The strong coupling from hadronic tau decays

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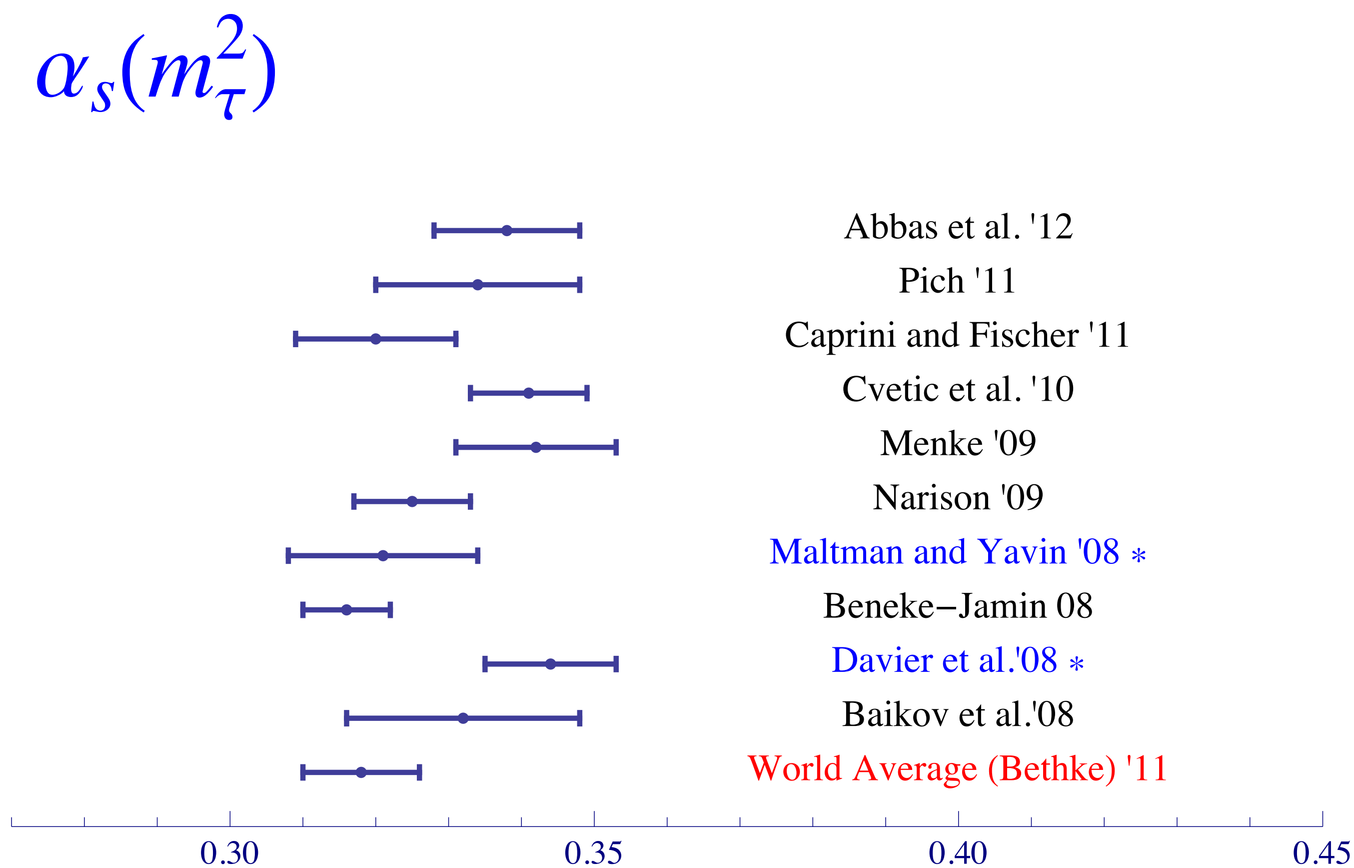
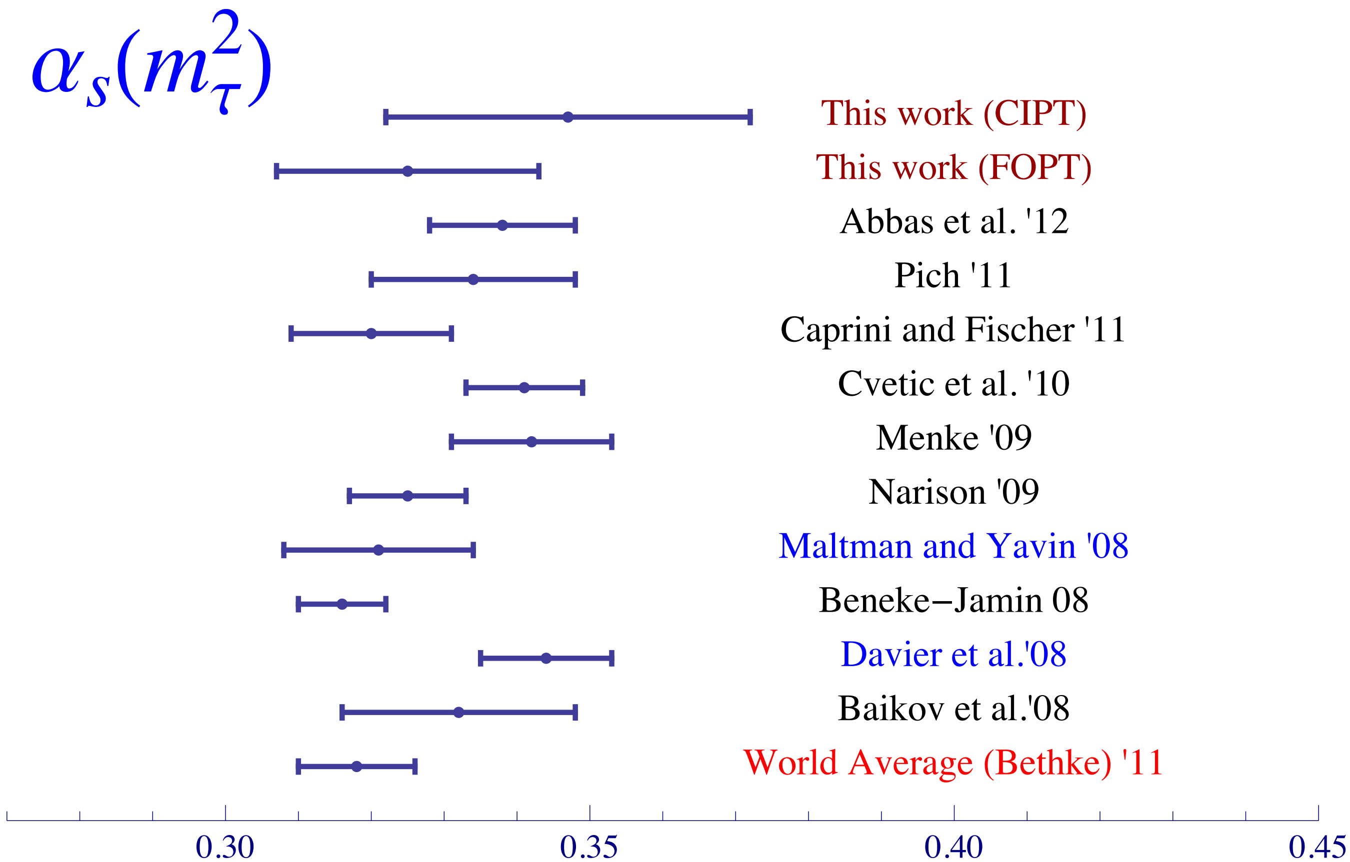
Status of $\alpha_s(m_\tau^2)$ determinations



* Complete analyses from (mostly) ALEPH data



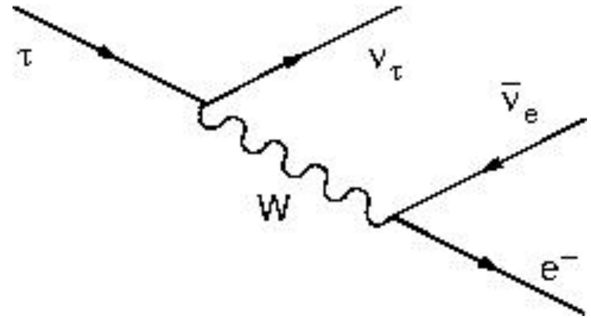
- Errors in non-perturbative part systematically underestimated
- Our analysis uses OPAL data (error in ALEPH correlations (Boito et al. '10))

Overview:

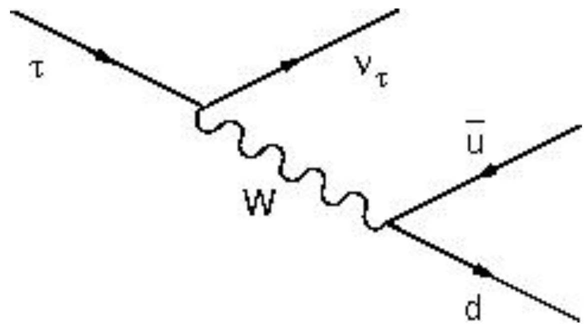
- Theory of tau decays
- Comparison of “standard” analysis with our analysis
- ALEPH vs. OPAL data
- Results of our analysis
- Conclusions

τ decays

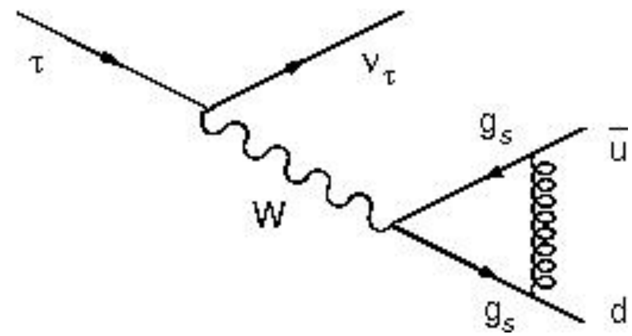
leptonic:



hadronic:



+



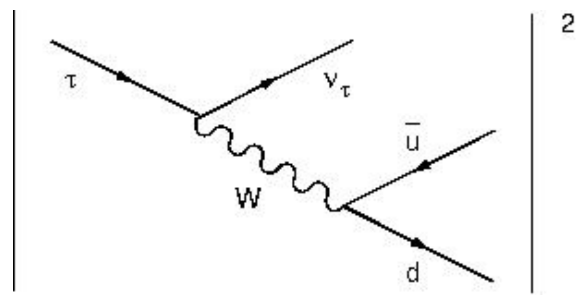
$$R_\tau = \frac{\Gamma(\tau \to \nu_\tau \, \mathrm{hadrons})}{\Gamma(\tau \to \nu_\tau e \overline{\nu}_e)} = 3 S_{EW} |V_{ud}|^2 \left[1 + \frac{\alpha_s}{\pi} + 5.2 \left(\frac{\alpha_s}{\pi} \right)^2 + \ldots \right]$$

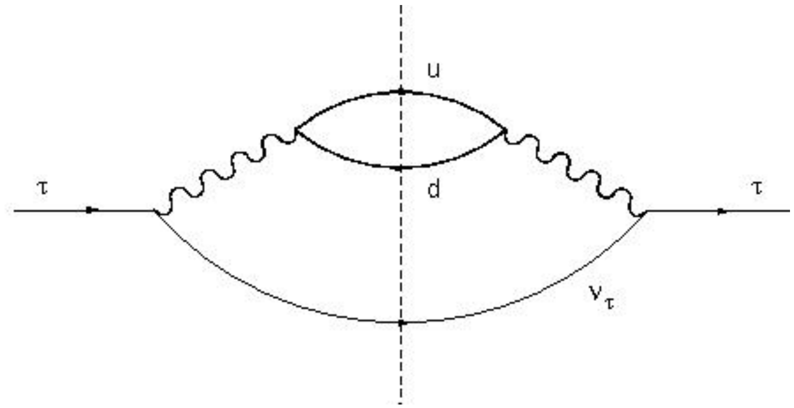
- use this to determine $\alpha_s(m_\tau)$ ($q_W \sim m_\tau$) from non-strange decays
- see $\tau \to \nu_\tau \rho^* \to \nu_\tau \, \mathrm{pions}$, not $\tau \to \nu_\tau \, \mathrm{jets}$

  $\rho^* = \rho(770), \; \rho(1450), \; \rho(1700)$ (and others: incl. axial, kaons, …)

$\longrightarrow$ relation with perturbative regime?

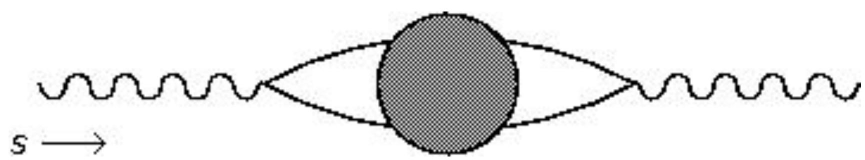
$\Gamma(\tau \to \nu_\tau \text{ hadrons})$

$\propto$ 

$\propto$ Im 

(optical theorem)

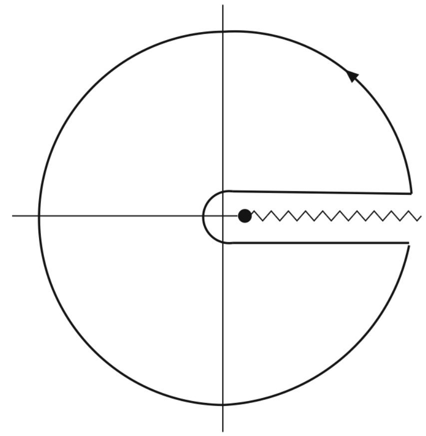
$$R_\tau(s_0) = 12\pi S_{EW} |V_{ud}|^2 \int_0^{s_0} \frac{ds}{s_0} \left(1 - \frac{s}{s_0}\right)^2 \left(1 + 2\frac{s}{s_0}\right) \text{Im } \Pi(s)$$

$\Pi(s) =$ 

$s = (q_W)^2$

$0 \leq s \leq s_0 = m_\tau^2$ depending on how much momentum ν_τ carries away

Relating α_s to τ decay data:



complex s plane
circle has radius s_0

- positive real axis: spectral data ($\mathrm{Im}\,\Pi(s)$)
- $\Pi(s)$ analytic everywhere except for real $s > 0$

(Shankar '77, …., Braaten, Narison, Pich '92)

$\longrightarrow$ Cauchy, with a polynomial weight $w(s)$:

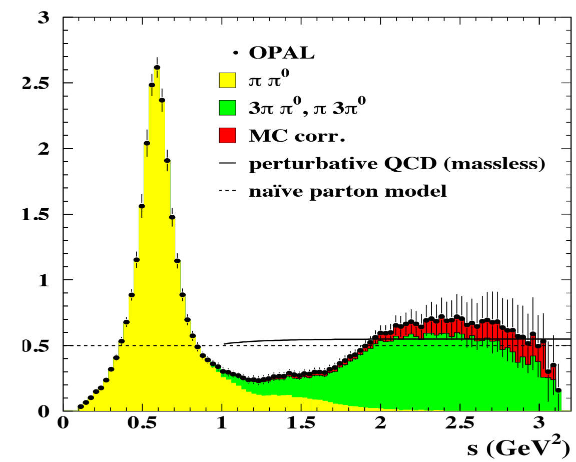
$$\int_0^{s_0} ds\ w(s)\ \frac{1}{\pi}\ \mathrm{Im}\ \Pi(s) = -\frac{1}{2\pi i}\oint_{|z|=s_0} dz\ w(z)\ \Pi(z)$$

“master” equation (FESR):

$$\int_0^{s_0} ds\, w(s)\, \rho_{\rm exp}(s) = -\frac{1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\, \Pi_{\rm OPE}(z) - \frac{1}{\pi}\int_{s_0}^{\infty} ds\, w(s)\, {\rm Im}\, \Pi_{\rm DV}(s)$$

$w(s)$ polynomial weight (which should we choose?)

$\rho_{\rm exp}(s)$ inclusive spectral function from experiment: (non-strange)

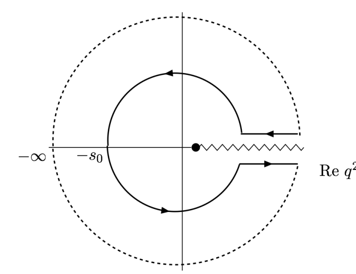


$\Pi_{\rm OPE}(z)$ perturbation theory ($\alpha_s(m_\tau^2)$: $\log z$)
plus OPE condensates ($1/z^k$)

$\Pi_{\rm DV}(z) = \Pi_{\rm QCD}(z) - \Pi_{\rm OPE}(z)$ Duality Violations not small near Minkowski axis! (OPE ≠ spectral function)

Assume $\Pi_{\rm DV}(z)$ decays exponentially for $|z| \to \infty$:

$$\frac{1}{2\pi i}\oint_{|z|=s_0} dz\ w(z)\ \Pi_{\rm DV}(z) = \frac{1}{\pi}\int_{s_0}^{\infty} ds\ w(s)\ {\rm Im}\ \Pi_{\rm DV}(s)$$



Ansatz: $\frac{1}{\pi}\,{\rm Im}\,\Pi_{\rm DV}(s) = e^{-\gamma s-\delta}\sin(\alpha+\beta s)$ for $s > s_{min}$

Compare "standard" vs. our analysis:

Standard: (OPAL '99, ALEPH '05, Davier et al. '08)

- Assume no duality violations, weights with pinching factor $(1-s/s_0)^n, \;\; n=2,3$
- 5 weights of degree 3 to 7 at $s_0 = m_\tau^2$ ⇒ assume dim. 10-16 condensates vanish
- Fit with 4 parameters ($\alpha_s(m_\tau^2)$, dim. 4,6,8 condensates) to 5 data points
- Inconsistent dependence on s_0 for other pinched weights (Maltman & Yavin '08)
- 4 out of 5 moments have bad behavior in pert. theory (Beneke, Boito, Jamin '12)

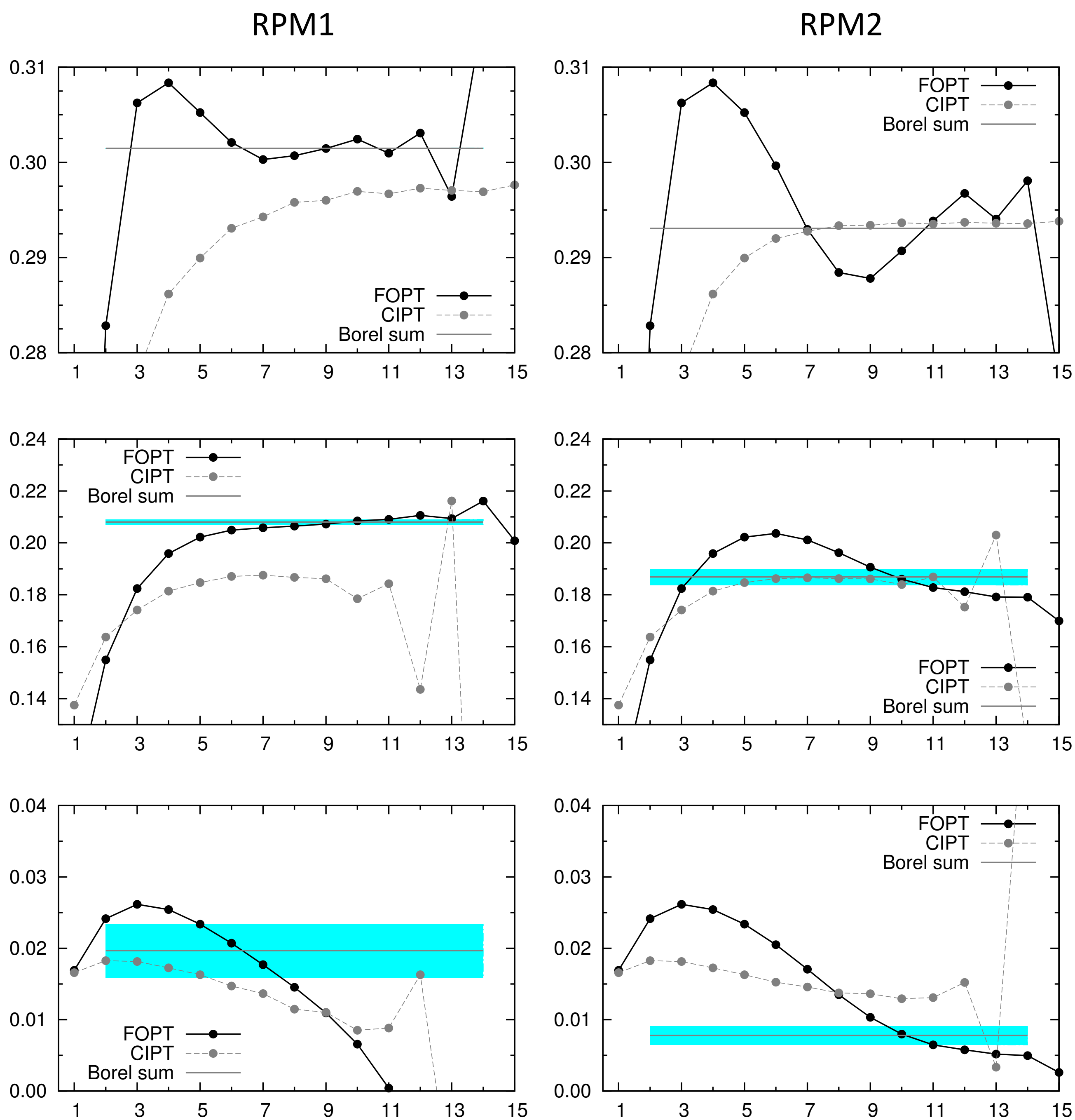
This work: ($\oint_{|z|=s_0} dz\, z^n \frac{1}{z^k} \propto \delta_{n,k-1}$)

- Main fit: $w=1$, hence no OPE condensates, but DV contribution instead
- Take $s_0 \in [s_{\rm min}, m_\tau^2]$, determine $s_{\rm min}$ from quality and stability of fit
- Fit with 5 parameters ($\alpha_s(m_\tau^2)$, DV parameters) to more (correlated) data
- Vector channel only in main fit
- Test with other weights (treat OPE consistently) and including axial channel

Both: perturbation theory to order α_s^4, CIPT and/or FOPT (Baikov, Chetyrkin, Kühn '08)

Moments in perturbation theory CIPT and FOPT

(Boito, Beneke, Jamin '12)

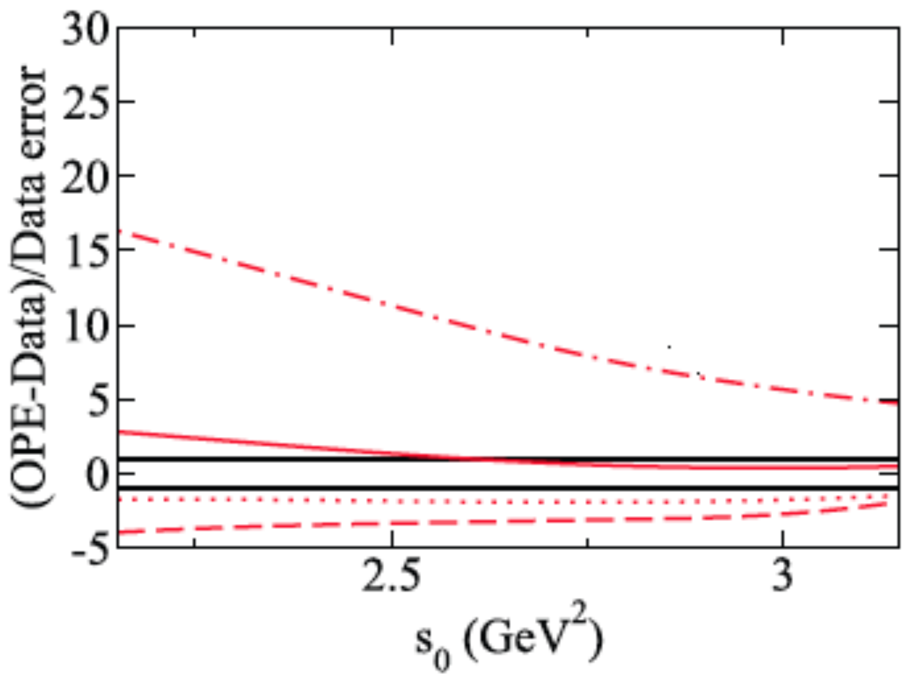


$w(z) = 1$

$(1-z)^2(1+2z)$

$(1-z)^3 z(1+2z)$

OPE: $\sum_{k=0}^{\infty} \frac{C_{2k}}{(-s)^k}$ (up to logarithms)



Non-strange vector channel
(Maltman & Yavin ‘08)

ALEPH ‘08 analysis
(Davier et al. ‘08)

- neglect duality violations
- choose weights with

- double zero at s_0
- use only data at $s_0 = m_\tau^2$
- OPE terms up to dim. 16
  but assume
  $C_{10} = C_{12} = C_{14} = C_{16} = 0$
- fit $\alpha_s,\ C_4,\ C_6,\ C_8$

Weights shown (top to bottom)

$$z(1-z)^2$$
$$(1-z)^2(1+2z)$$
$$(1-z)^2(1+z/2)$$
$$(1-z)^2$$

(all with degree ≤ 3)

Ansatz: $$\frac{1}{\pi}\,\mathrm{Im}\,\Pi_{\mathrm{DV}}(s) = e^{-\gamma s-\delta}\sin\left(\alpha+\beta s\right)$$

Oscillatory: duality violations due to resonances

Exponential decay: finite width $\Rightarrow$ $\gamma \propto 1/N_c$, small

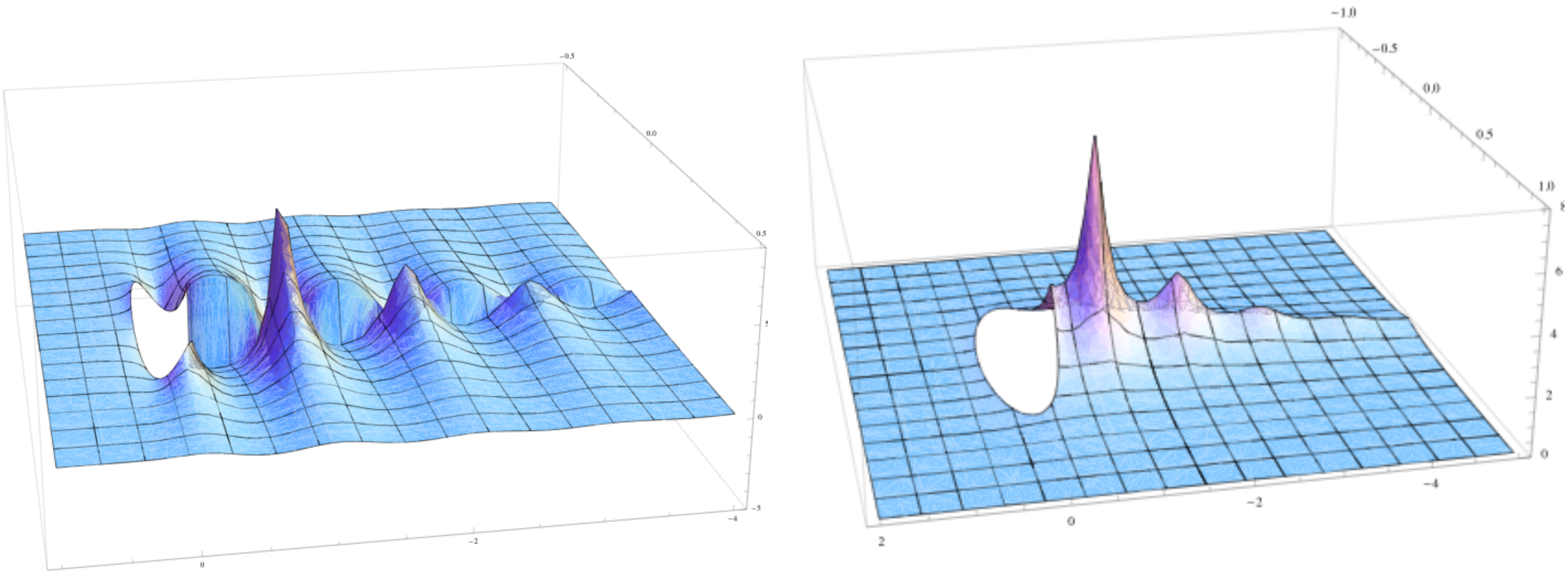
Argument of sine linear in s: Regge-like (daughter) trajectories

Model for $\Pi(s)$ with such behavior does exist! For example:

$$\Pi(z) \propto \psi(z) = \frac{d\log\Gamma(z)}{dz} = -\sum_{n=0}^{\infty}\frac{1}{z+n} + \text{constant}$$

$$z = s^{\zeta}$$

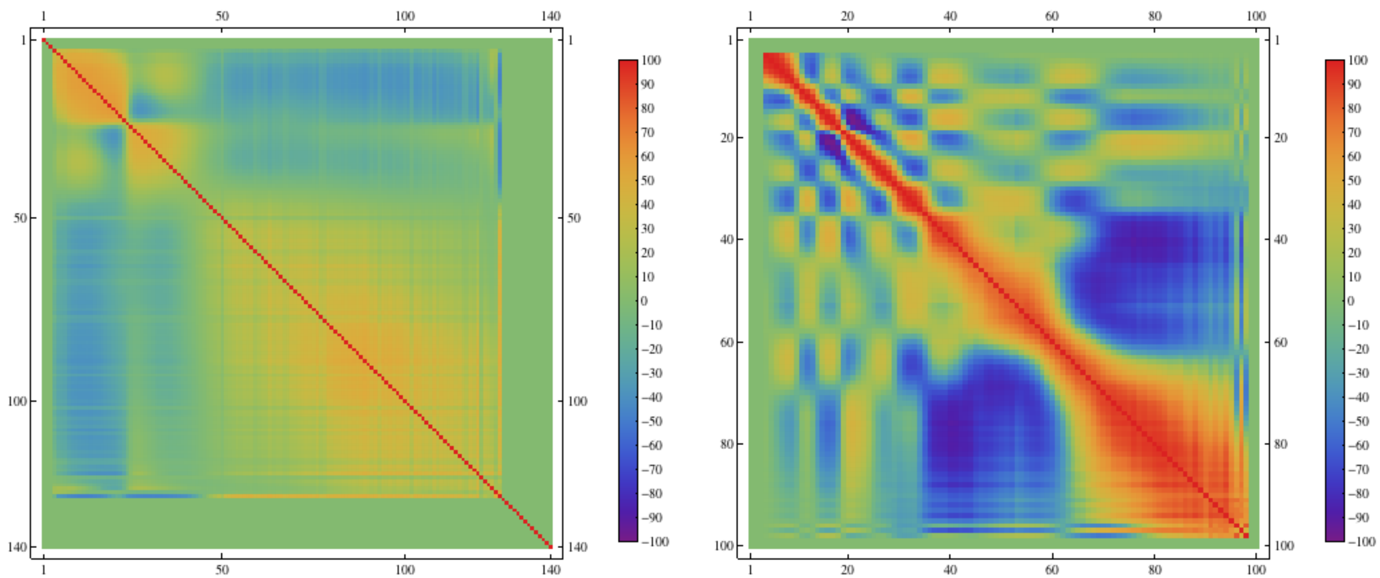
Analytic for $0 < \zeta < 1$, except for cut on (negative) real s axis



imaginary part and absolute value of $\Pi_{\rm DV}(z) = \Pi(z) - \Pi_{OPE}(z)$
asymptotic behavior along real axis has the form of the *ansatz* ,
with $\gamma = 2\pi^2(1-\zeta) \propto 1/N_c$ small.

(Blok, Shifman & Zhang ‘98, Bigi et al. ‘99, Catà, MG & Peris ‘05 & ‘08, Gonzalez-Alonso, Pich & Prades ’10, Jamin ‘11)

ALEPH vs. OPAL correlation matrices



(a) ALEPH (b) OPAL

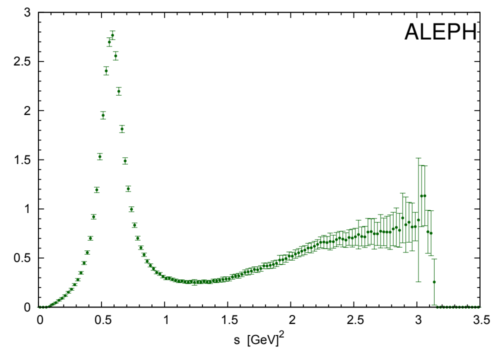
Figure E.1: Correlation matrices for the vector spectral functions from ALEPH (a) and OPAL (b). The correlations are given in %.

(Boito, Ph.D. thesis '11)

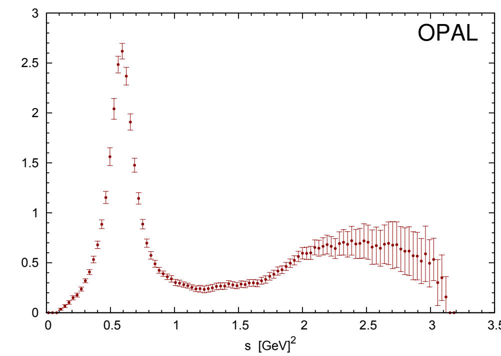
ALEPH vs. OPAL spectral function data (vector channel)

top: experimental data

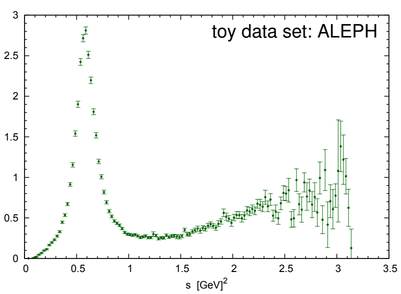
bottom: Monte Carlo data generated with covariance matrix



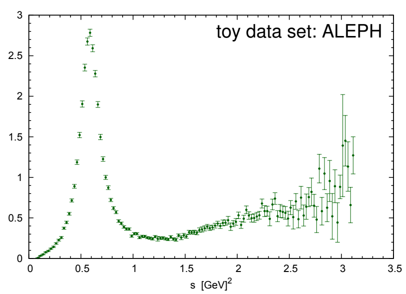
(a)



(a)

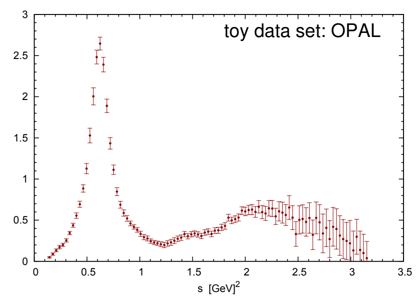


(b)

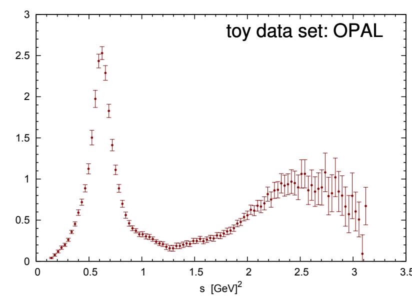


(c)

ALEPH



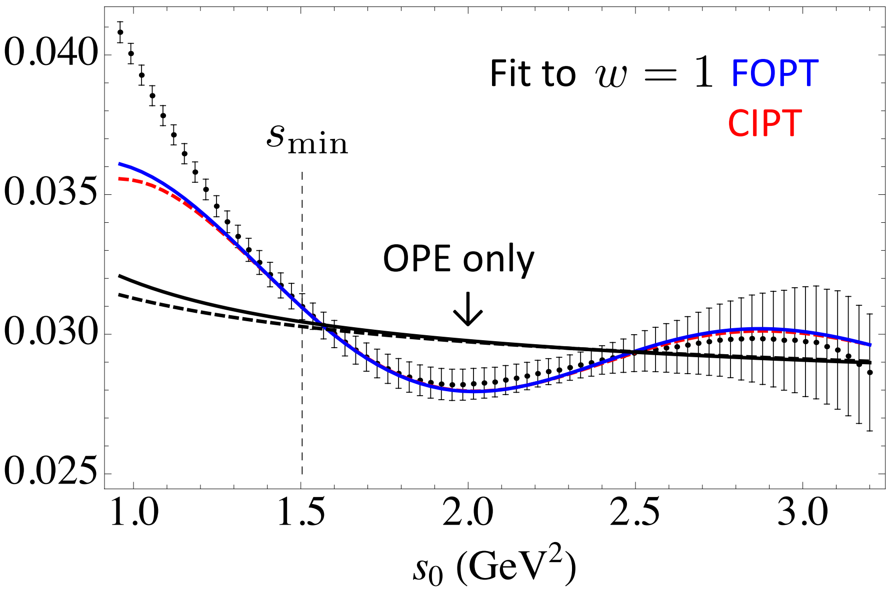
(b)

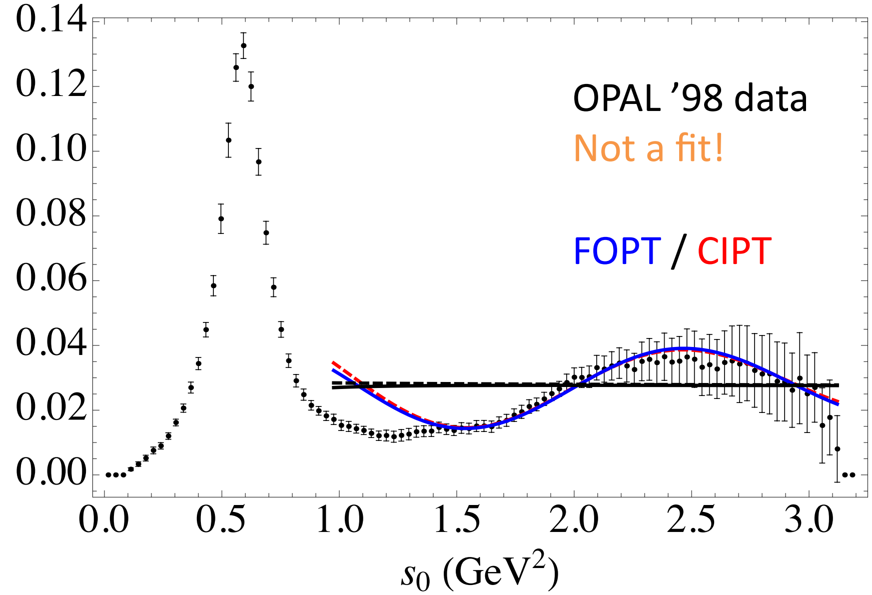


(c)

OPAL

RESULTS





Vector channel fit with weight $w(s) = 1$ and $s_{\min} = 1.5\ \mathrm{GeV}^2$

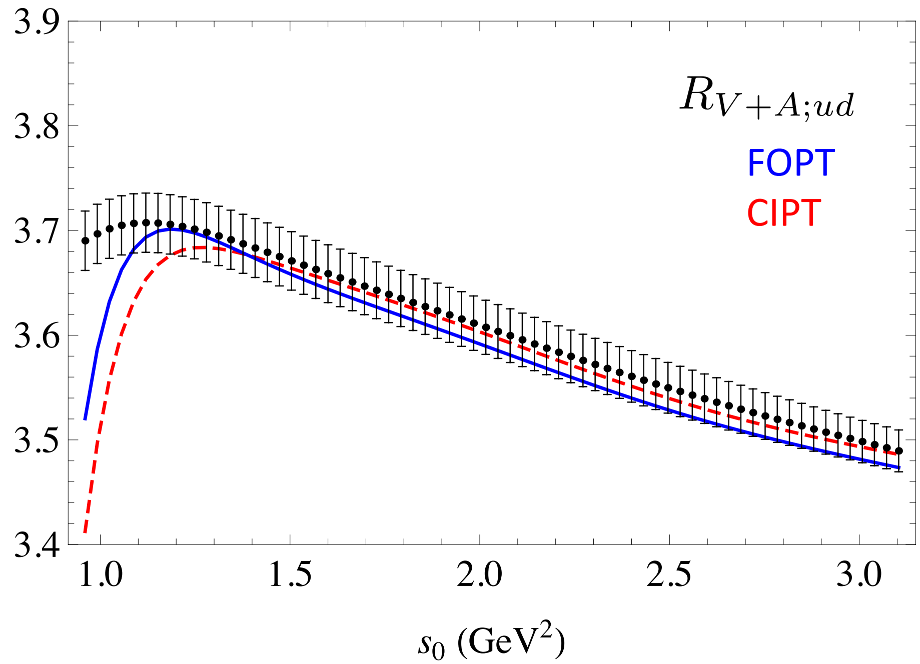
Result: $\alpha_s(m_\tau^2) = 0.307 \pm 0.018 \pm 0.004 \pm 0.005 \quad (\mathrm{FOPT})$

$$= 0.322 \pm 0.025 \pm 0.004 \pm 0.005 \quad (\mathrm{CIPT})$$

Errors: (1) fit error, (2) stability wrt $s_{\min}$, (3) truncation of pert. theory

$$\chi^2/\mathrm{dof} = 0.36 \ , \quad e^{-\delta} = 0.02 \pm 0.01$$

Checks:



- Fits with weights $w = 1\ ,\ 1-(s/s_0)^2\ ,\ (1-s/s_0)^2\,(1+2s/s_0)$ and axial channel data give completely consistent results
- Weinberg sum rules and DGMLY sum rule satisfied within errors
- Fits describe $R_{V+A,ud}$ extremely well

$\Rightarrow$ Take vector channel with $w=1$ as main result (only 5 parameters)

Compare with OPAL:

OPAL's original results: $\alpha_s(m_\tau^2) = 0.324 \pm 0.014 \quad \text{(FOPT)}$

$\alpha_s(m_\tau^2) = 0.348 \pm 0.021 \quad \text{(CIPT)}$

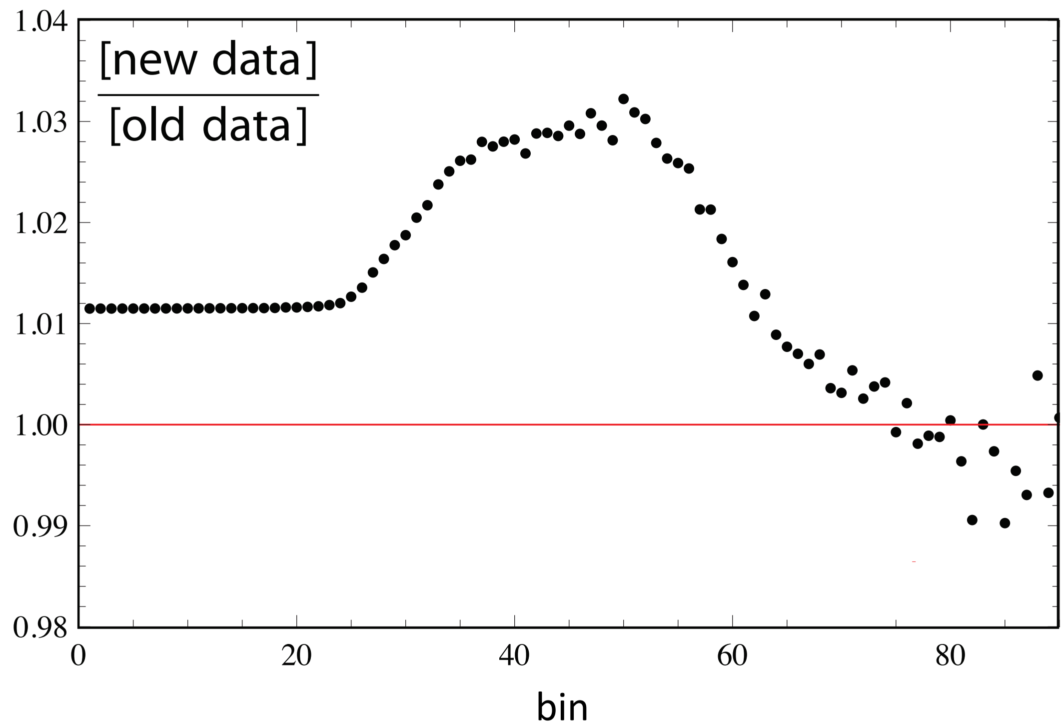
This work: $\alpha_s(m_\tau^2) = 0.307 \pm 0.019 \quad \text{(FOPT)}$

$\alpha_s(m_\tau^2) = 0.322 \pm 0.026 \quad \text{(CIPT)}$

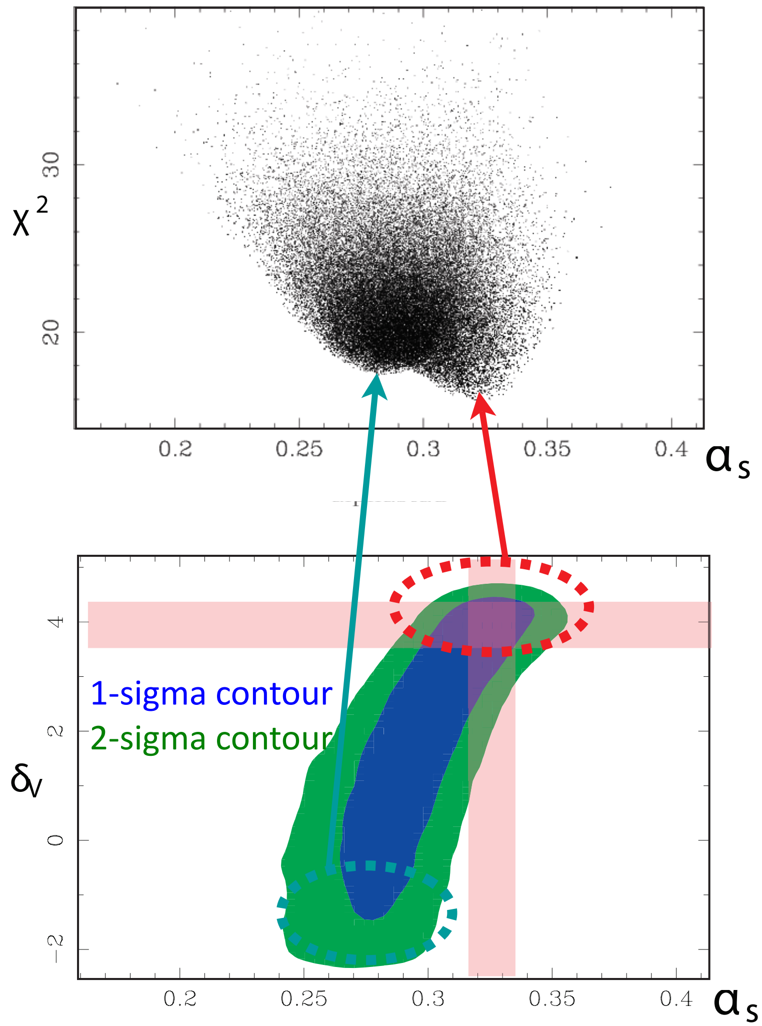
- Same data , central values shifted downward by about 0.02
- Errors previously underestimated; larger than difference CIPT and FOPT

Update OPAL data

- '98 OPAL spectral functions constructed by summing over exclusive modes, normalized with '98 PDG values for branching fractions
- Rescale by using current branching fractions from HFAG '11
- Vector channel update:



Markov-chain Monte Carlo analysis of χ^2 distribution



- vector channel, $w = 1$
- projection from 6d to 2d plot
- complicated landscape!
  χ^2 has two minima, with
  $\delta \approx 4$ or $\delta \approx -2$
- model favors (Catà, MG & Peris '08)

$$\delta \sim -\log \frac{F^2}{M_\rho^2} \approx -\log\left(0.12^2\right) = 4.2$$

(also absolute minimum)

- at the edge!

Results and comparison:

OPAL '99:
$$\alpha_s(m_\tau^2) = 0.324 \pm 0.014 \quad \text{(FOPT)}$$
$$\alpha_s(m_\tau^2) = 0.348 \pm 0.021 \quad \text{(CIPT)}$$

This work, updated data:
$$\alpha_s(m_\tau^2) = 0.325 \pm 0.018 \quad \text{(FOPT)}$$
$$\alpha_s(m_\tau^2) = 0.347 \pm 0.025 \quad \text{(CIPT)}$$

Agreement of central values purely coincidental!

Note **larger** errors; for instance compare:

This work:
$$\delta^{\rm NP} = -0.004 \pm 0.012 \quad \text{(FOPT)}$$
$$\delta^{\rm NP} = -0.002 \pm 0.012 \quad \text{(CIPT)}$$

vs. previous estimate: $\delta^{\rm NP} = -0.0059 \pm 0.0014$ (Pich '11, from ALEPH analysis)

in which $R_{V+A,ud}(m_\tau^2) = N_c |V_{ud}|^2 S_{\rm EW} \left(1 + \delta^{\rm pert.th.} + \delta^{\rm NP}\right)$

Conclusions

- New value of $\alpha_s(m_\tau^2)$ from hadronic tau decays
  Larger error (±0.02) than previously assumed because of non-perturbative uncertainties (OPE and DVs); supersedes earlier values

- Fits to OPAL data at the edge of being possible
  Best fit with 5 parameters and good χ^2 values

- Expect that significant progress (more stringent tests!) is possible if errors are reduced by a factor 2 or 3 –
  BaBar and BELLE: please produce inclusive spectral functions!

- Theory: better understanding of CIPT vs. FOPT? Duality violations?