Estimated calculation of the hadronic light-by-light contribution to the (g-2) of the muon

Pere Masjuan
Institut für Kernphysik
Johannes Gutenberg-Universität Mainz
Germany

Work done in collaboration with M.Vanderhaeghen
(hep-ph:1212.0357)

ECT*, Trento, 11th April 2013

Outline

- The anomalous magnetic moment of the muon
  - state-of-the-art: the 3σ deviation
- The Hadronic light-by-light contribution:
  - an estimated calculation
- Conclusions and Outlook

The anomalous magnetic moment of the muon

- gyromagnetic ratio: g

$$\vec{\mu} = g \frac{e}{2m} \cdot \vec{S}$$

spin $\frac{1}{2}$ $\rightarrow$ Dirac theory: $g = 2$

QFT: $g \neq 2$

- Deviation from the Dirac value $g = 2$ is:

$$a_\mu = \frac{g_\mu - 2}{2}$$

- BNL E821: 11659208.9±6.4 10^{-10} Bennet et al, PRD73,072003 (2006)

The anomalous magnetic moment of the muon

Anomalous magnetic moment a_μ (anomaly):

$$g_\mu = 2\left(1 + a_\mu = \frac{\alpha}{2\pi} + \cdots\right)$$

$$a_\mu^{th} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{had}$$

Contribution	Result in 10^{-10} units	
QED(leptons)	11658471.885 ± 0.004	Kinoshita *et al* 2012
HVP(leading order)	692.3 ± 4.2	Davier *et al* 2011
HVP(higher order)	-9.84 ± 0.07	Hagiwara *et al* 2009
HLBL	11.6 ± 4.0	Jegerlehner and Nyffeler 2009
EW	15.4 ± 0.2	Czarnecki *et al* 2003
Total	11659181.3 ± 5.8	

The anomalous magnetic moment of the muon

Anomalous magnetic moment a_μ (anomaly):

$$g_\mu = 2\left(1 + a_\mu = \frac{\alpha}{2\pi} + \cdots\right)$$

$$a_\mu^{th} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{had}$$

Contribution	Result in 10^{-10} units	
QED(leptons)	11658471.885 ± 0.004	Kinoshita *et al* 2012
HVP(leading order)	692.3 ± 4.2	Davier *et al* 2011
HVP(higher order)	-9.84 ± 0.07	Hagiwara *et al* 2009
HLBL	11.6 ± 4.0	Jegerlehner and Nyffeler 2009
EW	15.4 ± 0.2	Czarnecki *et al* 2003
Total	11659181.3 ± 5.8	

$$a_\mu^{exp} - a_\mu^{SM} = 27.6(8.0) \times 10^{-10} \Rightarrow 3.4\sigma$$

The anomalous magnetic moment of the muon

Anomalous magnetic moment a_μ (anomaly):

$$g_\mu = 2\left(1 + a_\mu = \frac{\alpha}{2\pi} + \cdots\right)$$

$$a_\mu^{th} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{had}$$

Contribution	Result in 10^{-10} units
QED(leptons)	11658471.885 ± 0.004
HVP(leading order)	692.3 ± 4.2
HVP(higher order)	-9.84 ± 0.07
HLBL	11.6 ± 4.0
EW	15.4 ± 0.2
Total	11659181.3 ± 5.8

New g-2 experiment at Fermilab with error

$$1.6 \times 10^{-10}$$

$$a_\mu^{exp} - a_\mu^{SM} = 27.6(8.0) \times 10^{-10} \Rightarrow 3.4\sigma$$

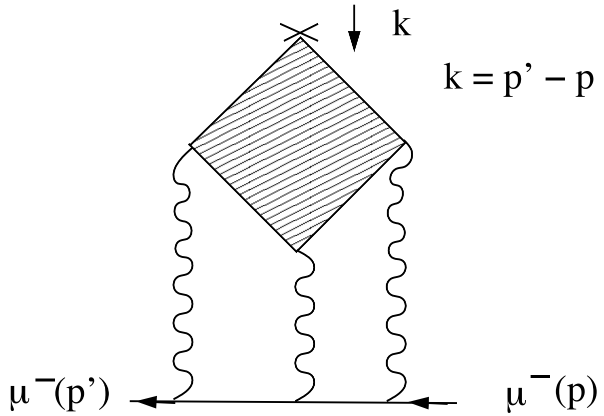
The anomalous magnetic moment of the muon

Anomalous magnetic moment a_μ (anomaly):

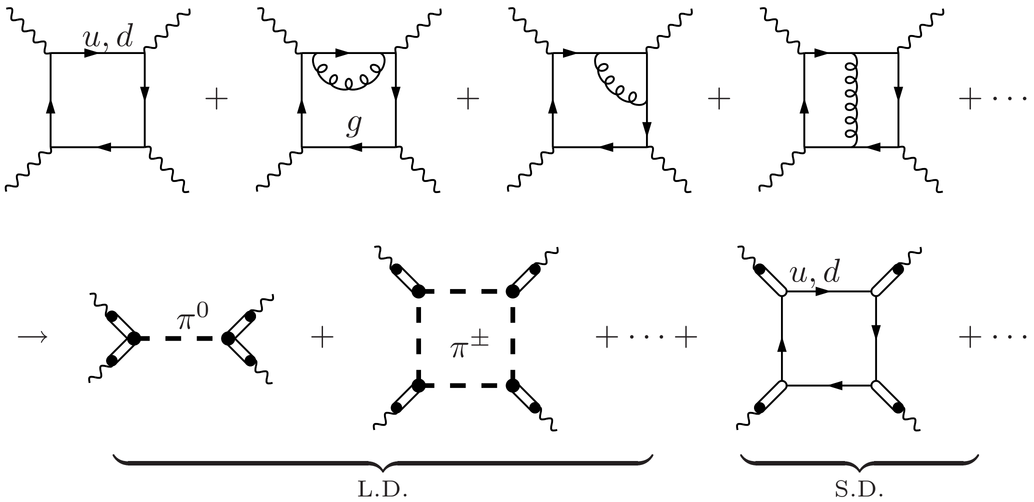
Contribution	Result in 10^{-10} units
QED(leptons)	11658471.885 ± 0.004
HVP(leading order)	692.3 ± 4.2
HVP(higher order)	-9.84 ± 0.07
HLBL	11.6 ± 4.0
EW	15.4 ± 0.2
Total	11659181.3 ± 5.8

$$a_\mu^{exp} - a_\mu^{SM} = 27.6(8.0) \times 10^{-10} \Rightarrow 3.4\sigma$$

Hadronic light-by-light scattering in the muon g-2



order $O(\alpha^3)$ hadronic contribution

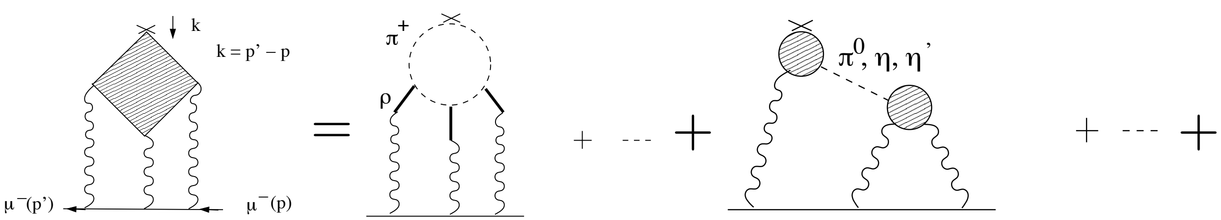


Model at low energies
(with exchange of resonances)

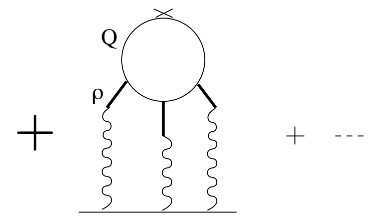
Model at high energies
(quark-loop)

Classification proposal by Eduardo de Rafael '94

Chiral Perturbation Theory counting (p²)+large-Nc counting

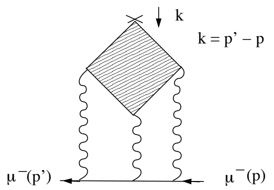


Exchange of other resonances $(f_0, a_1, \ldots)$

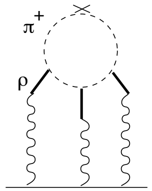


Chiral counting:	p^4	p^6	p^8	p^8
N_C-counting:	1	N_C	N_C	N_C

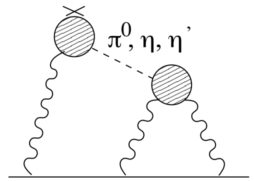
Pesudoscalars: numerically dominant contribution (according to most models)



=

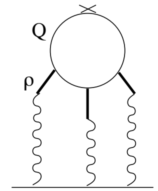


+ ··· +



+ ··· + Exchange of other resonances ($f_0, a_1, \ldots$)

+



+ ···

Chiral counting:	p^4	p^6	p^8	p^8
N_C-counting:	1	N_C	N_C	N_C

Contribution to $a_\mu \times 10^{11}$:

	Total				
BPP:	+83 (32)	-19 (13)	+85 (13)	-4 (3) [f_0, a_1]	+21 (3)
HKS:	+90 (15)	-5 (8)	+83 (6)	+1.7 (1.7) [a_1]	+10 (11)
KN:	+80 (40)		+83 (12)		
MV:	+136 (25)	0 (10)	+114 (10)	+22 (5) [a_1]	0
2007:	+110 (40)				
PdRV:	+105 (26)	-19 (19)	+114 (13)	+8 (12) [f_0, a_1]	+2.3 [c-quark]
N,JN:	+116 (40)	-19 (13)	+99 (16)	+15 (7) [f_0, a_1]	+21 (3)
GFW:	+217 (91)		+81 (12)		+136 (59)
GdR:	+150 (3)		+68 (3)		+82 (6)
		ud.: -45	ud.: $+\infty$		ud.: +60

ud. = undressed, i.e. point vertices without form factors

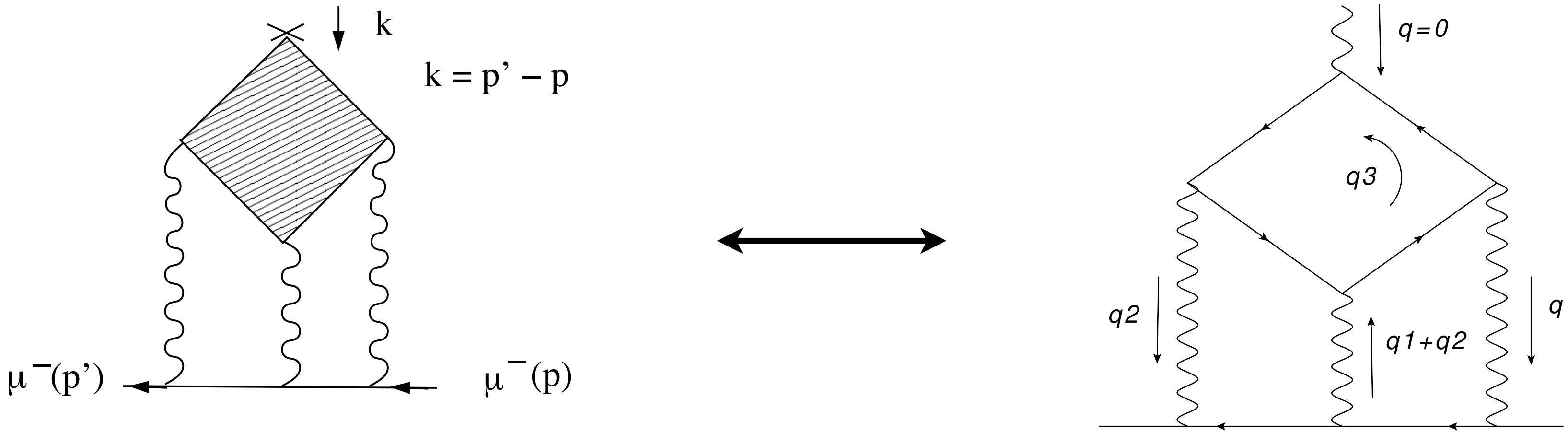
BPP = Bijnens, Pallante, Prades '96, '02; HKS = Hayakawa, Kinoshita, Sanda '96, '98, '02; KN = Knecht, Nyffeler '02; MV = Melnikov, Vainshtein '04; 2007 = Bijnens, Prades; Miller, de Rafael, Roberts; PdRV = Prades, de Rafael, Vainshtein '09; N,JN = Nyffeler '09; Jegerlehner, Nyffeler '09; GFW = Goecke, Fischer, Williams '11 (total includes estimate of "other contributions" = 0 (20)); GdR = Greynat, de Rafael '12 (given error only reflects variation $M_Q = 240 \pm 10$ MeV, estimated 20%-30% systematic error)

Recall (in units of 10^{-11}): δa_μ(had. VP) ≈ 45; δa_μ(exp [BNL]) $= 63$; δa_μ(future exp) $= 15$

Ballpark prediction for HLBL

P.M and M. Vanderhaeghen 2012

Duality argument between the hadronic degrees of freedom and the well-known quark loop contribution



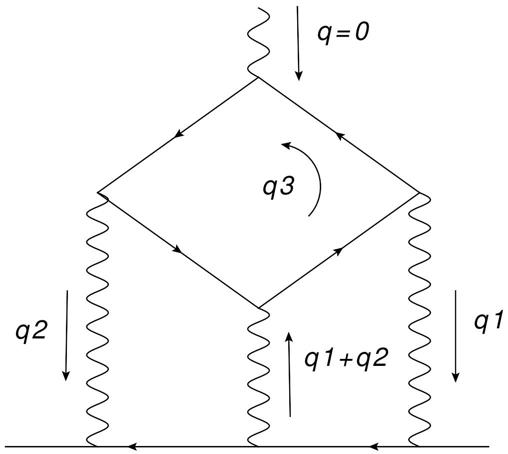
$$a_\mu^{HLBL}(M(Q)) = \left(\frac{\alpha}{\pi}\right)^3 N_c \left(\sum_{q=u,d,s} Q_q^4\right)\left[\left(\frac{3}{2}\zeta(3) - \frac{19}{16}\right)\frac{m_\mu^2}{M(Q)^2} + \mathcal{O}\left(\frac{m_\mu^4}{M(Q)^4}\log^2\frac{m_\mu^2}{M(Q)^2}\right)\right]$$

Laporta and Remiddi 1996

Ballpark prediction for HLBL

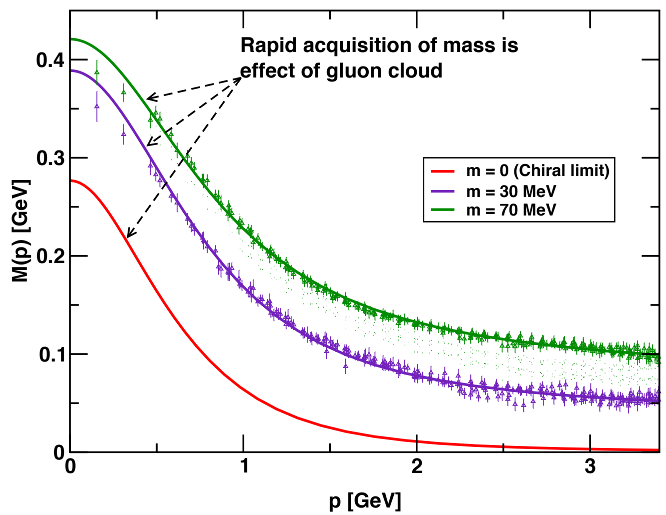
P.M and M.Vanderhaeghen 2012

Duality argument between the hadronic degrees of freedom and the well-known quark loop contribution



↔

For the duality estimate:
- quark: running momentum dependent mass
- use lattice calculation
(extrapolated at chiral limit using
 Dyson-Schwinger equation framework)

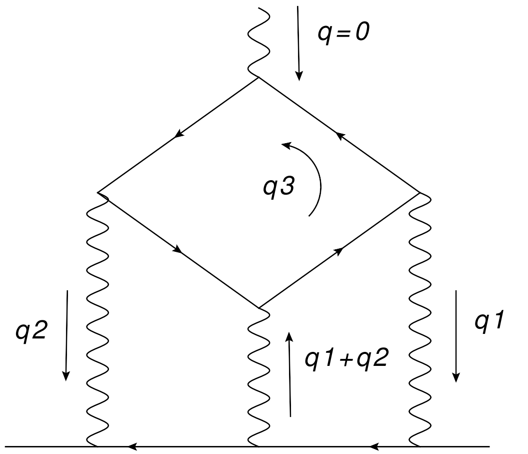


Bhagwat *et al* 2007

Ballpark prediction for HLBL

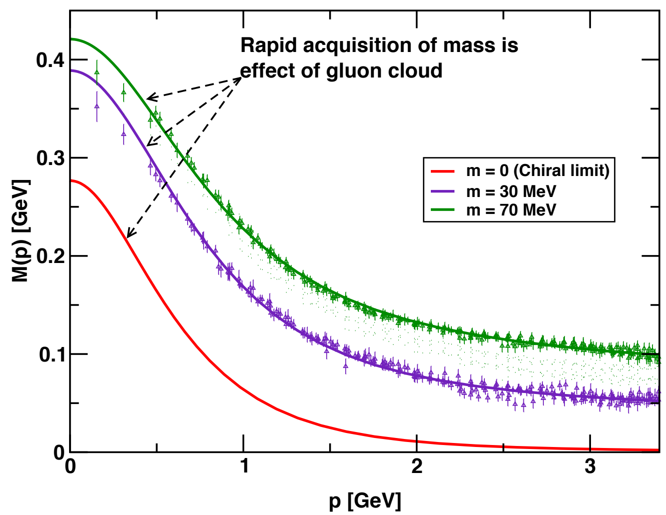
P.M and M.Vanderhaeghen 2012

Duality argument between the hadronic degrees of freedom and the well-known quark loop contribution



For the duality estimate:
- quark: running momentum dependent mass
- use lattice calculation
(extrapolated at chiral limit using
Dyson-Schwinger equation framework)

what momentum?

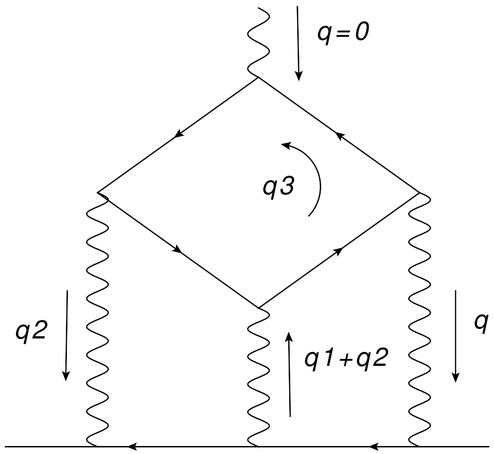


Bhagwat *et al* 2007

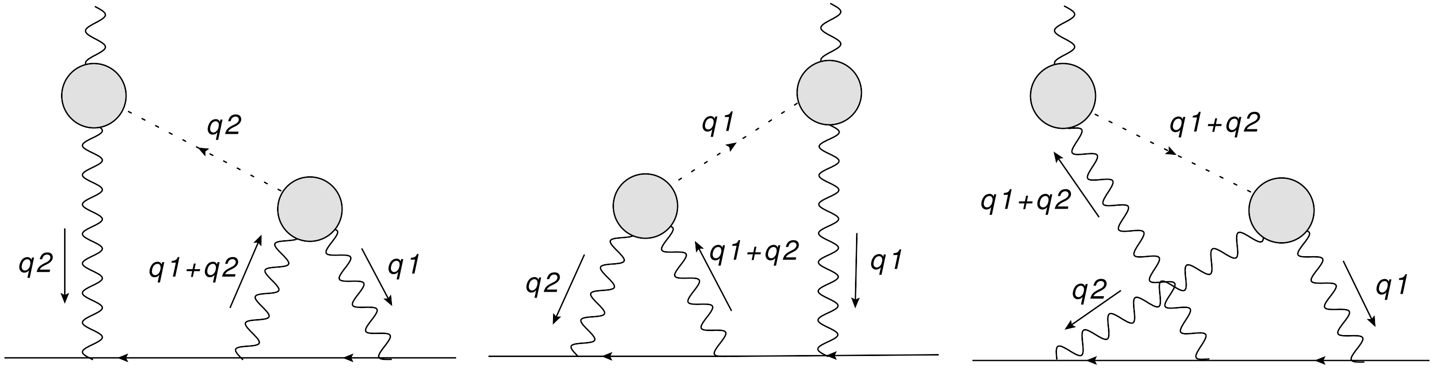
Ballpark prediction for HLBL

P.M and M. Vanderhaeghen 2012

Duality argument between the hadronic degrees of freedom and the well-known quark loop contribution



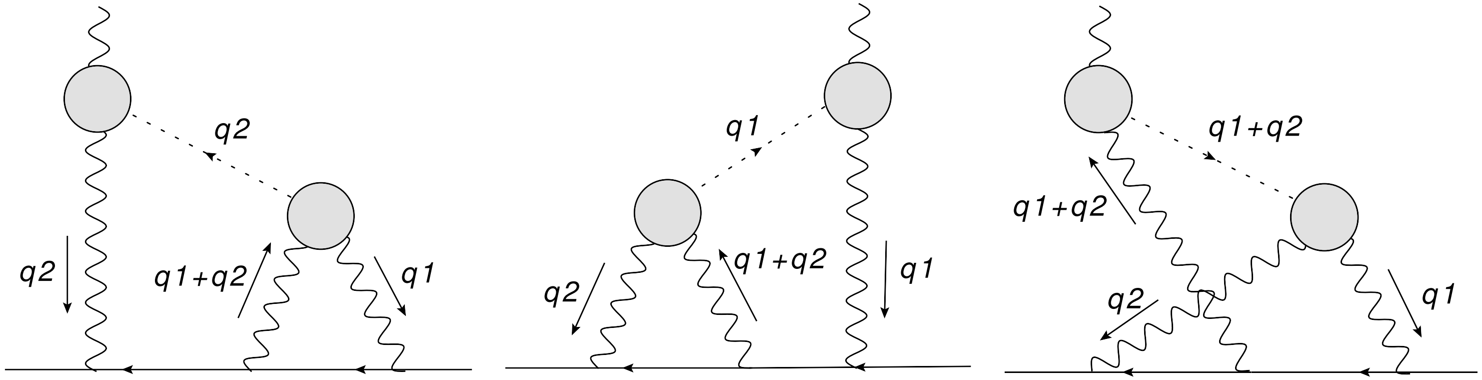
For the duality estimate:
- use ChPT and large-Nc counting [de Rafael '94]
- dominant piece: PS exchange (suggested by most models)



Obtain average momenta $M(Q_1)$ and $M(Q_2)$

Dissection of the HLBL contribution

(using the hyperspherical approach developed in [Knecht and Nyffeler '01])



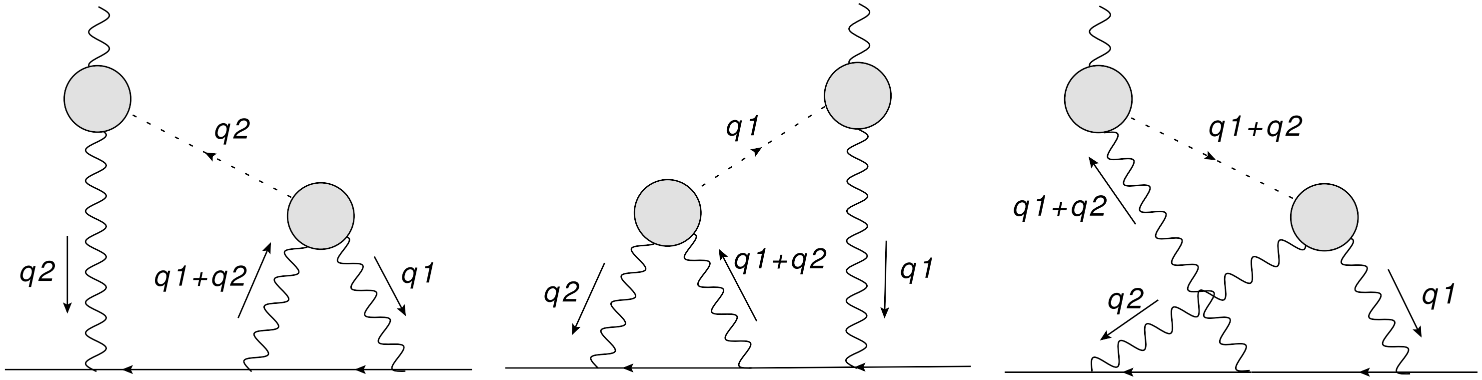
$$a_\mu^{LbyL;\pi^0} = -e^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (q_1+q_2)^2 [(p+q_1)^2 - m^2][(p-q_2)^2 - m^2]}$$

$$\times \left(\frac{F_{\pi^0\gamma^*\gamma^*}(q_1^2, (q_1+q_2)^2) F_{\pi^0\gamma^*\gamma^*}(q_2^2, 0)}{q_2^2 - M_\pi^2} T_1(q_1, q_2; p) \right.$$

$$\left. + \frac{F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) F_{\pi^0\gamma^*\gamma^*}((q_1+q_2)^2, 0)}{(q_1+q_2)^2 - M_\pi^2} T_2(q_1, q_2; p) \right)$$

Dissection of the HLBL contribution

(using the hyperspherical approach developed in [Knecht and Nyffeler '01])



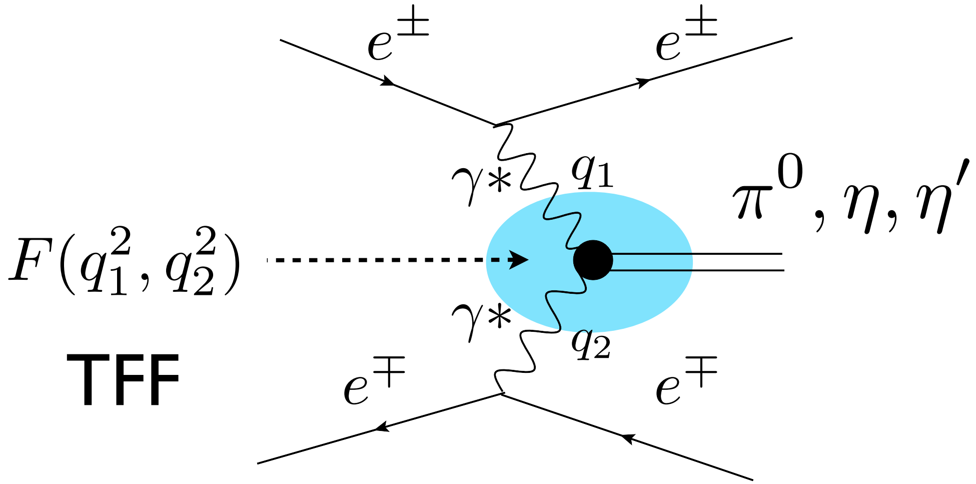
$$a_\mu^{LbyL;\pi^0} = -e^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (q_1+q_2)^2 [(p+q_1)^2 - m^2][(p-q_2)^2 - m^2]}$$

$$\times \left(\frac{F_{\pi^0\gamma^*\gamma^*}(q_1^2, (q_1+q_2)^2) F_{\pi^0\gamma^*\gamma^*}(q_2^2, 0)}{q_2^2 - M_\pi^2} T_1(q_1, q_2; p) \right.$$

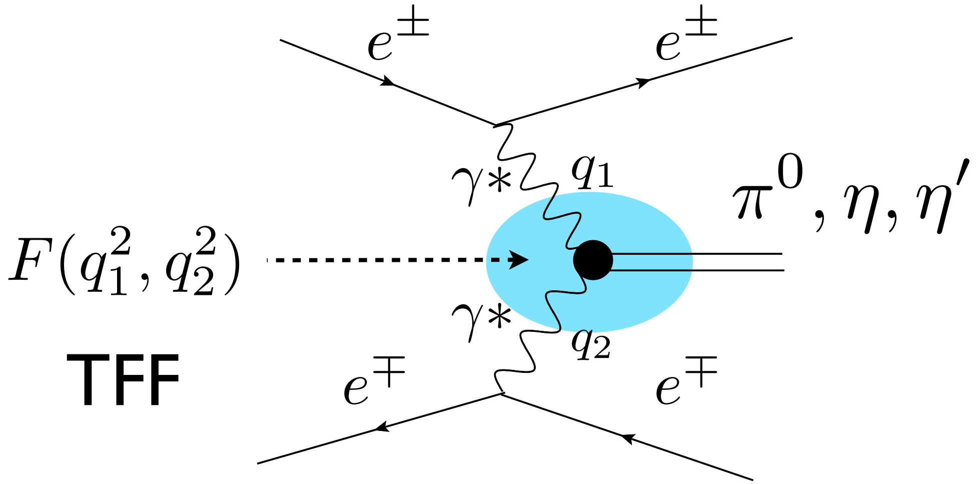
Use data from the π-Transition Form Factor

$$\left. + \frac{F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) F_{\pi^0\gamma^*\gamma^*}((q_1+q_2)^2, 0)}{(q_1+q_2)^2 - M_\pi^2} T_2(q_1, q_2; p) \right)$$

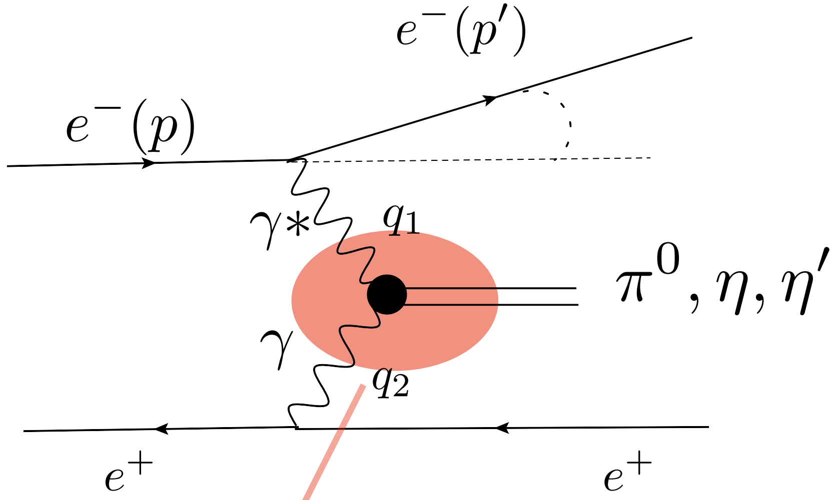
Dissection of the HLBL contribution

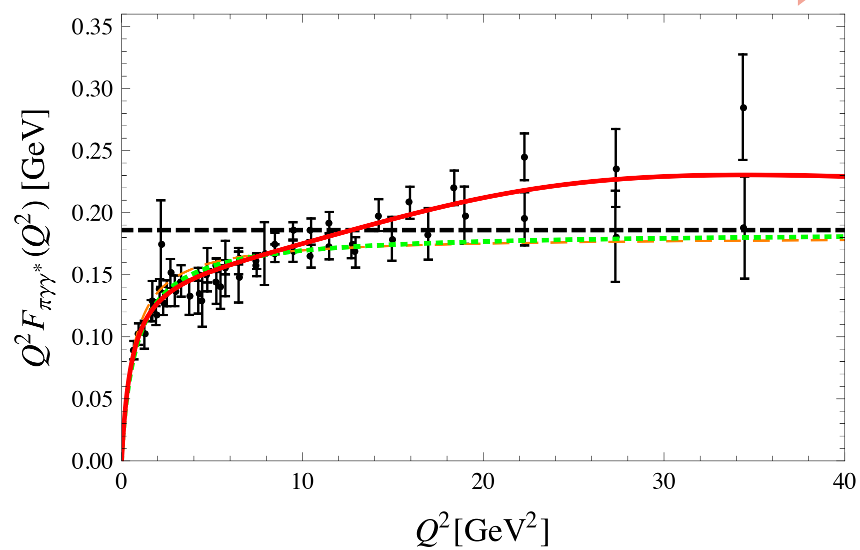


Dissection of the HLBL contribution



TFF

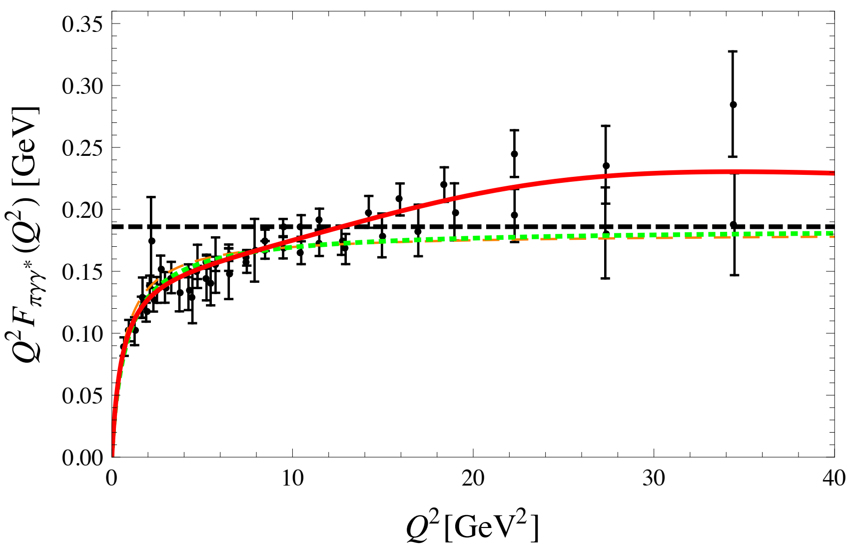




P.M. PRD 86, 094021, 2012

Dissection of the HLBL contribution

- Extraction of meson TFF (example π^0)
  - Using CLEO, CELLO, BaBar and Belle to obtain the TFF Low-energy Constants, constrain hadronic model and estimation of π^0-HLBL



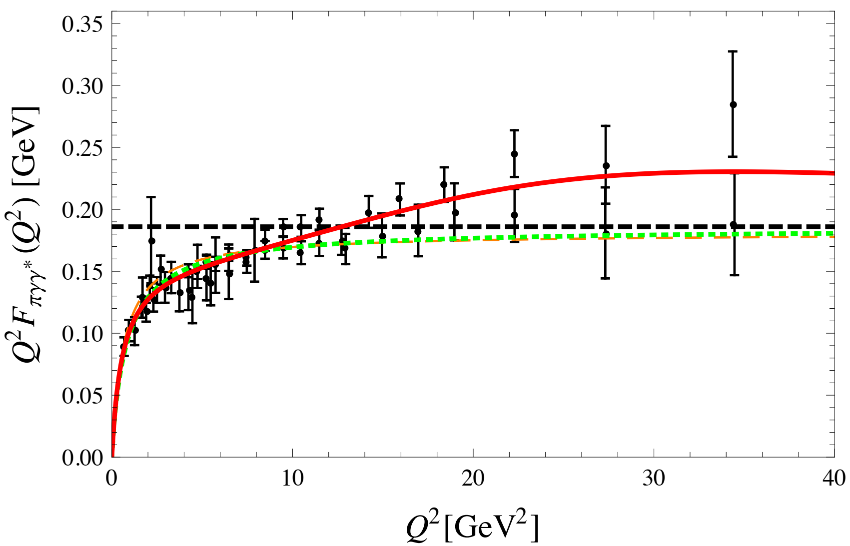
$$F_{\pi^0\gamma*\gamma}(Q^2,0) = a_0\left(1 + a_\pi \frac{Q^2}{m_\pi^2} + b_\pi \frac{Q^4}{m_\pi^4} + ...\right)$$

Using Padé Approximants
(*a la* BL'80, with correct fall-off at high Q^2)
P.M, PRD86,094021,2012

$$P_1^0(Q^2) = \frac{a_0}{1 + a_\pi \frac{Q^2}{m_\pi^2}}$$

Dissection of the HLBL contribution

- Extraction of meson TFF (example π^0)
  - Using CLEO, CELLO, BaBar and Belle to obtain the TFF Low-energy Constants, constrain hadronic model and estimation of π^0-HLBL



$$F_{\pi^0\gamma*\gamma}(Q^2,0) = a_0(1 + a_\pi \frac{Q^2}{m_\pi^2} + b_\pi \frac{Q^4}{m_\pi^4} + \ldots)$$

Using Padé Approximants
(*a la* BL'80, with correct fall-off at high Q^2)

P.M, PRD86,094021,2012

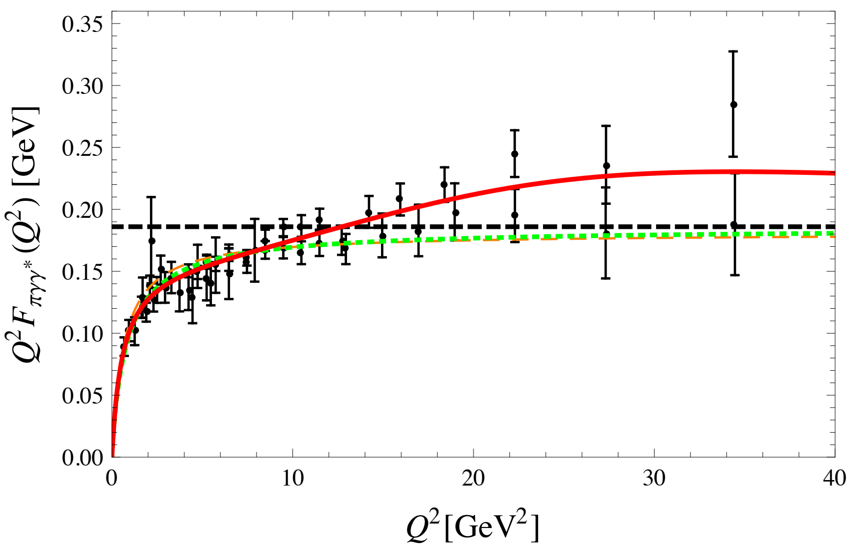
$$P_1^0(Q^2) = \frac{a_0}{1 + a_\pi \frac{Q^2}{m_\pi^2}}$$

$$P_1^0(Q^2), P_2^1(Q^2), P_3^2(Q^2), \ldots$$

by reexpansion we find $a_\pi,\ b_\pi$

Dissection of the HLBL contribution

- Extraction of meson TFF (example π^0)
  - Using CLEO, CELLO, BaBar and Belle to obtain the TFF Low-energy Constants, constrain hadronic model and estimation of π^0-HLBL



$$F_{\pi^0\gamma*\gamma}(Q^2,0) = a_0(1 + a_\pi \frac{Q^2}{m_\pi^2} + b_\pi \frac{Q^4}{m_\pi^4} + \ldots)$$

Using Padé Approximants
(*a la* BL'80, with correct fall-off at high Q^2)
P.M, PRD86,094021,2012

$$P_1^0(Q^2), P_2^1(Q^2), P_3^2(Q^2), \ldots$$

by reexpansion we find $a_\pi,\ b_\pi$

$$a_\pi = 0.0324(12)_{stat}(19)_{sys}$$
$$b_\pi = 1.06(9)_{stat}(25)_{sys}$$

to be compared with:

- ChPT: $a_\pi = 0.036$ [Bijnens,Bramon,Cornet'90]
- Dalitz decay: $a_\pi = 0.029(5)$ [Kampf,Knecht,Novotny '06]
- Regge theory: $a_\pi = 0.032(1)$ [Arriola,Broniowski '10]
- Ads/QCD: $a_\pi = 0.031$ [Grigoryan,Radyushkin '08]

Dissection of the HLBL contribution

- Extraction of meson TFF (example π^0)
  - Using CLEO, CELLO, BaBar and Belle to obtain the TFF Low-energy Constants, constrain hadronic model and estimation of π^0-HLBL

$$F_{\pi^0\gamma*\gamma}(Q^2,0) = a_0(1 + a_\pi \frac{Q^2}{m_\pi^2} + b_\pi \frac{Q^4}{m_\pi^4} + \ldots)$$

$$a_\pi = 0.0324(12)_{stat}(19)_{sys}$$
$$b_\pi = 1.06(9)_{stat}(25)_{sys}$$

+

$\Gamma_{\pi^0\gamma\gamma} \sim F_{\pi^0\gamma\gamma}(0,0)$ and M_ρ

$$F^{P01}_{\pi\gamma^*\gamma^*}(Q_1^2,Q_2^2) = P_1^0(Q_1^2,Q_2^2) = a\frac{b}{Q_1^2+b}\frac{b}{Q_2^2+b}$$

$$F^{P12}_{\pi\gamma^*\gamma^*}(Q_1^2,Q_2^2) = P_2^1(Q_1^2,Q_2^2) = \frac{a+bQ_1^2}{(Q_1^2+c)(Q_1^2+d)}\frac{a+bQ_2^2}{(Q_2^2+c)(Q_2^2+d)}$$

Dissection of the HLBL contribution

- Extraction of meson TFF and HLBL
  - Using CLEO, CELLO, BaBar and Belle to obtain the TFF Low-energy Constants, constrain hadronic model and estimation of π^0-HLBL

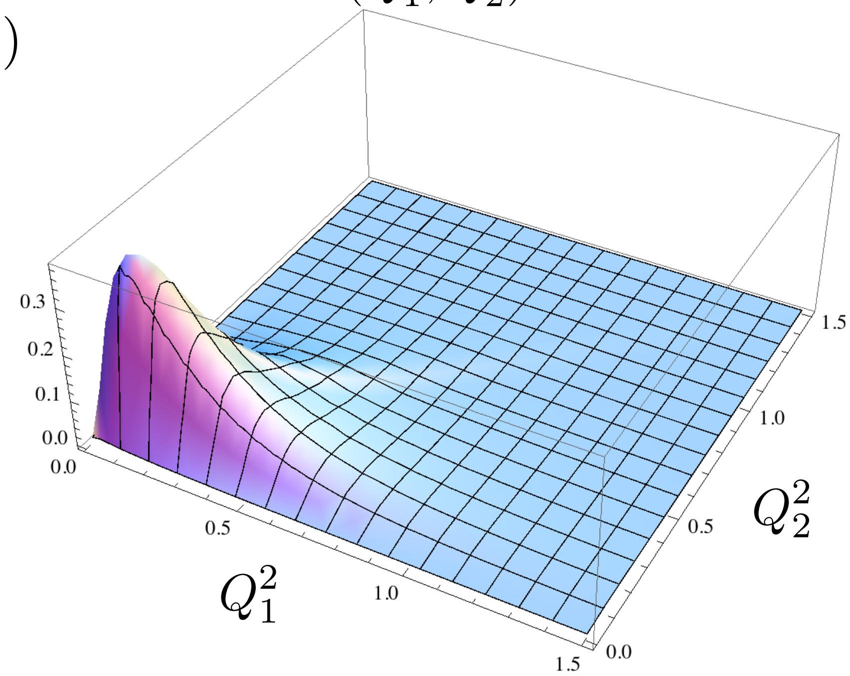
$$a_\mu^{HLBL;\pi^0} = e^6 \int \frac{d^4Q_1}{(2\pi)^4} \int \frac{d^4Q_2}{(2\pi)^4} K(Q_1^2, Q_2^2)$$

Using $F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2) \sim P_1^0(Q_1^2, Q_2^2)$

(main energy range from 0 to 1 GeV^2)

(using the hyperspherical approach developed in [Knecht and Nyffeler '01])

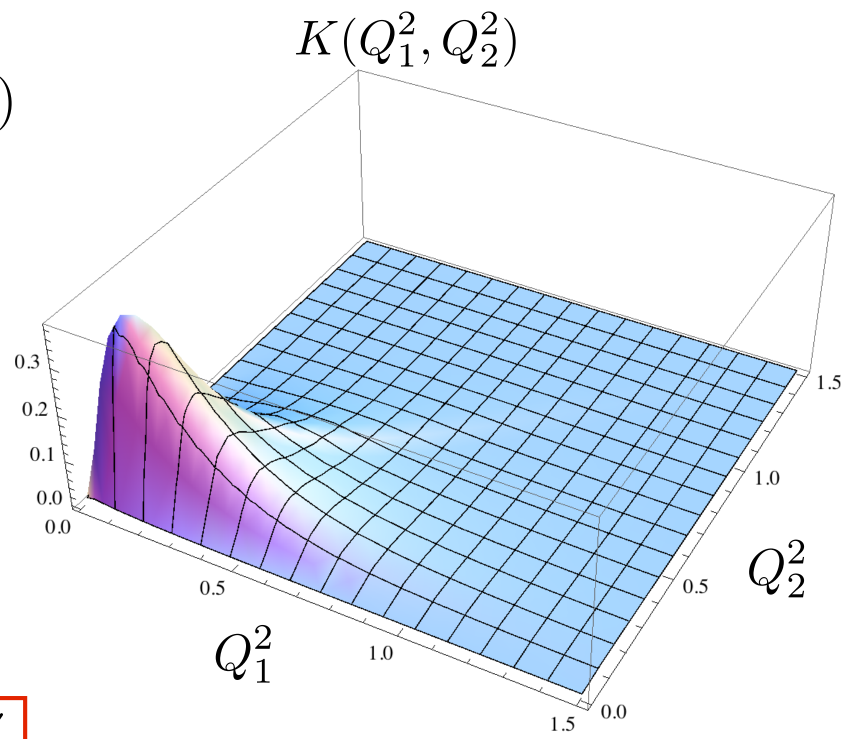
$K(Q_1^2, Q_2^2)$



Dissection of the HLBL contribution

- Extraction of meson TFF and HLBL
  - Using CLEO, CELLO, BaBar and Belle to obtain the TFF Low-energy Constants, constrain hadronic model and estimation of π^0-HLBL

$$a_\mu^{HLBL;\pi^0} = e^6 \int \frac{d^4Q_1}{(2\pi)^4} \int \frac{d^4Q_2}{(2\pi)^4} K(Q_1^2, Q_2^2)$$



Using $F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2) \sim P_1^0(Q_1^2, Q_2^2)$

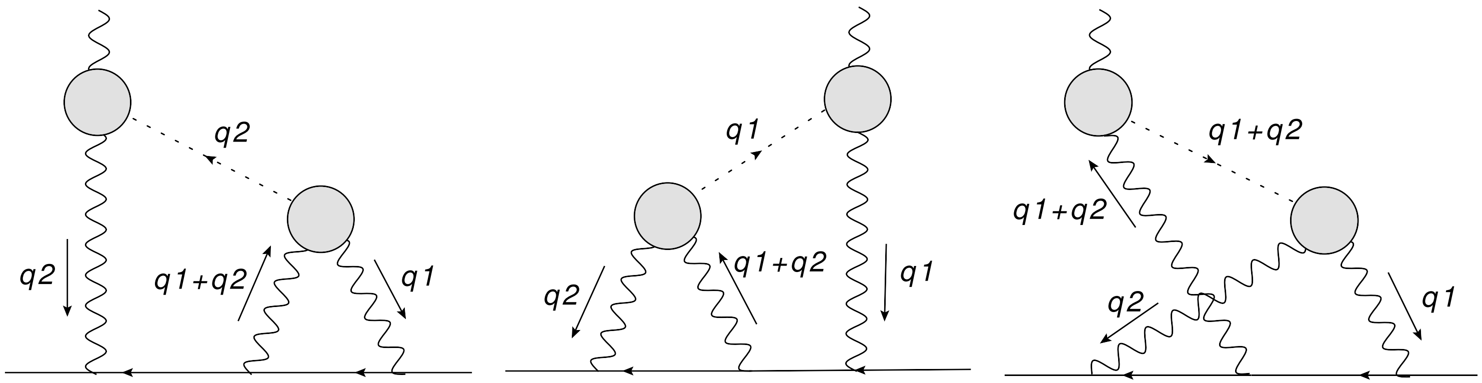
(main energy range from 0 to 1 GeV^2)

Systematic error from approach:

$P_1^0(Q_1^2, Q_2^2)$ vs $P_2^1(Q_1^2, Q_2^2)$ → 5%

(convergence guaranteed by Pomerenke's theorem) [P.M.,S.Peris,'07]

Dissection of the HLBL contribution



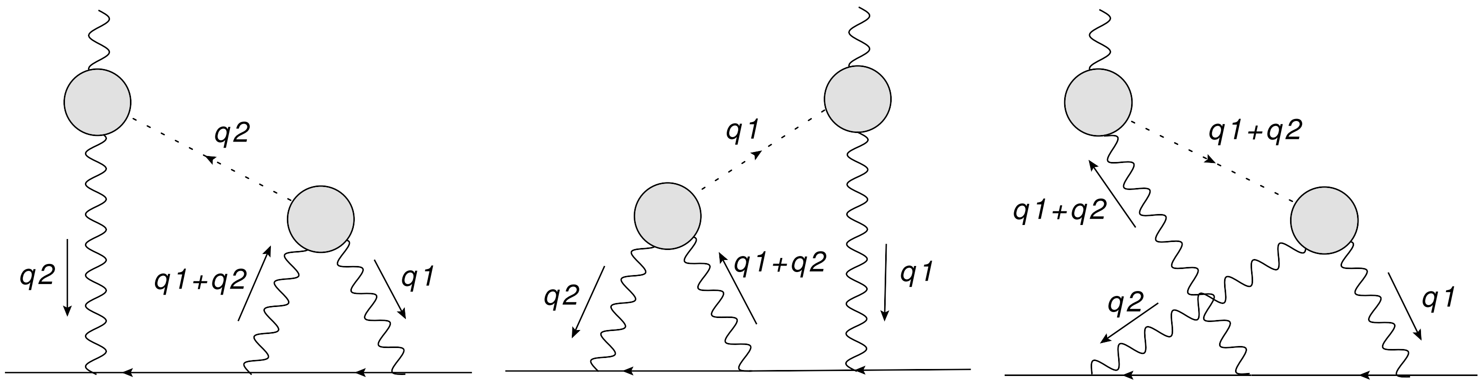
$$a_\mu^{LbyL;\pi^0} = -e^6 \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (q_1+q_2)^2[(p+q_1)^2-m^2][(p-q_2)^2-m^2]}$$

$$\times \left(\frac{F_{\pi^0\gamma^*\gamma^*}(q_1^2,(q_1+q_2)^2)F_{\pi^0\gamma^*\gamma^*}(q_2^2,0)}{q_2^2-M_\pi^2} T_1(q_1,q_2;p) \right.$$

Use data from
the pion Transition Form Factor

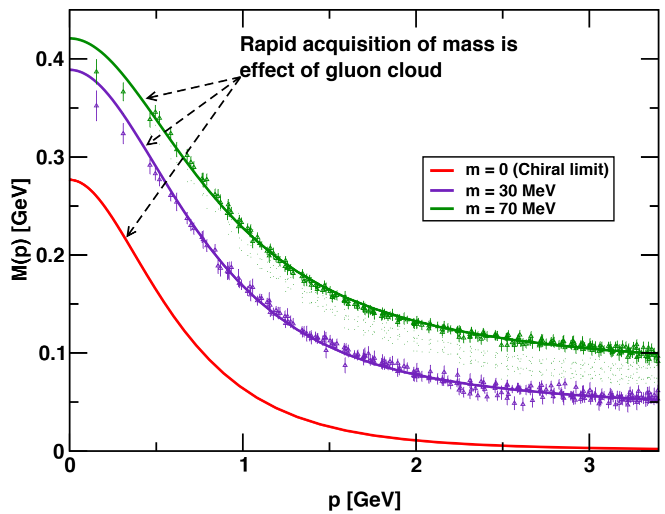
$$\left. + \frac{F_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2)F_{\pi^0\gamma^*\gamma^*}((q_1+q_2)^2,0)}{(q_1+q_2)^2-M_\pi^2} T_2(q_1,q_2;p) \right)$$

Dissection of the HLBL contribution



Obtain average momenta, thus M(Q$_1$) and M(Q$_2$)

		$\langle Q_i \rangle$	$M(Q_i)$GeV
$P_1^0(Q_1^2, Q_2^2)$	$\frac{Q_1+Q_2}{2}$	0.358(8)GeV	0.205(2)
	$\sqrt{Q_1 Q_2}$	0.323(7)GeV	0.216(2)
$P_2^1(Q_1^2, Q_2^2)$	$\frac{Q_1+Q_2}{2}$	0.358(11)GeV	0.205(3)
	$\sqrt{Q_1 Q_2}$	0.323(9)GeV	0.216(3)

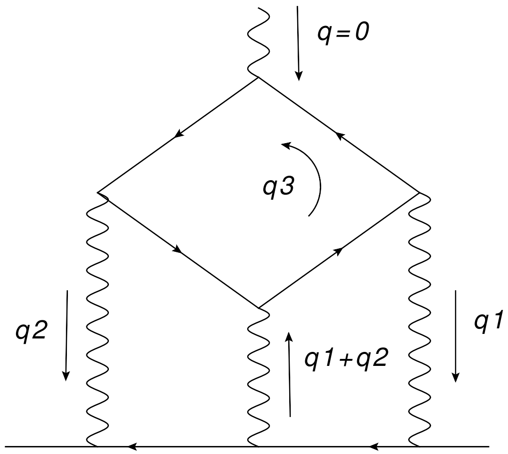


Bhagwat *et al* 2007

Ballpark prediction for HLBL

Masjuan and Vanderhaeghen 2012

Duality argument between the hadronic degrees of freedom and the well-known quark loop contribution



For the duality estimate:
- Using average momenta $M(Q_1)$ and $M(Q_2)$
- Formulae for spin 1/2 fermions

$$a_\mu^{HLBL}(M(Q)) = \left(\frac{\alpha}{\pi}\right)^3 N_c \left(\sum_{q=u,d,s} Q_q^4\right)\left[\left(\frac{3}{2}\zeta(3) - \frac{19}{16}\right)\frac{m_\mu^2}{M(Q)^2} + \mathcal{O}\left(\frac{m_\mu^4}{M(Q)^4}\log^2\frac{m_\mu^2}{M(Q)^2}\right)\right]$$

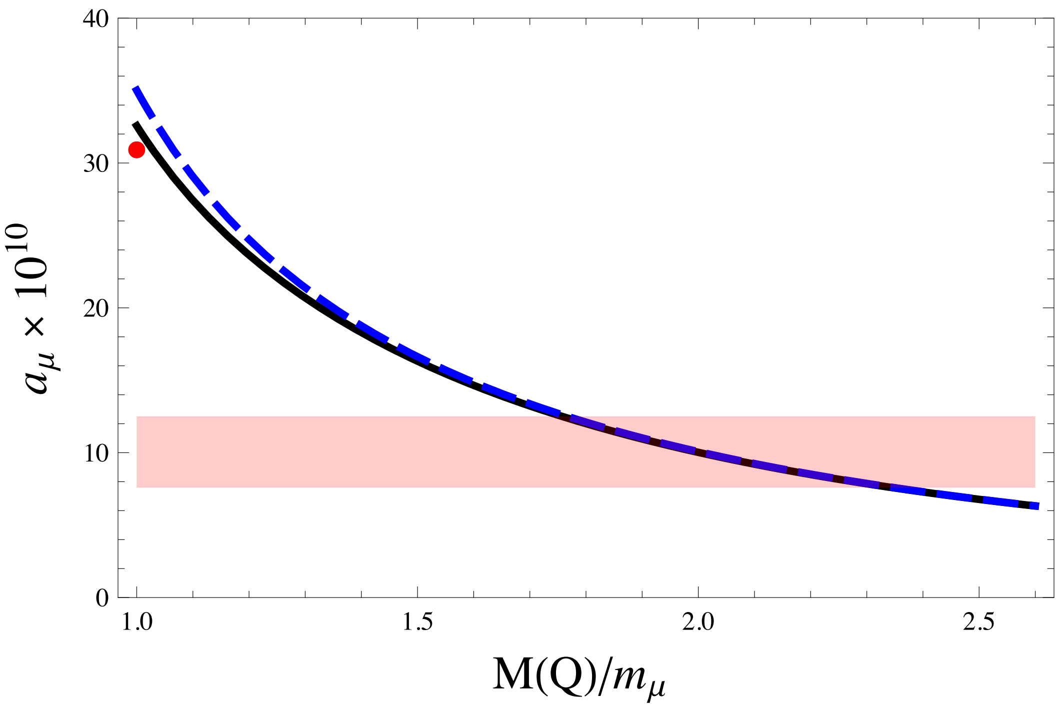
Laporta and Remiddi 1996

Ballpark prediction for HLBL

Masjuan and Vanderhaeghen 2012

Duality argument between the hadronic degrees of freedom and the well-known quark loop contribution

$$a_\mu^{HLBL}(M(Q)) = \left(\frac{\alpha}{\pi}\right)^3 N_c \left(\sum_{q=u,d,s} Q_q^4\right)\left[\left(\frac{3}{2}\zeta(3) - \frac{19}{16}\right)\frac{m_\mu^2}{M(Q)^2} + \mathcal{O}\left(\frac{m_\mu^4}{M(Q)^4}\log^2\frac{m_\mu^2}{M(Q)^2}\right)\right]$$



Ballpark prediction for HLBL

Masjuan and Vanderhaeghen 2012

Duality argument between the hadronic degrees of freedom and the well-known quark loop contribution

Ballpark prediction

		$\langle Q_i \rangle$	$M(Q_i)$GeV	$a_\mu^{HLBL} \times 10^{10}$
$P_1^0(Q_1^2, Q_2^2)$	$\frac{Q_1+Q_2}{2}$	$0.358(8)$GeV	$0.205(2)$	$10.52(21)$
	$\sqrt{Q_1 Q_2}$	$0.323(7)$GeV	$0.216(2)$	$9.68(15)$
$P_2^1(Q_1^2, Q_2^2)$	$\frac{Q_1+Q_2}{2}$	$0.358(11)$GeV	$0.205(3)$	$10.51(29)$
	$\sqrt{Q_1 Q_2}$	$0.323(9)$GeV	$0.216(3)$	$9.67(21)$

Ballpark prediction for HLBL

Masjuan and Vanderhaeghen 2012

Ballpark prediction

	$\langle Q_i \rangle$		$M(Q_i)$GeV	$a_\mu^{HLBL} \times 10^{10}$
$P_1^0(Q_1^2, Q_2^2)$	$\frac{Q_1+Q_2}{2}$	0.358(8)GeV	0.205(2)	10.52(21)
	$\sqrt{Q_1 Q_2}$	0.323(7)GeV	0.216(2)	9.68(15)
$P_2^1(Q_1^2, Q_2^2)$	$\frac{Q_1+Q_2}{2}$	0.358(11)GeV	0.205(3)	10.51(29)
	$\sqrt{Q_1 Q_2}$	0.323(9)GeV	0.216(3)	9.67(21)

Error estimation from:

- Exp data to build up FF: ~2% (smaller @BES-III)
- Error from approach at FF: 5%
- Departure from chiral limit: 15% (reduced when lattice at physical mass)
- Off-shellness is poorly known for π (not even the sign) and unknown for others, models for π-TFF suggest ± 5%-10% effect.

$$a_\mu^{HLBL} = [8.2(1) \div 12.6(2)] \times 10^{-10}$$

Conclusions

- Review of g-2 factors
- Emphasis on Hadronic light-by-light:
  - HLBL: New estimated calculation

$$a_\mu^{HLBL} = [8.2(1) \div 12.6(2)] \times 10^{-10}$$

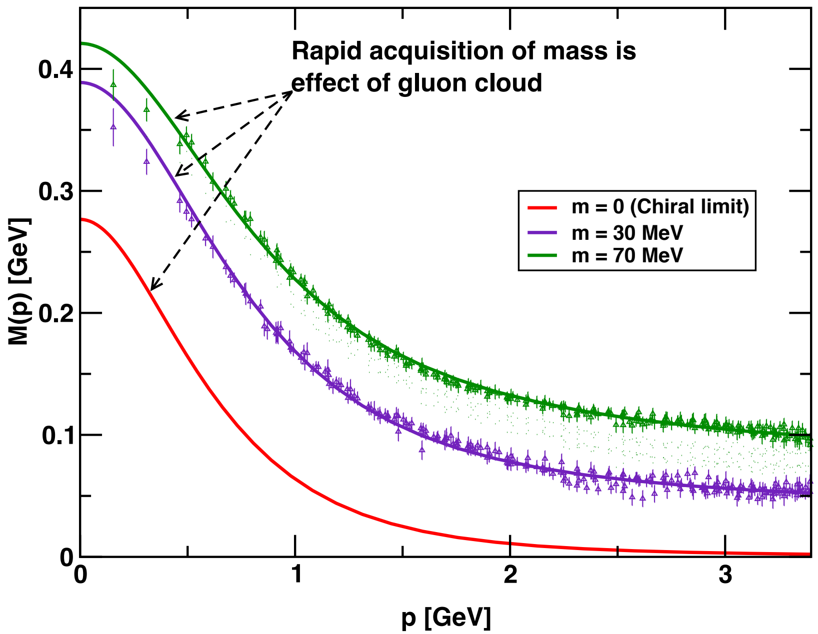
  - the 3σ still persists:
    - indication of NP?
    - what about off-shellness?

Outlook

- Concerning the πTFF (with P. Sanchez):
  - we'll test our approach using $\pi^0 \to e^+e^-$
- Concerning the running of the quark-mass (with V. Pascalutsa, V. Pauk and M. Vanderhaeghen):
  - instead of an averaged mass, numeric computation

Outlook

- Concerning the πTFF (with P. Sanchez):
  - we'll test our approach using $\pi^0 \to e^+e^-$
- Concerning the running of the quark-mass (with V. Pascalutsa, V. Pauk and M. Vanderhaeghen):
  - instead of an averaged mass, numeric computation



Bhagwat *et al* 2007

[Furui and Nakajima,'05]

$$M(q) = \frac{a}{q^2 + b}$$

with parameters from MILC fit

Thanks!

Dissection of the HLBL contribution

- Matching low- and high- energies

P.M., Sanchez-Puertas, in preparation

  - Low-energy description + pQCD: determine the matching point

The QCD model [Noguera,Vento'12]:
- assume πDA flat at Q_0
- apply QCD evolution at high energies
- phenomenological model at low energies (VMD)

- how to determine Q_0?

- the authors fixed

$$Q_0^2 = 1\text{GeV}^2$$

Dissection of the HLBL contribution

- Matching low- and high- energies

P.M., Sanchez-Puertas, in preparation

  - Low-energy description + pQCD: determine the matching point

The QCD model [Noguera,Vento'12]:

- assume πDA flat at Q_0
- apply QCD evolution at high energies
- phenomenological model at low energies (VMD)

- how to determine Q_0?

- the authors fixed

$$Q_0^2 = 1\mathrm{GeV}^2$$

Our approach:

- use the same pQCD
- at low energies ⟶ $P_3^2(Q^2)$
- determine Q_0 by matching
- Q_0 is fixed by data:

$$Q_0^2 = 5\mathrm{GeV}^2$$