Nucleons form factors and the final state radiation for the process $e^+e^- \to p\bar{p}\gamma$

H. CZYŻ, IF, UŚ, Katowice,

in collaboration with

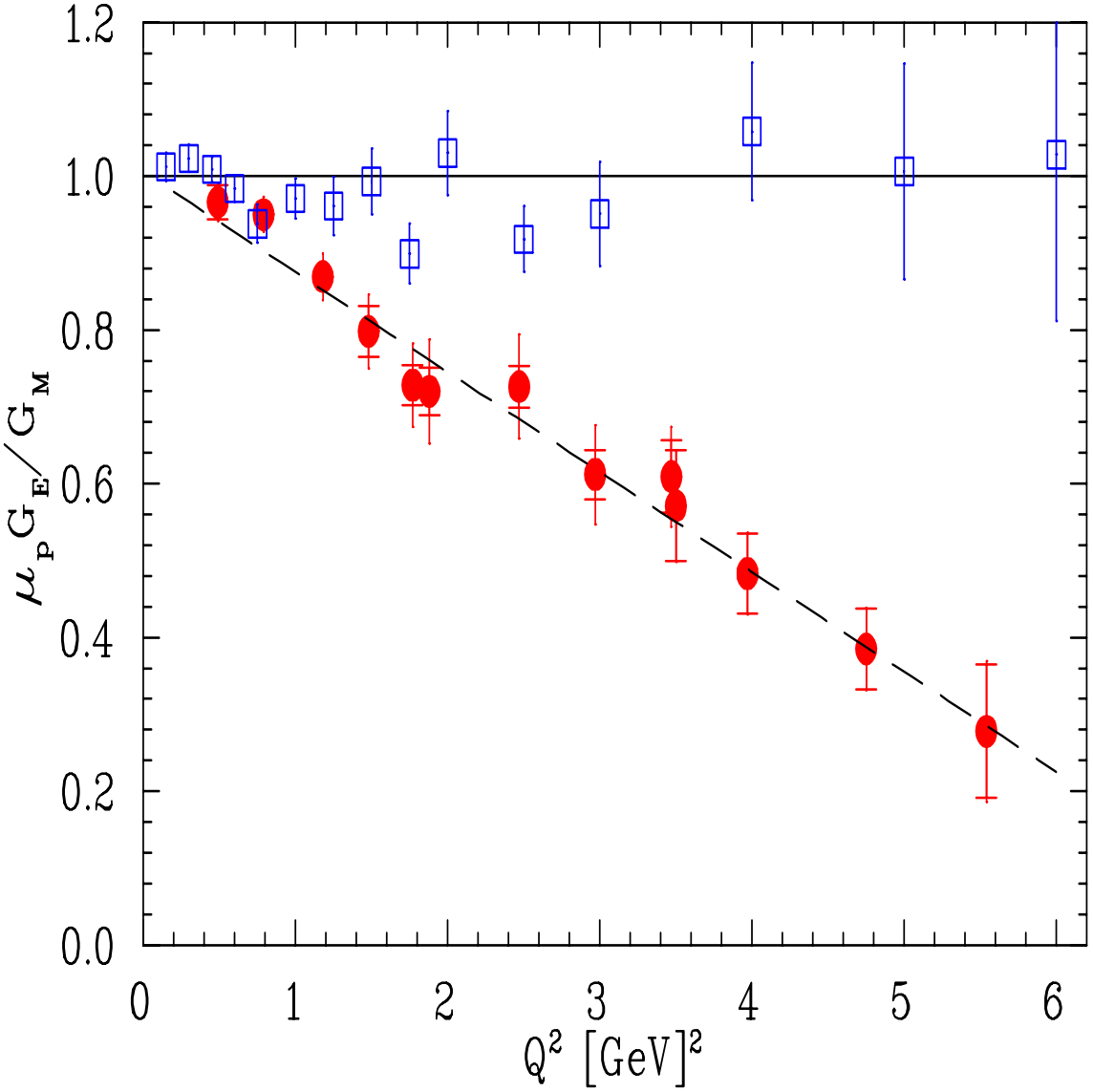
J.H.Kühn and Sz.Tracz

RMC WG Meeting, Trento 2013

Outline of the talk

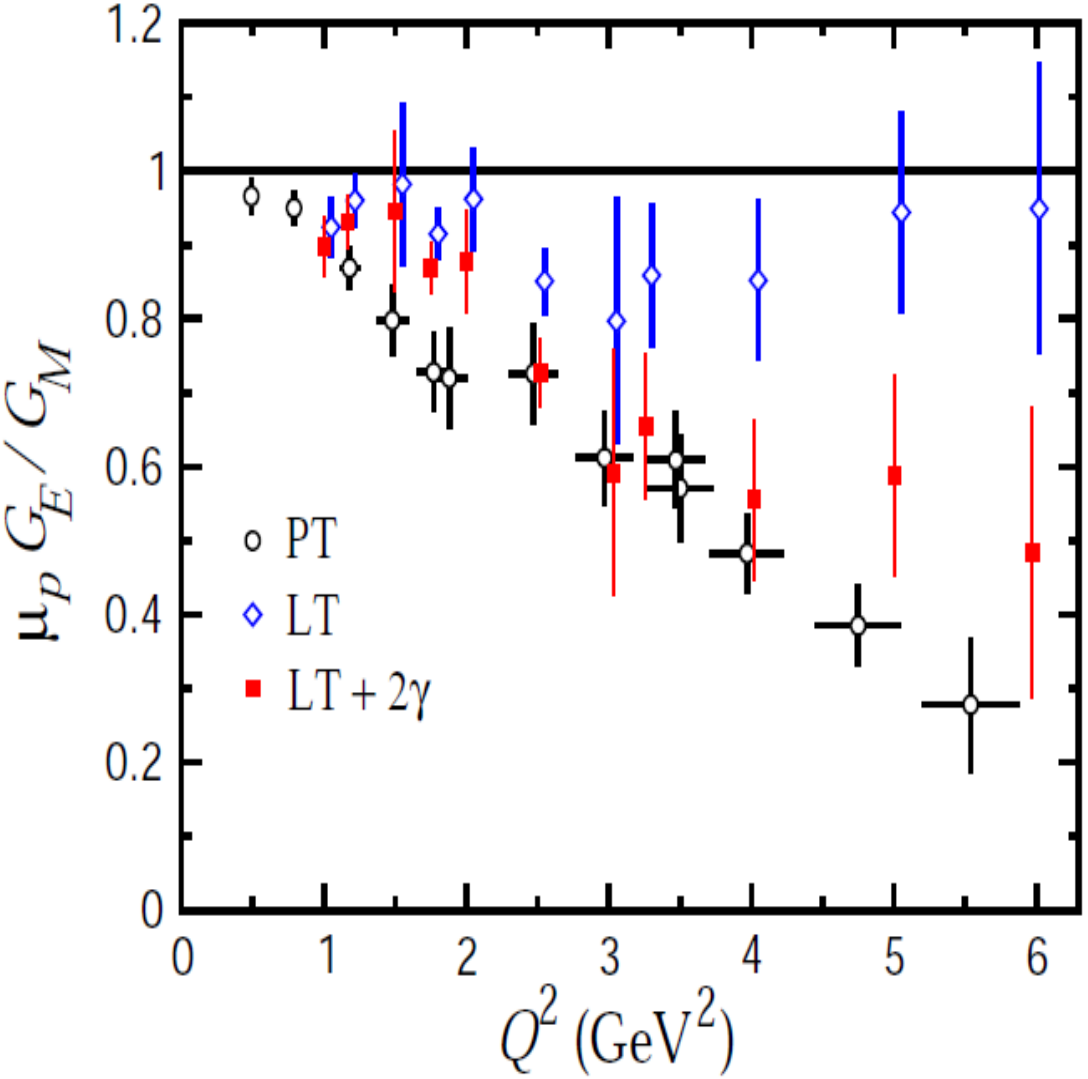
- $\Rightarrow$ Motivation
- $\Rightarrow$ Nucleon form factors
- $\Rightarrow$ FSR radiative corrections
- $\Rightarrow$ Implementation in PHOKHARA
- $\Rightarrow$ Importance of the FSR corrections
- $\Rightarrow$ Final remarks

Why proton FFs



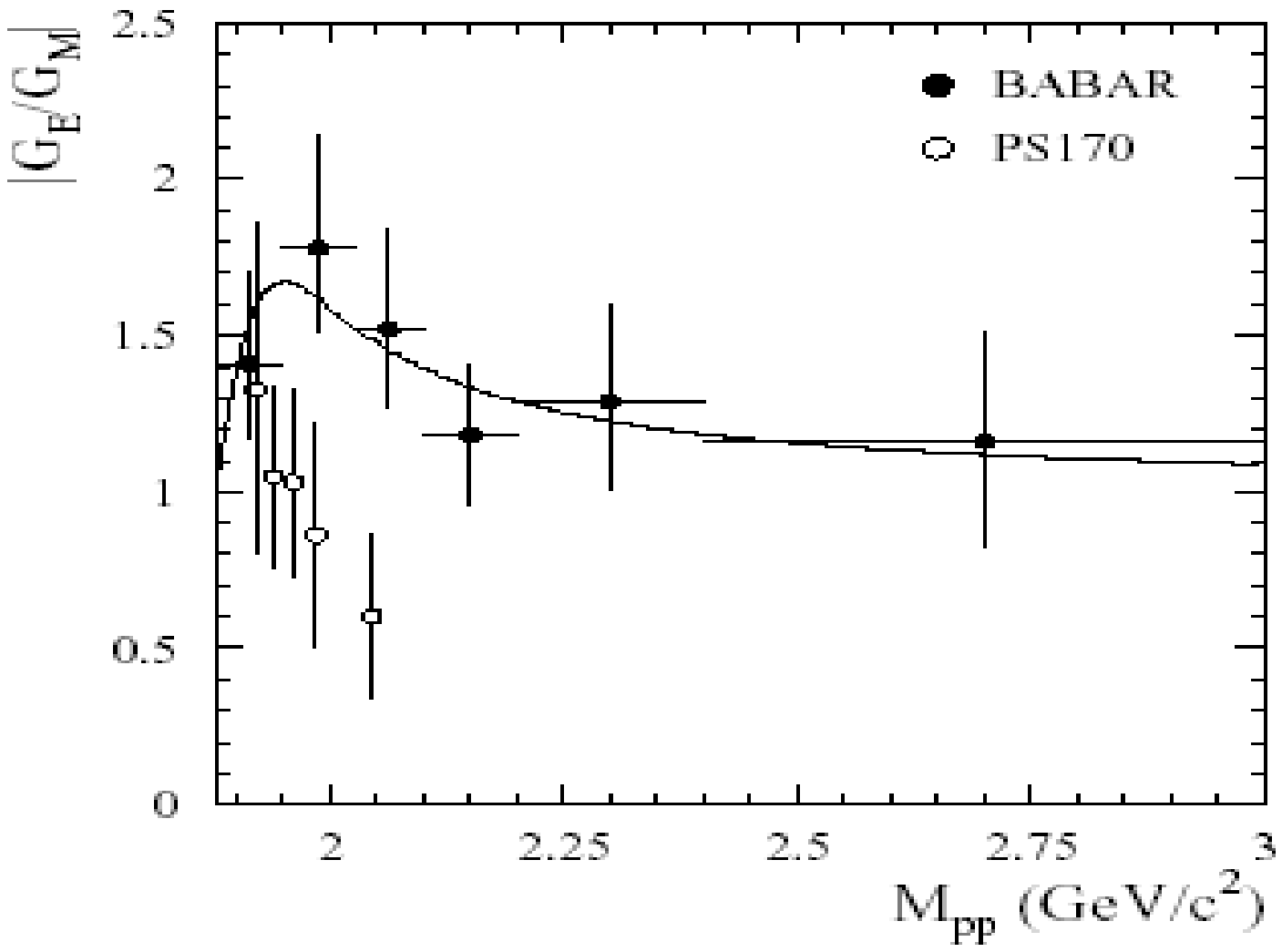
Arrington et al., Phys. Rev. **C 68** (2003) 034325

Why proton FFs



Carl E. Carlson , Marc Vanderhaeghen, Ann.Rev.Nucl.Part.Sci. 57 (2007) 171-204

nucleon FF

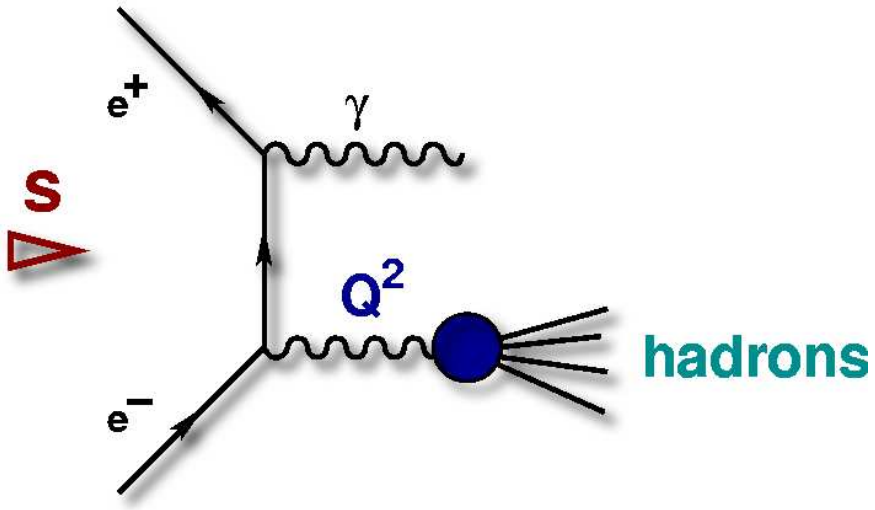


BaBar: Phys.Rev.D73:012005,2006.

THE RADIATIVE RETURN METHOD

$$d\sigma(e^+e^- \to \text{hadrons} + \gamma(ISR)) =$$

$$H(Q^2, \theta_\gamma)\; d\sigma(e^+e^- \to \text{hadrons})(s = Q^2)$$



- measurement of $R(s)$ over the full range of energies, from threshold up to $\sqrt{s}$
- large luminosities of factories compensate α/π from photon radiation
- radiative corrections essential (NLO,...)

High precision measurement of the hadronic cross-section at meson-factories

MC generators needed

EVA: $e^+e^- \to \pi^+\pi^-\gamma$

- tagged photon ($\theta_\gamma > \theta_{cut}$)
- ISR at LO + Structure Function
- FSR: point-like pions

[Binner et al.]

$e^+e^- \to 4\pi + \gamma$

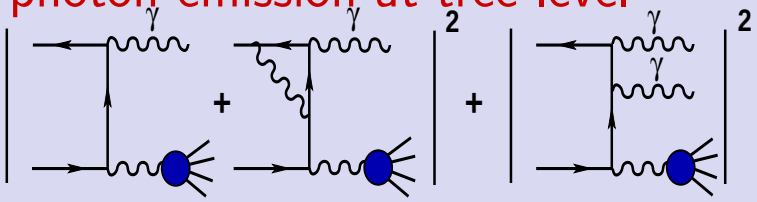
- ISR at LO + Structure Function

[Czyż, Kühn,2000]

H.C., A. Grzelińska,
J. H. Kühn, E. Nowak-Kubat,
G. Rodrigo, A. Wapienik

PHOKHARA 7.0: $\pi^+\pi^-$, $\mu^+\mu^-$, 4π, $\bar{N}N$, 3π, KK, $\Lambda(\to\cdots)\bar{\Lambda}(\to\cdots)$, $J/\psi, \psi(2S)$

- **ISR at NLO:** virtual corrections to one photon events and two photon emission at tree level



- FSR at NLO:$\pi^+\pi^-$, $\mu^+\mu^-$,K^+K^-
- tagged or untagged photons
- Modular structure

http://ific.uv.es/~rodrigo/phokhara/

Form factors i PHOKHARA

$$G_M^N = F_1^N + F_2^N, \;\; G_E^N = F_1^N + \tau F_2^N$$

$$F_{1,2}^p = F_{1,2}^s + F_{1,2}^v, \;\; F_{1,2}^n = F_{1,2}^s - F_{1,2}^v$$

Form factors i PHOKHARA

$$F_1^s = \frac{1}{2}\frac{\sum_{n=0}^{3} c_n^1 BW_{\omega_n}(s)}{\sum_{n=0}^{3} c_n^1}$$

$$F_1^v = \frac{1}{2}\frac{\sum_{n=0}^{3} c_n^2 BW_{\rho_n}(s)}{\sum_{n=0}^{3} c_n^2}$$

$$F_2^s = -\frac{1}{2}b\frac{\sum_{n=0}^{3} c_n^3 BW_{\omega_n}(s)}{\sum_{n=0}^{3} c_n^3}$$

$$F_2^v = \frac{1}{2}a\frac{\sum_{n=0}^{3} c_n^4 BW_{\rho_n}(s)}{\sum_{n=0}^{3} c_n^4}$$

Form factors i PHOKHARA

$$BW_i = \frac{m_i^2}{m_i^2 - s - i m_i \Gamma_i},$$

$$a = \mu_p - \mu_n - 1 \qquad b = -\mu_p - \mu_n + 1$$

$$F_1 \sim \frac{1}{(Q^2)^2}, \quad F_2 \sim \frac{1}{(Q^2)^3}.$$

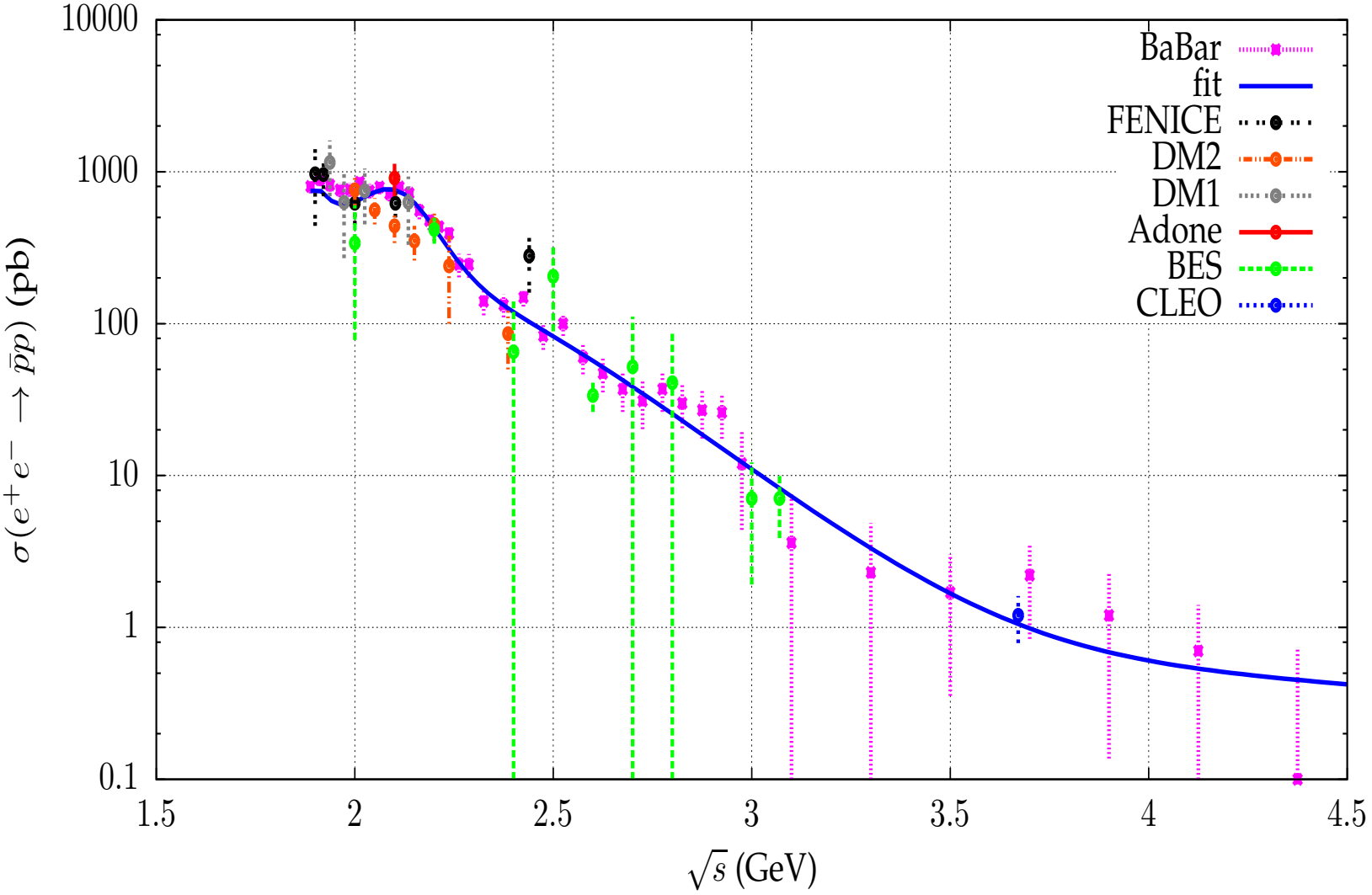
G. P. Lepage,S. J. Brodsky,Phys. Rev. D 22, 2157 (1980)

Fits

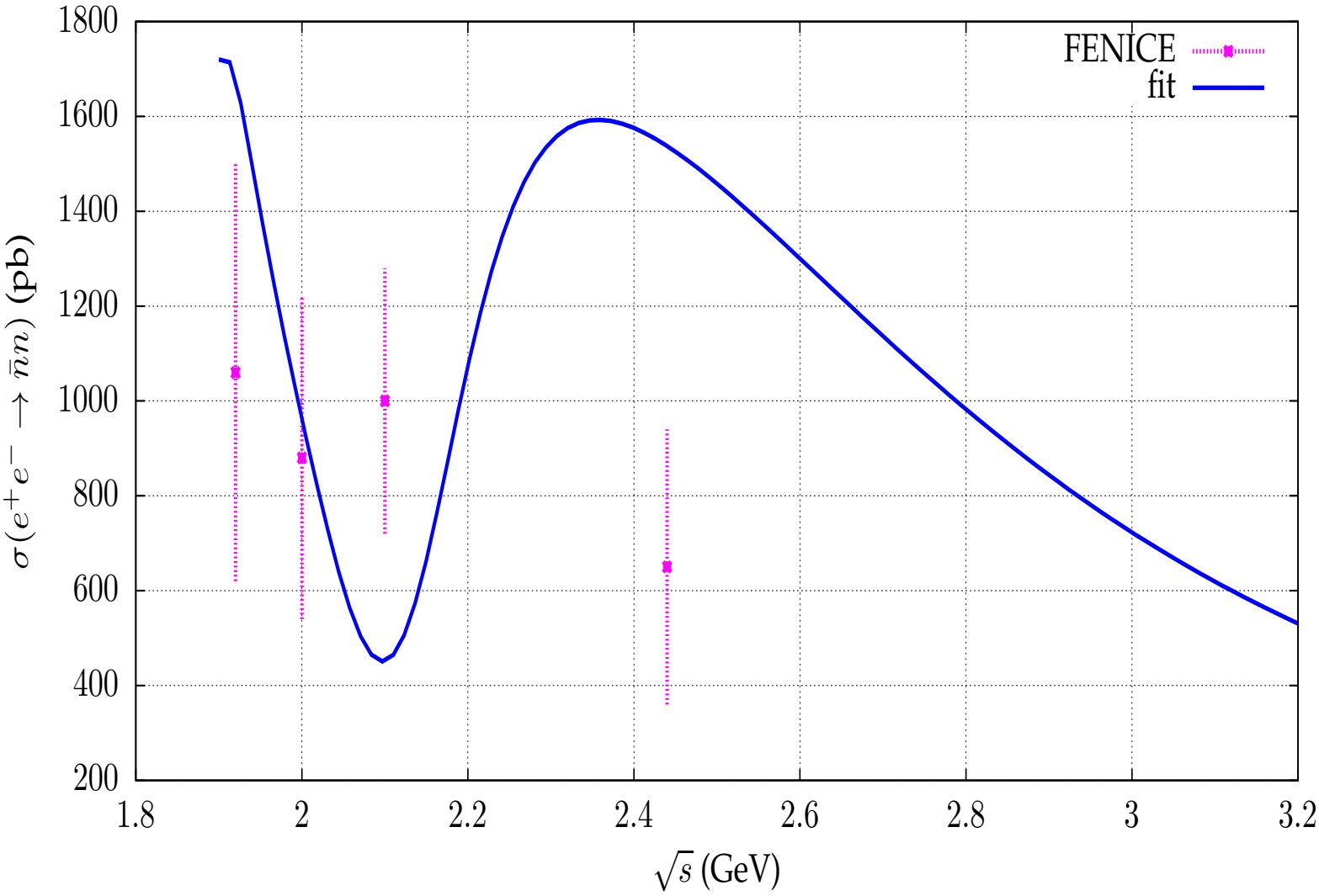
Experiment	number of points	chi-squared value	Experiment	number of points	chi-squared value
BaBar cross section	38	39.69	FENICE pp cross section	5	4.42
DM2 cross section	7	24.52	DM1 cross section	4	1.23
Adone cross section	1	0.46	BES cross section	8	13.58
CLEO cross section	1	0.127	JLab 2005 proton ratio	10	18.47
JLab 2002 proton ratio	4	5.32	JLab 2001 proton ratio	13	9.52
MAMI proton ratio	3	2.08	JLab 2010 proton ratio	3	3.63
PS 170 ratio	5	5.98	BaBar ratio	6	22.27
PS170 cross section	8	8.07	PS170 cross section	3	1.8
E760 cross section	3	1.05	E835 cross section	5	3.51
E835 cross section	2	0.08	JLab n ratio	3	3.64
BLAST n ratio	4	6.07	FENICE n cross section	4	15.26

Table 1: $N_{par} = 12$, $N_{points} = 136$, $\chi^2 = 191$

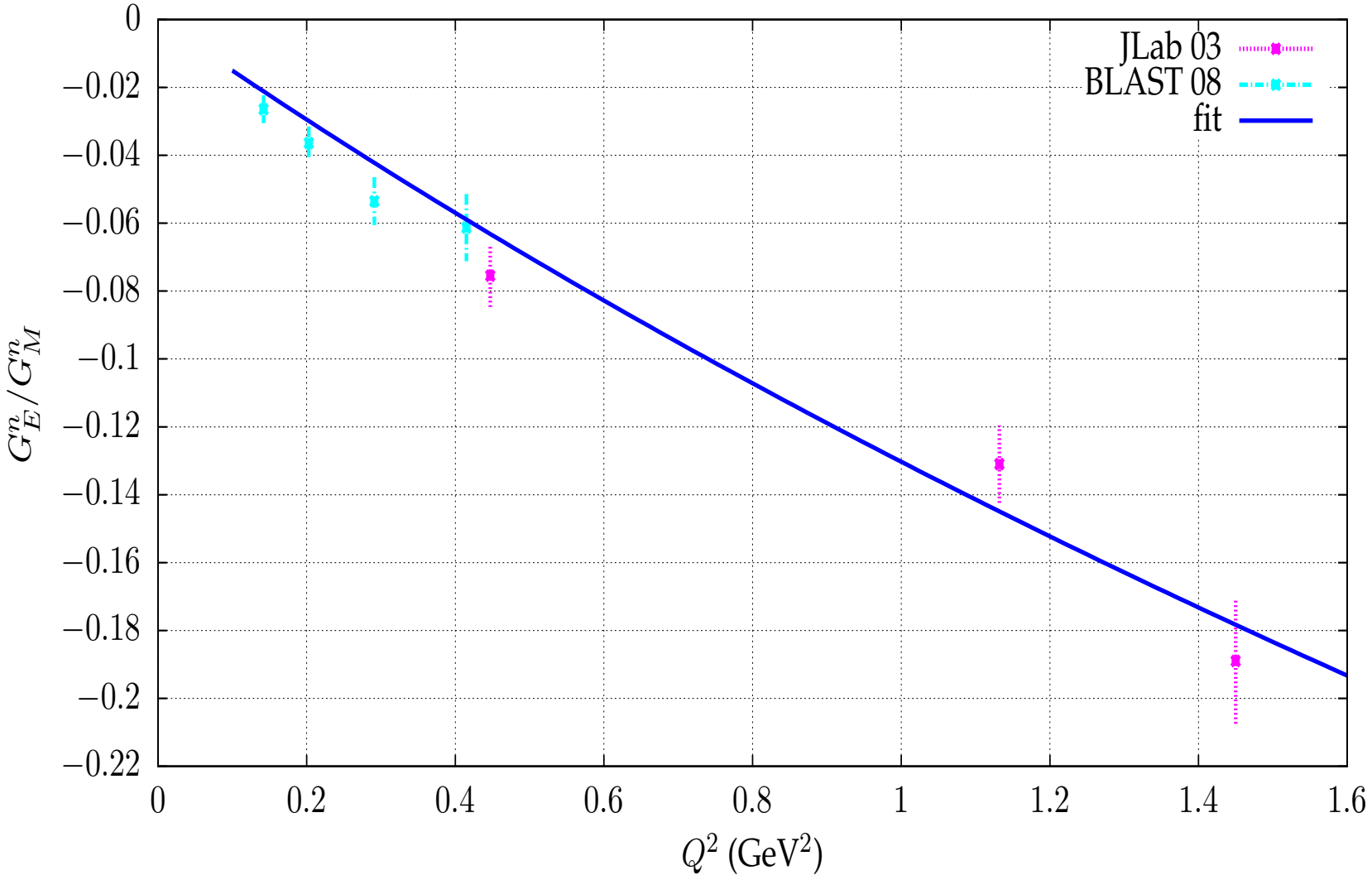
Fits


10000
1000
100
10
1
0.1
1.5
2
2.5
3
3.5
4
4.5
$\sigma(e^+e^- \to \bar{p}p)$ (pb)
$\sqrt{s}$ (GeV)
BaBar
fit
FENICE
DM2
DM1
Adone
BES
CLEO

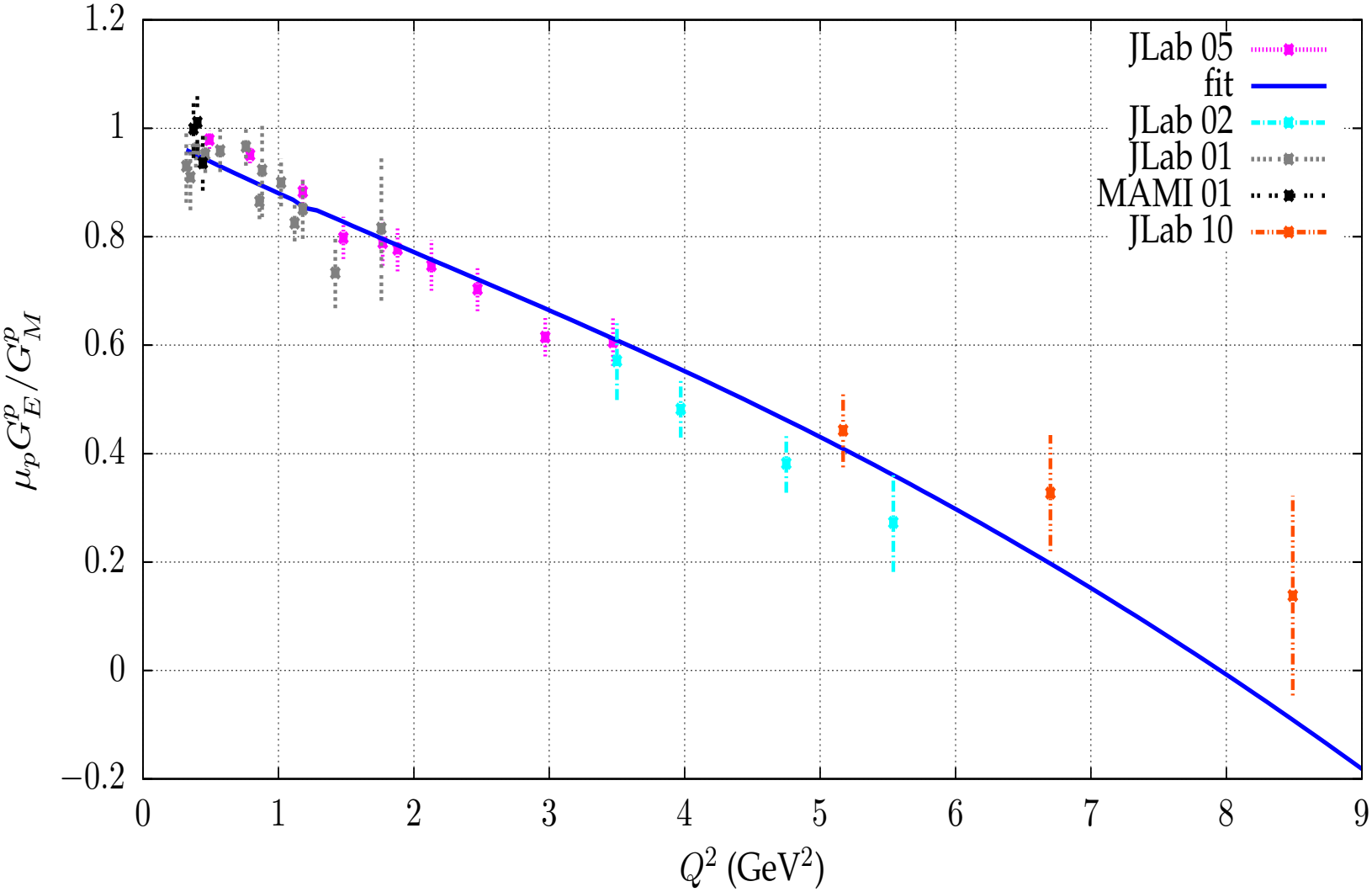
Fits


FENICE
fit
$\sigma(e^+e^- \to \bar{n}n)$ (pb)
$\sqrt{s}$ (GeV)
1800
1600
1400
1200
1000
800
600
400
200
1.8
2
2.2
2.4
2.6
2.8
3
3.2

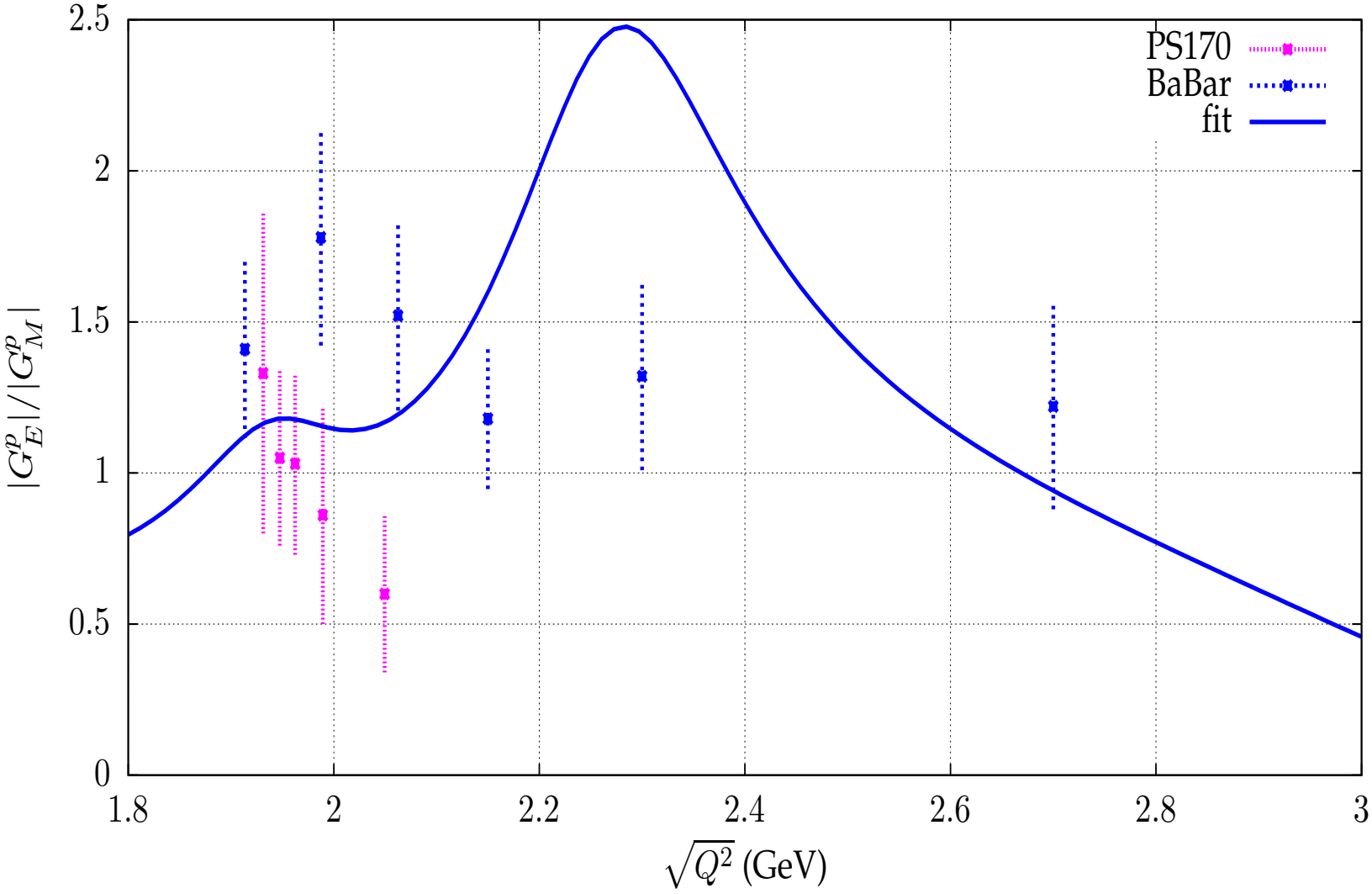
Fits


JLab 03
BLAST 08
fit
G_E^n/G_M^n
Q^2 (GeV^2)
0
−0.02
−0.04
−0.06
−0.08
−0.1
−0.12
−0.14
−0.16
−0.18
−0.2
−0.22
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6

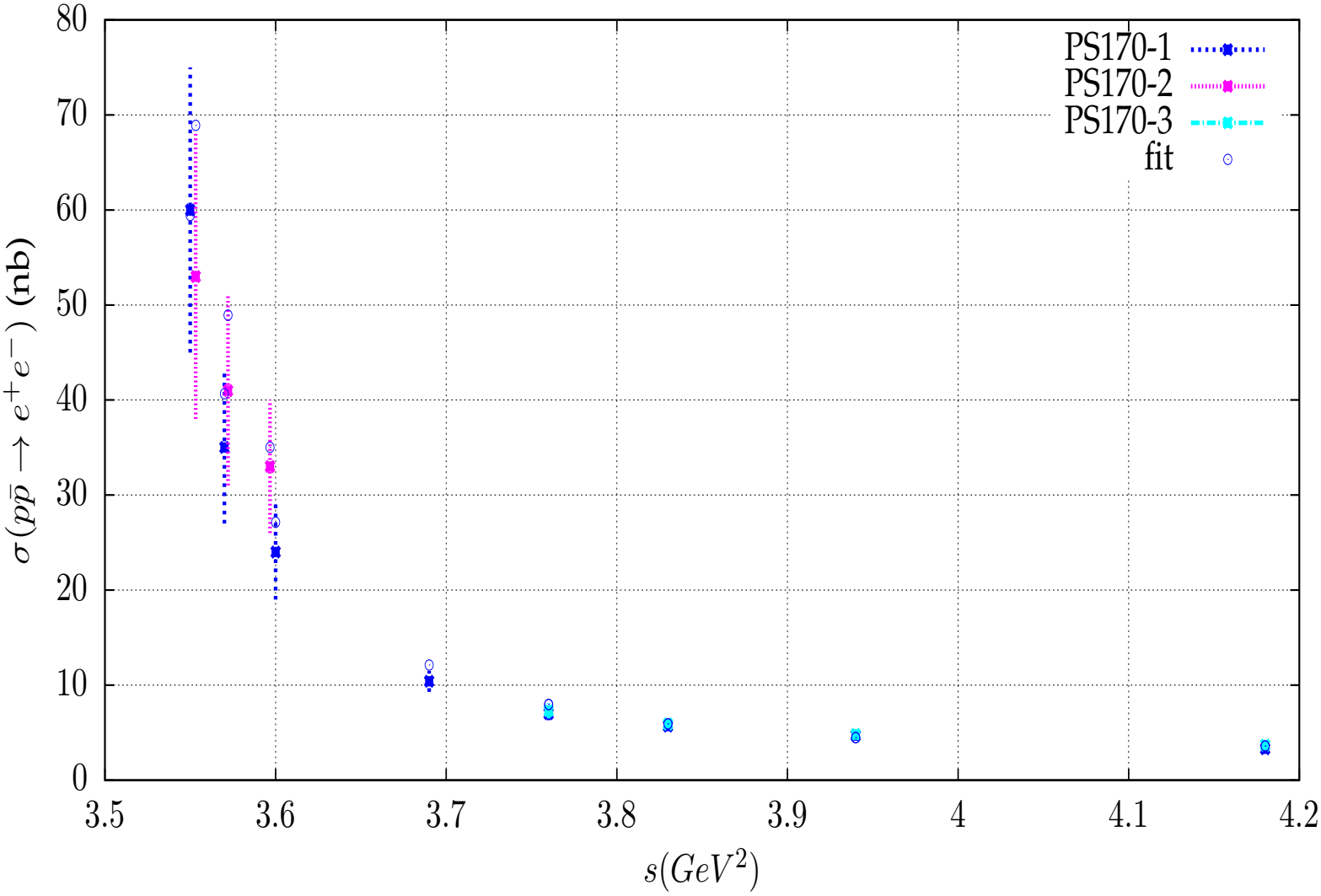
Fits


JLab 05
fit
JLab 02
JLab 01
MAMI 01
JLab 10
$\mu_p G_E^p / G_M^p$
Q^2 (GeV^2)
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
0
1
2
3
4
5
6
7
8
9

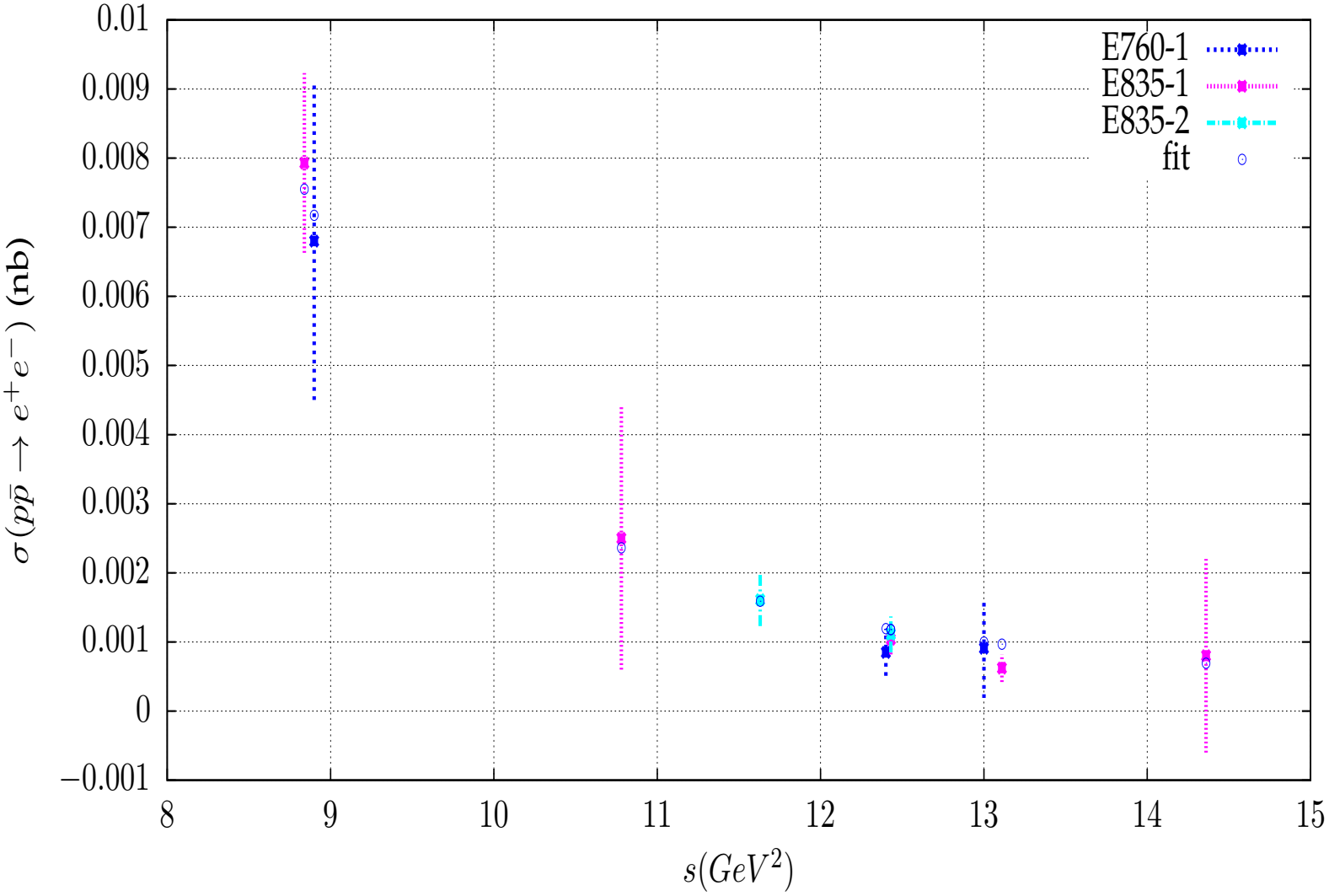
Fits



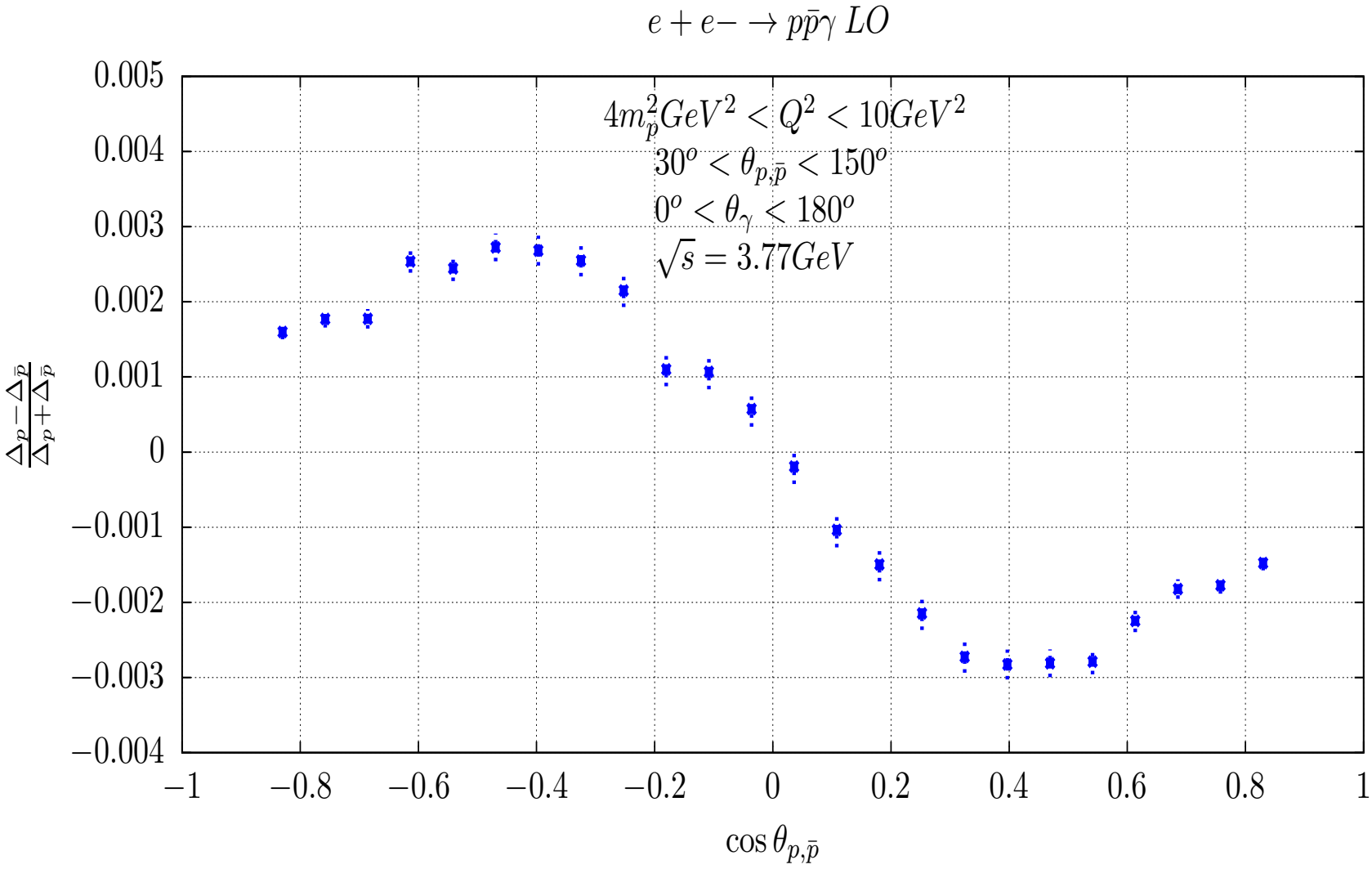
Fits


PS170-1
PS170-2
PS170-3
fit
$\sigma(p\bar{p} \to e^+e^-)$ (nb)
$s(GeV^2)$
80
70
60
50
40
30
20
10
0
3.5
3.6
3.7
3.8
3.9
4
4.1
4.2

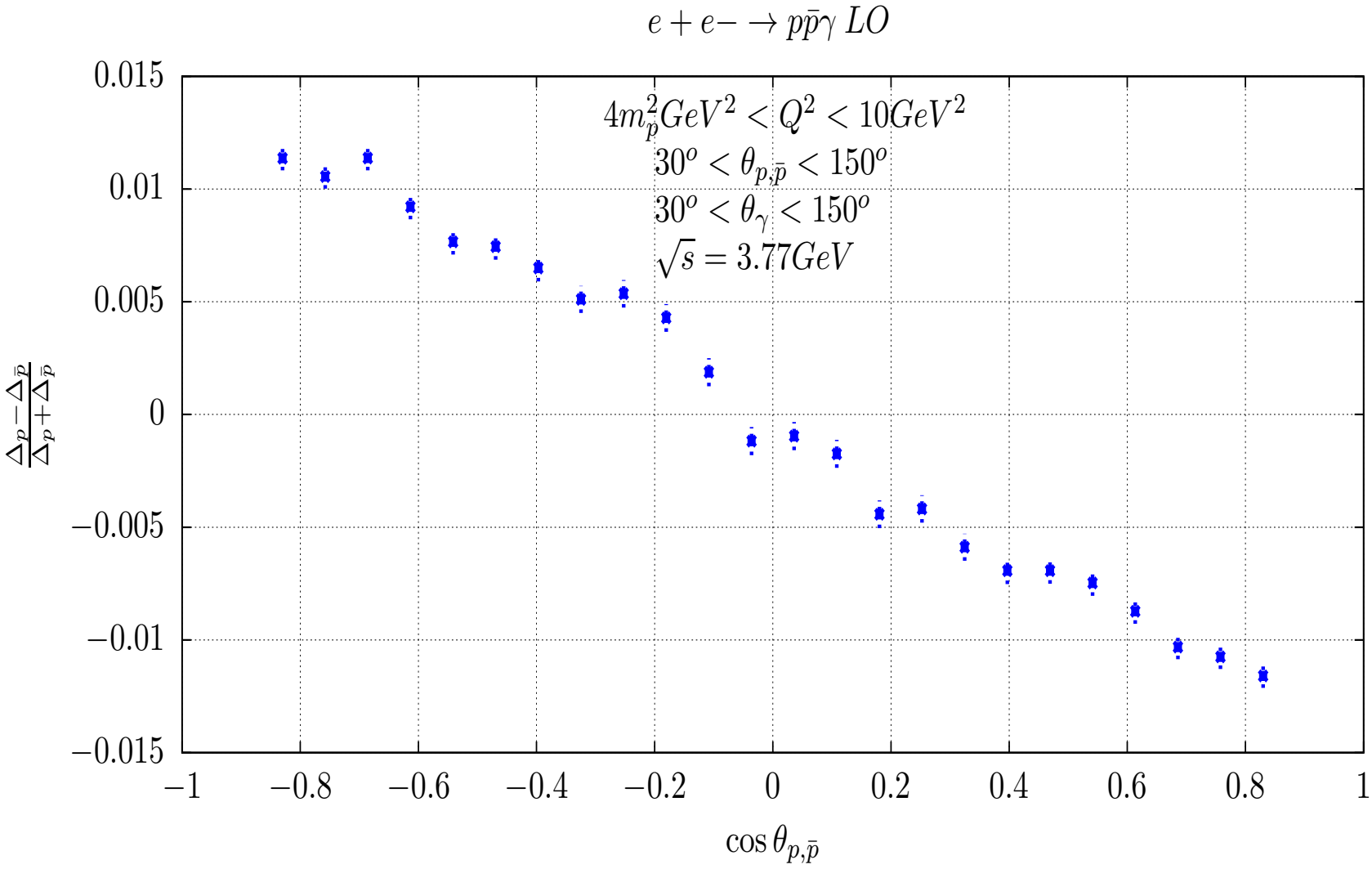
Fits


E760-1
E835-1
E835-2
fit
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
−0.001
8
9
10
11
12
13
14
15
$\sigma(p\bar{p} \to e^+e^-)$ (nb)
$s(GeV^2)$

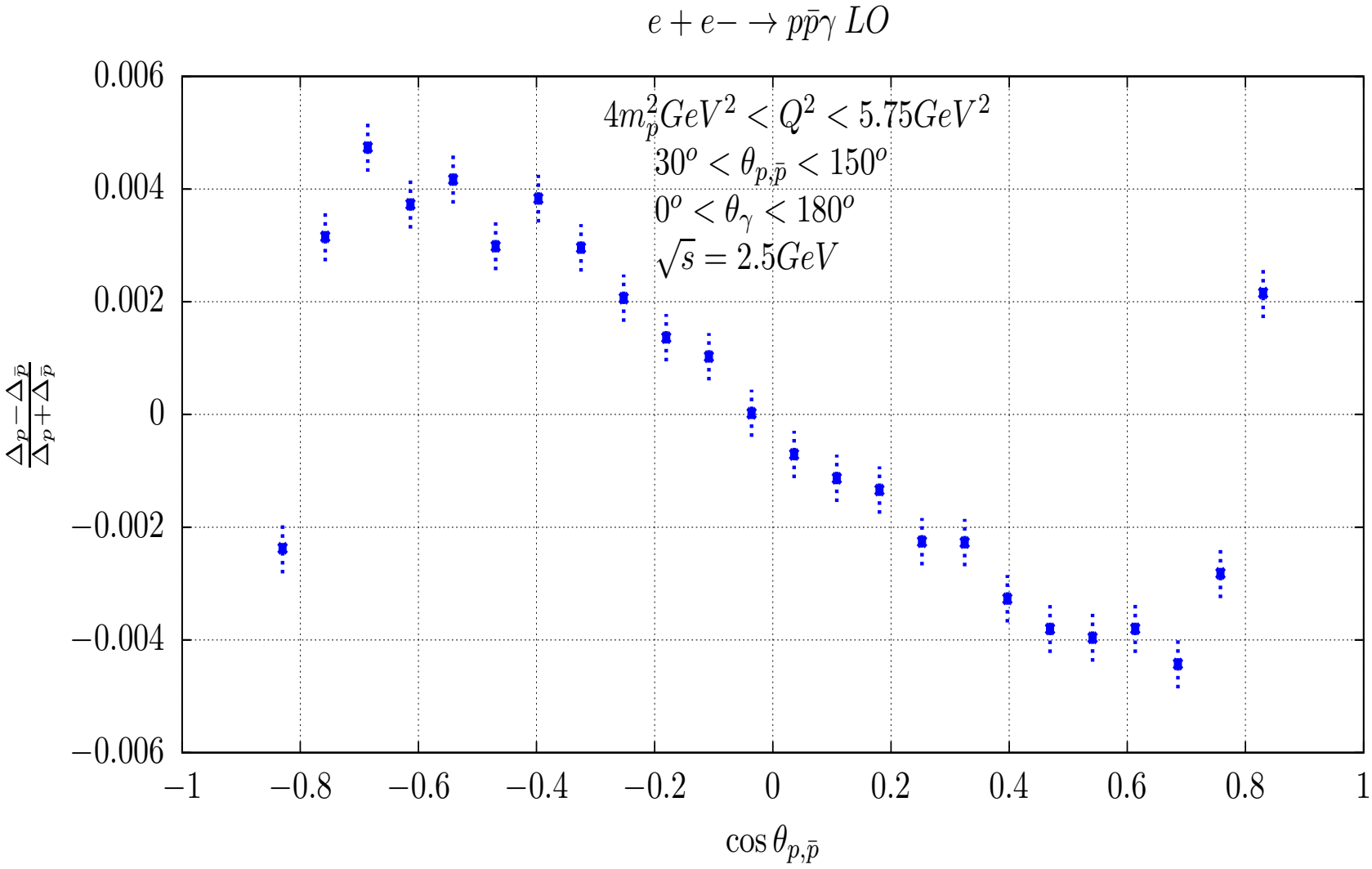
FSR


$e + e- \to p\bar{p}\gamma\ LO$
$4m_p^2 GeV^2 < Q^2 < 10 GeV^2$
$30^o < \theta_{p,\bar{p}} < 150^o$
$0^o < \theta_\gamma < 180^o$
$\sqrt{s} = 3.77 GeV$
$\frac{\Delta_p - \Delta_{\bar{p}}}{\Delta_p + \Delta_{\bar{p}}}$
$\cos\theta_{p,\bar{p}}$
0.005
0.004
0.003
0.002
0.001
0
−0.001
−0.002
−0.003
−0.004
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1

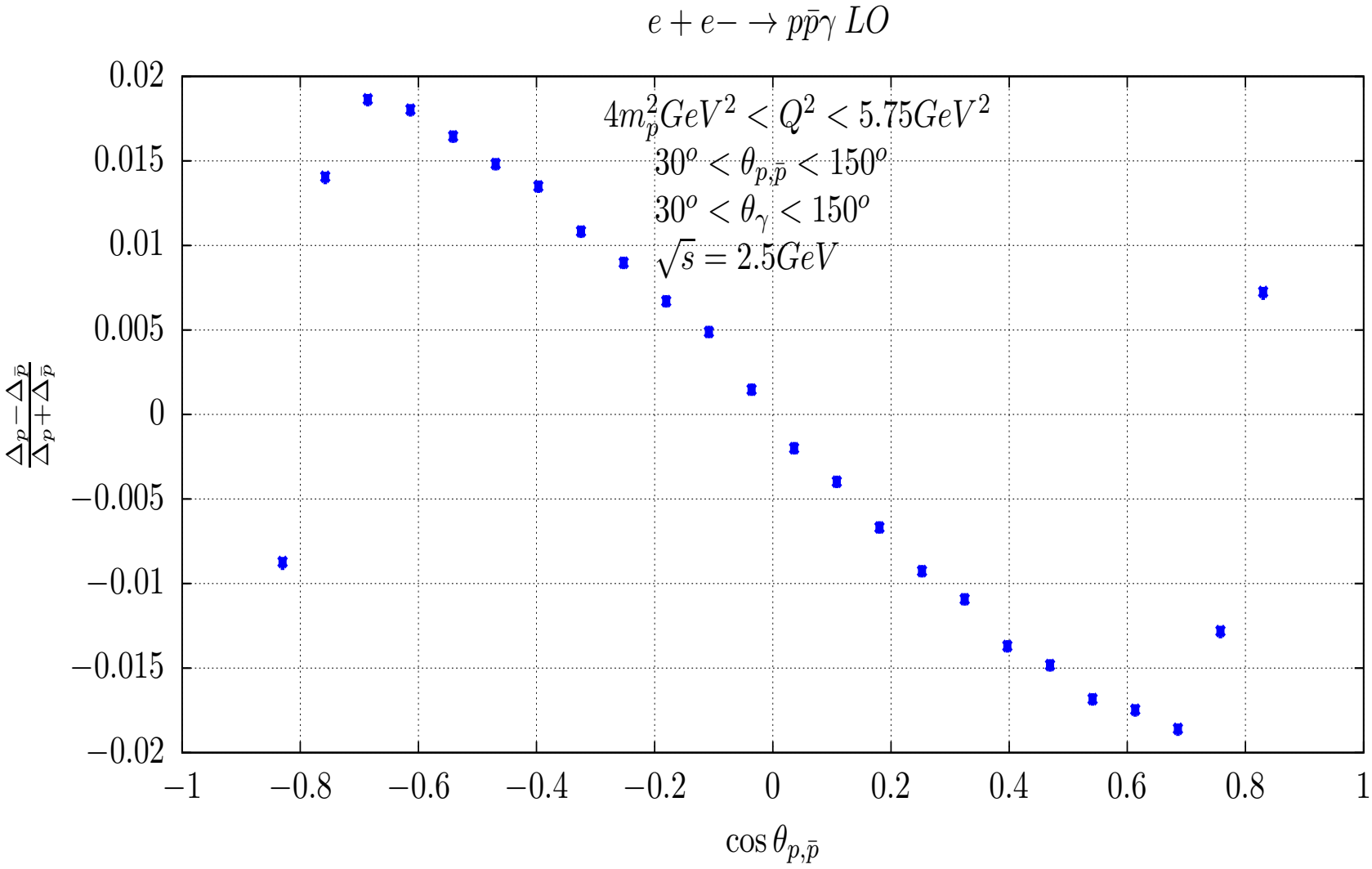
FSR


$e + e- \to p\bar{p}\gamma\ LO$
$4m_p^2 GeV^2 < Q^2 < 10 GeV^2$
$30^o < \theta_{p,\bar{p}} < 150^o$
$30^o < \theta_\gamma < 150^o$
$\sqrt{s} = 3.77 GeV$
$\frac{\Delta_p - \Delta_{\bar{p}}}{\Delta_p + \Delta_{\bar{p}}}$
0.015
0.01
0.005
0
−0.005
−0.01
−0.015
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
$\cos\theta_{p,\bar{p}}$

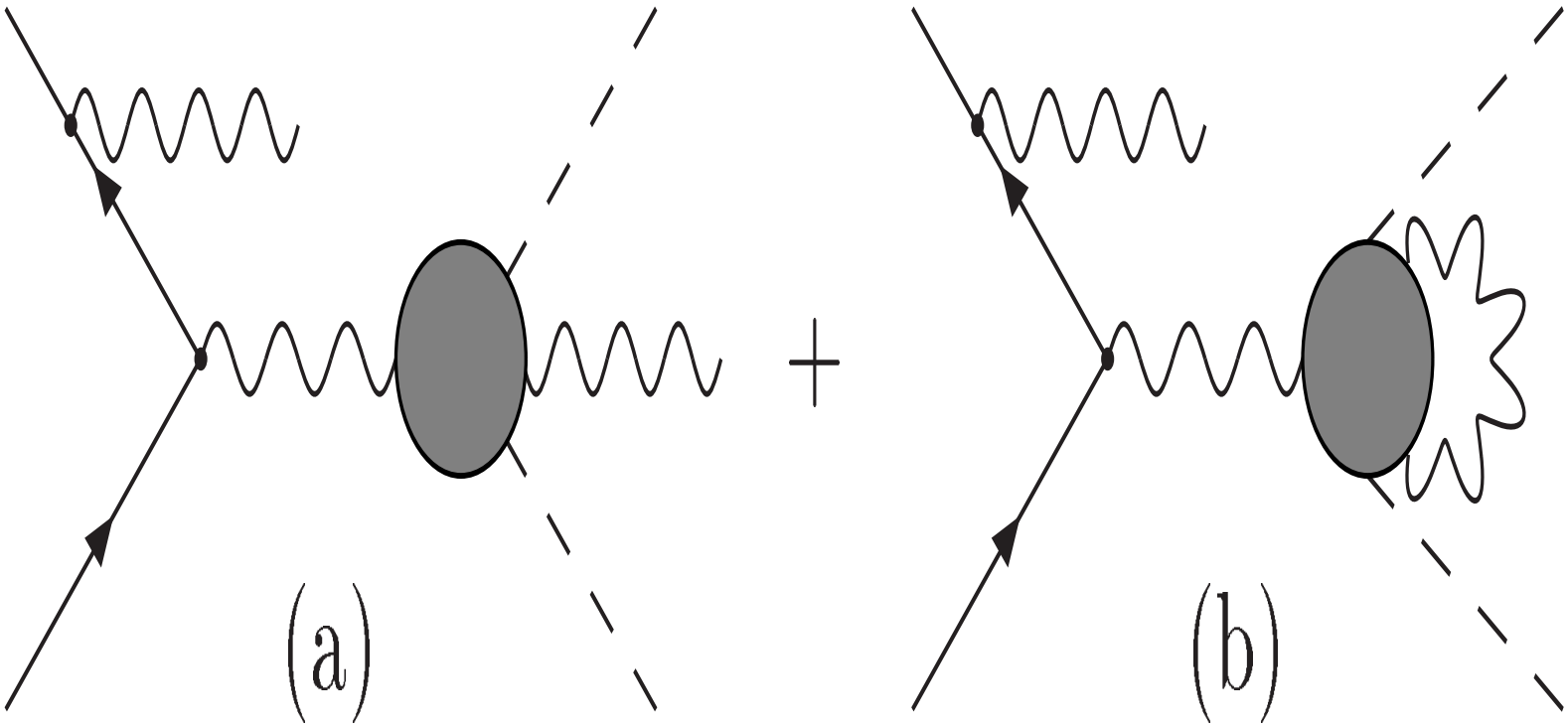
FSR


$e + e- \to p\bar{p}\gamma\ LO$
$4m_p^2 GeV^2 < Q^2 < 5.75 GeV^2$
$30^o < \theta_{p,\bar{p}} < 150^o$
$0^o < \theta_\gamma < 180^o$
$\sqrt{s} = 2.5 GeV$
$\frac{\Delta_p - \Delta_{\bar{p}}}{\Delta_p + \Delta_{\bar{p}}}$
$\cos\theta_{p,\bar{p}}$
0.006
0.004
0.002
0
−0.002
−0.004
−0.006
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1

FSR


$e + e- \to p\bar{p}\gamma\ LO$
$4m_p^2 GeV^2 < Q^2 < 5.75 GeV^2$
$30^o < \theta_{p,\bar{p}} < 150^o$
$30^o < \theta_\gamma < 150^o$
$\sqrt{s} = 2.5 GeV$
$\frac{\Delta_p - \Delta_{\bar{p}}}{\Delta_p + \Delta_{\bar{p}}}$
$\cos\theta_{p,\bar{p}}$
0.02
0.015
0.01
0.005
0
−0.005
−0.01
−0.015
−0.02
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1

FSR



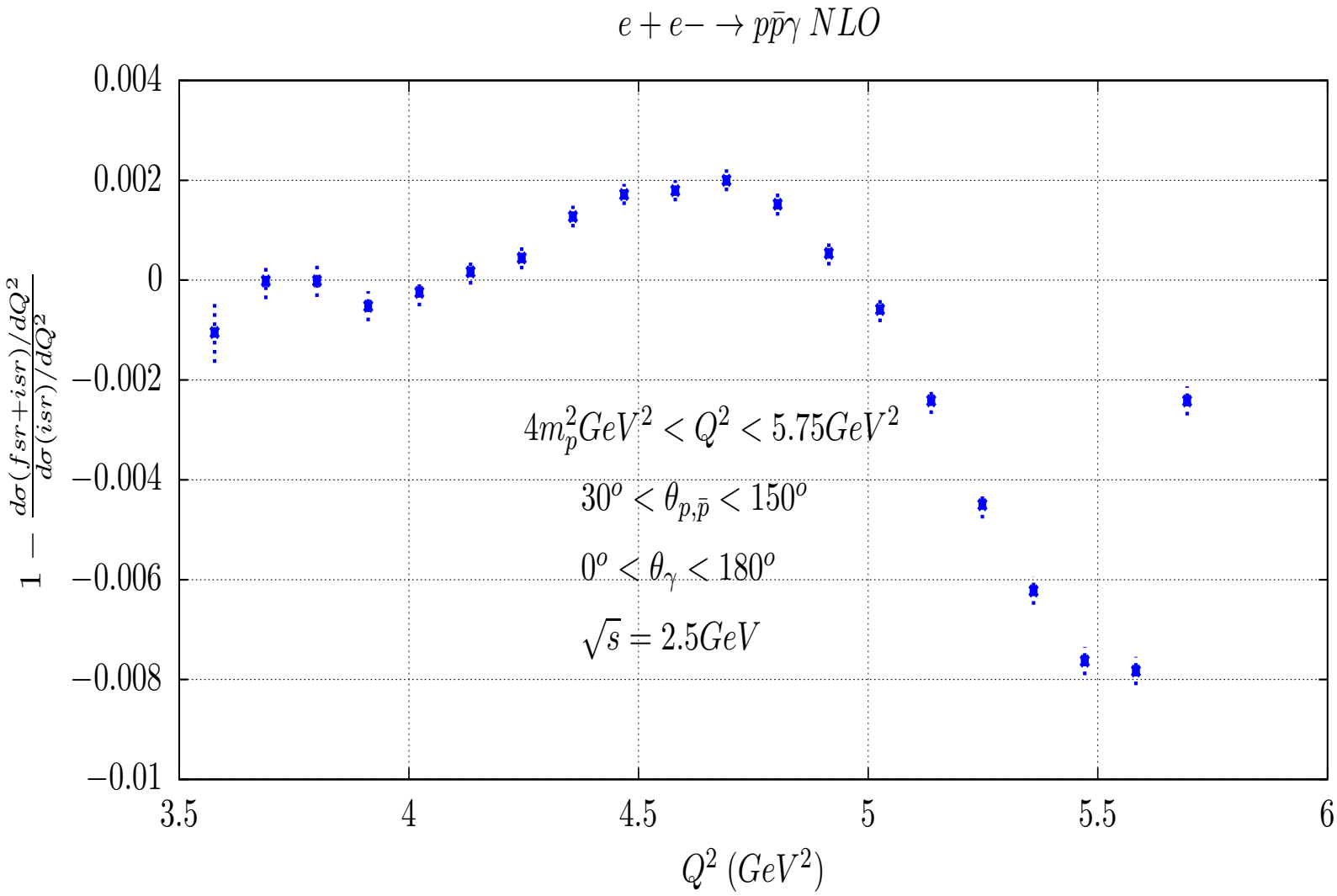
Radiative corrections

$$\Delta_{vert} = \frac{2\alpha}{\pi}[\frac{(1+\beta^2)}{2\beta}\log\frac{Q^2(1+\beta)^2}{4m_p^2} - 1]\log 2w$$

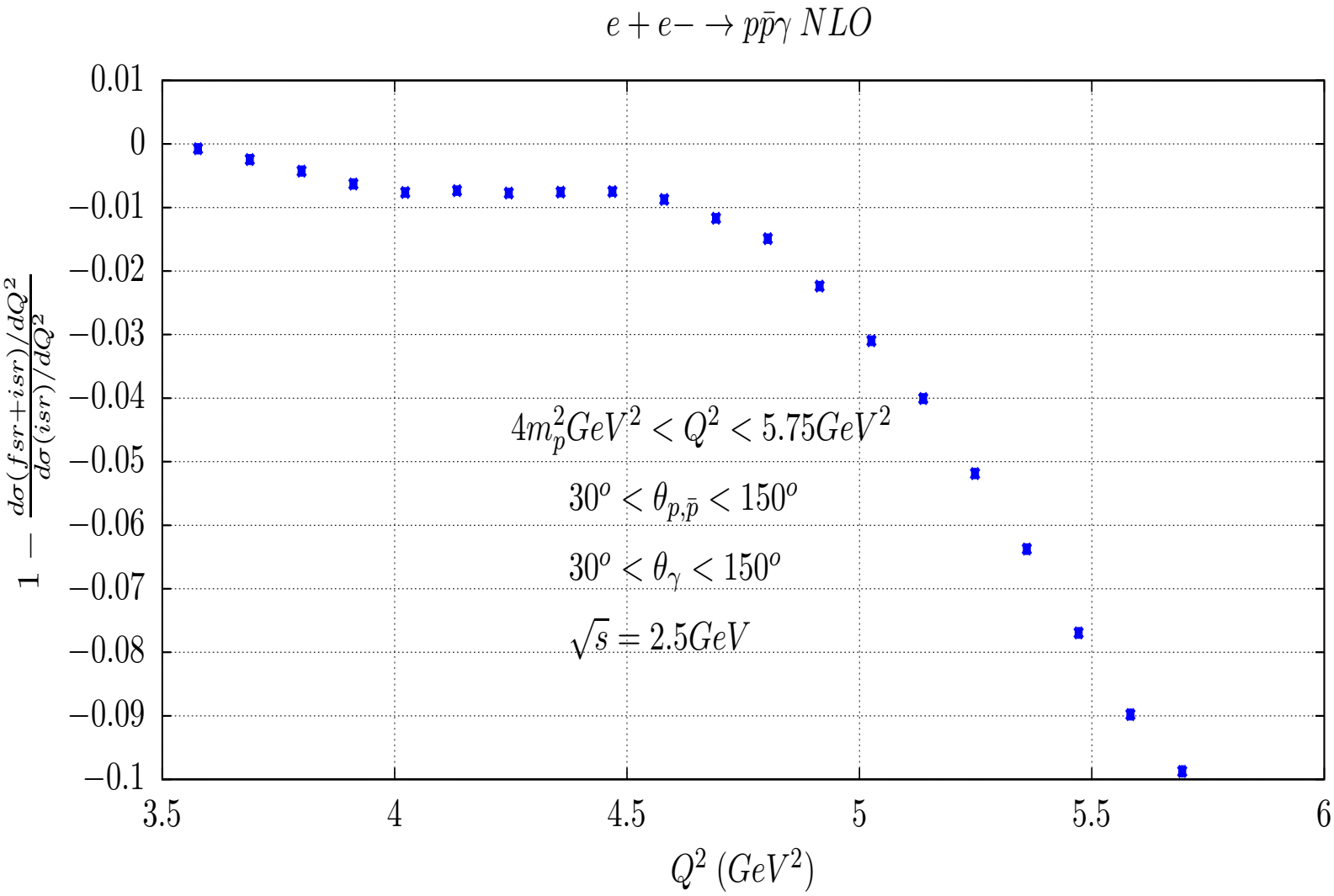
$$F(yy) = f(yy) - f(\pi\alpha) + \frac{3\alpha}{4\pi} + 1$$

$$f(yy) = \frac{yy}{1-\exp(-yy)}, \qquad yy = \pi\alpha/\beta$$

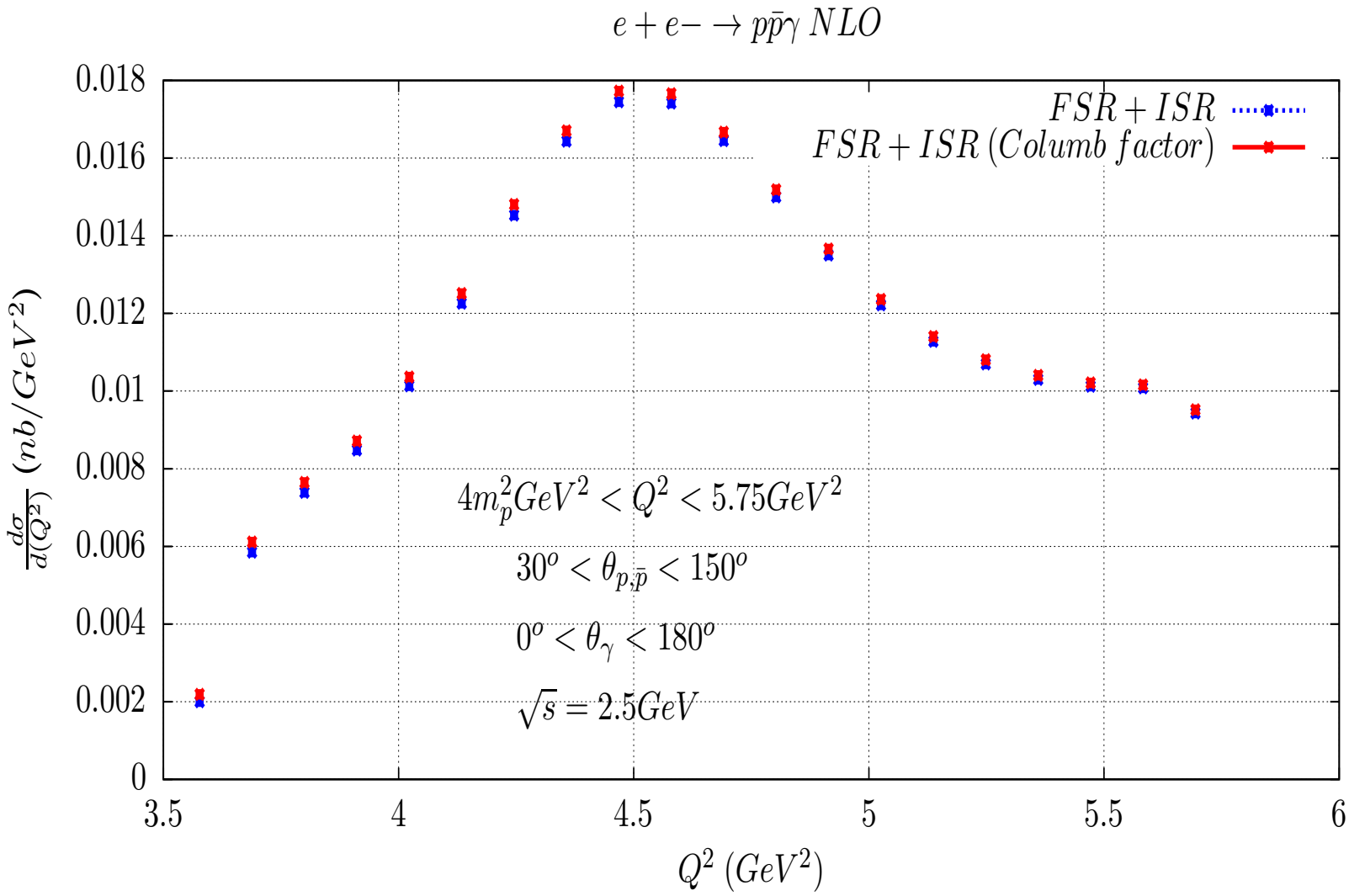
FSR


$e+e- \to p\bar{p}\gamma\ NLO$
$1 - \frac{d\sigma(fsr+isr)/dQ^2}{d\sigma(isr)/dQ^2}$
0.004
0.002
0
−0.002
−0.004
−0.006
−0.008
−0.01
3.5
4
4.5
5
5.5
6
$Q^2\ (GeV^2)$
$4m_p^2 GeV^2 < Q^2 < 5.75 GeV^2$
$30^o < \theta_{p,\bar{p}} < 150^o$
$0^o < \theta_\gamma < 180^o$
$\sqrt{s} = 2.5 GeV$

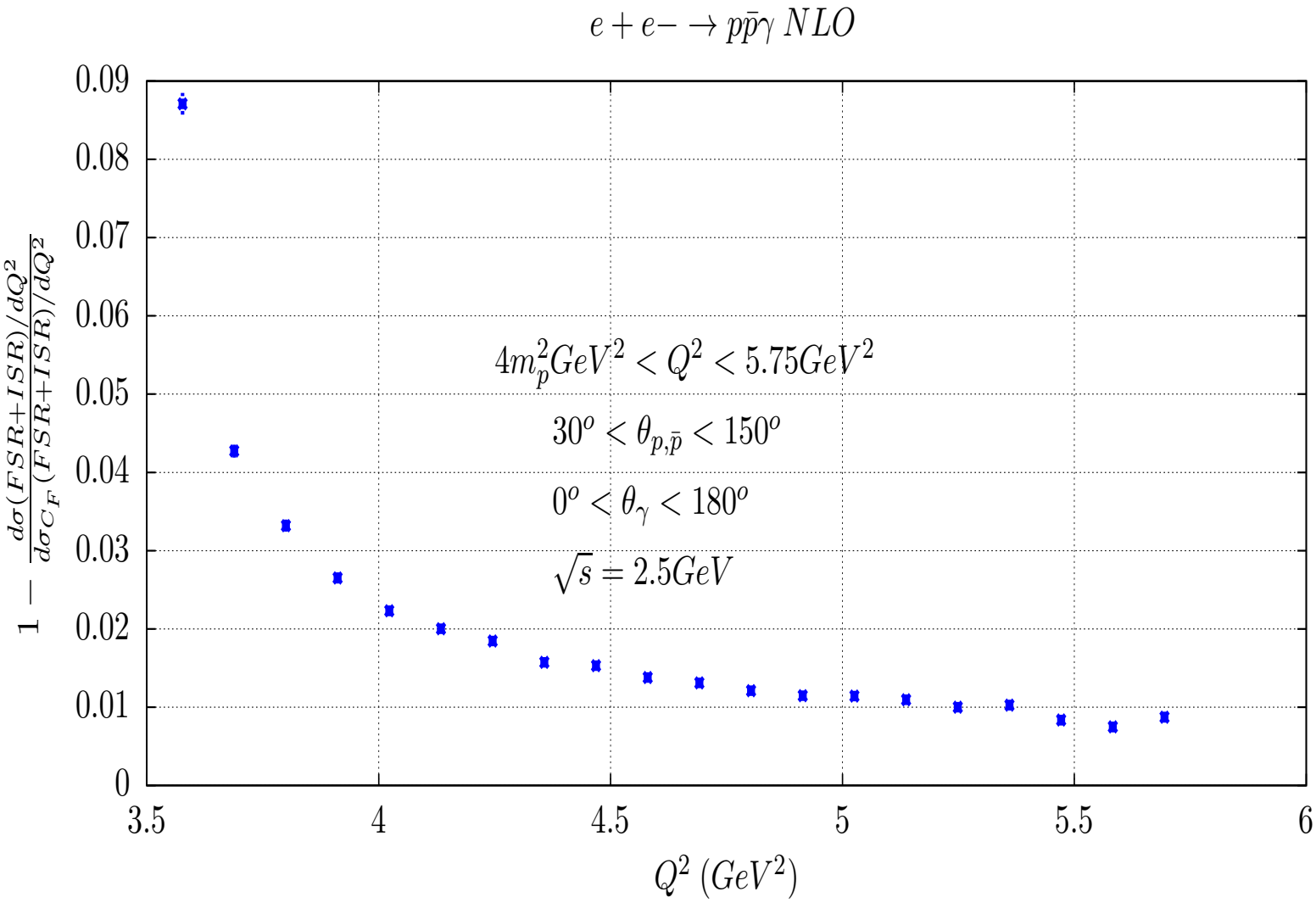
FSR


$e + e- \to p\bar{p}\gamma\ NLO$
$1 - \frac{d\sigma(fsr+isr)/dQ^2}{d\sigma(isr)/dQ^2}$
0.01
0
−0.01
−0.02
−0.03
−0.04
−0.05
−0.06
−0.07
−0.08
−0.09
−0.1
3.5
4
4.5
5
5.5
6
$Q^2\ (GeV^2)$
$4m_p^2 GeV^2 < Q^2 < 5.75 GeV^2$
$30^o < \theta_{p,\bar{p}} < 150^o$
$30^o < \theta_\gamma < 150^o$
$\sqrt{s} = 2.5 GeV$

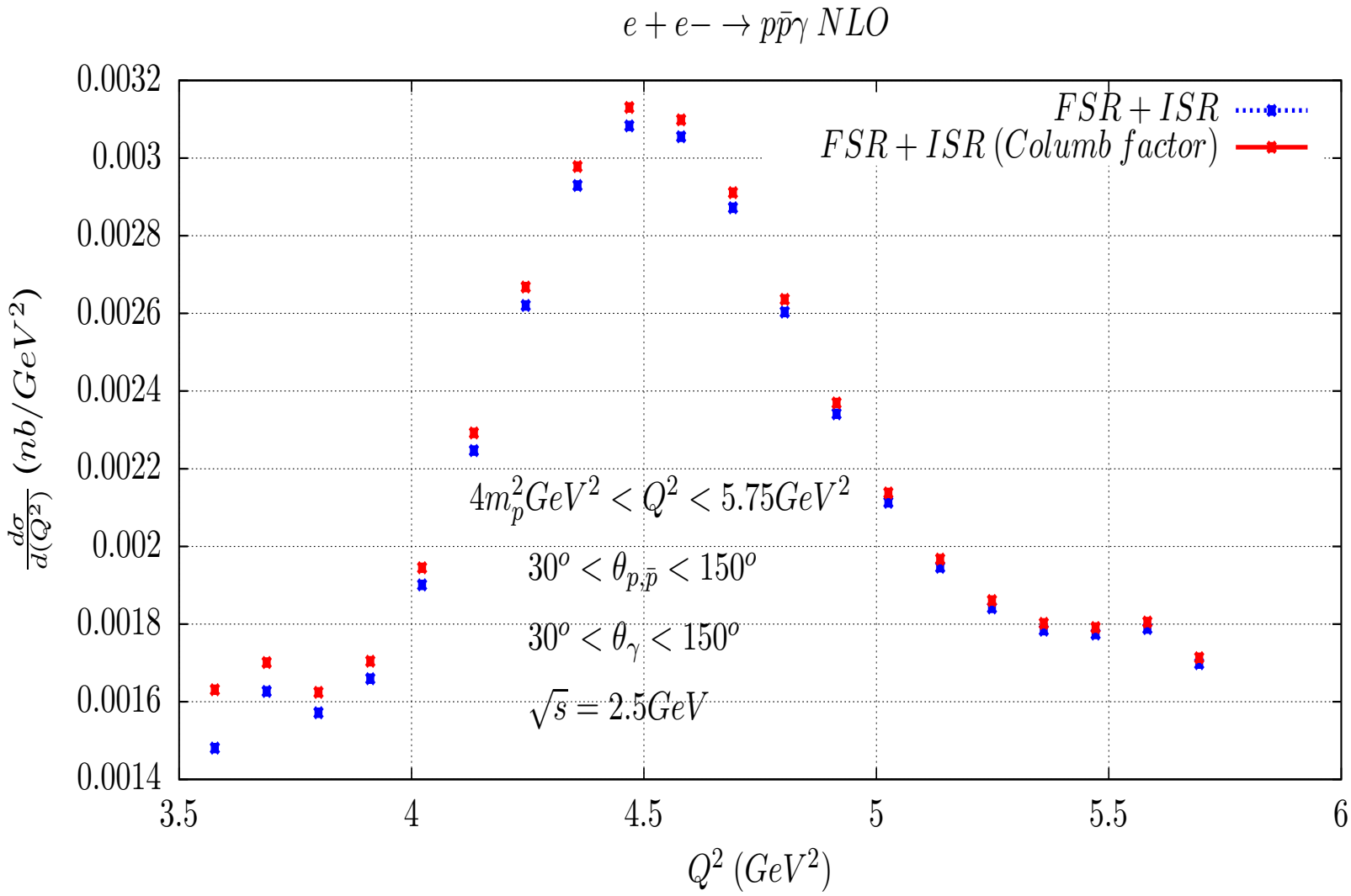
FSR


$e+e- \to p\bar{p}\gamma\ NLO$
$FSR+ISR$
$FSR+ISR\,(Columb\ factor)$
$4m_p^2 GeV^2 < Q^2 < 5.75 GeV^2$
$30^o < \theta_{p,\bar{p}} < 150^o$
$0^o < \theta_\gamma < 180^o$
$\sqrt{s} = 2.5 GeV$
$\frac{d\sigma}{d(Q^2)}\ (nb/GeV^2)$
$Q^2\ (GeV^2)$
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
3.5
4
4.5
5
5.5
6

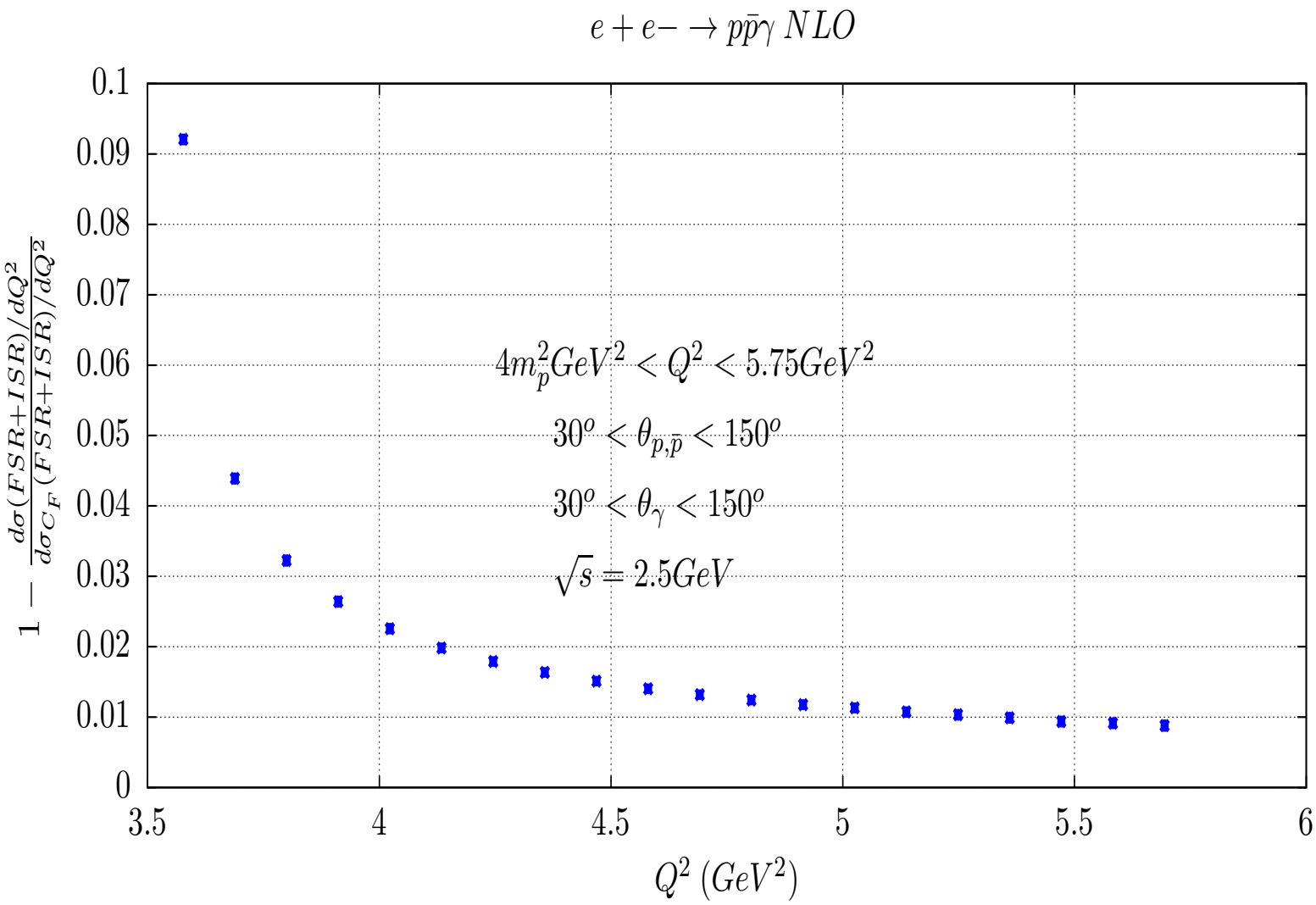
FSR


$e + e- \to p\bar{p}\gamma\ NLO$
$1 - \frac{d\sigma(FSR+ISR)/dQ^2}{d\sigma_{C_F}(FSR+ISR)/dQ^2}$
$4m_p^2 GeV^2 < Q^2 < 5.75 GeV^2$
$30^o < \theta_{p,\bar{p}} < 150^o$
$0^o < \theta_\gamma < 180^o$
$\sqrt{s} = 2.5 GeV$
$Q^2\ (GeV^2)$
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
3.5
4
4.5
5
5.5
6

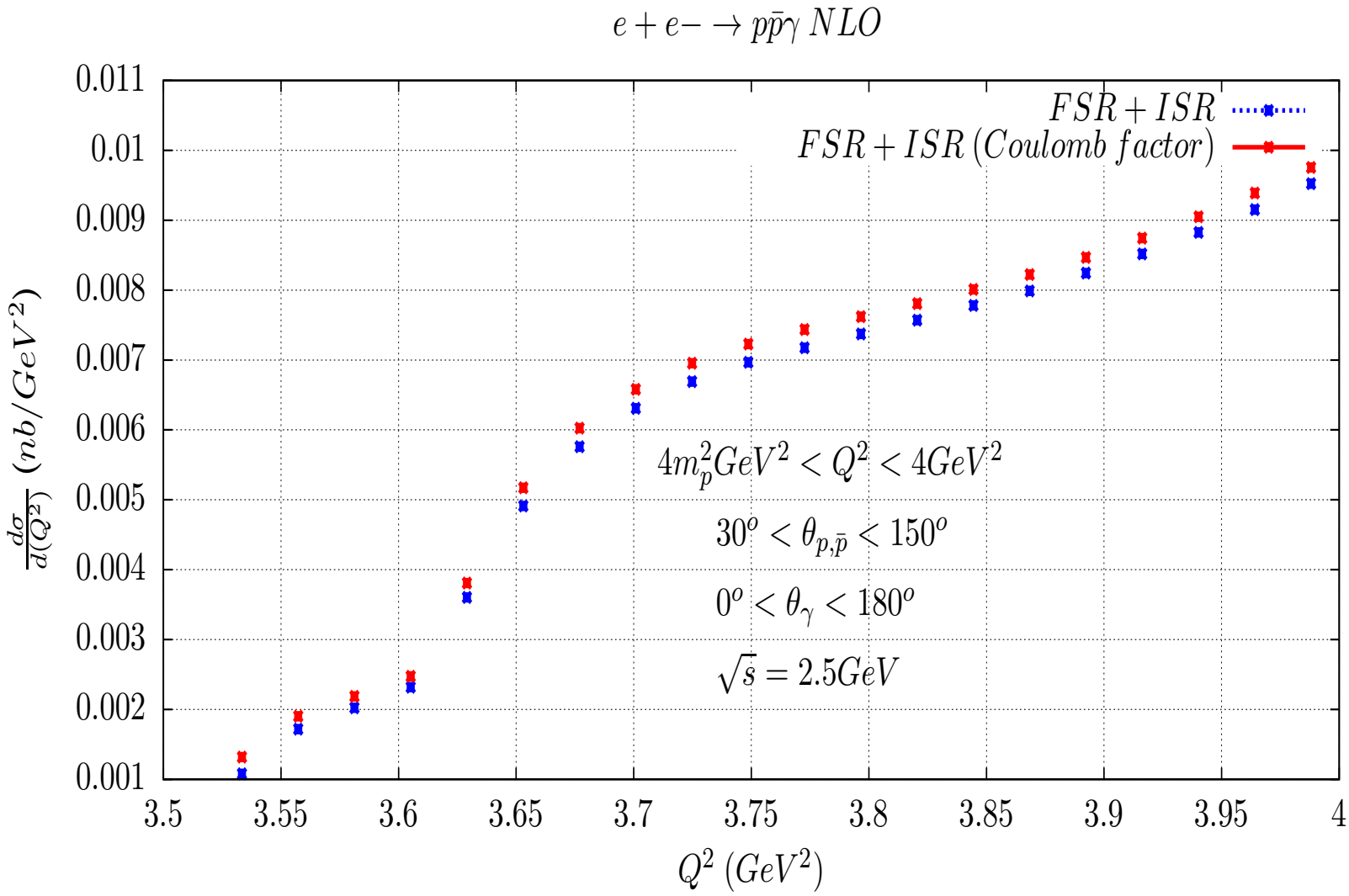
FSR


$e+e- \to p\bar{p}\gamma\ NLO$
$FSR+ISR$
$FSR+ISR\,(Columb\ factor)$
$4m_p^2 GeV^2 < Q^2 < 5.75 GeV^2$
$30^o < \theta_{p,\bar{p}} < 150^o$
$30^o < \theta_\gamma < 150^o$
$\sqrt{s} = 2.5 GeV$
$\frac{d\sigma}{d(Q^2)}\ (nb/GeV^2)$
$Q^2\ (GeV^2)$
0.0032
0.003
0.0028
0.0026
0.0024
0.0022
0.002
0.0018
0.0016
0.0014
3.5
4
4.5
5
5.5
6

FSR


$e + e- \to p\bar{p}\gamma\ NLO$
$1 - \frac{d\sigma(FSR+ISR)/dQ^2}{d\sigma_{C_F}(FSR+ISR)/dQ^2}$
$4m_p^2 GeV^2 < Q^2 < 5.75 GeV^2$
$30^o < \theta_{p,\bar{p}} < 150^o$
$30^o < \theta_\gamma < 150^o$
$\sqrt{s} = 2.5 GeV$
$Q^2\ (GeV^2)$
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
3.5
4
4.5
5
5.5
6

FSR


$e+e- \to p\bar{p}\gamma\ NLO$
$FSR+ISR$
$FSR+ISR\,(Coulomb\ factor)$
$4m_p^2 GeV^2 < Q^2 < 4GeV^2$
$30^o < \theta_{p,\bar{p}} < 150^o$
$0^o < \theta_\gamma < 180^o$
$\sqrt{s} = 2.5GeV$
$\frac{d\sigma}{d(Q^2)}\ (nb/GeV^2)$
$Q^2\ (GeV^2)$
0.011
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
3.5
3.55
3.6
3.65
3.7
3.75
3.8
3.85
3.9
3.95
4

Final remarks

$\Rightarrow$ New model of the nucleon FFs developed

$\Rightarrow$ FSRNLO corrections implemented in PHOKHARA for $e^+e^- \to \bar{p}p\gamma$

$\Rightarrow$ FSR radiative corrections important for precision measurements