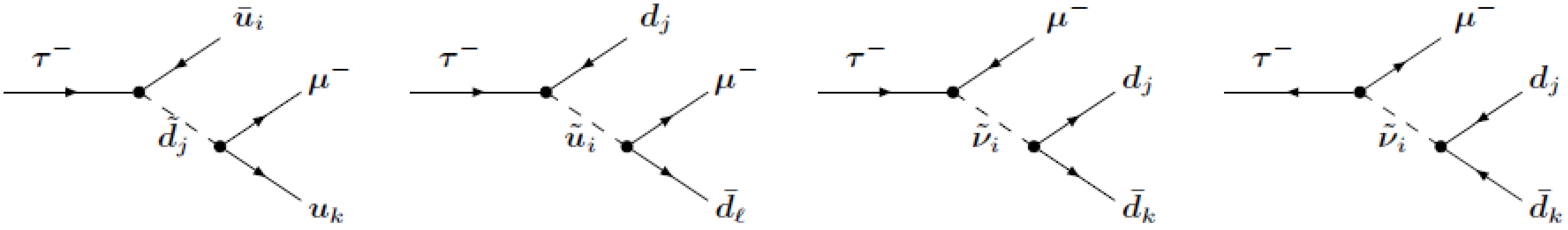
*Taus* + *ISR*

# 3.5 Lepton-flavour violating τ → μππ decays

- Leptonic decays : τ → μγ golden channel of Tau-Charm factory

- τ → μππ decays interesting probe as well
  Ex: R-parity violating SUSY operators

*Herrero, Portoles',Rodriguez'08,09*
*Dreiner, Hanart, Kubis, Meissner'13*



⇨ effective operators generated by heavy SUSY particle exchanges

$$\mathcal{L}_{\text{eff}} = \underbrace{\frac{\lambda'_{31j}\lambda'^{*}_{21j}}{2m^2_{\tilde{d}_j}}}_{\text{eff. coupling}} \underbrace{(\bar{\mu}\gamma_\alpha P_L\tau)(\bar{u}\gamma^\alpha P_L u)}_{\text{vector}\times\text{vector}} + \ldots + \underbrace{\frac{\lambda_{3i2}\lambda'^{*}_{i11}}{2m^2_{\tilde{\nu}_i}}}_{\text{eff. coupling}} \underbrace{(\bar{\mu}P_L\tau)(\bar{d}P_R d)}_{\text{scalar}\times\text{scalar}} + \ldots$$

- Problem : Have the hadronic part under control
  Huge model uncertainties e.g. in $\tau \to \mu f_0(980)$ strenght of scalar couplings to quark currents depends of controversial nature of scalar resonances
  ⇨ To avoid that use *form factors* and *dispersion relations*

# 3.5 Lepton-flavour violating $\tau \to \mu\pi\pi$ decays

- Hadronisation into ππ given by scalar/vector form factors

$$\langle \pi^+\pi^- | \tfrac{1}{2}(\bar{u}\gamma^\alpha u - \bar{d}\gamma^\alpha d) | 0 \rangle = F_\pi^V(s)(p_{\pi^+} - p_{\pi^-})^\alpha$$

$$\langle \pi^+\pi^- | \tfrac{1}{2}(\bar{u}u + \bar{d}d) | 0 \rangle = \mathcal{B}^n \Gamma^n(s) \qquad \langle \pi^+\pi^- | \bar{s}s | 0 \rangle = \mathcal{B}^s \Gamma^s(s)$$

- Vector ff can be extracted from $\tau \to \pi\pi\nu_\tau$ decay spectrum measured by *Belle*

$$\boxed{\frac{d\Gamma_{\pi\pi}}{d\sqrt{s}} = \frac{G_F^2 m_\tau^3}{32\pi^3 s} C_K^2 S_{EW} |V_{ud}|^2 \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + \frac{2s}{m_\tau^2}\right) q_{K\pi}^3(s) \left| f_+(s)\right|^2\right]}$$

with a dispersive parametrisation for the vector form factor

*Guerrero, Pich'98*
*Pich, Portolés'08*
*Gomez, Roig'13*

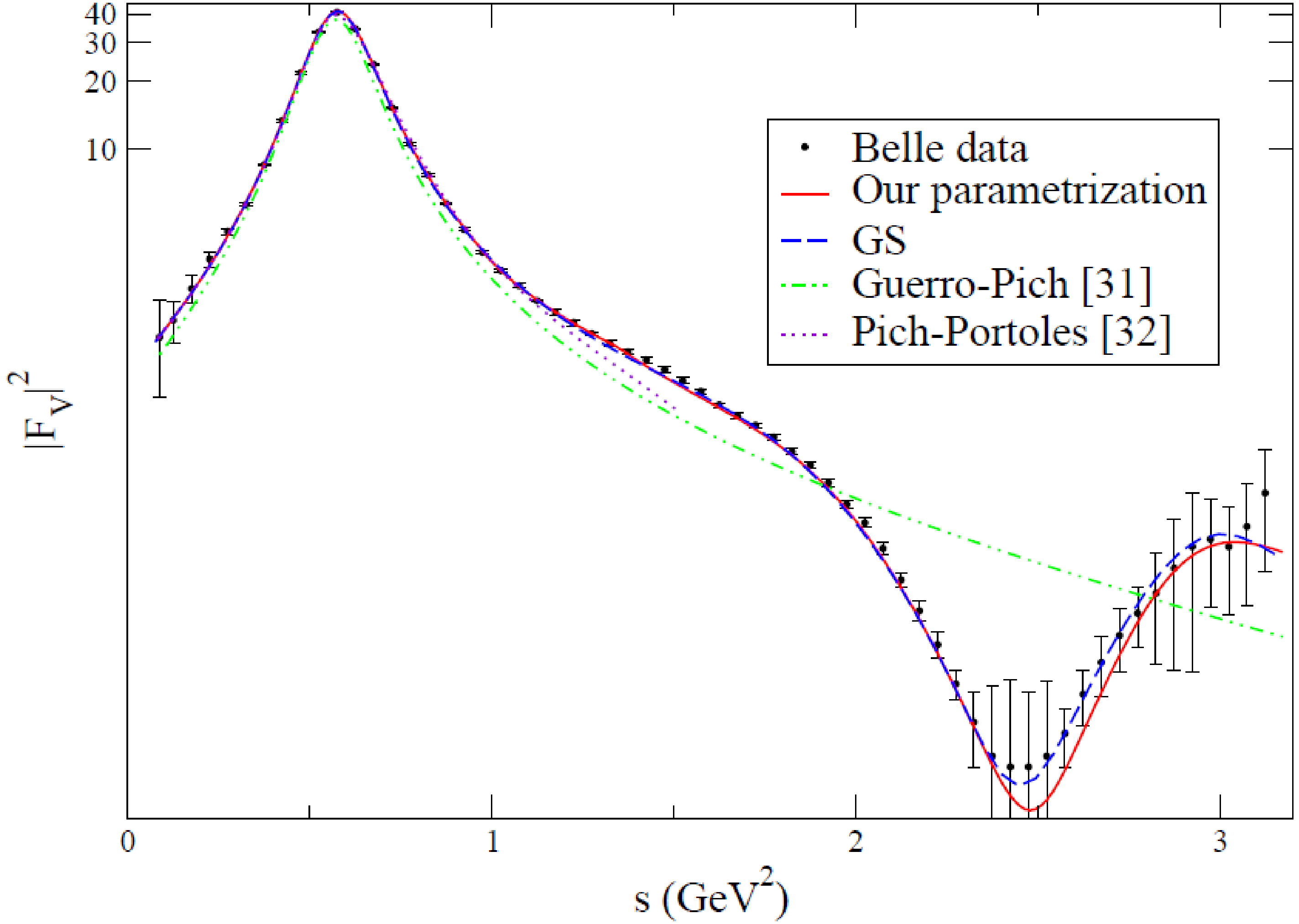
$$\boxed{\bar{f}_+(s) = \exp\left[\lambda_+^{'} \frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda_+^{''} - \lambda_+^{'2}\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^3} \frac{\phi_+(s')}{(s' - s - i\varepsilon)}\right]}$$

Extracted from a model including
3 resonances ρ(892) and K*(1414)
fitted to the data

# 3.5 Lepton-flavour violating $\tau \to \mu\pi\pi$ decays

- Vector ff can be extracted from $\tau \to \pi\pi\nu_\tau$ decay spectrum measured by *Belle*

*Gomez, Roig'13*



# 3.5 Lepton-flavour violating $\tau \to \mu\pi\pi$ decays

*Dreiner, Hanart, Kubis , Meissner'13*

- Results : $\boxed{BR\left(\tau \to \rho_0(770)\mu\right) < 1.2\ 10^{-8}}$ and $\boxed{BR\left(\tau \to \pi\pi\mu\right) < 2.1\ 10^{-8}}$

*Belle'08'11'12*

| product of couplings | bound | susy mass | eff. coupling |
|---|---|---|---|
| $\lambda'^*_{21i}\lambda'_{31i}$ | $2.1 \cdot 10^{-4}$ | $m_{\tilde{d}_i}$ | $\lambda_V$ |
| $\lambda'^*_{2i1}\lambda'_{3i1}$ | $2.1 \cdot 10^{-4}$ | $m_{\tilde{u}_i}$ | $\lambda_V$ |
| $\lambda_{3i2}\lambda'^*_{i11},\ \lambda_{2i3}\lambda'^*_{i11}$ | $1.3 \cdot 10^{-4}$ | $m_{\tilde{\nu}_i}$ | $\lambda^n_S$ |
| $\lambda_{3i2}\lambda'^*_{i22},\ \lambda_{2i3}\lambda'^*_{i22}$ | $1.5 \cdot 10^{-4}$ | $m_{\tilde{\nu}_i}$ | $\lambda^s_S$ |

- Previously $\lambda'_{21i}\lambda'_{31i} < 7.2 \cdot 10^{-3}\left(\frac{m_{\text{susy}}}{100\,\text{GeV}}\right)^2$

- The rigorous treatment of hadronic part ⇨ bound improved by a factor of *30* !

# 3.6 Constraint on NP from $\tau \to \eta\pi\nu_\tau$

*Descotes-Genon, Kou, Moussallam'Tau12*

- $\tau \to \eta\pi\nu_\tau$ decays
  - suppressed in the SM $\propto (m_d - m_u)$ ⇨ sensitive to *NP*
  - $f_0^{\eta\pi}$ probes the matrix element of scalar operator $\langle 0 | \bar{u}d | \eta\pi \rangle$
    ⇨ access to the coupling

- Decay rate :

$$\frac{d\Gamma}{ds} = \frac{G_F^2 V_{ud}^2 S_{EW}\, m_\tau^3}{384\,\pi^3} \frac{\sqrt{\lambda_{\eta\pi}(s)}}{s^3} \left(1 - \frac{s}{m_\tau^2}\right)^2 \times \left\{ |f_+^{\eta\pi}(s)|^2 \lambda_{\eta\pi}(s) \left(1 + \frac{2s}{m_\tau^2}\right) + 3 |f_0^{\eta\pi}(s)|^2 \Delta_{\eta\pi}^2 \right\}$$

- Dispersive approach for the ηπ form factors as for the Kπ form factors + constraints from ChPT and $\eta \to 3\pi$ decays

$$f_+^{\eta\pi}(s) = f_+^{\eta\pi}(0) + \dot{f}_+^{\eta\pi}(0)s + \frac{s^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\operatorname{Im} f_+^{\eta\pi}(s')}{(s')^2 (s' - s)} ds'$$

# 3.6 Constraint on NP from $\tau \to \eta\pi\nu_\tau$

*Descotes-Genon, Kou, Moussallam'Tau12*

- $\tau \to \eta\pi\nu_\tau$ decays
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- Dispersive approach for the ηπ form factors as for the Kπ form factors + constraints from ChPT and $\eta \to 3\pi$ decays

$$\boxed{f_0^{\eta\pi}(s) = f_0^{\eta\pi}(0) \left( \frac{f_0^{\eta\pi}(\Delta_{\eta\pi})}{f_0^{\eta\pi}(0)} \right)^{\frac{s}{\Delta_{\eta\pi}}} \times \exp\left( \frac{s(s-\Delta_{\eta\pi})}{\pi} \int_{(m_\eta+m_\pi)^2}^{\infty} ds' \frac{\phi^{\eta\pi}(s')}{s'(s'-\Delta_{\eta\pi})(s'-s)} \right)}$$

# 3.6 Constraint on NP from $\tau \to \eta\pi\nu_\tau$

*Descotes-Genon, Kou, Moussallam'Tau12*

- Results : Predicted $\tau \to \eta\pi\nu_\tau$ spectrum

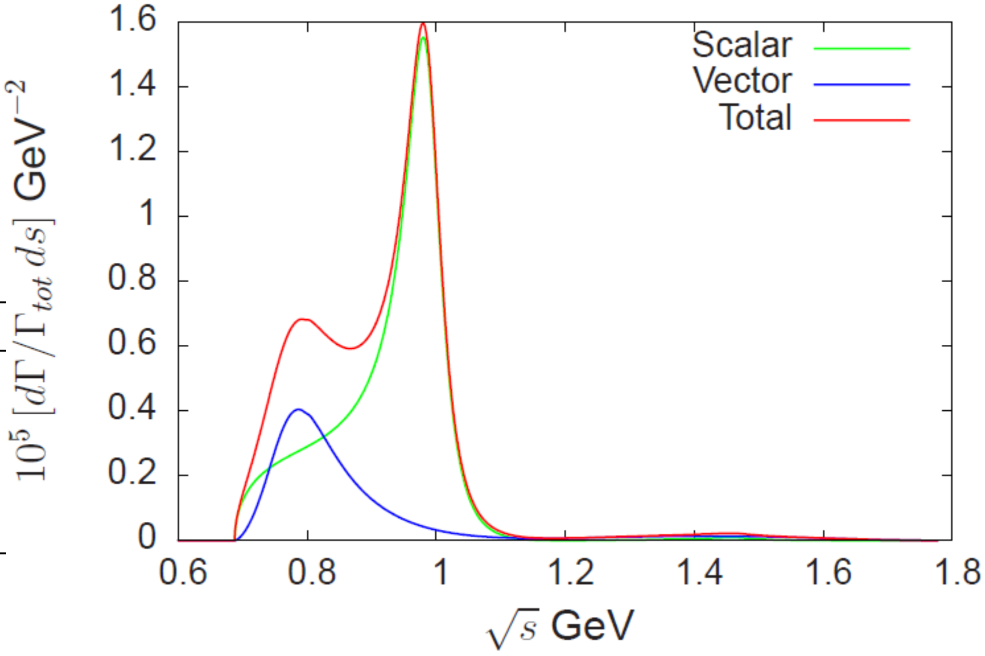
*Preliminary*

$$BF_{vect} \simeq 0.11 \times 10^{-5}$$

$$BF_{scal} \simeq 0.37^{+0.30}_{-0.20} \times 10^{-5}$$

is on the low side of previous ones : ($\times 10^5$)

| V | S | total | ref. |
|---|---|---|---|
| 0.25 | 1.60 | 1.85 | Tisserant, Truong (1982) |
| 0.12 | 1.38 | 1.50 | Pich (1987) |
| 0.15 | 1.06 | 1.21 | Neufeld, Rupertsberger(1995) |
| 0.36 | 1.00 | 1.36 | Nussinov, Soffer (2008) |
| [0.2-0.6] | [0.2-2.3] | [0.4-2.9] | Paver, Riazuddin (2010) |



Exp.: $BF \leq 9.9 \times 10^{-5}$

- It would be very interesting to have the experimental measurement of $\tau \to \eta\pi\nu_\tau$ spectrum !

# 4. Conclusion and outlook

# 4.1 Conclusion and Outlook

- Hadronic $\tau$-decays very interesting to study
  - Very precise determination of $\alpha_S$
    But error assignment and treatment of the NP part and new data needed

- Test of electroweak couplings very promising
  - New physics in $R_\tau$ : analyses in progress but it would be nice to have more data and a precise separation between V and A.
    Hadronic uncertainties have to be under control

  - Extraction of $V_{us}$ : the $\tau$ could give a very precise determination of $V_{us}$ but difference between inclusive/exclusive modes:
    Data normalization, unmeasured modes? New Physics?

  - CP violating asymmetry: very interesting measurements to constrain new physics:
    Experimentally: BaBar & Belle agreement?
    Theoretically: Hadronic form factors precisely described
    ⇨ measurement of $A_{FB}$ would help!
    Model of new physics to investigate

# 4.2 Conclusion and Outlook

- g-2 : improvement in the estimate of the hadronic part needed
  ⇨ the different experimental analyses don't agree

- Interesting LFV tests with $\tau \to \mu\pi\pi$ decays

- And many more very interesting tests allowed with hadronic $\tau$ decays : second class current in $\tau \to \eta\pi\nu_\tau$ , etc

- High precision era in $\tau$:
  - more precise data with LHC-B, Belle II, Tau-Charm
  - theoretically: ffs parametrizations, EM, IB corrections

# 5. Back-up

# Kπ form factors from $\tau \to K\pi\nu_\tau$ and $K_{l3}$ decays

- Several applications :
  - Callan-Treiman (CT) theorem :

*Bernard, Oertel, E.P., Stern'06*

$$C = \bar{f}_0(\Delta_{K\pi}) = \frac{F_K}{F_\pi f_+(0)} + \Delta_{CT} = \underbrace{\frac{F_K |V^{us}|}{F_\pi |V^{ud}|} \, \frac{1}{f_+(0)|V^{us}|} \, |V^{ud}|}_{} \, r + \Delta_{CT}$$

$$m_K^2 - m_\pi^2$$

Very precisely known
from $Br(K_{l2}/\pi_{l2})$, $\Gamma(K_{e3})$ and $|V_{ud}|$

– In the Standard Model : $\boxed{r = 1}$ $\left(\ln C_{SM} = 0.2141(73)\right)$

– In presence of new physics, new couplings : $\boxed{r \neq 1}$

⇨ Fit : $\boxed{\ln C = 0.2035(88)}$ in agreement with the SM

– Alternatively test of the lattice calculations of $F_K/F_\pi$ and $f_+(0)$

# $K\pi$ form factors from $\tau \to K\pi\nu_\tau$ and $K_{l3}$ decays

- $V_{us}$ from $\tau \to K\pi\nu_\tau$

$$\boxed{\Gamma_{\tau\to K\pi\nu_\tau} = N\left|f_+(0)V_{us}\right|^2 I_K^\tau}$$ with $$\boxed{I_K^\tau = \int ds\, F\left(s, \bar{f}_+(s), \bar{f}_0(s)\right)}$$

$$\boxed{\left|f_+(0)V_{us}\right| = 0.2110 \pm 0.0037} \Rightarrow \boxed{\left|V_{us}\right| = 0.2201 \pm 0.0040}$$

Not competitive yet!

- Precise extraction of $K\pi$ scattering phase and good determination of K*

$$\boxed{m_{K*} = 892.02 \pm 0.21 \text{ MeV}}$$ and $$\boxed{\Gamma_{K*} = 46.300 \pm 0.426 \text{ MeV}}$$

PDG : $m_{K*} = 891.66 \pm 0.26$ MeV and $\Gamma_{K*} = 50.8 \pm 0.9$ MeV

⇒ Tau-Charm: $\boxed{m_{K*} = 892.02 \pm 0.02 \text{ MeV}}$ and $\boxed{\Gamma_{K*} = 46.300 \pm 0.044 \text{ MeV}}$

- Prediction of the strange Brs and $V_{us}$

- Use of the form factors for CPV tests, etc.

# 3.3 Application : τ → Kπν$_\tau$ CP violating asymmetry

- Analysis of the angular CP violating asymmetry

$$\frac{d\Gamma\left(\tau^{-}\to K\pi^{-}\nu_{\tau}\right)}{dq^{2}d\cos\theta\; d\cos\beta} = \Big[A(q^{2}) - B(q^{2})\,\left(3\cos^{2}\psi - 1\right)\left(3\cos^{2}\beta - 1\right)\Big]\left|f_{+}(s)\right|^{2} + m_{\tau}^{2}\left|\tilde{f}_{0}(s)\right|^{2} - C(q^{2})\cos\psi\cos\beta\,\mathrm{Re}\left(f_{+}(s)\,\tilde{f}_{0}^{*}(s)\right)$$

  - $A(Q^2)$, $B(Q^2)$, $C(Q^2)$ kinematic functions

CP violating term
S-P interference

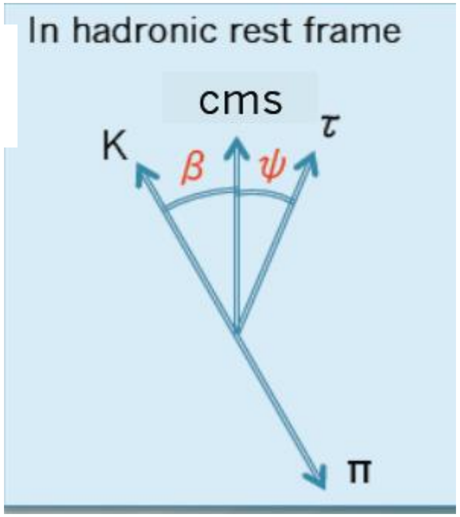
  - With a charged Higgs

$$\boxed{\tilde{f}_{0}(s) = f_{0}(s) + \frac{\eta^{2}}{m_{\tau}^{2}}\, f_{H}(s)}$$

with

$$f_{H}(s) = \frac{s}{m_{u} - m_{s}}\, f_{0}(s)$$

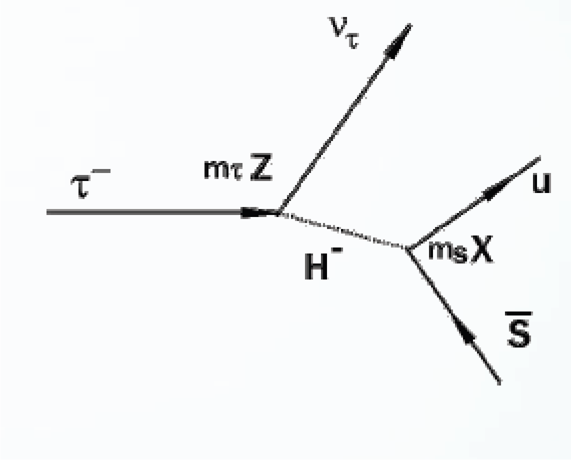
*Khün & Mirkes'05*



  - Measurement of CP violating parameter

$$\Delta \equiv \frac{d\Gamma\left(\tau^{+}\to K_{S}^{0}\pi^{+}\nu_{\tau}\right)}{ds\; d\cos\theta\; d\cos\beta} - \frac{d\Gamma\left(\tau^{-}\to K_{S}^{0}\pi^{-}\nu_{\tau}\right)}{ds\; d\cos\theta\; d\cos\beta} = C^{'}(s)\,\mathrm{Im}(\eta_{S})\,\frac{\mathrm{Im}\left(f_{+}(s)\,f_{H}^{*}(s)\right)}{m_{\tau}}\cos\beta\cos\Psi$$

# 3.3 Application : $\tau \to K\pi\nu_\tau$ CP violating asymmetry



- In NP scenarios with charged Higgs

$$\boxed{\Delta = C^{'}(s)\,\mathrm{Im}(\eta_S)\,\frac{\mathrm{Im}\left(f_+(s)\,f_H^*(s)\right)}{m_\tau}\cos\beta\cos\Psi}$$

- Measurement: $\boxed{\left|\mathrm{Im}\,\eta_S\right| < 0.19}$ $\Rightarrow$ $\boxed{\left|\mathrm{Im}\,\eta_S\right| < 0.026}$

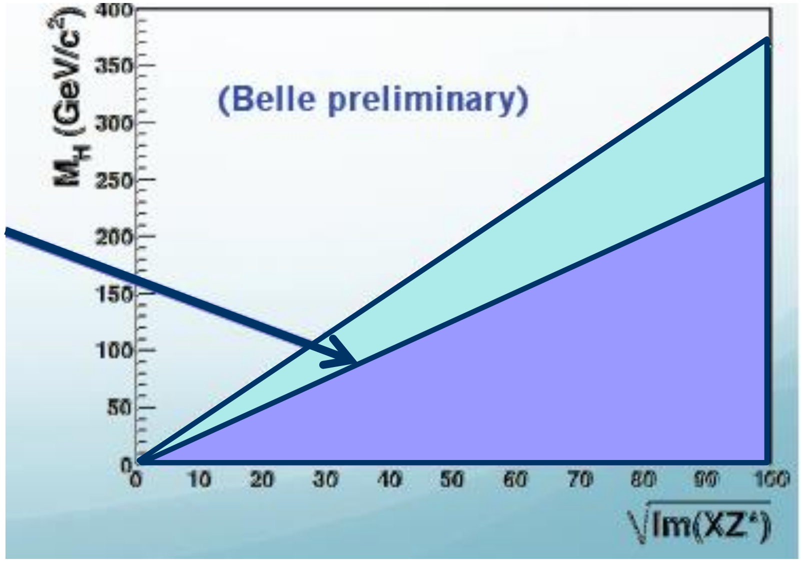
*CLEO'02* *Belle'11*

- Constraints on the couplings and $M_H$

$$\eta_S \simeq \frac{m_\tau m_s}{M_{H\pm}^2} X^* Z$$

$$\boxed{\left|\mathrm{Im}\,\eta_S\right| < 0.026} \Rightarrow |\Im(XZ^*)| < 0.15\frac{M_{H\pm}^2}{1\,\mathrm{GeV}^2/c^4}$$

*Bishchofberger Tau2010*

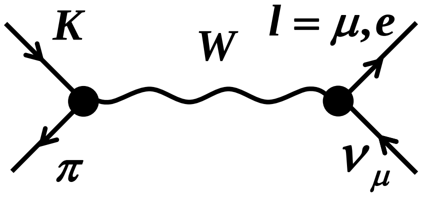


# Determination of the Kπ form factors

- Parametrization to analyse both $K_{l3}$ and $\tau \to K\pi\nu_\tau$ decays

- Indeed $K_{l3}$ $\left(K \to \pi l \nu_l\right)$ crossed channel,

$$\underset{\uparrow}{} (l = e, \mu)$$



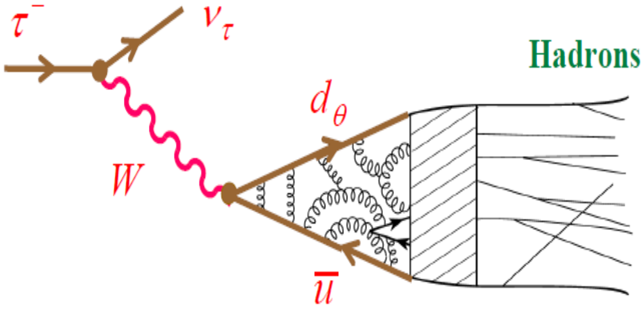
⇒ same form factors

$$\boxed{\langle \pi(p_\pi)|\, \bar{s}\gamma_\mu u \,|K(p_K)\rangle = \left[(p_K + p_\pi)_\mu - \frac{\Delta_{K\pi}}{t}(p_K - p_\pi)_\mu\right] \underset{\substack{\uparrow \\ \text{vector}}}{f_+(t)} + \frac{\Delta_{K\pi}}{t}(p_K - p_\pi)_\mu \underset{\substack{\uparrow \\ \text{scalar}}}{f_0(t)}}$$

$$t = q^2 = (p_\mu + p_{\nu_\mu})^2 = (p_K - p_\pi)^2$$

- Use a *dispersive parametrization* to combine experimental information on $K_{l3}$ and $\tau \to K\pi\nu_\tau$

# 1.3 Experimental situation



- A lot of effort for precise measurements :
  LEP (ALEPH, OPAL, L3), CLEO, BaBar, Belle, etc

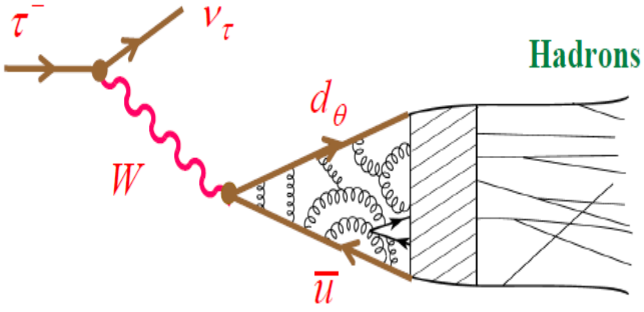
*S. Banerjee'Tau10*

| Experiment | Number of $\tau$ pairs |
|---|---|
| LEP | $\sim 3x10^5$ |
| CLEO | $\sim 1x10^7$ |
| BaBar | $\sim 5x10^8$ |
| Belle | $\sim 9x10^8$ |

- New experiments :
  LHCB, Belle II, *Tau-Charm*, etc

⇨ *HFAG* provides average of all these measurements!

# 1.4 Theory



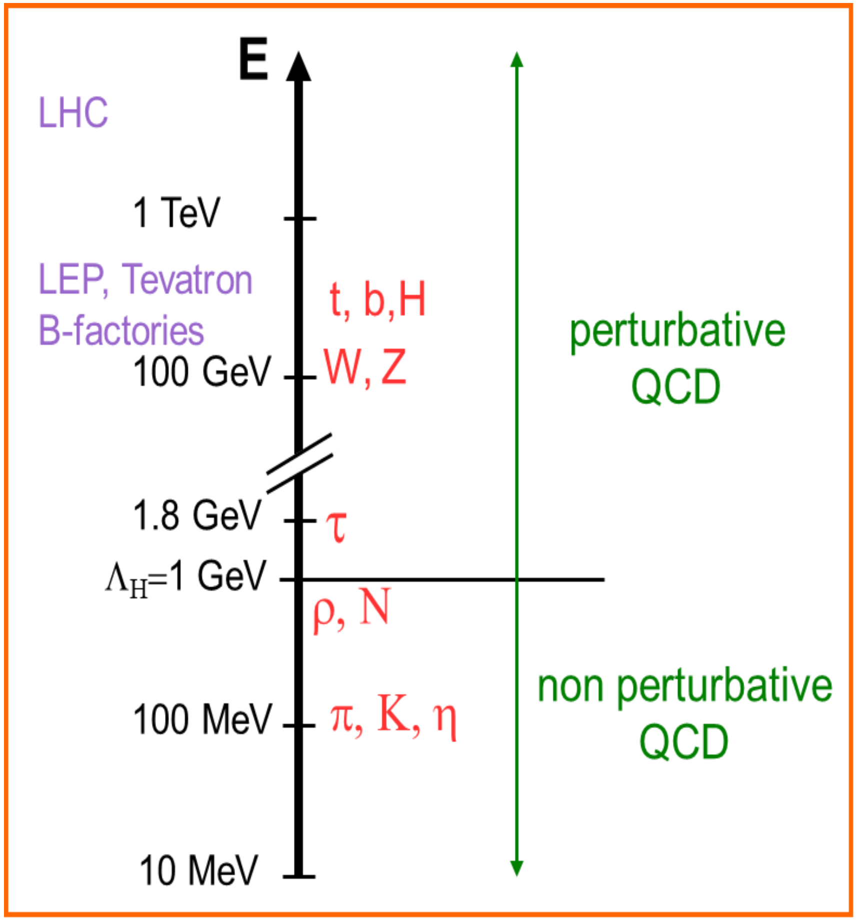
τ decays just at the border $\boxed{m_\tau \sim 1.77 \text{ GeV} > \Lambda(1 \text{ GeV})}$

- If E (~$m_\tau$) > Λ : High energies, short distance, *$\alpha_S$ small*

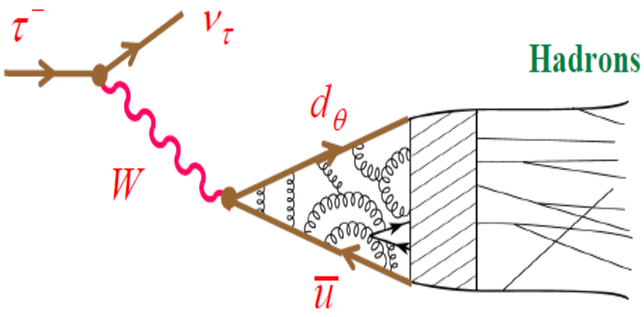
  ⇨ *Perturbative QCD*

  Order-by-order expansion in $\frac{\alpha_S(\mu)}{\pi}$

$$\sigma = \sigma_0 + \underbrace{\frac{\alpha_S}{\pi}\sigma_1}_{\text{small}} + \underbrace{\left(\frac{\alpha_S}{\pi}\right)^2\sigma_2}_{\text{smaller}} + \underbrace{\left(\frac{\alpha_S}{\pi}\right)^3\sigma_3}_{\text{negligible ?}} + \ldots$$



# 1.4 Theory



$\tau$ decays just at the border $\boxed{m_\tau \sim 1.77 \text{ GeV} > \Lambda(1 \text{ GeV})}$

- If E < Λ : Low energies, long distance, *$\alpha_S$ large !* ⇒ *Non-perturbative QCD*
  - Lattice QCD
  - Effective field theory: ChPT

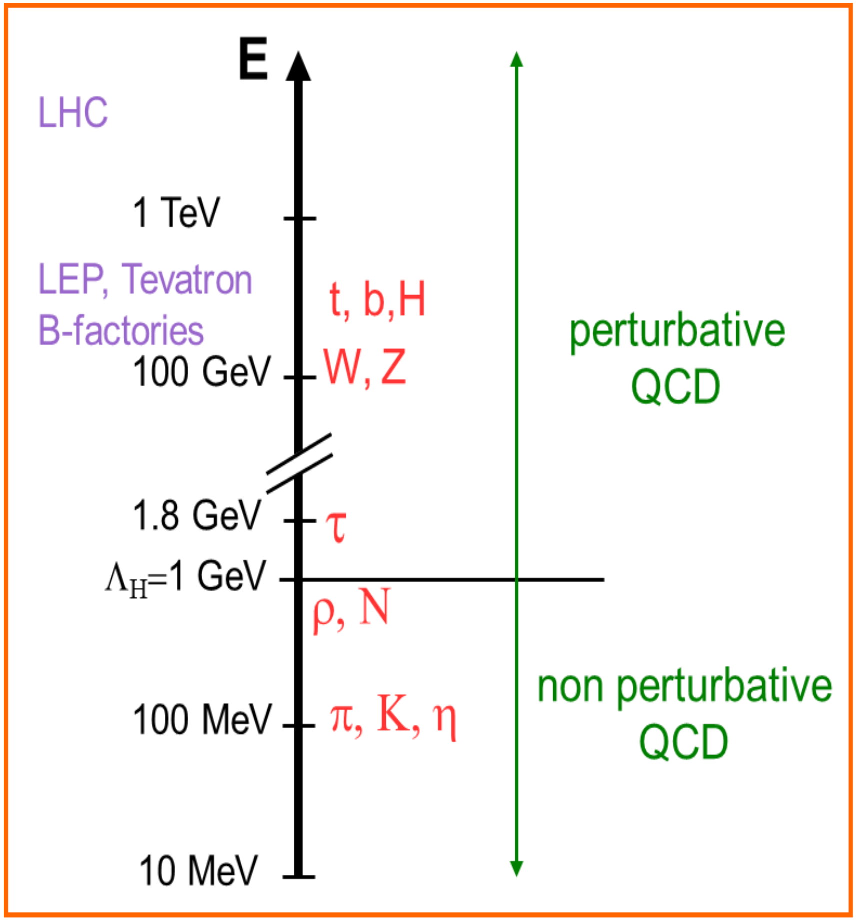
  Order-by-order expansion in $\frac{p}{\Lambda_H}$

  $\boxed{p \ll \Lambda \sim 1 \text{ GeV}}$

$$\sigma = \sigma_0 + \left(\frac{p}{\Lambda_H}\right)^2 \sigma_2 + \left(\frac{p}{\Lambda_H}\right)^4 \sigma_4 + \dots$$

At this energy, ChPT + resonances

⇒ RChPT



E

LHC

1 TeV

LEP, Tevatron

B-factories

t, b,H

100 GeV

W, Z

perturbative QCD

1.8 GeV

$\tau$

$\Lambda_H$=1 GeV

ρ, N

non perturbative QCD

100 MeV

π, K, η

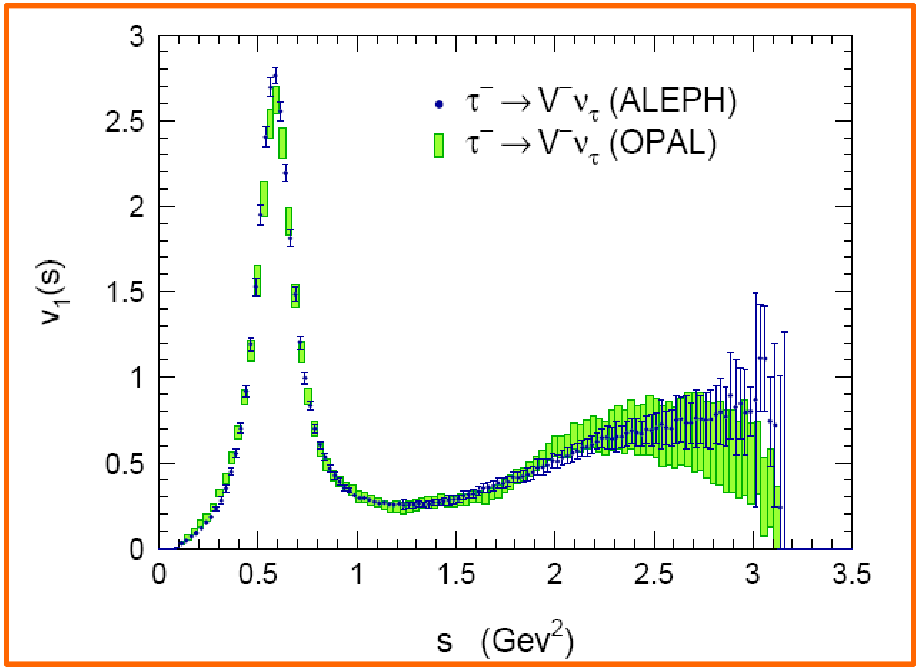
10 MeV

# 2.1 Introduction

- naïve QCD prediction

⇨ Experimentally $R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086$

- In tau decays mixing between
  - Perturbative QCD
  - Non-perturbative QCD: resonance structure



- Decomposition as a function of observed and separated final states

$$R_\tau = R_{\tau,V}^{NS} + R_{\tau,A}^{NS} + R_\tau^{S}$$

- Spectral functions only extracted from LEP data
- A lot of experimental and theoretical activities ⇨ $\alpha_S(m_\tau)$, $|V_{us}|$, $m_s$

# 2.1 Introduction

- Partonic QCD prediction :

Difficulty ⇒ QCD corrections : $R_\tau = |V_{ud}|^2 N_C + |V_{us}|^2 N_C + O(\alpha_s)$

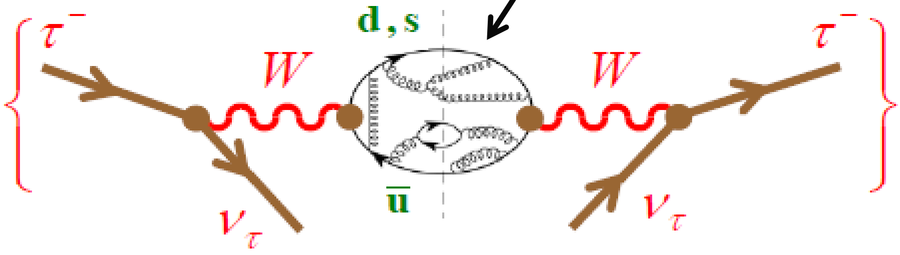
- Extraction of the strong coupling constant : ⇒ $\alpha_s$

measured calculated

- Determination of V$_{us}$ : $$|V_{us}|^2 = \frac{R_\tau^S}{\frac{R_\tau^{NS}}{|V_{ud}|^2} - \delta R_{\tau,th}}$$

- Calculation of $R_\tau$ :

$\Gamma_{\tau\to\nu_\tau+\text{had}} \sim \text{Im}\{\ \}$



⇒ $$R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\text{Im}\Pi^{(1)}(s+i\varepsilon) + \text{Im}\Pi^{(0)}(s+i\varepsilon)\right]$$

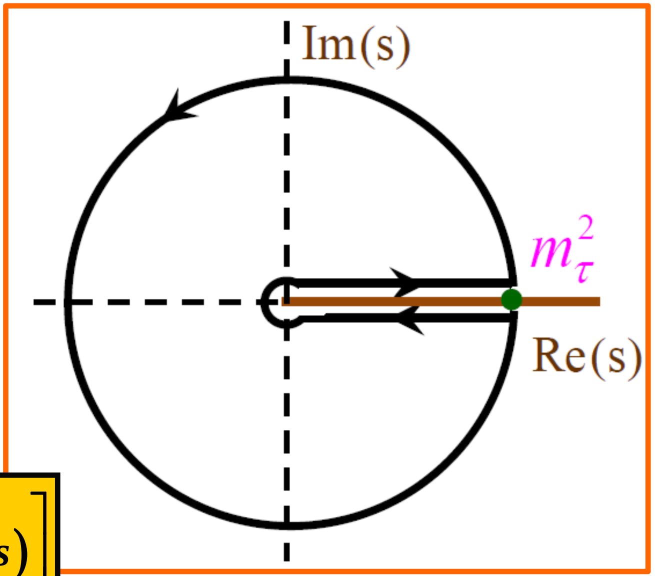
# 2.2 Theoretical Method

*Braaten, Narison, Pich'92*

- Analyticity: Π analytic in the entire complex plane except for s real positive

⇒ Cauchy theorem:

$$\frac{1}{\pi}\int_0^{s_0} ds\, g(s)\ \mathrm{Im}\,\Pi(s) = -\frac{1}{2\pi i}\oint_{|s|=s_0} ds\, g(s)\,\Pi(s)$$



⇒

$$R_\tau(m_\tau^2) = 6i\pi S_{EW}\oint_{|s|=m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\Pi^{(1)}(s)+\Pi^{(0)}(s)\right]$$

- Sufficient high energy for *Operator Product Expansion*
  Kinematic factor ⇒ decreases weight close to the real axis where Π has a cut

$$\Pi^{(J)}(s) = \sum_{D=0,2,4\ldots}\frac{1}{(-s)^{D/2}}\sum_{\dim O=D}\mathcal{C}^{(J)}(s,\mu)\,\langle O_D(\mu)\rangle$$

Wilson coefficients

Operators

$\mu$ separation scale between short and long distances

# 2.2 Theoretical Method

Braaten, Narison, Pich'92

- $$\boxed{R_\tau\left(m_\tau^2\right) = N_C\, S_{EW}\left(1+\delta_P+\delta_{NP}\right)}$$

  $S_{EW} = 1.0201(3)$ Marciano &Sirlin'88, Braaten & Li'90, Erler'04

- Perturbative part : (D=0) $$\boxed{\delta_P = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + 127\, a_\tau^4 + \ldots \approx 20\%}$$ $$a_\tau = \frac{\alpha_s(m_\tau)}{\pi}$$

  Baikov, Chetyrkin, Kühn'08

- D=2 : quark mass corrections

  ⇨ neglected for $R_\tau^{NS}$ $\left(\propto m_u, m_d\right)$ but not for $R_\tau^S$ $\left(\propto m_s\right)$

- Non perturbative part :
  - D=4: Non perturbative physics operators, $\left\langle \frac{\alpha_s}{\pi} GG \right\rangle$, $\left\langle m_j \bar{q}_i q_i \right\rangle$
  - D=6: 4 quarks operators, $\left\langle \bar{q_i}\Gamma_1 q_j \bar{q_j}\Gamma_2 q_i \right\rangle$
  - D≥8: Neglected terms, supposed to be small…

  ⇨ Not known, fitted from the data $\boxed{\delta_{NP} = -0.0059 \pm 0.0014}$
  Use of weighted distributions

  Davier et al'08

- Small unknown NP part $\left(\delta_{NP} \sim 3\%\ \delta_P\right)$ ⇨ very precise extraction of $\alpha_S$

# 2.7 New Physics in $R_\tau$

- Models with modifications of the couplings:
  - Tensor & scalar interactions ex: leptoquarks

*Cirigliano, Filipuzzi, Gonzalez-Alonso, E.P. in progress*

$$\boxed{\begin{aligned}R_\tau^{NS}(s_0) = {}& 6\pi i\left|V^{ud}\right|^2 \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left\{\left|\kappa_V\right|^2\left[\left(1+\frac{2s}{m_\tau^2}\right)\Pi^{(1)}_{ud,VV}(s)+\Pi^{(0)}_{ud,VV}(s)\right]\right.\\ &+\left|\kappa_A\right|^2\left[\left(1+\frac{2s}{m_\tau^2}\right)\Pi^{(1)}_{ud,AA}(s)+\Pi^{(0)}_{ud,VV}(s)\right]+2\,\mathrm{Re}\left(\kappa_V\kappa_S^*\right)\frac{\Pi_{ud,VS}(s)}{m_\tau}+2\,\mathrm{Re}\left(\kappa_A\kappa_P^*\right)\frac{\Pi_{ud,AP}(s)}{m_\tau}\\ &\left.+12\,\mathrm{Re}\left(\kappa_V\kappa_T^*\right)\frac{\Pi_{ud,VT}(s)}{m_\tau}\right\}\left[1-2\tilde{v}_L\right]\end{aligned}}$$

- But also charged Higgs, little Higgs, SUSY…

# 2.7 New Physics in $R_\tau$

- Disentangle New Physics from QCD effects:
  - Take QCD observables from other sources or more data :

    Inputs for $\alpha_s(m_\tau)$, $\left\langle \frac{\alpha_s}{\pi} GG \right\rangle$, $m_{u,d,s}$, $\langle \bar{q}_i q_i \rangle$, $\langle \bar{q}_i \Gamma_1 q_j \bar{q}_j \Gamma_2 q_i \rangle$

    Lattice QCD, SCET, moments…

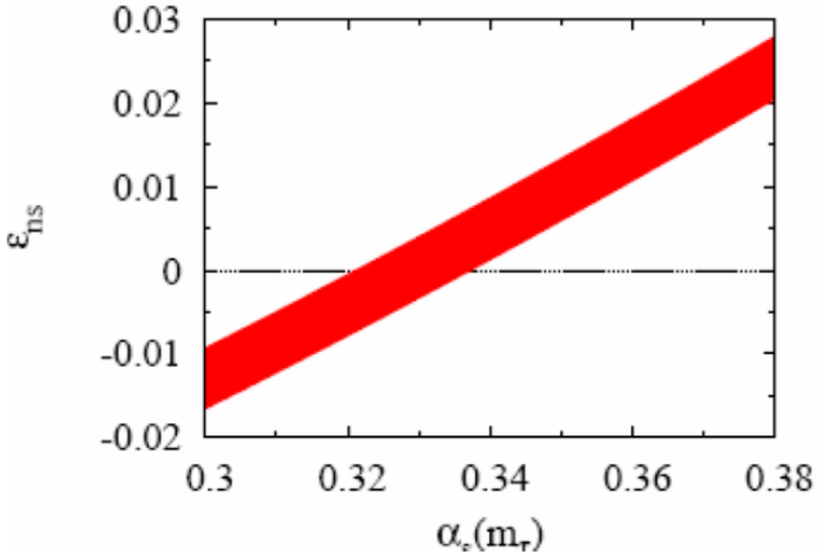
  - Experimental separation V/A very important

    ⇨ only data from OPAL, need more data

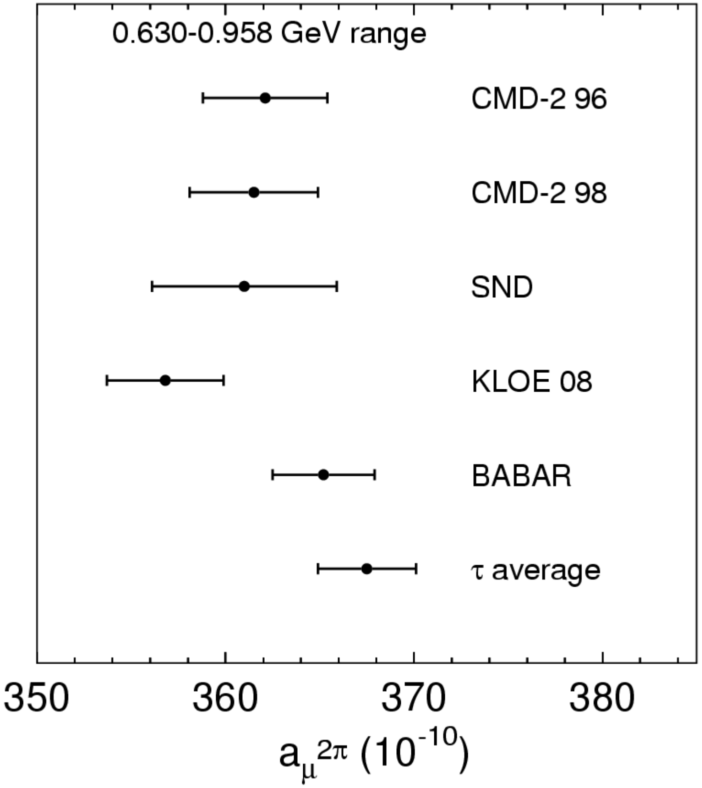
- Possible constraint on NP parameters
  Ex: RHCs

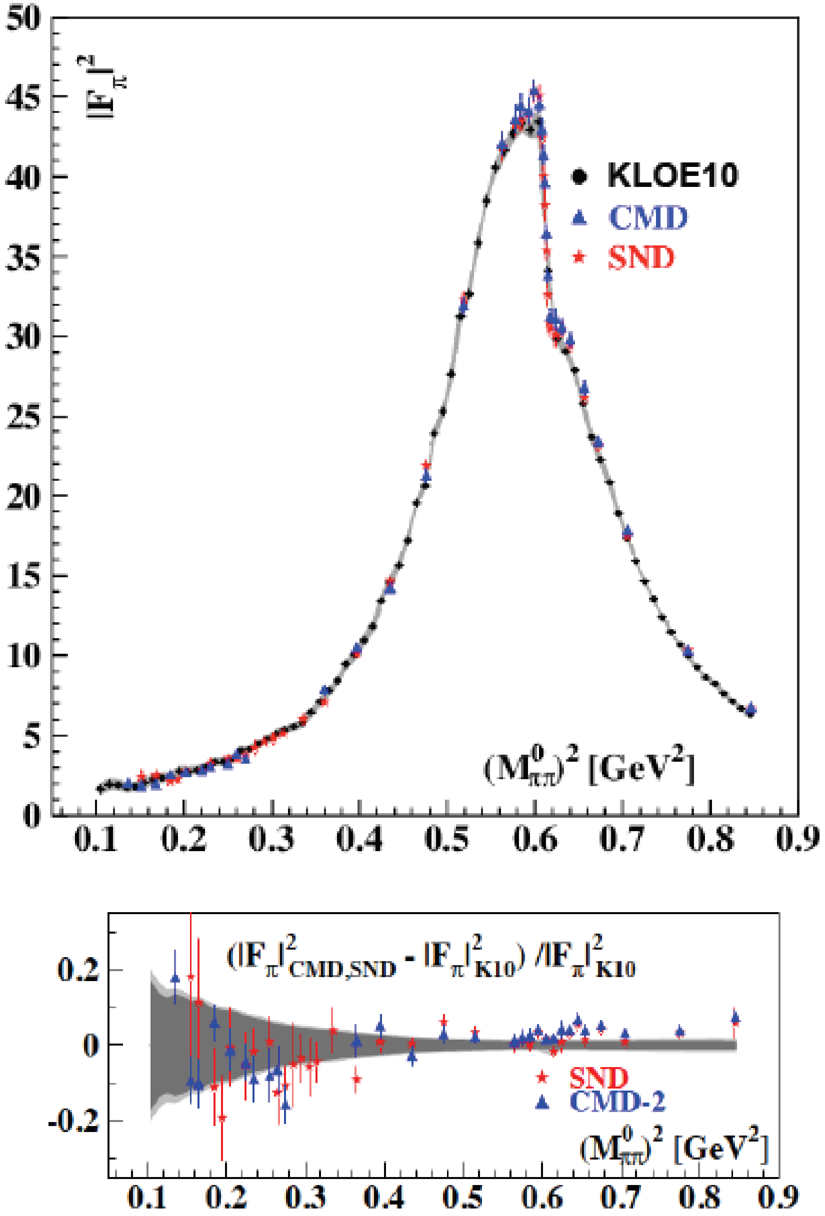
  *Bernard, Oertel, E.P., Stern'07*

⇨ Could explain the difference in the values for $V_{us}$



# 3.4 Anomalous magnetic moment of the muon





# 3.4 Anomalous magnetic moment of the muon