Two-Loop Electroweak Corrections to High-Energy Wide-Angle Bhabha Scattering

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Preface

- Linear Collider Zoo
  - *ILC*
  - *CLIC*
  - $\mu^+\mu^-$, $\gamma\gamma$, *etc.*

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  - *ILC* ⇨ *Higgs factory in Japan*
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$\left.\begin{array}{l}\text{CLIC}\\ \mu^+\mu^-,\ \gamma\gamma,\ \text{etc.}\end{array}\right\}$ *still science fiction*

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  - $\mu^+\mu^-$, $\gamma\gamma$, *etc.*

  } *still science fiction* (CLIC; $\mu^+\mu^-$, $\gamma\gamma$, etc.)

- High precision machine
  - *Luminosity calibration*

  ↪ *Bhabha scattering cross section to permill accuracy*

Bhabha scattering at Linear Collider

Permill accuracy:

- Small angle (luminosity)
  - *two-loop QED form factor*
  - *electroweak corrections are suppressed*

Bhabha scattering at Linear Collider

Permill accuracy:

- Small angle (luminosity)
  - *two-loop QED form factor*
  - *electroweak corrections are suppressed*

- Wide angle (luminosity spectrum)
  - *generic two-loop QED corrections*

➥ *two-loop electroweak logarithms* $\ln^4(s/M_{Z,W}^2) \sim 600$

Two-loop QED corrections

- Two-loop SA scattering

  A.B. Arbuzov, V.S. Fadin, E.A. Kuraev, L.N. Lipatov, N.P. Merenkov, L. Trentadue

- Logarithmic corrections to LA scattering

  E.W. Glover, J.B. Tausk, J.J. van der Bij

- Full massless result for virtual correction

  Z. Bern, L. Dixon, A. Ghinculov

- $m_e \neq 0$, fermion loop insertions

  R. Bonciani, A. Ferroglia, P. Mastrolia, E. Remiddi, J.J. van der Bij

- Photonic corrections, leading order in m_e^2/s

  A. Penin

- Heavy flavor corrections

  T. Becher, K. Melnikov; R. Bonciani, A. Ferroglia, A. Penin

  H. Kühn, S. Uccirati; S. Acris, M. Czakon, J. Guza, T. Riemann

Electroweak Sudakov Logarithms

- For annihilation $f\bar{f} \to f'\bar{f}'$:
  - LL: $\alpha_{ew}^2 \ln^4(s/M_{Z,W}^2)$ (Fadin, Lipatov, Martin, Melles)
  - NLL: $\alpha_{ew}^2 \ln^3(s/M_{Z,W}^2)$ (Kühn, Penin, Smirnov)
  - NNLL: $\alpha_{ew}^2 \ln^2(s/M_{Z,W}^2)$ (Kühn, Moch, Penin, Smirnov)
  - N^3LL: $\alpha_{ew}^2 \ln(s/M_{Z,W}^2)$ (Feucht/Jantzen, Kühn, Penin, Smirnov)
- Topics discussed in this talk:
  - *Generalization to* $\boxed{f\bar{f} \to f\bar{f}}$
  - *Application to Bhabha phenomenology*
  - *Based on:* *A. Penin, G. Ryan, JHEP 1111 (2011) 081*

General idea

- Many scales: $M_Z,\ M_W,\ M_H,\ \lambda,\ m_f, m_{f'}$
- Scales hierarchy: $s,\ t,\ u \gg \underbrace{M_Z \approx M_W \sim M_H}_{M} \gg \lambda,\ \underbrace{m_f,\ m_{f'}}_{0}$
- Leading asymptotics in: $M^2/s, \quad \lambda/M$

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Two types of large logs

Electroweak	*QED*
$\ln(s/M^2)$	$\ln(s/\lambda^2)$

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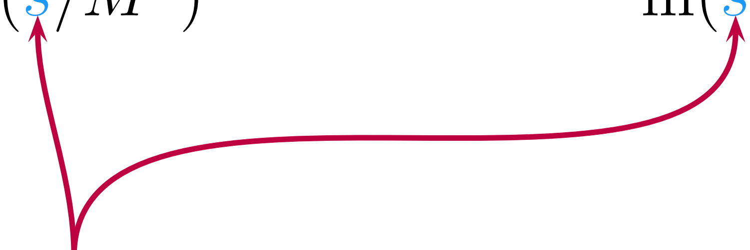

hard evolution equation

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Two types of large logs

Electroweak $\ln(s/M^2)$

QED $\ln(s/\lambda^2)$

hard evolution equation *infrared evolution equation*

Hard evolution

(Mueller; Collins; Sen; Sterman,...)

Form factor

$$\frac{\partial}{\partial \ln Q^2}\mathcal{F} = \left[\int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x}\gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2))\right]\mathcal{F}$$

Reduced amplitude

$$\frac{\partial}{\partial \ln Q^2}\tilde{\mathcal{A}} = \chi(\alpha(Q^2))\tilde{\mathcal{A}}$$

Amplitude decomposition ⇨ $\mathcal{A}(f\bar{f} \to f'\bar{f}') = -\frac{ig^2(Q^2)}{Q^2}\mathcal{F}^2\tilde{\mathcal{A}}$

Hard evolution

Solution

$$\mathcal{F} \quad = \quad F_0(\alpha(M^2))\exp\left\{\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\left[\int_{M^2}^{x}\frac{\mathrm{d}x'}{x'}\gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2))\right]\right\}$$

$$\tilde{\mathcal{A}} \quad = \quad \tilde{\mathcal{A}}_0(\alpha(M^2))\mathrm{Pexp}\left[\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\chi(\alpha(x))\right]$$

Anomalous dimensions ⇨ $\gamma, \quad \zeta, \quad \chi$

Initial conditions ⇨ $\xi, \quad F_0, \quad \mathcal{A}_0$

Two-loop logarithms

Two-loop log^{4,3,2}

$$\gamma^{(2)}, \quad \zeta^{(1)}, \quad \chi^{(1)}, \quad \xi^{(1)}, \quad F_0^{(1)}, \quad \mathcal{A}_{0i}^{(1)}$$

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$\chi^{(2)}$ ⇨ *two-loop massless amplitudes*

(Anastasiou, Glover, Oleari, Tejeda-Yeomans; Glover)

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$$\gamma^{(2)}, \quad \zeta^{(1)}, \quad \chi^{(1)}, \quad \xi^{(1)}, \quad F_0^{(1)}, \quad \mathcal{A}_{0i}^{(1)}$$

Two-loop linear log

$\chi^{(2)}$ ⇨ *two-loop massless amplitudes*

(Anastasiou, Glover, Oleari, Tejeda-Yeomans; Glover)

$\zeta^{(2)} + \xi^{(2)}$ ⇨ *two-loop massive form factor*

(Feucht/Jantzen, Kühn, Penin, Smirnov)

Generalization to Bhabha scattering

- Annihilation
- Scattering
  - *Crossing symmetry* $s \leftrightarrow t$
  - *Analytical continuation from* $s > 0$ *to* $t < 0$
- Interference

Bhabha scattering cross section

Logarithmic expansion

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^{\text{Born}}}{d\Omega} + \frac{\alpha^2}{4\,s\,x^2}\left[\frac{\alpha}{4\pi}\left(d\sigma_2^{(1)}\ln^2\left(\frac{s}{M_W^2}\right) + d\sigma_1^{(1)}\ln\left(\frac{s}{M_W^2}\right) + d\sigma_0^{(1)}\right)\right.$$
$$+\left(\frac{\alpha}{4\pi}\right)^2\left(d\sigma_4^{(2)}\ln^4\left(\frac{s}{M_W^2}\right) + d\sigma_3^{(2)}\ln^3\left(\frac{s}{M_W^2}\right)\right.$$
$$\left.\left. + d\sigma_2^{(2)}\ln^2\left(\frac{s}{M_W^2}\right) + d\sigma_1^{(2)}\ln\left(\frac{s}{M_W^2}\right) + \mathcal{O}(1)\right) + \mathcal{O}(\alpha^3)\right].$$

Born cross section

$$\frac{d\sigma^{\text{Born}}}{d\Omega} = \frac{\alpha^2}{64\,s\,x^2}\Big((1-x)^4(1+\tan^2_W)^2 + 8\tan^4_W(3-8x+12x^2-8x^3+3x^4)\Big)$$

with $x = \frac{1-\cos\theta}{2}$

Two-loop linear logarithms

Massive $SU(2)$ *model,* $M_H = M$, *six left-handed massless doublets*

Annihilation

$$
\mathrm{d}\sigma^{(2)}_{S\ 1} = \frac{28411}{216} - 122\zeta(3) + 26\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right) + 15\sqrt{3}\pi - \frac{(199-127x)\pi^2}{6(1-x)}
+ \left(\frac{125}{3} + \frac{50\pi^2}{3} + \frac{10\ln(x)}{1-x} - \frac{5(1-2x)\ln^2(x)}{(1-x)^2}\right)\ln(1-x)
+ \left(-\frac{1075-1250x}{6(1-x)} + \frac{(74-148x+38x^2)\pi^2}{3(1-x)^2}\right)\ln(x)
- \frac{(631-806x)\ln^2(x)}{4(1-x)^2} + \frac{19(1-2x)\ln^3(x)}{(1-x)^2}\,.
$$

Two-loop linear logarithms

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Scattering

$$\mathrm{d}\sigma^{(2)}_{T\,1} = \frac{28411}{216} - 122\zeta(3) + 26\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right) + 15\sqrt{3}\pi + \frac{(8+4x)\pi^2}{1-x}$$
$$+ \left(\frac{125}{3} + \frac{8\pi^2}{3} + \frac{(17-7x)\ln(x)}{1-x} - \frac{(10-30x+15x^2)\ln^2(x)}{(1-x)^2}\right)\ln(1-x)$$
$$+ 4\ln(x)\ln^2(1-x) + \left(\frac{1030-505x}{18(1-x)} + \frac{(80-88x+44x^2)\pi^2}{3(1-x)^2}\right)\ln(x)$$
$$+ \frac{(414-502x+263x^2)\ln^2(x)}{12(1-x)^2} + \frac{(10-22x+11x^2)\ln^3(x)}{(1-x)^2}\,,$$

Two-loop linear logarithms

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Interference

$$
\mathrm{d}\sigma^{(2)}_{ST\ 1} = \frac{28411}{216} - 122\zeta(3) + 26\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right) + 15\sqrt{3}\pi - \frac{67\pi^2}{12}
+ \left(\frac{125}{3} + \frac{11\pi^2}{3} + \frac{(27-7x)\ln(x)}{2(1-x)} - \frac{(27-64x+27x^2)\ln^2(x)}{2(1-x)^2}\right)\ln(1-x)
+ 2\ln(x)\ln^2(1-x) - \left(\frac{2195-3245x}{36(1-x)} + \frac{(13+41x)\pi^2}{3(1-x)}\right)\ln(x)
- \frac{(49+32x-431x^2)\ln^2(x)}{24(1-x)^2} - \frac{(8+2x^2)\ln^3(x)}{(1-x)^2}\,.
$$

$SU(2) \times U(1)$ model

Factorization of infrared singularities

$$\mathcal{A}_{f\bar{f}\to f'\bar{f}'} = \exp\left[-\frac{\alpha_e}{4\pi}(Q_f^2 + Q_{f'}^2)\ln^2\left(\frac{s}{\lambda^2}\right) + \ldots\right]\bar{\mathcal{A}}(M^2/s) + \mathcal{O}(\lambda/M)$$

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N^{3}LL approximation ➪ $\xi^{(2)}(\lambda/M)$

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No-mixing $SU(2)_M \times U(1)_\lambda$ model

- *Higgs boson of zero hypercharge, no mixing*

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- *For* $U(1)_M \times U(1)_\lambda$ *model* $\xi^{(2)} = 0$ (Feucht/Jantzen, Kühn, Penin, Smirnov)

  In fact $\xi^{(n)} = 0, \; n > 1$ ⇨ *QED Bhabha* (Penin)

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  In fact $\xi^{(n)} = 0,\ n > 1$ ⇨ *QED Bhabha* (Penin)
- *For* $SU(2)_M \times U(1)_\lambda$ *model* $\xi^{(2)}_{\lambda=0} = \xi^{(2)}_{\lambda=M}$

naïve factorization of infrared logs holds to N ^{3}*LL*

$SU(2) \times U(1)$ **model**

Two-loop $\log^{4,3,2}$

$SU(2) \times U(1)$ model

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- *Symmetric phase calculation,* $M_W = M_Z$

$SU(2) \times U(1)$ model

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- $M_W \neq M_Z$ *through one-loop result* ⇨ *5% effect*

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- *Approximation: Higgs boson of zero hypercharge,* $M_H = M_W = M_Z$

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- *Approximation: Higgs boson of zero hypercharge,* $M_H = M_W = M_Z$
- *No mixing* ⇨ *naïve factorization of infrared logs*
- *Mixing effects are suppressed by* $\sin^2\theta_W$ ⇨ *20% error,* $M_H \neq M_W$ *effect is negligible*

Two-loop quadratic to quartic logarithms

$$d\sigma_4^{(2)} = 0.34 - 1.21x + 1.81x^2 - 1.21x^3 + 0.34x^4\,,$$

$$\begin{aligned} d\sigma_3^{(2)} = & -1.43 + 3.51x - 4.81x^2 + 2.91x^3 - 1.07x^4 \\ & - \left(0.16 - 0.81x + 1.22x^2 - 0.81x^3 + 0.16x^4\right)\ln(1-x) \\ & - \left(0.18 + 0.74x - 4.23x^2 + 4.91x^3 - 1.71x^4\right)\ln(x)\,, \end{aligned}$$

$$\begin{aligned} d\sigma_2^{(2)} = & \, 5.78 + 6.42x - 20.04x^2 + 8.19x^3 + 3.98x^4 \\ & + \left(0.05 - 0.21x + 0.30x^2 - 0.21x^3 + 0.05x^4\right)\ln^2(1-x) \\ & + \left(0.97 - 3.27x + 3.62x^2 - 1.48x^3 - 0.13x^4\right)\ln(1-x) \\ & - \left(0.29 + 0.29x - 4.06x^2 + 5.22x^3 - 1.97x^4\right)\ln^2(x) \\ & - \left(3.74 - 7.56x + 9.03x^2 - 4.45x^3 + 0.43x^4\right)\ln(x) \\ & - \left(0.88 - 2.76x + 2.13x^2 - 0.10x^3 - 0.28x^4\right)\ln(1-x)\ln(x) \end{aligned}$$

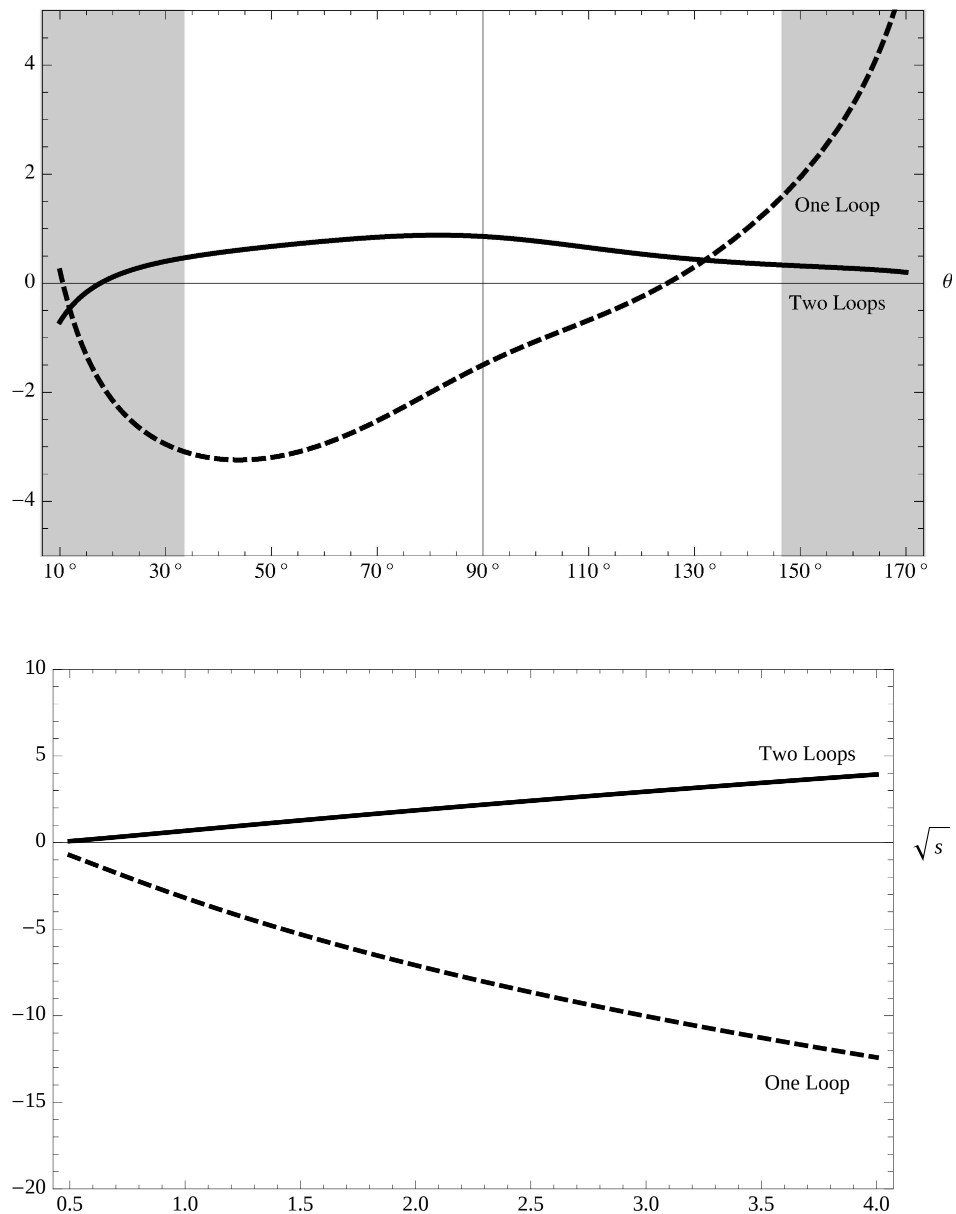
Two-loop linear logarithm

$$
\begin{aligned}
\mathrm{d}\sigma_1^{(2)} = &-0.30 + 0.87x + 0.41x^2 - 2.21x^3 + 1.24x^4 \\
&+ \left(4.25 - 18.23x + 36.60x^2 - 35.50x^3 + 12.89x^4\right) \ln(1-x) \\
&+ \left(0.63 - 0.38x + 1.88x^2 - 2.13x^3\right) \ln^3(x) \\
&+ \left(0.18 - 0.38x - 1.75x^2 - 0.03x^3\right) \ln^2(x) \\
&+ \left(18.97 - 10.32x + 6.39x^2 - 9.83x^3 - 5.21x^4\right) \ln(x) \\
&+ \left(0.25 - 0.75x + 0.75x^2 - 0.25x^3\right) \ln^2(1-x) \ln(x) \\
&- \left(0.63 - 3.56x + 5.25x^2 - 2.31x^3\right) \ln(1-x) \ln^2(x) \\
&+ \left(0.27 - 1.60x + 2.40x^2 - 1.06x^3\right) \ln(1-x) \ln(x)
\end{aligned}
$$

One and two-loop electroweak corrections

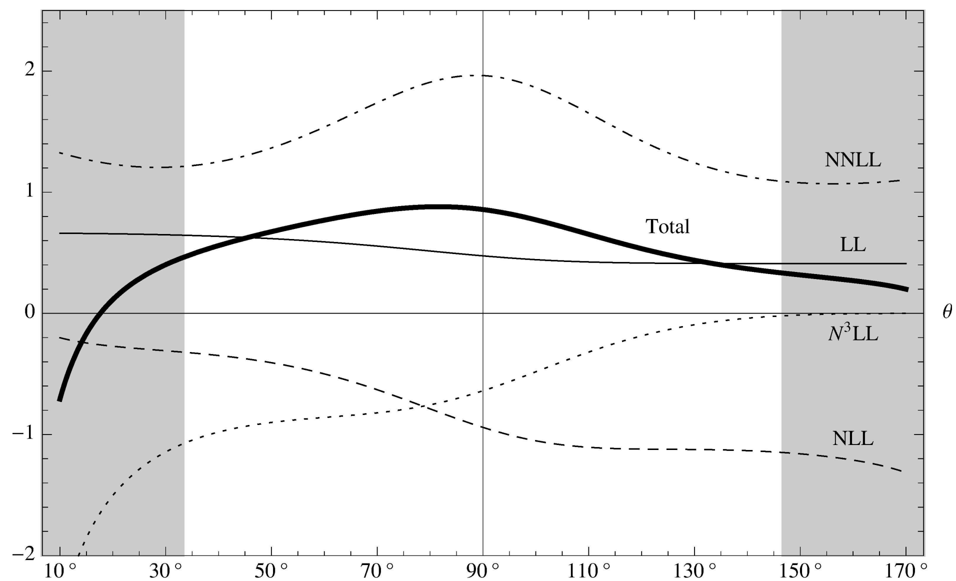
$$s = 1\ \text{TeV}$$

$$\theta = 50^\circ$$

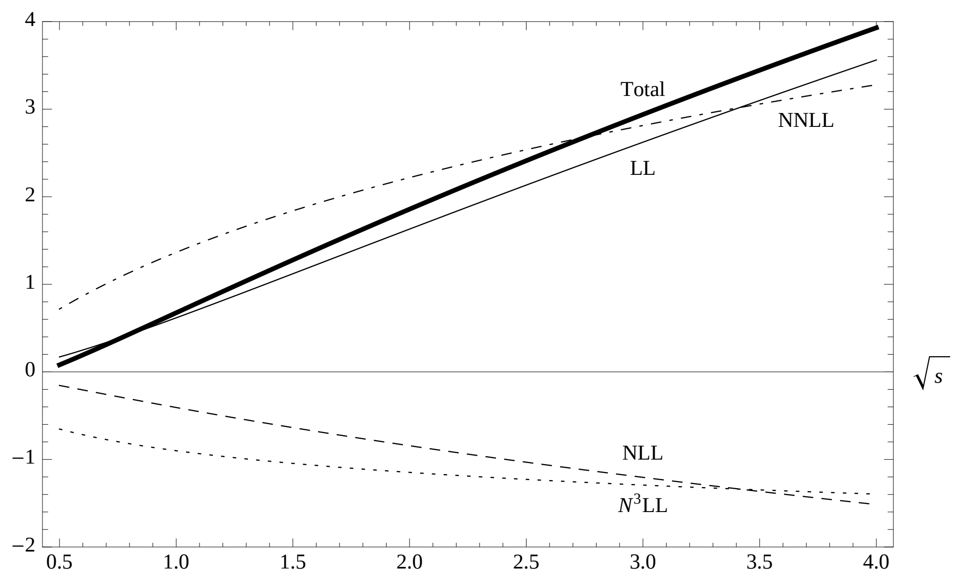


Decomposition of two-loop logarithms

$s = 1\ \text{TeV}$



$\theta = 50^\circ$



Numerics

For $s = 1\ \mathrm{TeV}$ *and* $\theta = 50^\circ$

One loop: -3.20% $\begin{cases} \text{LL:} & -9.45\% \\ \text{NLL:} & 7.35\% \\ \text{N}^2\text{LL:} & -1.09\% \end{cases}$

Two loops: 0.67% $\begin{cases} \text{LL:} & 0.62\% \\ \text{NLL:} & -0.41\% \\ \text{N}^2\text{LL:} & 1.36\% \\ \text{N}^3\text{LL:} & -0.90\% \end{cases}$

Summary

- Last missing part of numerically relevant two-loop corrections is here

- Wide-angle high-energy Bhabha cross section is known to a few permill accuracy

- We are ready for ILC/CLIC/MC